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Performance Analysis and Resource Allocation for Device-to-Device Video Transmission

A Dissertation submitted in partial satisfaction of the requirements for the degree Doctor of Philosophy in Electrical Engineering (Communication Theory and Systems)

by

Peizhi Wu

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2016
The Dissertation of Peizhi Wu is approved, and it is acceptable in quality and form for publication on microfilm and electronically:

Co-Chair  

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University of California, San Diego  

2016
DEDICATION

To my family.
EPIGRAPH

No one knew for certain where the limits of reality lay.

— Gabriel Garcia Marquez
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ABSTRACT OF THE DISSERTATION

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Doctor of Philosophy in Electrical Engineering
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Device-to-device (D2D) communication allows direct transmission between multiple transmitter-receiver pairs in cellular systems by reusing the spectrum, and offloads local traffic from the base station.

In resource allocation for a D2D video transmission system in a block fading environment, the performance improvement by applying the exact symbol error rate (SER) is compared with the conventional signal-to-interference-plus-noise-ratio (SINR) based SER evaluation method that uses a Gaussian approximation (GA) for the
aggregated interference. An analytical SER expression for a D2D system using multicarrier bandlimited QAM is derived and then used in the resource allocation algorithm. Centralized resource allocation for the D2D system is considered, given knowledge of the channel state information and the rate distortion information of the video streams, and an iterative algorithm for subcarrier assignment and power allocation is proposed. Bit-level simulations for different numbers of D2D pairs demonstrate a considerable improvement on user capacity and video peak signal-to-noise ratio by incorporating the proposed SER expression compared to the GA. By invoking the conditions under which the central limit theorem holds, and comparing these conditions with the number of interferers and the power ratio of the dominant interferer in the simulated D2D system, the reason why the GA for the interference results in a pessimistic performance is also studied.

Resource allocation algorithms for D2D video transmission with a filter bank multicarrier waveform in a Rayleigh fading environment are investigated. The co-channel interference between D2D pairs is analyzed, and a cross-layer algorithm with a subcarrier assignment outer loop and a power control inner loop, which aims to optimize the overall video quality, is proposed. Unlike the non-convexity in physical layer power control for maximizing the total throughput, the cross-layer power control problem is convex under certain conditions, so a high quality solution for power control can be efficiently found. Simulation results demonstrate a higher overall video quality by the proposed cross-layer algorithm compared to baseline algorithms.
Chapter 1

Introduction

1.1 Device-to-device Communication

In conventional cellular systems, all data traffic is transmitted or received via a base station (BS). Even if a transmitter-receiver pair is located in close proximity, the data traffic still needs to be forwarded by the BS, as shown in Fig. 1.1. Device-to-device (D2D) communication [1] is a paradigm of cellular systems that allows direct communications between cellular users in close proximity. As illustrated in Fig. 1.2, D2D communication allows multiple transmitter-receiver pairs to share the same spectrum, thereby improving the spectral efficiency and offloading traffic from the base station. A frequency band can be reused by multiple D2D pairs to improve the spectral efficiency. D2D communication is likely to be significant for many situations (airports, shopping centers, stadiums, amusement parks) where densely located users may wish to communicate with each other, or with merchants or local infrastructure.

Some literature on D2D communications proposes to use the licensed spectrum for both D2D and cellular communications, namely underlay D2D [1, 2, 3, 4].
Figure 1.1: Conventional cellular systems with multiple transmitter-receiver pairs.

Figure 1.2: Cellular systems with D2D communication.

An example of D2D as an underlay to cellular uplink is shown in Fig. 1.3. These works usually study the problem of interference mitigation between D2D and cellular communication.

In order to avoid this interference between D2D and cellular users, some literature [5] proposes to dedicate part of licensed spectrum exclusively to D2D, namely overlay D2D, as shown in Fig. 1.4. In this case, the resource allocation is of utmost importance such that dedicated licensed band can be efficiently shared by D2D users. In this dissertation, the overlay D2D model is investigated.
Figure 1.3: D2D communication as an underlay to cellular uplinks.

Figure 1.4: D2D communication as an overlay.

At present, Orthogonal Frequency Division Multiplexing (OFDM) [6] is a widely adopted waveform due to its simple implementation and robustness against multipath. However, the application scenarios predicted for D2D communications present transmission delays between multiple D2D pairs [7], making the strict syn-
chronization to maintain orthogonality between subcarriers infeasible. Also, the high out-of-band (OOB) emission of OFDM [8] poses a challenge. Therefore, OFDM is not the most promising waveform for D2D communications. In this context, filter bank multicarrier (FBMC) based waveforms, such as filtered multi-tone (FMT) [9], are currently being evaluated as candidates for the waveforms of D2D communications.

1.2 Interference in D2D Communication

Interference is a key consideration in the design of a D2D communication system, since a D2D user operating on shared spectrum receives not only the desired signal but also signals from other D2D users. Signals from other D2D pairs on shared spectrum produce cochannel interference (CCI) at the D2D receiver. A well-known approach to treating CCI is to approximate the aggregated interference as a Gaussian random variable, based upon a Gaussian approximation (GA) of the interference component of the test statistic. However, the GA heavily relies on the conditions that validate the central limit theorem, which may not be suitable for D2D systems. First, the frequency band of D2D systems is directly shared by users without the use of spread spectrum, so the number of interfering users can be small [10]. Also, some interferers may have significantly larger power than other interferers at a D2D receiver.

To circumvent the inaccuracy of the GA, an analytical expression of the exact symbol error rate (SER) of digital systems under CCI is of great interest. Early endeavors to calculate the probability of error in a bandlimited digital system with interference were devoted to phase-shift keying (PSK), such as binary phase-shift keying [11, 12, 13] and quadrature phase-shift keying [14, 15]. Current multimedia
applications often use quadrature amplitude modulation (QAM) for rate-adaptive transmission [16], due to its practicality and bandwidth efficiency. A vast literature has been dedicated to the performance analysis of QAM, such as [17, 18], but few papers were related to the performance analysis of bandlimited QAM corrupted by CCI. Unlike PSK, the symbols of high-order QAM, such as 16-QAM or 64-QAM, have multiple power levels. Also, the in-phase and quadrature components of a QAM CCI signal are correlated if a random phase shift is introduced in the interfering signal, so it is necessary to jointly consider the detection in the in-phase and quadrature paths. Therefore, the treatments for PSK modulated CCI in [11, 12, 14, 13, 15] are not applicable to QAM modulated CCI. For bandlimited QAM under CCI, [19] proposed an upper bound to the probability of error based on the Chernoff bound, but the bound was not tight. In [20], the authors presented a saddlepoint integration method to numerically evaluate the error probability of a bandlimited QAM system with interference. The authors of [21] derived a bit error rate (BER) expression for QAM systems with rectangular pulses corrupted by asynchronous CCI in Nakagami-\(m\) fading channels. However, the BER expression was given in an integral form, which limited its usefulness. In summary, an analytical expression for the error probability for asynchronous QAM CCI with bandlimited pulse shaping does not appear to be available in the literature.
1.3 Resource Allocation for D2D Video Transmission

In order to improve the overall video quality of a D2D video transmission system with multicarrier waveforms, it is essential to properly assign subcarriers to D2D pairs, and control their transmission power. Centralized resource allocation for D2D systems was investigated in [4, 22, 23, 24]. These papers aimed to maximize the weighted sum rate based on the capacity of the additive white Gaussian noise (AWGN) channel, by assuming that the GA was valid for the interference. In addition, [4, 22, 23, 24] divided the spectrum into frequency-flat subcarriers and restricted every D2D user’s access to the spectrum of a single subcarrier. However, to fully exploit frequency diversity, D2D users could be assigned to multiple subcarriers. A broader class of these physical layer optimization problems was investigated in [25] as a dynamic spectrum management (DSM) problem, and was shown to be non-convex. The convexity of several extensions for the DSM problem was investigated in [26]. Among these extensions discussed in [26], for the case of multiple users with a single subcarrier, the minimization of a harmonic utility function, which took the capacity of AWGN channel as the argument, was shown to be convex. For the multicarrier, multiuser setting, the discrete DSM problem was shown to be non-convex and thereby to be NP-hard [26].

A practical wireless channel usually experiences fading, so this AWGN model is only applicable when the BS, which is usually the resource allocator, is able to collect the instantaneous channel state information (CSI) in a timely manner. We are concerned with situations in which the base station only knows average CSI and
not instantaneous CSI. This might arise for some users because the coherence time is small, or because there are too many users (for example, large crowds in an airport or amusement park). In either case, the overhead for transmitting instantaneous CSI is burdensome. Average CSI is known at the BS either by letting D2D receivers report the average channel gains to the BS, or letting the BS estimate the average channel gains, which includes path loss and shadowing, according to location information of D2D users. This differs from [4, 22, 23, 24], in which the weighted sum rate is maximized based on the capacity of the AWGN channel.

For video delivery with delay constraints, forward error correction (FEC)-based transmission with no retransmission is usually used [27]. Since video quality is dependent on both the rate-distortion function of the video and the error resilience of the video codec [28], the system optimization must involve not only the video encoding rate but also the error probability of the received bits, which demands a low packet loss rate [29] or a low symbol error rate [30, 31]. A number of articles in the literature have studied cross-layer design for video transmission in multi-carrier communication systems, e.g. [30], in which an iterative resource allocation algorithm for video transmission on cellular uplinks was proposed, and every subcarrier was assigned to a single user. However, for D2D systems, a subcarrier can be assigned to multiple users to improve spectral efficiency. An extension of the iterative algorithm to a fast fading environment, including the pilot design, was given in [31]. The algorithm designs for video quality optimization in [30] and [31] are subject to an appropriate SER target, so that the video performance is not corrupted by the excessive number of errors in transmission. Literature on resource allocation for D2D video delivery included optimization for energy efficiency, subject to a QoS constraint [32], and for a
utility function that was jointly determined by the throughput and transmission power [33]. Optimization for overall video quality of D2D transmission does not appear to have been addressed.

This dissertation is organized as follows. Chapter 2 describes the system model. Chapter 3 derives an analytical expression for the SER of D2D receivers under CCI. For block fading channels, I propose an iterative resource allocation algorithm that uses this proposed SER expression. In Chapter 4, I propose a subcarrier assignment and power control algorithm for D2D video transmission in Rayleigh fading channels, and study the condition under which the cross-layer power control problem is convex. Chapter 5 contains the conclusion.
Chapter 2

System Model

2.1 Physical Layer Model

We consider a single cell D2D video transmission system served by a BS. There are $K$ pairs of D2D users in the cell. Each D2D pair consists of a transmitter and a receiver, with a D2D link from the transmitter to the receiver, as shown in Fig. 1.2. The system has a total frequency band of $W$ (Hz), which is equally divided into $M_c$ subcarriers, exclusively available to these D2D pairs [5].

2.1.1 Transceiver Architecture

The transceiver adopts a multicarrier architecture, whose block diagrams are given in Fig. 2.1. The FEC-protected video bit sequences are mapped to the subcarriers as complex-valued modulated symbol sequences at a symbol rate of $1/T$. The signal on the $m$-th subcarrier of the $k$-th transmitter is modulated by $M_{k,m}$-ary QAM, where the $\sqrt{M_{k,m}}$ are positive even integers. The $s$-th complex modulated symbol on the $m$-th subcarrier of the $k$-th transmitter is denoted by $X_{k}^{(m)}[s]$, where
\[ \text{Var}[X_k^{(m)}[s]] = E[|X_k^{(m)}[s]|^2]/2 \] is normalized to unity. The transmitted pulse is denoted by \( \hat{g}(t) \), and the pulse energy is normalized to unity, i.e. \( \int_{-\infty}^{\infty} [\hat{g}(t)]^2 = 1 \). The lowpass equivalent signal for the \( k \)-th D2D transmitter is given by

\[
x_k(t) = \sum_{m=1}^{M_c} \sqrt{p_{k,m}T} \sum_s X_k^{(m)}[s] \hat{g}(t - sT) e^{j2\pi f_m t}, \quad k = 1, 2, \ldots, K
\]

where \( p_{k,m} \) is the transmission power on the \( m \)-th subcarrier of the \( k \)-th D2D transmitter, and the central frequency of the \( m \)-th subcarrier is denoted by \( f_m = mF_0 \), where \( F_0 \triangleq W/M_c \) and \( m = 1, 2, \ldots, M_c \). Let the frequency response of \( \hat{g}(t) \) be \( \hat{G}(f) \). The cascade of the pulse shaping and matched filter \( G(f) = |\hat{G}(f)|^2 \) belongs to a
parametric family of Nyquist pulses [34, 5(a)], given by

\[
G(f) = \begin{cases} 
T, & 0 \leq f \leq \frac{1-\beta}{2T}, \\
TG \left( \gamma_r \left[ f - \frac{1-\beta}{2T} \right]^r \right), & \frac{1-\beta}{2T} < f \leq \frac{1}{2T}, \\
T \left\{ 1 - G \left( \gamma_r \left[ \frac{1+\beta}{2T} - f \right]^r \right) \right\}, & \frac{1}{2T} < f \leq \frac{1+\beta}{2T}, \\
0, & f > \frac{1+\beta}{2T}, 
\end{cases}
\] (2.2)

where \( r \) is a positive, finite integer, \( 0 < \beta \leq 1, \gamma_r \triangleq (2T/\beta)^r G^{-1}(1/2), \) and \( G(f) \) is a continuous function that possesses derivatives of all orders and satisfies \( G(0) = 1 \). The pulse has conjugate symmetry in the frequency domain, namely \( G(f) = G^*(-f) \). The conjugate frequency symmetry implies that the time pulse of \( G(f) \), denoted by \( g(t) \), is real valued. This parametric family of pulses in (2.2) includes many pulses that are candidates for 5th generation communications [35], such as the raised-cosine pulse [36], obtained by setting \( r = 1, \gamma_r = \pi T/(2\beta) \) and \( G(f) = \cos^2(f) \), and the conjugate-root pulses [34, 37], obtained by setting \( r = 1, \gamma_r = T/\beta \) with \( G(f) = \cos^2(\pi f/2) \) or \( G(f) = \cos^2((\pi/2)f^4(35 - 84f + 70f^2 - 20f^3)) \). The bandwidth of \( G(f) \) is \((1+\beta)/(2T)\).

The system adopts a FMT waveform [9], in which the spectra of adjacent subcarriers do not overlap, namely \((1+\beta)/T \leq W/M_c\), so inter-carrier interference does not exist. Since \( G(f) \) is a Nyquist pulse, inter-symbol interference does not exist if there is perfect bit synchronization.
2.2 Application Layer Model

2.2.1 Scalable Video Codec

The D2D videos are encoded with the scalable video coding extension of H.264/AVC with medium granular scalability (MGS) [38]. The video bitstream is organized in the unit of group of pictures (GOP). A short GOP has better protection against error propagation, and using long GOPs improves the compression rate. In each bitstream, the most important video information is conveyed by the base layer, and other video information of diminishing importance is contained in the successive enhancement layers. With a scalable video codec, the decoder only needs a portion of the encoded bitstream to display the video. The decoded fidelity of the video depends on the length of the bitstream that is correctly received, as well as the rate distortion characteristics of the video content. The mean square error (MSE) of the video diminishes as more enhancement layers are received. The video packets are transmitted in the order of descending priority. If an error occurs during transmission, that packet and all successive packets in the GOP will be dropped, but previous packets will be used for decoding the GOP.

2.2.2 Video Rate-distortion Characteristics

The rate-distortion (RD) function of the video characterizes the tradeoff between the video distortion and the number of bits used to compress the raw video data. Since the video is compressed on a GOP-by-GOP basis, this RD function is also measured for each GOP. The MSE distortion $D_k$ can be approximated as a function
of the number of correctly received video bits $B_k$ in the GOP, written as [28]

$$MSE_k = a_k + \frac{b'_k}{B_k + c'_k},$$

(2.3)

where $a_k$, $b'_k$, and $c'_k$ are positive constants determined by curve fitting and are dependent on video content. The system is assumed to transmit one GOP in each time slot, thus $T_{GOP}$ equals the duration of the time slot. The video bits are protected by a FEC code with fixed rate $u$. The average number of coded bits per symbol transmitted on the $m$-th subcarrier of the $k$-th link is $R_{k,m}$, and $R_{k,m}$ can be zero for some $k$ and $m$. The number of video bits $(B_k)$ transmitted on the $k$-th link in a time slot is given by

$$B_k = \frac{uT_{GOP}}{T_0} \sum_{m=1}^{M_c} R_{k,m}.$$  

(2.4)

For simplicity, define $\tilde{b}_k \triangleq \frac{b'_k T_0}{(uT_{GOP})}$, $\tilde{c}_k \triangleq \frac{c'_k T_0}{(uT_{GOP})}$. Substituting (2.4) into (2.3), the MSE of the GOP at the video decoder can be further written as

$$MSE_k = a_k + \frac{\tilde{b}_k}{\sum_{m=1}^{M_c} R_{k,m} + \tilde{c}_k}.$$  

(2.5)
Chapter 3

Exact Symbol Error Rate Analysis for D2D systems and Its Application in Resource Allocation

In this chapter, we use a block fading channel model, in which channel gains are assumed to be constant in the duration of a GOP, and are independent over different GOPs. At the beginning of each GOP, the BS collects instantaneous CSI and the RD information from D2D pairs, and assigns subcarriers and allocates transmission to D2D pairs.

An exact expression for the SER of D2D receivers under CCI is first derived, and it is compared with that by using a GA for CCI. Then an iterative resource allocation algorithm for D2D video transmission that uses the proposed SER expression is proposed. Finally, the performance gain by using the proposed SER expression over that using GA is shown by simulations.
3.1 Interference at D2D Receivers

The interference from the $i$-th transmitter arrives at the $k$-th receiver with a channel gain of $|h_{i,k}^m|$, a time delay $\tau_{i}^{(k)}$ and a phase delay $\phi_{i}^{(k)}$ compared to the desired signal from the $k$-th transmitter ($i, k = 1, 2, \cdots, K$), where $\phi_{i}^{(k)}$ is uniformly distributed in $[0, 2\pi)$ and $\tau_{i}^{(k)}$ modulo $T$ is uniformly distributed in $[0, T)$, if $i \neq k$. Also, $|\tau_{i}^{(k)}| \ll T_{\text{GOP}}$. A coherent receiver is assumed to be implemented, and the receiver maintains perfect bit synchronization and phase recovery for the desired signal, so we set $\tau_{k}^{(k)} = 0$ and $\phi_{k}^{(k)} = 0$. In addition, a quasi-static channel model for D2D is considered [4, 22], so the channel gain $|h_{i,k}^m|$ and the transmission power are assumed to be unchanged for the duration of a GOP. Therefore, the lowpass equivalent signal at the $k$-th receiver is given by

$$y_k(t) = \sum_{m=1}^{M_c} \sum_{i=1}^{K} \sqrt{P_i^{(k,m)} T \sum_s X_k^{(m)}[s] \hat{g}(t - \tau_{i}^{(k)} - sT) e^{j(2\pi f_m t + \phi_{i}^{(k)})} + n_k(t)}, \quad (3.1)$$

where $P_i^{(k,m)} = p_{i,m} |h_{i,k}^m|^2$ is the power of the $i$-th signal received on the $m$-th subcarrier of the $k$-th D2D receiver prior to demodulation, and $n_k(t)$ is complex AWGN at the $k$-th receiver with two-sided power spectral density $N_0$. As shown in Fig. 2.1, a matched filter is used at subcarrier $m$ of the $k$-th receiver, so the output for the $s$-th symbol is given by

$$Y_k^{(m)}[s] = \int_{-\infty}^{\infty} y_k(t) \hat{g}^*(t - sT) \exp(-j2\pi f_m t) dt. \quad (3.2)$$

Without loss of generality, the reception of the 0-th symbol is considered. The
The decision statistic for the 0-th symbol at the output of the matched filter is given by

\[ Y_{k}^{(m)}[0] = \sqrt{P_{k}^{(k,m)}} T X_{k}^{(m)}[0] + \sum_{i=1, i \neq k}^{K} \sqrt{P_{i}^{(k,m)}} T e^{j \phi_{i}^{(k)}} \sum_{s} X_{i}^{(m)}[s] g(-\tau_{i}^{(k)} - sT) + N_{k}^{(m)}[0]. \]  

In (3.3), the first term is the desired signal component, and the second term represents the CCI components. The noise term is given by

\[ N_{k}^{(m)}[s] = \int_{-\infty}^{\infty} n_{w}(t) \hat{g}(t - sT) dt, \]

which is a zero-mean circularly symmetric complex Gaussian random variable (RV) with power

\[ P_{N} = N_{0} \int_{-\infty}^{\infty} [\hat{g}(t)]^{2} dt = N_{0}. \]

Therefore, the signal-to-noise power ratio (SNR) and the signal-to-interference power ratio (SIR) on the \( m \)-th subcarrier of the \( k \)-th D2D receiver can be expressed as

\[ \text{SNR}_{k,m} = 10 \log_{10} \left[ \frac{p_{k,m} |h_{m}^{k,k}|^{2} T}{N_{0}} \right], \]

and

\[ \text{SIR}_{k,m} = 10 \log_{10} \left[ \frac{p_{k,m} |h_{m}^{k,k}|^{2}}{\sum_{i=1, i \neq k}^{K} p_{i,m} |h_{m}^{i,k}|^{2}} \right]. \]
3.2 Symbol Error Rate for D2D Receivers

This section considers the SER on the $m$-th subcarrier of the $k$-th D2D receiver. In this section, the indices $m$ and $k$ in the superscripts of variables are omitted for simplicity of notation unless otherwise stated. The aggregated CCI in the decision statistics for the $m$-th subcarrier of the $k$-th D2D receiver in (3.3) is denoted by

$$ I = \sum_{i=1,i\neq k}^{K} I_i, $$

(3.8)

where the CCI from the $i$-th transmitter is represented by

$$ I_i = \sqrt{P_i T} e^{j\phi_i} \sum_{s=-\infty}^{\infty} X_i[s] g(-\tau_i - sT). $$

(3.9)

3.2.1 SER Evaluation by Gaussian Approximation for the CCI

The GA treats the aggregated CCI as a zero-mean circularly complex Gaussian RV with the same variance. Since $E(I_i) = 0$, it is well known that $\text{Var}(I_i) = E(|I_i|^2)/2$. The variance of the aggregated CCI is given by

$$ \text{Var}(I) = \sum_{i=1,i\neq k}^{K} \text{Var}(I_i) = \sum_{i=1,i\neq k}^{K} P_i T \cdot E \left[ \sum_{s=-\infty}^{\infty} g^2(-\tau_i - sT) \right] = \gamma T \sum_{i=1,i\neq k}^{K} P_i, $$

(3.10)

where $\gamma$ is defined by

$$ \gamma \triangleq E \left[ \sum_{s=-\infty}^{\infty} g^2(-\tau_i - sT) \right]. $$

(3.11)
Invoking the well-known Poisson summation formula [39, p.47, (3-56)], \( \gamma \) is given by

\[
\gamma = E \left[ \frac{1}{T} \sum_{n=-\infty}^{\infty} G_2 \left( \frac{n}{T} \right) e^{jn \pi T} \right] = \frac{G_2(0)}{T} = \frac{1}{T} \int_{-\infty}^{\infty} |G(f)|^2 df, \tag{3.12}
\]

where the Fourier transform of \( g^2(t) \) is denoted by \( G_2(f) \), which equals \( G(f) \ast G(f) \), and \( \ast \) stands for convolution. Specially, \( \gamma = 1 - \beta/4 \) for a raised-cosine pulse with roll-off factor \( \beta \) [13, (55)].

