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# MESON SCATTERING IN QUANTUM CHROMODYNAMICS IN TWO DIMENSIONS

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# Meson Scattering in Quantum Chromodynamics in Two Dimensions

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## ABSTRACT

We report on some properties of meson collisions in quantum chromodynamics in two space-time dimensions. To leading order in 1/N, where N is the number of colors, the high energy behavior of two-body, non-diffractive scattering amplitudes (quark exchange) is power-behaved; $s^{\alpha}$. The power $\alpha$ is additive in the exchanged quarks, i.e., the amplitude corresponding to the exchange of quark a and antiquark $\bar{b}$ has power $\alpha_{ab} = -\beta_a - \beta_b$. Turning to diffractive scattering, we have calculated the high energy behavior of the "twisted loop" or "cylinder" graph. In general, it vanishes more rapidly than $s^{\alpha}$. In other words, the theory does not possess a bare Pomeron to leading order in 1/N. We conclude with a number of phenomenological speculations.

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Quantum chromodynamics (QCD), the theory of colored quarks interacting via colored gauge fields, may be the fundamential theory of strong interactions.[1] Its properties at short distances can be calculated from perturbation theory, as if the virtual quanta are almost non-interaction. Thus, it possesses the approximate scaling behaviors characteristic of certain inelastic processes, such as deep inelastic lepton scattering and electron-position annihilation. Attempts are being made[2] to show that the theory confines quarks and gluons to form the color singlet bound states observed [3] as hadrons in the world. If indeed it does describe hadron physics, one should be able to derive certain general features of experimental data and to relate them to concepts previously employed to explain them. One striking characteristic of high energy collisions is the transverse momentum damping: when hadrons collide along an axis, nearly all the particles produced lie near the axis. This observation motivated Feynman to assume that, in a hadron moving with very large momentum, the transverse momentum distribution of its virtual constituents is also sharply damped.[4]. Two other prominent phenomenological properties of scattering should also be noted: (1) two-body, non-diffractive amplitudes manifest power law behavior at high energies, the power depending on the quantum numbers exchanged and simply related to Regge trajectories of particles which can be exchanged. (2) Elastic and diffractive amplitudes grow approximately linearly with energy, corresponding to approximately constant total cross sections.

Some time ago, 't Hooft proposed [5] a classification of the Feynman diagrams of QCD based on an expansion in the inverse of the number N of colors. He showed that, in each order, the diagrams could be put into one-to-one correspondence with the dual perturbation theory.[6] Thus, to leading order, meson-meson scattering amplitudes consist of planar graphs with no internal quark loops. Assuming this approximation is sufficient to confine quarks and gluons, then, to this order, all channels should be pole-dominated, just as in the Veneziano model.[7] If these amplitudes manifest Regge-asymptotic behavior, all the usual phenomenology based on duality diagrams can be anticipated, such as exoticity criteria and exchange degeneracy.[8] Unfortunately, no progress has been made toward solving QCD in four dimensions, even in this extreme approximation.

However, in two space-time dimensions, 't Hooft showed[9] that the 1/N expansion may be implemented and the properties of the solution demonstrated. The two-dimensional theory is prototypical of a non-trivial field theory which is both asymptotically free and confining.[10] A number of interesting theoretical and phenomenological questions may be analyzed in this model.[11,12] Experimentally, transverse momenta are observed to be strongly damped in high energy hadron collisions, so it may even be that certain results obtained in two-dimensions may be abstracted and usefully applied to the real world. Be that as it may, the two dimensional model poses a well-defined problem wherein the properties of high energy scattering of bound-state mesons may be evaluated. Obviously, questions related specifically to crossing symmetry, to spin, or to large transverse momentum behavior cannot be approached in two dimensions.

