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Shaping Land Use along an Agricultural Frontier: A Dynamic Household Model for Early Small-Scale Settlers in the Brazilian Amazon

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Shaping Land Use along an Agricultural Frontier: A Dynamic Household Model
for Early Small-Scale Settlers in the Brazilian Amazon
by

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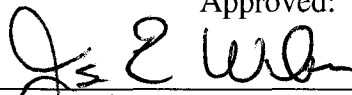Agricultural and Resource Economics

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Acknowledgments

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Abstract

This dissertation uses a dynamic household utility optimization model to examine small-scale settler behavior in colonization projects within the Brazilian Amazon's arc of deforestation. The aim is to illuminate factors influencing the typical trajectory from forest to pasture – a process with implications for settler welfare and the environment. Model dynamics derive from herd growth and degradation, in a frontier-like context of constrained access to capital and labor markets.

Using numerical simulations, the analysis shows competition for the household's scarce factor labor among pasture investment, annual cropping, and off-farm wage work playing out in different trajectories for household time allocation and land use with different model parameters. The investment approach bridges two often opposing views of the small-scale farmer – struggling for subsistence, or rising out of poverty – placing them as potential snapshots along a trajectory.

In the long run, the household devotes all its labor to the activity with the higher return to that labor – maintaining a fixed amount of pasture and living off sales from herd growth, or working off farm. Herd growth and the discount rate determine optimal pasture stocking rates. The household's trajectory shifts with additional activities in ways dependent on their returns-to-labor profiles and parameter levels. Dynamic patterns change under decreasing marginal returns to labor over time (with annuals), constant returns (with wage work), and rising returns (pasture investment). Because the shadow wage changes over time, participation in a labor market can become attractive due to pasture investment.

Simulations highlight dynamic trade-offs, and include scenarios with a prolonged annuals phase, and volatile behavior such as a one-year spike in pasture labor, or overshooting the long-run herd stocking rate. In an attempt to examine frontier labor market integration, the dissertation also includes simulations with a changing exogenous wage and labor trading between two households. Expectation of off-farm work in the future encourages annuals and slows pasture growth prior to the wage phase, but speeds the pasture transition thereafter. Trading labor between farms with different household sizes favors annuals along the trajectory, but does not affect the steady state. Labor trades between farms with different time preferences favors pasture.

Chapter 1 Deforestation and Frontier Settlement in the Brazilian Amazon

The Brazilian Amazon grabbed international attention in the 1980s, when government incentives sparked an influx of farmers, and the rainforest started to fall rapidly. Of policy concern were broad environmental consequences of deforestation such as higher carbon emissions and lost biodiversity in the largest remaining expanse of tropical rainforest – spanning some 6 million km^2, about 70% of it in Brazil (Faminow and Vosti 1998) – and economic growth and poverty alleviation goals that were among the motivations for government land giveaways in the Amazon.

From less than 5% in the 1970s, accumulated deforestation in Brazil's Legal Amazon jumped to around 10% by the end of the 1980s. It is close to 15% today (Wood 2002; Pfaff et al. 2007), with between 1 and 2 million ha of forest falling each year (Walker 2003; Pfaff et al. 2007).[1] As forests have fallen, area in pasture has grown. Pasture accounted for just over 50% of cleared area in the forest eco-zone of the Legal Amazon in the 1995 agricultural census (Thomas 2003).[2] Most clearing occurs along a wide swath cutting across the southern Brazilian Amazon dubbed the "arc of deforestation."

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[1] Deforestation totals can vary considerably due to differences in methods (e.g., spatial resolution of satellite images), what's being measured (e.g., treatment of secondary forest) and coverage (e.g., including or excluding the roughly 0.8 million km^2 of savannah bordering the rainforest eco-zone but inside the administrative unit called the "Legal Amazon") (Faminow 1998; Thomas 2003). The figures used here, based on data from the Brazilian Space Agency INPE, are for the Legal Amazon and do not include secondary forest as part of the 'forest' category. Secondary forest can be the result of a management decision, but also appears when land is abandoned. Its extent, not well documented, could be considerable (Faminow 1998; Thomas 2003).

[2] Soybeans are growing in importance in the southern portion of the zone (Thomas 2003); perennial cropping occupies large areas only in particular niches within the region. In the high deforestation areas of interest here, small-scale farmers who plant perennials still have pasture as their dominant land use (Thomas 2003).

This area encompasses a large number of colonization projects – areas set aside by the government (before, and sometimes, after, settler in-migration) for small-scale settlement. Colonization projects cover about 4% of the Legal Amazon and account for an accumulated deforestation of 15% of the region's total (Brandão and Souza 2006). Figure 1 illustrates, for a sizeable subset of the projects, the overlap with areas of relatively high deforestation and proximity to the region's major road arteries.[3]

Figure 1. Deforestation in Settlement Projects, 2004
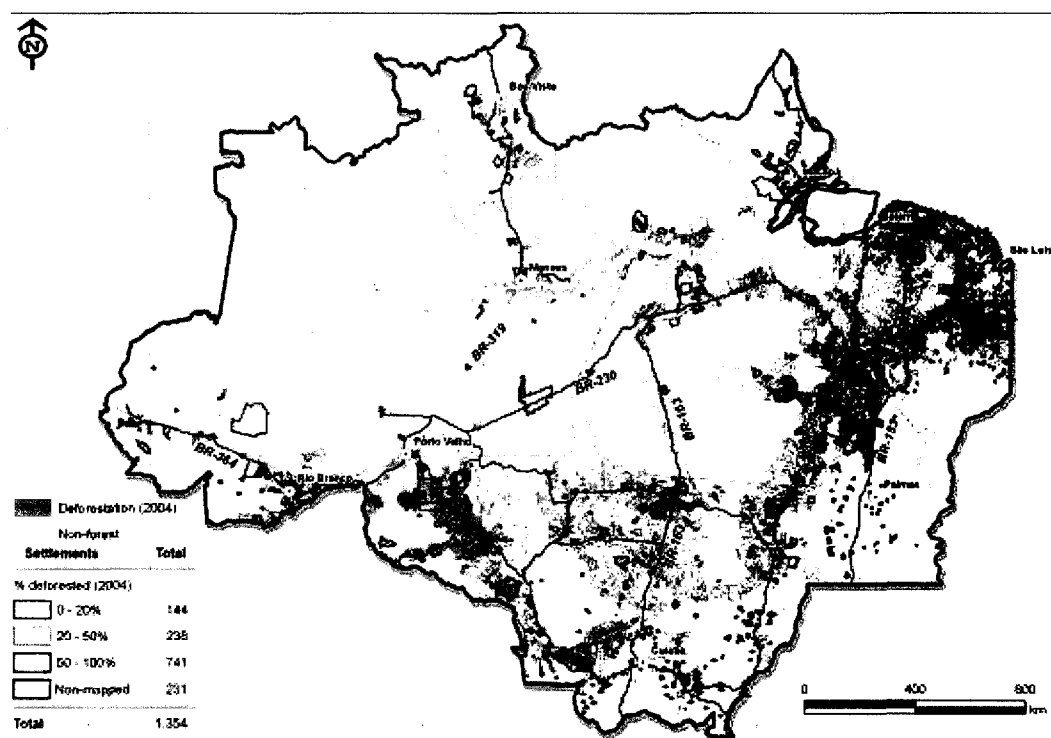
Source: Brandão and Souza 2006.

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[3]This map, made by the Amazon Forest Monitoring Program (Prodes), highlights projects with recent deforestation: those in the forest ecozone with more than 50% forest cover in 1994. This amounts to 82% of the 1354 projects in the Legal Amazon created between 1970 and 2002 (Brandão and Souza 2006). The map's non-forested area represents savannah within the Legal Amazon. The savannah eco-zone is more favorable to annual cropping, and easier to clear for agriculture than is the rainforest eco-zone (Thomas 2003).

