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Publication Date

1963-04-29

UCRL-10775
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Berkeley, California

Rept. for meeting of 3rd Conf. on Reactions Between Complex Nuclei, Asilomar.

This rept. also included in the pub. proceedings of the meeting.

UCRL-10775

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Contract No. W-7405-eng-48

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ABSTRACT

Equilibrium configurations of an idealized charged rotating drop have been studied by using an electronic computer. The symmetric families of saddle-point shapes, and two families of stable equilibrium shapes (flat shapes and cylinders) have been traced as functions of the fissionability parameter x, and of the amount of rotational energy for all nuclei in the periodic table with any angular momentum. It was found that in the case of heavy nuclei ($x \geqslant 0.7$), rotation distorts the spherical nucleus into a flat shape (rotating about its axis of symmetry) and that further rotation results in instability against fission. For $x \leqslant 0.7$, as rotation increases, the flat shape first undergoes a transition to a cylinder-like shape before becoming unstable against fission. The results are illustrated by means of impact parameter versus bombarding energy diagrams for the case of Ne^{20} ions reacting with Al^{27}, Pr^{141}, and U^{238}. Expected regions of formation of stable cylindrical shapes and of breakup due to large angular momentum are shown. It was found that relatively favorable conditions for forming a stable cylindrical shape exist for very light targets, and that breakup reactions caused by large angular momentum become important for heavy targets. Variations of the fission threshold with the amount of rotational energy for different values of x is given. The fission threshold is found to decrease rapidly with increasing rotation.

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I. INTRODUCTION

The study of collisions between complex nuclei, made possible by the introduction of heavy-ion accelerators, necessitates the discussion of nuclear systems possessing very large angular momenta. In some situations, already accessible with presently available accelerators, the amount of rotational energy in the system may become sufficiently large compared to the nuclear binding energy that a spontaneous disruption of the system would take place. In other less extreme situations, the centrifugal force, even though not sufficiently large to cause disruption, may alter qualitatively the shape of the most stable nuclear configuration.

Several of the more important effects of large angular momenta have been pointed out in the past. Various approximate treatments of the problems have been discussed, among others, by Pik-Pichak,[1] Beringer and Knox,[2] Hiskes,[3] Sperber,[4] and Carlson and Pao Lu.[5] The model used in many of these investigations has been that of a charged, rotating liquid drop; various approximations, restricting the range of validity of the results, had to be introduced. Our object in the present work is to provide a comprehensive survey of the qualitative and quantitative features of the configurations of equilibrium of idealized rotating charged liquid drops, for conditions that may be met in bombardments of any target in the periodic table, with a projectile of arbitrary mass and energy. In this report we present only the principal qualitative features of the results. A detailed quantitative presentation of the equilibrium configurations and their properties will be described in a later paper.

The formulation of the mathematical problem in its most general way, as an investigation of idealized equilibrium shapes for any amount of rotation and any amount of charge, has led to one not unexpected consequence; namely, that certain of the results extend beyond the domain of nuclear physics into the domain of astronomy. This is so by virtue of the circumstance that the mathematical expression for the electrostatic energy, if taken with a negative sign, becomes identical with the Newtonian gravitational energy. In this way the rotating nuclear charged drop may be made to go over into a gravitating liquid mass corresponding (in the limit of vanishing surface tension) to the classical model of planets or stars in uniform rotation, familiar from the investigations of Jacobi, Poincaré, Jeans, and others (see for example, reference 6).

The study of the astronomical and nuclear problems as special cases of a more general formalism has proved useful in interpreting the nuclear situation in terms of concepts familiar in astronomical investigations, and vice versa.

The techniques used in the present work are a straight-forward generalization of those used in reference 7 (see Appendix).

The principal results of the mathematical analysis are described in Sec. II. They lead to a certain classification of collisions between complex nuclei into several groups. This classification is illustrated in Sec. III with the aid of examples involving neon-induced reactions.

