Title

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Abstract

Production and cost frontiers of a firm in an industry are directly affected by the uncertainty of market demand and the uncertainty of input availability. The nonparametric approach of data envelopment analysis is generalized here in both static and dynamic directions by incorporating these uncertainties.

Keywords: production and cost efficiency; demand and supply uncertainty; growth and level efficiency

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1. Introduction

The nonparametric analysis of input-output efficiency by the data envelopment analysis (DEA) generally assumes a deterministic framework with no uncertainty. In the real world however the input output decisions by a firm or decision-making unit (DMU) are constrained by both market uncertainty and technological uncertainty. The former arises when firms have to sell their outputs in a market where expected demands may not always be realized by actual or observed demand. Technological uncertainty occurs when inputs required for current production at time t may not be completely known to the firm; the firm has then to forecast the availability of inputs at t from past information at times t-1, t-2 and so on.

Our object here is two-fold. We consider first an extension of the convex hull method of DEA for determining a production frontier in the presence of two types of uncertainty as above. Secondly, we consider some dynamic extensions of the efficiency model under uncertainty, when the firms have to consider inventory costs if supply of output exceeds demand.

2. Static models of uncertainty

We consider here two versions of the nonparametric approach known as the data envelopment analysis (DEA): the production frontier and the cost frontier approach. The former deals with the decision problem of a firm or a decision-making unit (DMU) when it has to produce the maximum possible output from a given set of inputs. The latter measures the firm's success in choosing an optimal set of inputs by minimizing total input costs. Two types of uncertainty have been generally considered in the DEA literature. First, we have the case of uncertainty in the output and input prices, when future demand is unknown. In such cases the DMUs consider the riskiness of alternative production plans due to price fluctuations and allow for these in terms of the means and variances of prices to define an optimal risk adjusted production plan. Sengupta [1,2] has dealt with this type of *risk averse efficiency frontier* approach in both static and dynamic forms and explored their relationship with the deterministic approach.

Secondly, the incompleteness of the information structure available to different DMUs has been recently used by Bogetoft [3] to consider the post productivity analysis problem of deciding which production plans to choose in the future given information from past and current production analysis.

We consider here another important class of uncertainty arising in DEA framework in both optimal and post optimal phase. In the optimal phase we consider specifically the role of demand and supply uncertainty of output and inputs respectively. In the post optimal phase we consider the role of demand and supply forecasts of output and inputs respectively, when information evolves over larger samples.

We start with a production model with one output (y), m inputs (x_i) and N firms or DMUs. We compare the relative efficiency of firms in choosing the inputs optimally by minimizing total input costs (TC) with given input prices (q_i). The DEA model takes the form

$$\text{Min TC} = \sum_{i=1}^{m} q_i x_i$$

subject to (s.t.)

$$\sum_{j=1}^{N} x_{ij}\lambda_j \le x_i;\ i=1,2,\ldots,m \tag{1}$$

$$\sum_{j=1}^{N} y_j\lambda_j \ge y;\ \sum_j \lambda_j = 1;\ \lambda_j \ge 0$$

Let x_i^*, y* and λ_j^* denote the optimal values, where a cost efficient DMU_j must be on the production frontier

$$y^* = \gamma_0^* + \sum_{i=1}^{m} \gamma_i^* x_{ij};\ \gamma_0^* = \beta_0^*/\alpha^*,\ \gamma_i^* = \beta_i^*/\alpha^* \tag{2}$$

obtained from the Lagrangean

$$L = -\sum_i q_i x_i + \sum_{i=1}^{m} \beta_i \left(x_i - \sum_{j=1}^{N} x_{ij}\lambda_j \right) + \alpha((\sum_j y_j \lambda_j - y) + \beta_0 (1- \Sigma\lambda_j)$$

with nonnegative optimal values of α^*, β_i^* and β_0^*. This type f cost minimization model is appropriate for public sector enterprises, where output prices are not market determined but input prices are. For private sector firms the objective function in (1) may be replaced by a profit function π

$$\text{Max } \pi = py - \sum_{i=1}^{m} q_i x_i$$

$$\text{s.t.} \quad (x,y) \in R$$

where R is the constraint set defined by the linear constraints of (1) and p is the output price given for each firm.