Inside colonization projects, settlers occupy lots along feeder roads off a main artery leading towards a market. They receive land by government lottery or by squatting. Deforestation typically proceeds in an expanding fishbone pattern along feeder roads. Figure 2 depicts an example of this process, providing an overhead perspective for one colonization project (Theobroma, Rondônia) in the western Brazilian Amazon within the arc of deforestation (Imbernon and Branthomme 2001). Looking over time from left to right, the forested (dark) area increasingly gives way to cleared (gray) area. Deforestation tends to extend along lines (feeder roads) first (the parallel lines visible in 1978), and expand as individual lots (rectangles) along the roads are cleared (1987). Eventually, deforested areas previously separated become contiguous (1993), and remaining forest inside these areas dwindles (1994 and 1996).

Figure 2. Deforestation in Theobroma, Brazil
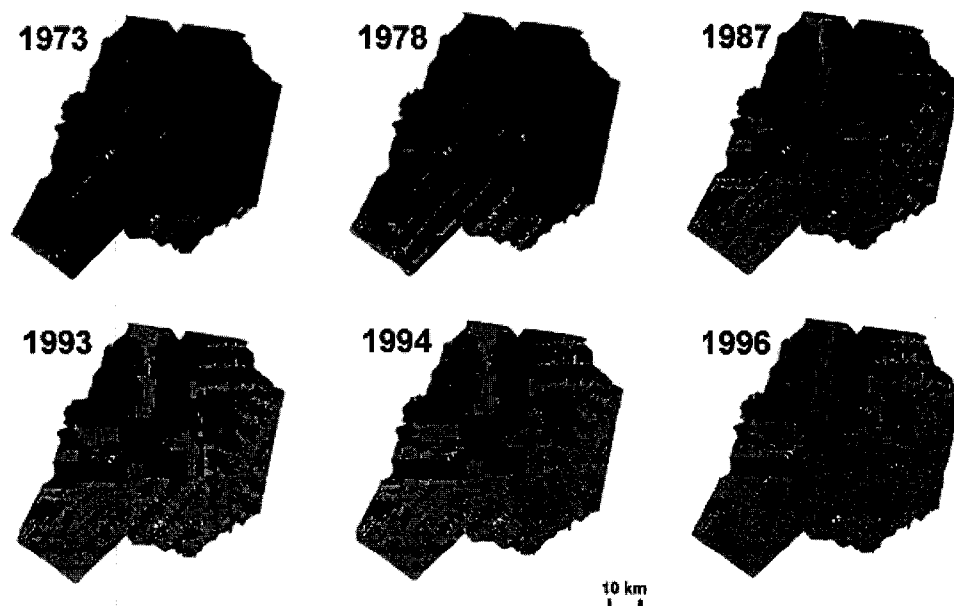
Source: Imbernon and Branthomme 2001.

This dissertation seeks to shed light on factors influencing the speed of a typical local trajectory from forest to pasture within the Brazilian Amazon's arc of deforestation,

and ultimate size of cleared area. The dynamic spatial pattern of deforestation and changing land use could have important ecological consequences in terms of habitat fragmentation, prospects for forest regrowth, provision of ecosystem services by various land uses, and policy targeting for land use and other economic development goals (Vance and Geoghegan 2004; Ferraz et al. 2005; Albers 1996). More broadly, a better understanding of micro-level behavior at the frontier could help shed light on the frontier development process itself, and provide guidance as to how public policy might better manage it.

This research uses economics against the backdrop of biophysical realities to examine the behavior of small-scale settlers in colonization projects – important actors in frontier development in the Brazilian Amazon since the 1970s (Almeida 1992; Walker 2004). The focus is on early arriving small-scale migrants to frontier areas, and how they move from a reliance on annual cropping towards land uses requiring more investment – here, primarily pasture. Despite pasture's continued dominance in the area, there has been a dearth of theoretical models outlining an optimal trajectory for small-scale households seeking to establish a herd under frontier conditions, specifically the usually tenuous links to existing market centers.[4]

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[4]The only dynamic household model for the Brazilian Amazon to account for inseparability of production and consumption decisions in the frontier context is Walker (2003). There, though, the dynamics derive from the decision to fallow land (allow secondary forest to grow) – that is, shifting cultivation. Models including pasture either analyze the production activity outside the context of household decisionmaking (Faminow 1998), or assume household utility is strictly profit maximizing (that is, do not examine preference for leisure time) (Vosti et al. 2002). This last model, geared toward policy analysis, is structured to reflect a particular context – so is tightly constrained – and doesn't examine long-run behavior (existence and nature of a steady state). Thomas (2003) presents a profit-maximizing model whose dynamics center on the decision to deforest, but which abstracts away from the details of pasture/cattle production (using a generalized diminishing returns production function).

The dissertation helps fill that gap by developing a simple dynamic household utility optimization model, where the dynamics derive principally from investment in pasture and cattle (but also involve degradation) to capture the key economic trade-offs colonists face along the frontier at the fringe of markets' reach. An investment model can provide a common framework to encompass two often competing versions of smallholder agriculture and deforestation in the Brazilian Amazon, with subsistence needs or capitalization out of poverty alternatively driving farmer behavior. The model should be able to replicate either outcome, depending on economic conditions and household resources (as captured in model parameters).

After analytically examining aspects of the theoretical model's trajectory, focus turns to numerical simulations to highlight factors that influence a farm's optimal path. Throughout, the emphasis is on how household use of the frontier's scarce factor – labor – determines deforestation and land use patterns over time. While the bulk of the dissertation uses single-household analysis, some simulations are included that look at early settlers as potential trading partners, examining how labor trading might affect two households' land use trajectories. In this way, the spatial pattern of settlement may affect overall area outcomes. The aim is to add to emerging research on frontier areas seeking to move beyond a false dichotomy characterizing settlers as either totally autonomous households beyond the reach of markets (making interdependent consumption and production – including deforestation – decisions) or smoothly linked to existing markets (with deforestation purely a byproduct of production decisions).[5]

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[5]This research on frontier areas includes: Walker (2003), whose model includes an initial period with the farmer beyond the reach of markets, an exogenous market arrival time, and a period post market arrival; and Vance and Geoghegan (2004) find that empirically accounting for market participation or lack thereof

The rest of this chapter provides some background on deforestation in the Brazilian Amazon and smallholders' role in it. It presents a rationale for the research topic, summarizing some of the existing literature. It describes how the research approach fills some of the gaps in the literature. It ends by laying out the structure of the dissertation.

1.1 Recent migration to the Brazilian Amazon – a brief history

From the 1960s, Brazil's national government looked to settlement of the country's interior for reasons ranging from national security, to economic growth, to alleviation of poverty (for a good overview tracing the shifting foci of national policies via a sequence of government slogans, see Campari 2005).

In early days, government subsidies created incentives for cattle ranching on a large scale (Mahar 1989). Starting in the 1970s and accelerating in the 1980s, the government encouraged small-scale agriculture in the Amazon via colonization projects where poor migrants from elsewhere in Brazil could claim land practically for free. At the same time, road construction and paving started to connect parts of the Amazon with the rest of Brazil. Reports of poor migrants streaming into the Amazon hoping for a better life boosted the image of a modern-day "gold rush." Once arrived, new migrants faced the necessity of deforesting to feed their families, and a difficult life fraught with a variety of risks – from malaria, to low soil fertility, to difficult access to markets and other public infrastructure (Vosti and Witcover 1996).

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in examining land use determinants in tropical forests in southern Mexico matters for results. Thomas (2003) adds an innovation that seeks to capture not just market reach, but time since market arrival, by looking at distance from the frontier in empirical regressions at census tract level in the Brazilian Amazon.

Because of the large role government played in getting farmers to the Amazon, doubts were raised about whether large- or small-scale farming could survive absent government support: large-scale cattle ranching might not be profitable without large subsidies, and small-scale farmers would struggle to meet subsistence needs (Hecht 1985).

Yet booming pasture growth by small- and large-scale farmers continued even after most subsidies were retracted in the latter part of the 1980s. Adapted pasture varieties coupled with improved herd management helped extend the productive life of pasture beyond what was initially thought possible (Valentim and Andrade 2004). Cattle products, moreover, seemed to open a path out of poverty for many (Schneider 1995; Andersen et al. 2003; Vosti et al. 2002). A combination of growing populations, especially in urban areas, and rising incomes boosted regional demand for cattle products to an extent that even the phenomenal growth in regional supply (favored because of the vast distance of these demand centers from non-Amazon production areas) barely kept pace (Faminow 1997). While government policy played a major role in the timing of the start of the push to colonize the Brazilian Amazon, frontier development eventually generated its own momentum (Andersen et al. 2002).

