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G PARITY AND THE BREAKING OF EXCHANGE DEGENERACY*

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ABSTRACT

Exact exchange degeneracy, a property of the planar S matrix, is not a feature of the physical world. G parity, an exact symmetry of strong interactions, fails to be maintained by the discontinuities of the planar S matrix. This paper examines the connection between these two conflicting concepts and shows how calculations of exchange-degeneracy breaking may be based on G parity. The relation of G parity to the cylinder and torus components of the topological expansion is a central feature. We apply our arguments to ρ-A_2 splitting and justify the use of an unsubtracted dispersion relation for $\alpha_{A_2}-\alpha_\rho$. Our analysis of the dispersion relation reveals a dominant role for the $\pi\pi$ channel.

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I. INTRODUCTION

The planar approximation to the strong-interaction S matrix is distinguished by a number of characteristic simple features, including isospin degeneracy (e.g. ρ-ω and f-A_2) and Regge-trajectory exchange degeneracy (e.g. ρ-A_2 and f-ω). At the same time the discontinuity formulas that define the planar S matrix--formulas sometimes described as "planar unitarity"--do not fulfill the general requirements of Bose symmetry and charge conjugation invariance. Conservation of G parity, in consequence, is not obeyed by the intermediate states that appear in planar discontinuities. A striking illustration of such G-parity violation has been discussed in ref. [1] where it was pointed out that the planar ω has the same 2π decay width as the planar ρ. An essential task fulfilled by the nonplanar components of the topological expansion is to restore G parity, and it was explicitly shown in ref. [1] how the cylinder eliminates the 2π width of the ω. Quite generally, in the course of repairing G parity the nonplanar terms produce trajectory shifts that break exchange and isospin degeneracy. This paper expands on the remarks in ref. [1] and develops a procedure for calculating the violation of exchange degeneracy from G-parity considerations.

In order to treat nonplanar corrections of trajectories like ρ and A_2 that are not isosinglets, one must go beyond the cylinder to the torus (single-handle) level of the topological expansion. Other authors [2-5] have recently considered torus degeneracy-breaking but without explicit recognition of the connection with G parity. Our approach indicates that important aspects of the torus have been overlooked. In particular, we find ρ-A_2 splitting to be dominated by intermediate channels containing low-mass unnatural-parity mesons-- channels ignored in previous calculations.

II. G-PARITY VIOLATION FOR THE LEADING PLANAR TRAJECTORIES

Consider the two-particle channels that contribute to the discontinuities of the four degenerate leading natural-parity planar trajectories--f, ω, A_2, ρ. All our considerations can be extended to unnatural-parity mesons through the correspondence $\rho \leftrightarrow B$, $A_2 \leftrightarrow \pi$, $\omega \leftrightarrow H$, $f \leftrightarrow \eta$. The planar t-discontinuity has the form shown in fig. 1, where the boundaries i, j each take on two values, corresponding in quark language to "n" and "p" or "up" and "down". The boundary k is to be summed and we are principally concerned with the contribution from $k = n$ and $k = p$, that is, from nonstrange intermediate mesons where G parity can be defined. By appropriate superpositions of (n, p) combinations one may form planar trajectories of well-defined isospin, signature and G parity. For example, the planar ρ^+ and A_2^+, both with $I = 1$ but with opposite signature, are

$$A_2^{+(Pl)} = \frac{1}{\sqrt{2}} \left(\frac{\overrightarrow{p}}{\overleftarrow{n}} + \frac{\overleftarrow{n}}{\overrightarrow{p}} \right) \text{, even signature, odd G,}$$

$$\rho^{+(Pl)} = \frac{1}{\sqrt{2}} \left(\frac{\overrightarrow{p}}{\overleftarrow{n}} - \frac{\overleftarrow{n}}{\overrightarrow{p}} \right) \text{, odd signature, even G ,}$$

while the $I = 0$ planar f and ω are

$$f^{(P\ell)} = \frac{1}{2}\left\{ \begin{matrix} p \\ \longrightarrow \\ \longleftarrow \\ p \end{matrix} + \begin{matrix} n \\ \longrightarrow \\ \longleftarrow \\ n \end{matrix} + \begin{matrix} p \\ \longleftarrow \\ \longrightarrow \\ p \end{matrix} + \begin{matrix} n \\ \longleftarrow \\ \longrightarrow \\ n \end{matrix} \right\}, \text{ even signature, even G}$$

$$\omega^{(P\ell)} = \frac{1}{2}\left\{ \begin{matrix} p \\ \longrightarrow \\ \longleftarrow \\ p \end{matrix} + \begin{matrix} n \\ \longrightarrow \\ \longleftarrow \\ n \end{matrix} - \begin{matrix} p \\ \longleftarrow \\ \longrightarrow \\ p \end{matrix} - \begin{matrix} n \\ \longleftarrow \\ \longrightarrow \\ n \end{matrix} \right\}, \text{ odd signature, odd G .}$$