Next we consider a cost frontier model where all costs are combined as C_j for firm j with components C_{ij} denoting labor costs, material cost and cost of capital services. The input oriented DEA model may then be set up as

$$\text{Min } \theta$$

$$\text{s.t.} \quad \sum_{j=1}^{N} C_{ij}\lambda_j \le \theta C_{ih};\ \sum_{j=1}^{N} C_j \lambda_j = C_h$$

$$\Sigma \lambda_j = 1,\ \lambda_j \ge 0;\ j=1,\ldots,N \tag{3}$$

where DMU_h is tested for efficiency relative to all other DMUs. If the optimal values λ_j^*, θ^* are such that $\theta^* = 1.0$ and all slack variables are zero, then DMU_h is cost efficient in the sense

$$\sum_{j=1}^{N} C_{ij}\lambda_j^* = C_{ih};\ i=1,2,\ldots,m$$

and $\quad \Sigma\, C_j \lambda_j^* = C_h$

By duality this implies that if DMU_j is on the cost frontier it must satisfy the condition

$$C_j = x_0^* + \sum_{i=1}^{m} \gamma_i^* C_{ij}\ ;\ x_0^* = \beta_0^*/b^*,\ x_i^* = \beta_i^*/b^* \tag{4}$$

where b is the Lagrange multiplier associated with the constraint $\sum_j C_j \lambda_j = C_h$.

One may also rewrite the model in a form where each DMU is choosing the inputs x_i optimally. Let $c_{ij}x_{ij}$ denote C_{ij}. The model then takes the form

$$\text{Min TC} = \Sigma\, q_i x_i$$

$$\text{s.t.} \quad \sum_{j=1}^{N} c_{ij} x_{ij} \le x_i;\ \sum_j C_j \lambda_j = C_h \tag{5}$$

$$\Sigma\, \lambda_j = 1,\ \lambda_j \ge 0;\ j=1,2,\ldots,N$$

where it is assumed that the input price q_i equals the average input cost $\bar{c}_i$. The cost frontier in this case for DMU_j must satisfy the conditions:

$$\beta_i^* = \bar{c}_i = q_i$$

$$C_j = \gamma_0^* + \sum_{i=1}^{m} \gamma_i^* c_{ij} x_{ij} \tag{6}$$

Another form of the cost frontier is generated when the cost output relation is considered, e.g.,

$$\text{Min } \theta$$

$$\text{s.t.} \quad \sum_{j=1}^{N} C_j \lambda_j \le \theta\, C_h; \; \sum_j y_j \lambda_j \ge y_h$$

$$\sum_j \lambda_j = 1; \; \lambda_j \ge 0, \; j=1,2,\ldots,N \qquad (7)$$

If DMU_j is efficient here, then it follows by duality that it is on the cost output frontier defined by

$$C_j^* = (\beta_0^* / \beta^*) + (\alpha^* / \beta^*)\, y_j \qquad (8)$$

where the Lagrangean is

$$L = -\theta + \beta(\theta\, C_h - \sum_j C_j \lambda_j) + \alpha(\sum_j y_j \lambda_j - y_h) + \beta_0\, (\Sigma\beta\, \lambda_j - 1)$$

If we add a second order output constraint to (7) as

$$\sum_{j=1}^{N} y_j^2 \lambda_j \ge y_h^2 \qquad (9)$$

then the cost output frontier becomes quadratic

$$C_j^* = \gamma_0^* + \gamma_1^* y_j + \gamma_2^* y_j^2 \qquad (10)$$

where $\gamma_0^* = \beta_0^* / \beta^*, \gamma_1^* = \alpha^* / \beta^*$ and $\gamma_2^* = a^* / \beta^*$, with a* as the nonnegative Lagrange multiplier of the output constraint (9).

The quadratic cost frontier (10) has two advantages over the linear frontier (8). First, it is more flexible since marginal cost varies as output varies. Secondly, one may further minimize the average cost for the efficient DMU_j:

$$AC_j^* = C_j^* / y_j = \frac{\gamma_0^*}{y_j} + \gamma_1^* + \gamma_2^* y_j \qquad (11)$$

On minimizing this AC_j one obtains the optimal size of output (y_j^{**}) defining the most efficient scale as:

$$y_j^{**} = (\gamma_0^* / \gamma_2^*)^{1/2}$$

with $$c_j^{**} = \min AC_j^* = \gamma_1^* + (\gamma_2^* \gamma_0^*)^{1/2} \qquad (12)$$

Clearly the observed average cost (c_j^*) and output (y_j^*) on the frontier would satisfy the inequalities:

$$y_j^{**} > y_j^* \text{ and } c_j^{**} < c_j^* \qquad (13)$$

This implies that demand for firm j must be high enough for producing y_j^{**} to meet demand. Thus with a lower demand the firm would supply y_j^*, but a higher demand would generate a higher supply y_j^{**}.

