Title

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Evidence against a strong association between numerical symbols and the quantities they represent.

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Abstract

Are numerals divorced from a sense of the actual quantities they represent? We show that accessing a sense of how much a numerical symbol actually represents is a surprisingly difficult and non-trivial process. Irrespective of numerical size or distance, direct comparison of the relative quantities represented by symbolic and non-symbolic quantities leads to performance markedly worse than when comparing two non-symbolic quantities. Experiment 2 shows that this effect cannot be attributed to differences in perceptual processing streams. Experiment 3 shows that there is no additional cost of mixing two formats that are both symbolic; that is, the decrement in mixing formats is specific to mixing *symbolic* and *non-symbolic* representations. Our data are consistent with the view that numerical symbols operate primarily as an associative system in which relations between symbols come to overshadow those between symbols and their quantity referents.

Keywords: Numerical Cognition; Symbolic Representation.

Introduction

Does one really have a meaningful sense of very large quantities, like a million or a billion? Or does representing quantities in exact, symbolic form come to change the way we think about (and with) these numerical symbols themselves? In recent years, evidence has accumulated in favor of a strong overlap between symbolic and non-symbolic number-representation systems (Dehaene, 1997, 2008; Dehaene et al., 2003; Nieder & Dehane, 2009; Piazza et al., 2007, 2010; Fias et al., 2003; Condry & Spelke, 2008; McCrink & Spelke 2010; Gilmore et al., 2010; Halberda et al., 2010; Santens et al., 2010). Considerable attention has been paid to the notion that complex mathematical concepts are grounded in an evolutionarily ancient and developmentally fundamental sense of quantity (e.g., which basket contains more apples, which tribe comprises more members, which of two bushes contains more berries?) (Nieder & Dehaene, 2009; Halberda et al., 2008; Pica et al., 2004). Furthermore, this view proposes that an intuitive sense of approximate quantity (i.e., the approximate number system – ANS) should be a fundamental aspect of any numerical symbol – that is, there should be considerable overlap between symbolic and non-symbolic numerical processes (Dehaene, 2008). Thus, accessing this sense of quantity from a symbol should be a relatively fast and effortless process.

On the other hand, it may be that through repeated use and mastery of numerical symbols, the ties between exact numerical symbols (e.g., Arabic numerals) are weakened to the point that these symbols are often used with very little access to a sense of the actual quantities they presumably represent. For example, it is hard to conjure a sense of what a million actually looks or feels like – one's intuitive sense of what 1,000,000 actually means seems divorced from the symbol that is meant to represent that quantity. Of course, we can still use 1,000,000 in myriad ways; for example, it should be easy enough to understand that 999,999 < 1,000,000 < 1,000,001. In other words, the symbol 1,000,000 makes perfect sense in terms of its relative (ordinal) position with respect to other numerical symbols (Verguts & Fias, 2004), even if it is possibly divorced from the quantity it represents.

In sum, a crucial facet of numerical symbols is how they relate to other symbols; indeed, it may even be the case that with repeated exposure to numerical symbols, symbol-symbol relations in literate adults come to usurp symbol-quantity relations. As has been found in abstract semantic representation more generally (Crutch & Warrington, 2010), how a (numerical) symbol relates to other symbols may thus become more central to that symbol's meaning than how it relates to the quantity it supposedly represents (Deacon, 1997; Nieder, 2009). If so, then eliciting a sense of the actual quantity represented by a numerical symbol may be an onerous process, in that it is not typically necessary when using such symbols in a normal mathematical context. That is, the link between numerical symbols and the quantities they represent (at least in terms of the ANS) may actually be considerably weaker than previously assumed.

One way to distinguish these hypotheses directly is to ask participants to use numerical symbols in a context that forces them to access how much a given symbol represents explicitly. In the current study, we asked participants to compare quantities represented either in symbolic (Arabic numeral or written number-word) or non-symbolic format (an array of dots flashed too briefly to be counted). In Experiments 1-2, we had adult participants make comparison judgments (decide which item depicts the greater quantity) in three different conditions: numeral/numeral judgments, dot/dot judgments, and mixed dot/numeral (or numeral/dot) judgments. In Experiment 3, participants compared quantities in numeral-numeral, number-word/number-word, and number-word/numeral (or numeral/number-word) conditions.