The neutral $\rho^{(P\ell)}$ and $A_2^{(P\ell)}$ have similar forms with appropriate sign changes. It is now straightforward to show that the discontinuities of both planar ρ and planar A_2 get contributions from two-particle states in the sixteen combinations shown in the left-hand column of Table I, while the discontinuities of both planar f and planar ω get contributions from another set of sixteen combinations, also shown in Table I. Although isospin is conserved, there is no observance of G parity. Only those combinations indicated in the right-hand column of Table I are G parity allowed.

The degeneracy of all four trajectories at the planar level corresponds to the appearance in each planar discontinuity of $4 + 4$ channels built from planar particles of the same signature (++ and --) and $4 + 4$ from planar particles of the opposite signature (+- and -+). Were all signature combinations equivalent, overall degeneracy might be compatible with G parity, because the total number of physical channels allowed to communicate with each trajectory is seen uniformly to be 8. The contributions from different signature combinations, however, are manifestly different, so the degeneracy is unavoidably broken for the physical trajectories.

TABLE I

(a) Two-particle planar channels that contribute to discontinuities of planar trajectories.

(b) G-parity allowed channels.

Trajectories	Signature Combinations	(a) Planar Channels		(b) G-Parity Allowed Channels	
$A_2^{(+)}$	+ +	fA_2^+, A_2^+f, $A_2^0A_2^+$, $A_2^+A_2^0$	(4)	fA_2^+, A_2^+f	(2)
	- -	$\omega\rho^+$, $\rho^+\omega$, $\rho^0\rho^+$, $\rho^+\rho^0$	(4)	$\omega\rho^+$, $\rho^+\omega$	(2)
	- +	$\rho^0A_2^+$, $\rho^+A_2^0$, ρ^+f, ωA_2^+	(4)	$\rho^0A_2^+$, $\rho^+A_2^0$	(2)
	+ -	$A_2^+\rho^0$, $A_2^0\rho^+$, $f\rho^+$, $A_2^+\omega$	(4)	$A_2^+\rho^0$, $A_2^0\rho^+$	(2)
ρ^+	+ +		(4)	$A_2^0A_2^+$, $A_2^+A_2^0$	(2)
	- -	"	(4)	$\rho^0\rho^+$, $\rho^+\rho^0$	(2)
	- +		(4)	ρ^+f, ωA_2^+	(2)
	+ -		(4)	$f\rho^+$, $A_2^+\omega$	(2)

Table I continued.

TABLE I (Cont.)

Trajectories	Signature Combinations	(a) Planar Channels		(b) G-Parity Allowed Channels	
f	+ +	ff, $A_2^+A_2^-$, $A_2^-A_2^+$, $A_2^oA_2^o$	(4)	ff, $A_2^+A_2^-$, $A_2^-A_2^+$, $A_2^oA_2^o$	(4)
	- -	ωω, $\rho^+\rho^-$, $\rho^-\rho^+$, $\rho^o\rho^o$	(4)	ωω, $\rho^+\rho^-$, $\rho^-\rho^+$, $\rho^o\rho^o$	(4)
	+ -	fω, $A_2^+\rho^-$, $A_2^-\rho^+$, $A_2^o\rho^o$	(4)	---	(0)
	- +	ωf, $\rho^-A_2^+$, $\rho^+A_2^-$, $\rho^oA_2^o$	(4)	---	(0)
ω	+ +		(4)	---	(0)
	- -		(4)	---	(0)
	+ -	"	(4)	fω, $A_2^+\rho^-$, $A_2^-\rho^+$, $A_2^o\rho^o$	(4)
	- +		(4)	ωf, $\rho^-A_2^+$, $\rho^+A_2^-$, $\rho^oA_2^o$	(4)

One sees from Table I that there is no immediate breaking of ρ-A_2 degeneracy. The first-order effect will be to shift f and ω. Within the topological expansion such a distinction corresponds to the cylinder's communicating only with isosinglets. Let us now identify the precise correspondence between the cylinder and the right-hand column of Table I.