II. RESULTS

The idealization of a charged liquid drop in uniform rotation implies that in searching for the equilibrium configurations there are just three energies to consider; the electrostatic (or gravitational) energy, the surface energy, and the rotational energy. As a result there are in the problem two dimensionless ratios in terms of which all results may be discussed. We shall take the conventional fissionability parameter x as a measure of the charge on the drop; it may be defined as the ratio of half the electrostatic (or gravitational) energy of the sphere to the surface energy of the sphere. As a measure of the angular momentum we shall take the parameter y, defined as the ratio of the rotational energy that a rigid sphere with the given angular momentum would have to the surface energy. The problem now is to describe the sequence of equilibrium shapes that one would observe if, starting with a given positive or negative x (a given charge or intensity of gravitation), the value of y (the amount of angular momentum) were gradually increased from zero. The results we found are as follows:

For small rotation the originally spherical drop is flattened by the centrifugal force, at first into an oblate spheroid value of x (i.e., independently of whether we discuss a planet or a nucleus).

The flattening increases gradually with increasing rotation up to a certain critical value of y (a function of x). At this critical value of y the flat pseudospheroids become unstable and a qualitative change takes place in the configuration. The nature of the change depends on whether x is below or above a certain critical value of x_c, which we estimate to be about 0.7, corresponding to fairly heavy nuclei in the general neighborhood of thorium.

If $x > x_c$, the flat pseudospheroid becomes unstable against spontaneous disintegration, the drop becomes unable to support any more rotation, and no further configurations of equilibrium exist.

If $x < x_c$, and this includes the rest of the periodic table as well as the astronomical cases with negative x, the flat pseudospheroid, on becoming unstable, goes over into a non-axially-symmetric shape. This new configuration has the symmetry of an ellipsoid with three unequal axes. The axis of rotation is along the shortest axis of the figure. In the astronomical case of large negative x these shapes are exact ellipsoids; in the nuclear case they are only approximately so, being somewhere between an ellipsoid and a cylinder (or even a dumb-bell) with rounded ends and an elliptical cross section. We shall refer to these shapes as pseudoellipsoids. They exist only if y exceeds a certain critical amount and if x is less than x_c.

As the angular momentum is increased beyond the critical value, the pseudoellipsoids become more and more elongated under the influence of the centrifugal force until a second critical value of y is reached. At this point the pseudoellipsoids become unstable against spontaneous disintegration and no further configurations of stable equilibrium exist. In the astronomical case, the instability is toward an asymmetric necking-in of the pseudoellipsoid; in the nuclear case into a symmetric necking-in.

The various possibilities are summarized in Fig. 1. The upper solid line in the plot of y vs x corresponds to the absolute upper limit of the amount of rotation that can be supported by a drop. The lower solid line corresponds to the value of y at which the transition from pseudospheroids to pseudoellipsoids takes place. The two lines coalesce beyond x_c.

In the astronomical case, the transition from spheroidal shapes (the Maclaurin spheroids) to ellipsoidal configurations (the Jacobi ellipsoids) is well known. In the nuclear case we would like to suggest the name Hiskes[3] configurations for the pseudospheroidal shapes of equilibrium of a charged rotating drop with surface tension, and Beringer-Knox[2] configurations for the pseudoellipsoids.

To summarize our survey of the shapes of stable equilibrium regarded as functions of angular momentum, we may say that, looking apart from the heaviest nuclei with $x > x_c$, the "normal" sequence of equilibrium shapes is

$$\text{pseudospheroids} \longrightarrow \text{pseudoellipsoids} \longrightarrow \text{disintegration.}$$

For the heavy nuclei the pseudoellipsoids are absent.

In addition to the above stable equilibrium configurations, there exist shapes of unstable equilibrium representing saddle-point configurations associated with the barrier for the disintegration of the system. In the astronomical case, the saddles are the famous pear-shaped figures of Poincaré. In the nuclear case, the saddle shapes have an appearance similar to the pseudoellipsoids, but are more elongated and have a deeper waist. We would like to call these shapes the Pik-Pichak[1] configurations. These shapes exist in the same part of the x-y diagram as the stable (pseudospheroidal or pseudoellipsoidal) configurations. On the critical curve which defines the maximum amount of rotation (upper solid curve in Fig. 1) the Pik-Pichak shapes and the shapes of stable equilibrium become identical and the threshold energy vanishes.

The quantitative tabulation of the thresholds as functions of x and y will be given in a later paper. The general features are that the barrier decreases rapidly as the amount of rotation is increased. This is in qualitative agreement with the relative ease of inducing fission with heavy ions.