With the GA, the SER evaluation for QAM in CCI plus AWGN resembles the well-known SER expression in AWGN [40], which depends on the signal-to-interference-plus-noise ratio (SINR). The SER of \( M_{k,m} \)-ary QAM under the CCI using the GA is thus given by

\[
\text{SER}_{k,m}^{GA} = 4 \left( 1 - \frac{1}{\sqrt{M_{k,m}}} \right) Q \left( \sqrt{\frac{3 \text{SINR}_{k,m}}{M_{k,m} - 1}} \right) - 4 \left( 1 - \frac{1}{\sqrt{M_{k,m}}} \right)^2 Q^2 \left( \sqrt{\frac{3 \text{SINR}_{k,m}}{M_{k,m} - 1}} \right), \tag{3.13}
\]

where the received SINR is given by

\[
\text{SINR}_{k,m} = \frac{p_{k,m} |h_{m}^{k,k}|^2}{\gamma \sum_{i=1, i \neq k}^{K} p_{i,m} |h_{m}^{i,k}|^2 + N_0/T}. \tag{3.14}
\]

### 3.2.2 Characteristic Function of CCI

To demonstrate the inaccuracy of the GA in the evaluation of the SER for multicarrier D2D systems, we derive the exact expression for the SER of the bandlimited QAM under CCI and AWGN. We start with the characteristic function (CF) of the CCI. The minimum distance between two adjacent constellation points of the \( i \)-th
QAM signal at the $k$-th D2D receiver is denoted by $2A_i$. Since $P_i$ is the power of the $i$-th QAM signal at the receiver, $A_i$ is given by [17, (19)]

$$A_i = \sqrt{\frac{3P_i T}{M_{i,m} - 1}}. \quad (3.15)$$

We also denote the in-phase and quadrature components of the $i$-th CCI signal by $I_i^I = \text{Re}(I_i)$ and $I_i^Q = \text{Im}(I_i)$. To derive the joint CF of $I_i^I$ and $I_i^Q$, we first consider their conditional joint CF, given $\phi_i$ and $\tau_i$, which is presented in Proposition 1.

**Proposition 1.** Given $\tau_i$ and $\phi_i$, the conditional joint CF of $I_i^I$ and $I_i^Q$ is

$$\varphi_{I_i^I,I_i^Q|\tau_i,\phi_i}(u,v|\tau_i,\phi_i) = \prod_{s_1=-\infty}^{\infty} \frac{2}{\sqrt{M_{i,m}}/2} \sum_{j_1=1}^{M_{i,m}/2} \cos\left(A_i(u \cos \phi_i + v \sin \phi_i)(2j_1 - 1)g(-\tau_i - s_1 T)\right)$$

$$\cdot \prod_{s_2=-\infty}^{\infty} \frac{2}{\sqrt{M_{i,m}}/2} \sum_{j_2=1}^{M_{i,m}/2} \cos\left(A_i(v \cos \phi_i - u \sin \phi_i)(2j_2 - 1)g(-\tau_i - s_2 T)\right). \quad (3.16)$$

**Proof.** See Appendix A. 

Averaging over $\phi_i$ and $\tau_i$, we obtain the joint CF of $I_i^I$ and $I_i^Q$,

$$\varphi_{I_i^I,I_i^Q}(u,v) = \frac{1}{2\pi T} \int_0^T \int_0^{2\pi} \varphi_{I_i^I,I_i^Q|\tau_i,\phi_i}(u,v|\tau_i,\phi_i) d\phi_i d\tau_i. \quad (3.17)$$

Using the assumption that all CCI signals are independent and $I_i^I = \sum_{i=1,i \neq k}^{K} I_i^I$, $I_i^Q = \sum_{i=1,i \neq k}^{K} I_i^Q$, the joint CF of the in-phase and the quadrature components of the
aggregated CCI is given by

$$\varphi_{I^I, I^Q}(u, v) = \prod_{i=1, i \neq k}^{K} \varphi_{I^I_i, I^Q_i}(u, v).$$

(3.18)

Substituting (3.16) and (3.17) into (3.31), the CF of the aggregated CCI can be obtained in an integral form. However, the integral in (3.17) does not appear to have a closed-form expression due to the implicit dependence on the pulse shape. To evaluate an integral with this form, [13] numerically integrates the result without providing an analytical expression for the error rate. In contrast, the analysis in this paper proceeds by expanding $\varphi_{I^I_i, I^Q_i}(u, v)$ in a power series, averaging over $\tau_i, \phi_i$ term by term, and finally deriving an analytical expression for the error rate in the form of a power series. The convergence analysis for the resulting power series is also provided in a later section.

Proposition 2 presents a power series expansion for the joint CF of the in-phase and quadrature components of the aggregated CCI.

**Proposition 2.** The power series expansion of the joint CF of $I^I$ and $I^Q$ is given by

$$\varphi_{I^I, I^Q}(u, v) = \sum_{n=0}^{\infty} b_n^{(k,m)} (-N_0)^n (u^2 + v^2)^n,$$

(3.19)

where $b_0^{(k,m)} = 1$ and a recursive relation of $b_n^{(k,m)}$ for any positive integer $n$ is given by

$$b_n^{(k,m)} = \sum_{p=1}^{n} \frac{p}{n} b_{n-p}^{(k,m)} \sum_{i=1, i \neq k}^{K} \left( \frac{P_i^{(k,m)} T}{N_0} \right)^p d_p^{(M_i, m)}.$$

(3.20)

In (3.20), $d_0^{(M_i, m)} = 0$ and the following recursive relationship holds for any positive
integer $n$:

$$d_n^{(M,m)} = \alpha_n^{(M,m)} - \sum_{p=1}^{n-1} \frac{p}{n} d_p^{(M,m)} \alpha_{n-p}^{(M,m)},$$

(3.21)

where the $\{\alpha_n^{(M,m)}\}$ are the coefficients of the power series by expanding the joint CF of $I_i^I$ and $I_i^Q$, namely

$$\varphi_{I_i^I,I_i^Q}(u, v) = \sum_{n=0}^{\infty} \alpha_n^{(M,m)} [-P_{i}^{(k,m)} T(u^2 + v^2)]^n.$$

(3.22)

The expression for $\{\alpha_n^{(M)}\}$, as a function of alphabet size $M$, is given by

$$\alpha_n^{(M)} = \left(\frac{3}{M-1}\right)^n \sum_{p=0}^{n} B_{2p,2(n-p)} \sum_{q=0}^{2n} \sum_{q_1=0}^{q} B_{q_1,q-q_1} \sum_{q_2=\max(0,q+2p-2n)}^{\min(q,2p)} \sum_{q_3=\max(0,q_1+q_2-q)}^{\min(q_1,q_2)} u_p^{(M)}(q_2,q_3) w_{n-p}^{(M)}(q-q_2,q_1-q_3),$$

(3.23)

where $B_{m,n} = (m-1)!! (n-1)!!/[(m+n)!!]$ if $m \geq 0$, $n \geq 0$, $m,n$ are both even, and $B_{m,n} = 0$ otherwise. $(2n-1)!! = 1 \cdot 3 \cdots (2n-1)$, $(2n)!! = 2 \cdot 4 \cdots 2n$ for any positive integer $n$, and $0!! = (-1)!! = 1$.

For $n \geq 1$, $0 \leq q_1 \leq q \leq 2n$, the coefficients $\{w_n^{(M)}(q,q_1)\}$ in (3.23) are given by

$$w_n^{(M)}(q,q_1) = \sum_{p=1}^{n} \frac{p}{n} c_p^{(M)} \sum_{q_2=\max(0,q+2p-2n)}^{\min(q,2p)} \sum_{q_3=\max(0,q_1+q_2-q)}^{\min(q_1,q_2)} R_p(q_2,q_3) w_{n-p}^{(M)}(q-q_2,q_1-q_3),$$

(3.24)

where the initial condition is given by $w_0^{(M)}(0,0) = 1$, and the $\{c_n^{(M)}\}$ are given by
\[ c_0^{(M)} = 0 \text{ and } c_n^{(M)} = a_n^{(M)} - \sum_{p=1}^{n-1} \left( \frac{p}{n} \right) c_p^{(M)} a_{n-p}^{(M)} \text{ for } n = 1, 2, \cdots, \]

where the \( \{ a_n^{(M)} \} \) are given by

\[
a_n^{(M)} = \frac{1}{(2n)!} \frac{2}{\sqrt{M}} \frac{\sqrt{M/2}}{\sum_{l=1}^{\infty} (2l - 1)^2}.
\]

Lastly, the \( \{ R_n(s, s_1) \} \) in (3.24) are defined by

\[
R_n(s, s_1) = \begin{cases} 
\frac{2 - \delta_s}{T} \text{Re}\left[ G_{2n} \left( \frac{s}{T} \right) \right] \left( \begin{array}{c} s \\ s_1 \end{array} \right) (-1)^{s_1/2}, & \text{if } s_1 \text{ even, } 0 \leq s_1 \leq s \leq 2n, \\
\frac{2}{T} \text{Im}\left[ G_{2n} \left( \frac{s}{T} \right) \right] \left( \begin{array}{c} s \\ s_1 \end{array} \right) (-1)^{(s_1+1)/2}, & \text{if } s_1 \text{ odd, } 0 \leq s_1 \leq s \leq 2n, \\
0, & \text{otherwise}, 
\end{cases}
\]

where the Fourier transform of \([g(t)]^{2n}\) is denoted by

\[
G_{2n}(f) = \int_{-\infty}^{\infty} [g(t)]^{2n} \exp(j2\pi ft) \, dt,
\]

and \( \delta_s \) is the Kronecker delta function.

Proof. See Appendix B.

Among the coefficients in Proposition 2, only \( \{ b^{(k,m)}_n \} \) need to be calculated online using (3.20). The other coefficients are determined only by the alphabet size and pulse shape, and can be calculated offline.
3.2.3 Exact SER in the Form of a Power Series

We use Proposition 2 to derive the exact expression for the SER. The in-phase and quadrature components of the noise in the decision statistics are denoted by $n^I = \text{Re}(N_k[0])$ and $n^Q = \text{Im}(N_k[0])$. Let the sum of the aggregated CCI and the noise be $Z = I + N_k[0]$. We also denote $I^I = \text{Re}(I)$, $I^Q = \text{Im}(I)$, $Z^I = \text{Re}(Z) = I^I + n^I$, and $Z^Q = \text{Im}(Z) = I^Q + n^Q$. The event that $Z^I$ and $Z^Q$ both fall in the range $(-A_k, A_k)$ is denoted by $B_1$, whose probability is written as

$$\Pr(B_1) = \Pr(|Z^I| < A_k, |Z^Q| < A_k).$$  \hfill (3.28)$$

Let $B_2$ be the event that $Z^I$ falls in the range $(-A_k, A_k)$, whose probability is given by

$$\Pr(B_2) = \Pr(|Z^I| < A_k).$$  \hfill (3.29)$$

One can see from (3.9) that the CCI in the decision statistics is a complex circularly symmetric RV, since the phase shifts $\{\phi_i\}$ are assumed to be uniformly distributed in $[0, 2\pi)$. Also recall that the noise $N_w[0]$ is a complex circularly symmetric RV. Therefore, $Z = I + N_k[0]$ is a complex circularly symmetric RV. Making use of the circular symmetry of $Z$, the constellation points of the transmitted desired signal can be classified into three categories [18]: 1) points at the four corners; 2) points at the edge of the constellation but not corners; 3) interior points. We denote the event of the transmitted symbol being in Category 1, 2 and 3 by $C_1$, $C_2$ and $C_3$, respectively. Let $c$ be the event of a symbol being correctly received. Making use of the circular
symmetry of $Z$, the expressions for $\Pr(c|C_j)$ are given by

$$\Pr(c|C_j) = \begin{cases} 
\Pr(Z^I < A_k, Z^Q < A_k) &= \frac{1}{4} + \frac{1}{4} \Pr(B_1) + \frac{1}{2} \Pr(B_2), \\
\Pr(Z^I < A_k, |Z^Q| < A_k) &= \frac{1}{2} \Pr(B_1) + \frac{1}{2} \Pr(B_2), \\
\Pr(|Z^I| < A_k, |Z^Q| < A_k) &= \Pr(B_1), 
\end{cases} \quad (3.30)$$

Since each constellation point is assumed to appear with equal probability, we obtain

$$\Pr(C_1) = \frac{4}{M_{k,m}}, \quad \Pr(C_2) = 4(\sqrt{M_{k,m}} - 1)/M_{k,m} \quad \text{and} \quad \Pr(C_3) = (\sqrt{M_{k,m}} - 2)^2/M_{k,m}.$$ 

The SER is thus given by

$$\text{SER}_{k,m} = 1 - \sum_{j=1}^{3} \Pr(C_j) \cdot \Pr(c|C_j)$$

$$= 1 - \frac{1}{M_{k,m}} \left[ (\sqrt{M_{k,m}} - 1)^2 \Pr(B_1) + 2(\sqrt{M_{k,m}} - 1) \Pr(B_2) + 1 \right]. \quad (3.31)$$

Invoking [41, p.101, (10.6.2)] yields

$$\Pr(|Z^I| < A_k, |Z^Q| < A_k) = \frac{1}{\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\sin(A_k u) \sin(A_k v)}{u} \varphi_{Z^I,Z^Q}(u, v) \, du \, dv,$$ 

$$\varphi_{Z^I,Z^Q}(u, v) = \varphi_{I^n,Q^n}(u, v) \varphi_{n^n,I^n}(u, v),$$

where the joint CF of $Z^I, Z^Q$ is given by $\varphi_{Z^I,Z^Q}(u, v) = \varphi_{I^n,Q^n}(u, v) \varphi_{n^n,I^n}(u, v)$, since the interference and noise are independent. The joint CF of $n^I$ and $n^Q$ is $\varphi_{n^n,I^n}(u, v) = \exp[-N_0(u^2 + v^2)/2]$ since $n^I, n^Q$ are i.i.d zero-mean jointly Gaussian RVs with variance $N_0$. Substituting (3.19) and (3.32) into (3.28) yields

$$\Pr(B_1) = \frac{1}{\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\sin(A_k u) \sin(A_k v)}{u} \left[ \sum_{n=0}^{\infty} b_{n,k,m} (-N_0)^n (u^2 + v^2)^n \right] e^{-N_0(u^2+v^2)/2} \, du \, dv.$$ 

$$\quad (3.33)$$
Using the binomial theorem to expand \((u^2 + v^2)^n\), (3.33) can be rewritten as

\[
\Pr(B_1) = \sum_{n=0}^{\infty} \sum_{p=0}^{n} b_n^{(k,m)} (-N_0)^n \left( \frac{n}{p} \right) \int_{-\infty}^{\infty} \sin(A_k u) u^{2p-1} e^{-\frac{u^2 N_0}{2}} \, du \\
\cdot \int_{-\infty}^{\infty} \sin(A_k v) v^{2(n-p)-1} e^{-\frac{v^2 N_0}{2}} \, dv.
\]  

(3.34)

From [42, 3.952 (6)] and [42, 3.952 (10)], respectively, we obtain

\[
\int_{-\infty}^{\infty} \sin(A_k u) u^{-1} e^{-\frac{u^2 N_0}{2}} \, du = \pi \left[ 1 - 2Q \left( \frac{A_k}{\sqrt{N_0}} \right) \right]
\]  

(3.35)

and

\[
\int_{-\infty}^{\infty} \sin(A_k u) u^{2l-1} e^{-\frac{u^2 N_0}{2}} \, du = (-1)^{l-1} \frac{2\sqrt{\pi}}{N_0^l} e^{-\frac{A_k^2}{2N_0}} H_{2l-1} \left( \frac{A_k}{\sqrt{2N_0}} \right)
\]  

(3.36)

for positive integer \(l\), where \(H_n(x)\) is the Hermite polynomial of the \(n\)-th order with the following recursive relation [42, 8.95]: \(H_0(x) = 1, H_1(x) = 2x\), and for \(n = 2, 3, \ldots \)

\[
H_{n+1}(x) = 2x H_n(x) - 2n H_{n-1}(x).
\]  

(3.37)

Therefore, (3.34) reduces to

\[
\Pr(B_1) = \left[ 1 - 2Q \left( \frac{A_k}{\sqrt{N_0}} \right) \right]^2 - \frac{4}{\sqrt{\pi}} \left[ 1 - 2Q \left( \frac{A_k}{\sqrt{N_0}} \right) \right] e^{-\frac{A_k^2}{2N_0}} \sum_{l=1}^{\infty} b_l^{(k,m)} H_{2l-1} \left( \frac{A_k}{\sqrt{2N_0}} \right) \\
+ \frac{4}{\pi} e^{-\frac{A_k^2}{2N_0}} \sum_{n=1}^{\infty} \sum_{p=1}^{n-1} b_n^{(k,m)} \left( \frac{n}{p} \right) H_{2p-1} \left( \frac{A_k}{\sqrt{2N_0}} \right) H_{2(n-p)-1} \left( \frac{A_k}{\sqrt{2N_0}} \right).
\]  

(3.38)

Using a similar procedure, and applying [41, p.101, (10.6.2)] to (3.29), \(\Pr(B_2)\)
is given by

\[
\Pr(B_2) = \Pr\left( |Z|^2 < A_k \right) \\
= \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\sin(A_k u)}{u} \varphi_{Z} (u) \, du \\
= \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\sin(A_k u)}{u} \varphi_{I, IQ} (u, 0) e^{-u^2N_0/2} \, du \\
= \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\sin(A_k u)}{u} \left[ \sum_{l=0}^{\infty} b_l^{(k,m)} (-N_0) u^{2l} \right] e^{-u^2N_0/2} \, du, \tag{3.39}
\]

where the third equality is due to \( \varphi_{I, IQ} (u) = E[e^{j(uI^2 + uIQ)}]|_v=0 = \varphi_{I, IQ} (u, 0) \). Similar to \( \Pr(B_1) \), (3.39) reduces

\[
\Pr(B_2) = 1 - 2Q \left( \frac{A_k}{\sqrt{N_0}} \right) - \frac{2}{\sqrt{\pi}} e^{-\frac{A_k^2}{2N_0}} \sum_{l=1}^{\infty} b_l^{(k,m)} H_{2l-1} \left( \frac{A_k}{\sqrt{2N_0}} \right). \tag{3.40}
\]