Let us consider the Feynman diagrams corresponding to meson-meson scattering. To leading order in 1/N, the scattering amplitude consists of planar graphs with no internal quark loops. In any gauge in which there is no self-coupling of the gluon field, the scattering amplitude may be depicted as in Fig. 1, where we have drawn one amplitude[13] for $1+2\to n+m$. In the figure, T denotes the quark-antiquark scattering amplitude, which has been explicitly calculated in the light-cone gauge, $A_- = 0$.[11,12] To be general, we have supposed all mesons have different flavors and have labelled the quark lines by their flavors a,b,c,d. For example, meson 1 is composed of quark a and antiquark $\bar{d}$. Its wave function is $\varphi_1^{a\bar{d}}(x)$, where x is the momentum fraction carried by quark a. The kinematics of this process has been described by Feynman[4] and will not be repeated here. However, in this model, we may go further. Recalling that[8] $\varphi_1^{a\bar{d}}(x) \to x^{\beta_a}$ as $x \to 0$, we can say that the probability amplitude to find quark a in meson 1 with wee momentum goes as $P_1^{-\beta_a}$, where $P_1$ is the total momentum of the meson. Similarly, the amplitude to find antiquark $\bar{a}$ in meson 2 with wee momentum goes as $P_2^{-\beta_a}$. Hence, the amplitude for exchanging a wee quark of flavor a between meson 1 and meson 2 goes as $s^{-\beta_a}$. An analogous discussion applies to the exchange of quark b, so the asymptotic behavior of this amplitude is $s^{\alpha_{ab}}$, where $\alpha_{ab} = -\beta_a - \beta_b$. (It is noteworthy that the power is additive in the quarks and will be returned to later.) This result can be verified by direct calculation, and one can show that the asymptotic behavior of all three diagrams in Fig. 1 is the same.[14] Recall that $\beta_i$ lies between zero and one, being determined by the equation [9]

$$\pi\beta_i \cot\pi\beta_i = -\pi^2\alpha'\tilde{m}_i^2 \qquad (1)$$

where the (asymptotic) level spacing $\alpha'^{-1}$ is related to the coupling constant by $\alpha'^{-1} = \pi g^2 N$ and $\tilde{m}_i$ is the renormalized quark mass, related to the bare quark mass $m_i$ according to $\tilde{m}_i^2 = m_i^2 - g^2 N/\pi$. ($\tilde{m}_i^2$ can be negative!)

It is also interesting to explore unitarity in this model. Similar to the discussion of virtual Compton scattering,[12] it can be shown [14] that the disconnected diagram, Fig. 1a, is cancelled by a piece of Fig. 1b and that there are no quark discontinuities coming from the remaining contributions to Fig. 1b or from Fig. 1c. Of course, this is to be expected of a field theory with confinement. The only dicontinuities in s coming from Fig. 1 are meson poles, so this much of duality is retained by the two-dimensional model: the sum of s-channel poles leads to a Regge-like asymptotic power behavior $(-s)^{\alpha}$. (Similarly, the sum of u-channel poles leads to $s^{\alpha}$.) The fact that we apparently must sum over s- and t-channel exchanges, Figs. 1b and 1c, is gauge-dependent. In any gauge in which the self-coupling of gluons does not vanish, this decomposition of the planar diagram does not occur.[15] Unitarity is quite simple: to this leading order in 1/N, the effective theory of mesons may be built from a phenomenological Lagrangian involving only three-point couplings of mesons.[16]

To next order, $O(N^{-2})$, many diagrams contribute corresponding to "Regge-Regge cuts", renormalizations of the "intercept" $\alpha_{ab}$, etc. In this note, we concentrate on the imaginary part of the class of planar diagrams having two quark boundaries and no handles, which is usually identified with the "bare" pomeron.[17] (Fig. 2a) (To this graph must be added all planar gluon exchanges, just as Fig. 1b and 1c were added to Fig. 1a.) Topologically, this graph with all its gluonic corrections may also be depicted as a cylinder, Fig. 2b, or, alternatively, as the twisted loop graph, Fig. 2c. In four dimensions, one would describe this diagram in terms of gluon exchange between the quarks in each meson, as may be easily seen from Fig. 2b. Indeed, one expects these gluons to form quarkless mesonic bound states ("glueballs") which would be expected to lie on the pomeron trajectory.[a] In two dimensions, the gluon field contains no dynamical degrees of freedom; there are no gluons, hence no glueballs to support the bare pomeron. Nevertheless the quarks can scatter via the potential represented by the gluon field. Might this lead to an asymptotic behavior which dominates the Regge-like behavior discussed above and which might therefore by identified as the "bare" pomeron in two-dimensions?