In ref. [6] there was introduced a cylinder "operator" in the space of planar trajectories, an operator whose matrix elements may be represented by the diagram in fig. 2. The diagonal elements correspond to trajectory shifts if we ignore flavors beyond isospin. The cylinder operator vanishes when applied to isotriplets such as ρ and A_2, while the diagonal elements corresponding to f and ω are equal but opposite in sign, at $t = 0$ being positive for f and negative for ω. Thus, in the notation of ref. [6], the cylinder-induced trajectory shifts are

$$\Delta_{cyl.}\, \alpha_{A_2}(t) = \Delta_{cyl.}\, \alpha_{\rho}(t) = 0 \tag{1}$$

$$\Delta_{cyl.}\, \alpha_{f}(t) = -\Delta_{cyl.}\, \alpha_{\omega}(t) = 2k(t)\,, \tag{2}$$

where $k(t)$ is real and positive near $t = 0$. At the same time $k(t)$ is an analytic function with threshold branch points on the positive t axis [7]. The two-particle discontinuity of $k(t)$ may be written schematically as

$$\text{disc}_t\, k(t) = (T - V)^{P\ell}(T - V)^{P\ell}\,, \tag{3}$$

where $f^{P\ell} = A_2^{\;P\ell} = T^{P\ell}$ and $\omega^{P\ell} = \rho^{P\ell} = V^{P\ell}$, corresponding to the twisted planar link in fig. 2 whose pole residues carry a factor $(-1)^J$.

The discontinuity of $k(t)$ implies a corresponding discontinuity in trajectory functions. Comparing formulas (1), (2) and (3) to the differences in the right-hand column of Table I shows that the cylinder is correcting the G-parity defect of the planar discontinuity through a planar approximation to the intermediate particles. That is, to the extent that intermediate physical particles are approximated by their planar counterparts, one calculates from column (b) of Table I

$$\mathrm{disc}_t(\alpha_{A_2} - \alpha_\rho) = 0 , \tag{4}$$

$$\begin{aligned} \mathrm{disc}_t(\alpha_f - \alpha_\rho) &= -\,\mathrm{disc}_t(\alpha_\omega - \alpha_\rho) \\ &= 2(T^{P\ell} - V^{P\ell})(T^{P\ell} - V^{P\ell}) , \end{aligned} \tag{5}$$

in precise correspondence with eqs. (1), (2) and (3).

So long as ρ-ω and f-A_2 degeneracy is maintained in the intermediate states, G parity requires no relative displacement between ρ and A_2 trajectories. Once the cylinder has shifted f and ω, however, G parity demands a second-order splitting between ρ and A_2. (There will also be second-order shifting of f and ω, in addition to the cylinder shift.) These "second order" shifts tend to be small even if the first-order shifts are large because of the low statistical weight of isosinglets compared to isotriplets. A glance at column (b) of Table I shows that only one state in four involves f or ω. (This is Veneziano's $1/N^2$ factor, with $N = 2$. See refs. [8,1].) We shall see, however, that there is another, more important, reason for the second-order shifts to be small.

With f split from A_2 and ω split from ρ, Table I instructs us to replace eqs. (4) and (5) by

$$\mathrm{disc}_t(\alpha_{A_2} - \alpha_\rho) = [(f - A_2) + (\rho - \omega)](A_2 - \rho) + (A_2 - \rho)[(f - A_2) + (\rho - \omega)], \tag{6}$$

$$\mathrm{disc}_t(\alpha_f - \alpha_\omega) = 3(A_2 - \rho)(A_2 - \rho) + (f - \omega)(f - \omega), \tag{7}$$

$$\mathrm{disc}_t(\alpha_f + \alpha_\omega - \alpha_\rho - \alpha_{A_2}) = [(\rho - \omega) - (f - A_2)][(\rho - \omega) - (f - A_2)]. \tag{8}$$