The area was also affected by ebbs and flows of the macroeconomic environment. From the mid-1980s to the mid-1990s, when the national economy experienced weakness, migration *within* the Brazilian Amazon grew more important than migration *to* it. While the 1970s saw migration into the area at a level close to 19,000 people per year (netting out inflows and outflows), the trend reversed in the 1980s, when about 38,000 people per year migrated out of the region (Perz 2002a). Figure 3 categorizes counties by

levels of net migration for 1991; it shows the spatial pattern of counties losing people (blue-shaded areas) and those gaining (red-shaded areas), notably in the arc of deforestation (Perz 2002b).

Figure 3. Intraregional migration in the Brazilian Amazon, 1991

Net Mig. per 100 Pop. 1991

- -166.92 to -40.00
- -40.00 to -20.00
- -20.00 to -.00
- .00 to 19.99
- 20.00 to 39.99
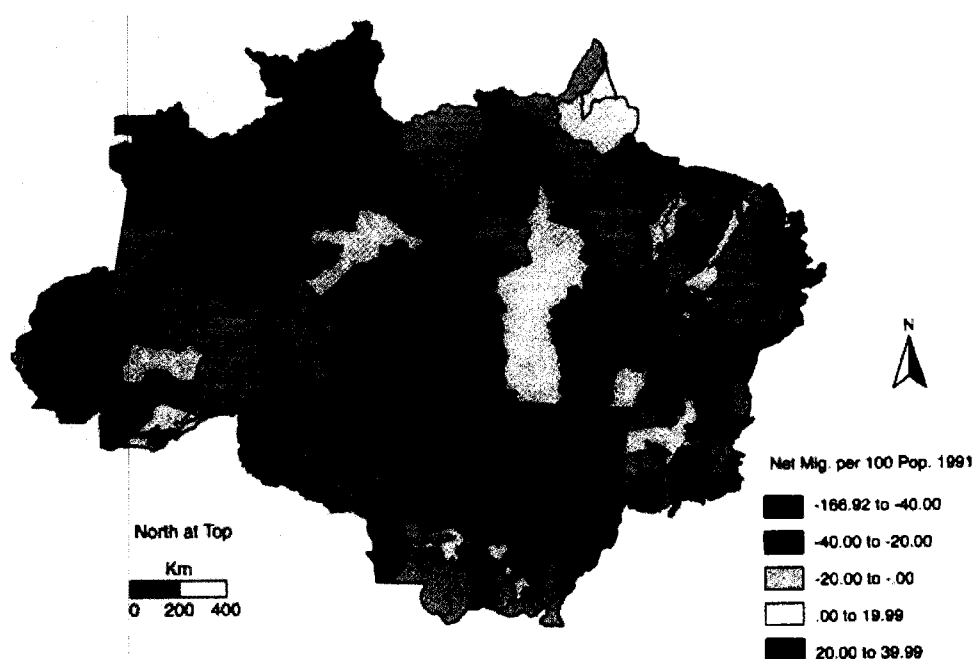
Source: Perz 2002a.

Urbanization proceeded, but robust rural growth in population and GDP per capita within the region continued only in some areas, notably within the arc of deforestation (Perz 2002b; Andersen et al. 2003). The period since an economic stabilization plan of the mid-1990s has seen a general trend toward higher deforestation rates, although figures vary substantially from year to year (Wood 2002; Pfaff et al. 2007). In that time, the government has returned to a focus on infrastructure development including paving existing roads (Andersen et al. 2003; Pfaff et al. 2007; Thomas 2003). In addition, the number of settlement projects – areas of land designated by the government for small-scale farming by landless migrants – has surged: nearly 90% of these have been created since 1995 (Brandão and Souza 2006).

1.2 Smallholders and deforestation in the Brazilian Amazon – some background

The prominent deforestation agent varies considerably from place to place (and can vary over time in any given place), but smallholders dominate in many locales (Walker et al. 2000; Wood 2002).[6] Figure 4 (Alves 2002a) based on satellite data, is suggestive of this, depicting the spatial pattern of deforestation dominated by smaller vs. larger clearing sizes for 1990s deforestation hotspots.[7] Note the considerable range of clearings over 100 ha but less than 500 ha abutting the smaller clearing areas.

Figure 4. Highest Deforestation Rate Areas 1991-7 by Predominant Clearing Size
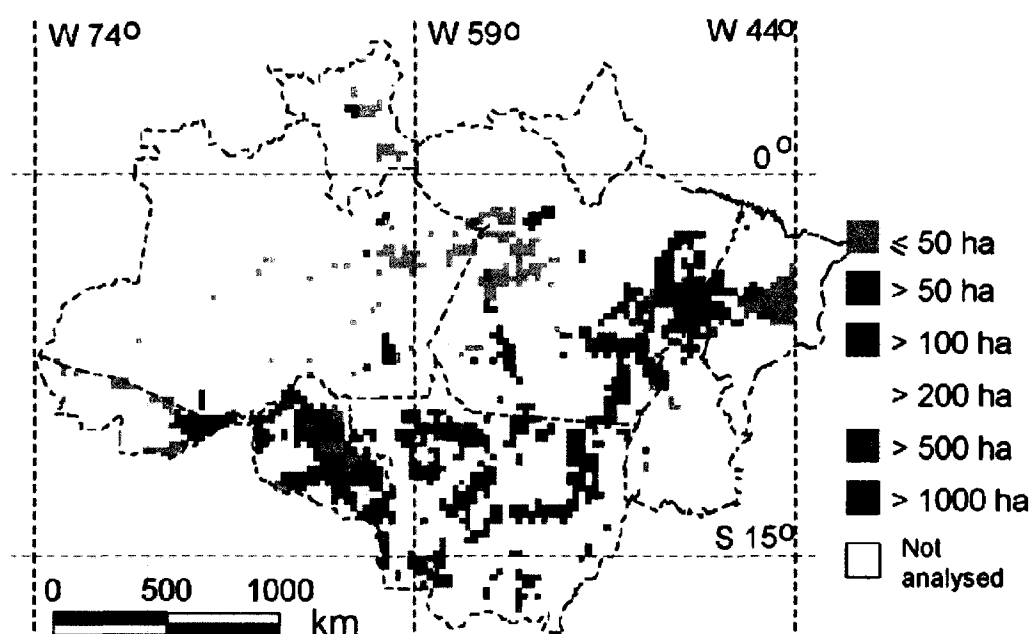
Source: Alves 2002a.

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[6]Assigning responsibility for deforestation among various actors – loggers, small-scale farmers, large-scale farmers – has been difficult and controversial. This is because different types of actors have access to the same areas over time in a complicated dynamic (Sunderlin 1996), and because land cover does not match up precisely to land use. Recent estimates range from between a quarter and a third attributable to each of two farming categories – large-scale farmers (holdings over 1000 ha) and small-scale farmers (holdings under 100 ha) (Walker, Moran, and Anselin 2000). According to census data, establishments with 2000 ha or more hold nearly 53% of the farmland and just under 47% of the cleared area (Chomitz and Thomas 2003).

[7]In the figure, each pixel is a 1/4° gridcell from INPE satellite images. Pixels are only depicted under a clearing size if that size category accounted for over half the cell's deforestation from 1991-7. Clearing size may not always indicate farm size of the deforester, however, since satellite images do not pick up the smallest clearings; clouds can obscure the view; largeholders can deforest small amounts; and smallholders can jointly clear larger forest patches.

The smaller clearing sizes at the eastern and western edges of the deforestation arc and in the region's interior seen in Figure 4 overlap with colonization project areas – the focus of this research (Figure 1). As already seen, colonization projects tend to fall within areas of relatively higher deforestation close to the region's major road arteries.

The deforestation pattern – starting at roadside and working perpendicular to this away from the roads – repeats within colonization projects. Figure 2 showed a western Amazonian site; Figure 5 shows, for the Altamira project in Pará, [8] eastern Amazon, a similar overhead view, the longest deforested areas appearing along the road grid, with deforestation expanding and extending in later years as settlers arrive and clear in cohorts.

