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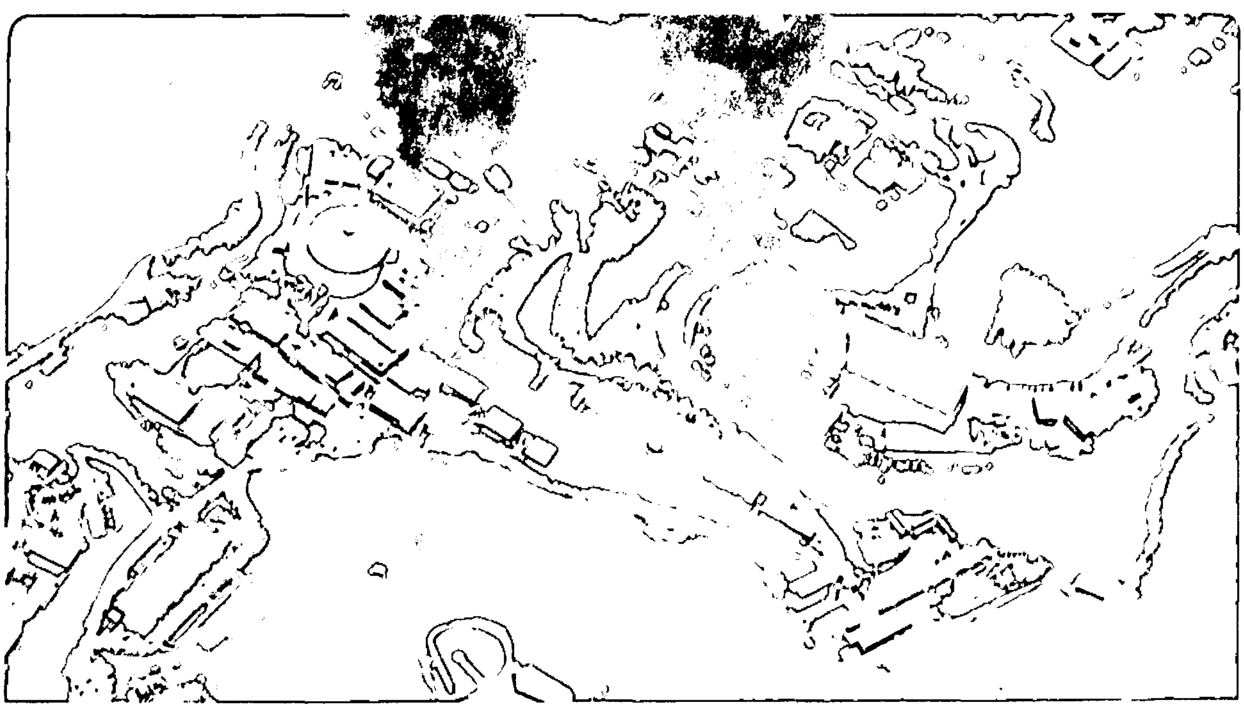

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BOOTSTRAPPING THE PHOTON*

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ABSTRACT

A nontechnical review is given of topological bootstrap theory, with emphasis on the raison d'être for an electromagnetism whose fine-structure constant is of order 10^{-2}.

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I. INTRODUCTION

Physicists have achieved for electromagnetism a description acclaimed for beauty and power as among the foremost human achievements. Over several centuries there has evolved a chain of electromagnetic insights interwoven with space-time, causality and quantum mechanics. Contributing to the latter connection was Dirac's equation relating relativistic electrons to electromagnetism. More recently, building on Dirac's discovery, connections have been found between electromagnetism and short-range weak interparticle forces. Paradoxically, throughout this remarkable history interweaving with the fabric of physics, electromagnetism in one sense remained aloof; physicists failed to achieve understanding of electromagnetism's strength. Until recently no physical principles have precluded a zero value for the corresponding dimensionless parameter. "Turning off" electromagnetism is allowed by the Newtonian-Cartesian view of a universe governed by equations of motion for "objects" in a space-time continuum. The 19th century Maxwelliam view of electromagnetic fields was "objective" -- a viewpoint left unchanged by the 20th century's Lagrangian-based quantum electrodynamics (QED). Photons, that is to say, are regarded in QED as objects in the same sense as electrons.

A nonobjective aspect of electromagnetism does seem to preclude exact vanishing for the elementary electric charge. This is the circumstance that any "observation" of one macroscopic object by another such object involves electromagnetic interaction. The zero photon mass and the associated long-rang coupling between electric charges provides a "gentle" communication between large, almost isolated and electrically almost-neutral systems; such gentle interaction, which underlies the meaning of localized "reproducible" experiments, is uniquely achievable through electromagnetism. (Gravitational interaction also is of long range but is not screenable through the cancellation of opposite charges and not suitable as an agent of measurement.) One may therefore argue the nonvanishing of electromagnetism as essential to an observable "real world" of distinguishable objects, but the numerical value of the elementary charge remains unexplained.

Slowly, over recent decades, there has been developing recognition that physics need not, and perhaps should not, be based on an a priori space-time continuum; Newtonian-Cartesian objectivity may be not an underpinning but rather one approximate aspect of a "larger reality". Since the beginning of this century, observation of quantized physical phenomena has been suggesting that apparent continua are but approximations to an underlying discrete structure, even though it has turned out possible to understand an extraordinary assortment of atomic and nuclear observations through continuous equations of motion supplemented by quantized rules of interpretation. (Dirac's electron-electromagnetic equation is a famous example.) Starting in 1939, on the other hand, with recognition by Wheeler of the S-matrix concept -- a notion generalizing Heisenberg's original matrix approach to quantum mechanics and elaborated during the forties by Heisenberg himself -- the possibility has been appreciated of achieving the successful predictive content of quantum theory from a discrete basis without equations of motion.[1]

Recently there has become recognized an S-matrix framework based on embellished Feynman graphs which contacts observed discrete phenomena and at the same time promises explanation of the apparently-continuous Newtonian-Cartesian "classical" world in which we recognize our existence. The expected link between quantum (discrete) and classical (continuous) worlds lies in the "soft" photon ("infrared") aspects of Feynman graphs; the raison d' être of quantized electromagnetism thereby looms as the raison d' être of Newtonian-Cartesian reality with its space-time continuum. From such a viewpoint space-time is expected to be "built" from soft photons.

This paper will describe considerations implying not only the inevitability of electromagnetism in a consistent S-matrix but the numerical value of the elementary electric charge. This latter number emerges from combinatorial topology describing an entropy-carrying S matrix. Introduction of entropy into S-matrix theory via embellished Feynman graphs is the key addition to the structure recognized by Heisenberg and Wheeler.

The approach to be described is called "bootstrap" theory because of its assumption that elementary particles are not arbitrarily postulable but uniquely determined by S-matrix consistency; each particle is supposed to owe existence to interaction with other particles.[2] The term "bootstrap", introduced in 1959 in the foregoing sense before entropy became appreciated, is now developing broader significance -- synonymous with the more general and much older idea that no aspect of nature is arbitrary. Bootstrappers expect eventually to understand our familiar space-time world of "real" objects as one aspect of a self-sustaining universal "mosaic". This paper will nevertheless restrict itself to the quantum world -- the S matrix. The posed question is, "why must a consistent S matrix include photons coupled to an elementary charge whose squared value corresponds to the celebrated fine structure constant -- $e^2/4\pi \approx 1/137$? Although a brief account of this work has been written,[3] the lengthier treatment here should be easier for most readers to follow.

II. THE TOPOLOGICAL ENTROPY EXPANSION

Feynman graphs are usually seen as devices convenient for perturbative calculations based on a Lagrangian; topological particle theory on the other hand ascribes to Feynman's invention a more profound significance associated with a notion of entropy. Any S-matrix connected part (scattering amplitude) S_{fi}, joining an ingoing set i of elementary particles to an outgoing set f, is represented by an infinite "topological expansion"[4]

$$S_{fi} = \sum_{\gamma} S_{fi}^{\gamma} \tag{II.1}$$

in which each term, labeled by the index γ, belongs to an embellished (connected) Feynman graph. The embellishment, which is gradually described in the course of this review, attaches to each graph a well-defined discrete measure of complexity. When graphs are combined ("added" together) to form new graphs, an appropriately-defined graphical complexity cannot decrease and correspondingly may be called "entropy". Terms within the topological expansion (II.1) are to be understood as arranged in order of increasing entropy.*

There is no underlying continuous Lagrangian in topological particle theory. The set of elementary particles -- i.e., distinguishable Feynman lines -- is to be determined by S-matrix consistency within the discrete entropy expansion (II.1). A principal aspect of

* One entropy index easily appreciated is the total number of Feynman-graph vertices.

such consistency is unitarity, which determines through a "Feynman rule" the complex number S^{γ}_{fi} belonging to the embellished graph γ. Because the expansion extends to infinite entropy, a continuum of some kind is implicit; but it remains to be discovered what continuum will emerge. At the start no meaning is assumed for physical space-time or equations of motion.