To simplify the calculation, we proceed as follows: Because quarks are confined, we are assured that the only discontinuities come from mesonic intermediate states, so that the imaginary part is given by the square of the planar (ut)-amplitude, Fig. 3:

$$\mathrm{Im}\, P = \sum_{m,n} |A_{12 \to nm}(s)|^2 \rho_{nm}(s) \qquad (2)$$

Here, P denotes the amplitude corresponding to Fig. 2; $A_{12 \to mn}$, the amplitude represented by Fig. 3. The phase space $\rho_{nm}(s)$ may easily be shown to be proportional to $\lambda^{-1/2}(s, \mu_m^2, \mu_n^2)$, where $\lambda$ is the familiar triangular function, and $\mu_m$ and $\mu_n$ are the masses of meson m and n, respectively. The sum extends over all mesons m,n allowed by momentum conservation ($\lambda > 0$.) We wish to discuss the asymptotic behavior as $s \to \infty$ of Eq. 2, to determine whether a new power emerges which dominates the "Regge" behavior determined previously. One would anticipate that the dominant contribution would come from the region where both $\mu_m^2$ and $\mu_n^2$ are of order s. This may be easily understood intuitively, for example, in the center-of-mass frame with meson 1 (2) moving to the right (left). There is a finite, scaling amplitude $\varphi_1(x_1)$ to find a right-moving quark in meson 1 carrying momentum fraction $x_1$ and, similarly, a scaling amplitude $\varphi_2(x_2)$ to find a left-moving antiquark in meson 2 carrying momentum fraction $1-x_2$. Interacting via the long-range Coulomb potential (in four dimensions, we would say "wee gluon exchange"), this quark-antiquark pair bind to form meson n with $\mu_n^2 \cong x_1(1-x_2)s$. Similarly, the remaining quark-antiquark pair bind to form meson m with $\mu_m^2 \cong x_2(1-x_1)s$. Thus, one might <u>guess</u> that the production amplitude would scale as $s \to \infty$ for fixed $x_1, x_2$:

$$\lim_{s \to \infty} A_{12 \to nm}(s) \xrightarrow{??} A(x_1, x_2). \qquad (3)$$

the sum in Eq. (1) may then be represented as an integral over quark momentum fractions:

$$\sum \rho_{nm}(s) \approx \int d\mu_m^2 \, d\mu_n^2 \, \lambda^{-1/2}\left(s,\mu_n^2,\, \mu_m^2\right) \approx s\iint dx_1 \, dx_2. \qquad (4)$$

Combining these two expressions, Eq. (3) and (4), would then lead to Im P growing linearly with energy. We have performed the calculation of $A_{12\to nm}(s)$ in the $A_- = 0$ gauge. Because the result is somewhat surprising, we display the exact form of the (ut)-amplitude here in an Appendix.[18] The result is that the asymptotic behavior of $A_{12\to nm}(s)$ does not scale as described by Eq. (2), in fact, it vanishes at least as rapidly as $s^{-1}$ which, in turn, leads to a contribution to Im P which vanishes at least as rapidly as $s^{-1}$. In general, this will be more rapidly than the "Regge" term vanishes and cannot be interpreted as a pomeron.[19] If we conjecture that the absence of the pomeron is related to the absence of gluons in two-dimensions, it could be that, when one calculates to higher order in 1/N and begins to build the quark-antiquark "sea", their exchange will lead to pomeron-like behavior. Such speculations we leave for future investigations.

In four dimensions, where the transverse gluon field is an independent dynamical degree of freedom, the mesonic wave function must also describe the probability amplitude to find any number of gluons in addition to the valence quark-antiquark pair. It may well be that the analogous calculation involving the exchange of these wee gluons will produce a bare pomeron already in order $1/N^2$.[20]