If numerical symbols retain a strong link to an approximate sense of the quantities they represent, then mixing formats should be akin to comparing two entities that ostensibly differ only in representational quality (i.e., sharpness of approximate tuning curves; Piazza et al., 2004; Nieder & Merten, 2007). Adults are faster and more accurate when comparing two numeral-stimuli than two dot-stimuli (Buckley & Gillman, 1974; Lyons & Beilock, 2009). Thus, replacing one dot-stimulus with a (superior) numeral-stimulus should improve mixed-format comparison (relative to dot-dot comparison) performance. According to the hypothesis that symbolic and non-symbolic quantities draw from the same neural populations (Dehaene, 2008; Santens, 2010), mixed-format comparisons (which combine a broadly tuned dot-stimulus with a finely tuned numeral-stimulus) in Experiments 1-2 should yield performance somewhere in between that of numeral-numeral (two finely tuned stimuli) and dot-dot comparisons (two broadly tuned stimuli); or more conservatively, mixed performance should at least be no worse than dot-dot comparisons.

By contrast, if symbolic numbers have become detached from an intuitive sense of the non-symbolic quantities to which they presumably refer, accessing this sense of quantity directly from a numerical symbol may incur an additional processing cost. According to this hypothesis, mixed-format comparisons should lead to performance significantly worse than either numeral-numeral or dot-dot comparisons.

In Experiment 3, we tested whether the potential cost of mixing formats observed in Experiments 1 and 2 might simply be due to mixing representational or visual format, rather than to asymmetric accessing of quantity information. We expected quantities presented as number words to be represented symbolically, as in the case of numerals. We thus predicted that directly comparing a numeral with a number-word should not yield performance worse than number-word/number-word comparisons (which were expected to yield less efficient performance than numeral-numeral comparisons; Damian, 2004). Such a result would suggest that the performance degradation seen for mixed comparisons is not simply due to mixing representational or visual formats.

Methods

In Experiment 1 (N=21 University of Chicago students), subjects decided which of two simultaneously presented visual stimuli (dots and/or numerals) was numerically larger (Figure 1a-c). In Experiment 2 (N=21 Dartmouth College students), the two stimuli (dots and/or numerals) were presented sequentially (Figure 1d-f). In Experiment 3 (N=21 University of Chicago students), stimuli (numerals and/or number-words) were presented sequentially (timing was the same as in Experiment 2).