III. APPLICATIONS

Any nuclear reaction, with a specified angular momentum, may be classified into one of three groups, depending on whether the angular momentum is such as could lead to a pseudospheroid, a pseudoellipsoid, or to disintegration. We shall call such collisions Hiskes collisions, Beringer-Knox collisions, and escape collisions, respectively. We shall combine this classification with the usual classification of collisions according to the closest distance of approach. If this distance is greater than the sum of the radii of the two complex nuclei, we shall refer to distant collisions; if it is comparable, we shall speak of peripheral collisions; and if less, we shall speak of close collisions. The significance of these two classifications according to angular momentum and distance of closest approach is illustrated in Fig. 2, which refers to the collision of a Ne^{20} ion with an Al^{27} target. Each point in the diagram specifies a set of conditions for such a collision by defining a lab. energy E_{lab} and the square of the impact parameter b. (The reason for taking b^2 rather than b is that the ordinate in Fig. 2 is then related to relative cross sections.)

There are two sets of curves dividing the diagram into several areas corresponding to different combinations of angular momenta and distances of closest approach. The unshaded portion corresponds to collisions where the distance of closest approach exceeds $(1.5)10^{-13}(A_1^{1/3} + A_2^{1/3})$cm. Only Coulomb scattering and Coulomb excitation would take place here. The boundary of this region is specified by the Coulomb barrier, V_c, which makes b^2 on the boundary proportional to $(1 - \frac{V_c}{E_{c.m.}})$, where $E_{c.m.}$ is the center-of-mass energy.

The other similarly shaped curve corresponds to taking the distance of closest approach to be $(1.2)10^{-13}(A_1^{1/3} + A_2^{1/3})$cm. In the band between the two lines, nuclear forces begin to come into play -- we have here peripheral collisions. For still smaller impact parameters, the collisions become more nearly head-on and we have close collisions. Except for the effects of angular momentum, such collisions would lead to compound nucleus formation. (The choice of the definite values $(1.2)10^{-13}$cm and $(1.5)10^{-13}$cm for the radius constant to illustrate approximately the region of peripheral collisions is not to be understood as implying sharp boundaries for this region.)

The effect of angular momentum is indicated by the other set of curves. The loci of constant angular momentum in a plot of b^2 vs E are rectangular hyperbolae with the coordinate axes as asymptotes. The two curves shown correspond, in the case of $Ne^{20} + Al^{27}$, to the two critical values of angular momentum that separate the Hiskes from the Beringer-Knox collisions, and the latter from escape collisions. Thus, in the part of the diagram closest to the coordinate axes, the angular momentum is sufficiently low so that a compound nucleus, <u>if formed</u>, would rotate as a pseudospheroid (Hiskes shape). In the band between the two hyperbolae, the equilibrium shape would be a pseudoellipsoid. In the last region the angular momentum is so high that even for close collisions no stable equilibrium configuration exists and the system must break up at once.

The above example shows how the combination of the angular momentum and distance classifications leads to several qualitatively different regions in a b^2 vs E plot, indicated by different shadings in Fig. 2. The intensity of the shadings corresponds roughly to the relative stability of the configurations. The cross-hatched areas correspond, approximately, to compound nucleus formation. We note that for energies up to about 180 MeV, compound nucleus formation is limited on the upper side by the requirement that the collision be sufficiently close; but for energies greater than 180 MeV, it is limited by the requirement that the angular momentum be sufficiently low. Thus the boundary of the compound nucleus region changes, in its dependence on E, from a $1-\frac{V_c}{E_{c.m.}}$ law at low energies, to a $\frac{1}{E_{c.m.}}$ law at high energies. The parallel hatched and the dotted portions of Fig. 2 correspond to various types of peripheral collisions in which the nuclear forces come into play only to a moderate extent. The small region with crosses corresponds to violent collisions where nuclear forces are fully in evidence; but the system is, nevertheless, forced to break up under the influence of the centrifugal force.

In the particular case of $Ne^{20} + Al^{27}$, the size of the Beringer-Knox region is especially noteworthy. Thus, at 180 MeV, more than half of the close collisions are likely to result in elongated pseudoellipsoidal shapes. These shapes, having abnormally large moments of inertia, would be of great interest if detected experimentally and would be a valuable check on the theory.