Figure 5. Deforestation in Altamira project
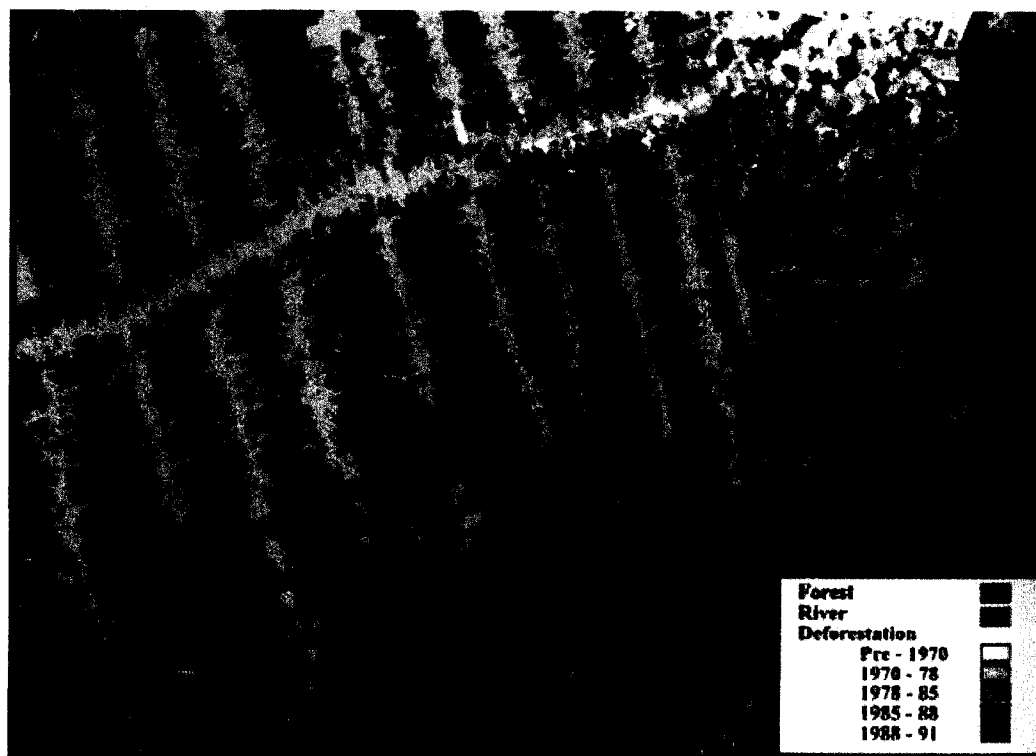
Source: Moran et al. 2002.

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[8]Generally, the eastern Amazonian sites have been settled for longer than those in the west.

1.2.1 Land use in western Brazilian Amazon settlements – from forest to pasture

In the western Brazilian Amazon, the conversion of forest to pasture takes place

incrementally on a given farm. Figure 6 traces the land use trajectory for a typical plot of

land in the study region in the western Brazilian Amazon. The time spent in each land use

indicates length of continuous use under commonly used technologies. For 'end' land

uses (pasture and perennials), this is the length of time before replanting or abandonment.

Figure 6. Land use trajectories and deforestation
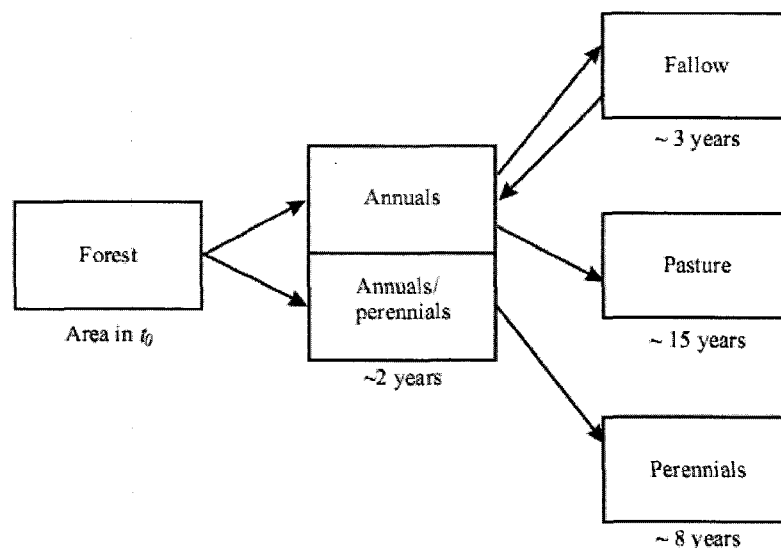
Source: modified from Vosti et al. 2002.

Any planned fallow/secondary forest regrowth usually occurs only after a first

cycle of annuals and supports just one additional year of annuals before being relegated

to pasture (Fujisaka et al. 1996; Vosti et al. 2002); the practice is disappearing.[9] The land

use trajectory – the progression from forest through annuals to pasture – repeats on

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[9]Not necessarily so in the eastern Brazilian Amazon. In a study of projects along the TransAmazon
Highway, deforestation was more highly correlated with later secondary growth than with later production
(Brondizio et al. 2002). The data covered the period (late 1980s) when the region was undergoing
economic hardship, however (and it is impossible to track the extent to which fallow was an active
management decision or due to inaction). See also Boerner et al. (2007).

different plots of land within the farm. Pasture predominates even among the small proportion of farmers who plant perennials (Vosti et al. 2002).

Survey data from two colonization projects in the area showed the average plot deforested was close to 4.5 ha in 1994 and 1996, and deforestation took place on average every other year (Vosti et al. 2002).[10] In that household sample, area in forest between 1994 and 1996 fell from an average of 61% to 56% of farm area, and the percentage of cleared area devoted to pasture grew from 52% to 61% (Vosti et al. 2002). Figure 7 shows how, the longer a farm is settled, the more forest is converted to pasture using the 1996 sample.

Figure 7. Land use upon arrival, in 1993/4, and in 1995/6, 1996 survey data (n=228)
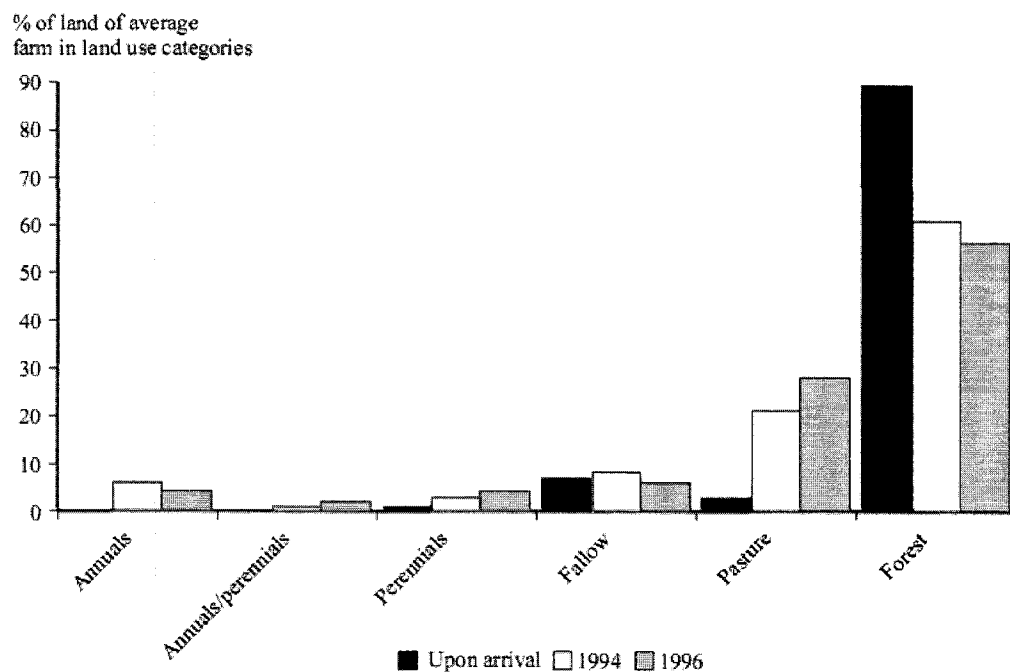
Source: modified from Vosti et al. 2002.