Substituting (3.38) and (3.40) into (3.31), after some manipulations and denoting

\[
\nu_{k,m} \triangleq A_{k,m} = \sqrt{\frac{3p_{k,m} |h_{m,k}|^2 T}{2N_0(M_{k,m} - 1)}}, \tag{3.41}
\]

\[
\mu_{k,m} \triangleq 1 - \frac{1}{\sqrt{M_{k,m}}}, \tag{3.42}
\]

the exact SER on the \( m \)-th subcarrier of the \( k \)-th D2D receiver is given by

\[
\text{SER}^{\text{exact}}_{k,m} = 4\mu_{k,m} Q(\sqrt{2\nu_{k,m}}) - 4\mu_{k,m}^2 Q(\sqrt{2\nu_{k,m}})^2 \\
+ \frac{4\mu_{k,m}^2}{\sqrt{\pi}} e^{-\nu_{k,m}^2} \left[ 1 - 2\mu_{k,m} Q(\sqrt{2\nu_{k,m}}) \right] \sum_{l=1}^{\infty} b_l^{(k,m)} H_{2l-1}(\nu_{k,m}) \\
- \frac{4\mu_{k,m}^2}{\sqrt{\pi}} e^{-2\nu_{k,m}^2} \sum_{n=1}^{\infty} \sum_{p=1}^{n-1} b_n^{(k,m)} \binom{n}{p} H_{2p-1}(\nu_{k,m}) H_{2(n-p)-1}(\nu_{k,m}).
\]
In practice, only a finite number of terms from the power series of the CF of CCI are used in the SER calculation. Retaining the terms with order less than $2L$ from $\varphi_{M,Iq}(u,v)$ in the evaluation of the SER, where $L$ is a positive integer, one can formally define the truncated evaluation for SER as

$$
\text{SER}^{(2L)}_{k,m} = 4\mu_{k,m}Q(\sqrt{2}\nu_{k,m}) - 4\mu_{k,m}^2 \left[Q(\sqrt{2}\nu_{k,m})\right]^2
+ \frac{4\mu_{k,m}}{\sqrt{\pi}} e^{-\nu_{k,m}^2} \left[1 - 2\mu_{k,m}Q(\sqrt{2}\nu_{k,m})\right] \sum_{l=1}^{L-1} b_{l}^{(k,m)} H_{2l-1}(\nu_{k,m})
- \frac{4\mu_{k,m}^2}{\pi} e^{-2\nu_{k,m}^2} \sum_{n=1}^{L-1} \sum_{p=1}^{n-1} b_{n}^{(k,m)} \left(\begin{array}{c} n \\ p \end{array}\right) H_{2p-1}(\nu_{k,m}) H_{2(n-p)-1}(\nu_{k,m}),
$$

(3.43)

where the coefficients $\{b_{n}^{(k,m)}\}$ are given in (3.20). If CCI is absent, then $b_{n}^{(k,m)} = 0$ for $n = 1, 2, \cdots$, thereby reducing (3.43) to the SER for $M_{k,m}$-ary QAM in AWGN [40]. By comparing $\text{SER}^{(2L)}_{k,m}$ in (3.43) and $\text{SER}^{\text{exact}}_{k,m} = \lim_{L \to \infty} \text{SER}^{(2L)}_{k,m}$, a truncation error is introduced. The truncation error is denoted by $e(2L) = |\text{SER}^{\text{exact}}_{k,m} - \text{SER}^{(2L)}_{k,m}|$.

Appendix C gives an upper bound for the truncation error in (C.19), which is repeated below:

$$
e(2L) < \frac{1}{M_{k,m}} \sqrt{\frac{6p_{k,m} |h_{k,m}^{k,k}|^2}{\pi N_0/T}} \left(\sum_{i=1, i \neq k}^{K} \sqrt{\frac{6p_{i,m} |h_{i,m}^{k,k}|^2}{N_0/T}}\right)^{2L}
\cdot \left(\sqrt{\frac{6p_{k,m} |h_{k,m}^{k,k}|^2}{\pi N_0/T}} \frac{1}{(2L-1)!!} + \frac{2}{(2L)!!}\right),
$$

(3.44)

where $L$ is a positive integer, and $C = \sup_{0 < \tau < T} \left\{\sum_{s=-\infty}^{\infty} |g(\tau - sT)|\right\}$. Using the results in [34], the constant $C$ can be shown to be finite for the family of Nyquist pulses that are considered in (2.2). For example, $C = 10\sqrt{2}/(3\pi) \approx 1.5005$ for a raised-cosine pulse with roll-off factor $\beta = 0.5$, given by Proposition 4 in Appendix C.
Notice that $e(2L) \to 0$ as $L \to \infty$. Therefore, the truncated series in (3.43) converges to the exact SER as more terms are used in the calculation.

### 3.3 Iterative Cross-layer Resource Allocation

The optimization problem for resource allocation in multicarrier D2D video transmission is formulated in this section, and an iterative algorithm for subcarrier assignment and power allocation that uses the analytical result in (3.43) is then proposed.

#### 3.3.1 Problem Formulation

The objective for resource allocation is to minimize the total video MSE for all D2D pairs. The BS collects the RD information and the channel conditions of all D2D pairs, and allocates subcarriers and transmission power to D2D pairs based on both the RD functions and channel conditions.

Recall that the transmission power and the alphabet size of the $k$-th D2D pair on subcarrier $m$ are denoted by $p_{k,m}$ and $M_{k,m}$, respectively. The number of supported alphabet sizes is denoted by $N_A$ and the set of supported alphabet sizes is denoted by $\mathcal{M}_A = \{M_1, \cdots, M_{N_A}\}$, where $M_1 < M_2 < \cdots < M_{N_A}$. Only square QAM is considered in this dissertation, so the number of bits per symbol ($\log_2(M_n)$) is a non-negative even integer. We set $M_1 = 1$, which indicates that the number of bits per symbol is $\log_2 (M_1) = 0$ and corresponds to the case of no transmission.

Each D2D transmitter is subject to a total power constraint $P$. In the search procedure, the BS sets a fixed target for the SER on every D2D link and assumes
error-free transmission in the power allocation algorithm if the SER target is satisfied\(^1\).

The video MSE minimization problem is formally written as

\[
\begin{align*}
\text{minimize} & \quad \sum_{i=1}^{K} \frac{b_k}{\sum_{m=1}^{M_c} \log_2(M_{k,m}) + c_k} \\
\text{subject to} & \quad (C1) \quad \sum_{m=1}^{M_c} p_{k,m} \leq P, \quad \forall k \in \{1, 2, \cdots, K\}, \\
& \quad (C2) \quad \text{SER}_{k,m} \leq \text{SER}_{\text{target}}, \quad p_{k,m} \geq 0, \quad M_{k,m} \in \mathcal{M}_A, \\
& \quad \forall k \in \{1, 2, \cdots, K\}, \quad \forall m \in \{1, 2, \cdots, M_c\}.
\end{align*}
\]

We omit the \(\{a_k\}\) in (3.45), since \(\{a_k\}\) are constant terms in the objective function for optimization. The constraint (C1) is the total power constraint for each D2D transmitter, and (C2) is the SER constraint for each subcarrier of each D2D pair that is demanded by video delivery with no retransmission. Since constraint (C2) is not a convex set and the optimization problem in (3.45) is NP-hard, we propose an iterative algorithm as a suboptimal solution, which updates both the subcarrier assignments and the power allocation strategy based on the CSI and the RD function. The details of the proposed algorithm are given in the next section.

### 3.3.2 Proposed Iterative Resource Allocation Algorithm

Our proposed algorithm is initialized by multi-user diversity (MUD), where each subcarrier is assigned to the D2D pair with the best channel gain. Next, we consider the D2D pair which has the steepest slope on the RD curve, and attempt to assign one additional subcarrier to this chosen D2D pair. The next step is to iterate

\(^1\)This error-free assumption is only used in determining the subcarrier assignment and power allocation. In the simulation, an error may still occur even if the SER target is satisfied.
over all subcarriers and iterate over all supported alphabet sizes for the chosen D2D pair. In the iteration, the minimal transmission power of the chosen D2D pair is calculated for each supported alphabet size on the current subcarrier via a bisection search, using either the proposed SER expression (3.43) or the SER obtained by the GA (3.13). For other D2D pairs on the current subcarrier, we also run an exhaustive search based on (3.43) or (3.13) for the largest alphabet sizes that satisfy the SER target under the interference from the chosen D2D pair. Afterwards, if the total MSE decreases, and the total transmission power of the chosen D2D pair does not exceed the power constraint, we update the transmission power, alphabet sizes and subcarrier assignment. This procedure is repeated iteratively until the total MSE of the video will not decrease by assigning one more subcarrier to any D2D pair.

The following notation will be used in the algorithm.

1. Let $\sigma^{(l)}_m$ be the set of D2D pairs that are allocated with subcarrier $m$ in the $l$-th iteration;

2. The potential set $\Omega^{(l)}$ is the set of D2D pairs that are still possible to decrease the total video MSE by receiving an additional subcarrier at the $l$-th iteration.

3. $p^{(l)}_{k,m}$ and $M^{(l)}_{k,m}$ are the transmission power and alphabet size on subcarrier $m$ of the $k$-th D2D pair at the $l$-th iteration, respectively.

The details of our proposed algorithm are elaborated as follows.

**Step 1: Initialization**

The potential set $\Omega^{(0)}$ is initialized as the set of indices of all D2D pairs, i.e. $\Omega^{(0)} = \{1, 2, \cdots, K\}$. We first assign each subcarrier to the D2D pair who has the
best channel response, i.e., $\sigma_m^{(0)} = \{\arg\max_{k \in \Omega^{(0)}} (h_{m}^{k,k})\}$ for $m = 1, 2, \ldots, M_c$. After the initial subcarrier assignment, each D2D pair maximizes its own bit rate by the conventional water-filling power allocation [30], namely $\hat{p}_{k,m} = \left[1/\lambda_k - 1/(\eta|h_{m}^{k,k}|^2)\right]^+$, where $\eta = (3/N_0)[Q^{-1}(\text{SER}_{\text{target}}/4)]^{-2}$ [30] and $[x]^+ = \max(x, 0)$. The parameters $\{\lambda_k\}$ can be found numerically such that the total transmission power of each D2D pair is equal to the power constraint, i.e., $\sum_{m=1}^{M_c} \hat{p}_{k,m} = P$. Given the water-filling power allocation, the alphabet sizes for initialization are given by [30, (4)]:

$$M_{k,m}^{(0)} = \min \left(4\left[\log_4(1+\eta\hat{p}_{k,m}|h_{m}^{k,k}|^2)\right], M_{N_A}\right),$$

(3.46)

where $M_{N_A}$ is the largest alphabet size supported by the system. The initial transmission power is allocated as the minimal transmission power for alphabet size $M_{k,m}^{(0)}$, given by

$$p_{k,m}^{(0)} = \frac{M_{k,m}^{(0)} - 1}{\eta|h_{m}^{k,k}|^2}.$$  

(3.47)

The iteration indicator $l$ is initialized as $l = 0$.

**Step 2: Slope Calculation**

The slope of the RD curve of the $k$-th D2D pair is given by

$$S_k^{(l)} = -\frac{c_k}{(\sum_{m=1}^{M_c} \log_2(M_{k,m}^{(l)} + b_k)^2)}.$$  

(3.48)

In the $l$-th iteration, the index of the D2D pair with the steepest slope in the
potential set \( (\Omega^{(l)}) \) is

\[
k^* = \arg \min_{k \in \Omega^{(l)}} \{ S_k^{(l)} \}.
\] (3.49)

The BS iterates over the subcarriers that are not currently allocated to D2D pair \( k^* \), and the subcarrier index in the iteration is denoted by \( m^* \). Note that \( m^* \) is only a candidate, and the decision for subcarrier assignment is not made until Step 3.

**Step 3: Video MSE Calculation**

In this step, the BS lets the alphabet size on the current subcarrier \( m^* \) of the \( k^* \)-th D2D pair iterate over the set of all supported alphabet sizes \( (\mathcal{M}_A) \). The BS considers each supported modulation format \( A_n \in \mathcal{M}_A \), where \( n = 1, 2, \cdots, N_A \). For the \( n \)-th supported alphabet size \( (\mathcal{M}_n) \), a minimal transmission power for D2D pair \( k^* \) on subcarrier \( m^* \), denoted by \( p_{k^*,m^*}[n] \), is calculated by a bisection search using either the proposed SER expression (3.43) or the SER under the GA (3.13), corresponding to interference power \( \{ p_{j,m^*}^{(l)} \} \), where \( j \in \sigma_{m^*}^{(l)} \). Due to the interference caused by the newly added transmission power \( (p_{k^*,m^*}[n]) \) from the \( k^* \)-th D2D pair, the alphabet sizes of D2D pairs in set \( \sigma_{m^*}^{(l)} \) on subcarrier \( m^* \) are updated as \( \{ \hat{M}_{j,m^*}[n] \} \), where \( j \in \sigma_{m^*}^{(l)} \), via an exhaustive search within \( \mathcal{M}_A \) using either the proposed SER expression (3.43) or the SER under the GA (3.13).

Notice that the alphabet sizes of all D2D pairs that are assigned to subcarrier \( m^* \) and the \( k^* \)-th D2D pair are subject to change in the iterative search procedure. If the \( n \)-th supported alphabet size \( \mathcal{M}_n \) is allocated to subcarrier \( m^* \) of D2D pair \( k^* \) as
\( \hat{M}_{k^*,m^*}[n] = \mathcal{M}_n \), the corresponding total video MSE is given by

\[
\text{MSE}^{(l)}_{\text{total}}[n] = \sum_{j \in \sigma_{m^*}^* \cup k^*} c_j \log_2(\hat{M}_{j,m^*}[n]) + \sum_{m=1, m \neq m^*}^{M_c} \log_2(M^{(l)}_{j,m}) + b_j
\]
\[
+ \sum_{k=1, k \notin \sigma_{m^*}^* \cup k^*}^{K} c_k \sum_{m=1}^{M_c} \log_2(M^{(l)}_{k,m}) + b_k.
\] (3.50)

This step will be repeated for each supported alphabet size, i.e., \( n = 1, 2, \ldots, N_A \).

**Step 4: Subcarrier Assignment and Power Allocation**

This step aims to determine whether the total video MSE will decrease by allocating D2D pair \( k^* \) with subcarrier \( m^* \) and to make the subcarrier assignment and power allocation decision for the \( (l + 1) \)-th iteration. Among all supported modulation formats on subcarrier \( m^* \) of D2D pair \( k^* \), the index of the modulation format that leads to the minimal total MSE is given by

\[
n^* = \arg \min_{n \in \{1, 2, \ldots, N_A\}} \text{MSE}^{(l)}_{\text{total}}[n],
\] (3.51)

where \( \text{MSE}^{(l)}_{\text{total}}[n] \) is given by (3.50). We check whether the minimal total MSE is smaller than the total MSE at the start of the \( l \)-th iteration, namely \( \text{MSE}^{(l)}_{\text{total}}[n^*] < \sum_{k=1}^{K} c_k / [\sum_{m=1}^{M_c} \log_2(M^{(l)}_{k,m}) + b_k] \), and check if the \( k^* \)-th D2D pair satisfies the total power constraint, namely \( \sum_{m=1, m \neq m^*}^{M_c} p_{k^*,m}^{(l)} + p_{k^*,m^*}[n] \leq P \).

If both conditions are satisfied, then we assign D2D pair \( k^* \) to subcarrier \( m^* \) with alphabet size \( \mathcal{M}_{n^*} \). Let \( \sigma_{\text{req}} = \{ k \mid M_{k,m^*}[n^*] = \mathcal{M}_1, k \in \sigma_{m^*}^* \} \) be the indices of the D2D pairs that were previously allocated to subcarrier \( m^* \) whose alphabet size becomes \( \mathcal{M}_1 \). We let D2D pairs in \( \sigma_{\text{req}} \) relinquish subcarrier \( m^* \) and
set their transmission power on subcarrier $m^*$ to zero. We allocate the corresponding transmission power and alphabet sizes to D2D pairs $\sigma_m^{(l)}$ and D2D pair $k^*$ for the next iteration. The specific update procedure is as follows,

$$
\begin{align}
P^{(l+1)}_{k,m} &= \begin{cases} 
  p_{k,m^*}[n^*], & \text{if } m = m^*, \ k = k^* \\
  0, & \text{if } m = m^*, \ k \in \sigma_{\text{req}} \\
  p_{k,m}^{(l)}, & \text{otherwise,}
\end{cases} \\
M^{(l+1)}_{k,m} &= \begin{cases} 
  \tilde{M}_{k,m^*}[n^*], & \text{if } m = m^*, \ k \in \sigma_m^{(l)} \cup k^* \\
  M_{k,m}^{(l)}, & \text{otherwise,}
\end{cases} \\
\sigma_m^{(l+1)} &= (\sigma_m^{(l)} \cup k^*) \setminus \sigma_{\text{req}}, \\
\Omega^{(l+1)} &= \Omega^{(l)} \cup \sigma_{\text{req}}.
\end{align}
$$

(3.52a)  
(3.52b)  
(3.52c)  
(3.52d)

Otherwise, index $k^*$ is removed from the potential set by letting $\Omega^{(l+1)} = \Omega^{(l)} \setminus k^*$, and the BS keeps the power and alphabet sizes the same for the next iteration, namely

$$
P^{(l+1)}_{k,m} = p_{k,m}^{(l)}, \ M^{(l+1)}_{k,m} = M_{k,m}^{(l)}, \ \sigma_m^{(l+1)} = \sigma_m^{(l)}.$$

**Step 5: Termination**

If the potential set $\Omega$ is not empty, the iteration indicator $l$ is incremented by 1 and the algorithm returns to Step 2. Otherwise, the algorithm terminates.

As a summary, to compare the proposed SER expression (3.43) and the SER obtained by the GA (3.13) for resource allocation, the majority of the proposed algorithm is kept the same. Each of these two SER evaluation methods is applied in the bisection search for the minimal transmission power and the exhaustive search for the largest alphabet size in Step 3.
3.4 Results and Discussion

3.4.1 Numerical Results for the SER of a D2D Receiver

In this section, the SER of D2D receivers in CCI using the proposed exact SER expression is investigated. First, the proposed SER expression is corroborated by simulation. Next, the SER under multiple interferers of equal received interference power is considered. Then the impact of a dominant interferer on the SER is studied. Finally, the accuracy of the GA for the interference is assessed by comparing the resultant SER with the exact SER result.

Fig. 3.1 shows the SER versus SNR for the 4-QAM desired signal and a 16-QAM CCI signal, both using a raised-cosine pulse with roll-off factor $\beta = 0.5$ at SIR = 10dB and SIR = 15dB. The solid lines stand for the SERs obtained using the analytical expression (3.43) with $L = 20$, and the crosses denote the SERs obtained by Monte Carlo simulation. In the simulation, each data point is generated by averaging over $10^9$ symbols. We observe that the analytical results are in excellent agreement with the simulation. Therefore, we only show the results obtained by the analytical expression with $L = 20$ in the remaining examples.

Fig. 3.2 presents how SER changes as the number of interferers increases, given a fixed received SIR and letting the interference power be equally distributed among all received interference signals. The desired signal and CCI signals are modulated by 4-QAM and 16-QAM, respectively. We observe from Fig. 3.2 that the SER increases as the number of CCI signals increases, and the gap between the analytical SER result and the SER obtained by the GA becomes smaller, which is the consequence of the central limit theorem.
Figure 3.1: SER versus SNR for the 4-QAM desired signal and one 16-QAM CCI signal at SIR = 10dB and SIR = 15dB.

Figure 3.2: SER versus SNR for one interferer and multiple interferers with equal received interference power at SIR = 15dB.
Figure 3.3: SER versus SNR for 8 interferers at SIR = 15dB. The received interference power is dominated by one interferer, whereas other interferers have equal received power. The desired and CCI signals use 4-QAM and 16-QAM, respectively.

We also consider the interference dominated by one interferer. Fig. 3.3 presents the SER for 8 interferers as the received interference power becomes more dominated by a single interferer. As the power of the dominant interferer accounts for a larger proportion of the total received interference power, the exact SER becomes smaller. It is seen that the GA is more pessimistic in evaluating the SER when the received interference power is more dominated by a single interferer.

It is observed in Figures 3.1 - 3.3 that the GA overestimates the SER for QAM in CCI. The gap between the GA and the analytical SER expression is negligible at low SNR but is significant at high SNR. For example, when the SER target equals $10^{-3}$ and SIR=10dB, a SNR gap that is larger than 5dB is observed in Fig. 3.1 between the exact SER expression and the SER obtained by the GA. This behavior can be
explained by the unbounded tail of Gaussian RVs. The QAM CCI signal at the output of the matched filter is bounded, if a pulse from the parametric family defined in (2.2) is used. However, the GA generates an unbounded tail for the CCI, which increases the estimated SER at high SNR.