Associating an elementary particle to each Feynman line means attachment of energy-momentum to the line with a conservation law at each graph vertex. Definition of an S matrix requires energy-momentum even though physical space-time at the start has been given no meaning. Later we shall also consider particle spin. It is assumed that spin and momentum transform in the usual way under the Poincaré group.[1]

No serious attempt has yet been made in topological particle theory to explain the physical relevance of the Poincaré group, although a bootstrap approach is obligated eventually to do so. Human limitations demand temporary compromises with the bootstrap assumption that consistency is all determining. At some point in the future the physical relevance of the Poincaré group should become understandable,* but currently-proceeding consistency analysis accepts this group as part of the also-accepted S-matrix notion. There is nevertheless no assumption of any "internal" group structure such as the celebrated SU(3) × SU(2) × U(1) which dominates current Lagrangian theories. To the extent that such structure is relevant, that fact is to be explained from the consistency of an S-matrix topological expansion based on

* Certain ideas of Penrose[5] look promising in this connection.

discrete graphs.

Readers unfamiliar with the S matrix concept may find puzzling the assumption of meaning for momentum-energy without an a priori significance for space-time. One needs to remember that, with no limitation on the scale of measuring apparatus, it is possible to measure particle velocity as accurately as desired without need for local space-time. We anticipate an association of "size" with topological complexity, so the inclusion of unbounded entropy within the topological expansion (1) looms as essential to the "asymptotic" measurements which the S matrix is supposed to describe. These measurements are anticipated to be understandable as highly-complex patterns of "gentle" discrete events involving soft photons.

Although one may associate the term "elementary event" with any vertex of a Feynman graph, the meaning of "physical event" is unclear at the start and must await elucidation of space-time through high entropy. Similar remarks apply to the notion of "physical particle" as opposed to the "elementary particle" which is synonymous with a Feynman-graph line. There has developed nevertheless, a useful working hypothesis about the latter relationship; this hypothesis is based on experience with the Feynman expansions that represent perturbative treatment of a Lagrangian.

The first aspect of the hypothesis is the correspondence between particles and poles of the S matrix, a correspondence supported by extensive study of general S-matrix conditions such as unitarity and causality.[1] It has been shown that S-matrix elements are analytic functions of energy-momentum variables apart from isolated singularities. Poles of the S matrix correspond to individual particles, and pole location in the complex energy-momentum Riemann surface corresponds to particle mass. It turns out that if a connected Feynman graph disconnects when one of its internal lines is cut, then this internal line corresponds to a pole of the associated topological-expansion component S^{γ}_{fi}.[6] The position of the pole is given by the mass of the elementary particle associated with the internal line. The pole residue is a product of the S-matrix connected parts belonging to the two separated graphs. This rule is a special case of Landau's general rules for associating Riemann-surface singularities with graphs.[6]

It is assumed that physical particles similarly correspond to poles of the matrix S_{fi}-- which is an infinite sum over topological components S^{γ}_{fi}. Experience with Lagrangian-based infinite Feynman expansions suggests that the topological expansion diverges at isolated points of the complex-momentum Riemann surface, creating new poles for S_{fi} not present in the individual components S^{γ}_{fi} and extinguishing elementary-particle poles.[7] The mass of a physical particle generally, then, does not coincide with any elementary-particle mass (the photon may be the unique exception to this rule, as discussed in Sec. XI). Physical protons, electrons, etc., have masses corresponding to the location of poles in S_{fi} that do not coincide with any pole of an S^{γ}_{fi}.

For reasons developed below it is necessary to attach topological embellishments to Feynman (momentum-energy carrying) graphs. One consequence is a collection of conserved "internal" quantum numbers which label a "Feynman space" of lines -- distinguishing one type of elementary particle from another. There will be described in this

review embellishments that distinguish elementary particles as carrying different quantities of topological quantum numbers called "baryon number", "lepton number", and "isospin". The Feynman space may be labeled by such topological attributes. Because conservation of these quantum numbers is preserved by all components of the infinite topological expansion, the poles of S_{fi} -- corresponding to physical particles -- also carry topological quantum numbers. A matching may then be made between certain physical particles and elementary particles carrying the same internal quantum numbers. We speak, for example, of an "elementary proton", an "elementary neutron", an "elementary electron" and an "elementary neutrino", since poles of S^{γ}_{fi} appear with internal quantum numbers matching physical particles with these names.

Certain topological embellishments needed for labeling of the Feynman elementary-particle space correspond to approximate quantum numbers -- conserved only for "orientable" components of the topological expansion. The distinction between orientable and nonorientable components will be discussed in Sec. VIII. Because non orientable components appear only with small magnitude, one also may meaningfully attach approximately-conserved quantum numbers to physical particles. An example is the attribute that has been called "strangeness"; attributes in this category have been generally described by a "generation index", as will be explained in Sec. VIII.

The bootstrap idea was spawned more than two decades ago by experimental measurements of strong interactions -- involving those physical particles like neutrons and protons that have been called "hadrons".[2] The observed order of magnitude of the interaction between hadrons was found to be understandable from S-matrix unitarity once the assumption was made that all hadrons are "bound states of each other". What is meant by such a "bound state" notion?

The most satisfactory statement depends on "dualizing" Feynman graphs so that contractions become possible. By "dualizing" we mean embellishing each graph so as to give it a well-defined complexity or "entropy". One then postulates that certain adjacent lines and adjacent vertices of a graph may be contracted into one another to achieve an equivalent graph; a necessary condition for contraction is that entropy is not thereby changed. It was discovered about 14 years ago by Harari and Rosner[8] that duality (contractability) constitutes a mathematically-precise graphical representation of the statement that each elementary hadron is a bound state of other elementary hadrons. Subsequent works developed the needed embellishments on the basis of S-matrix consistency and found a unique set of elementary hadrons, labeled by topological quantum numbers that explained the essential features of observed strong-interaction pheonomena.[9,10] We shall describe this Feynman hadron space in some detail below, since its understanding is prerequisite to understanding the photon -- which is not a hadron.

III. THE CLASSICAL SURFACE; COMPLEXITY AND CONTRACTION

Attribution to a connected Feynman graph of a complexity which consistently controls contraction requires embedding the graph in a connected bounded 2-dimensional surface -- discussed already in 1974 by Veneziano[4] and now called the "classical" surface.[9] The vertices of such an embedded graph are cyclically ordered and, because a 1-dimensional object has been extended to 2 dimensions, the embedded Feynman graph is said to be "thickened".[9]

The boundary of the classical surface is important.* All Feynman-graph ends lie along this boundary as do the ends of certain electric-charge carrying lines that nowhere touch the momentum-carrying Feynman graph. The latter "isospin" lines, due to J. Finkelstein,[12] will be described in Sec. VI; they are vital to the photon bootstrap. We further anticipate (following section) that spin flows along the Σ_C boundary. It must be understood that the classical surface of topological bootstrap theory is an abstract surface, without metric and not embedded in space time.**

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* The Harari-Rosner graphs discovered in 1969[8] lie along the classical-surface boundary and, to the extent that string models[11] relate to Σ_C, the "string" is the Σ_C boundary.

** The 1969 discovery of Harari-Rosner graphs[8] led to two distinct theoretical branchings: Lagrangian string models[11] and the topological expansion. Both branches interpret Harari-Rosner graphs as surface boundaries, but string models give the surface a space-time underpinning.

We shall find below that the complexity (or "entropy") classification of Feynman graphs attributes the lowest levels of complexity to single-vertex graphs.* More complex graphs are achieved by joining lines from originally-disconnected vertices. This joining is accomplished by gluing together appropriate classical-surface boundary segments -- segments containing the ends of lines to be joined. In this way "connected sums" of Feynman graphs are achieved through classical-surface "connected sums along the boundary". Any multivertex graph thereby finds itself automatically embedded in an appropriate connected bounded classical surface.

Allowable graph contractions respect the topology of the embedding classical surface. In particular the surface genus,** the number of boundary components and the cyclic order of Feynman-graph ends along each boundary component must not change. But certain internal Feynman lines may be removed -- identifying the two vertices at the line end points -- without changing the topology of the classical surface. When this line erasure is possible, it is understood that the topological index γ in the expansion (II.1) refers to the contracted graph. Equivalence of different thickened Feynman graphs that contract to the same graph is called "duality".

Veneziano[4] noted in 1974 that the genus of the classical surface may increase when connected sums are performed but never decreases.

* The "elementary vertices" of topological theory play a role analogous to the interaction terms of a Lagrangian. Lagrangian theory, however, does not permit graph contraction.

** For an orientable surface, the genus is twice the number of handles.

The genus -- which counts the number of independent handles and Möbius bands contained in the surface -- is in this sense an "entropy" index. Components of the topological expansion (II.1) may be ordered according to increasing value of genus, with the property that, given zero-genus topologies, others may be built therefore by connected sums. Corresponding Feynman rules specify the value of higher-genus amplitudes given zero-genus amplitudes.

