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# Truck Scheduling for Ground to Air Connectivity 

Randolph W. Hall<br>University of Southern California<br>California PATH Working Paper UCB-ITS-PWP-2001-8

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# Truck Scheduling for Ground to Air Connectivity 

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#### Abstract

A critical link in the overnight package business is the on-time arrival of trucks at airport terminals. Truck delays can delay the package sorting and transfer process, which can in turn delay aircraft departures from the local terminal, as well as aircraft departures from hub terminals that depend on timely aircraft arrivals. This paper models the airport terminal as a queueing process with random bulk arrivals. Predictions are provided for expectation, and standard deviation, of arrived work. From these predictions performance measures are developed for sort end-time and sort starvation. Particular attention is given to scheduling the start time for the sorting process, to balance a tradeoff between maximizing sort productivity and minimizing the end time of the sort. The methods are being implemented as a web-based scheduling tool.


## EXECUTIVE SUMMARY

This is an interim report for the project "ATIS for Ground to Air Connectivity," which is investigating schedule coordination between ground transportation and aircraft departures at airport cargo terminals. $U P S$, Federal Express and other air express companies operate many local terminals within the Los Angeles region. At the end of each day, trucks depart from these terminals carrying express shipments, which are processed at Los Angeles International Airport, Ontario Airport and other local airports in Southern California. Aircraft depart from these airports according to a rigid schedule, so it is important for trucks to arrive on time and for shipments to be processed on time. Otherwise, aircraft cannot be fully loaded by their scheduled departure time, and they will either have to be held for late arrivals or depart without all of their shipments (thus creating late deliveries for the affected shipments).

This paper models the sorting process at the terminal and the effects of truck arrival time on the completion of the sort. This paper also models the effects of sorting capacity and the start time for the sort. Additional work is underway to create a web-based scheduling tool that incorporates the principles found in this paper. Work is also underway on methods for forecasting truck arrival times at the airport. This second phase of work is currently waiting for funding from PATH, and will be documented upon completion.

## 1. INTRODUCTION

In recent years the fastest growing segments of the goods movement industry in the United States have been small shipments and air shipments (Bearth, 2000). Federal Express, United Parcel Service, DHL, Emery, Airborne Express and the US Postal Service have prospered in this environment by creating integrated groundair networks. Air cargo terminals have also developed a capability to rapidly unload trucks, sort shipments, load these shipments into air containers, and load the air containers onto aircraft. These steps can sometimes be completed within a time span of 1 hour or less. At a destination airport the steps are reversed, allowing aircraft to be unloaded, and trucks to be loaded, within a short time span. High efficiency in sorting and loading has made it economical to send shipments across the country with next morning delivery (Analla and Helms, 1996; Chan and Ponder, 1979;Hansen and Kiesling, 1993; Jannah and Wilder, 1999; Larson, 1998; and Oster et al, 1995, examine the economic structure of the industry.)

Taken as a whole, express transportation can often be divided into the 11 steps listed in Table 1. To meet time commitments, it is desirable to make all of these steps as fast as possible. And it is also desirable to expedite some of the steps in order to provide more flexibility in others (e.g., to allow later pick-up times for shipments).

The sorting process at the origin airport is a particularly critical step, as it is susceptible to random delays in the arrival of work, and because it demands relatively large investments in facilities and labor. The facilities and labor are only needed within concentrated time periods, which sometimes makes it uneconomical to provide sufficient capacity to process shipments as quickly as they arrive. Unfortunately, late truck arrivals
can also delay sorting, with significant repercussions. The issue is especially critical in Southern California for three reasons: (1) west coast shipments have a $\mathbf{3}$ hour time lag relative to east coast, due to the difference in time zones, (2) Southern California is the dominant population center on the west coast, and (3) congestion in Southern California has both elongated travel times and made them less predictable. The importance of these shipments is magnified by the fact that major hub terminals (such as Federal Express' Memphis hub) cannot release their outbound aircraft until all inbound flights have arrived and been processed. Thus a single delay to a Southern California flight (due to a few late trucks) can translate into systemwide delays.

From a customer service perspective, systemwide delays can force an airline to alter its delivery commitment and pickup cutoff times. Thus, airlines that are better at managing their ground operations can offer more competitive service to their customers, and capture a larger share of the express shipment market.

## Analysis Aprroach

The fundamental unit of analysis in this paper is a "sort" at an air cargo terminal. Some air cargo terminals schedule multiple sorts at different times, and some terminals have multiple lines that simultaneously complete sorts. Each sort ends when all the packages have been processed for an individual aircraft, or for a group of aircraft that share a sort. Because different sorts process different inbound trucks, and because different sorts feed different aircraft, they can be analyzed independently of each other.

This paper is based on a project that was completed in cooperation with the two largest private express package companies in the United States: Federal Express and

United Parcel Service. One objective of the project was to create a real-time/web-based tool for scheduling trucks and managing the sort. To do this, models were developed for predicting the performance of a sort. These models are being specified and tested with empirical data, including travel time measurements obtained from the airlines and realtime information on highway congestion.