Figures 3 and 4 are similar to Fig. 2, but correspond to the collisions of Ne^{20} with heavier target nuclei. In the case of Pr^{141}, Fig. 3, the region of pseudoellipsoids has almost disappeared, and little breakup is expected. Nearly all close collisions (in the energy range up to 210 MeV) should result in a flat compound nucleus (Hiskes shape). In the case of U^{238} (Fig. 4), the region of pseudoellipsoids has disappeared entirely, but the breakup reactions have reappeared and have become relatively important. Considering only close collisions, about one third of the impact parameters at 210 MeV bombarding energy bring in too much angular momentum for compound nucleus formation to be possible. Some evidence for the presence of an appreciable number of breakup reactions in the bombardment of U^{238} with 210 MeV Ne^{20} ions is provided by the experiments of Sikkeland and Viola.[8] Using a careful angular correlation technique, they found that about 40% of the measured cross section involves a mechanism of incomplete momentum transfer; i.e., a compound nucleus is not produced. This is, in fact, of the order of magnitude expected on the basis of Fig. 4, but we would like to stress that, in view of the arbitrary definition of peripheral collisions, undue weight should not be attached to the precise quantitative features of Fig. 2 to 4.

Figure 5 shows for different types of reactions, the relative cross sections as functions of the fissionability parameter x for 210 MeV Ne^{20} ions. The cross sections are expressed as fractions of the number of close collisions. At low x values, breakup reactions are present, though not to a large extent, and they soon disappear entirely with increasing x. The most prominent feature in this region is the large relative cross section for the formation of pseudoellipsoids (which, for low x values, are actually somewhat dumb-bell shaped). As x increases, the pseudoellipsoids disappear (at about $x = 0.7$). On the other hand, breakup reactions reappear and their relative importance increases rapidly.

A more complete account of these calculations is in preparation.

ACKNOWLEDGMENTS

We would like to thank Dr. S. G. Thompson for his continued interest in this investigation. We are grateful to Dr. J. Mollenauer for making available to us the results of his unpublished calculation on the energy of a charged rotating ellipsoid.

This work was done under the auspices of the U. S. Atomic Energy Commission.

APPENDIX

The method used in calculating the equilibrium shapes was identical with that described in the investigation of the equilibrium configurations of a nonrotating drop (reference 7). The principal difficulty in treating the rotating case is not so much the presence of the rotational energy (which is a relatively simple expression proportional to the square of the angular momentum divided by the appropriate moment of inertia), but the necessity of considering, in the case of the Beringer-Knox and Pik-Pichak configurations, shapes without axial symmetry. Fortunately, except for certain x values in the neighborhood of the transition from the Hiskes to the Beringer-Knox configurations, two of the three axes of the pseudoellipsoids are nearly equal, which made it possible to use the computer code from reference 7 essentially unchanged. In the critical regions where none of the three axes are even approximately equal, the shapes appear to be close to ellipsoids, and we have found it possible to supplement our calculations with available studies of exactly ellipsoidal shapes (Mollenauer,[9] and Carlson and Pao Lu[5]) in which no assumptions are made about the relative size of the three axes.

REFERENCES

1. G. A. Pik-Pichak, J. Exptl. Theoret. Phys. (USSR) 34, 341 (1958).
2. R. Beringer and W. J. Knox, Phys. Rev. 121, 1195 (1961).
3. J. R. Hiskes, The Liquid Drop Model of Fission: Equilibrium Configurations and Energetics of Uniform Rotating Charged Drops, Lawrence Radiation Laboratory Report UCRL-9275 (1960).
4. D. Sperber, Nuclear States With High Angular Momenta and Their Stability, Princeton University Technical Report NYO 2961 (1961).
5. B. C. Carlson and Pao Lu, Proceedings of the Rutherford Jubilee International Conference, 291 (1962).
6. R. A. Lyttleton, The Stability of Rotating Liquid Masses,(Cambridge University Press, Cambridge, England, 1958).
7. S. Cohen and W. J. Swiatecki, The Deformation Energy of a Charged Drop: Part V: Results of Electronic Computer Studies, Lawrence Radiation Laboratory Report UCRL-10450 (1962).
8. T. Sikkeland and V. E. Viola, Jr., Momentum Transfer in Heavy-Ion-Induced Reactions, Proceedings of Third Conference on Reactions Between Complex Nuclei, Asilomar, California (1963).
9. J. Mollenauer (Brookhaven National Laboratories) private communication.