Classical-surface genus was not the only characteristic of Feynman-graph complexity recognized by Veneziano; the number of disconnected components of the classical-surface boundary is also significant. (Each boundary component contains at least one end of the graph.) Of special interest are graphs embedded in zero-genus classical surfaces with either one or two boundary components. Those with one boundary component are called "planar" and those with two are called "cylindrical"; we are here speaking of spherical surfaces with either one or two disks cut out. A sphere minus a single disk is topologically equivalent to a bounded plane; a sphere minus two disks is equivalent to a cylinder. Figure (1) gives examples of planar and cylindrical thickened Feynman graphs. The classical surfaces here are topologically inequivalent; a cylindrical surface may not be contracted to a planar surface even though both have zero genus.

It is possible to show that planar and cylindrical classical surfaces are the only "self-building" categories -- i.e., the only topologies achievable through connected sums where all ingredients are of their own topological type.[9] A toroidal bounded surface (sphere plus handle, genus = 2) cannot, for example, be built from the connected sum of two toruses. But a planar surface can be built from the connected sum of two planes and a cylindrical topology built from the connected sum of two cylinders. This fact will emerge as of deep significance for the topological bootstrap. Only at planar and cylinder levels is a bootstrap mechanism operative. All higher topological amplitudes are determined by Feynman rules once planar and cylinder levels are established. The photon will turn out to be related to the cylinder.

Complexity of momentum flow has been seen in this section to be described through the genus and boundary structure of the classical surface which thickens the Feynman graph. We turn attention next to the flow of spin. Consistency with the contraction idea will lead us locally to orient the classical surface and to associate such orientation with "chirality" along the surface boundary.

IV. CHIRAL FERMION LINES

The idea that spin "flows" along the classical-surface boundary was suggested by the 1969 "quark-line" diagrams of Harari and Rosner[8] and formally realized by subsequent work of Mandelstam[13] and Stapp,[10] who succeeded in attaching spin 1/2 to boundary lines by recognizing a 2-fold "chiral" degree of freedom for boundary lines associated with massive elementary particles. Subsequently this "chirality" was associated with agreement or disagreement between boundary orientation and local orientation of the classical surface near the boundary.[14]

Chiral doubling is the same doubling recognized by Dirac through his 4-component spinors that represent spin 1/2.* In the Weyl basis the upper two components of the space in which a Dirac 4 × 4 matrix operates may be said to have "ortho" chirality while the lower two components have "para" chirality.[15] The parity operation interchanges ortho and para, but proper Lorentz transformations do not mix the two "chiralities".

In Dirac's theory of electron-electromagnetic interactions, ortho and para chiralities are superposed but separate consideration of ortho and para is required for dealing with topological contraction. Erasure of an intermediate line in a Feynman graph is possible only if each of the parallel fermion lines is unambiguously ortho or para.

To understand this point consider the Dirac 4 × 4 matrix

* Dirac doubling corresponds to the fact that the complex conjugate of a finite - dimensional representation of the Lorentz group is an inequivalent representation.

$$1 + \frac{\gamma \cdot p}{m_0} \qquad \text{(IV.1)}$$

which appears in Feynman's spin - 1/2 propagator. Here p is the momentum flowing along the Feynman line and m_0 is the elementary-particle mass. In the Weyl basis the matrices $\gamma_\mu = (\gamma_0, \vec{\gamma})$ relate to 2 × 2 Pauli matrices $\sigma_\mu = (1, \vec{\sigma})$ by

$$\gamma_\mu = \begin{pmatrix} 0 & \sigma_\mu \\ \tilde{\sigma}_\mu & 0 \end{pmatrix} \qquad \text{(IV.2)}$$

where $\tilde{\sigma}_\mu \equiv (1, -\vec{\sigma})$, so the second term of (IV.1) produces ortho-para transitions and simultaneously couples spin to momentum. By contrast the first term of (IV.1) -- the part diagonal in chirality as here defined -- is the unit matrix in spin space. So long as "chirality" does not change, spin is decoupled from momentum and there is no spin-impediment to erasure of the intermediate Feynman line.

The second term of (IV.1) cannot be discarded, so one associates "chirality" with a topological orientation and permits this orientation to reverse, accepting that such reversal brings an entropy increase irremovable by contraction.

Consider the example in Fig. (2) of a cubic thickened Feynman graph. The boundary here is topologically an oriented circle but has been drawn to suggest that each boundary segment runs "parallel" to a Feynman momentum line. Figure (2) emphasizes how each Feynman line is accompanied by a pair of oppositely-directed "fermion" lines, a fermion line being defined as an oriented boundary segment located between the ends of two momentum lines. Notice how the full boundary is globally oriented. In this example there are 3 fermion lines,

each with a pair of spinor indices implicitly attached at the ends. These indices describe in the sense of Stapp[16] the spin of the elementary particle belonging to the momentum-line end where the index locates.

The chiral orientation for each fermion line is that of the classical-surface portion which touches the line. The surface of Fig. 3 is divided by the Feynman graph into 3 portions, each touching exactly one fermion line. If the orientation of a surface portion agrees (disagrees) with its fermion-line orientation, we say that the fermion is ortho (para). Connecting Feynman lines between vertices (to form a "propagator") joins corresponding fermion lines, and along each separate fermion line there may or may not occur an ortho-para transition. Transition means reversal of local classical surface orientation, as in the upper portion of Fig. 3, and thereby a patch boundary in the surface. The number of such patch boundary lines (not momentum lines) counts the number of fermion "chirality" reversals" and constitutes an entropy index; this index can only increase in connected sums. An intermediate Feynman line along which "chirality reversal" occurs, such as in Fig. (3), may not be erased; the intermediate momentum affects the spin dependence.

For massive elementary particles, ortho-para chirality is not a physically-observable degree of freedom. The Feynman-Dirac matrix (IV.1) in the rest system of the propagating particle, where $p = (m_0, 0,0,0)$, becomes

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix},$$

which projects an arbitrary 2-component vector onto the vector

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

The Feynman-Dirac formalism is here recognizing that, in summing over the index γ in the topological expansion (II.1), one automatically makes the superposition, ortho + para, for each separate fermion line attached to a massive elementary particle.

This is perhaps an appropriate point to pause and call the reader's attention to a general feature of the topological expansion (II.1): the sum over γ for specified initial and final S-matrix channels (i, f) is a sum over physically <u>inaccessible</u> topological features. <u>Accessible</u> features are orientations preserved whenever elementary-particle lines are joined. Fermion-line direction, for example, is always maintained and one may correspondingly say that +1 unit of fermion number flows along each fermion line. When we come to consider oriented isospin lines it will be possible, because isospin direction is continuous, to say that electric charge flows along these lines. It follows that fermion number and number of electric-charge units are physical labels attached to any elementary particle. Such "accessible" labels are included in (i, f) and not summed over in (II.1). But ortho-para chirality is not an accessible label when the elementary particle is massive; here one makes the O + P sum.

For <u>massless</u> elementary particles fermion chirality associates with helicity. Chirality in this case becomes physically accessible. In both massive and massless instances the parity operation interchanges ortho with

para. Left-right physical symmetry results when ortho and para fermion lines appear symmetrically in the topological expansion.

Self-building topologies, in the sense described by the preceding section, must be completely free from fermion ortho-para reversals. Planar topologies (zero genus, 1 boundary component) without "chiral switches" are often called "zero-entropy";[9,10] all constituents of a connected sum that leads to zero entropy must individually be of zero entropy. A cylindrical topology (zero genus, 2 boundary components) without ortho-para reversals has been characterized as a "naked" cylinder. Any or all of the constituents of a connected sum leading to a naked cylinder may be either zero-entropy or naked cylinder; constituents with "chiral reversals" are inadmissable.

V. JUNCTION LINES

Zero-entropy connected sums where the embedding classical surface is smooth, invariably lead to single closed fermion loops accompanying each closed Feynman loop. The accompanying minus sign, first recognized by Stapp,[10] then conflicts at the base of the topological expansion with positivity aspects of S-matrix unitarity.* The way found to circumvent this problem has been to allow more than 2 fermion lines sometimes to accompany a Feynman line. With a smooth classical surface, the 2-fermion pattern of Fig. (2) is unavoidable, but with junction lines in the classical surface it is possible to accommodate 0, 1, 2, 3 or 4 fermion lines.[17] At zero entropy we shall see in Sec. VII, that the possibilities are 2, 3, 4 and one finds a unique and consistent structure in which every Feynman loop is accompanied either by 1 or 2 fermion loops. Because the trace of the unit Dirac matrix is 4, corresponding to each (massive) fermion line carrying 2 possible spin values and 2 possible chiralities,** the topology with two fermion loops not not only brings a positive sign but exceeds in magnitude the negative contribution from the single fermion loop (see Formula IV.1 below).

What is a junction line? Half a decade ago, before recognition of the fermion loop problem, the need for 3 quark lines to represent

* For example, the imaginary part of a forward elastic scattering amplitude must be positive.