Figure 1

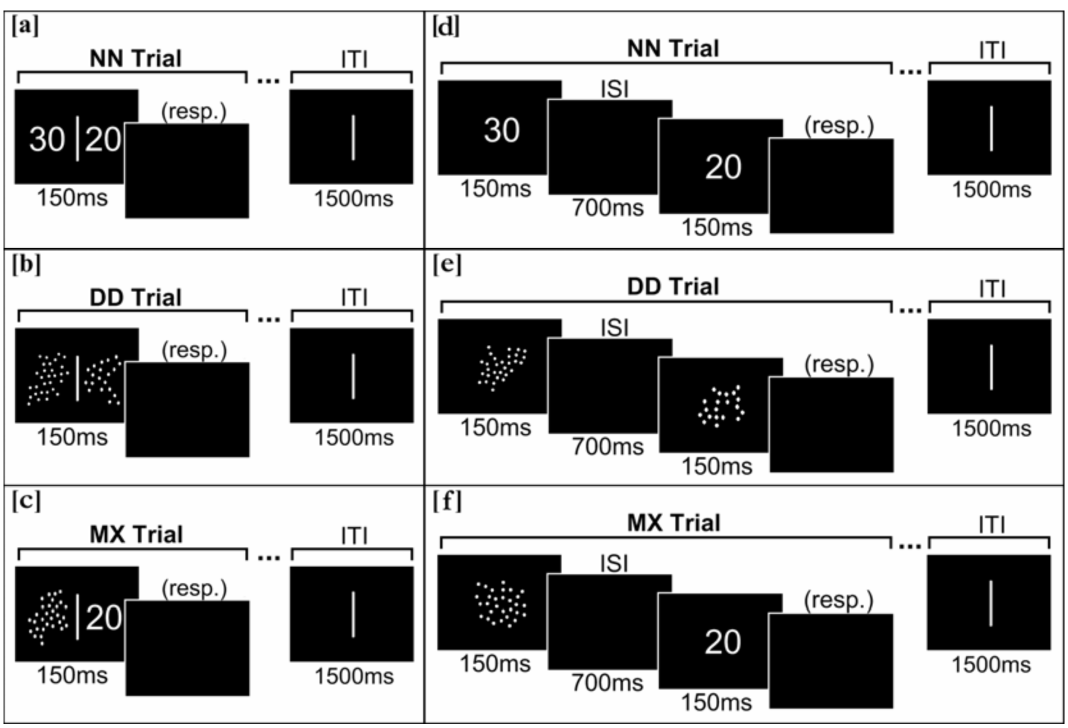

Figure 1 shows stimulus examples from Exps. 1 [a-c] and 2 [d-f] (Exp. 3 trial timing was the same as in Exp. 2). Note that for DD trials, the two dot-arrays in a given comparison trial were equated with respect to two of four continuous parameters (individual dot-size, total dot-area, inter-item density, total array perimeter), such that, across all trials, relying exclusively on any one of these parameters would lead to performance no better than chance. No array was ever presented to a subject twice.

In all Experiments, subjects' task was to decide which stimulus represented the greater quantity (Figure 1 depicts sample trials). In Experiments 1-2, there were three format conditions: numeral/numeral (NN), dot/dot (DD), mixed-format (MX) numeral/dot (or dot/numeral). In Experiment 3, there were three format conditions: numeral/numeral (NN), number-word/number-word (WW), mixed-format (MX) numeral/number-word (or number-word/numeral. Trials were always blocked by condition (with rest and instructions between blocks), so participants always knew which presentation-order to expect. The order of condition-blocks was randomized across participants. For MX trials in Experiment 1, which side (left or right) contained the dot-array was randomized across trials. For MX trials in Experiments 2 and 3, which stimulus-type was presented first in was also balanced across trials. Subjects were to press a key with their left middle finger if they thought the left (Experiment 1) or first (Experiments 2-3) stimulus was greater, press a key with their right middle finger if they thought the right (Experiment 1) or second (Experiments 2-3) stimulus was greater, or press a third key (space bar) with both index fingers if they thought the two stimuli were numerically equal (catch trials).

In all experiments, there were 48 critical trials and 16 catch trials in each condition. In all conditions, half of critical trials were numerically small (1,2,3,4), and half were large (10,20,30,40); orthogonally, half of critical trials were numerically close ($|n_1 – n_2|$ = 1 or 10) and half were far ($|n_1 – n_2|$ = 2,3,20 or 30). In this way, for each format condition, stimuli were subdivided into four categories: small-far, small-close, large-far, large-close. Performance differences were tested within each of these categories separately: all contrasts were two-tailed, within-subjects contrasts with 20 degrees of freedom.

Results

In all experiments, two behavioral measures were collected: response-times (RTs) and error-rates (ERs). RT and ER condition means are summarized in Table 1. Contrast results for RTs are shown in Figure 2. In all experiments, our hypotheses concerned the difference between the MX and the single-format condition (NN, DD or WW) that yielded the worst performance in that experiment.

In Experiment 1, RTs tended to be longer on DD than NN trials (large-close: p=.056; large-far: p=.245; small-close: p=.001; small-far: p=.138). ERs were higher on DD close (ps<.001) but not far (ps≥.336) trials. This result is unsurprising in that performance on numerical comparisons using symbols is typically better than on comparisons using non-symbolic quantities (e.g., arrays of dots) (Buckley & Gillman, 1974; Lyons & Beilock, 2009). Thus, our critical contrasts were between MX and DD trials. RTs were significantly longer for MX than DD trials in all categories (all ps<.001; Figure 2a). ERs were higher for MX than DD trials in all four categories as well (large-close: p=.137; large-far: p<.001; small-close: p=.023; small-far: p=.014).