Although these three discontinuity formulas superficially look similar to one another, according to the principle of asymptotic planarity [6,7] there are qualitative differences between eqs. (6), (7) and (8). Asymptotic planarity implies that the poles on the A_2 trajectory become more and more closely degenerate with the f-trajectory poles as their mass increases (isospin degeneracy). A similar trend is supposed to occur for the poles on the ω and ρ trajectories. In contrast, although the A_2 and ρ <u>trajectories</u> become asymptotically more and more closely degenerate, the poles on the two trajectories never coincide--the one sequence being at odd J and the other at even J. A similar remark applies to the f and ω trajectories. So the combinations $(f\text{-}A_2)$ and $(\rho\text{-}\omega)$ tend smoothly to zero with increasing mass while the combinations $(A_2\text{-}\rho)$ and $(f\text{-}\omega)$ oscillate indefinitely, being $+(-)$ according to whether the particle signature is $+(-)$. The discontinuity of formula (8) is built from the product of two strongly-converging combinations, the discontinuity of formula (7) from products of the oscillating type and the discontinuity of formula (6) from a product of the two types. It can be shown that the inclusion of channels involving K^* and K^{**} will alter this pattern for the discontinuity of formula (8) but (up to second order) not that of formula (6). We are left then with a qualitative difference between the cylinder t discontinuity--the type appearing in formulas (3) and (7)--and the type appearing in eq. (6), which we shall see later is the torus discontinuity.

It is well known that, when a discontinuity of an analytic function oscillates with a large amplitude, the analytic continuation of the function may be large in certain regions away from the cut. Such a situation obtains for the cylinder function $k(t)$, which has been shown to be large for $t \lesssim 0$ even though it decreases strongly for t growing in the positive direction. The torus t discontinuity (e.g., formula (6)) from intermediate natural-parity mesons is, however, everywhere small because already for the lowest physical states (1^- and 2^+) isospin degeneracy is accurate to a few percent. Therefore we conclude, even for $t \lesssim 0$, that the contribution to ρ-A_2 splitting from intermediate natural-parity mesons is extremely small. In contrast, because isospin degeneracy is badly broken for the low-mass unnatural-parity mesons π and η, we should look here for a significant torus contribution. It is proposed, then, that ρ-A_2 splitting should be calculated not from formula (6) but rather from

$$\mathrm{disc}_t(\alpha_{A_2} - \alpha_\rho) = [(\eta - \pi) + (B - H) + (f - A_2) + (\rho - \omega)][(\pi - B) + (A_2 - \rho)] + [(\pi - B) + (A_2 - \rho)][(\eta - \pi) + (B - H) + (f - A_2) + (\rho - \omega)]. \quad (9)$$

Here we have added the unnatural-parity contributions, anticipating a near vanishing of the components ρ-ω and f-A_2 . We do this in full awareness that unnatural-parity contributions to f and ω shifts are less important than those from purely natural parity. The cylinder and torus shifts must be recognized as having different origins.

III. ESTIMATE OF ρ-A_2 SPLITTING AT SMALL t

Although the cylinder function $k(t)$ cannot in practice be calculated from its discontinuity through a straightforward dispersion relation, it may be hoped that the difference $\alpha_{A_2} - \alpha_\rho$, whose discontinuity we have shown to be better behaved, is amenable to the elementary approach for small $|t|$. If such is the case we can estimate the sign and the order of magnitude of the difference from the lowest-lying contributions to the discontinuity.

According to formula (9) the first contribution to the discontinuity of $\alpha_{A_2} - \alpha_\rho$ is negative, arising from the 2π channel which communicates with ρ (not with A_2) and whose threshold is at $t = 4\,m_\pi^2$. From the 2π widths of the $\rho(1^-)$, $f(2^+)$ and $g(3^-)$ mesons, together with standard threshold considerations, one may deduce the 2π contribution to the imaginary part of $\alpha_{A_2} - \alpha_\rho$ that is shown in fig. 3. The next contribution is positive, arising from the $\pi\eta$ channel that communicates with A_2 (not with ρ). The $\pi\eta$ contribution, as shown in fig. 3 , is much smaller in magnitude than the $\pi\pi$, partly because the higher η mass reduces the phase space and partly because the η couplings have been strongly shifted away from ideal by the cylinder. The third threshold is associated with $\pi\omega$ and $\eta\omega$ channels and gives a net positive contribution, but the total discontinuity remains negative for $4\,m_\pi^2 \lesssim t \lesssim 2\ \mathrm{GeV}^2$, as shown in fig. 3. Following will be the [illegible] and the [illegible] contributions which we know are negative, even though we have here no experimental data. The $\pi\pi$ channel prevails throughout this low t interval, tending to raise the ρ trajectory above the A_2. An important part of ρ-A_2 splitting is thus a second-order consequence of a large first-order π-η splitting. Were the physical η as nearly degenerate with the π as the physical ω is with ρ, the ρ-A_2 difference at low $|t|$ would be even smaller than that observed.