## Background Research

Three categories of research are relevant to coordinating groundair operations.
First, the design of the network as a whole, including route structure and placement of terminals, has been studied by Hall (1989), Kim et al (1999), and Kuby and Gray (1993). Second, the operation and design of air terminal facilities, and the aircraft loading process, have been studied by Ashford and Fathers (1989), Cheung et al (1998), Cornett et al (1996), Geinzer and Meszaros (1990), Heidelberg et al (1998), Nobert and Roy (1998), Swip and Lee (1991) and Thomas et al (1998). A last area of research is the coordination of schedules for incoming and outgoing vehicles at a transfer terminal, which has been studied by Abkowitz et al (1987), Bookbinder and Desilets (1982), Hall (1985), and Lee and Schonfeld (1991). The focus of these papers was on ground-toground transfer, especially for mass transit systems.

Although many aspects of air freight operations have been examined, prior research has not addressed the interaction between the arrival of incoming trucks and the processing of shipments, as is the focus of this paper. Hall and Chong (1993) did investigate queueing interactions for banked arrivals of aircraft at a hub terminal, with
focus on aircraft-to-aircraft transfers, rather than ground-to-airtransfers. In this case queueing appeared as a consequence of runway capacity, rather than sortation capacity.

## Paper Organization

The paper is divided as follows. First, models are developed for the arrival of shipments (measured as "work") at a groundair terminal. Then the scheduling of the sorting process is examined, and methods for determining the start time for the sort are created. Simulations are provided next, showing the effects of different sorting schedules on the completion time of the sort. The paper ends with conclusions and a description of future work.

## 2. MODELS FOR ARRIVAL OF WORK

The air freight terminal is modeled as a work conserving single server queueing system. Work arrives in the form of truckloads of shipments, and is processed by a conveyor sorting line. The amount of work on a truck depends on the number of shipments and their characteristics. By definition, one unit of work can be processed in one unit of time. Trucks are scheduled to arrive at a reasonably constant rate with the goals of keeping the conveyor line productive and minimizing the queue of shipments awaiting processing. This section models the arrival process; the service process is examined later.

Let:
$\mathrm{n}=$ number of trucks scheduled for a sort
$X_{i}=$ amount of work on truck i
$I_{i}(t)=1$ if truck i has arrived by time $t, 0$ otherwise
$\mathrm{X}_{\mathrm{i}}$ and $\mathrm{I}_{\mathrm{i}}(\mathrm{t})$ are random variables that depend on the characteristics of the terminal sending shipments, distances, roadway speeds and congestion.

Measures of cumulative arrival of work can be derived from $\mathrm{X}_{\mathrm{i}}$ and $\mathrm{I}_{\mathrm{i}}(\mathrm{t})$. Let:
$\mathrm{W}_{\mathrm{i}}(\mathrm{t})=$ work arrived by time t on truck i .
$W(t)=$ cumulative work arrived by time $t$, among all trucks
Then:

$$
\begin{align*}
& W_{i}(t)=\sum_{i=1}^{n} I_{i}(t) X_{i}  \tag{1a}\\
& W(t)=\sum_{i=1}^{n} W_{i}(t)=\sum_{i=1}^{n} I_{i}(t) X_{i} \tag{1b}
\end{align*}
$$

We first wish to compute the expectation of $\mathrm{W}(\mathrm{t})$ :

$$
\begin{equation*}
E[W(t)]=\sum_{i=1}^{n} E\left[I_{i}(t) X_{i}\right] \tag{2}
\end{equation*}
$$

In the special case where $\mathrm{I}_{\mathrm{i}}(\mathrm{t})$ and $\mathrm{X}_{\mathrm{i}}$ are mutually independent (arrival time is independent of load size), Eq. 2 reduces to:

$$
\begin{equation*}
E[W(t)]=\sum_{i=1}^{n} p_{i}(t) E\left(X_{i}\right) \tag{3}
\end{equation*}
$$

Where:
$p_{i}(t)=$ probability that truck i has arrived by time $t$
Examples of $E[W(t)]$ are shown in Figure 1. Each figure represents 12 trucks, scheduled at 5 minute intervals beginning at time 0 and ending at time 55 (an arrival period of 55
minutes). The mean load size is 5 in each case. The actual arrival time is assumed to be normally distributed, with standard deviation shown ( $\sigma=0,5$ minutes or 20 minutes). As illustrated, small $\sigma$ produces a step pattern, with each step representing one scheduled arrival. But when $\sigma$ is large relative to the spacing between scheduled arrivals, random deviations in arrival times smooth $\mathrm{E}[\mathrm{W}(\mathrm{t})]$. It can also be seen that the slope is most nearly constant when $\sigma$ is also small relative to the arrival period ( 55 minutes in the example). For large values of $\sigma$ (e.g., when it equals 20 minutes in the figure), curvature in $\mathrm{E}[\mathrm{W}(\mathrm{t})]$ extends well beyond the end points (times 0 and 55). By comparison the slope is nearly constant at the mean arrival rate of 1 ( 5 units of work per 5 minute interval) when $\sigma=5$.