Figure 2

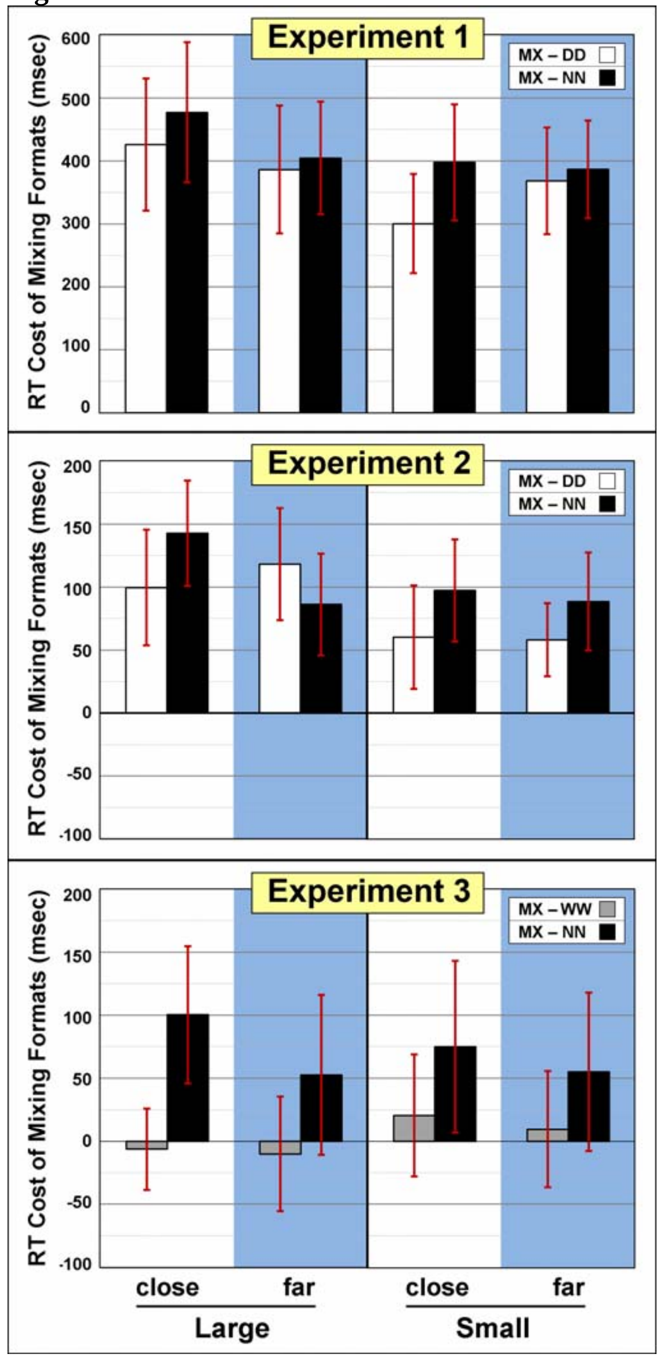

Figure 2 shows the cost – in milliseconds – of mixing formats. Red error bars are 95% confidence intervals. For Experiments 1-2, the y-axis is the mean difference between MX and DD conditions (white bars) and MX and NN conditions (black bars). The critical contrast (see text) was between MX and DD conditions. For Experiment 3, the y-axis is the mean difference between MX and WW conditions (grey bars) and MX and NN conditions (black bars). The critical contrast (see text) was between MX and WW conditions. Terms and abbreviations are the same as in Figure 1. Raw cell means for both response-times and error-rates can be found in Table 1.