Let us conclude with a few phenomenological speculations motivated by our results on "Regge" behavior. Applications to four dimensions depend on the nature of confinement there. So long as a hadron's wave function in the infinite momentum frame has the property that the probability to find a parton at large transverse momentum $\underline{K}$ falls faster than $\underline{K}^{-2}$, we expect transverse momenta to play a relatively innocuous role in many cases, such as the processes discussed here and in Ref. 12. In such cases, although quantitative differences will occur, the physical picture abstracted from two dimensions may continue to hold in four. It seems promising to identify the mesons of the two-dimensional model with the pseudoscalar mesons in the real world.[9] As the bare mass of the u and d quarks tend to zero, both the mass of the $u\bar{d}$ ground state (pion) and $\beta$ tend to zero, thus $\alpha_\pi = -2\beta$ behaves like the intercept of the pion Regge trajectory, more precisely, we find $\alpha_\pi \to -3\alpha' m_\pi^2$ as $m_\pi \to 0$. For heavy quarks ($m \to \infty$), $\beta \to 1$. Thus there is a lower bound $\alpha_{ab} > -2$. If satisfied in the real world, this would mean, for example, that the slope of charmed meson trajectories would be flatter than more familiar trajectories, an intriguing possibility. Finally, there is the property that the Regge intercept is related additively to the quarks exchanged. Certain relations among intercepts follow, reminiscent of $\alpha_\rho + \alpha_\varphi = 2\alpha_{K^*}$. Notice again, however, that $\beta$ is not proportional to the mass-squared of the mesons except for light mesons. Notice also that these additivity relations hold to leading order in 1/N despite whatever flavor-symmetry-breaking there may be.[21]

Since the asymptotic behavior of the pion form factor[12] is also determined by $\beta$, we find a simple relation between the Regge intercept and power law of the form factor. This correspondence is probably an artifice of the 1/N expansion, since form factors are related to the behavior of valence quarks as $x \to 1$, while Regge behavior depends on the behavior as $x \to 0$ of not only valence quarks, but also quarks in the "sea". Presumably, it is only to leading order in N that the behaviors as $x \to 1$ and as $x \to 0$ are so closely intertwined.

We would like to thank W.A. Bardeen and A. Mueller for conversations. S.N. and E.R. would like to acknowledge the hospitality of the high energy physics group at the University of Michigan, where a portion of the work was carried out. Most of this work was performed while one of us (E.R.) was still a member of the Theory Department at Fermilab.

Note added: While preparing this manuscript for publication, we received a preliminary copy of a preprint[22] in which conclusions similiar to those presented here have been reached. (We would like to thank R. Savit for informing us of this work and G.F. Chew for making this pre-preprint available to us.)

## APPENDIX

Here, we display the exact form of the (ut)-amplitude, depicted in Fig. 3. We find[23]

$$\frac{N}{4\pi^3}A_{12\to nm}(s) = p_2p_m \int\int du\, dv \frac{\left[\varphi_1\left(\frac{q+vp_2}{p_1}\right) - \varphi_1\left(\frac{q+up_m}{p_1}\right)\right]}{(vp_2 - up_m)^2} \varphi_n\left(\frac{q+vp_2}{p_n}\right)\varphi_2(v)\,\varphi_m(u)$$

$$+p_2p_n\int\int du\, dv \frac{\left[\varphi_1\left(\frac{q+vp_2}{p_1}\right) - \varphi_1\left(\frac{up_n}{p_1}\right)\right]}{\left(p_2(1-v)-p_n(1-u)\right)^2} \varphi_n(u)\varphi_2(v)\varphi_m\left(\frac{vp_2}{p_m}\right)$$

$$+ q^2p_2p_m\int\int\int\int dxdydudv\; G(x,y;U)\;\varphi_m(u)\varphi_2(v) \qquad \times$$

$$\times \frac{\left[\varphi_n\left(\frac{yq}{p_n}\right) - \varphi_n\left(\frac{q+p_2v}{p_n}\right)\right]}{\left(p_2v + q\,(1-y)\right)^2}\; \frac{\left[\varphi_1\left(\frac{xq}{p_1}\right) - \varphi_1\left(\frac{q+up_m}{p_1}\right)\right]}{\left(p_mu+q(1-x)\right)^2}$$

$$+\Delta^2p_2p_n\int\int\int\int dxdydudv\; G\,(x,y;T)\;\varphi_n(u)\varphi_2(v) \qquad \times \tag{A1}$$

$$\times \frac{\left[\varphi_m\left(\frac{p_2+y\Delta}{p_m}\right) - \varphi_m\left(\frac{vp_2}{p_m}\right)\right]}{\left(\Delta y+ p_2(1-v)\right)^2}\; \frac{\left[\varphi_1\left(\frac{p_n+x\Delta}{p_1}\right) - \varphi_1\left(\frac{up_n}{p_1}\right)\right]}{\left(\Delta x+p_n(1-u)\right)^2}$$

All momenta appearing in the formula refer to their minus components, ie, $q/p_1 \equiv q_-/p_{1-}$ all integrals run over (0,1).