Variance calculations can be more complicated. Let:
$\sigma_{i j}^{2}(t)=$ covariance between $\left[I_{i}(t) X_{i}\right]$ and $\left[I_{j}(t) X_{j}\right]$
Then:

$$
\begin{equation*}
\mathrm{V}[\mathrm{~W}(\mathrm{t})]=\sum_{\mathrm{i}, \mathrm{j}=1}^{\mathrm{n}} \sigma_{\mathrm{ij}}^{2}(\mathrm{t}) \tag{4}
\end{equation*}
$$

Computation of the covariance terms can be difficult in this general case, as detailed data are frequently unavailable for estimation of parameters. Consider the special case where all random variables are mutually independent. Then the variance can be expressed as:

$$
\begin{align*}
V[W(t)] & =\sum_{i=1}^{n} p_{i}(t)\left[E\left(X_{i}^{2}\right)-p_{i}(t) E^{2}\left(X_{i}\right)\right]  \tag{5a}\\
& =\sum_{i=1}^{n} p_{i}(t)\left[V\left(X_{i}\right)+\left(1-p_{i}(t)\right) E^{2}\left(X_{i}\right)\right] \tag{5b}
\end{align*}
$$

Figures 2 and $\mathbf{3}$ illustrate two cases, again based on 12 trucks scheduled over a 55 minute interval, with normally distributed arrival times. Figure 2 is based on a standard deviation in arrival time $\left(\sigma_{\mathrm{t}}\right)$ of 5 minutes and a standard deviation in load size $\left(\sigma_{\mathrm{x}}\right)$ of zero. Figure 3 is based on a standard deviation in arrival time of 20 minutes, with a standard deviation in load size of 1.25 . Each figure shows $E[W(t)]$, along with $E[W(t)] \pm$ one standard deviation. For very small values oft $\left(p_{i}(t)=0\right.$, for all $\left.i\right), V[W(t)]=0$, and for very large values oft $\left(p_{i}(t)=1\right.$, for all $\left.i\right)$, $\left.V[W(t))\right]$ equals the sum of $V\left(X_{i}\right)$. The variance can be larger for intermediate values, and in some cases stays nearly constant in an intermediate range.

## Model Extensions

The reality is that dependencies do exist among some of the variables $I_{i}(t)$. For instance, trucks that use the same route at similartimes also experience similar travel times as well as positively correlated arrival times (because they are exposed to similar levels of congestion). Accounting for these dependencies, but still assuming independence with respect to, and among, the $\mathrm{X}_{\mathrm{i}}$ random variables:

$$
\begin{equation*}
V[W(t)]=\left[\sum_{i=1}^{n} p_{i}(t)\left[E\left(X_{i}^{2}\right)-p_{i}(t) E^{2}\left(X_{i}\right)\right]+\left[\sum_{i \neq j} E\left(X_{i}\right) E\left(X_{j}\right) E\left\{\mathrm{I}_{\mathrm{i}}(t) \mathrm{I}_{j}(t)\right\}-p_{i}(t) E\left(X_{i}\right) p_{j}(t) E\left(X_{j}\right)\right]\right. \tag{6}
\end{equation*}
$$

where

$$
\begin{align*}
\mathrm{E}\left\{\mathrm{I}_{\mathrm{i}}(\mathrm{t}) \mathrm{I}_{\mathrm{j}}(\mathrm{t})\right\}= & \mathrm{P}\left(\mathrm{~T}_{\mathrm{i}}<\mathrm{t}\right) \mathrm{P}\left(\mathrm{~T}_{\mathrm{j}}<\mathrm{t} \mid \mathrm{T}_{\mathrm{i}}<\mathrm{t}\right) \\
& =\int_{0}^{\mathrm{t}} \mathrm{P}\left(\mathrm{~T}_{\mathrm{j}}<\mathrm{t} \mid \mathrm{T}_{\mathrm{i}}\right) \mathrm{f}\left(\mathrm{~T}_{\mathrm{i}}\right) \mathrm{dT}_{\mathrm{i}} \tag{7}
\end{align*}
$$

$T_{i}$ and $T_{j}$ are the arrival times of trucks $i$ and $j$.
$f\left(T_{i}\right)$ is the probability density function for $T_{i}$