** Isospin and generation orientations produce a further factor of 8, as discussed in Sec. IX.

baryons led several workers[18] to the idea of oriented lines along which there meet 3 smooth pieces ("feathers") of the classical surface. The idea is illustrated by Fig. (4) which gives the 3-sheeted (3-"feathered") classical surface corresponding to a zero-entropy baryon propagator. The junction line constitutes part of each sheet's boundary and is oriented accordingly but does not itself lie along the classical-surface boundary. Fermion lines, as before, correspond to classical-surface boundary segments located between the ends of "momentum" lines.

Only one of the three solid lines in Fig. 4 actually carries momentum; introduction later of isospin will tell us which one. The other two, called "momentum-copy" lines, delineate portions of the classical surface that touch individual fermion lines and that through local orientations give ortho-para chirality to their fermions -- as in Fig. (2).

There remain 3 portions of classical surface that touch the junction line. For reasons to be explained in Sec. VII, the local orientations of these portions are "frozen" so as to agree with that of the junction line. The frozen junction-line chirality turns out not to violate left-right fermion symmetry for strong or electromagnetic interactions but does produce asymmetry in weak interactions. In scanning the classical surface of Fig. 4, the reader's eye may have caught the (dotted) Y-shaped boundary segments which do not correspond to fermion lines; these segments will be discussed in Sec. VII in connection with Fig.7b.

Why not introduce junction lines where more than 3 smooth sheets meet? Reference (9) discusses consistency problems that then arise with respect to contraction rules. It has not been proved that more complex classical surfaces are inconsistent, but the "3-feathered" junction line has many special topological attributes not shared with other structures and apparently needed for a consistent S matrix.

Reference (9) explains how each of the 3 sheets attached to a junction line is topologically distinct, so that one may describe each region of the classical surface as carrying one of 3 topological "colors". Fermion lines within a given region may be said to inherit the "color" of that region, but nothing is added to the theory by so doing. Speaking of "color" is sometimes convenient for computations but is never necessary. Like quark "chirality", quark "color" is an inaccessible degree of freedom that is summed over in the topological expansion.

Because junction-line orientations are all frozen, the labels attached to elementary particles can always be determined by their fermion-line labels. To the extent that many computations can be performed without explicit reference to junction lines, the importance of junction lines to particle theory has not been as recognized as that of fermion lines -- which at zero entropy have been called "quark lines". We nevertheless shall find that junction lines are critical in determining electroweak properties.

VI. ISOSPIN LINES

We come now to a mysterious but beautiful and established aspect of topological bootstrap theory: oriented lines in the classical surface that begin and end on the boundary while touching neither the Feynman graph nor each other. These lines, located so that each end uniquely associates with the end ("head" or "tail") either of a fermion line or of a junction line, were proposed by J. Finkelstein and have been called isospin lines.[12,19] An esthetically-striking aspect, probably fundamental to the overall consistency of the theory, is the isospin line bundle to be described in Sec. VII. In this present section we motivate isospin lines by two considerations: (1) The need to distinguish in the baryon-propagator 3-sheeted classical surface (Fig. 4) a "principal sheet" which carries the Feynman graph -- through which momentum flows. (2) The need to correlate the orientations of fermion lines lying along disconnected components of the classical-surface boundary. Certain isospin lines will be found to have their two ends on different boundary components while, as always, connecting the "head" of a fermion or junction-line arrow to the "tail" of another arrow.

We give in Fig. 5 as first examples of isospin lines those embedded in the zero-entropy classical surface of Fig. 2, although neither of the above two motivations is relevant to Fig. 5 -- where there is a single boundary component and no junction line. Here each isospin line associates with a single fermion line and gives it a "yes" or "no" label according to whether the isospin orientation agrees or disagrees with the fermion direction.[19] Because isospin orientation is preserved in connections between ingoing and outgoing Feynman lines, it will be found that any fermion line or any junction line is either "yes" or "no" along its entire length. It will further eventually develop that a "yes" line carries +1 unit of electric charge while a "no" line is uncharged.

Finkelstein was originally led to isospin lines by the perceived phenomenological need to represent electric charge. From the bootstrap standpoint of this paper, of course, the existence of electric charge is not postulated but rather is to be deduced as a necessary component of a consistent S matrix. At this stage we mention electric charge so as to offer the physicist reader an alternative terminology in "charged", "neutral" more recognizable than "yes", "no".

If instead of "yes", "no" one were to speak of "up", "down" there would be contact with familiar particle-physics language used for decades to describe "strong isospin"; the best-known quarks have been characterized as "up", "down". In Sec. VIII the "generation" label for certain fermion lines will be described; one characteristic of a quark line (Sec. VII) is that it carries a generation label as well as "yes", "no". For one special value of the generation label, the quark "yes", "no" degree of freedom is equivalent to the standard "up", "down". But "yes", "no" applies not only to all generations but to fermion lines without generation index and also to junction lines. Topological isospin, an extraordinarily unifying feature of bootstrap theory is more similar to the "weak isospin" of Weinberg-Salam (Lagrangian) theory[20] than to "strong isospin", but it is not identical with either.

For topologies that can be built by connected sums of zero-entropy Feynman graphs, each isospin line associates at both ends with the same fermion or junction line. Figure 6 shows how this works for the zero-entropy baryon-propagator classical surface of Fig. 4, where there are 4 isospin lines -- one for each of 3 fermion lines and 1 for the junction line. The location of the latter distinguishes the "principal" sheet -- where the Feynman graph lives -- from secondary sheets.

In accord with the general rule that all junction-line orientations are frozen, the isospin of the junction line in Fig. 6 has been fixed at "yes" in contrast to that of each fermion line, which is either "yes" or "no". From the bootstrap standpoint one defines "yes" to be the isospin orientation of any hadron junction line. The fact developed in Sec. XI that only one member of an isospin quartet of vector bosons couples to hadron junction lines, will distinguish the photon member of this quartet and simultaneously identify electric charge with the number of "yes" lines.

So long as isospin lines all attach to individual fermion or junction lines, there is "yes" - "no" fermion symmetry: any fermion line may be changed from "yes" to "no" or vice versa without changing the amplitude. By reasoning first invoked by Chan and Paton,[21] it is then possible to infer a continuous SU(2) isospin symmetry. This symmetry is broken when isospin lines connect fermion lines with hadron junction lines because only "yes" fermion lines can so couple. We shall eventually find photons generating fermion-junction line interaction; electromagnetism thereby breaks SU(2) isospin symmetry.

VII. THE TRIANGULATED QUANTUM SURFACE

In some future review of topological particle theory the next ingredient in the story -- the closed "quantum surface" -- might well be placed at the beginning as the base space for an isospin line bundle. We have here tended to follow the historical sequence of developments which began with Feynman graphs and then expanded into the bounded classical surface with junction lines. Historically it was junction lines that first suggested a need for the quantum surface -- a structure subsequently recognized as profoundly significant for the integrity of the theory as a whole.

How did junction lines suggest the need for yet another ingredient in the topological soup? The problem was the connected sum associated with joining "in" and "out" baryon lines when all 3 baryon fermion (quark) lines carry the same labels. Six ways (3!) of joining are possible, but S-matrix unitarity at the zero-entropy level requires that only one joining be topologically "smooth" -- without complexity. To distinguish the 3 quark lines it was necessary to give a <u>cyclic order</u> to the three "feathers" of classical surface meeting at the junction line. Now the boundary of the classical surface is a cubic graph, graph vertices being located at the ends of junction lines; what was needed was to cyclically order these vertices. But we have seen that giving cyclic order to the vertices of a graph amounts to "thickening" the graph into a 2-dimensional object.

The 2-dimensional object here meets the classical surface transversally along the boundary of the latter, so we now have <u>two</u> surfaces that intersect along the boundary of one. The second -- the

thickening of the classical-surface boundary -- has been called the "quantum surface"; the reader will presently understand why. The quantum surface need not have a boundary because the boundary graph (in contrast to the Feynman graph) is closed, and it was discovered that at zero entropy there was no impediment to <u>closing</u> the quantum surface into a <u>sphere</u>.

The fact that the ends of the Feynman graph divide the closed classical-surface boundary graph into bounded subgraphs suggests dividing the closed quantum surface into bounded areas each associated with an elementary particle. Because the simplex for 2 dimensions is the triangle it is furthermore natural to <u>triangulate</u> the quantum surface and to associate combinations of triangles with elementary particles. At zero entropy, where the quantum surface is spherical, these particle areas are triangulated <u>disks</u> that fit smoothly together to cover the sphere. In more general topologies, elementary-particle areas need not be disks but they are always triangulated and combine into a closed surface which thickens the boundary of the classical surface.