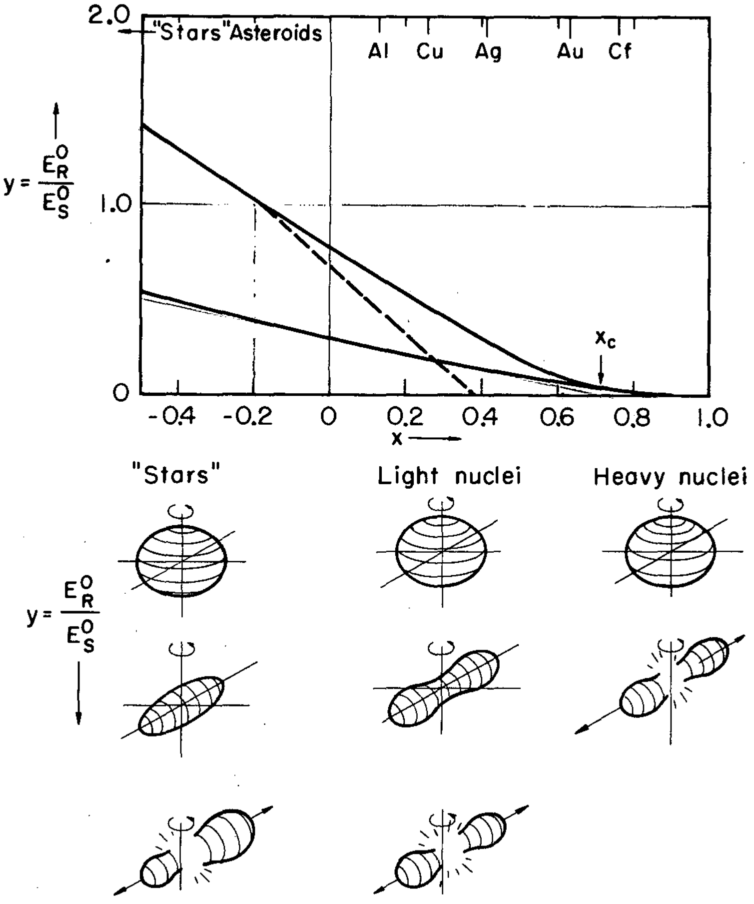

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Fig. 1. Illustration of equilibrium shapes of a rotating charged or gravitating liquid mass for different values of the fissionability parameter x and of the rotational parameter y. The lower part of the figure illustrates how, with increasing angular momentum, stars and the lighter nuclei go over from a pseudospheroidal to a pseudoellipsoidal shape before disintegrating. The x-y diagram shows the critical values of y corresponding to the limiting amount of rotation for any given x (upper solid curve) and the critical values of y marking the transition from pseudospheroids to pseudoellipsoids (lower solid curve). The dashed curve refers to the stability of the saddle point configurations. To the right of the line, the Pik-Pichak shapes are stable against small asymmetric distortions; to the left they are unstable.