Assuming that the maximum in $|\mathrm{Im}\,(\alpha_{A_2} - \alpha_\rho)|$ reached near $t = 1\ \mathrm{GeV}^2$ as a consequence of the $\pi\pi$ channel is not exceeded at higher t, we may estimate the real part of $\alpha_{A_2} - \alpha_\rho$ near $t = 0$ through a dispersion relation. Cutting off the integral at $t = 2\ \mathrm{GeV}^2$ leads to $\alpha_{A_2}(0) - \alpha_\rho(0) \sim -0.06$. This number probably is smaller in absolute value than the full integral because the next contributions are known to be negative. The experimentally measured value for this difference is -0.11 ± 0.01 [9].

Had we constructed $\mathrm{Im}(\alpha_f - \alpha_\omega)$ in the low-t region, this quantity would have looked similar to fig. 3 , but once past the $\rho\rho$ and ρA_2 thresholds, large oscillations are expected from channels built out of natural-parity mesons, oscillations which invalidate an unsubtracted dispersion relation. According to formula (6) these natural-parity contributions are very small in $\mathrm{Im}(\alpha_{A_2} - \alpha_\rho)$.

An interesting aspect of $\pi\pi$ dominance of the $A_2 - \rho$ difference, pointed out to us by C. Schmid, is that the derivative of the difference near $t = 0$ has the same sign as the difference itself. In other words the difference becomes smaller in absolute value as t becomes negative, in agreement with experiment. From fig. 3 we predict the derivative of the difference at $t = 0$ to be $-0.14\ \mathrm{GeV}^{-2}$. The experimentally measured derivative is -0.14 ± 0.07 [9]. Although the absolute value of the difference will increase temporarily as t increases in the positive direction, once past the $\pi\pi$ peak in the discontinuity there will be a change in sign of the real part of the difference, followed by a smooth approach toward zero in accord with asymptotic planarity.

IV. THE TORUS

It has been verified in sec. II above that the cylinder produces the major deviations from planarity needed to restore G parity. Let us now see how the torus corresponds to shifts of the next order. It will suffice to use a notation based on natural parity trajectories, as in Table I, even though we have found the dominant torus contributions to involve unnatural parity. Our task is to understand formulas (6), (7) and (8) from the torus viewpoint.

In the sense in which fig. 2 represents a cylinder operator, we may associate a torus "operator" with fig. 4, the diagonal elements of this operator corresponding to trajectory shifts. The "handle," which appears in the upper portion of the diagram, may be regarded as a cylinder whose two ends are internally attached. There exists a torus corresponding to fig. 4 without twists, but as discussed in ref. [10] such a torus produces a common shift of all trajectories.

One of the most important properties of the cylinder is that it represents the <u>difference</u> between the final, shifted states and the planar approximation thereto [6,11,12]. Since the cylinder communicates only with isosinglets its contribution can be written as

$$(f - \omega) - (T - V)^{P\ell} . \qquad (10)$$

The lower branch of fig. 4, being twisted planar, has the structure $(T - V)^{P\ell}$. Thus, if we represent the torus of fig. 4 as a twist operator times a function $\mathcal{J}(t)$, we have

$$\text{disc}_t \mathcal{J}(t) = [(f - \omega) - (T - V)^{P\ell}](T - V)^{P\ell}$$

$$+ (T - V)^{P\ell}[(f - \omega) - (T - V)^{P\ell}], \qquad (11)$$

a formula which should be contrasted with formula (3) for the discontinuity of the cylinder.

Including both cylinder and torus, the breaking of exchange degeneracy is given by

$$\alpha_f - \alpha_\omega = 4k(t) + \mathcal{J}(t) + \cdots \qquad (12)$$

$$\alpha_{A_2} - \alpha_\rho = \mathcal{J}(t) + \cdots . \qquad (13)$$

Let us now see whether the discontinuity formulas (3) and (11), for the cylinder and torus, respectively, are consistent with formulas (6) and (7). Since $f^{P\ell} = A_2^{P\ell} = T^{P\ell}$ and $\omega^{P\ell} = \rho^{P\ell} = V^{P\ell}$, one verifies immediately that the discontinuity of formula (13), using formula (11), is a consistent approximation to formula (6). The corresponding matching of formula (12) with formula (7), using both eqs. (3) and (11), is achieved if we recognize that the second term on the right-hand side of (7) may be expanded:

$$(f - \omega)(f - \omega) = (T - V)^{P\ell}(T - V)^{P\ell} + [(f - \omega) - (T - V)^{P\ell}](T - V)^{P\ell}$$

$$+ (T - V)^{P\ell}[(f - \omega) - (T - V)^{P\ell}] + \cdots .$$

$$(14)$$

the remainder being of higher order. Including the higher order term corresponds to representing all intermediate f and ω states by their cylinder-shifted approximations.