There is other evidence that the pace of deforestation varies over time on a given farm, and for different arrival cohorts. Older accounts describe the first several years

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[10]The survey covered 156 farms in two colonization projects – Pedro Peixoto in Acre and Theobroma in Rondônia – in 1994, and 228 farms in 1996, of which 142 were re-surveys of 1994 farms.

after settlement as the most intense in terms of deforestation (Fearnside 1984). Brondizio

et al. (2002) provide one of the clearest looks at on-farm deforestation dynamics, using

data from an eastern Amazonian project. Figure 8 from their study traces average

deforestation patterns by arrival cohort. Notable are the two deforestation pulses varying

in size by cohort but visible in all but the latest arrival cohorts. For these, a second pulse

may still be ahead, or the overall pattern may have shifted. This last period is the only

one in which all cohorts experienced an upswing in deforestation (in keeping with the

1996 'deforestation spike' seen throughout the Brazilian Amazon).

Figure 8. Deforestation by arrival cohort, eastern Amazon colonization project
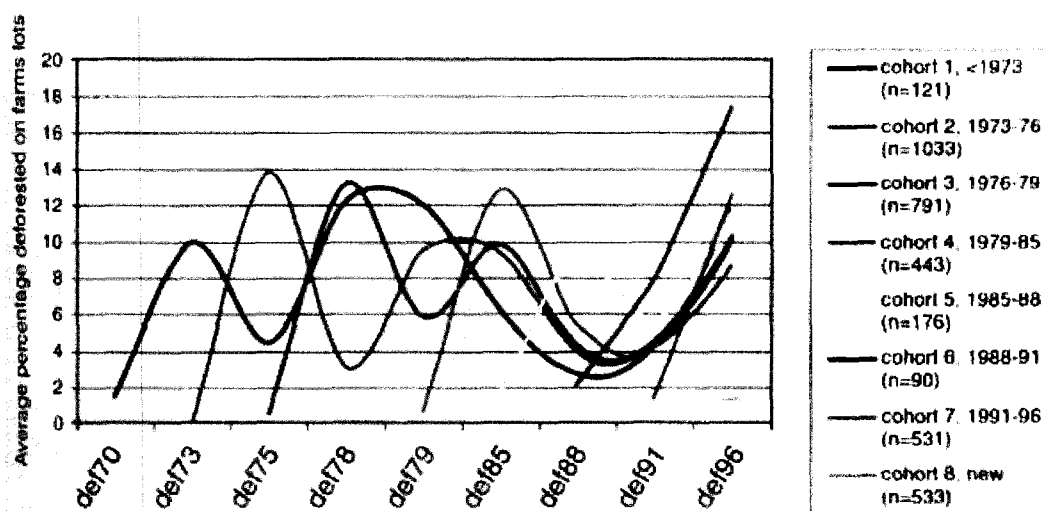
Source: Brondizio et al. 2002.

Thomas (2003) finds using satellite data that deforestation rate peaks in areas

approximately 30% deforested, with lower rates in areas with less, and areas with more,

deforestation. The rise is partly attributable to frontier regions, defined as having

deforestation but with a non-deforesting neighboring 'pixel' (area unit of analysis).

Geographically closer to markets from the forest frontier (that is, inside the frontier),

deforestation rates slow down as deforestation proceeds.

1.2.2 Lot settlement, turnover, and survival among settlers

The household-level forest-to-agriculture pattern repeats across farms in areas opened for settlement. Settlement tends to occur in waves, starting out closest to the main artery, then moving out along the feeder roads – the human occupation behind the expanding fishbone pattern of deforestation. Settlers are awarded blocks of land by government lottery and receive provisional title. Informal squatters who settle at the boundaries of areas already officially allocated usually receive provisional title after several years' residence.[11] Figure 9 shows, for one colonization project in the eastern Brazilian Amazon, the spatial pattern of lot settlement vis-à-vis feeder roads over time.

Figure 9. Arrival cohorts in Altamira colonization project
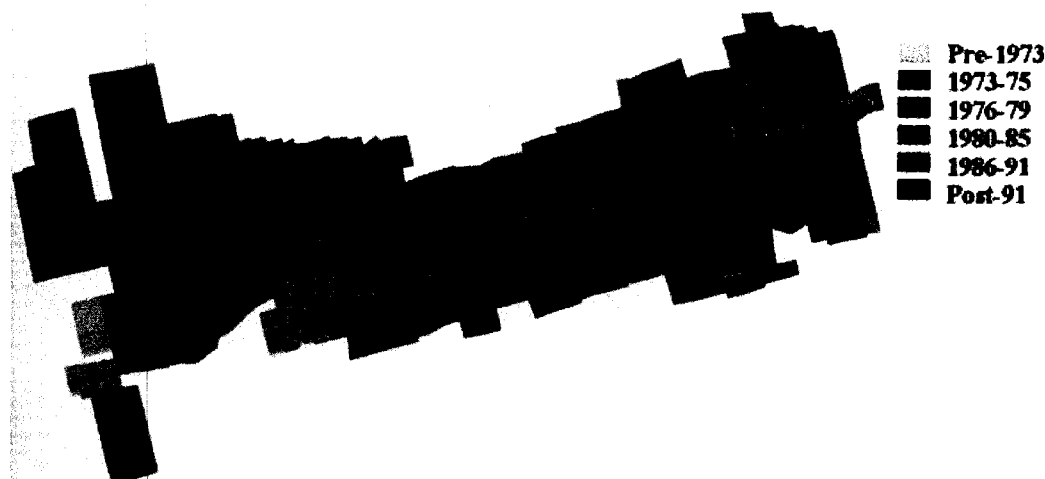
Source: McCracken et al. 2002.

Distributed lot sizes started at 80 to 100 ha, but with continued in-migration and squatting, newly distributed lots within older colonization projects became smaller (as low as 25 ha). The Vosti et al. (2002) study showed some increase in the average farm size (around 10 ha) from the time interviewees first arrived on lots to the 1994-6 period, with more of the sample net land buyers vs. sellers (around 10% vs. less than 5%). Still,

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[11]Law bars land transfers for several years after a lot is settled, but this measure is not enforced.

in 1996, a sizeable majority of respondents (80%) owned a single lot, 15% owned two lots, and the remainder owned more than two.

Surveys in the early 1990s in the western Brazilian Amazon found settlers with an average of about ten years on the same lot (Jones et al. 1995; Vosti et al. 2002). A survey that spanned colonization projects in the heart of the arc of deforestation and the eastern Amazon along the TransAmazon Highway concurred – on average, 71% of lots within projects had identical ownership throughout the decade 1981-91, apparently surviving the economic downturn of the mid to late 1980s (Campari 2005).

That said, lot ownership does often turn over from original settlers. In the Vosti et al. (2002) study, just under 30% of lots surveyed in 1996 still had their first owners. Time since lots opened, moreover, averaged six years longer than the average time of current owner on lot (15 years vs. 9 years).[12] This jibes with accounts of second waves of migrants arriving to buy out some first arrivers several years after a settlement project opens (Leña 1991).

Some research finds no difference in land use behavior by arrival status – meaning decade-long residents vs. newcomers in Campari (2005). Similarly Jones et al. (1995) find no link between length of residence and land use, controlling for other factors. Greater tenure *security* measured as progression to a definitive title, on the other hand, did have expected links to on-farm investment – higher proportions of land in perennials and pasture – controlling for time elapsed since settlement (Vosti et al. 2002).

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[12]This is one of the micro-level studies to make the critical distinction between lot history and current owner history. It also tracked recent changes in ownership and demographic household shifts. It did find that recent ownership (in the last two years) was significantly tied to greater investment in perennials, but as this coincided with a price spike in perennial crops may reflect investment flexibility on the part of new owners.

This study also found, though, that more recent arrivals had a greater propensity to start gearing up for perennial cropping in response to a recent price surge, and had lower proportions of their farm remaining in forest, controlling for other factors. *Lots* opened later, on the other hand, had relatively higher proportions of the farm in pasture, and less in forest.