If, for instance, travel times have a multivariate normal distribution, then

$$
\begin{equation*}
\mathrm{E}\left\{\mathrm{I}_{\mathrm{i}}(\mathrm{t}) \mathrm{I}_{\mathrm{j}}(\mathrm{t})\right\}=\int_{-\infty}^{\mathrm{t}} \Phi\left[\frac{\mathrm{t}-\mu_{\mathrm{j}}-\rho\left(\sigma_{\mathrm{j}} / \sigma_{\mathrm{j}}\right)\left(\mathrm{T}_{\mathrm{i}}-\mu_{\mathrm{i}}\right)}{\sigma_{\mathrm{j}} \sqrt{ } \sqrt{ }\left(1-\rho^{2}\right)}\right] \phi\left[\left(\mathrm{T}_{\mathrm{i}}-\mu_{\mathrm{i}}\right) / \sigma_{\mathrm{i}}\right] \mathrm{dT} \mathrm{~T}_{\mathrm{i}} \tag{8}
\end{equation*}
$$

Where:

$$
\mu_{\mathrm{i}}=\mathrm{E}\left(\mathrm{~T}_{\mathrm{i}}\right)
$$

$\sigma_{i}=$ standard deviation for arrival time of truck i
$\rho=$ correlation coefficient between arrival times of truck i and truckj

As a practical matter, the usefulness of Eqs. 6-8 is limited by the availability of data to estimate model parameters. For this reason, the models provided in the following sections will be demonstrated with the simpler case of Eq. 5.

## 3. SORT STARVATION AND SCHEDULING

Shipments are processed on a belt sortation system, which operates at a constant rate (defined by the pre-determined belt speed). The sorting process is assumed to begin at a time $\tau$, corresponding to the time employees arrive for work, and continue until all incoming work is processed. Without loss in generality, the maximum sort rate is assumed to be one unit of work per unit time, and the time of the first scheduled truck
arrival equals zero. We assume that incoming loads arrive instantaneously, and that the sorting process continues at the rate 1 whenever work is queued, and the rate 0 otherwise. An example arrival and departure diagram is shown in Figure 4.

Let T represent a practical upper bound on the time when a truck could arrive at a given sort. Then the end time for the sort can be computed as:

$$
\begin{align*}
\varepsilon(\tau) & =\quad \text { end time of sort } \\
& =\quad \tau+\mathrm{W}(\mathrm{~T})+\mathbf{S} \tag{9}
\end{align*}
$$

where $\mathbf{S}$ is the length of time that the sort is idled due to the absence of queued work. By making $\tau$ smaller, the expectation of $\varepsilon(\tau)$ can also be made smaller, thus allowing aircraft to depart earlier on average. However, because $\mathbf{S}$ is also a function of $\tau$ this effect is nonlinear. In fact, as $\tau$ becomes small, $\mathrm{E}[\varepsilon(\tau)]$ approaches a limiting value, which we denote by the sum $\mathrm{W}(\mathrm{T})+\mathrm{S}_{0}$. That is, when $\tau$ is sufficiently small, there is 0 likelihood that a truck will arrive earlier than $\tau$, so reducing $\tau$ further has no effect on $E[\varepsilon(\tau)]$.

When $\tau$ is sufficientlylarge, $\mathrm{E}(\mathrm{S})$ approaches 0 , so $\mathrm{E}[\varepsilon(\tau)]$ approaches $\tau+$ $\mathrm{E}[\mathrm{W}(\mathrm{T})]$. Combining these two limiting cases, the following bound is created:

$$
\begin{equation*}
\mathrm{E}[\varepsilon(\tau)]-\mathrm{E}[\mathrm{~W}(\mathrm{~T})] \geq \min \left\{\mathrm{S}_{0}, \tau\right\} \tag{10}
\end{equation*}
$$

The right-hand side of Eq. 10 can be viewed as the "excess end time," meaning the amount that $\mathrm{E}[\varepsilon(\tau)]$ exceeds the expected work, $\mathrm{E}[\mathrm{W}(\mathrm{T})]$. (For some distributions, it is possible for the excess end time to be negative.)

The performance of the sortation system depends on the rate at which work is scheduled to arrive, along with the start time of the sorting process. If work is scheduled to arrive at a fast rate relative to the sort rate, then work is likely to queue, which has the benefit of minimizing idle time. But beyond a certain point there is little benefit in increasing the arrival rate, as the end time will be dictated by the sorting rate and start time (and not by arrivals). Thus, it may be acceptable to hold back some trucks, reducing pressure on processing shipments at origin terminals and possibly extending the cutoff time for pickup and drop-off of shipments. On the other hand, delaying the start time also reduces idle time (again, because work queues), but has the negative effect of extending the end time. Overall, a desirable design would be to pace the arrival of work to roughly match the sorting rate, and to schedule the start of the sort at a time that balances the objectives of maximizing productivity and minimizing the end time. System performance is further defined in the following.