What about formula (8)? The effect here is quadratic in first-order shifts and consequently will not appear at the single-handle (simple torus) level of the topological expansion. One would have to examine two-handle components in order to find the shift given by eq. (8). We have remarked already that a much larger contribution to this particular shift will arise from off-diagonal elements of the cylinder operator corresponding to intermediate channels with strange particles.

Speaking of strange particles, what about the breaking of exchange degeneracy between K^* and K^{**}? Since the latter trajectories do not have well-defined G parity, we cannot discuss their relative shift in the framework of secs. I and II. The torus framework is appropriate as soon as one recognizes that off-diagonal elements of the cylinder are involved--those that mix f with f' and ω with ϕ. (Such elements vanish in the planar approximation.) We shall not pursue the K^*-K^{**} shift in this paper except to point out that the $K\pi$ intermediate channel seems as likely to be important here as the $\pi\pi$ channel was found to be for the ρ-A_2 difference.

Treatments of the ρ-A_2 torus shift by other authors have ignored the $\pi\pi$ contribution and have arrived at relatively large estimates of the contributions from channels involving only natural-parity trajectories. These other approaches have been based on s discontinuities rather than t discontinuities. In principle the two approaches should give the same answer, but in practice one may be more manageable than the other. We believe that for the torus, the t-discontinuity approach exploits more effectively the phenomenon of "precocious cylinder quenching," which leads to relatively accurate isospin degeneracy for <u>all</u> physical particles in the leading natural-parity family. This remarkable phenomenon is awkward to incorporate into an s-discontinuity calculation, although it is trivial for the t discontinuity.

For the case of the cylinder it has proven possible [6,7] to construct a model for $k(t)$ based on s-channel unitarity which at the same time exhibited the t-channel unitarity properties we have referred to in previous sections. The reason was the simple s-channel discontinuity structure of the cylinder operator as shown in fig. 5. The only nonvanishing s discontinuity is that labeled #1. More complicated s-channel discontinuities such as that labeled #2 vanish because one is cutting through a twisted reggeon. Furthermore, because all vertices are planar there will be no reggeon-reggeon cut. Neither of these simplifying conditions is present in the case of the torus. Discontinuities through the cylinder in fig. 4 are nonvanishing and, because the cylinder reggeon-reggeon vertex is nonplanar, we have reggeon cuts. These phenomena complicate s-channel models and obscure the connection with a t-channel description. Only for the cylinder is the structure sufficiently noncomplex as to permit simultaneously simple s- and t-channel descriptions.

There have been various attempts to calculate s discontinuities of the torus, each employing different approximation. Major difficulties have been to estimate the sign and magnitude of fixed pole residues, and to evaluate absorptive cuts. In a complete s-discontinuity calculation of the contribution from natural parity intermediate trajectories the various components would, according to our results here, cancel each other.

It is easy to identify one important s-channel contribution that has so far been ignored. Crucial to our analysis of the torus has been the realization that the cylinder is not just the physical f or ω but also contains a negative planar piece (see eq. 10). To date all s-channel calculations of the torus have included only the physical f (Pomeron) component of the cylinder link and have neglected the negative planar piece.

Our calculation of ρ-A_2 splitting has produced a small value. From the point of view of the topological expansion there are two sources of this smallness. One is the $1/N^2$ factor. But in addition we have found that the cylinder insertion from which tori are constructed, involves the cylinder-shifted trajectory minus the original planar trajectory. It is only when the difference between cylinder-shifted and planar objects are of order unity that handle corrections will even be as large as $1/N^2$. In the example we have considered the natural-parity handle contributions are small because in the relevant regions, the difference as expressed in eq. (10), is small. It is only the π,η system, where the cylinder shift is significant, that can make contributions as large as $1/N^2$.