The requirement that triangulated particle areas fit smoothly together to close a surface leads to conserved particle quantum numbers -- hence the name "quantum surface". We shall see in fact that quantum numbers attach to individual triangles, allowing these to be regarded as "particle constituents" in the same sense that fermion and junction lines are constituents.[17] Each end of any fermion or junction line attaches to its "own" quantum-surface triangle; there are as many quantum triangles as ends of fermion and junction lines. Orientations of fermion and junction lines may be regarded as "induced" by quantum-triangle orientations; at the "head" of a fermion or junction line there occurs a (+) triangle and at the "tail" a (-) triangle.

Exactly one end of an isospin line lies within the interior of each quantum triangle.* Hence the closed quantum surface is the base space for an isospin line bundle, and each triangle carries a "yes", "no" label. The two triangles located at the ends of any isospin line have opposite orientations but both are "yes" or both are "no".

The interior of each quantum triangle houses a piece of classical-surface boundary. Quantum triangles attached to fermion lines are cut by the boundary in the "I" pattern of Fig. 7a** while those attached to junction lines are cut in the "Y" pattern of Fig. 7b. We refer accordingly to "I" and to "Y" triangles, each of which has a (+, -) orientation, a "yes", "no" orientation, and inherits a chiral ortho-para orientation from the adjacent portion of classical surface. At zero entropy all Y triangles are "yes", ortho, in accord with the requirement that all hadron junction lines be "yes", ortho. We now are approaching the stage at which we shall be able to understand such a constraint.

The extensive redundancy between quantum triangles and lines on the classical surface may cause the reader to wonder why the quantum surface is needed. We have already emphasized the topological problem

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* Note that fermion lines lie <u>entirely</u> within the quantum surface, not just the line ends.

** The boundary segment inside an I triangle is "half" of a fermion line.

of 3-feathered baryon connections; this problem requires the quantum surface for a satisfactory resolution[22] although we do not here give details.* Another question requiring the quantum surface is the distinction between elementary electroweak bosons and elementary mesons. Both will be seen below to correspond to a pair of oppositely-directed fermion lines, that is, to an I^+, I^- triangle pair. The distinction will be that quantum I triangles building electroweak bosons are not individually disks but Möbius bands; one consequence is absence of <u>generation</u> index. We shall see immediately below how generation emerges from the quantum surface.

The principle of Feynman-graph contraction controls the triangle patterns acceptable on zero-entropy spheres. After lengthy study of unitarity constraints, the permissable zero-entropy elementary-particle areas have emerged as the disks of Fig. 8, where the cross designates the Feynman-line end belonging to the elementary particle.[9] Reversal of all (+, -) orientations is allowed. Other disk areas are not allowed. The disk of Fig. 8(a) corresponds to two oppositely-

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* In the course of this resolution one discovers that elementary particles may not contain more than 2 junction lines (more than 2 Y triangles) because one edge of each Y triangle must contain the end of the elementary-particle Feynman line. This feature, illustrated in figures immediately to follow, is equivalent to saying that every junction line is "close to" a parallel Feynman-graph line. A neighboring momentum-copy line does not suffice.

directed fermion lines (as in Fig. 2) and is describable as an elementary meson. That of Fig. 8(b) corresponds to 3 fermion lines plus an oppositely-oriented junction line (Fig. 6) and is an elementary baryon. The disk of Fig. 8(c) cannot be omitted from a consistent Feynman space of elementary particles and corresponds to 4 fermion lines plus 2 (oppositely-directed) junction lines; this disk is called an "elementary baryonium". The necessity for baryoniums once baryons are added to mesons was anticipated already in 1969 by Harari and Rosner.[8]

A general feature of any quantum surface is that each + triangle is uniquely "mated" to a − triangle in the sense of sharing all 3 vertices.[9] I^+ triangles mate with I^- and Y^+ with Y^-. Two conserved quantum numbers, N_I and N_Y, emerge, $N_{I(Y)}$ being defined as the number of $(+)N_{I(Y)}$ triangles minus the number of $(-)N_{I(Y)}$ triangles. One may similarly define N^{yes} and N^{no}, with an identity for the total triangle number,

$$N^{yes} + N^{no} = N_I + N_Y, \equiv N \tag{VII.1}$$

and there emerge finally 3 independent conserved quantities. It may help readers to develop physical feeling for the quantum surface if they are told already at this point that the 3 quantum numbers can be put into correspondence with electric charge$_Q$, baryon number$_B$ and lepton number L according to the rules

$$Q = N^{yes} \tag{VII.2}$$

$$B = -N_Y^{yes} \tag{VII.3}$$

$$L = -N_Y^{no} \tag{VII.4}$$

A collection of particle-area conditions, such as

$$4B = N_I - N_Y$$

$$-4L = N_I + 3N_Y$$

$$2(B-L) = N, \tag{VII.5}$$

constrain the total of independent quantum numbers. We are here anticipating that a lepton quantum area is built from a$(-)Y^{no}$ triangle together with a$(-)I^{\text{yes or no}}$ triangle.[23]

The foregoing remark may usefully be accompanied by the observation that <u>all</u> elementary-particle areas share with the disks of Fig. 8 the feature that the area disconnects into two "halves" when cut along the edge which carries the Feynman-line end. No elementary-particle area can consist of a single triangle; the Feynman line must end <u>within</u> the area interior and furthermore, because of its role in delineating "chirality", must end on an <u>edge</u> of the quantum-surface triangulation.

Disk I triangles are appropriately called "quark" triangles even though they have integral electric charge* and do not individually

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* The frozen orientations of Y triangles allow computation rules entirely in terms of quark lines, with no explicit reference to junction lines.[15] When such rules are used, quark lines carry the familiar fractional electric charges so as to include the electric charge carries by junction lines.[12]

carry momentum. Quark triangles are doublets both in isospin and in spin and will be seen in the following section to carry a generation index.

VIII. PARTICLE-AREA PERIMETERS: GENERATION

The disk elementary-hadron areas of Fig. 8 have perimeters that edgewise fit together when covering a quantum surface.[9] Each triangulation edge along a perimeter requires an orientation in order to specify the sense in which pairs of perimeter edges are to be identified; these special edges are seen in Fig. 8 to be invariably belonging to I triangles, never to Y triangles. Every disk I-triangle has 2 oriented edges that lie along a particle-area perimeter -- those 2 edges uncut by the classical-surface boundary, as shown in Fig. 9. Each disk I-triangle then carries a 4-valued perimeter index G = 1...4 -- associable with the particle label called "generation": topological bootstrap theory predicts 4 "quark" generations.

An elementary <u>lepton</u> area, described in detail in Ref. (23), contains a 4-generation I triangle which is not quite a simple disk. The two generation-carrying edges of a lepton I triangle constitute the entire perimeter of the lepton area, inasmuch as the two I-triangle vertices uncut by the classical-surface boundary are identified with each other -- as shown in Fig. 10 for G = 1.* Within an electroweak-boson area the two I-triangle <u>edges</u> which carry generation in quarks and leptons are <u>identified</u> with each other and thereby removed from the perimeter. At the same time there is identification of all 3 vertices -- as shown in Fig. 11. This generationless I triangle is

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* The lepton I-triangle edge cut by the classical-surface boundary is identified with the single-edge perimeter of a Möbius-band Y-triangle. The complete lepton area is then itself a Möbius band.[23]

a Möbius band and has been designated I_M. The edges of an I_M triangle do not influence the rules by which an electroweak boson interacts.

In the bootstrap reasoning that follows in Sec. X there appears for elementary electroweak bosons a raison d'être which controls the form of the quantum-surface areas. No such bootstrap basis has yet been found for elementary leptons; the structure of the lepton area proposed in Ref. (23) has been inferred from experimentally-observed properties of physical leptons. The bootstrapping of leptons is a chapter of the continuing bootstrap story that remains unwritten as this review is being prepared.

Connected sums of Feynman graphs are accompanied by connected sums of attached quantum surfaces as well as of classical surfaces.[9] All quantum surfaces generated by connected sums of zero-entropy quantum spheres are orientable and thereby maintain conservation of each separate quark generation. "Strong-interactions" fall into this category, by definition. If we define N^G to be the number of (+) disk I-triangles of generation G minus the number of matching (-) triangles (Fig. 9), then each N^G is separately conserved in any strong-interaction topology.

Because electroweak interactions involve nonorientable quantum surfaces, only the total number of quark triangles is guaranteed to be conserved. Reference (23) points out that, because the generation-carrying edges of a lepton I-triangle (Fig. 9) close into a circle, it is more difficult to break conservation of separate lepton generation than to break N^G conservation. The necessity for electroweak generation mixing, either of quarks or of leptons, at present accompanies the necessity for leptons among missing elements of the bootstrap mosaic.

Orientable quantum surfaces unavoidably maintain not only conservation of separate quark generations but a symmetry under exchange of one generation with another, similar to "yes" ↔ "no" isospin symmetry. Nonorientable quantum surfaces can break generation symmetry at the same time as breaking generation conservation.