Now we consider the role of demand uncertainty in the DEA models introduced above. Consider first the cost minimizing model (1) where demand for output is r and it is random; also the input supply is z_i and it is random. We have to replace the objective function in (7) by the expected total cost, since output demand and input supply contain random fluctuations. Let f(r) and $f(z_i)$ be the probability density functions of demand r and supply z_i with F(r) and $F(z_i)$ denoting their cumulative distributions. Then the DEA model can be transformed as

$$\text{Min ETC} = \sum_i q_i \left[a_i \int_{x_i}^{\infty} x_i f(z_i)\, dz_i + b_i \int_0^{x_i} z_i f(z_i)\, dz_i \right]$$

$$+ g \int_0^y (y - r) f(r)\, dr + h \int_y^{\infty} (r - y) f(r)\, dr$$

$$\text{s.t.} \quad \sum_{j=1}^{N} x_{ij} \lambda_j \le x_i;\ 1,2,\ldots m \qquad (14)$$

$$\sum_j y_j \lambda_j \ge y;\ \Sigma \lambda_j \;\; 1,\ \lambda_j \ge 0;\ j=1,2,\ldots,N$$

Here the unit inventory costs are a_i and g and the cost of lost sales and the input shortfall are h and b_i respectively. These parameters are assumed to be known by each firm. The optimal inputs and output are now determined as

$$x_i^* = F_{-1}(\delta_i), \delta_i = 1 - (\beta_i^* / a_i q_i)$$

$$y^* = F^{-1}(\phi), \phi = (g + h)^{-1} (\alpha^* + h) \tag{15}$$

$$\alpha^* y^* = \beta_0^* + \sum_{i=1}^{m} \beta_i^* x_{ij}$$

Clearly higher inventory costs would tend to reduce optimal levels of output and input use. The gaps $|x_{ij} - x_i^*|$ and $|y_j - y^*|$ would indicate inefficiency in input usage and output production respectively.

Two implications of these results are to be noted. First, the presence of expected inventory costs and costs of shortage would affect the DEA efficiency results derived under a deterministic framework. This may explain why some firms may carry large 'organizational slacks' on the average and may then be judged as inefficient in the deterministic DEA approach. Secondly, the form of the distribution of demand for output and supply of inputs would affect the level of expected inventory or shortage carried by firms.

Consider now the transformation of the quadratic cost model specified by (7) and (8) when only demand uncertainty is present:

$$\text{Max } E(p \min(y,r)) - w\,C$$

$$\text{s.t.} \quad \sum_j C_j \lambda_j \le C;\ \Sigma\, y_j \lambda_j \ge y,\ \Sigma\, \lambda_j = 1,\ \lambda_j \ge 0$$

where w = 1.0 and r is random demand with a distribution F(r). The optimal output and the cost frontier can be easily calculated as follows for an efficient DMU_j:

$$C_j = \gamma_0 + \gamma_1 + \gamma_2 y_j^2;\ \gamma_0 = \beta_0^* / \beta^*, \gamma_1 = \alpha^*/\beta^*, \gamma_2 = a^*/\beta^*$$

$$2a^*y^* + p\,F(y^*) + \alpha^* - p = 0$$

If the demand r is uniformly distributed with range $0 < r < k$, then the optimal output reduces to

$$y^* = (2a^* + p/k)^{-1} (p - \alpha^*);\ p > \alpha^* \qquad (15)$$

where the Lagrangean is

$$L = E[p \min(y,r)] - C + b(C - \Sigma C_j\lambda_j) + \alpha(\Sigma y_j\lambda_j - y) + \beta_0(\Sigma \lambda_j - 1)$$

For the linear cost output relation $\gamma_2 = 0$ and we obtain

$$F(y^*) = 1 - \alpha^*/p;\ y^* = F^{-1}(1 - \alpha^*/p) \qquad (16)$$

If for example the demand has an exponential distribution with parameter $\lambda = 1/E(r)$, then (16) reduces to

$$y^* = \ln(p/\alpha^*)\ \bar{r},\ \bar{r} = Er$$

Thus the level of optimal output rises as mean demand $\bar{r}$ or price rises and it falls when the implicit cost (α^*) of output inefficiency rises.