There are accounts of early settlers being driven to sell out because of indebtedness in a pattern that sometimes includes land consolidation (Leña 1991; Almeida 1992). But there are also accounts of early settlers surviving and even thriving on their lots or selling out to move to more favorable locations, accompanied by relatively weak upward pressure on average farm size (Vosti et al. 2002; Campari 2005).[13]

1.2.3 Household land use – a dynamic investment decision

The decision to deforest and how to use the cleared land are both part of the same underlying household investment plan. In this context, the constraints on household decisionmaking include the biophysical realities of the area (poor soil fertility and degradation with use), the labor scarcity in the area, and the poorly capitalized condition of early arrivals. How the investment actually plays out – its success or failure – can shed light on what happens to individual settlers.

Early settlers need to deforest and plant annual subsistence crops for survival. The government supplies arriving settlers with a 'basket' of goods meant to assist them

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[13]The economic shake-out of winners and losers happened on a larger geographical scale: recall the GDP and migration growth in the arc of deforestation compared with losses elsewhere in the Amazon in Figure 3. Intraregional migration in the Brazilian Amazon, 1991. This is reminiscent of other frontier areas that have undergone shake-ups in the first serious economic crisis post-migration, e.g., the string of bankruptcies and land consolidation in the wake of the first prolonged drought in the Great Plains of the U.S. after the arrival of homesteaders (Hansen and Libecap 2004).

through this period. In the study area in Vosti et al. (2002), settlers arrived with an average of 3 1/2 months of resources, but no cattle. They also already had social contacts within the colonization project, who might have lent some assistance during this precarious period.

Felling forests is a labor intensive activity, and hazardous. It requires at least two adult males with an axe a chainsaw (owned or rented) working between 2 and 3 days per hectare of forest (Vosti et al. 2002).[14] Land-clearing occurs in the dry season; felled forest is left to dry then burned before the rains begin, usually in September. The amount of labor available limits how much land can be cleared at any given time, because of the labor needed for not just deforesting itself, but also managing the cleared land. Land left untended reverts to forest, and land used too intensively (overmining soil nutrients derived from the forest burn) quickly loses productivity. Also affecting the size of area deforested at any one time is the need to control the subsequent burn: too large an area (with inadequate supervision), and the fire can escape the deforested area to burn crops on the farm and beyond. The burning itself also requires some skill – an inadequate burn not only fails to transfer critical biomass to the soil, but leaves debris (logs and stumps) that impede planting. In this study area, the average amount felled in a given year in the mid-1990s was approximately 4.5 ha, but ranging widely, from 0.5 ha to about 20 ha (Vosti et al. 2002).

Settlers move towards growing a herd, planting perennial crops, or some combination of these as soon as they can (Vosti et al. 2002; Brondizio et al. 2002; Browder 2002). Pasture's advantage lies in its profitability, price stability, its ability to

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[14]The workers first clear the area of the smaller trees and brush area using an axe, then cut the large trees using a chainsaw. So, the amount of labor required depends on sometimes varying forest characteristics.

act as a store of wealth, and its flexibility in allowing incremental adoption of management improvements (Faminow 1998). Above all, though, cattle/pasture production uses relatively little labor – an attractive feature along the frontier where manpower is the scarce factor.[15] Households typically allow the herd to expand through natural growth, managing the process through selective culling.

In the Vosti et al. (2002) sample, households engaged more frequently in output than in labor markets. Fieldwork also uncovered some trading among settlers, often involving labor, but sometimes other products (for example, outputs) as well. Some evidence showed labor trading more prevalent in the more newly opened (outlying) areas within a colonization project, with the incidence of wage work rising in longer settled areas (author, unpublished data from IFPRI fieldwork).

An investment framework ties together the decision of when and how much to deforest with the decision to expand pasture and grow the herd, with labor availability the critical resource constraint forcing economic trade-offs. A dynamic optimization framework provides a tool to generate hypotheses about settler behavior – not just in a single year but over time, as well as a metric for ranking outcomes for settlers' well-being under different conditions. It can also indicate which conditions give farmers incentives to trade locally, and with what impact on their land use trajectories.

Deforestation studies often embrace an investment perspective, but vary considerably in how explicitly they lay out an investment model against which farmer

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[15]Averaged over its productive lifetime, and including the typical land uses (deforesting and annual cropping) leading to pasture, pasture uses less than 15 person-days per hectare per year. This compares with just over 20 for annual cropping, and between 25 and 60 for coffee-based perennial systems for the area (using similar accounting, so that deforestation, itself a labor intensive process, is included in all systems) (Vosti et al. 2002).

behavior, especially its timing, could be compared. Less common still is an examination of the potential for local trading to affect investment trajectories. We now turn to a fuller discussion of the existing literature.

1.3 Modeling small-scale settlers at the frontier: research rationale

This dissertation looks at deforestation through a particular lens (household dynamic optimization of utility) in a particular locale (the frontier) for a particular actor (small-scale settler). The aim is to better understand the process of frontier development – how areas transform from forest to pasture. The process has economic as well as ecological determinants and consequences, and an understanding could lead to improved policy that better targets desired outcomes for the environment and the people who inhabit these areas.

This section examines the rationale behind this research focus, asking first why the frontier, and second why the small-scale settler. The importance of dynamics enters into each discussion. In the process, it reviews some of the existing literature, including studies with an analytical as well as an empirical bent.

1.3.1 Why the frontier? Conceptual and analytical models, empirical evidence

As the leading edge of agricultural expansion into tropical forests, and a delimiter of key land use zones, the frontier has received attention as a potent policy target for halting frontier expansion (Chomitz 2007). In areas beyond, at, and inside it, different behavioral models and policy prescriptions apply because each zone has a qualitatively

distinct position vis-à-vis markets, accompanied by different amounts and patterns of standing tropical forest (Chomitz 2007; Vance and Geoghegan 2004).[16]

1.3.1.1 Conceptual and analytical models

Conceptual frameworks for tropical deforestation have at their heart the idea of frontier development. They often invoke conditions that extend the spatial reach of profitable activities, building on a von Thünen-esque model, where distance from market center delineates a gradient of profitability that helps determine spatial extent of land use and zones of land use along that radius. Spatial extent shifts in response to a change in some underlying parameter (e.g., demand level, population, technology, transport cost) (Angelsen 1999), or as part of an ongoing adjustment towards an ideal steady state (the dynamic optimization framework seen in Lopez and Niklitschek 1991). Critical to the models is the linkage (or not) of the region under study to broader markets, with particular attention paid to the relatively scarce factor along the frontier (directly affected by in-migration) – labor. Of concern are the degree of labor mobility and locus of wage determination (Angelsen 1999; Jones and O'Neill 1992).

In contrast to this profit-based view, conceptual models of tropical deforestation arising to explain the 1980s surge in forest-felling centered on the slow but steady clearing by small-scale agriculturalists driven by subsistence needs and constrained by poor agricultural conditions. Such models are seen to apply in special situations like

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[16]That said, the spatial frontier is difficult to define precisely. It is an area where sustained deforestation is getting underway; inside the frontier, land-clearing for agriculture is reaching completion, and beyond the frontier, clearing is less systematic, sustained, or common. *Inside* frontiers, deforestation has led to forest fragmentation and shrinking fragments amid other agricultural uses (see Ferraz et al. 2005 for a site within Brazil's arc of deforestation; Chomitz 2007 describes these "mosaiclands" in tropical forest margins worldwide). *At* the frontier, most forest is still standing but may be falling at a rapid pace. *Beyond* the frontier, deforestation is patchy. Market reach is overlaid on this pattern, with more complete market access within the frontier, markets just reaching (perhaps incompletely) the frontier, and only selectively reaching beyond the frontier.

economically stagnant frontiers or areas beyond frontiers, or with some tweaks in frontier areas where some but not full market integration exists (e.g., Rudel 2005). At the extreme, each agent in this model behaves as an autonomous (Chayanovian) household, and non-profit objectives (e.g., leisure) come into play (see, e.g., Angelsen 1999).[17] Neither the subsistence-based model nor its profit-based counterpart, however, speaks to details of how the spatial filling out process occurs, or with what degree of "filling in."