		Large						Small					
		Close			Far			Close			Far		
		NN	DD	MX	NN	DD	MX	NN	DD	MX	NN	DD	MX
[a]	RT	448	498	924	407	426	812	401	499	799	367	385	754
		27	*28*	*56*	*26*	*23*	*43*	*27*	*30*	*52*	*20*	*15*	*51*
	ER	2.3	22.0	28.5	1.9	1.1	11.0	0.4	6.9	12.4	0.4	0.3	5.9
		1.3	*2.3*	*3.3*	*0.9*	*0.6*	*1.8*	*0.4*	*1.9*	*2.0*	*0.4*	*0.3*	*2.0*
[b]	RT	880	923	1023	873	841	959	842	879	939	813	843	901
		37	*29*	*37*	*40*	*30*	*38*	*37*	*33*	*38*	*41*	*34*	*37*
	ER	5.2	28.6	33.1	6.3	4.4	14.5	5.6	7.9	12.3	2.8	3.2	6.3
		1.8	*2.3*	*3.1*	*1.4*	*1.8*	*2.1*	*1.3*	*2.5*	*2.6*	*1.0*	*1.2*	*1.7*
[c]		NN	WW	MX	NN	WW	MX	NN	WW	MX	NN	WW	MX
	RT	648	754	748	666	728	718	650	704	725	653	699	708
		34	*30*	*28*	*41*	*36*	*30*	*38*	*30*	*31*	*38*	*41*	*26*
	ER	2.8	6.4	5.2	3.2	4.0	4.0	5.6	4.8	5.1	3.2	2.8	2.9
		1.7	*2.8*	*1.3*	*1.2*	*1.6*	*1.9*	*1.8*	*1.7*	*1.4*	*1.1*	*1.1*	*1.5*

Table 1 Condition means for both response times (RT: msec) and error rates (ER: percent errors) broken down by category for: [a] Experiment 1, [b] Experiment 2, [c] Experiment 3. Values in italics are standard errors of the mean.

In sum, RTs were significantly longer for MX than DD trials (and ERs showed a similar pattern). Accessing a sense of quantity entails additional processing costs over and above typical numerical or dot comparisons and thus does not appear to be an automatically accessible aspect of symbolic number representation.

However, it may be that the difference between DD and MX performance arises, not because of a weak link between the ANS and numerical symbols, but due to the cost of switching between different perceptual input streams (in the case of MX trials) (Dehaene et al., 2003; Santens, 2010; Dehaene & Cohen, 1995). In Experiment 1, (simultaneous presentation), the average difference between DD and MX conditions in terms of response-times was 370msec, with the maximum difference arising for large-close trials (M=426msec, Figure 2a). Thus, in Experiments 2-3, we chose an inter-trial-interval (ITI) that far exceeded (roughly doubled) this potential switch-cost window (700msec; total time between stimulus 1 and 2 onsets: 850msec; see Figure 1). In addition, 850msec is also well in excess of the maximum duration typically observed in visual attentional blink paradigms (Raymond et al., 1992; Kranczioch et al., 2005), which further reduces the possibility that any effect of mixing formats in Experiments 2-3 would be due to switching between input processing streams. Note that a still longer ISI was not chosen to avoid placing undue demands on working-memory maintenance processes.

In Experiment 2, RTs tended to be longer on DD than NN trials (large-close: p=.078; small-close: p=.111; small-far: p=.099), with the exception of large-far trials, which showed a trend in the opposite direction (p=.178). ERs were higher on DD than NN large-close trials (p<.001) but not in any other category (ps≥.329). Thus, excepting large-far trials, our critical contrasts were between MX and DD trials. Note that in Experiment 2, MX trials could be presented either dot-first (DN) or numeral-first (ND). The main effect of presentation-order and all interaction terms involving this factor were non-significant for both RTs and ERs (all Fs<1); Experiment 2 MX results were thus collapsed across presentation-orders.

RTs were significantly longer for MX than DD trials in all categories (all ps≤.006; Figure 2b); RTs were also significantly longer for MX than NN trials in all categories (all ps≤.001; Table 1b). ERs tended to be higher for MX than DD trials as well (large-close: p=.144; large-far: p=.001; small-close: p=.084; small-far: p=.046). In sum, RTs were significantly longer for MX than DD and NN trials (and ERs showed a similar pattern). The results from Experiment 2 thus provide further evidence that accessing a sense of quantity entails an additional processing cost, beyond any cost of switching between different perceptual input streams that may have been observed in Experiment 1.

In Experiment 3, we tested whether the potential cost of mixing formats observed in Experiments 1 and 2 might be due to mixing visual format. In contrast to the MX conditions above, we predicted that mixing symbolic visual formats (numerals and number-words) would not lead to performance significantly worse than that seen for the worst-performing single-format condition (NN or WW). Quantities used and other parameters were the same as in Experiment 2. Number-words were presented in English in the center of the screen (24-point Arial font).