Productivity: The productivity is represented by the amount of time that the sort is functioning (i.e., not idled due to the absence of work). To maximize productivity, it is desirable to schedule truck arrivals, and $\tau$, such that $S$ is made as small as possible. For instance, by increasing $\tau$, a queue can be built prior to the start of the sort process, making idle time smaller. For a given realization of $W(t)$, the following relationship holds:

$$
\mathrm{dS}(\tau) / \mathrm{d} \tau= \begin{cases}1, & \mathrm{~S}(\tau)>0  \tag{11}\\ 10, & \mathrm{~S}(\tau)=0\end{cases}
$$

where $S(\tau)$ is the idle time for a random realization of the process with start time of $\tau$. That is, either there is idle time, in which case a change in $\tau$ causes slack to decrease by an identical amount, or there is no idle time, in which case slack is unaffected by a delay in $\tau$. It can also be concluded that:

$$
\begin{align*}
& \mathrm{dE}[\mathrm{~S}(\tau)] / \mathrm{d} \tau=-\mathrm{P}[\mathrm{~S}(\tau)>0]  \tag{12a}\\
& \mathrm{E}[\mathrm{~S}(\tau)]=\int_{-\infty}^{\tau}-\mathrm{P}[\mathrm{~S}(\tau)>0] \mathrm{dt} \tag{12b}
\end{align*}
$$

where $\mathrm{P}[\mathrm{S}(\tau)>0]$ is the probability that the idle time $(\mathbf{S})$ exceeds zero when the sort begins at time $\tau$. It should be noted that for large $\tau, \mathrm{dE}[\mathrm{S}(\tau)] / \mathrm{d} \tau$ approaches 0 and for small $\tau, \mathrm{dE}[\mathrm{S}(\tau)] / \mathrm{d} \tau$ approaches -1 .

Expected Completion Time: The expected completion time is clearly a non-decreasing function of $\tau$. For a given realization of $\mathrm{W}(\mathrm{t})$, the following relationship holds:

$$
\mathrm{d} \varepsilon(\tau) / \mathrm{d} \tau= \begin{cases}0, & S(\tau)>0  \tag{13}\\ 1, & S(\tau)=1\end{cases}
$$

That is, either there is idle time, in which case a change in $\tau$ has no effect on the end time, or there is no idle time, in which case a delay in $\tau$ delays the end time by an identical amount. It can also be concluded that:

$$
\begin{align*}
& \mathrm{dE}[\varepsilon(\tau)] / \mathrm{d} \tau=1-\mathrm{P}[\mathrm{~S}(\tau)>0]  \tag{14a}\\
& \mathrm{E}[\varepsilon(\tau)]=\int_{-\infty}^{\tau}\{1-\mathrm{P}[\mathrm{~S}(\tau)>0]\} \mathrm{dt} \tag{14b}
\end{align*}
$$

It can be noted that $\mathrm{dE}[\mathrm{E}(\tau)] / \mathrm{d} \tau$ equals $1+\mathrm{dE}[\mathrm{S}(\tau)] / \mathrm{d} \tau$.

Percentile Completion Time: Aircraft are typically scheduled to depart at a set time that is held close to constant from day to day. To avoid unplanned schedule changes, the aircraft departure time must allow for a reasonable amount of slack, to cushion against random fluctuations in truck arrival times and truck load sizes. The performance can be measured from a percentile of the distribution for E , which we denote $\varepsilon_{\alpha}$, where $\mathbf{a}$ is the percentile. It should be noted that W and $S$ tend to be negatively correlated (when work is large, slack tends to be small). Thus, an increase in $\tau$ may have a smaller effect on $\varepsilon_{\alpha}$ than on $\mathrm{E}[\varepsilon(\tau)]$ (i.e., slack may be reduced on average, but reduced very little on the worst days, which define $\varepsilon_{\alpha}$ ).

## Trade-off Analysis and Selection of $\tau$

A simulation was created to illustrate the trade-offs involved in setting $\tau$. A similar example to before is used: 12 trucks scheduled at 5 minute intervals beginning at time 0 ; the arrival times in the simulation are independent normal random variables (each with standard deviation 2.5 minutes) and the load sizes are also normal, each with mean 5 and standard deviation 1.25. We again assume that arrival times of trucks and their load sizes are mutually independent (as in Eq. 5). With dependencies, queueing would be somewhat larger because $\mathrm{V}[\mathrm{W}(\mathrm{t})]$ is likely to increase, though the analysis of start times would not change.

As noted in Eq. $10, \mathrm{E}(\varepsilon)>\mathrm{E}[\mathrm{W}(\mathrm{T})]+\mathrm{S}_{0}$. Figure 5 exploits this fact, and plots the average values of $\varepsilon(\tau)$ in excess of the lower bound of $E[W(T)]+S_{0}$ (which we call the "excess end time"; $S_{0}$ was found to equal $41 / 6$ minutes in the example). The second lower bound on $E[\varepsilon(\tau)], \mathrm{E}[\mathrm{W}(\mathrm{t})]+\tau$, is also represented (again subtracting $E[W(T)]+\mathrm{S}_{0}$ ). The figure illustratesthat $\mathrm{E}[\varepsilon(\tau)]$ approaches the first lower bound for small $\tau$ and the second lower bound for large т. Figure 5 also plots the average idle time (S), which equals $S_{0}-\tau$ for large values of $\tau$, and approaches 0 for large values of $\tau$.