The emphasis in the present section on particle-area perimeters makes this the appropriate place to explain why the orientations of Y triangles and hence of junction lines are frozen. The reason is that Y triangles never appear on particle-area perimeters; originally Y triangles were for this reason called "core" triangles.[9] On zero-entropy quantum spheres, contraction rules are expressed through matching conditions on quark triangles along perimeters of elementary-hadron disks; all elementary-hadron attributes emanate from these matching rules. Because junction-line orientations are not "felt" along the hadron-disk perimeters and cannot affect the rules by which a hadron area is allowed to join with adjacent hadron areas, consistency of zero-entropy contraction rules forbids any junction-line orientation from being a hadronic degree of freedom. Junction-line orientations do contribute to the distinction between leptons and hadrons[23]-- for reasons that presumably will emerge as the lepton raison d'être becomes clarified.

The full set of quantum numbers, including spin and parity, carried by the quark triangles of the elementary meson and baryon disks of Fig. 8 match the observed quantum numbers of the least massive physical mesons and baryons.[9,10] To date there are no established physical baryoniums, but Ref. (24) has estimated such short baryonium lifetimes that the experimental elusiveness of baryonium is understandable.

IX. TOPOLOGIES BUILDABLE FROM ZERO-ENTROPY SUPERSYMMETRIC VERTICES

Zero-entropy components of the topological expansion are self-building. The zero-entropy bootstrap problem is nonlinear but self contained and simplified by a "supersymmetry" that ensures a single mass m_0 for all elementary mesons, baryons and baryoniums even though spin values 0, 1/2, 1, 3/2, 2 all occur.[25,26] Belonging to any cubic zero-entropy vextex is a unique dimensionless coupling constant g_0 whose order of magnitude is given by[26]

$$(N^2 - N)\ \frac{g_0^2}{16\pi^2} \sim 1, \tag{IX.1}$$

where N is the quark multiplicity controlling the number of zero-entropy closed quark loops:

$$\begin{aligned} N &= 2 \text{ spins} \times 2 \text{ chiralities} \times 2 \text{ isospins} \times 4 \text{ generations} \\ &= 32. \end{aligned} \tag{IX.2}$$

The factor $N^2 - N$ arises from the circumstance, described above in Sec. V, that each zero-entropy Feynman loop is accompanied by either one or two quark loops. It follows from (IX.1) and (IX.2) that

$$\frac{g_0^2}{4\pi} \sim 10^{-2}. \tag{IX.3}$$

<u>Physical</u> hadron coupling constants are much larger than g_0 because of summation over the inaccessible topological index γ in the expansion (II.1); "chiral" and "color" quark degrees of freedom

lead to large factors in passing from S^{γ}_{fi} to S_{fi}.[24] Because fermion lines attached to I_M triangles turn out to possess no inaccessible degrees of freedom the electroweak boson coupling constant is not similarly amplified. The large magnitude of strong-interaction amplitudes with respect to electromagnetic amplitudes stems from the inaccessible "color" and "chiral" quark degrees of freedom.

Zero-entropy Feynman graphs are "naked" single-vertex "trees". (any connected graph without closed loops is a "tree".) The adjective "naked" means absence of ortho-para or "color" mismatch along associated quark lines, "color" mismatch referring to a twisted joining of the 3 "feathers" along junction lines.*

When zero-entropy vertices are connected by intermediate Feynman lines, there may arise complexity of two general types:

(1) Mismatch, "chiral" or "color", along quark lines that accompany internal Feynman lines. We have noted already in Sec. IV that any such complexity prevents contraction of the Feynman line. For economy of language it is convenient to characterize either "chiral" or "color" mismatch as a "quark switch".[15] Any naked Feynman line -- free from quark switches -- that joins two <u>different</u> vertices, can be erased with the vertices becoming identified.

(2) Mismatch of Feynman-line <u>cyclic order</u> at vertices along a closed Feynman loop. Section III has explained how such mismatch generates classical-surface handles or (and) extra boundary components; contractions do not alter these topological characteristics.

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* "Color" mismatch is shown in Refs. (9) and (22) to generate Möbius bands across junction lines in the classical surface. Feynman-graph loops nevertheless continue to be orientation preserving, because momentum lines do not touch junction lines.

Contraction may lead to a naked tadpole Feynman line. (A "tadpole" is a line both of whose ends touch the same vertex.) Can a naked tadpole be erased? The answer is "yes" if the two ends of the tadpole line are adjacent in the cyclic sequence of lines around the vertex. Otherwise the tadpole must remain. Figure (12) shows a naked vertex with a nonerasable tadpole. The classical surface here, although of zero genus, has 2 boundary components; tadpole erasure would lead to a single boundary component. The topology of Fig. 12 -- a "naked cylinder" -- will play a leading role in the photon bootstrap.

The expectation of bootstrap theorists for more than two decades was that a consistent (unitary) S matrix could be achieved in a hadron, strong-interaction, basis.[2] This idea, with the advent of the topological expansion, translated into the expectation of consistency within topologies reached by connected sums of zero-entropy topologies. It was anticipated, specifically, that a complete space of Feynman lines could be labeled by the elementary-hadron disks of Fig. 8. The photon would not be generated by such a bootstrap; only strongly-interacting particles -- hadrons -- would be explained. We proceed in the following section, however, to show that a self-consistent world of hadrons is not possible. Unitarity in conjunction with a topological entropy expansion requires elementary particles beyond the hadron disks of Fig. 8, even though only hadrons appear at zero entropy.

X. THE NAKED CYLINDER

Systematic accommodation of unitarity, order by order in the topological expansion, requires correspondence between every real singularity of any connected part S^{γ}_{fi} and intermediate lines in some Feynman graph (which may contract to the graph γ). It is shown in Ref. (3) that, for any topology except zero entropy and the naked cylinder, all singularities correspond to lines already appearing in topologies of lower entropy. Thus a consistent bootstrap space of Feynman lines might possibly be limited to the real zero-entropy and naked-cylinder singularities. But if the naked cylinder contains a real pole not corresponding to a zero-entropy Feynman line, there must be an elementary-particle area on the quantum surface different from the disks of Fig. 8 and corresponding to this naked-cylinder pole.

The possibility of a naked-cylinder pole not belonging to a zero-entropy quantum disk relates to the naked-cylinder's self-generating character -- a feature emphasized already in Sec. III. The naked cylinder has the unique propety of being buildable from zero entropy and at the same time being self-buildable. The nonlinear unitarity equations associated with the latter aspect allow the naked cylinder to possess poles not already present at zero entropy; the position and residue of any such pole is nevertheless calculable from zero-entropy amplitudes.

In 1973 an approximate relation between cylinder and planar amplitudes was discovered independently by H. Lee and G. Veneziano[27] -- a relation based on unitarity at low momentum transfer and predicting a mass for the cylinder ground state lower than for the planar ground state. This work preceded understanding of "color" and "chiral" quark degrees of freedom but applies to naked topologies -- where there are no quark switches. Lee and Veneziano, that is to say, predict a naked-cylinder pole of mass <u>lower</u> than the elementary-hadron mass m_0.

Although the Lee-Veneziano reasoning was only approximate, depending on the so-called "multiperipheral" model, substantial success in understanding the phenomenology of high-energy hadron reactions at low momentum transfer ensued from this model.[4] The core of their argument lies in the positive definiteness of total cross sections and the connection through unitarity with forward-direction elastic amplitudes. I propose to take seriously the Lee-Veneziano prediction.

A statement not requiring any approximation is that the naked-cylinder ground state has "vacuum" quantum numbers (all zero) and couples equally to all members of the zero-entropy elementary-hadron supermultiplet. The order of magnitude of this coupling was predicted by the multiperipheral model to be the same as that of the zero-entropy coupling constant g_0. We thus need to augment the space of Feynman lines by a low mass neutral scalar boson ($Q = B = L = 0$) without generation index and with a universal dimensionless coupling constant of order 10^{-2}.

Assuming the quantum-surface area for the naked-cylinder ground state is to be built from I and Y triangles according to the principles sketched above in Secs. VII and VIII, one is led to an $I_M(+)$, $I_M(-)$ triangle pair. The $I_M(+)$ and $I_M(-)$ Möbius bands (Fig. 11) fit together along their perimeters to close a Klein-bottle quantum surface. It is not possible to employ Y triangles because of the frozen Y-isospin

orientation which precludes universal coupling.

Isospin-independent coupling is achieved through the linear superposition

$$\frac{1}{\sqrt{2}}\ \{I^{yes}_{M(+)} I^{yes}_{M(-)} + I^{no}_{M(+)} I^{no}_{M(-)}\}, \qquad (X.1)$$

which transforms under SU(2) isospin "rotations" as a singlet. Employing both "yes" and "no" I_M triangles means that the Feynman space contains not only an isosinglet but also an <u>isotriplet</u> of bosons -- which do not correspond to the naked cylinder; singlet and triplet together constitute an isoquartet. Although it remains for the future to develop detailed Feynman rules which ensure unitarity, we proceed here on the assumption that rules can be found.* The Feynman rules must attend to boson spin as well as to isospin.