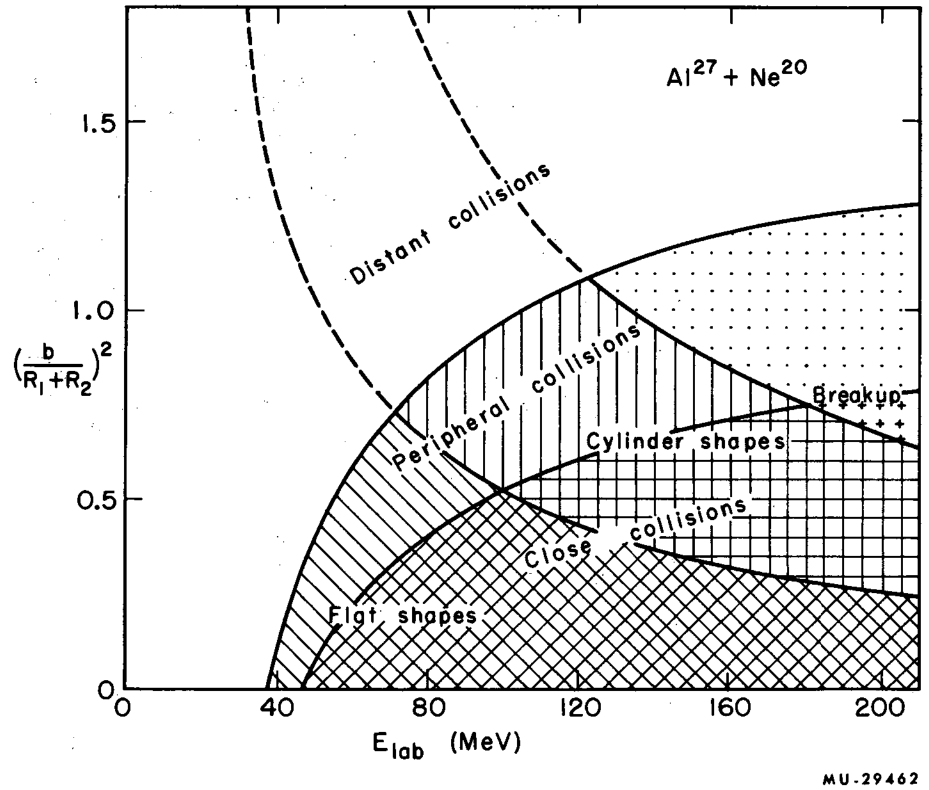

Fig. 2. Classification of different types of collisions in a plot of the square of the impact parameter against the bombarding energy. The impact parameter is expressed in units of the sum of the radii of the two nuclei (Al^{27} and Ne^{20}).

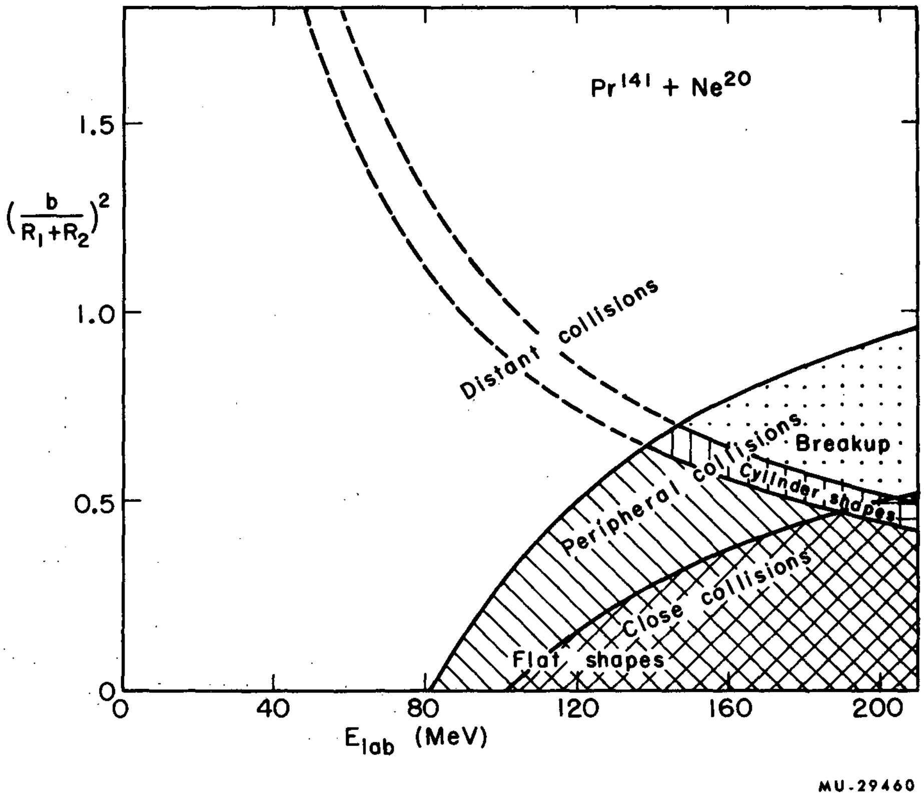

Fig. 3. Same as Fig. 2, but for Pr^{141} and Ne^{20}.