Conceptual models with more detail on frontier development dynamics, particularly those involving rapid deforestation, invoke critical interactions in space or time across actors or sectors within a framework of weak property rights and poor market development. In one construct, an interest with sufficient capital (here loggers, capitalized farmers, or the government) paves the way for settlement by small-scale farmers by opening access to new areas before their arrival, or providing a promise – implicit or explicit – of buy-outs at a later time (Rudel and Roper 1997; Sunderlin 1996).[18] Modeling efforts in this direction emphasize the role of uncertain property rights at the extensive margins of profitability. One study examines the race for property rights on the frontier as a game between large- and small-scale farming interests, with an important role for the government (Alston et al. 2001). Others explore how the profitability gradient from a market center is shifted by uncertain property rights,

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[17]More on this model appears in the subsection below on small-scale settlers.

[18]For the Amazon, these stories include: a) loggers opening access roads for agriculture to move in – this can lead to a 'boom-bust' cycle in areas unsuitable for agriculture if the initial capital generated is not carefully invested (Schneider 1995); b) small-scale farmers clear land then sell out to large-scale ranchers, moving on to newer frontiers or urban areas – dubbed "the hollow frontier" because of eventual farm consolidation (Almeida 1992). These are both variants of an argument regarding the presence of "growth coalitions" in rapidly expanding frontiers that supply the capital to push the process forward (Rudel and Roper 1997; Walker 2004).

generating incentives for actors with low opportunity costs to colonize the peripheries of settlements (Alston et al. 1996; Schneider 1995).

The current dearth of spatially explicit analytical models on frontier development is partly due to the difficulty of incorporating inter-agent interactions and incomplete market participation into existing analytical frameworks on spatial extent of production and land use (Walker 2004).[19] The above-cited Alston et al. (2001) study of frontier settlement as a game with large- and small-scale holder and the government as players is one notable step in this direction. Other studies take a first cut at understanding landscape processes by straightforwardly aggregating results of micro-level models (so assume no important feedback from agent interactions) (Vosti et al. 2002; Evans et al. 2001).[20]

The difficulty is, in a sense, how to aggregate across heterogeneous agents in a dynamic investment environment. The dynamics come at potentially two levels, the individual agent and some aggregation of agents: individual agents make decisions about their own paths given expectations about broader conditions, but the interplay across agents is a critical driver behind changes in those broader conditions, including market conditions. The frontier, of interest because it is a rapidly shifting environment, is a challenge to model for precisely that reason.

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[19]Other difficulties include capturing dynamics of land use paths to long-term equilibria, and accounting for linkages from rural hinterland to urban center as well as to the broader economy (Walker 2004).

[20]Vosti et al. (2002) identifies two stylized representative households emerging from cluster analysis on household survey data. The Evans et al. (2001) work uses random variation in demographic events by household to generate landscape-level scenarios.

1.3.1.2 Empirical studies with broad geographic coverage[21]

As Walker (2004) points out, the relative quiet on the analytical front of spatially explicit tropical deforestation study is contrasted with a great deal of activity on the empirical side. Econometric studies with broad geographic coverage often marry satellite imagery with data on agricultural potential (e.g., rainfall patterns and soil types) and socioeconomics (e.g., agricultural production, population, distance to market centers), at ever finer spatial levels (see, e.g., Pfaff 1999 and Andersen et al. 2002 using data at county level, and Thomas 2003 and Pfaff et al. 2007 with finer data at census-tract level). They use administrative units, which may encompass heterogeneous deforestation agents, as the level of observation.[22]

These econometric studies have uncovered statistical relationships between deforestation and various land-use determinants, but, due to insufficient temporal and spatial data resolution, only crudely capture indications of lack of market integration or disentangle the effects of the two levels of dynamics described above within observed outcomes. Acknowledging these shortcomings, authors take a profit-maximizing approach focusing on production with the dynamics relegated to the discounting term, so turn the maximization problem into a series of static optimizations (Nelson and

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[21]Empirical studies that focus on one agent – the small-scale farmer – are discussed below. Sector-specific studies – notably for logging and cattle production – exist that flesh out how spatial extent of production (with concomitant deforestation) adjusts to changing market conditions. They are not discussed in detail here because they exclude actors not participating in markets (by design). They find, as expected, clearing activity tends towards the margins of profitability shaped by available production technology (which helps determine whether increased production requires more area or more intensive use of given area) and given the strength and location of demand. Processing infrastructure adjusts to market conditions with some time lag involved (Stone 1998; Schneider 1995; Faminow 1997).

[22]There has been innovative work using correspondence between certain land covers and decisionmakers (e.g., pasture and ranchers) to distinguish to some degree among heterogeneous actors. Cattaneo (2001) does this in a spatially disaggregated computable general equilibrium model that separates out the Amazon region in a CGE model for Brazil, and treats deforestation as a separate sector. The study shows different results between short- and long-term deforestation effects of exogenous shocks (e.g., introduction of new technology) stemming from relaxed constraints on interregional labor mobility over the long term.

Hellerstein 1997). In longer settled areas, the current land use is studied as a steady state (see, e.g. Tachibana et al. 2001).[23] Only rarely is the aggregation across individual decisionmakers made explicit in the derivation of the higher spatial scale model (Pfaff 1999).

The studies partially control for market imperfections and other important dynamics through choice of independent variables.[24] In studies within the Brazilian Amazon, authors note the simplifying assumptions might make a difference to results, given: a) underlying economic conditions (including spatial reach of markets or local population density) are often undergoing rapid change; and b) the observed land use systems – establishment of a herd or plantation, or management of a fallow rotation – have important dynamic elements (Pfaff 1999).

The studies find complicated spatial spillovers in deforestation patterns and its socioeconomic determinants across adjoining administrative units. Authors have inferred frontier effects in interpreting results such as counterintuitive signs on coefficients for distance to markets (but cannot exclude other explanations or pinpoint specific pathways behind the spillover effects). Another study shows path dependency in frontier

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[23]The last decade has seen a similar model structure that involves biophysical and socioeconomic determinants of land use applied to the Brazilian Amazon and other tropical forests (Andersen et al. 2003; Chomitz and Gray 1996; Chomitz and Thomas 2003; Cropper et al. 2001; Nelson and Hellerstein 1997; Pfaff 1999; Thomas 2003; Pfaff et al. 2007). Walker (2004) outlines how the model structure emerged from profit-maximizing explanatory models in the social sciences (with error terms added for estimation of probabilities that land is in the use with the highest return, usually a static maximization) and Markov land use transition descriptive models in ecology and regional science (where a change in land cover is expressed in probabilistic terms, based on observation).

[24]For market imperfections, methods used include using some observable indicator – e.g., an output associated with subsistence farmers for landscape-level studies (Chomitz and Gray 1996), distance to market center, or factor constraint – as an explanatory variable (Walker et al. 2000; Vosti et al. 2002), or subsuming effects of market imperfections in assumptions about the production function's form (Thomas 2003). For dynamics, independent variables are measured with some lag (determined by data) or themselves track time (e.g., time since settlement on the lot). Another approach is to look cross-sectionally at observations of different ages, as though the older unit foretells the future of the younger; an approach that can mislead without applying theory about trajectories and what conditions they depend on.

development, with early settlers to an area having a disproportionate effect on longer run

deforestation patterns (Pfaff 1999).

The studies show that early settlement – an old frontier – provides a "seed" from

which subsequent deforestation proceeds (Alves 2002a; Thomas 2003). Figure 10 from

Alves (2002b) highlights spatial autocorrelation in accumulated deforestation by the

1970s (a single level), and ongoing deforestation for 1991-1997 (a single rate), against a

backdrop of 100km buffer zones around major roads.[25] Spatially concentrated

deforestation in the later period tended to occur close to (spatially concentrated) earlier

deforestation near roads in the southern deforestation arc.