More subtle than isospin is the topological representation of boson spin and chirality. The four chiral possibilities are interpreted as two scalar bosons:

$$S_0 = I^{ortho}_{M^+} I^{ortho}_{M^-} \text{ and } S_P = I^{para}_{M^+} I^{para}_{M^-}\ , \qquad (X.2)$$

and two vector bosons

$$V_R = I^{ortho}_{M^+} I^{para}_{M^-} \text{ and } V_L = I^{para}_{M^+} I^{ortho}_{M^-} \qquad (X.3)$$

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* Feynman rules for all strong-interaction topologies -- those without naked tadpoles that can be built from zero entropy -- are given in Ref. (15).

where R and L may be described as "right" and "left-handed". The form of coupling to fermions justifies this terminology. Chirality is now an accessible degree of freedom. The even-parity superposition

$$\frac{1}{\sqrt{2}}\,(S_0 + S_P) \qquad \text{(X.4)}$$

has ortho-para independent couplings, and the scalar simultaneously symmetric in (0, P) and in (yes, no) provides a suitable representation for the naked-cylinder ground state. At the same time the S_0 boson couples to hadron junction lines while the S_P boson does not -- thereby breaking parity symmetry.

In order to say something about the spectrum of <u>physical</u> electroweak bosons it is tempting to lean on the Weinberg-Salam type of model[20] by assuming that our elementary electroweak bosons have zero mass. The Lee-Veneziano cylinder model[27] admits zero mass without selecting this value. One may rationalize the deficiency by noting that the model enforces unitarity in only one of the channels connected by energy analytic continuation; models with greater attention to unitarity tend to select zero mass. We have here an insecure element of the photon bootstrap which must be strengthened.

XI. ELECTROWEAK BOSONS

Topological bootstrap theory contains a total of 16 elementary massless electroweak bosons: 4 isospins × 4 chiralities; 8 spin-zero plus 8 spin-one. Guided by Lagrangian gauge theories[20] we expect closed-loop higher-order graphs to give masses to all, apart from one $yes^+\ yes^-$ vector boson. The masslessness of one self-conjugate physical vector boson relates to the presence of exactly one independent conserved quantum number. Ward identities[7] are expected to produce closed-loop cancellations which sustain zero mass for a vector that is coupled to the conserved quantity N^{yes} while carrying zero amount thereof. In other words N^{yes} is electric charge and the photon is a $yes^+\ yes^-$ vector boson.

What distinguishes N^{yes} from N^{no}? We have remarked that lepton quantum areas contain Y triangles, but it can be shown that if elementary leptons are <u>massless</u> it is impossible to couple a vector boson to a lepton Y triangle. It is also known that the lepton Y triangle is "no" in isospin orientation, so N^{no} is not identifiable with electric charge (the photon must couple to electric charge wherever charge appears). The only possibility for electric charge is N^{yes} -- to which the photon couples whether carried by an I or by a Y triangle (it is essential that Y^{yes} triangles appear only in <u>massive</u> elementary particles). The observed hadron electric charges confirm that N^{yes} is indeed electric charge.[12]

In the notation of (X.3), supplemented by isospin, the photon is the neutral vector

$$\gamma = \frac{1}{\sqrt{2}} \left(V_R^{yes^+ yes^-} + V_L^{yes^+ yes^-} \right), \qquad \text{(XI.1)}$$

so one of the remaining neutral spin -1 bosons must have an axial-vector character orthogonal to (XI.1):

$$A^{yes^+ yes^-} = \frac{1}{\sqrt{2}} \left(V_R^{yes^+ yes^-} - V_L^{yes^+ yes^-} \right), \qquad \text{(XI.2)}$$

joining two other neutral spin -1 bosons,

$$V_R^{no^+ no^-}, \quad V_L^{no^+ no^-}. \qquad \text{(XI.3)}$$

The remaining four spin -1 bosons carry electric charge:

$$V_R^{yes^+ no^-}, \quad V_R^{no^+ yes^-} \qquad \text{(XI.4)}$$

and

$$V_L^{yes^+ no^-}, \quad V_L^{no^+ yes^-}. \qquad \text{(XI.5)}$$

Because parity is not a good symmetry, the physical left and right-handed vector bosons are expected to have different masses -- an asymmetry related to the circumstance that in acquiring mass each vector boson must "eat" a scalar of matching electric charge so as to achieve the requisite 3 helicity states for a spin -1 massive particle. (We have observed how scalar coupling to junction lines violates parity.) The only physical (massive) scalar boson will then be the photon's "uneaten" neutral partner.

Closed-loop corrections not only shift masses but mix states of the same electric charge. The physical photon must remain yes^+yes^-, but massive neutral vector bosons are expected to have a mixed yes^+yes^-, no^+no^- character. Experimental facts suggest substantially larger masses for right-handed vectors than for left handed, so the most significant mixing is expected to be between $A^{yes^+yes^-}$ and $V_L^{no^+no^-}$. Preliminary comparison with the "standard" model, which has only half as many vector bosons as bootstrap theory, indicates that some but not all of the same "low-energy" predictions will emerge. It should be possible for experiment to distinguish bootstrap theory from the standard model.

Because the photon appears in the same multiplet as the naked-cylinder ground state, the value of the elementary electric charge is equal to the naked-cylinder coupling constant. We have seen in Sec. IX that the latter has the order of magnitude appropriate to such an equality. It is possible to hope that, as dynamical models of zero entropy and naked cylinder improve, the naked-cylinder coupling constant will converge toward the observed value of e.

The standard model has a second arbitrary coupling constant, usually designated as the "Weinberg angle". The bootstrap value of this angle so far as the photon is concerned is 45° -- inferrable from Formula (X.1). Generally speaking, however, the pattern of neutral vector bosons is qualitatively different, and reference to such an angle should be avoided.

XII. OUTLOOK

The graphically-based bootstrap mosaic is presently far from completely exposed; only a fragment confined to the discrete S matrix has been uncovered. No serious effort has yet been launched to recognize within the mosaic the continuum of "ordinary reality" -- Newtonian objects moving in space-time -- and even at the S-matrix level important details remain obscure.

Although a complete grasp cannot presently be claimed of the photon's attachment to neighboring mosaic segments, the connection described in this review encourages expectation of further illumination. A unitary S matrix based on an entropy expansion has been found to require elementary particles beyond hadrons; the qualitative characteristics of the immediately-needed additions strikingly resemble those of the experimentally-observed photon and the partners implied by low-energy weak-interaction data.

Emphasis now is passing to leptons. Why should such ingredients be required in a consistent mosaic? The topology has been seen to provide a place for leptons, but a raison d'être is still lacking. In developing detailed Feynman rules for electroweak topologies a need for leptons is expected to become recognized.*

Beyond leptons will be "infrared" phenomena, stemming from the zero photon mass, which promise significance for continuous space time and simultaneously for the measurement event-clusters that define the S matrix. Large numbers of soft-photon Feynman-graph lines have

* Together with a need for weak quark generation mixing and CP violation.

potential for illuminating the meaning of "ordinary reality".

Eventually the bootstrap mosaic must be seen as encompassing gravitation although capacity to recognize this large-scale feature awaits the combinatorial understanding of space-time. And why, after all, do graphs constitute elements from which a universally consistent mosaic can be constructed? The questions to be faced by bootstrappers have no end.

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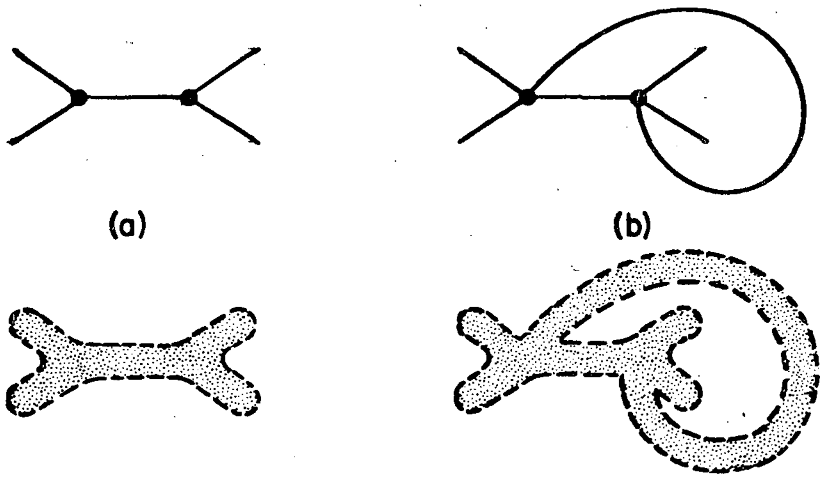
FIGURE CAPTIONS

1. (a) A planar 2-vertex thickened Feynman graph; the embedding classical surface is shown below the graph.
   (b) A cylindrical 2-vertex thickened Feynman graph.
2. Cubic Feynman graph (solid lines) embedded in a planar bounded locally-oriented classical surface. The (dashed) boundary consists of 3 fermion lines.
3. A 2-vertex thickened Feynman graph that is uncontractible because of a "chiral" reversal along a fermion (boundary) line that accompanies the intermediate Feynman line.
4. 3-feathered classical surface for an elementary-baryon "propagator".
5. Isospin lines on the classical surface of Fig. 2.
6. Isospin lines on the classical surface of Fig. 4.
7. Quantum-surface triangles
   (a) I triangle
   (b) Y triangle
8. Elementary-hadron disks.
9. Disk I triangles ("quarks") with (generation) edge orientations.
10. I triangle within lepton area.
11. Möbius-band I triangle within electroweak boson area.
12. Example of a naked noncontractible tadpole.