Two comments are in order here. First, the risk aversion factor for each firm can be included here by minimizing for example a linear combination of expected total cost and variance of total cost in the DEA formulation (14) for example. However this would impart a higher degree of nonlinearity in the efficiency frontier. Secondly, the firms have no control on the fluctuations of demand and input supply, since we are assuming a competitive market with firms as price takers. In imperfectly competitive markets however price p will vary in relation to demand and thus the price elasticity would affect the optimal levels of output and inputs for instance.

3. Dynamic models under uncertainty

We consider here two types of dynamic models under uncertainty. One follows a production schedule in model known as the HMMS [4] model well known in operations research literature. Here we assume demand uncertainty only and each firm is assumed to maximize an intertemporal net expected return function defined as expected revenue minus expected input and inventory costs over time. The second model assumes that each firm has incomplete knowledge of the inputs and output in the current period t and it has to decide on the levels of optimal input and output. Clearly the efficient firm in this framework has two options. One is to forecast the current levels of inputs and output on the basis of past levels and then use a DEA model to compute an efficiency frontier. The second option is to use last period's inputs and output as observed data and then estimate the optimal production frontier in the current under demand uncertainty. These two cases would be discussed here.

The first type of dynamic model in a DEA framework has the following intertemporal form:

$$\text{Max } J = \sum_{t=1}^{T}[E\,p(t)\{\min(y(t),\tilde{r}(t))\} - EC(I(t),I(t-1)) - \sum_{i=1}^{m} q_i(t)x_i(t)]$$

subject to the constraint set R defined by

$$I(t) = I(t-1) + y(t) - \bar{r}(t);\ I_0 \text{ given}$$

$$\sum_{j=1}^{N} x_{ij}(t)\,\lambda_j(t) \le x_i(t),\ i=1,\ldots,m$$

$$\sum_{j=1}^{N} y_j(t)\ \lambda_j(t) \ge y(t) \tag{17}$$

$$\sum_j \lambda_j(t)\, I_j(t) \le I(t)$$

$$\sum_j \lambda_j(t) = 1; \lambda_j(t) \geq 0 \text{ ; } j=1,2,\ldots,N,\ t=1,2,\ldots,T$$

Here $\bar{r}(t)$ is random demand for output with a fixed distribution F(r) with mean $\bar{r}$, E is expectation and the inputs $x_i(t)$, inventory I(t) and output y(t) are optimally chosen for the efficient firm. The prices p(t) and $q_i(t)$ are given. The expected inventory costs are of the HMMS form:

$$EC(I(t), I(t-1) = \tfrac{1}{2}(I(t) - \hat{I}(t))^2,\ \hat{I}(t) = k\ \bar{r}(t)$$

with $\hat{I}(t) = k\ \bar{r}(t)$ as the target level of inventory viewed as a proportion of mean demand. On using the Lagrangean function

$$L = J + s(t)\{I(t) - I(t\text{-}1) - y(t) + \bar{r}(t)\}$$

$$+ \sum_{i=1}^{m} \beta_i(t)\{x_i(t) - \sum_j x_{ij}(t)\lambda_j(t)\}$$

$$+ b(t)\{I(t) - \sum_j I_j(t)\,\lambda_j(t)\} + \alpha(t)\{\sum_j y_j(t)\,\lambda_j(t) - y(t)\}$$

$$+ \beta_0(t)\{1 - \sum_j \lambda_j(t)\}$$

and applying the Euler-Lagrange conditions it follows that the optimal efficiency frontier for firm j must satisfy for each t=1,2,…,T the following necessary conditions for positive levels of y*(t), $x_i^*(t)$ and I*(t):

$$p(1 - F(y^*)) = s^*(t) + \alpha^*(t);\ \beta_i^*(t) = q_i(t)$$

$$I^*(t) = k\bar{r}(t) + b^*(t) - s^*(t) + s^*(t) \qquad (18)$$

$$\alpha^*(t)\ y_j(t) = \beta_0^*(t) + \sum_{i=1}^{m} \beta_i^*(t)\ x_{ij}(t) + b^*(t)\ I_j^*(t)$$