## Trade-off Between Productivity and End-Time

Figure 5 is typical of the trade-off between the two objectives of maximizing productivity (i.e., making $\mathrm{E}(\mathrm{S})$ small), and minimizing the end time (i.e., making $\mathrm{E}[\varepsilon(\tau)]$ small). We express these objectives with the following function:

$$
\begin{equation*}
\min _{\tau} \alpha E[\varepsilon(\tau)]+E(S) \tag{15}
\end{equation*}
$$

where a represents the weight of the end-time objective relative to the idle time objective. Larger values of a push the optimal $\tau$ smaller.

As noted earlier, the derivative of $E(S)$ equals the derivative of $E[\varepsilon(\tau)]-1$. Thus, the optimal $\tau^{*}$ is defined by the following equation:

$$
\begin{equation*}
P\left[S\left(\tau^{*}\right)>0\right]=\alpha /(1+a) \tag{16}
\end{equation*}
$$

For instance, if the two objectives have equal weight ( $a^{=} .5$ ), then $50 \%$ of the realizations should produce some idle time and $50 \%$ of realizations should produce no
idle time. As a increases, the probability that there will be some idle time during the duration of the sort should become larger, making the optimal start time ( $\tau$ ) smaller.

Unfortunately $\mathrm{P}\left[\mathrm{S}\left(\tau^{*}\right)>0\right]$ is difficult to model analytically, so we propose the following heuristic. In an optimal solution we expect that the likelihood of starvation should be kept reasonably small at most times, and this can be accomplished by ensuring the $E[W(t)]$ substantially exceeds $t-z$ at all times, as follows:

$$
\begin{equation*}
\operatorname{Min}(\tau) \text { s.t. } E(W(t))-k \sqrt{V[W(t)}] \geq(t-z), \quad \text { for all } T \geq t \geq \tau \tag{17}
\end{equation*}
$$

$\mathrm{t}-\mathrm{z}$ represents the cumulative departure of work in the absence of idle time. Eq. 17 sets $\tau$ as early as possible, while ensuring that the expected arrival of work exceeds t-z by at least k standard-deviations up until time T. For the purposes of Eq. 17, we use the last scheduled arrival time for T . (More conservative standards can be set by increasing the upper bound in Eq. 17 beyond the time of the last scheduled arrival, and extending to the last possible arrival time.)

By varying $k$, the likelihood of starvation can be increased or decreased. For instance, with $\mathrm{k}=0$, starvation is highly likely, as there will be times when the expected arrival of work exactly equals the completion of work; when $\mathbf{k}$ is $\mathbf{3}$ or larger, starvation is fairly rare. Figure 6 provides an example, in which trucks are not scheduled to arrive at uniform intervals (a gap occurs around time 30). $\tau$ is set at 16 in the example, ensuring that a one standard deviation gap exists between $E[W(t)]$ and the line $(t-\tau)$, up until time 55 , when the last truck is scheduled to arrive.

A third, even simpler, method, might also be followed. It can be noted that the "excess end time" is identical to $\mathrm{E}(\mathrm{S})$ when $\tau=\mathrm{S}_{0}$ in all cases. This point occurs when both objectives are reasonably small, thus providing a $\tau$ value in the vicinity of the optimum (see Figure 5, for example.). Thus, the value $\tau=S_{0}$ provides another heuristic approximation.

The last two methods for setting $\tau$ also provide the basis for approximating the function $\mathrm{E}[\mathrm{S}(\tau)]$, representing the expected idle time as a function of $\tau$ :

$$
\begin{align*}
& \mathrm{E}[\mathrm{~S}(\tau)] \approx \mathrm{S}_{0}-\tau  \tag{18a}\\
& \mathrm{E}[\mathrm{~S}(\tau)] \approx \tau(\mathrm{k})-\tau \tag{18b}
\end{align*}
$$

where:
$\tau(\mathrm{k})$ is the start time resulting from Eq. 17, for $\mathbf{k}$ standard deviations.

Illustrations of the approximation are provided in Figure 7, for the same example introduced at the start of this section, and using $\tau$ (1). It should be noted that Eq. 18a provides a lower bound.

To conclude, three methods are provided for setting $\tau$ : (1) $\mathrm{P}\left[\mathrm{S}\left(\tau^{*}\right)>0\right]=$ $\alpha /(1+a)$, and (3) $\tau$ set by optimizing Eq. 17., (3) $\tau=S_{0}$. Only the first is exact. The usefulness of the two approximations is evaluated in the following section.