Here, $\varphi_i(z)$ is the wave function of hadron $i(=1,2,n,m)$, $q^\mu \equiv p_1^\mu - p_m^\mu$, $\Delta^\mu \equiv p_1^\mu - p_n^\mu$, $U \equiv q_\mu^2$, $T \equiv \Delta_\mu^2$; G is the Green's function[12]

$$G(x,y;S) \equiv \sum_n \frac{\varphi_n(x)\varphi_n(y)}{\mu_n^2 - S}\,.$$

The first term comes from

Fig. 3a and the Born term of Fig. 3b; the third comes from the rest of Fig. 3b. The second term comes from Fig. 3a plus the Born term of Fig. 3c; the fourth term, from the rest of Fig. 3c. Unfortunately, the intuitive discussion given above for the center-of-mass frame does not apply in the $A_- = 0$ gauge where parity invariance is not manifest. However, one can consider the limit $s \to \infty$ for fixed $\mu_n^2/s$, $\mu_m^2/s$. (In this limit, one has also U/s and T/s fixed also.) Using scaling relations derived earlier,[12] one may easily determine the dominant contribution of each term. In each case, the dominant behavior comes from the infrared region for the gluon "propagator". Multiplying out the factors in Eq. A1 gives a sum of 12 terms, each of which <u>does</u> scale, and the argument of $\varphi$ becomes, in every case, $p_{n-}/p_{1-} \equiv x_1$, as was anticipated. However, recombining terms again, it is easy to see that this contribution cancells leaving a contribution which falls by (at least) $s^{-1}$. (We have not been able to show that this coefficient is non zero, so the rate of vanishing may be even faster. We have not found any simple explanation for this cancellation and, although we are not surprised that there is no pomeron, we are surprised at the subtle way in which it appears to decouple. Clearly a deeper understanding of this is desirable.)

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(7) See the collection in Dual Theory, Physics Reports Reprints, (M. Jacob, ed), 1976

(8) An interesting discussion based on 't Hooft's work plus extensions to a "topological expansion" has been given by G. Veneziano, Ref. TH.2200 CERN (1976).

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(10) These properties emerge almost trivially in two dimensions because (a) the theory is super-renormalizable, and (b) the Coulomb potential is linear.

(11) C.G. Callan, N. Coote, and D.J. Gross, Phys. Rev. D13, 1649 (1976).

(12) M.B. Einhorn, Fermilab-Pub-76/22-THY (Phys. Rev., to be published).

(13) Fig. 1 is often referred to as the (st)-diagram in discussions of duality and/or the Veneziano model (see ref. 7). To Fig. 1 must be added the two cyclicly inequivalent permutations of the external mesons, commonly called (su) and (ut). The (ut)-diagram (Fig. 3) will be discussed below in connection with the pomeron.

(14) Full details and further discussion will be presented elsewhere: M.B. Einhorn, in preparation. It can also be shown that the coefficient manifests t-channel factorization. (A word of caution must be added, however, since we have not proved that the coefficient of $s^{\alpha_{ab}}$ is non-zero.) The (ut)-diagram behaves as $u^{\alpha}$ which, when added to the (st)-diagram may be interpreted as giving "signature" to the exchange. Finally, the (su)-diagram behaves as $s^{\alpha-2}$ for forward scattering.

(15) In four dimensions, one cannot gauge away the self-coupling of the gluon field, and we anticipate that full duality will be recovered: The planar amplitude will have meson poles in both the s- and t- channels and manifest true Regge-asymptotic-behavior.

(16) Assuming ordinary analyticity for meson amplitudes, there can be no "contact

term," contrary to the claim of Ref. 11. See ref. 14 for further discussion.

(17) In the context of the topological expansion, this correspondence has been made more precise recently, where it has been identified with the "bare" pomeron which must be iterated (eg. via Gribov's reggeon calculus) to obtain a fully unitary S-matrix. For a summary with references to the original literature, see Ref. 8. Its identification with the pomeron in some general sense goes back as far as P.G.O. Freund, Lett. Nuovo Cimento 4, 147 (1970); P.G.O. Freund and R.J. Rivers, Phys. Lett. 29B, 510 (1969).