$$I^*(t) = I^*(t-1) + y^*(t) - \bar{r}(t)$$

where asterisks indicate optimal values. Several implications follow. First of all, price equals the marginal costs of lost sales, inventories and output in the form of shadow prices. Optimal inventories at time t depend on the target or desired level $\hat{I}(t)$ of inventories, the shadow price of inventories and the change $\Delta s^*(t) = s^*(t+1) - s^*(t)$ in the shadow price of incremental inventories $\Delta I(t) = I(t) - I(t-1)$. This implies that firms with higher desired levels of $\hat{I}_j(t)$ will carry higher optimal inventories and hence have higher inventory costs. Secondly, if these conditions (18) hold for any j over all t, then an optimal recursive decision rule for each of the optimizing variables $z^*(t) = (y^*(t),\ x_i^*(t),\ I^*(t))$ can be constructed as a linear function of lagged $z^*(t-1)$ and $\sum_{i=0}^{T-1} u_i \bar{r}_{t+i}$, where u_i is a function of the parameters already computed and $\bar{r}_{t+i}$ is the forecast level of mean demand at time t+i. The advantage of this optimal linear decision rule is that it allows a sequential revision of policies over time as the precision of forecasting of demand improves. Finally, the relative inefficiency of any firm h can be estimated as in the DEA model, i.e., output shortfall below the optimal or excess inputs over the optimal level.

Next we consider the uncertainty associated with incomplete information available to each DMU at time t. Here we assume that each DMU wants to select the optimal inputs $x_i(t)$ and output y(t) at the current time t, given the information at time t-1. The DEA model is of the form

$$\text{Min TC} = \sum_{i=1}^{m} \hat{q}_i(t)\, x_i(t)$$

$$\text{s.t.} \quad \sum_{j=1}^{N} \hat{x}_{ij}(t)\, \lambda_j(t) \le x_i(t) \qquad (19)$$

$$\sum_{j=1}^{N} \hat{y}_j(t)\, \lambda_j(t) \ge y(t)$$

$$\Sigma\, \lambda_j(t) = 1;\ \lambda_j(t) \ge 0;\ j=1,2,\ldots,N$$

Here the hat over a variable denotes its forecast from the past levels. Assuming a Markovian framework we assume the forecasts to be generated by an exponentially weighted scheme (EWS) as follows:

$$\hat{z}(t) = \hat{z}(t\text{-}1) + v[z(t\text{-}1) - \hat{z}(t\text{-}1)];\ 0 < v < 1 \tag{20}$$

where z(t) may denote any of the variables $q_i(t)$, $x_{ij}(t)$ and $y_j(t)$ above. It is important to note two important features of the EWS of generating forecasts. First, the forecast value $\hat{z}(t)$ may be viewed as the weighted average of past value z(t-1) and its forecast at t-1, i.e., $\hat{z}(t)$ = vz(t-1) + (1-v) $\hat{z}(t\text{-}1)$. Secondly, one could rewrite (20) in the form

$$\hat{z}(t) = \sum_{i=1}^{\infty} v(1-v)^{i-1} z(t-i)$$

$$= \sum_{k=1}^{\infty} w_k\, z(t-k), w_k = v(1-v)^{k-1} \tag{21}$$

where the weights w_k attached to past values decrease exponentially. Note that if v is close to one, the recent observations get more weight so that in the limit v=1.0 the past observations have no influence on the forecast values. If v is small, then the past values are important. Clearly the EWS yields smoothed input output data with reduced noise when compared with the observed data containing fluctuations. Hence the DEA model (19) with optimal estimates of $x_i^*(t)$, y(t) would yield a more stable production frontier. The degree of stability of this production frontier may be assessed if the observed time series z(t) can be decomposed into two independent additive components, e.g., the permanent component $\hat{z}(t)$ and the transitory component n(t). Assume the permanent component to follow a linear stochastic process

$$\hat{z}(t) = \hat{z}(t\text{-}1) + \varepsilon(t) = \sum_{i=1}^{t} \varepsilon(i),\ \ \hat{z}(0) = 0$$

where the ε's are serially independent with mean zero and constant variance σ_ε^2. The forecasting problem is then to find the coefficients w_k in (21) which minimize the error variance v

$$v = E[z(t) - \hat{z}(t)]^2$$

Following Muth [5] one can show that the optimal weights w_k^* are given by

$$w_k^* = (1-\mu_1)\ \mu_1^{k-1}$$

where μ_1 is the smaller characteristic root less than unity satisfying the characteristic equation

$$(\sigma_\varepsilon^2 / \sigma_n^2) - (1-\mu)^2 / \mu = 0$$

where σ_n^2 is the variance of the transitory component. With the optimal values of w_k^*, the optimal weights v* can be easily determined from (21). Thus if the fluctuations or noise in the observed input output data are large, then a smaller value of v* helps. The reverse holds for smaller fluctuations in observed data.