V. SUMMARY AND CONCLUSION

The chief new idea in this paper has been our conjecture that the torus of fig. 4, in contrast to the cylinder, satisfies an unsubtracted t-dispersion relation. Although we were led to this conjecture by the difference in structure of formulas (6) and (7) based on G-parity considerations, the conjecture might have been arrived at by straightforward topological-expansion analysis, coupled with the principle of asymptotic planarity. One striking physical consequence is that isotriplet trajectory splittings such as ρ-A_2, that are controlled by the torus of fig. 4, should approach zero at both large positive t and large negative t. The maximum difference should occur at moderately small positive t where the $\pi\pi$ channel plays a dominant role; at larger positive t the difference should change sign. In contrast, an isosinglet trajectory splitting such as f-ω, that is controlled by the cylinder, grows indefinitely as $t \to -\infty$ even though it decreases as $t \to +\infty$.

Our conjecture is encouraged by the low-t discontinuity of $\alpha_{A_2} - \alpha_\rho$, which has a maximum in the $\pi\pi$ dominated region, $t \lesssim 1\ \mathrm{GeV}^2$. This maximum has the appropriate magnitude and sign to explain both the ρ-A_2 splitting and the t derivative of the splitting near $t = 0$.

An historical comment may be in order. For many years attempts were made to explain the ρ as a dynamical $\pi\pi$ composite, but the attempts were quantitatively unsuccessful, and recently it has come to be believed that channels such as $\rho\rho$ are more important than $\pi\pi$. Roughly speaking, what we are suggesting in this paper is that the $\pi\pi$ contribution, present in the ρ but not in the A_2, is largely responsible for ρ-A_2 splitting. The observed smallness of this splitting confirms that the $\pi\pi$ contribution to ρ dynamics is relatively unimportant. At the same time it is agreeable to discover that in the experimental domain of isotriplet exchange-degeneracy breaking the extensive theoretical investment in $\pi\pi$ dynamics seems to be relevant.

There have been previous, qualitative attempts to explain deviations from exchange degeneracy in terms of t-channel dispersion relations [13]. Our investigation has put this approach on a firm theoretical and quantitative footing. The use of an unsubtracted dispersion relation for ρ-A_2 splitting was justified and satisfactory numerical results that are in accord with recent experiments were found. We have seen that the t-channel approach is not well suited for the f-ω splitting, but we have previously given a satisfactory treatment of this problem from an s-channel point of view [6]. The topological expansion provides a coherent and consistent framework for discussing both aspects of the problem and correctly describes the observed pattern of broken exchange degeneracy.

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FIGURE CAPTIONS

1. Two-particle planar discontinuity diagram.
2. Diagram of the cylinder operator.
3. The low-t contributions to the discontinuity of $\alpha_{A_2}-\alpha_\rho$. Interpolation between the experimental points at the resonance masses was done by using standard centrifugal barrier and threshold factors. The thresholds for the $\eta\rho$, πA_2 and πf contributions are also indicated.
4. Diagram of the torus that breaks exchange degeneracy.
5. Two different discontinuities of the cylinder operator.