(18) The full derivation will be given elsewhere: M.B. Einhorn, E. Rabinovici, in preparation.

(19) The surprising aspect of the calculations, as discussed in greater detail in the Appendix, is not so much that we do not find a pomeron, but that, even though each "time"-ordered graph does satisfy the scaling property, Eq. (3), their sum does not. We do not have a simple explanation of this fact.

(20) For approaches in four dimensions, see F.E. Low, Phys. Rev. D12, 163 (1975) and S. Nussinov, Phys. Rev. Letters 34, 1286 (1975) and Phys. Rev D14, 246 (1976).

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(22) R.C. Brower, J. Ellis, M.G. Schmidt, and J.H. Weis, Ref. TH.2233-CERN (1976).

(23) We have assumed, without loss of generality, the $p_{1-} \geq p_{n-}$, $p_{m-} \geq p_{2-}$

## FIGURE CAPTIONS

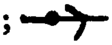
Fig. 1 The (st)-amplitude to leading order in 1/N ( = hadron;  = dressed quark)

Fig. 2 Equivalent representations of the bare pomeron graph: (a) planar graph with two quark boundaries, (b) cylinder or tube showing gluonic exchanges in the t-channel, (c) twisted loop displaying mesonic intermediate states.

Fig. 3 The (ut)-amplitude to leading order in 1/N.