3.4.2 Results of Resource Allocation for D2D Video Transmission

We simulate a D2D system with $M_c = 8$ subcarriers, each with bandwidth $15\text{kHz}$. The roll-off factor for the raised-cosine pulse is $\beta = 0.5$, and the symbol rate is $1/T = 10\text{kHz}$. The channel response consists of path loss, shadowing and multipath fading. The path loss is $46.8 + 18.7\log_{10}(d[m])$ and the shadowing follows the log-normal distribution with standard deviation of $3\text{dB}$ [43]. The subcarriers experience independent Rayleigh fading due to multipath, and the channel response is assumed to be flat within a subcarrier. The maximal transmission power for each D2D pair is $100\text{mW}$. The SER target is $10^{-3}$ in the simulation. The supported modulation formats are 4-QAM, 16-QAM and 64-QAM.

For each realization of geographical location, we first initialize an empty D2D pair list. We repeatedly generate a transmitter and a receiver that are each uniformly distributed in a cell of a radius of 500 meters. The D2D distance is set between 10 and 50 meters. If the distance between the newly generated pair is within this range of 10 to 50 meters, the newly generated pair is considered suitable for D2D transmission, and therefore it is added to the list. This procedure is repeated until the number of D2D pairs in the list reaches the desired number $K$. A realization of the cell with $K = 32$ D2D pairs is shown in Fig. 3.4.
A video sequence with a resolution of $640 \times 480$, encoded using H.264/SVC reference software JSVM version 9.19.15, is used in simulation. The total length of the video is 30 seconds at 30 frames per second, and the video is organized in GOPs of 15 frames (I-P-P-P). By assigning random starting points of the same cyclic video to different D2D pairs, application layer diversity is created among D2D pairs and yet the videos have the same average complexity over time for different D2D pairs. The $4 \times 4$ DCT coefficients for the MGS layer of each macroblock are split with MGS vector $[1, 1, 2, 2, 2, 8]$ [38]. The video contents are protected by a rate $2/3$ punctured turbo code. Each packet consists of 300 bytes of FEC plus data bits. The video quality is evaluated by peak signal-to-noise ratio (PSNR), which is defined by
Figure 3.5: Video PSNR versus the number of D2D pairs for the proposed algorithms using the GA or exact SER, compared to baseline algorithms.

PSNR = \(10 \log_{10} \frac{255^2}{\text{E}[\text{MSE}]}\).

The performance of the proposed cross-layer algorithm combined with either the exact SER expression or the GA is obtained by simulations. In Fig. 3.5, the PSNR gap between the proposed SER expression and the GA is approximately 1dB. When the proposed resource allocation algorithm with the GA will admit 24 users with an average PSNR of approximately 27dB, the same algorithm with the proposed SER expression will be able to admit 30 users. Two baseline algorithms are also used for comparison. The first uses MUD, namely Step 1 of the proposed algorithm; the second baseline algorithm is the iterative orthogonal subcarrier assignment algorithm with subcarrier swapping from [30]. The proposed resource allocation algorithm outperforms the baseline algorithms by a PSNR gap that is larger than 5dB for 20 to 32 D2D pairs.
Figure 3.6: Distribution of the number of D2D pairs assigned to a subcarrier, by the proposed algorithm using the exact SER.

To explain the underlying reasons for the gap between the video PSNR using the exact SER and the GA for resource allocation, the distribution of the number of D2D pairs in a subcarrier is presented in Fig. 3.6. More than 95% of subcarriers simulated are assigned to between 2 and 6 D2D pairs. This simulation result indicates that the number of D2D pairs sharing a subcarrier is typically small, even if the number of D2D pairs in the system is much larger than the number of subcarriers, which is one reason that the GA is not applicable for this scenario.

The other reason that undermines the accuracy of the GA is that it is highly possible that the received interference power is dominated by a single interferer. Fig. 3.7 shows the cumulative distribution of the power ratio of the dominant interferer over the total interference power at D2D receivers. The cumulative distribution is calculated at each D2D receiver for the cases that at least 4 D2D pairs are allocated.
in a subcarrier, using the proposed algorithm with the exact SER. As $K$ ranges from 8 to 32, the probability of the dominant interferer accounting for more than half of the total received interference power is larger than 0.75. For the case of 8 D2D pairs in the cell, there is a probability of 0.5 that the dominant interferer takes larger than $3/4$ of the total received interference power.

3.5 Summary

This section makes two main contributions: (1) We derived an analytical and exact SER expression for D2D systems using multicarrier bandlimited QAM signaling, and (2) we developed a novel resource allocation algorithm for multicarrier D2D video transmission. The new algorithm outperforms MUD resource allocation as well as
a previous method in the literature when using the conventional method of SER evaluation involving approximating the interference as Gaussian. Further gains were demonstrated from using the newly derived SER expression in the resource allocation algorithm. The small number of interferers in the multicarrier D2D system and the high probability of a dominant interferer both undermine the accuracy of the GA.

3.6 Acknowledgement

This chapter, in part, is a reprint of the paper, P. Wu, P. C. Cosman and L. B. Milstein, “Resource Allocation for Multicarrier Device-to-Device Video Transmission: Symbol Error Rate Analysis and Algorithm Design,” *IEEE Transaction on Communications*, accepted. The dissertation author is the primary researcher and author.
In this chapter, resource allocation algorithms for D2D video transmission in Rayleigh fading channels is investigated. The difference between this chapter and Chapter 3 is as follows: The heuristic algorithm in Chapter 3 optimizes overall video quality of D2D transmission using instantaneous CSI in a block fading environment, and uses discrete bit loading according to instantaneous CSI. Therefore, in Chapter 3, the transmission power takes on discrete values, which formulates a discrete optimization problem. In contrast, in this chapter, the D2D pairs are assumed to be able to update the alphabet size according to the instantaneous channel gains under Rayleigh fading, so the transmission power can be chosen from a continuous range, and hence the
optimization problem based on average channel gain and video RD is continuous.

4.1 Adaptive Modulation at D2D Receivers

The block diagram of the transceiver is the same as Fig. 2.1. The transceiver has a multicarrier architecture, where the symbol rate is $1/T_0$ and the frequency spacing of subcarriers is $F_0 = W/M_c$. The complex envelope of the lowpass equivalent signal for the $i$-th D2D transmitter is given by

$$x_i(t) = \sum_{n=1}^{M_c} \sum_s X_{i,n}[s] g(t - sT_0) e^{j2\pi nF_0 t},$$  \hspace{1cm} (4.1)

where $g(t)$ is the transmitted pulse, whose energy is normalized to unity, namely,

$$\int_{-\infty}^{\infty} |g(t)|^2 = 1. \hspace{1cm} (4.2)$$

The FEC protected video bit sequences are mapped to data symbols, where the $s$-th modulated symbol on the $n$-th subcarrier of the $i$-th transmitter is denoted by $X_{i,n}[s]$. The data symbols $\{X_{i,n}[s]\}$ are complex and modulated by square QAM, given by

$$X_{i,n}[s] = X_{i,n}^I[s] + jX_{i,n}^Q[s]. \hspace{1cm} (4.3)$$

The energy per symbol for D2D pair $i$ on subcarrier $n$ is denoted by $E_{i,n}$, and $\delta_z$ is the Kronecker delta function. The in-phase and quadrature components of the $l$-th complex data symbol on the $n$-th subcarrier of the $i$-th transmitter are denoted by
$X_{i,n}[l]$ and $X_{i,n}^Q[l]$, respectively, where

$$E[X_{i_1,n_1}[l_1]X_{i_2,n_2}^Q[l_2]] = E[X_{i_1,n_1}^I[l_1]X_{i_2,n_2}^Q[l_2]] = \mathcal{E}_{i_1,n_1}\delta_{i_1-i_2}\delta_{n_1-n_2}\delta_{l_1-l_2},$$

(4.4)

and the in-phase and quadrature components are uncorrelated, namely

$$E[X_{i_1,n_1}[l_1]X_{i_2,n_2}^Q[l_2]] = 0.$$  

(4.5)

A Rayleigh fading channel model is used, in which the coherence bandwidth $f_{coh}$ is larger than the bandwidth of a subcarrier. The coherence time $T_{coh}$ is larger than the duration of a symbol but much smaller than the duration $T_{GOP}$ of a GOP. The complex channel response on subcarrier $m$ from the $i$-th transmitter to the $k$-th receiver is denoted by

$$h_{i,k,m} = \sqrt{H_{i,k}}\tilde{h}_{i,k,m},$$

(4.6)

where $H_{i,k}$ denotes the average channel gain due to large scale fading, and $\tilde{h}_{i,k,m}$ is the channel response due to multipath fading. The large scale fading includes path loss and shadowing and is constant over a GOP. The multipath fading is modeled as Rayleigh fading, so $\tilde{h}_{i,k,m}$ is circularly symmetric complex Gaussian with zero mean and unit variance. In addition, multipath fading is assumed to be independent for different D2D pairs.

The signal from the $i$-th transmitter arrives at the $k$-th receiver with a time delay $\tau_{i,k}$ compared to the desired signal from the $k$-th transmitter ($i, k = 1, 2, \cdots, K$), where $\tau_{i,k}$ modulo $T_0$ is uniformly distributed in $[0, T_0)$, if $i \neq k$. Also, $|\tau_{i,k}| \ll T_{GOP}$. 

A coherent receiver is assumed to be implemented and the receiver maintains perfect bit synchronization and phase recovery for the desired signal, so \( \tau_{k,k} = 0 \). Therefore, the lowpass equivalent signal at the \( k \)-th receiver is given by

\[
y_k(t) = \sum_{i=1}^{K} \sum_{n=1}^{M_c} \sum_{s} h_{i,k,n} X_{i,n}[s] g(t - \tau_{i,k} - sT_0) e^{j2\pi n F_0(t \tau_{i,k})} + n_k(t), \quad (4.7)
\]

where \( n_k(t) \) is complex AWGN at the \( k \)-th receiver with two-sided power spectral density \( N_0 \). A matched filter is used at subcarrier \( m \) of the \( k \)-th receiver. The output for the \( l \)-th received symbol is shown to be

\[
Y_{k,m}[l] = \int_{-\infty}^{\infty} y_k(t) e^{-j2\pi m F_0 t} g^*(t - lT_0) dt. \quad (4.8)
\]

Substituting (4.1), (4.7) into (4.8), the output for the \( l \)-th received symbol is given by

\[
Y_{k,m}[l] = \sum_{i=1}^{K} \sum_{n=1}^{M_c} \sum_{s} h_{i,k,m} X_{i,m}[s] e^{-j2\pi \tau_{i,k}} \int_{-\infty}^{\infty} g(t - \tau_{i,k} - sT_0) g^*(t - lT_0) dt + N_{k,m}[l], \quad (4.9)
\]

where the noise term is given by

\[
N_{k,m}[l] = \int_{-\infty}^{\infty} n_k(t) e^{-j2\pi m F_0 t} g^*(t - lT_0) dt, \quad (4.10)
\]

which is a zero-mean circularly symmetric complex Gaussian random variable (RV) with variance \( N_0 \int_{-\infty}^{\infty} |g(t)|^2 dt = N_0 \). Substituting \( \tau_{k,k} = 0 \) and \( \int_{-\infty}^{\infty} |g(t)|^2 dt = 1 \) into
(4.9), it is obtained that

\[ Y_{k,m}[l] = h_{k,k,m} X_{k,m}[s] \]

\[ + \sum_{i=1, i \neq k}^{K} h_{i,k,m} \sum_{s} X_{i,m}[s] e^{-j2\pi \tau_{i,k}} \int_{-\infty}^{\infty} g(t - \tau_{i,k} - sT_0) g^*(t - lT_0) dt \]

\[ + N_{k,m}[l]. \] (4.11)

From [44],

\[ E \left[ \left| \sum_{s} X_{i,m}[s] e^{-j2\pi \tau_{i,k}} \int_{-\infty}^{\infty} g(t - \tau_{i,k} - sT_0) g^*(t - lT_0) dt \right|^2 \right] = \mathcal{E}_{i,m} \left( 1 - \frac{\beta}{4} \right). \] (4.12)

Therefore, the power of the co-channel interference is given by

\[ P_I = \sum_{i=1, i \neq k}^{K} E[|h_{i,k,m}|^2] \mathcal{E}_{i,m} \left( 1 - \frac{\beta}{4} \right) = \left( 1 - \frac{\beta}{4} \right) \sum_{i=1, i \neq k}^{K} H_{i,k} \mathcal{E}_{i,m}. \] (4.13)

A fixed power, variable rate scheme is considered, in which the transmission powers of D2D transmitters are constant over a GOP, and D2D receivers adaptively select the alphabet sizes according to the instantaneous SINR. The transmission power varies for different D2D transmitters. Given the channel response of the desired channel \( h_{k,k,m} \), the instantaneous SINR on the \( m \)-th subcarrier of the \( k \)-th receiver is given by

\[ \gamma_{k,m}(h_{k,k,m}) = \frac{|h_{k,k,m}|^2 \mathcal{E}_{k,m}}{(1 - \frac{\beta}{4}) \sum_{i=1, i \neq k}^{K} H_{i,k} \mathcal{E}_{i,m} + N_0}. \] (4.14)
Given $h_{k,k,m}$, the SER of square QAM with alphabet size $M_{k,m}$ for the instantaneous SINR $\gamma_{k,m}$ is approximated by [30]

$$\text{SER} \approx 4Q\left(\frac{3 \gamma_{k,m}(h_{k,k,m})}{M_{k,m} - 1}\right) = 4Q\left(\frac{3}{M_{k,m} - 1} \cdot \frac{|h_{k,k,m}|^2 E_{k,m}}{(1 - \frac{\beta}{4}) \sum_{i=1,i \neq k}^{K} H_{i,k} E_{i,m} + N_0}\right). \quad (4.15)$$

Therefore, given $h_{k,k,m}$, the number of bits per symbol for a SER target is obtained as

$$\tilde{R}_{k,m} = \min \left\{ \left\lfloor \log_2 \left(1 + \frac{\eta |h_{k,k,m}|^2 E_{k,m}}{(1 - \frac{\beta}{4}) \sum_{i=1,i \neq k}^{K} H_{i,k} E_{i,m} + N_0}\right) \right\rfloor, \quad \log_2(M_{\text{max}}) \right\}, \quad (4.16)$$

where $\eta \triangleq 3/\left[Q^{-1}(\text{SER}_{\text{target}}/4)\right]^2$ is a positive constant determined by the SER target, and $M_{\text{max}}$ is the largest alphabet size allowed by the system.

Since $h_{k,k,m} = \sqrt{H_{k,k}} \tilde{h}_{k,k,m}$ and $\tilde{h}_{k,k,m}$ is a circularly symmetric complex Gaussian random variable with unity variance, $\gamma \triangleq |\tilde{h}_{k,k,m}|^2$ satisfies an exponential distribution with unity variance. If the flooring operation is ignored, the average number of bits per symbol is given by

$$R_{k,m} \triangleq E[\tilde{R}_{k,m}] \approx \min \left\{ \int_{0}^{\infty} \log_2 \left(1 + \frac{\eta H_{k,k} E_{k,m} \gamma}{(1 - \frac{\beta}{4}) \sum_{i=1,i \neq k}^{K} H_{i,k} E_{i,m} + N_0}\right) e^{-\gamma d\gamma}, \quad \log_2(M_{\text{max}}) \right\}$$

$$= \min \left\{ \int_{0}^{\infty} \log_2 \left(1 + \eta \gamma_{k,m} \gamma\right) e^{-\gamma d\gamma}, \quad \log_2(M_{\text{max}}) \right\}, \quad (4.17)$$
where $\gamma_{k,m}$ is the average SINR of D2D pair $k$ on subcarrier $m$, given by

$$
\gamma_{k,m} \triangleq \frac{H_{k,k}E_{k,m}}{(1 - \frac{\beta}{4}) \sum_{i=1,i \neq k}^{K} H_{i,k}E_{i,m} + N_0}.
$$

(4.18)

From [45, (34)], it is obtained that

$$
f(x) \triangleq \int_{0}^{\infty} \log_2 \left(1 + \frac{x}{\gamma} e^{-\gamma} d\gamma = \log_2(e) e^x E_1(x),
$$

(4.19)

where $f(\cdot)$ maps a power ratio (the reciprocal of average SINR) to the average number of bits per symbol. $E_1(x)$ is the exponential integral of the first order, defined by $E_1(x) = \int_{x}^{\infty} e^{-t}/t dt, x > 0$. The average number of bits per symbol is finally given by

$$
R_{k,m}(\gamma_{k,m}) \approx \min \left\{ f \left( \frac{1}{\eta \gamma_{k,m}} \right), \log_2(M_{\text{max}}) \right\}.
$$

(4.20)

Since the integrand in (4.19), $\log_2 (1 + \gamma/x) e^{-\gamma}$, is positive and monotonically decreasing in $x$ for $\gamma > 0$, $f(x)$ is positive and monotonically decreasing in $x$. Therefore, $f(1/(\eta \gamma_k))$ is positive and monotonically increasing in $\gamma_k$.

4.2 Subcarrier Assignment and Power Control

4.2.1 Subcarrier Assignment

Subcarriers are divided into groups to simplify the subcarrier assignment procedure. The frequency band of the system, $W$ (Hz), is equally divided into $M_c = N M_g$ subcarriers, where there are $N$ groups of subcarriers and a group consists of $M_g$ interleaved subcarriers, as shown in Fig. 4.1. More specifically, group $n$ consists
of subcarriers $n$, $n + N, \cdots, n + (M_g - 1)N$, where $n = 1, 2, \cdots, N$. Each D2D pair can be assigned to at most one group of subcarriers. The groups of subcarriers are determined in advance and are not optimization parameters.

The system operates in a time-slotted manner, with a single GOP transmitted in each time slot. The duration of a GOP, $T_{\text{GOP}}$, is a constant. The subcarrier assignment and power allocation decision is made by the BS at the beginning of each GOP, and is fixed for the entire GOP. The BS is able to collect the average channel gain ($H_{i,k}$) of the desired and interference signals and video RD information from the D2D pairs via the control channel. Each transmitter is assigned to at most one group by the BS. Each transmitter allocates equal power into each assigned subcarrier. The indices of D2D pairs are randomized at the beginning of each GOP. While groups 1 to $N$ stand for sets of subcarriers, group 0 is also used for simplicity of notation: a D2D pair assigned to group 0 does not transmit. The proposed subcarrier assignment algorithm is as follows:

- **Step 1: Initialization.**
  - Step 1.1: Assign D2D pairs 1, 2, $\cdots$, $N$ to groups 1, 2, $\cdots$, $N$, respectively.
  - Step 1.2: Sequentially consider D2D pairs $N + 1, N + 2, \cdots, K$. For each user $i$, iterate over groups $n = 0, 1, 2, \cdots, N$, and consider assigning user $i$ to group $n$. Notice that the subcarrier assignments of D2D pairs 1, 2, $\cdots$, $i - 1$ have already been made, so the BS runs the cross-layer power control.
algorithm based on the subcarrier assignments of D2D pairs $1, 2, \cdots, i - 1$.

Assign D2D pair $i$ to the group that minimizes the total video MSE.

- **Step 2: Subcarrier reassignment.** Sequentially consider D2D pairs $1, 2, \cdots, K$:
  For each group $n = 0, 1, 2, \cdots, N$ to which D2D pair $i$ is not assigned, consider reassigning D2D pair $i$ into group $n$ instead, and run the cross-layer power control algorithm. Reassign D2D pair $i$ into group $n$ if the total video MSE decreases.

  Every time the power control algorithm is called in the search for group $n$, the temporary subcarrier assignment in group $n$ and the index of the D2D pair newly added to group $n$ are passed to the cross-layer power control algorithm as the input. The indices of D2D pairs in group $n$ are denoted by $\mathcal{J}_n = \{j^{(n)}_{1}, j^{(n)}_{2}, \cdots, j^{(n)}_{K_n}\}$, where $K_n$ is the number of D2D pairs in group $n$ and $n = 0, 1, 2, \cdots, N_g$. Also, $\mathcal{K}_n$ denotes $\mathcal{K}_n \triangleq \{1, 2, \cdots, K_n\}$.