## 4. SIMULATIONS

We now present the results from a series of simulations in which system performance is estimated as a function of $\tau$. The analysis compares different methods for determining $\tau$, as follows:

1) $\tau=S_{0}$
2) $\operatorname{Min}(\tau)$ s.t. $E(W(t))-\sqrt{ } \mathrm{V}[W(t)] \geq(t-\tau), \quad$ for all $T \geq t \geq \tau$
3) $\operatorname{Min}(\tau)$ s.t. $E(W(t))-3 \sqrt{ }[W[W(t)] \geq(t-z)$, for all $T \geq t \geq \tau$

For each value of $\tau, \mathrm{E}[\varepsilon(\tau)], \mathrm{E}(\mathrm{S})$ and $\varepsilon_{.95}$ are estimated. In addition, Eq. (15) is evaluated for values of $\mathbf{a}=.1,1$ and 10 . These comparisons help determine whether any of the heuristics is sufficiently robust to produce a near-optimal solution for a variety of cases.

The evaluations were completed for a series of cases in which 12 trucks are scheduled to arrive over a 55 to 70 minute period, with 2.5 to 5 minute intervals between scheduled times. The cases are further defined by: (1) the standard deviation in truck arrival times $\left(\sigma_{t}=5\right.$ or 10 minutes $),(2)$ standard deviation of the load size $\left(\sigma_{x}=1.25\right.$ or .25), (3) the length of the time interval over which trucks arrive ( $\mathrm{T}=55,61$ or 70 minutes), (4) whether or not the scheduled interarrival times are constant. In the case of non-constant interarrivaltimes, we evaluated a bi-modal pattern, where trucks are scheduled to arrive in two periods, each of length .5. The two periods are separated by a gap g during which no trucks are scheduled to arrive ( $\mathrm{g}=\mathbf{6}$ or 15 minutes). 50 trials were completed for each case, producing standard errors on the order of .3 minutes.

Table 2 presents results for six cases. It should be noted that the three methods result in a fairly tight range of values for average idle time, with Method 1 producing values between 1.6 and 5.7 minutes, Method 2 producing values between 1.1 and $\mathbf{6 . 4}$ minutes and Method 3 producing values between 1.1 and $\mathbf{4 . 6}$ minutes. It can be seen that

Methods 2 and 3 are most conservative(i.e., produce smallest idle time) for the case where $T=25$ minutes. This is because $E[W(t)]$ has a much larger slope than the 1 , meaning that beyond the first few minutes there is little chance of idle time. Methods 2 and $\mathbf{3}$ are least conservative when $\sigma_{\mathbf{x}}=.25$ minutes (a small value), as the slopes of $\mathrm{E}[\mathrm{W}(\mathrm{t})] \mathbf{- k}$ standard deviations is most nearly constant and most nearly equal to 1 , meaning that the sort can be idled at almost any time. In most cases, it appears that the solutions produced by the three methods bracket around the optimum for different values of $a$.

In all of the cases, the $95^{\text {th }}$ percentile for the end time of the sort is substantially larger than average - varying from about 10 to 15 minutes later. Variations in load sizes, in particular, add to the variability in the end time.

## 5. SUMMARY AND EXTENSIONS

The models developed in this paper for evaluatingperformance are being implemented in a web-based tool. Multiple users, located at different sites, can enter and edit data on planned schedules, and can adjust the data on a daily basis to reflect current conditions. The tool produces graphs of expected cumulative arrival, and completion, of work, and also indicates when there is a substantial likelihood of starvation. It also provides what-if capability, so that the effects of a change in schedule can be evaluated. For instance, the user may alter the departure time, and then see whether the change is likely to cause starvation and a delay in the completion of the sort. This provides some guidance in whether a truck should be released immediately or held-over for late
shipments. Lastly, the tool provides a means for information sharing among multiple sites.

An added feature under development is a real-time travel time forecasting capability. This will entail monitoring current traffic conditions, and incorporating the conditions in predictions for travel time. This enables the program to more accurately forecast the arrival of work, and more accurately predict whether starvation will occur. The program also provides access to a variety of real-time traffic services so that the dispatcher can gather additional information on traffic problems.

The key concept behind the model is conversion of truck schedules into forecasts for the expected arrival of work, and forecasts for the standard-deviationin the arrival of work. These pieces of information make it possible to predict the occurrence of starvation and the end-time for a sort. Combined, the methods develop in this paper provide a tool for representing the trade-off between sort productivity and the objective of completing the sort as early as possible.