In Experiment 3, responses on WW trials tended to be slower than on NN trials (large-close: p<.001; large-far: p=.072; small-close: p=.145; small-far: p=.203). ERs were higher on WW than NN large-close trials (p=.047) but not in any other category (ps≥.605). Thus, our critical contrasts were between MX and WW trials.

Crucially, performance did not significantly differ between MX and WW conditions for either RTs (all ps≥.390) or ERs (all ps≥.636). In sum, Experiment 3 results are consistent with the hypothesis that switching between visual numerical formats – so long as both formats point to symbolic representations – does not incur the same cost that arises when switching between symbolic and non-symbolic numerical formats, as was seen in Experiments 1 and 2.

Discussion

Experiments 1 and 2 provide clear evidence that numerical comparisons between a symbolic and a non-symbolic quantity are considerably more difficult than comparing two non-symbolic quantities. This is surprising in that one might expect the comparison of a highly accurate stimulus (numeral) and an inaccurate stimulus (dot-array) to be easier (or at least no worse) than comparison of two inaccurate stimuli (two dot-arrays). Our data reject this view and suggest instead that symbolic numbers do not provide automatic access to an approximate sense of the quantity that they represent. Rather, it appears that additional, seemingly inefficient processing is required to compare symbolic with non-symbolic quantities.

Interestingly, MX performance was worse than DD performance even for small numbers. First, this indicates that inefficiency of directly comparing symbolic and non-symbolic numbers is present even for highly familiar quantities. Furthermore, the small numbers used here (1-4) are within the subitizing range for adults (Mandler & Shebo, 1982; Revkin et al., 2008; Demeyere et al., 2010). Non-symbolic quantities in this range tend to be represented in exact (as opposed to approximate) fashion since they do not exceed the limited capacity of visual short-term memory (Luck & vogel, 1997; Pylyshyn, 2001; Ansari et al., 2007). Inefficient mixed-format comparisons for small quantities suggests that neither familiarity nor representational acuity (i.e. sharpness of approximate tuning curves; Piazza et al., 2004; Nieder & Merten, 2007) is the primary cause of this cost. Instead, it appears that symbolic and non-symbolic representations of quantity are incompatible seemingly across the board.

At a theoretical level, our results are partially consistent with the place-code model of symbolic number representation proposed by Verguts and Fias (2004), in which a numerical symbol is represented in terms of its relative ordinal position. Our results go beyond this model, however, in that they suggest that numerical symbols operate primarily as an associative system in which relations between symbols come to overshadow those between symbols and their quantity referents, and may even become devoid of a strong sense of quantity *per se* (Deacon, 1997; Nieder, 2009). Thus, an important step for future research will be to understand symbolic representation of number in a way that is not necessarily tied explicitly to actual quantity referents. This may be especially interesting to consider in a developmental context and with respect to the individual differences that limit exactly how and when numerical symbols are best understood in conjunction with or separate from one's more intuitive number sense (Lyons & Beilock, 2009; Santens et al., 2010; Ansari, 2008; Holloway & Ansari, 2010).

Here it is important to note that one potentially simple explanation for the current results is that number sense and numerical symbols were simply never associated with one another in the first place. We do not believe this to be the case, however, for two important reasons. First, considerable neural evidence has accrued suggesting that the neural substrates underlying the ANS do overlap at least to some extent with those thought to underlie symbolic representations of number (Dehaene et al., 2003; Nieder & Dehane, 2009; Piazza et al., 2007; Fias et al., 2003; Santens et al., 2010). Furthermore, recent developmental evidence suggests that individual differences in ANS acuity are linked with symbol-based math abilities from a relatively young age (Piazza et al., 2010; McCrink & Spelke 2010; Gilmore et al., 2010; Halberda et al., 2010). Therefore, our assertion is that, perhaps via years of practice with and over-learned associations between symbols, these representations may have become functionally distinguishable from the ANS, at least in that the link between symbol and quantity has been weakened or perhaps relegated to a more indirect status. In conclusion, the data reported here plainly call into question the strength of the link between numerical symbols and a sense of the quantities they are meant to represent in literate adults. Therefore, future studies aimed at understanding the cognitive and neural basis of more complex math skills in particular should consider not only the commonalities across systems, but also the unique properties that symbolic representations of number bring to the table.

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