### 4.2.2 Analysis for Cross-layer Power Control

The cross-layer power control is formulated as a total video MSE minimization problem, given the subcarrier assignment. The optimization parameter is the transmission power of each D2D pair. The average channel gains, the video RD and the largest transmission power of a D2D pair are known constants in the optimization.
The cross-layer power control problem for D2D pairs assigned to a given group \( n \) \((1 \leq n \leq N)\) is formally written as

\[
\begin{align*}
    \text{minimize} & \quad \sum_{j \in \mathcal{J}_n} \left( a_j + \frac{\tilde{b}_j}{\sum_{m \in \{n, n+N, \cdots, n+(M_g-1)N\}} r_{j,m} + \tilde{c}_j} \right) \\
    \text{subject to} & \quad 0 \leq r_{j,m} \leq R_{j,m}(\bar{\gamma}_{j,m}), \quad 0 \leq M_g \mathcal{E}_j / T_0 \leq P, \\
                                     & \quad \forall j \in \mathcal{J}_n, \forall m \in \{n, n+N, \cdots, n+(M_g-1)N\},
\end{align*}
\]

where \( R_{j,m}(\bar{\gamma}_{j,m}) \) is the average number of bits per symbol on subcarrier \( m \) of D2D pair \( j \), and is given by (4.20) as a function of the average SINR \( (\bar{\gamma}_{j,m}) \). \( \mathcal{E}_j \) is the symbol energy of D2D pair \( j \) on each assigned subcarrier, so \( M_g \mathcal{E}_j / T_0 \) is the total transmission power of D2D transmitter \( j \), and the total transmission power of a D2D transmitter is upper bounded by the total transmission power constraint \( P \).

For each D2D pair in group \( n \), since equal power is allocated to assigned subcarriers \( m \in \{n, n+N, \cdots, n+(M_g-1)N\} \), the average SINR \( \bar{\gamma}_{j,m} \) from (4.18) is equal for each assigned subcarrier. Therefore, the subscript \( m \) can be omitted in the notation of \( \bar{\gamma}_{j,m} \) and the average SINR of D2D pair \( j \) on each subcarrier is written as

\[
\bar{\gamma}_j = \frac{H_{j,j} \mathcal{E}_j}{(1 - \frac{\beta}{4}) \sum_{i \in \mathcal{J}_n \setminus j} H_{i,j} \mathcal{E}_i + N_0}, \quad \forall j \in \mathcal{J}_n. \quad (4.22)
\]

Similarly, the subscript \( m \) can be omitted in the notation of \( r_{j,m} \) and it can be written as \( r_j \). Substituting (4.20) into (4.21), (4.21) can be rewritten as

\[
\begin{align*}
    \text{minimize} & \quad \sum_{j \in \mathcal{J}_n} \frac{\tilde{b}_j}{M_g r_j + \tilde{c}_j} \\
    \text{subject to} & \quad 0 \leq r_j \leq \min \left\{ f \left( \frac{1}{\eta \bar{\gamma}_j} \right), \log_2(M_{\text{max}}) \right\}, \quad 0 \leq M_g \mathcal{E}_j / T_0 \leq P, \ \forall j \in \mathcal{J}_n,
\end{align*}
\]
where \( f(\cdot) \) is a mapping from a power ratio (the reciprocal of average SINR) to the average number of bits per symbol given by (4.19), and \( f(\cdot) \) monotonically decreases, so the inverse function of \( f(\cdot) \), denoted by \( f^{-1}(\cdot) \), exists. Specifically, \( f(e^{-x}) \) monotonically increases in \( x \), and the inverse function of \( y = f(e^{-x}) \) is \( x = -\ln[f^{-1}(y)] \).

Consider variable transformations \( b_k \triangleq \tilde{b}_j^{(n)} / M_g \), \( c_k \triangleq \tilde{c}_j^{(n)} / M_g \), \( z_k \triangleq \ln(E_j^{(n)}) \), and \( s_k \triangleq -\ln[f^{-1}(r_j^{(n)})] \), where \( K_n \). Since \( r_j^{(n)} \geq 0 \) is the average number of bits per symbol, \( 1/f^{-1}(r_j^{(n)}) \geq 0 \) is the average SINR and hence \( s_k \in (-\infty, +\infty) \) is proportional to the average SINR in dB. With this variable transformation, (4.23) can be rewritten as

\[
\begin{align*}
\text{minimize} & \quad \sum_{k=1}^{K_n} \frac{b_k}{f(e^{-s_k})} + c_k \\
\text{subject to} & \quad (C1) \quad s_k \leq s_{\text{sat}} \triangleq -\ln[f^{-1}(\log_2(M_{\text{max}}))], \quad \forall k \in K_n, \\
& \quad (C2) \quad s_k \leq \ln(\eta \tilde{\gamma}_j^{(n)}), \quad z_k \leq \ln(PT_0/M_g), \quad \forall k \in K_n,
\end{align*}
\]

which is a constrained minimization problem with variables \( \{s_1, \ldots, s_{K_n}, z_1, \ldots, z_{K_n}\} \).

**Property of the Feasible Region**

The feasible region \( S \) is defined as the set of all vectors \( \underline{s} = (s_1, \ldots, s_{K_n}) \) that satisfy constraints (C1) and (C2) in (4.24).

To prove constraint (C2) is a convex region in \( (s_1, \ldots, s_{K_n}) \), one can substitute \( E_j^{(n)} = \exp(z_k) \) into (4.22), which yields

\[
\tilde{\gamma}_j^{(n)} = \frac{H_j^{(n)} e^{z_k}}{(1 - \frac{\beta}{\delta}) \sum_{i=1, i \neq k}^{K_n} H_{j_i}^{(n)} e^{z_i} + N_0}, \quad \forall k \in K_n.
\]
Substituting this equation into constraint (C2) in (4.24) yields

\[ s_k \leq \ln \left[ \frac{\eta H_{j_k^{(n)}} j_k^{(n)} e^{z_k}}{\left(1 - \frac{\beta}{4}\right) \sum_{i=1, i \neq k}^{K_n} H_{j_i^{(n)}} j_k^{(n)} e^{z_i} + N_0} \right], \]

which reduces to

\[
\ln \left\{ \left(1 - \frac{\beta}{4}\right) \sum_{i=1, i \neq k}^{K_n} H_{j_i^{(n)}} j_k^{(n)} e^{z_i - z_k} + N_0 e^{-z_k} \right\} + s_k \leq \ln(\eta H_{j_k^{(n)}} j_k^{(n)}), \\
\]

\[ z_k \leq \ln(PT_0/M_g), \quad \forall k \in K_n. \]

(4.26)

Denote \( \underline{z} \triangleq (z_1, z_2, \cdots, z_{K_n}) \) and \( \underline{x}(k) \triangleq (x_1^{(k)}, x_2^{(k)}, \cdots, x_{K_n}^{(k)}) \). The first term in (4.26) is the composition of a log-sum-exp function \( \ln[\sum_{i=1}^{K_n} e^{x_i^{(k)}}] \) and an affine mapping:

\[
x_i^{(k)} = \begin{cases} 
    z_i - z_k + \ln \left[ (1 - \beta/4) H_{j_i^{(n)}} j_k^{(n)} \right], & \text{if } i \neq k, \quad \forall k \in K_n, \\
    -z_k + \ln(N_0), & \text{if } i = k, \quad \forall k \in K_n.
\end{cases}
\]

(4.27)

The log-sum-exp function \( \ln[\sum_{i=1}^{K_n} e^{x_i^{(k)}}] \) is convex in \( \underline{x}(k) \) [46]. Therefore, the first term in (4.26) is a composition of a convex function with an affine mapping, and thereby is a convex function in \( \underline{z} \) [46]. Consequently, (4.26) is convex in \((s, z)\) and thereby is convex in \(s\), which indicates that constraint (C2) is convex.

In addition, the halfspaces specified by constraint (C1) are convex regions in \((s_1, \cdots, s_{K_n})\) [46]. Now that both constraints (C1) and (C2) in (4.24) are proved to be convex regions, the feasible region \(S\) is a convex region in \((s_1, \cdots, s_{K_n})\).

To give a general idea of the shape of region \(S\), one can easily identify a rectangular region that contains \(S\). From (4.22), the average SINR of D2D pair \(j_k^{(n)}\) is
upper bounded by

\[ \gamma_{j_k}^{(n)} \leq \frac{H_{j_k}^{(n)} j_{j_k}^{(n)} PT_0}{M_g N_0} \triangleq \gamma_{k,\max}. \tag{4.28} \]

Therefore,

\[ S \subset \{(s_1, \cdots, s_{K_n})|s_k \leq s_{k,\max}, \forall k \in \mathcal{K}_n\}, \tag{4.29} \]

where \( s_{k,\max} \triangleq \min\{\ln(\eta \gamma_{k,\max}), s_{sat}\}. \)

The equality in (4.28) is achieved when D2D pair \( k \) is transmitted with the largest power and all other users sharing the subcarriers with D2D pair \( k \) use no transmission power. Let \( s_{k,\max} = (-\infty, \cdots, -\infty, s_{k,\max}, -\infty, \cdots, -\infty) \), which indicates that D2D pair \( j_k^{(n)} \) transmits at the largest power and other D2D pairs do not transmit, thereby \( s_{k,\max} \in S \).

The boundary of \( S \) does not have an explicit expression, however, it is convenient to check if a vector \( \underline{s} = (s_1, s_2, \cdots, s_{K_n}) \) satisfies \( \underline{s} \in S \). Let \( s_k = \ln(\eta \gamma_{j_k}^{(n)}) \) according to (C2) in (4.24), then \( \gamma_{j_k}^{(n)} = e^{s_k}/\eta \). It is obtained from (4.22) that

\[ \frac{H_{j_k}^{(n)} j_{j_k}^{(n)} \mathcal{E}_{j_k}^{(n)}}{(1 - \beta/4) \sum_{i=1, i \neq k}^{K_n} H_{j_i}^{(n)} j_{j_i}^{(n)} \mathcal{E}_{j_i}^{(n)} + N_0} = \gamma_{j_k}^{(n)} = \frac{e^{s_k}}{\eta}, \forall k \in \mathcal{K}_n, \tag{4.30} \]

which yields a set of linear equations of symbol energy \( \{\mathcal{E}_{j_k}^{(n)}\} \):

\[ \left(1 - \frac{\beta}{4}\right) \sum_{i=1, i \neq k}^{K_n} H_{j_i}^{(n)} j_{j_i}^{(n)} \mathcal{E}_{j_i}^{(n)} + N_0 = \eta e^{-s_k} H_{j_k}^{(n)} j_{j_k}^{(n)} \mathcal{E}_{j_k}^{(n)}, \forall k \in \mathcal{K}_n. \tag{4.31} \]

If \( \underline{s} \in S \), the corresponding symbol energy \( \{\mathcal{E}_{j_k}^{(n)}\} \) should satisfy the power
constraint. Therefore, the following steps can determine whether $s \in S$:

- First step is to check the simple bounds: if $\exists k \in \mathcal{K}_n$ such that $s_k > s_{k, \text{max}}$, then $s \notin S$; otherwise, proceed to next step.

- The second step is substituting $\{s_k\}$ into (4.31) and solving for $\{\mathcal{E}_{j_k^{(n)}}\}$. If $0 \leq \mathcal{E}_{j_k^{(n)}} \leq PT_0/M_g$, $\forall k \in \mathcal{K}_n$, then the vector $s \in S$; if there exists $k \in \mathcal{K}_n$ such that $\mathcal{E}_{j_k^{(n)}} < 0$ or $\mathcal{E}_{j_k^{(n)}} > PT_0/M_g$, then the vector $s \notin S$.

### Property of the Objective Function

Proposition 3 characterizes a key property of the objective function in (4.24):

**Proposition 3.** Let $y_k^*$ be the smallest root of

$$e^y E_1(y) = \left[ (y + 1) c_k \ln(2) + 3 \right] y - \sqrt{y[(\ln(2)c_k)^2y(y + 1)^2 + y + 2 \ln(2)c_k y(y + 5) + 8]} \over 2(y - 1)$$

in $y > 0$, and denote the video MSE of D2D pair $j_k^{(n)}$ by

$$g_k(s_k) \triangleq b_k \frac{e^{-s_k}}{f(e^{-s_k}) + c_k}, \quad (4.32)$$

then: 1) $y_k^*$ always exists and is only dependent on $c_k$; 2) $g_k''(s_k)$ has a real root $s_k^* \triangleq -\ln(y_k^*)$, and $g_k''(s_k) > 0$ for $s_k > s_k^*$.

**Proof.** The proof is given in Appendix D.

As a result of Proposition 3, the video MSE of D2D pair $j_k^{(n)}$, $g_k(s_k)$ is convex in $s_k > s_k^*$, where $s_k^*$ is the inflection point, as presented in Fig. 4.2. $s_k^*$ is a function of $c_k$ and their relation is shown in Fig. 4.3. In the proposed algorithm, $s_k^*$ is found for a
Figure 4.2: An example of the video MSE as a function of $s_k$. The inflection point $s_k^*$ is marked as a red $x$.

Figure 4.3: Inflection point $s_k^*$ as a function of $c_k$. 
given value of $c_k$ offline by a bisection search, and a one-dimensional look-up table is created for $s_k^*$ as a function of $c_k$.

Let $\mathcal{C}$ be the set of all vectors $\underline{s} = (s_1, s_2, \cdots, s_{K_n})$ over which the objective function of (4.24) is convex. From Proposition 3, $\mathcal{C} = \{(s_1, \cdots, s_{K_n})| s_k > s_k^*, \forall k \in \mathcal{K}_n\}$, which is the intersection of half-spaces, thereby being a convex set. Since $s_k^*$ always exists $\forall k \in \mathcal{K}_n$, region $\mathcal{C}$ also always exists. The objective function is convex in region $\mathcal{C}$, and is non-convex outside region $\mathcal{C}$.

### 4.2.3 Proposed Cross-layer Power Control Algorithm

The proposed power control algorithm aims to minimize the total video MSE based on the video RD and average channel gain. Two scenarios need to be considered for the cross-layer power control, and an example of both scenarios for a two-user case is shown in Fig. 4.4. The proposed power control algorithm searches for a local optimum in $\mathcal{S} \cap \mathcal{C}$, if $\mathcal{S} \cap \mathcal{C} \neq \emptyset$. If $\mathcal{S} \cap \mathcal{C} = \emptyset$, the algorithm keeps the transmission power of the existing D2D pairs in the current group unchanged and searches for a locally optimal transmission power for the newly added D2D pair. Notice that $\mathcal{S} \cap \mathcal{C} = \emptyset$ is equivalent to $\underline{s}^* \triangleq (s_1^*, s_2^*, \cdots, s_{K_n}^*) \notin \mathcal{S}$. Therefore, to determine whether $\mathcal{S} \cap \mathcal{C} = \emptyset$, one only needs to check if $\underline{s}^* \in \mathcal{S}$.

If $\underline{s}^* \in \mathcal{S}$, the aim is to find the local optimum in $\mathcal{S} \cap \mathcal{C}$. Since $\mathcal{S} \cap \mathcal{C}$ is convex and the objective function of (4.24) is also convex in $\mathcal{S} \cap \mathcal{C}$, many well-established algorithms can be implemented to find the optimal solution to (4.24) in $\mathcal{S} \cap \mathcal{C}$. For example, the interior point method [46] can be used, whose complexity is proportional to $\sqrt{K_n}$.

If $\underline{s}^* \notin \mathcal{S}$, while keeping the transmission power of all existing D2D pairs in the
Figure 4.4: Region C and feasible region S for a two-user case. The left and right figures show the cases of $S \cap C \neq \emptyset$ and $S \cap C = \emptyset$, respectively. The boundary of C is given by $s_k > s^*_k$, $\forall k \in K_n$, where $s^*_k$ is determined by $c_k$ using a look-up table.

current group unchanged, the BS searches for a local optimum for the transmission power of the newly added user using coordinated descent [47].

Here the proposed cross-layer power control algorithm is summarized as follows:

- Step 1: For each $k \in K_n$, find the values of $s^*_k$ from the entries in the look-up table (Fig. 4.3) corresponding to $c_k$.

- Step 2: Check if $s^*$ is feasible. If so, go to Step 3a; otherwise, go to Step 3b.

- Step 3a: First let $s_k := s_{k, \text{max}}$, $\forall k \in K_n$. Repeatedly update $s_k := (s_k + s^*_k)/2$ until $(s_1, \cdots, s_{K_n})$ is feasible. With $(s_1, \cdots, s_{K_n})$ as the initialization, the interior point method can be applied to (4.24) with (C2) replaced by (4.26). It will converge to a local optimum in region $S \cap C$ for the cross-layer power control problem (4.24), and the cross-layer power control algorithm terminates.

- Step 3b: Keep the transmission power of all existing D2D pairs in the current group unchanged. Initialize the transmission power of the newly added D2D
pair to zero, and repeatedly update the transmission power of the newly added D2D pair using gradient descent [47], subject to the power constraint, until the decrease of total MSE is less than a threshold, and then terminate the cross-layer power control algorithm.

4.3 Baseline Algorithms

4.3.1 Other Methods for Local Search in Subcarrier Assignment

Step 2 of the subcarrier assignment uses one round of subcarrier reassignment for local search. This approach is compared with other local search methods in terms of complexity and performance.

First, one may consider replacing one round of subcarrier reassignment by one round of subcarrier swapping. Specifically, in Step 2 of the subcarrier assignment algorithm, the following operations are used instead:

- [One round of swapping] For each D2D pair \(i = 1, 2, \cdots, K\), the BS considers each D2D pair \(j\) that is not assigned to D2D pair \(i\)'s group, and assesses the total MSE of swapping the subcarrier assignment of D2D pairs \(i\) and \(j\) by calling the cross-layer power control algorithm. The BS swaps the group assignment if the total video MSE decreases.

Another alternative approach is to replace Step 2 of the original subcarrier assignment algorithm by repeatedly reassigning a D2D pair into a different group, until the total MSE does not decrease. The detailed local search step is given as follows:
• [Multiple rounds of reassignment] The BS first creates a list of all D2D pairs. The BS repeats the following steps until all D2D pairs are removed from the list: For each D2D pair $i = 1, 2, \ldots, K$ on the list, let $n$ iterate over the indices of all groups that D2D pair $i$ is not assigned to, and assess the total MSE of reassigning D2D pair $i$ into group $n$ by calling the cross-layer power control algorithm. If the total MSE decreases, the BS reassigns D2D pair $i$ into group $n$, puts all D2D pairs in group $n$ on the list, and removes the duplicated indices on the list. If the total MSE does not decrease after iterating over all $n$, the BS removes D2D pair $i$ from the list.

Similarly, another potential approach is to replace Step 2 of the original subcarrier assignment algorithm by repeatedly swapping the assignment of two D2D pairs, until the total MSE does not decrease:

• [Multiple rounds of swapping] The BS first creates a list of all D2D pairs. The BS repeats the following steps until all D2D pairs are removed from the list: For each D2D pair $i = 1, 2, \ldots, K$ on the list, the BS lets $j$ iterate over the indices of all D2D pairs that are not in D2D pair $i$'s group, and assesses the total MSE of swapping the subcarrier assignment of D2D pairs $i$ and $j$ by calling the cross-layer power control algorithm. If the total MSE decreases, the BS swaps the subcarrier assignment of D2D pairs $i$ and $j$, puts all D2D pairs in these two groups on the list, and removes the duplicated indices on the list. If the total MSE does not decrease after iterating over all $j$, the BS removes D2D pair $i$ from the list.
4.3.2 Cross-layer Subcarrier Assignment with No Power Control

To demonstrate the performance improvement of our proposed cross-layer power control, another baseline algorithm is considered: in this baseline algorithm, all D2D pairs transmit with the largest transmission power, with power equally allocated on all assigned subcarriers, while the subcarrier assignment algorithm remains the same as the original one.

4.3.3 Heuristic Subcarrier Assignment without Subcarrier Grouping

In Section 4.2.2, given that a set of subcarriers is assigned to a set of D2D pairs exclusively and equal power is allocated to subcarriers of each D2D pair, the cross-layer power control problem is shown to have a tractable convex structure. In contrast, if D2D pairs are allowed to use arbitrary sets of subcarriers, such as overlapping but non-identical sets of subcarriers, the cross-layer power control problem is non-convex [44], and the algorithm to search for the global minimum, such as exhaustive search, has a complexity that is exponential in $K$ and $M_c$, and thereby the resultant complexity of searching for the global minimum is prohibitive, even for offline computation. Therefore, a heuristic algorithm with a polynomial complexity in $K$ and $M_c$ is of interest. The following heuristic algorithm is considered as a baseline, in which equal transmission power is allocated to all assigned subcarriers of each D2D pair and the total transmission power of each D2D pair is set to $P$ for simplicity:

- Step 1: Each subcarrier is initially assigned to the D2D pair with the best
instantaneous channel response.

- Step 2: The BS chooses the D2D pair with the steepest RD slope.

- Step 3: The BS iterates over all subcarriers that are not assigned to the chosen D2D pair. If the total MSE decreases when the chosen user acquires the current subcarrier, the BS assigns this subcarrier to the chosen user.

- Step 4: The BS lets \( i \) iterate over the subcarriers that are assigned to the chosen D2D pair, and lets \( j \) iterate over the subcarriers that are not assigned to the chosen D2D pair. The chosen D2D pair relinquishes subcarrier \( i \) and gains subcarrier \( j \) if the total MSE decreases.