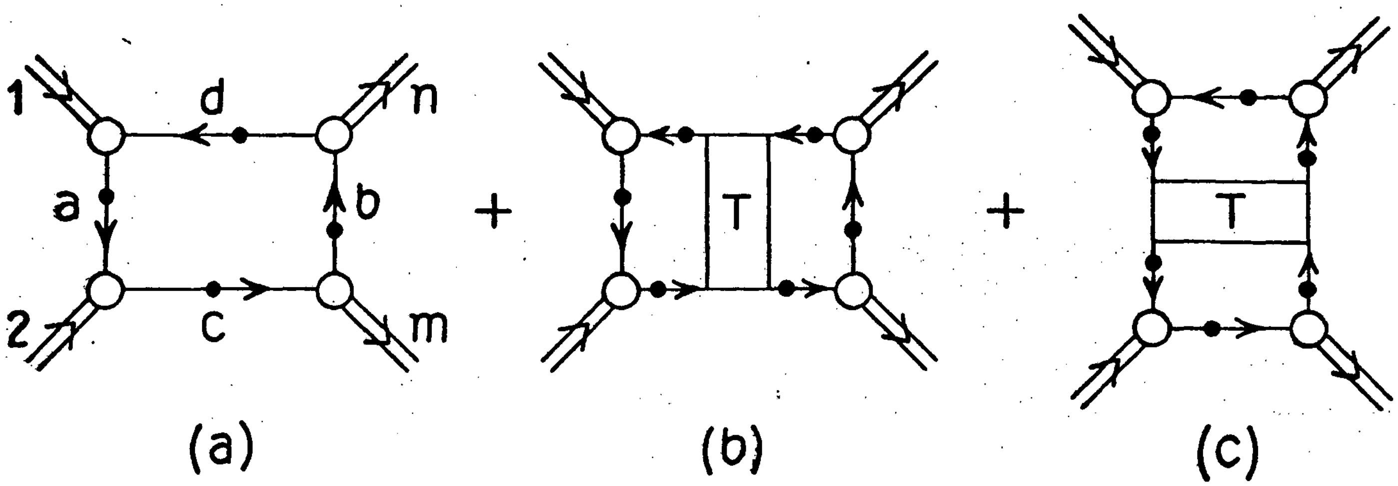



Fig. 1

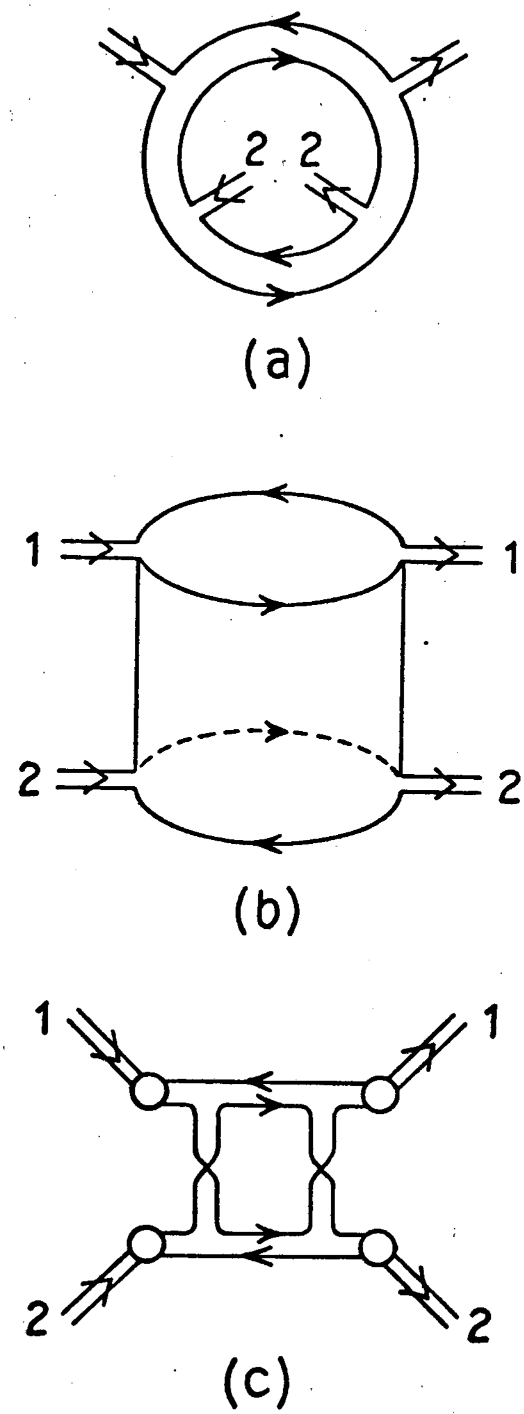



Fig. 2

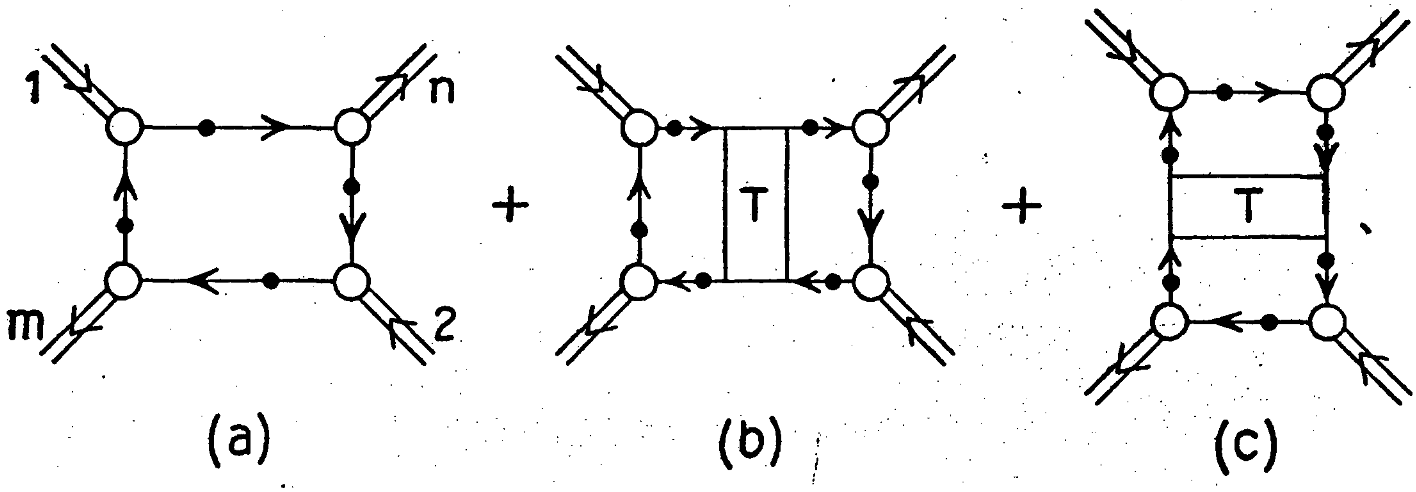



Fig. 3

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