The generality of this smoothing approach may be seen by applying this EWS to the cost efficiency model defined by (7) and (9) by replacing C_j, y_j and λ_j by $\hat{C}_j(t), \hat{y}_j(t)$ and $\lambda_j(t)$ respectively. The smoothed efficiency frontier for DMU_j may then be derived as:

$$\hat{C}_j^*(t) = \tilde{\gamma}_0^* + \tilde{\gamma}_1^* \hat{y}_j(t) + \tilde{\gamma}_2^* \hat{y}_j^2(t)$$

This frontier may be compared with the frontier (10) derived from observed data. Secondly, the optimal weights v* may be different from the inputs and output and one could select different values of v to simulate the cost efficiency frontier. Thus the sensitivity of the DEA efficiency frontier may be directly evaluated.

Finally, one could apply this method to estimate a stable growth efficiency frontier by the DEA approach. Let $\hat{X}_{ij}(t)$ and $\hat{Y}_j(t)$ denote the relative growth $\Delta\hat{x}_{ij}(t)/\hat{x}_{ij}(t)$ and $\Delta\hat{y}_j(t)/\hat{y}_j(t)$

of inputs and outputs in their smoothed values. In order to test the relative efficiency of DMU_h we set up the linear programming model:

$$\text{Min } \sum_{i=1}^{m} \beta_i \hat{X}_{ih}(t) + \beta_0 \tag{22}$$

$$\text{s.t.} \quad \beta_0 + \sum_{i=1}^{m} \beta_i \hat{X}_{ij}(t) \geq \hat{Y}_j(t)$$

$$\sum_{i=1}^{m} \beta_i = 1, \ \beta_i \geq 0; \ \beta_0 \text{ free in sign}$$

This assumes constant returns to scale since $\sum_{i=1}^{m} \beta_i = 1$. Without this constraint we would have variable returns to scale. Let the optimal values of β_i and β_0 be β_i^* and β_0^*. If DMU_h is efficient, then its growth efficiency frontier is

$$\Delta\hat{y}_h(t)/\hat{y}_h(t) = \sum_{i=1}^{m} \beta_i^* (\Delta\hat{x}_{ij}(t)/\hat{x}_{ij}(t) + \beta_0^*$$

$$\sum_{i=1}^{m} \beta_i^* = 1, \beta_0^* \text{ free in sign.} \tag{23}$$

When β_0^* is positive, it measures according to Solow [7] the rate of technological progress. Solow assumed the Cobb-Douglas production function

$$y(t) = A(t)\, x_1^{\beta_1}(t)\, x_2^{\beta_2}(t); \ \beta_1 + \beta_2 = 1$$

with two inputs: labor $x_1(t)$ and capital $x_2(t)$ and derived by differentiation

$$\Delta y(t)/y(t) = \Delta A(t)/A(t) + \sum_{i=1}^{2} \beta_i (\Delta x_i(t)/x_i(t))$$

where $\Delta A(t)/A(t) = \beta_0$ is termed the rate of technological progress ($\beta_0>0$) or regress ($\beta_0<0$). Clearly the growth efficiency frontier (23) above can be used to estimate both scale economies (e.g., variable returns to scale) and technological progress. The smoothing technique would permit a more stable estimate of the growth efficiency frontier of a typical firm or DMU_h and this may be compared with the level efficiency frontier, when only the input and output levels are considered. Sengupta [6] has recently applied this method to the computer industry over the period 1987-2000.

4. Conclusions

Models of data envelopment analysis have been generalized here in both static and dynamic versions under conditions of uncertainty of demand and the uncertainty of input supply. The role of inventory cost and the loss due to lost sales are captured in this framework. In a dynamic setting the uncertainty of the information structure is captured through an exponentially weighted forecasting technique, which allows us to characterize a more stable production and cost frontier. This characterization can also be applied to estimate nonparametrically a stable growth efficiency frontier, which can measure the scale economies and the rate of technological progress.

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