- Step 5: Steps 2 to 4 are repeated until all D2D pairs have been considered.

### 4.3.4 PHY-layer Greedy Assignment with Power Control

A greedy subcarrier assignment and power control algorithm aiming to maximize the total bit rate of all D2D pairs was proposed in [4]. The subcarrier assignment in [4] was initialized by assigning all D2D pairs to subcarriers in a greedy approach, and did not have any additional steps for local search. As for the power control, although the PHY-layer power control problem is non-convex [25], local maxima can be found using the bisection search algorithm proposed in [4]. This algorithm was based on orthogonal "subchannels", and details of waveform design were not provided. For the purpose of comparison, the algorithm from [4] is adapted as a baseline by replacing "subchannels" by subcarrier groups. Also, all constraints related to the cellular users in [4], such as the throughput constraints of cellular users, are removed, as this dissertation aims to investigate resource sharing among D2D pairs in a dedicated spectrum. Also,
the objective function used in [4] is based on the capacity of Gaussian channels in
AWGN. For a fair comparison in Rayleigh fading channels, the objective function in
the baseline algorithm from [4] is replaced by the average number of bits per symbol
from (4.20).

4.4 Simulation Results and Discussion

A D2D system with 24 subcarriers, each with 15kHz bandwidth, is simulated. Each subcarrier uses a raised-cosine pulse with roll-off factor 0.5 for pulse shaping and the symbol rate is 10kHz. As in Chapter 3, the channel response consists of path loss, shadowing and multipath fading. The path loss is $46.8 + 18.7 \log_{10}(d[m])$, and the shadowing follows the log-normal distribution with standard deviation of 3dB. The subcarriers experience independent Rayleigh fading due to multipath, and the channel response is assumed to be flat within a subcarrier. The two-sided noise power spectral density is $-174$ dBm/Hz. The SER target is $10^{-3}$ in the simulation. The supported modulation formats are 4-QAM, 16-QAM, 64-QAM and 256-QAM. We consider a single cell whose radius is 1000 meters. Similar to Chapter 3, D2D pairs are uniformly distributed in the cell, and the distance between transmitter and receiver in each D2D pair is between 10 and 50 meters.

We use 6 video sequences with a resolution of $640 \times 480$, encoded by H.264/SVC reference software JSVM version 9.19.15. The length of each video sequence is 10 seconds at 30 frames per second, and the video is organized in GOPs of 15 frames (I-P-P-P). The $4 \times 4$ DCT coefficients for the MGS layer of each macroblock are split with MGS vector $[1, 1, 2, 2, 2, 8]$ [27]. The video contents are protected by a rate $2/3$ low density parity check code from DVB/C2 [16]. Each packet consists of 300 bytes of
FEC plus data bits. The video quality is evaluated by PSNR. For the transmission of
the video, the subcarriers are divided into 4 groups, where each group has 6 subcarriers
with a total bandwidth 90kHz.

4.4.1 Simulation Results for Different Methods of Local Search
in Subcarrier Assignment

A cell with \( K = 40 \) D2D pairs is simulated to compare the different methods
of local search. The power constraint for each D2D pair is 10 mW. To evaluate the
complexity of different search methods in subcarrier assignment, 20 realizations of
geographical locations are simulated, each with independent channels and randomly
selected video sequences with random starting points. Under this setting, the tradeoff
between the average video PSNR and the complexity of different local search methods
can be seen in Fig. 4.5. The plot shows the number of times that the power control
algorithm is called, and the corresponding average PSNR in each realization. The
cross-layer initialization generates acceptable average PSNR, but is still more than
0.8dB inferior in the worst case and more than 0.4dB inferior in the best case than
other subcarrier assignment methods. After one round of subcarrier reassignment, the
average PSNR is already very close to the local optimum. Running multiple rounds of
subcarrier reassignment delivers negligible gains in average PSNR, but almost doubles
the complexity. Subcarrier swapping requires a significantly larger computational
effort than subcarrier reassignment, but only leads to marginal gains in average PSNR.
Based on the results at Fig. 4.5, the cross-layer initialization together with a single
round of subcarrier assignment is chosen as the outer loop in our proposed algorithm.
4.4.2 Simulation Results of the Proposed Algorithm Compared to Baseline Algorithms

A cell with $K = 10$ to 80 D2D pairs is simulated. The performance of the proposed cross-layer algorithm is compared with baseline algorithms, as shown in Figures 4.6 - 4.8. The power constraint for each D2D pair is 100, 10 and 1 mW in Figures 4.6 - 4.8, respectively. In Figures 4.6 - 4.8, the proposed algorithm outperforms baseline algorithms by more than 1 dB in the average PSNR. The cross-layer subcarrier assignment with maximal transmission power has the highest average PSNR among the baseline algorithms, but is still more than 1 dB inferior to the proposed algorithm, due to the absence of cross-layer power control. The average PSNR of the heuristic subcarrier assignment algorithm is approximately 2 dB lower than the proposed algorithm. The average PSNR of the PHY-layer algorithm from [4] is more than 2 dB lower than the
Figure 4.6: Average PSNR for the proposed algorithm in comparison with baseline algorithms. The power constraint of each D2D transmitter is 100mW.

The relationship between the average PSNR and the power constraint for 10 D2D pairs is shown in Fig. 4.9. As the power constraint increases, the average PSNR will increase at the beginning, and gradually become flat. The proposed algorithm achieves higher average PSNR than the baseline algorithms over the entire range of...
Figure 4.7: Average PSNR for the proposed algorithm in comparison with baseline algorithms. The power constraint of each D2D transmitter is 10mW.

Figure 4.8: Average PSNR for the proposed algorithm in comparison with baseline algorithms. The power constraint of each D2D transmitter is 1mW.
power constraints simulated. At a low power constraint, e.g., $10^{-5}$ W, the gap in average PSNR between the proposed algorithm and the baseline algorithms is larger than 5dB. For a large power constraint, such as $10^{-1}$ W, under which the average PSNR curves for all algorithms becomes almost flat, the proposed algorithm still outperforms the baseline algorithms by at least 1dB.

### 4.5 Summary

A cross-layer resource allocation algorithm is proposed to optimize the overall quality of D2D video transmission, with an outer loop for subcarrier assignment and an inner loop for power control. The condition under which the cross-layer power control problem for D2D video transmission is convex is derived. The proposed
subcarrier assignment scheme achieves a balance between complexity and performance, when compared to other subcarrier assignment schemes in the simulation. Simulation results also demonstrate a considerable improvement by the proposed cross-layer algorithm over baseline algorithms, including an algorithm that uses only physical layer information.

4.6 Acknowledgement

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Chapter 5

Conclusion and Future Work

In this dissertation, performance analysis and resource allocation in D2D video transmission systems are investigated. Chapter 3 starts with the performance analysis of D2D systems by quantifying the impact of co-channel interference on error probability. These results are compared to those obtained by using a Gaussian approximation (GA) for the aggregated interference. By invoking the conditions under which the central limit theorem holds, and comparing these conditions with the number of interferers and the power ratio of the dominant interferer to the total interference in simulated D2D systems, the reason why the GA for the interference results in pessimistic performance is also studied.

In Chapter 4, resource allocation algorithms for D2D video transmission with a FBMC waveform in a Rayleigh fading environment is investigated. A cross-layer algorithm is proposed with a subcarrier assignment outer loop and a power control inner loop, which aims to optimize the overall video quality. Unlike the non-convexity in physical layer power control for maximizing the total throughput, the cross-layer power control problem is shown to be convex under certain conditions, such as at high
signal-to-interference-plus-noise ratio. Based on the convexity, a high quality solution for power control can be efficiently found. Simulation results demonstrate a significant improvement in overall video quality by the proposed algorithm compared to baseline algorithms.

Future work may be as follows. In contrast to the FMT waveforms illustrated in Chapter 4, in which the spectra of adjacent subcarriers do not overlap, other waveforms can be considered, such as OFDM, or more general FBMC waveforms, e.g., staggered multitone. In these waveforms, the spectra of adjacent subcarriers overlap, so inter-carrier interference exists without synchronization, since synchronization causes overwhelming overhead in D2D systems. With the inter-carrier interference, it is necessary to consider an extension of the cross-layer resource allocation problem in Chapter 4. In addition, with the inter-carrier interference, it becomes interesting to compare the performance of interleaved grouping of subcarriers with that of contiguous grouping of subcarriers. This research topic is currently been investigated.
Appendix A

Proof of Proposition 1

Define random variables

\[ Y_i^I \triangleq \sqrt{P_i T} \sum_{s=-\infty}^{\infty} X_i^I[s] g(-\tau_i - sT), \quad (A.1) \]

and

\[ Y_i^Q \triangleq \sqrt{P_i T} \sum_{s=-\infty}^{\infty} X_i^Q[s] g(-\tau_i - sT), \quad (A.2) \]

where \( X_i^I[s] \triangleq \text{Re}(X_i[s]) \) and \( X_i^Q[s] \triangleq \text{Im}(X_i[s]) \) are the in-phase and quadrature components of the \( s \)-th symbol in the \( i \)-th transmitted QAM signal. Since \( X_i^I[s_1] \) and \( X_i^Q[s_2] \) are independent, \( Y_i^I \) and \( Y_i^Q \) are conditionally independent, given \( \tau_i \). Given \( \tau_i \), the conditional CF of \( Y_i^I \) is

\[
\varphi_{Y_i^I|\tau_i}(u|\tau_i) = E[e^{juY_i^I}|\tau_i] \\
= \prod_{s=-\infty}^{\infty} E\left[ e^{ju\sqrt{P_i T}X_i^I[s]g(-\tau_i - sT)}|\tau_i \right]
\]
\[
\begin{align*}
    &= \prod_{s=-\infty}^{\infty} \frac{1}{\sqrt{M_{i,m}}} \sum_{n=-(\sqrt{M_{i,m}/2}-1)}^{\sqrt{M_{i,m}/2}} e^{j(n-1)g(\tau_i-sT)A_i} \\
    &= \prod_{s=-\infty}^{\infty} \frac{2}{\sqrt{M_{i,m}}} \sum_{n=1}^{\infty} \cos \left( (2n-1)g(\tau_i-sT)A_i \right), \\
\end{align*}
\]

(A.3)

where \(A_i\) is given by (3.15). Let \(I_i^I = \text{Re}(I_i)\) and \(I_i^Q = \text{Im}(I_i)\). From (3.9), \(I_i^I\) are given by

\[
I_i^I = \sqrt{P_i T} \sum_{s=\infty}^{\infty} (X_i^I[s] \cos \phi_i - X_i^Q[s] \sin \phi_i) g(\tau_i-sT) \\
= Y_i^I \cos \phi_i - Y_i^Q \sin \phi_i. \\
\]

(A.4)

Similarly, we obtain that \(I_i^Q = Y_i^I \sin \phi_i + Y_i^Q \cos \phi_i\), and \(\varphi_{Y_i^Q|\tau_i}(u|\tau_i)\) is identical to \(\varphi_{Y_i^Q|\tau_i}(u|\tau_i)\) in (A.3). Since \(X_i^I[s_1]\) and \(X_i^Q[s_2]\) are independent, \(Y_i^I\) and \(Y_i^Q\) are conditionally independent, given \(\tau_i\). Given \(\phi_i\) and \(\tau_i\), the conditional joint CF of \(I_i^I\) and \(I_i^Q\) is given by

\[
\varphi_{I_i^I,I_i^Q|\tau_i,\phi_i}(u,v|\tau_i,\phi_i) \\
= E \left[ \exp \left( j(uI_i^I + vI_i^Q) \right) \bigg| \tau_i, \phi_i \right] \\
= E \left[ \exp \left( jY_i^I(u \cos \phi_i + v \sin \phi_i) + jY_i^Q(v \cos \phi_i - u \sin \phi_i) \right) \bigg| \tau_i, \phi_i \right] \\
= E \left[ \exp \left( jY_i^I(u \cos \phi_i + v \sin \phi_i) \right) \bigg| \tau_i, \phi_i \right] \\
\cdot E \left[ \exp \left( jY_i^Q(v \cos \phi_i - u \sin \phi_i) \right) \bigg| \tau_i, \phi_i \right] \\
= \varphi_{Y_i^I|\tau_i}(u \cos \phi_i + v \sin \phi_i|\tau_i) \cdot \varphi_{Y_i^Q|\tau_i}(v \cos \phi_i - u \sin \phi_i|\tau_i). \\
\]

(A.5)

Substituting (A.3) and \(\varphi_{Y_i^Q|\tau_i}(u|\tau_i) = \varphi_{Y_i^I|\tau_i}(u|\tau_i)\) into (A.5) finishes the proof.
Appendix B

Proof of Proposition 2

The aim of this section is to expand the joint CF of $I^I$ and $I^Q$ ($\varphi_{I^I,I^Q}(u,v)$) into a power series. To proceed, the following lemma is useful:

**Lemma 1.** Consider power series $a(y) = \sum_{n=0}^{\infty} a_n y^n$ and $c(y) = \ln[a(y)]$. The power series expansion of $c(y)$ is denoted by $c(y) = \sum_{n=0}^{\infty} c_n y^n$. If $a_0 = 1$, then a recursive relation for the $\{c_n\}$ is given by $c_0 = 0$ and

$$c_n = a_n - \sum_{p=1}^{n-1} \frac{p}{n} c_p a_{n-p}, \quad \text{(B.1)}$$

for positive $n$. Conversely, if $c_0 = 0$, then $a_0 = 1$ and the following recursive relation for the $\{a_n\}$ holds for positive $n$:

$$a_n = \sum_{p=1}^{n} \frac{p}{n} c_p a_{n-p}. \quad \text{(B.2)}$$

**Proof.** To compute the recurrence relations for the coefficient $\{c_n\}$, we need to differ-
entiate $c(y) = \ln[a(y)]$ with respect to $y$ as follows:

$$\frac{d}{dy}a(y) = a(y) \cdot \frac{d}{dy} \ln[a(y)] = a(y) \cdot \frac{d}{dy}c(y). \quad (B.3)$$

We further differentiate (B.3) with respect to $y$ by using the well known Leibniz’s rule [42, 0.42],

$$\frac{d^n}{dy^n}a(y) = \sum_{q=0}^{n-1} \binom{n-1}{q} \frac{d^q}{dy^q}a(y) \cdot \frac{d^{n-q}}{dy^{n-q}}c(y), \quad n \geq 1. \quad (B.4)$$

It can be shown that $\frac{d^n}{dy^n}a(y)|_{y=0} = n!a_n$. Therefore, let $y = 0$ in (B.4), (B.4) yields

$$n!a_n = \sum_{q=0}^{n-1} \frac{(n-1)!}{q!(n-1-q)!} q!a_q (n-q)!c_{n-q}, \quad n \geq 1, \quad (B.5)$$

which can be further simplified to

$$a_n = \sum_{q=0}^{n-1} \frac{n-q}{n} a_q c_{n-q} = \sum_{p=1}^{n} \frac{p}{n} a_{n-p} c_p, \quad n \geq 1, \quad (B.6)$$

where we substitute $q$ by $n-p$ in the last equality. A simple manipulation gives

$$a_0c_n = a_n - \sum_{p=1}^{n-1} \frac{p}{n} a_{n-p} c_p, \quad n \geq 1. \quad (B.7)$$

Substituting $a_0 = 1$ finishes the proof. \qed

We denote $g_s(\tau_i) = g(-\tau_i - sT)$, $\rho = \sqrt{u^2 + v^2}$ and $\theta = \tan^{-1}(v/u)$. Therefore, $u = \rho \cos \theta$ and $v = \rho \sin \theta$. From Theorem 1, the conditional joint CF of $I^I_i$ and $I^Q_i$,
given $\phi_i$ and $\tau_i$, can be written as

$$
\varphi_{I_i^I, t_i^Q | \tau_i, \phi_i}(u, v | \tau_i, \phi_i) = \prod_{s_1 = -\infty}^{\infty} \frac{2}{\sqrt{M_{i,m}}} \sqrt{M_{i,m}/2} \sum_{j_1 = 1} \cos \left( A_i \rho \cos(\theta - \phi_i)(2j_1 - 1)g_{s_1}(\tau) \right) 
$$

$$
\cdot \prod_{s_2 = -\infty}^{\infty} \frac{2}{\sqrt{M_{i,m}}} \sqrt{M_{i,m}/2} \sum_{j_2 = 1} \cos \left( A_i \rho \sin(\theta - \phi_i)(2j_2 - 1)g_{s_2}(\tau) \right).
$$

To proceed, two auxiliary functions are defined as follows:

$$
f^{(M)}(x, \tau, s) = \frac{2}{\sqrt{M}} \sqrt{M/2} \sum_{j=1} \cos \left( x(2j - 1)g_s(\tau) \right), \quad \text{(B.8)}
$$

$$
\Phi^{(M)}(x, \tau) = \sum_{s = -\infty}^{\infty} \ln \left[ f^{(M)}(x, \tau, s) \right]. \quad \text{(B.9)}
$$

Therefore, the conditional joint CF satisfies

$$
\varphi_{I_i^I, t_i^Q | \tau_i, \phi_i}(u, v | \tau_i, \phi_i) = \prod_{s_1 = -\infty}^{\infty} f^{(M_{i,m})}(A_i \rho \cos(\theta - \phi_i), \tau_i, s_1) 
$$

$$
\cdot \prod_{s_2 = -\infty}^{\infty} f^{(M_{i,m})}(A_i \rho \sin(\theta - \phi_i), \tau_i, s_2) 
$$

$$
= \exp \left[ \Phi^{(M_{i,m})}(A_i \rho \cos(\theta - \phi_i), \tau_i) \right] 
$$

$$
\cdot \exp \left[ \Phi^{(M_{i,m})}(A_i \rho \sin(\theta - \phi_i), \tau_i) \right]. \quad \text{(B.10)}
$$

Using the power series $\cos(y) = \sum_{n=0}^{\infty} (-1)^n / [(2n)!] y^{2n}$ from [42, 1.411 (3)] in (B.8), we obtain

$$
f^{(M)}(x, \tau, s) = \frac{2}{\sqrt{M}} \sqrt{M/2} \sum_{i=1} \sum_{n=0}^{\infty} (-1)^n \frac{1}{(2n)!} (x(2i - 1)g_s(\tau))^{2n} 
$$

$$
= \sum_{n=0}^{\infty} \left[ \frac{1}{(2n)!} \frac{2}{\sqrt{M}} \sqrt{M/2} \sum_{j=1} (2j - 1)^{2n} \right] (-1)^n g_s^{2n}(\tau) x^{2n}. \quad \text{(B.11)}
$$
Let
\[ a_n^{(M)} \triangleq \frac{2}{(2n)!\sqrt{M}} \sum_{j=1}^{\sqrt{M}/2} (2j - 1)^{2n}, \]  
so that from (B.11)
\[ f^{(M)}(x, \tau, s) = \sum_{n=0}^{\infty} a_n^{(M)} (-1)^n g_s^{2n}(\tau) x^{2n}. \]  
It can be verified that \( a_0^{(M)} = 1 \). Invoking Lemma 1 in (B.11), we have
\[ \ln[f^{(M)}(x, \tau, s)] = \sum_{n=0}^{\infty} c_n^{(M)} (-1)^n g_s^{2n}(\tau) x^{2n}, \]  
where \( c_0^{(M)} = 0 \) and the following recursive relation holds for positive integer \( n \):
\[ c_n^{(M)} = a_n^{(M)} - \sum_{p=1}^{n-1} \frac{p}{n} c_p^{(M)} a_{n-p}^{(M)}. \]  
Substituting (B.14) into (B.9) yields
\[ \Phi^{(M)}(x, \tau) = \sum_{s=-\infty}^{\infty} \sum_{n=0}^{\infty} c_n^{(M)} (-1)^n g_s^{2n}(\tau) x^{2n} \]
\[ = \sum_{n=0}^{\infty} \left[ c_n^{(M)} \sum_{s=-\infty}^{\infty} g_s^{2n}(\tau) \right] (-1)^n x^{2n}. \]  
Invoking Lemma 1 in (B.16), we obtain
\[ \exp[\Phi^{(M)}(x, \tau)] = \sum_{n=0}^{\infty} c_n^{(M)} (\tau) (-1)^n x^{2n}. \]
where $\zeta_0^{(M)}(\tau) = 1$ and the following recursive relation holds for positive integer $n$:

$$
\zeta_n^{(M)}(\tau) = \sum_{p=1}^{n} \frac{P}{n} \left[ \zeta_p^{(M)} \sum_{s=-\infty}^{\infty} g_{2p}^2(\tau) \right] \zeta_{n-p}^{(M)}(\tau).
$$

(B.18)

Because (B.18) is a recursive expression, we cannot directly compute the average of $\zeta_n^{(M)}(\tau)$ over $\tau$. We use a method inspired by [12, (45)] to write $\zeta_n^{(M)}(\tau)$ as a function of two components: one is only dependent on $M$ and $g(t)$, and the other one is only dependent on $\tau$. Recall that $g(t)$ is a Nyquist pulse, and let $G_m(f)$ be the Fourier transform of $[g(t)]^m$, $m \geq 1$, given by $G_m(f) = \int_{-\infty}^{\infty} [g(t)]^m e^{-j2\pi ft} dt$.