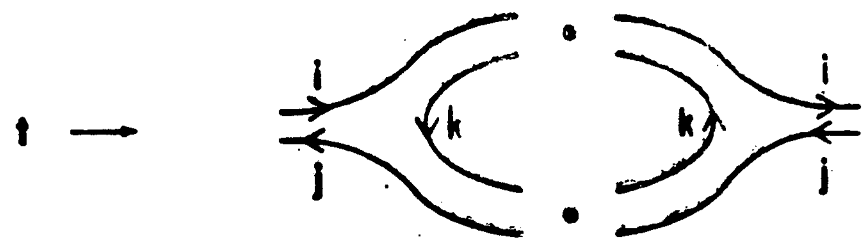


FIG. 1

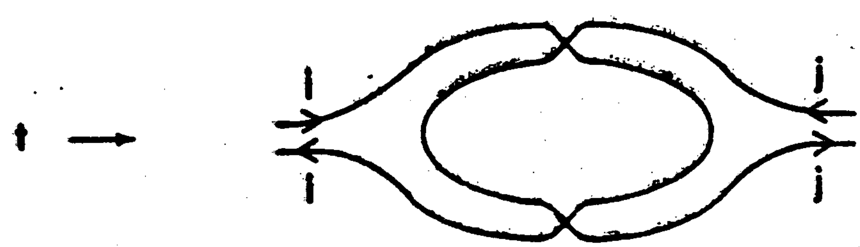


FIG. 2

FIG. 3

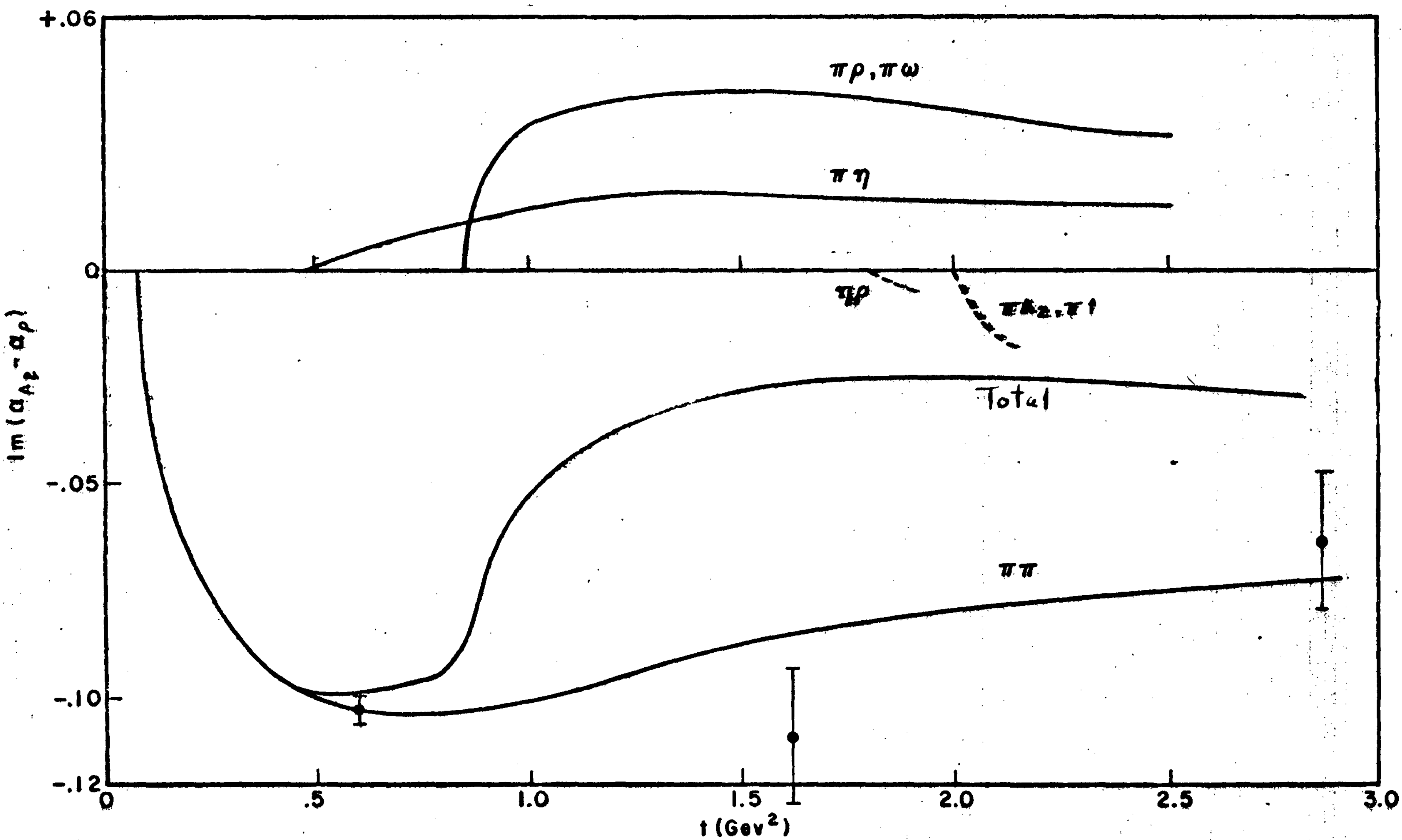
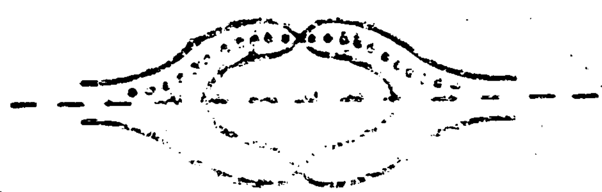


FIG. 4

FIG. 5



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