XBL829-4638

FIGURE 1

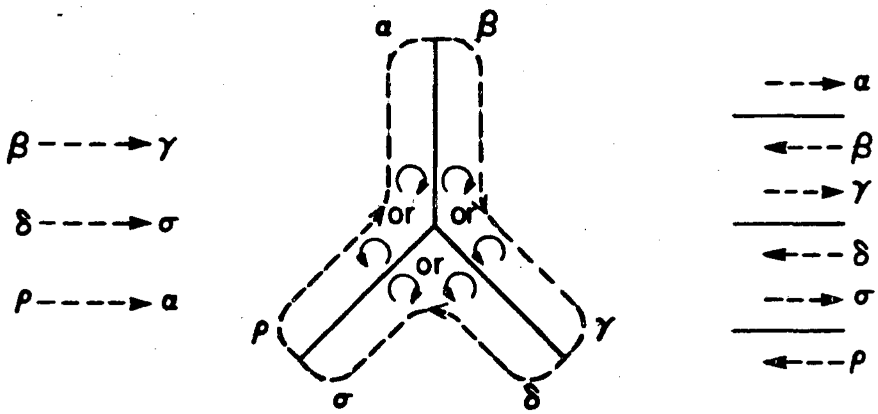


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FIGURE 2

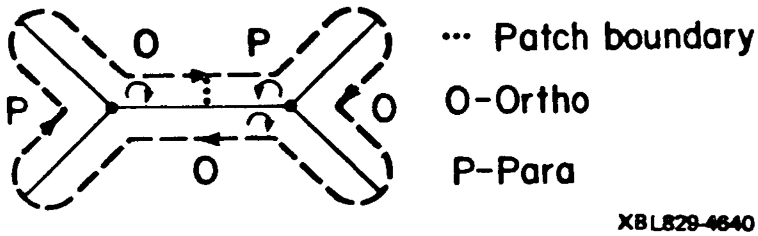

FIGURE 3

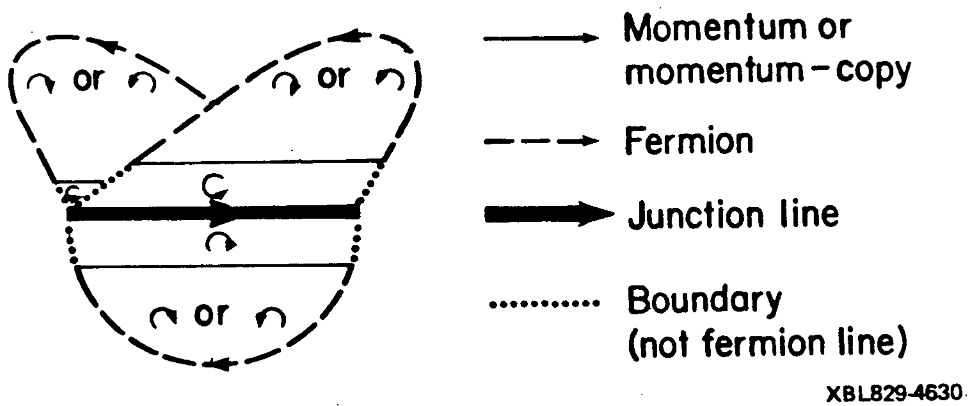

FIGURE 4

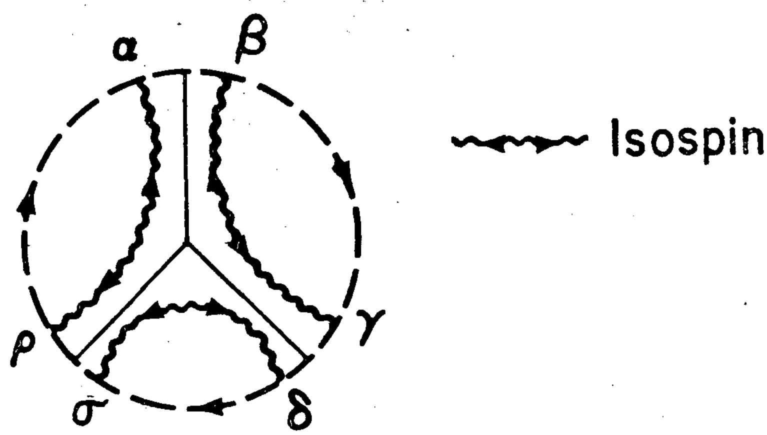

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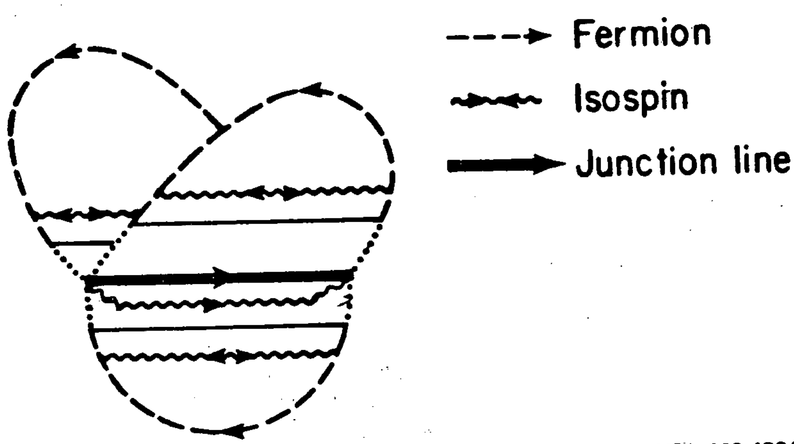
FIGURE 5

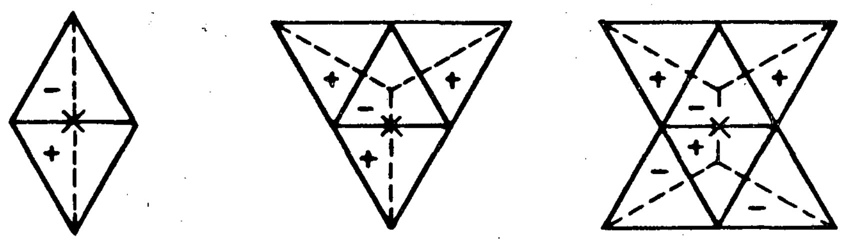


XBL 829-4624

FIGURE 6

XBL 829-4625

FIGURE 7



(a) (b) (c)

XBL 829-4626

FIGURE 8

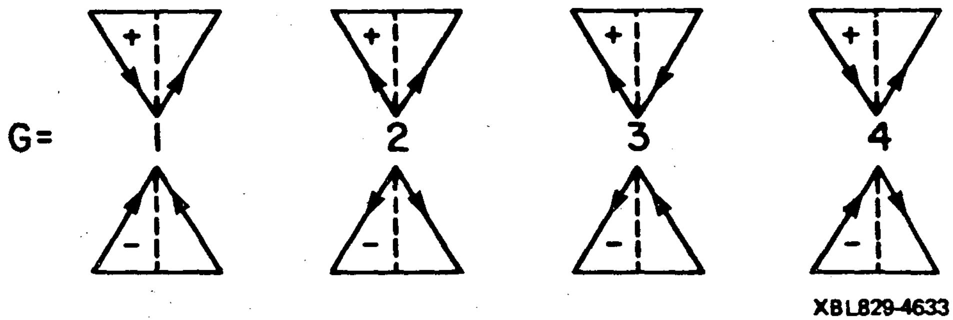


FIGURE 9

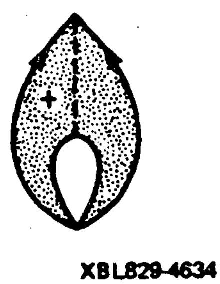
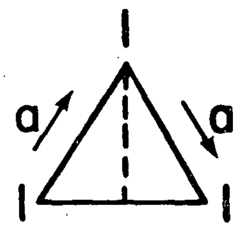


FIGURE 10



XBL829-4635

FIGURE 11

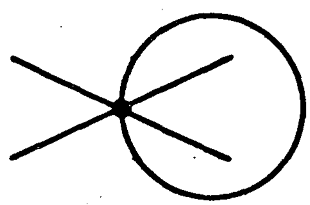


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FIGURE 12

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