Regarding $G_m(f)$, we have the following lemma:

**Lemma 2.** $G_m(f) = 0$ for $|f| > m/T$, where $m \geq 1$.

**Proof.** Note that $G_1(f) = G(f)$ is zero for $|f| > 1/T$ according to (2.2). Assume $G_{m_0-1}(f) = 0$ for $|f| > (m_0 - 1)/T$. Since $G_{m_0}(f) = G_{m_0-1}(f) * G_1(f)$, where $*$ stands for convolution, we know $G_{m_0}(f)$ is real-valued, $G_{2n}(-f) = G_{2n}^*(f)$. Therefore, $\sum_{s=-\infty}^{\infty} [g_s(\tau)]^{2n}$ is further given by

$$
\sum_{s=-\infty}^{\infty} [g_s(\tau)]^{2n} = \sum_{s=-\infty}^{\infty} [g(-sT - \tau)]^{2n} = \frac{1}{T} \sum_{s=-\infty}^{\infty} \left[ \frac{G_{2n}^2(sT)}{T} \exp \left( j\frac{2\pi \tau s}{T} \right) \right].
$$

(B.19)

Since $g^{2n}(t)$ is real-valued, $G_{2n}(-f) = G_{2n}^*(f)$. Therefore, $\sum_{s=-\infty}^{\infty} [g_s(\tau)]^{2n}$ is further given by

$$
\sum_{s=-\infty}^{\infty} [g_s(\tau)]^{2n} = \sum_{s=-\infty}^{\infty} \frac{2 - \delta_s}{T} \mathcal{R} \left[ G_{2n} \left( \frac{s}{T} \right) \exp \left( j\frac{2\pi \tau s}{T} \right) \right],
$$

(B.20)
where $\delta_s$ is the Kronecker delta function. Invoking Lemma 2, (B.20) reduces to

$$\sum_{s=-\infty}^{\infty} [g_s(\tau)]^{2n} = \sum_{s=0}^{2n} \frac{2 - \delta_s}{T} \left\{ \text{Re} \left[ G_{2n} \left( \frac{s}{T} \right) \right] \cos \left( \frac{2\pi \tau s}{T} \right) - \text{Im} \left[ G_{2n} \left( \frac{s}{T} \right) \right] \sin \left( \frac{2\pi \tau s}{T} \right) \right\}. \tag{B.21}$$

From [42, 1.331 (1),(3)], $\sin sx = \sum_{l=0}^{s} \binom{s}{l} (-1)^{(l-1)/2} \cos^{s-l} x \sin^l x$ and $\cos sx = \sum_{l=0}^{s} \binom{s}{l} (-1)^{l/2} \cos^{s-l} x \sin^l x$. Therefore, (B.21) yields

$$\sum_{s=-\infty}^{\infty} [g_s(\tau)]^{2n} = \sum_{s=0}^{2n} \sum_{s_1=0}^{s} \cos^{s-s_1} \left( \frac{2\pi \tau}{T} \right) \sin^{s_1} \left( \frac{2\pi \tau}{T} \right) R_n(s, s_1), \tag{B.22}$$

where $R_n(s, s_1)$ is given by (3.26). Comparing (B.18) and (B.22), $\zeta_n^{(M)}(\tau)$ can be written as

$$\zeta_n^{(M)}(\tau) = \sum_{s=0}^{2n} \sum_{s_1=0}^{s} \cos^{s-s_1} \left( \frac{2\pi \tau}{T} \right) \sin^{s_1} \left( \frac{2\pi \tau}{T} \right) w_n^{(M)}(s, s_1), \tag{B.23}$$

where $\{w_n^{(M)}(s, s_1)\}$ is a set of coefficients defined for $0 \leq s_1 \leq s \leq 2n$ whose recursion is to be determined. For $n \geq 1$, substituting (B.23) and (B.22) into the righthand side of (B.18), we have

$$\zeta_n^{(M)}(\tau) = \sum_{p=1}^{n} \frac{p}{n} c_p^{(M)} \left[ \sum_{s=-\infty}^{\infty} g_s^{2p}(\tau) \right] \zeta_{n-p}^{(M)}(\tau) = \sum_{p=1}^{n} \frac{p}{n} c_p^{(M)} \left[ \sum_{s=0}^{2p} \sum_{s_1=0}^{s} \cos^{s-s_1} \left( \frac{2\pi \tau}{T} \right) \sin^{s_1} \left( \frac{2\pi \tau}{T} \right) R_p(s, s_1) \right]$$

$$\cdot \left[ \sum_{s_2=0}^{2(n-p)} \sum_{s_3=0}^{s_2} \cos^{s_2-s_3} \left( \frac{2\pi \tau}{T} \right) \sin^{s_3} \left( \frac{2\pi \tau}{T} \right) w_{n-p}^{(M)}(s_2, s_3) \right]. \tag{B.24}$$
Substituting \( s + s_2 \) with \( q \), \( s_1 + s_3 \) with \( q_1 \), \( s \) with \( q_2 \), and \( s_1 \) with \( q_3 \) in (B.24), we obtain

\[
\zeta_n^{(M)}(\tau) = \sum_{q=0}^{2n} \sum_{q_1=0}^{q} \cos^{q-q_1} \left( \frac{2\pi \tau}{T} \right) \sin^{q_1} \left( \frac{2\pi \tau}{T} \right) 
\cdot \sum_{p=1}^{n} \frac{P_n^{(M)}}{n} \sum_{q_2=\max(q+2p-2n)}^{\min(q_2,q_2)} \sum_{q_3=\max(q_1+q_2-q)}^{\min(q_1+q_2)} R_p(q_2,q_3) w_{n-p}^{(M)}(q-q_2,q_1-q_3).
\]

(B.25)

Comparing (B.23) and (B.25), for \( n \geq 1 \), \( 0 \leq q_1 \leq q \leq 2n \), we have

\[
w_n^{(M)}(q,q_1) = \sum_{p=1}^{n} \frac{P_n^{(M)}}{n} \sum_{q_2=\max(q+2p-2n)}^{\min(q_2,q_2)} \sum_{q_3=\max(q_1+q_2-q)}^{\min(q_1+q_2)} R_p(q_2,q_3) w_{n-p}^{(M)}(q-q_2,q_1-q_3).
\]

(B.26)

Substituting \( n = 0 \) and \( \zeta_0^{(M)}(\tau) = 1 \) into (B.23), we obtain \( w_0^{(M)}(0,0) = 1 \). Therefore, the recursive relation of \( \{w_n^{(M)}(q,q_1)\} \) is given by \( w_0^{(M)}(0,0) = 1 \) and (B.26).

Also, substituting (B.17) into (B.10) yields

\[
\varphi_{I^I,t^Q | \tau_i,\phi_i}(u,v|\tau_i,\phi_i) = \sum_{p=0}^{\infty} \zeta_p^{(M)}(\tau_i) \cos^{2p}(\theta - \phi_i)(-A_i^2 \rho^2)^p 
\cdot \sum_{m=0}^{\infty} \zeta_m^{(M)}(\tau_i) \sin^{2m}(\theta - \phi_i)(-A_i^2 \rho^2)^m.
\]

(B.27)

Using the Cauchy product [48, p.10] of two power series, \( \varphi_{I^I,t^Q | \tau_i,\phi_i}(u,v|\tau_i,\phi_i) \) can be further written as

\[
\varphi_{I^I,t^Q | \tau_i,\phi_i}(u,v|\tau_i,\phi_i) = \sum_{n=0}^{\infty} \xi_n^{(M,\nu)}(u,v,\tau_i,\phi_i)(-A_i^2 \rho^2)^n.
\]

(B.28)
where $\xi^{(M_i,m)}_n(u, v, \tau_i, \phi_i)$ is given by

$$
\xi^{(M_i,m)}_n(u, v, \tau_i, \phi_i) = \sum_{p=0}^{n} \xi^{(M_i,m)}_p(\tau_i) \xi^{(M_i,m)}_{n-p}(\tau_i) \cos^{2p}(\theta - \phi_i) \sin^{2(n-p)}(\theta - \phi_i). \quad (B.29)
$$

Substituting (B.23) into (B.29) yields

$$
\xi^{(M_i,m)}_n(u, v, \tau_i, \phi_i) = \sum_{p=0}^{n} \cos^{2p}(\theta - \phi_i) \sin^{2(n-p)}(\theta - \phi_i)
\cdot \sum_{s=0}^{2p} \sum_{s_1=0}^{s} \cos^{s-s_1}\left(\frac{2\pi \tau_i}{T}\right) \sin^{s_1}\left(\frac{2\pi \tau_i}{T}\right) w^{(M_i,m)}_p(s, s_1)
\cdot \sum_{s_2=0}^{2(n-p)} \sum_{s_3=0}^{s_2-s_1} \cos^{s_2-s_3}\left(\frac{2\pi \tau_i}{T}\right) \sin^{s_3}\left(\frac{2\pi \tau_i}{T}\right) w^{(M_i,m)}_{n-p}(s_2, s_3). \quad (B.30)
$$

Substituting $s + s_2$ with $q$, $s_1 + s_3$ with $q_1$, $s$ with $q_2$ and $s_1$ with $q_3$ yields

$$
\xi^{(M_i,m)}_n(u, v, \tau_i, \phi_i)
= \sum_{p=0}^{n} \cos^{2p}(\theta - \phi_i) \sin^{2(n-p)}(\theta - \phi_i) \sum_{q_1=0}^{2n} \sum_{q=0}^{q_1} \cos^{q_1-q_2}\left(\frac{2\pi \tau_i}{T}\right) \sin^{q_1}\left(\frac{2\pi \tau_i}{T}\right)
\cdot \sum_{q_2=\max(0,q+2p-2n)}^{\min(q,2p)} \sum_{q_3=\max(0,q_1+q_2-q)}^{\min(q_1,q_2)} w^{(M_i,m)}_p(q_2, q_3) w^{(M_i,m)}_{n-p}(q-q_2, q_1-q_3).
$$

(B.31)
Notice that $2\pi \tau_i/T$ and $\phi_i$ are both uniformly distributed in $[0, 2\pi)$. We define
\[ B_{m,n} = \frac{1}{2\pi} \int_0^{2\pi} \cos^m \phi \sin^n \phi d\phi, \]
whose value is given by [42, 3.621 (5)]
\[ B_{m,n} = \begin{cases} 
(\frac{m-1}{2})!(\frac{n-1}{2})! & , \ m,n \geq 0, \ m,n \text{ even}, \\
0 & , \text{otherwise}. 
\end{cases} \quad (B.32) \]

With the definition of \{\(B_{m,n}\)\}, the expectation of (B.31) is given by
\[ E_{\tau_i,\phi_i}[\xi(M_i,m)_{n}(u,v,\tau_i,\phi_i)] = \sum_{p=0}^{n} B_{2p,2(n-p)} \sum_{t=0}^{2n} \sum_{q_1=0}^{q} B_{q_1,q-q_1} \cdot \sum_{q_2=\max(0,q+2p-2n)}^{\min(q,2p)} \sum_{q_3=\max(0,q_1+q_2-t)}^{\min(q_1,q_2)} w^{(M_i,m)}_{p}(q_2,q_3) \cdot w^{(M_i,m)}_{n-p}(q-q_2,q_1-q_3). \quad (B.33) \]

From (B.28), we can express the joint CF of $I_i^I$ and $I_i^Q$ as
\[ \varphi_{I_i^I,I_i^Q}(u,v) = E_{\tau_i,\phi_i}[\varphi_{I_i^I,I_i^Q}\mid \tau_i,\phi_i](u,v|\tau_i,\phi_i) ] \]
\[ = \sum_{n=0}^{\infty} E_{\tau_i,\phi_i}[\xi^{(M_i,m)}(u,v,\tau_i,\phi_i)](-A_i^2 \rho^2)^n \]
\[ = \sum_{n=0}^{\infty} \left[ \sum_{p=0}^{n} B_{2p,2(n-p)} \sum_{q=0}^{2n} \sum_{q_1=0}^{q} B_{q_1,t-q_1} \cdot \sum_{q_2=\max(0,q+2p-2n)}^{\min(q,2p)} \sum_{q_3=\max(0,q_1+q_2-t)}^{\min(q_1,q_2)} w^{(M_i,m)}_{p}(q_2,q_3)w^{(M_i,m)}_{n-p}(t-q_2,q_1-q_3) \right] \cdot \left( -\frac{3P_i T}{M_i,m - 1} (u^2 + v^2) \right)^n, \quad (B.34) \]

where the last equality is from (3.15), (B.33) and $\rho = \sqrt{u^2 + v^2}$. 

Invoking Lemma 1 in (B.34), the logarithm of the joint CF is given by

\[
\ln \left[ \phi_{I_i I_i \tilde{Q}}(u, v) \right] = \sum_{n=0}^{\infty} d_n^{(M_{i,m})} \left[ -P_i T (u^2 + v^2) \right]^n.
\] (B.35)

The recursive relations of coefficients \( \{d_n^{(M_{i,m})}\} \) are given by invoking Lemma 1 in (B.35) and (B.34). Substituting (B.35) into (3.18), the joint CF of the aggregated CCI is given by

\[
\phi_{I_i I_i \tilde{Q}}(u, v) = \exp \left\{ \sum_{n=0}^{\infty} \left[ \sum_{i=1, i \neq k}^{K} \left( -P_i T \right)^n d_n^{(M_{i,m})} \right] (u^2 + v^2)^n \right\}
\]

Applying Lemma 1 to the last equality finishes the proof.
Appendix C

Convergence Analysis for Proposed SER Expression

We derive an upper bound for the magnitude of the truncation error for the proposed SER expression in (3.43). Suppose we use the terms with less than 2L-th order in the power series expansion of the total CCI’s CF for SER calculation. According to the Taylor theorem in multivariate functions [49, p.360, Theorem 13.3], the magnitude of error incurred by truncation is given by

$$\left| \varphi_{II, IQ}(u, v) - \sum_{n=0}^{L-1} b^{(k,m)} (-N_0)^n (u^2 + v^2)^n \right|$$

$$\leq \frac{(u^2 + v^2)^L}{(2L)!} \max_{l=0,1,2L, \ldots} \left| \frac{\partial^{2L}}{u_0^{l}v_0^{2L-l}} \varphi_{II, IQ}(u_0, v_0) \right|.$$  \hspace{1cm} (C.1)

The joint probability density function of $I^I$ and $I^Q$ is denoted by $p_{II, IQ}(x, y)$. Since $p_{II, IQ}(x, y)$ is the Fourier transform of $\varphi_{II, IQ}(u_0, v_0)$, it is well known that $(jx)^l(jy)^{2L-l}p_{II, IQ}(x, y)$ is the Fourier transform of $\frac{\partial^{2L}}{u_0^{l}v_0^{2L-l}} \varphi_{II, IQ}(u_0, v_0)$ [39, p.16,
Making use of this Fourier transform pair, we have

\[
\left| \frac{\partial^{2L}}{\partial u_0 \partial v_0^{2L-1}} \varphi_{P^I, I^Q}(u_0, v_0) \right|
\leq \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |x|^{2L-1} p_{P^I, I^Q}(x, y)\, dydx
\]

\[
= E \left[ |I^I|^l |I^Q|^{2L-l} \right]
\leq E \left[ |I|^{2L} \right],
\]

(C.2)

where the last inequality follows from \(|I^I| = |\text{Re}(I)| \leq |I| and |I^Q| = |\text{Im}(I)| \leq |I|.

Recall that, in (3.9), the aggregated CCI is given by

\[
I = \sum_{i=1, i \neq k}^{K} \sqrt{P_i T} \sum_{s=-\infty}^{\infty} X_i[s] g(-\tau_i - sT),
\]

and the magnitude of the received QAM symbol is bounded using (3.15) by

\[
\sqrt{P_i T} |X_i[s]| \leq \sqrt{2}(\sqrt{M_i,m} - 1)A_i = \sqrt{2}(\sqrt{M_i,m} - 1) \sqrt{\frac{3P_i T}{M_i,m - 1}} \leq \sqrt{6P_i T},
\]

(C.3)

so the magnitude of the aggregated CCI is bounded by

\[
|I| \leq \sum_{k=1}^{K} \sqrt{P_k T} \sum_{s=-\infty}^{\infty} |X_k[s]| \cdot |g(-\tau_k - sT)|
\leq \sum_{i=1, i \neq k}^{K} \max_{s'}(\sqrt{P_i T} |X_i[s']|) \sum_{s=-\infty}^{\infty} |g(-\tau_i - sT)|
\leq \sum_{i=1, i \neq k}^{K} \sqrt{6P_i T} \cdot C,
\]

(C.4)
where the maximum inter-symbol interference for the Nyquist pulse is defined as

\[
C = \sup_{0 < \tau < T} \left\{ \sum_{s=-\infty}^{\infty} |g(\tau - sT)| \right\}.
\]  

Using the results in [34], \(C\) is shown to be a finite constant for the family of Nyquist pulses that are defined in (2.2). A special case of \(C\) is given by Proposition 4.

**Proposition 4.** For a raised-cosine pulse with roll-off factor \(\beta = 0.5\), \(C = 10\sqrt{2}/(3\pi)\).

**Proof.** From [34], the raised-cosine pulse is given by

\[
g(t) = \text{sinc} \left( \frac{\pi t}{T} \right) \cdot \cos \left( \frac{\pi \beta t}{T} \right) \cdot \frac{1}{1 - \frac{4\beta^2}{T^2}}.
\]  

(C.6)

For \(\beta = 0.5\),

\[
g(-\tau - sT) = g((a + s)T) = \{\sin(\pi a)/(\pi(a + s))\} \cdot \{\cos((\pi/2)(a + s))/[1 - (a + s)^2]\},
\]  

(C.7)

where \(a = \tau/T\) and \(0 < a < 1\). Letting

\[
\psi(a, p) \triangleq \frac{1}{(a + p)[(a + p)^2 - 1]},
\]  

(C.8)

we have

\[
\sum_{s=-\infty}^{\infty} |g((a + s)T)| = \frac{\sin(\pi a)}{\pi} \left[ \cos \left( \frac{\pi a}{2} \right) \sum_{n=-\infty}^{\infty} |\psi(a, 2n)| + \sin \left( \frac{\pi a}{2} \right) \sum_{l=-\infty}^{\infty} |\psi(a, 2l + 1)| \right].
\]  

(C.9)
Notice that
\[
\psi(a, p) = \frac{1}{2(a + p - 1)} - \frac{1}{a + p} + \frac{1}{2(a + p + 1)}. \tag{C.10}
\]

For \( n \geq 1, \psi(a, 2n) > 0 \) and \( \psi(a, 2n) < 0 \) otherwise. For \( l \geq -1, \psi(a, 2l + 1) > 0 \) and \( \psi(a, 2l + 1) < 0 \) otherwise. Therefore, (C.9) reduces to
\[
\sum_{s=-\infty}^{\infty} |g((a + s)T)| = \frac{\sin(\pi a)}{\pi} \left[ \cos \left( \frac{\pi}{2} a \right) \left( \sum_{n_1=0}^{\infty} \frac{(-1)^{n_1}}{a - n_1} - \sum_{n_2=2}^{\infty} \frac{(-1)^{n_2}}{a + n_2} \right) \right. \\
+ \left. \sin \left( \frac{\pi}{2} a \right) \left( \sum_{l_1=-1}^{\infty} \frac{(-1)^{l_1}}{a + l_1} - \sum_{l_2=3}^{\infty} \frac{(-1)^{l_2}}{a - l_2} \right) \right]. \tag{C.11}
\]

We can verify that the maximum of (C.11) is achieved at \( a = 1/2 \). Substituting \( a = 1/2 \), (C.11) yields
\[
C = \sum_{s=-\infty}^{\infty} \left| g \left( \left( \frac{1}{2} + s \right) T \right) \right| \\
= \frac{1}{\pi} \left[ \frac{\sqrt{2}}{2} \sum_{n=0}^{2} \frac{(-1)^n}{\frac{1}{2} - n} + \frac{\sqrt{2}}{2} \sum_{l=-1}^{1} \frac{(-1)^l}{\frac{1}{2} + l} \right] \\
= \frac{10\sqrt{2}}{3\pi}. \tag{C.12}
\]

Further substitute (C.2) and (C.4) into (C.1) to obtain
\[
\left| \varphi_{\mu, \nu}(u, v) - \sum_{n=0}^{L-1} b_n^{(k, m)} (-N_0)^n (u^2 + v^2)^n \right| \leq \frac{(u^2 + v^2)^L}{(2L)!} \left( C \sum_{i=1, i \neq k}^{K} \sqrt{6P_i T} \right)^{2L}. \tag{C.13}
\]