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## Table 1. Express Transportation Phases

1. Pickup at shipper and transportation to local terminal
2. Processing shipment at local terminal (unload, sort, reload)
3. Truck transportation from local terminal to airport terminal
4. Processing shipment at airport terminal (unload, sort, load aircraft)
5. Air transportation to hub airport
6. Processing shipment at hub airport (unload, sort, load aircraft)
7. Air transportation to destination airport
8. Processing shipment at destination airport (unload, sort, load truck)
9. Truck transportationto local terminal
10. Processing shipment at local terminal (unload, sort, load truck)
11. Transportation to receiver and delivery

Table 2. Comparison of Performance Measures with Differing Start Times

| $\sigma_{\mathrm{t}}=5, \sigma_{\mathrm{x}}=1.25, \mathrm{~T}=55, \mathrm{~g}=0$ | Objective Function |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Method | $\tau$ | $\mathrm{E}(\varepsilon)$ | $\mathrm{E}(\mathrm{S})$ | $\varepsilon .95$ | 0.1 | 0.5 | 0.9 |
| 1 | 6 | 67.8 | 3.2 | 72.0 | 10.0 | 37.1 | 64.2 |
| 2 | 11 | 69.8 | 1.7 | 76.0 | 8.6 | 36.6 | 64.5 |
| 3 | 7 | 68.9 | 2.7 | 74.0 | 9.6 | 37.2 | 64.7 |


| $\sigma_{\mathrm{t}}=10, \sigma_{\mathrm{x}}=1.25, \mathrm{~T}=55, \mathrm{~g}=0$ |  |  |  |  | Objective Function |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Method | $\tau$ | $\mathrm{E}(\varepsilon)$ | $\mathrm{E}(\mathrm{S})$ | $\varepsilon_{95}$ | 0.1 | 0.5 | 0.9 |  |
| 1 | 9 | 71.6 | 2.8 | 83.0 | 9.9 | 38.6 | 67.2 |  |
| 2 | 14 | 75.4 | 1.3 | 87.0 | 8.9 | 39.0 | 69.2 |  |
| 3 | 9 | 71.6 | 2.8 | 83.0 | 9.9 | 38.6 | 67.2 |  |


| $\sigma_{\mathrm{t}}=10, \sigma_{\mathrm{x}}=.25, \mathrm{~T}=55, \mathrm{~g}=0$ | Objective Function |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Method | $\tau$ | $\mathrm{E}(\varepsilon)$ | $\mathrm{E}(\mathrm{S})$ | $\varepsilon .95$ | 0.1 | 0.5 | 0.9 |
| 1 | 4 | 68.6 | 5.7 | 79.0 | 12.5 | 39.9 | 67.4 |
| 2 | 6 | 69.6 | 4.6 | 83.0 | 11.6 | 39.4 | 67.2 |
| 3 | 8 | 69.9 | 3.1 | 82.2 | 10.1 | 38.0 | 66.0 |


| $\sigma_{\mathrm{t}}=10, \sigma_{\mathrm{x}}=1.25, \mathrm{~T}=27.5, \mathrm{~g}=0$ | Objective Function |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Method | $\tau$ | $\mathrm{E}(\varepsilon)$ | $\mathrm{E}(\mathrm{S})$ | $\varepsilon .95$ | 0.1 | 0.5 | 0.9 |
| 1 | 2 | 62.2 | 1.1 | 76.0 | 7.4 | 32.2 | 57.1 |
| 2 | 3 | 62.7 | 1.1 | 77.0 | 7.4 | 32.4 | 57.5 |
| 3 | -2 | 60.4 | 2.6 | 75.0 | 8.6 | 32.8 | 57.0 |


| $\sigma_{\mathrm{t}}=10, \sigma_{\mathrm{x}}=.25, \mathrm{~T}=61, \mathrm{~g}=6$ | Objective Function |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Method | $\tau$ | $\mathrm{E}(\varepsilon)$ | $\mathrm{E}(\mathrm{S})$ | $\varepsilon_{.95}$ | 0.1 | 0.5 | 0.9 |
| 1 | 10 | 73.4 | 4.2 | 84.0 | 11.5 | 40.9 | 70.2 |
| 2 | 15 | 75.4 | 1.6 | 86.0 | 9.1 | 39.3 | 69.4 |
| 3 | 15 | 75.4 | 1.6 | 86.0 | 9.1 | 39.3 | 69.4 |


| $\sigma_{\mathrm{t}}=10, \sigma_{\mathrm{x}}=.25, \mathrm{~T}=70, \mathrm{~g}=15$ |  |  |  |  | Objective Function |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Method | $\tau$ | $\mathrm{E}(\varepsilon)$ | $\mathrm{E}(\mathrm{S})$ | $\varepsilon_{.95}$ | 0.1 | 0.5 | 0.9 |  |
| 1 | 18 | 83.4 | 6.4 | 96.0 | 14.7 | 48.1 | 81.5 |  |
| 2 | 23 | 86.0 | 3.7 | 98.0 | 12.3 | 46.6 | 81.0 |  |
| 3 | 19 | 83.5 | 5.7 | 92.0 | 14.0 | 47.5 | 80.9 |  |






> ミnd
> Idle
> LB on ₹nd
> ミnd + Idle
> Start Sort Time


$\rightarrow$ Simulated

- 1 sd Prediction SO Prediction


## Sort Start Time