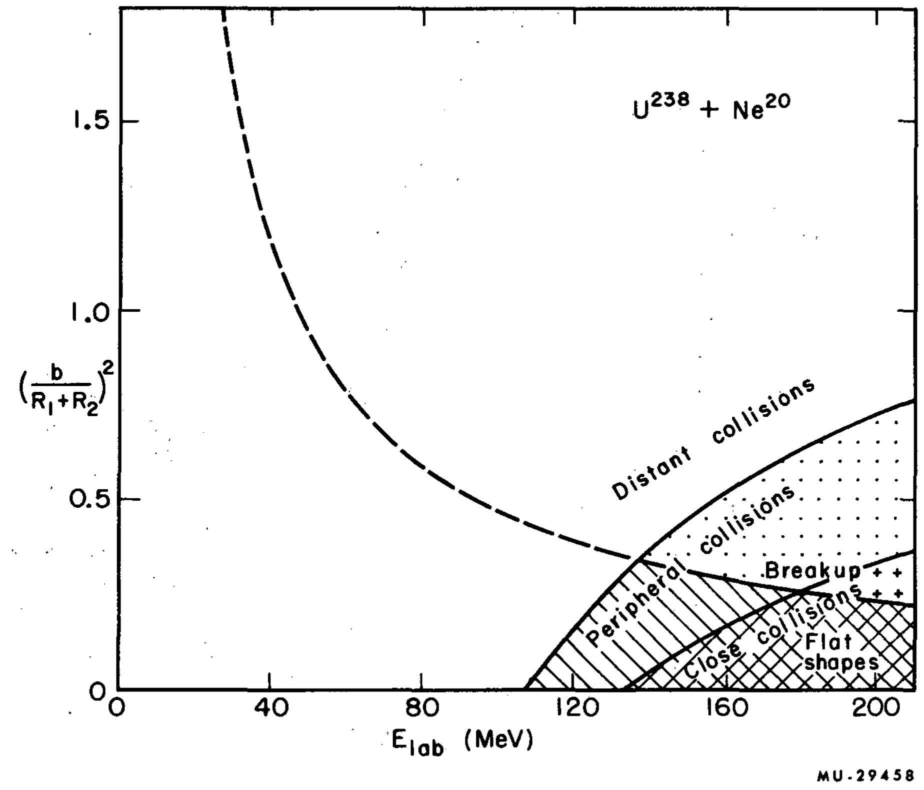

Fig. 4. Same as Fig. 2, but for U^{238} and Ne^{20}.

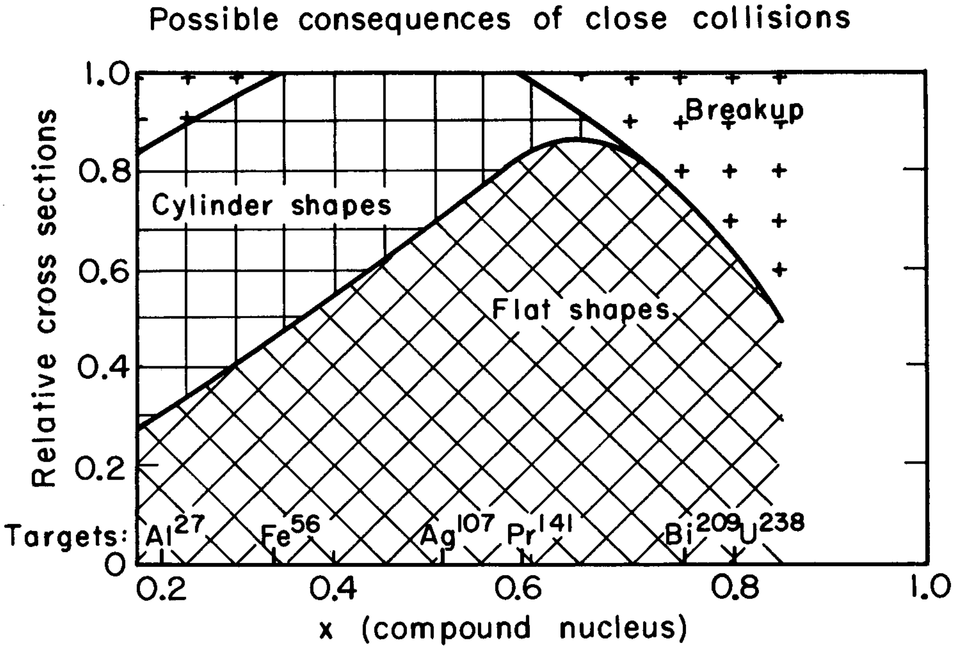

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Fig. 5. The variation of relative cross sections for different types of collisions of a 210 MeV Ne^{20} ion with various targets.