Let \( \Pr(B_1; 2L) \) be the truncated expression obtained by retaining the terms
with order less than $2L$ in the expression $\Pr(B_1)$ in (3.38). The magnitude of the truncation error is upper bounded by

\[
\left| \Pr(B_1) - \Pr(B_1; 2L) \right| = \frac{1}{\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\sin(A_ku) \sin(A_kv)}{uv} \left[ \varphi_{1,1/0}(u,v) - \sum_{n=0}^{L-1} b_n^{(k,m)} (-N_0)^n (u^2 + v^2)^n \right] e^{-\frac{(u^2+v^2)N_0}{2}} \, du \, dv
\]

\[
< \frac{A_k^2}{\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left| \varphi_{1,1/0}(u,v) - \sum_{n=0}^{L-1} b_n^{(k,m)} (-N_0)^n (u^2 + v^2)^n \right| e^{-\frac{(u^2+v^2)N_0}{2}} \, du \, dv
\]

\[
\leq \frac{A_k^2}{\pi^2} \left( \frac{1}{(2L)!} \right) \left( C \sum_{i=1, i \neq k}^{K} \sqrt{6P_i T} \right)^{2L} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (u^2 + v^2)^L e^{-\frac{(u^2+v^2)N_0}{2}} \, du \, dv,
\]

(C.14)

which follows from $|\sin(ax)/x| \leq |a|$ and (C.13). By substituting $u$ with $\sqrt{r} \cos \varphi$ and $v$ with $\sqrt{r} \sin \varphi$, we have

\[
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (u^2 + v^2)^L \exp \left[ -\frac{(u^2 + v^2)N_0}{2} \right] \, du \, dv
\]

\[
= \int_{0}^{2\pi} \int_{0}^{\infty} r^L \exp \left( -\frac{rN_0}{2} \right) \frac{1}{2} \, dr \, d\varphi
\]

\[
= \frac{2^{L+1} \pi L}{N_0^{L+1}},
\]

(C.15)

where the last equality is from [42, 3.381(4)]. Then (C.14) reduces to

\[
|\Pr(B_1) - \Pr(B_1; 2L)| < \frac{2A_k^2}{\pi N_0} \left( C \sum_{i=1, i \neq k}^{K} \sqrt{6P_i T} \frac{1}{N_0} \right)^{2L} \frac{1}{(2L-1)!!}
\]

(C.16)
Similarly, the truncation error for \( \text{Pr}(B_2) \) from (3.39) is bounded by

\[
\left| \text{Pr}(B_2) - \text{Pr}(B_2; 2L) \right| = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\sin(A_k u)}{u} \left( \varphi_{M, Q}(u, 0) - \sum_{m=0}^{L-1} b_m (-N_0)^m u^{2m} \right) e^{-\frac{u^2 N_0}{2}} \, du
\]

\[
< \frac{A_k}{\pi (2L)!} \left( C \sum_{k=1}^{K} \sqrt{6 P_k T} \right)^{2L} \int_{-\infty}^{\infty} u^{2L} e^{-\frac{u^2 N_0}{4}} \, du
\]

\[
= \frac{A_k}{\pi (2L)!} \left( C \sum_{i=1, i \neq k}^{K} \sqrt{6 P_i T} \right)^{2L} \int_{0}^{\infty} z^{L-\frac{1}{2}} e^{-\frac{z N_0}{2}} \, dz
\]

\[
= \frac{A_k}{\pi (2L)!} \left( C \sum_{i=1, i \neq k}^{K} \sqrt{6 P_i T} \right)^{2L} \left( \frac{2}{N_0} \right)^{L+\frac{1}{2}} \left( 2L - 1 \right)!! \sqrt{\pi}
\]

\[
= \sqrt{2 A_k^2 \pi N_0} \left( C \sum_{i=1, i \neq k}^{K} \sqrt{\frac{6 P_i T}{N_0}} \right)^{2L} \left( \frac{1}{(2L)!!} \right), \tag{C.17}
\]

which follows from \( |\sin(ax)/x| \leq |a| \) and (C.13). We also substitute \( u^2 \) with \( z \) and use [42, 3.381(4)] in (C.17). From (3.31), the truncation error on the SER is bounded by

\[
\left| \text{SER}_{k,m}^{\text{exact}} - \text{SER}_{k,m}^{(2L)} \right|
\]

\[
\leq \left( \sqrt{M_{k,m} - 1} \right)^2 \left| \text{Pr}(B_1) - \text{Pr}(B_1; 2L) \right| + \frac{2 (\sqrt{M_{k,m} - 1})}{M_{k,m}} \left| \text{Pr}(B_2) - \text{Pr}(B_2; 2L) \right|
\]

\[
\leq \frac{M_{k,m} - 1}{M_{k,m}} \left| \text{Pr}(B_1) - \text{Pr}(B_1; 2L) \right| + \frac{2 (\sqrt{M_{k,m} - 1})}{M_{k,m}} \left| \text{Pr}(B_2) - \text{Pr}(B_2; 2L) \right|, \tag{C.18}
\]

where the last inequality is due to \( (\sqrt{M_{k,m} - 1})^2 \leq M_{k,m} - 1 \) for positive \( M_{k,m} \).

Substituting (C.16), (C.17) and (3.15) into (C.18), we obtain

\[
\left| \text{SER}_{k,m}^{\text{exact}} - \text{SER}_{k,m}^{(2L)} \right| < \frac{1}{M_{k,m}} \sqrt{\frac{6 P_k T}{\pi N_0}} \left( C \sum_{i=1, i \neq k}^{K} \sqrt{\frac{6 P_i T}{N_0}} \right)^{2L} \cdot \left[ \sqrt{\frac{6 P_k T}{\pi N_0}} + \frac{2}{(2L)!!} \right]. \tag{C.19}
\]
Appendix D

Proof of Proposition 3

We show the convexity of $1/[f(e^{-s}) + c]$ in $s$ by deriving its second derivative.

Define $y$ as

$$y \triangleq e^{-s},$$  \hspace{1cm} (D.1)

where $y > 0$, so that $dy/ds = -y$ and $d^2y/ds^2 = y$. Since $f(y)$ is monotonically decreasing in $y$, $f'(y) < 0$. From (B.8), the first derivative of $f(y)$ with respect to $y$ is given by

$$f'(y) = \log_2(e) \frac{d}{dy}[e^y]E_1(y) + \log_2(e) e^y \frac{d}{dy}[E_1(y)] = f(y) - \log_2(e) \frac{1}{y},$$  \hspace{1cm} (D.2)

since $\frac{d}{dy}E_1(y) = -e^{-y}/y$. The second derivative of $f(y)$ with respect to $y$ is given by

$$f''(y) = \frac{d}{dy} \left[ f(y) - \log_2(e) \frac{1}{y} \right] = f'(y) + \log_2(e) \frac{1}{y^2} = f(y) + \log_2(e) \frac{1 - y}{y^2}.$$  \hspace{1cm} (D.3)

Ignoring the constant terms, the first and second derivatives of the objective
function with respect to $y$ are given by

$$
\frac{d}{dy} \left\{ \frac{1}{f(y) + c} \right\} = -\frac{f'(y)}{[f(y) + c]^2}
$$

(D.4)

and

$$
\frac{d^2}{dy^2} \left\{ \frac{1}{f(y) + c} \right\} = -\frac{d}{dy} \left[ \frac{f'(y)}{[f(y) + c]^2} \right]
= \frac{2[f(y) + c][f'(y)]^2 - f''(y)[f(y) + c]^2}{[f(y) + c]^4}
= \frac{2[f'(y)]^2 - f''(y)[f(y) + c]}{[f(y) + c]^3}.
$$

(D.5)

Therefore, the second derivative of the objective function with respect to $s$ is given by

$$
\frac{d^2}{ds^2} \left\{ \frac{1}{f(y) + c} \right\} = \frac{d^2y}{ds^2} \cdot \frac{1}{f(y) + c} + \left( \frac{dy}{ds} \right)^2 \cdot \frac{d^2}{dy^2} \left\{ \frac{1}{f(y) + c} \right\}
= -y \cdot \frac{f'(y)}{[f(y) + c]^2} + y^2 \cdot \frac{2[f'(y)]^2 - f''(y)[f(y) + c]}{[f(y) + c]^3}.
$$

(D.6)

Thus, $1/[f(e^{-s}) + c]$ is convex in $s$ if and only if

$$
\frac{d^2}{ds^2} \left\{ \frac{1}{f(y) + c} \right\} > 0,
$$

which is equivalent to

$$
2y[f'(y)]^2 > [f(y) + c] \cdot [f'(y) + yf''(y)].
$$

(D.8)
according to (D.6). We can rewrite (D.8) using (D.2) and (D.3) as

\[
2y \left[ f(y) - \log_2(e) \frac{1}{y} \right]^2 > \left[ f(y) + c \right] \cdot \left[ f(y) - \log_2(e) \frac{1}{y} + y \left( f(y) + \log_2(e) \frac{1 - y}{y^2} \right) \right].
\]

(D.9)

For simplicity, let

\[
u \triangleq \frac{f(y)}{\log_2(e)} = \ln(2) f(y) = e^y E_1(y),
\]

(D.10)

where \( f(y) > 0 \) according to (B.8) and the last equality is from (B.8). Also, we let

\[
\tilde{c} \triangleq \frac{c}{\log_2(e)} = c \ln(2).
\]

(D.11)

With (D.10) and (D.11), we can rewrite (D.9) as

\[
2y \left( u - \frac{1}{y} \right)^2 > (u + \tilde{c}) \cdot \left[ u - \frac{1}{y} + y \left( u + \frac{1 - y}{y^2} \right) \right],
\]

(D.12)

which is equivalent to

\[
h(u, \tilde{c}) \triangleq y(y - 1)u^2 - [3 + (y + 1)\tilde{c}]yu + \tilde{c}y + 2 > 0.
\]

(D.13)

For \( y \neq 1 \), \( h(u, \tilde{c}) \) is a quadratic function in \( u \), whose discriminant is given by

\[
\Delta = [3 + (y + 1)\tilde{c}]^2 y^2 - 4y(y - 1)(\tilde{c}y + 2)
\]

\[= y(\tilde{c}^2 y^3 + \tilde{c}^2 y + y + 2\tilde{c}y^2 + 10\tilde{c}y + 2\tilde{c}^2 y^2 + 8).\]

(D.14)
Since \( y > 0 \) and \( \tilde{c} > 0 \), the discriminant \( \Delta > 0 \).

The roots of \( h(u, \tilde{c}) = 0 \) in \( u \) are given by

\[
\begin{align*}
  u_1 &= \frac{[(y + 1)\tilde{c} + 3]y - \sqrt{\Delta}}{2y(y - 1)}, \\
  u_2 &= \frac{[(y + 1)\tilde{c} + 3]y + \sqrt{\Delta}}{2y(y - 1)},
\end{align*}
\]  

(D.15)

where \( u_1 \) and \( u_2 \) are both functions of \( y \).

From (D.13), \( h(u, \tilde{c}) \) can be written as

\[ h(u, \tilde{c}) = Au^2 + Bu + C, \]

where \( A \triangleq y(y - 1), B \triangleq -[(y + 1)\tilde{c} + 3]y \) and \( C \triangleq \tilde{c}y + 2 \). Since \( u = e^yE_1(y) > 0 \), we are interested in the range of \( u \) which satisfies not only \( h(u, \tilde{c}) > 0 \) but also \( u > 0 \). Therefore, the range of \( u \) where \( h(u, \tilde{c}) > 0, u > 0 \) is given as follows:

1. If \( y > 1 \), then \( A > 0, B < 0 \) and \( C > 0 \), so \( u_1 + u_2 = -B/A > 0 \) and \( u_1u_2 = C/A > 0 \) [50, pp. 17, 3.8.1]. Therefore, \( u_1 \) and \( u_2 \) are positive. Since \( 0 < u_1 < u_2 \) in this case, the range of \( u \) where \( h(u, \tilde{c}) > 0, u > 0 \) is \( \{u | 0 < u < u_1 \text{ or } u > u_2 \} \);

2. If \( 0 < y < 1 \), then \( A < 0 \) and \( C > 0 \), so \( u_1u_2 = C/A < 0 \) [50, pp. 17, 3.8.1]. Therefore, \( u_1 \) and \( u_2 \) have different signs. Since \( u_1 > u_2 \) for \( 0 < y < 1 \), so \( u_1 > 0 > u_2 \) in this case. Therefore, the range of \( u \) where \( h(u, \tilde{c}) > 0, u > 0 \) is \( \{u | 0 < u < u_1 \} \);

3. If \( y = 1 \), \( h(u, \tilde{c}) > 0 \) simplifies to \( u < (\tilde{c} + 2)/(2\tilde{c} + 3) \). We can show that \( \lim_{y \to 1} u_1 = (\tilde{c} + 2)/(2\tilde{c} + 3) \), so when \( y = 1 \), the range of \( u \) where \( h(u, \tilde{c}) > 0, u > 0 \) is \( \{u | 0 < u < u_1 \} \).

Therefore, the solution set of \( u \) for \( h(u, \tilde{c}) > 0, u > 0 \) is \( \{u | 0 < u < u_1 \} \cup \{u | y > 1, u > u_2 \} \).

The next step is to convert the solution set of \( u \) to the solution set of \( y \), using the fact that \( u, u_1 \) and \( u_2 \) are functions of \( y \). In fact, from [50, pp. 229, 5.1.19], the
following inequality holds for \( y > 0 \):

\[
  u = e^y E_1(y) \leq \frac{1}{y}.
\]  

(D.16)

Also, the following inequality holds for \( y > 1 \):

\[
  u_2 = \frac{[(y + 1)c + 3]y + \sqrt{\Delta}}{2y(y - 1)} > \frac{3y}{2y(y - 1)} > \frac{3(y - 1)}{2y(y - 1)} = \frac{3}{2y} > \frac{1}{y}.
\]

(D.17)

Combining (D.16) and (D.17), \( u < u_2 \) always holds \( \forall y > 1 \), so \( \{ u | y > 1, u > u_2 \} = \emptyset \). Consequently, the range of \( u \) where \( h(u, \tilde{c}) > 0, u > 0 \) equals to

\[
\{ u | h(u, \tilde{c}) > 0, u > 0 \} \\
= \{ u | 0 < u < u_1, y > 1 \} \cup \{ u | u > u_2, y > 1 \} \\
\cup \{ u | 0 < u < u_1, 0 < y < 1 \} \cup \{ u | 0 < u < u_1, y = 1 \} \\
= \{ u | 0 < u < u_1, y > 1 \} \cup \emptyset \cup \{ u | 0 < u < u_1, 0 < y < 1 \} \cup \{ u | 0 < u < u_1, y = 1 \} \\
= \{ u | 0 < u < u_1, y > 0 \}.
\]

(D.18)

Let \( y^* \) be the smallest root of \( u(y) = u_1(y) \) in \( y > 0 \). From \( u \triangleq e^y E_1(y) \) and (D.15), \( y^* \) is equivalently the smallest root of

\[
e^y E_1(y) = \frac{[(y + 1)c \ln(2) + 3]y - \sqrt{\Delta}}{2y(y - 1)}
\]

(D.19)

in \( y > 0 \). Such a root \( y^* \) always exists for the following reason:

1. \( u < u_1 \) for \( y \to 0^+ \), where \( y \to 0^+ \) means that \( y \) approaches 0 from above. On the one hand, \( e^y E_1(y) \leq \ln(1 + 1/y) \) from [50, pp. 229, 5.1.20] and \( u \triangleq e^y E_1(y) \),
therefore,

\[
\lim_{y \to 0^+} (\sqrt{y}u) \leq \lim_{y \to 0^+} [\sqrt{y} \ln(1 + 1/y)]^{z = \ln(1+1/y)} = \lim_{z \to +\infty} \frac{z}{\sqrt{e^z - 1}} = \lim_{z \to +\infty} \frac{2\sqrt{e^z - 1}}{e^z} = 0. \tag{D.20}
\]

On the other hand, \(\lim_{y \to 0^+} \sqrt{y}u_1 = \sqrt{2}\) by substituting (D.14) into (D.15):

\[
\lim_{y \to 0^+} (\sqrt{y}u_1) = \lim_{y \to 0^+} \sqrt{y} \cdot \frac{\tilde{c}y^2 + (\tilde{c} + 3)y}{2y(-1)} - \lim_{y \to 0^+} \sqrt{y} \cdot \frac{-\sqrt{y[\tilde{c}^2y^3 + 2(\tilde{c} + \tilde{c}^2)y^2 + 10\tilde{c}y + \tilde{c}^2y + y + 8]}}{2y(-1)}
= \sqrt{2}. \tag{D.21}
\]

Therefore, \(\sqrt{y}u < \sqrt{y}u_1\) for \(y \to 0^+\), and thereby \(u < u_1\) for \(y \to 0^+\).

2. For \(y \to +\infty\), \(u > u_1\). On the one hand, from [50, pp. 229, 5.1.19], \(u = e^yE_1(y) > 1/(y + 1)\). Therefore, for \(y \to +\infty\), \(u\) is bounded by

\[
u > \frac{1}{y + 1} = \frac{y - 1}{y^2 - 1} > \frac{y - 1}{y^2} = 1 - \frac{1}{y^2}. \tag{D.22}
\]

On the other hand, for \(y \to +\infty\), substituting (D.14) into (D.15), \(u_1\) is given by

\[
u_1 = \frac{\tilde{c}y^2 + (\tilde{c} + 3)y - \sqrt{y[\tilde{c}^2y^3 + 2(\tilde{c} + \tilde{c}^2)y^2 + (1 + \tilde{c})^2y + 8\tilde{c}y + 8]}}{2y(y - 1)}
= \frac{\tilde{c}y^2 + (\tilde{c} + 3)y - \sqrt{y^2(\tilde{c}y + 1 + \tilde{c})^2 + 8y(\tilde{c}y + 1)}}{2y(y - 1)}
= \frac{\tilde{c}y^2 + (\tilde{c} + 3)y - y(\tilde{c}y + 1 + \tilde{c})\sqrt{1 + \frac{8y(\tilde{c}y + 1)}{y^2(\tilde{c}y + 1 + \tilde{c})^2}}}{2y(y - 1)}. \tag{D.23}
\]
Because \([8y(\tilde{c}y + 1)]/y^2(\tilde{c}y + 1 + \tilde{c})^2] \ll 1\),

\[
\begin{align*}
\frac{\tilde{c}y^2 + (\tilde{c} + 3)y - y(\tilde{c}y + 1 + \tilde{c})(1 + \frac{1}{2} \cdot \frac{8(\tilde{c}y+1)}{y(\tilde{c}y+1+\tilde{c})^2})}{2y^2} &
\approx \frac{2y - 4}{2y^2} \\
&= \frac{1}{y} - \frac{2}{y^2}.
\end{align*}
\]  

(D.24)

Combining (D.22) and (D.24), we have \(u > u_1\) for \(y \to \infty\).

Because \(u > u_1\) for \(y \to +\infty\) and \(u < u_1\) for \(y \to 0^+\), there must exist a \(y^* > 0\) such that \(u(y^*) = u_1(y^*)\), and \(u(y) < u_1(y)\) for any \(y\) satisfying \(0 < y < y^*\). Since \(c\) is the only variable besides \(y\) in (D.19), \(y^*\) is only dependent on \(c\). From (D.10), \(u(y) = \ln(2) f(y)\), where \(f(y) > 0\) is defined in (B.8), so \(0 < u(y) < u_1(y)\) for any \(y\) satisfying \(0 < y < y^*\). Denoting \(s^* \triangleq -\ln(y^*)\), and recalling that \(y \triangleq e^{-s}\) from (D.1), \(0 < y < y^*\) is equivalent to \(s > s^*\). Therefore, for any \(s\) satisfying \(s > s^*\), the range of \(u\) satisfies \(0 < u(y) < u_1(y)\) and \(y > 0\), where \(y > 0\) is guaranteed by its definition in (D.1) and always holds.

From (D.18), the range of \(u\) such that \(y > 0\), \(0 < u(y) < u_1(y)\) holds is equivalent to the range of \(u\) such that \(h(u(y), \tilde{c}) > 0\), \(u(y) > 0\) holds, where \(u(y) > 0\) is guaranteed by (D.10) and always holds. According to (D.7) - (D.9), (D.12) and (D.13), \(h(u(y), \tilde{c}) > 0\) is equivalent to the second derivative of \(1/[f(e^{-s}) + c]\) with respect to \(s\) being positive, which equals to \(1/[f(e^{-s}) + c]\) being convex in \(s\). Therefore, \(1/[f(e^{-s}) + c]\) is convex in \(s\) over \(s > s^*\).
Bibliography