Figure 10. Spatial Autocorrelation (SA-C) in Deforestation
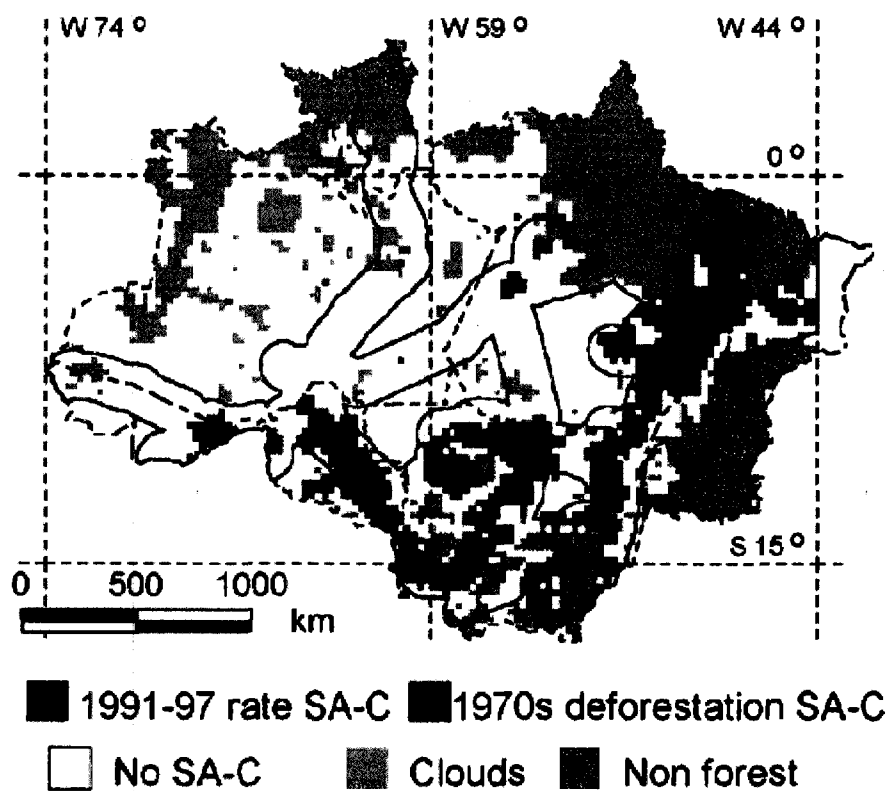
Source: Alves 2002b.

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[25]Alves uses a local indicator of spatial association based on similarities in rates for adjacent $1/4°$ gridcells from INPE satellite images to establish spatial autocorrelation, and a 5% significance level cut-off.

Proximity to roads does not tell the whole story, however. Thomas (2003) shows in econometric work that the "seeding" effect is behind a good deal of road infrastructure's impact on deforestation. Once early settlement is taken into account, regression coefficients for roads shrink in both explanatory power and magnitude – an important policy finding.[26] Within part of the arc of deforestation, the likelihood of imminent (within a three-year period) deforestation beyond the frontier was highest closest to the frontier, falling off sharply within 15km (Thomas 2003).[27] Again, though, the data are not up to isolating mechanisms.

Summaries of studies at aggregate spatial scales highlight the need, given heterogeneity within these observational units, for more micro-level studies – where the unit of observation and decisionmaking unit coincide (Kaimowitz and Angelsen 1998). The next subsection examines a subset of micro-level studies focusing on small-scale settlers.

1.3.2 Why small-scale settlers? Importance and modeling tractability

Many micro-level studies on tropical deforestation in the Brazilian Amazon have taken small-scale settlers as their focus. The attention is partly due to the substantial numbers of small farmers in the region, some half million by the early 1990s, so even a

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[26]The road effect, and probability of deforestation more generally, declined the higher the rainfall in an area (Chomitz and Thomas 2003; Thomas 2003). There has been considerable debate in the wake of Brazil's planned infrastructure improvements in the Amazon (*Avança Brasil*) about how road construction, and road improvement, either in pristine forest areas or already settled zones, affects deforestation rates. Econometric work at county level showing some easing of pressure on forest with roadbuilding (Andersen et al. 2002) did not hold up in two econometric analyses at census-tract level (Pfaff et al. 2007; Thomas 2003). Thomas (2003) includes a discussion of what might have been behind the anomalous result. This debate lies outside the focus of this paper.

[27]This work represents a significant advance in that it singles out a 'frontier' among land use pixels and uses it as a reference point to understand trends and test hypotheses. 'Distance to frontier,' an about-face from the usual focus on distance to market, may provide, in a setting with an expanding frontier, some proxy for time since market arrival. It thus moves empirical study closer in line with conceptual frameworks.

little deforestation by each adds up (Faminow 1998; Wood 2002; Walker et al. 2000), and partly due to the fact that poverty concerns here mesh with a potential environmental threat. It also stems from a realization that the higher-level studies that assume market integration may mischaracterize small-scale farmers' behavior, and consequently miss a big part of the deforestation story. From a modeling perspective, this group is tractable in that an analytical household model (Singh et al. 1986a) exists that can be tailored to fit varied situations as regards market integration.

Perhaps more importantly, the small-scale settler figures prominently in the conceptual stories told of frontier development processes, both past (as the primary actors, largely cut off from markets) and present (as contributors to a more complicated temporal and spatial process, or of paramount importance in particular situations and areas). Indeed, the literature is filled with studies that, focusing on different settings, describe forest margin farming as either a poverty trap or opportunity for improved welfare (extensions of the debate in the early literature about the economic profitability of the enterprise).

1.3.2.1 Empirical micro-level studies: market imperfections and dynamics

The micro-level empirical studies, while limited geographically (one or several settlement areas), add detail in terms of agent behavior and its determinants. The relative labor scarcity in land abundant frontier areas makes labor availability and alternatives for how labor is used a natural area for study (Kaimowitz and Angelsen 1998; Tomich et al. 1998). Econometric studies usually mirror the approach taken at higher spatial scales: they focus on the production side, and include as independent variables observables that

could control for market imperfections and dynamics.[28] These include household/farm characteristics, lagged as data allow, variables that track time since an important (household or farm-specific) event, and, where possible, location dummies to control for other unspecified effects particular to the locale, including those changing over time.

There are exceptions, though. Shively and Pagiola (2004), for instance, use a utility-based approach in a study in the Philippines on deforestation effects of links between lowland irrigated farms and the labor-supplying upland farms. They elaborate a dynamic utility-maximizing model that makes different separability assumptions about upland and lowland households. Key to the estimation strategy is the observation that a change in shadow wages (sparked by rising agricultural productivity due to lowland irrigation) will prompt a re-allocation of labor across activities for the upland farms.[29]

For the Brazilian Amazon, several studies confirm the idea that deforestation at the frontier is shaped by market forces but does not conform to a fully market-integrated approach. Ease of market access captured in distance or time to market center matter to deforestation and land use patterns (Vosti et al. 2002; Jones et al. 1995).[30] So does family structure, providing evidence of nonseparability of consumption and production decisions (Vance and Geoghegan 2004; Vance and Geoghegan 2002; Vosti et al. 2002).[31]

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[28] The econometric studies also, like higher level studies, include information on the biophysical resource base regarding agricultural potential, usually using a static measure. We focus here on findings related to market access and on-farm dynamics.

[29] They use observed labor input and output to estimate shadow wages, then include these estimates as an independent variable in a system of equations including determinants of land-clearing among the (labor-supplying) upland households. Higher off-farm wages, they conclude, prompt upland intensification and take pressure off forests.

[30] The Vosti et al. (2002) study showed, as expected, more forest and less pasture the greater the time to market, and the longer the additional time to market in the rainy vs. the dry season. Other measures of accessibility linked to nearby traffic flow (volume and types of vehicles) were associated with perennial cropping, in keeping with the idea of better market access fostering this type of investment.

[31] Interestingly, this implication is often not explicitly mentioned. Vance and Geoghegan (2004) show, in an innovative household study from Mexico using switching regressions, that evaluating land use

Household demographic structure on arrival continued to matter to land use patterns years later in a relatively recently settled areas in the western Brazilian Amazon in the deforestation arc (Vosti et al. 2002). For a longer settled area in the eastern Brazilian Amazon, household demographics mattered less than subsequent changes to farm labor or contemporaneous hired labor (Walker et al. 2000). The same studies found similar differences in effects of household wealth at arrival: it mattered to deforestation outcomes in the more recently settled site (interestingly associated with higher proportions of farms left forested), but not the longer settled one.

Other household characteristics found to be significant include age and literacy of household head, the former mattering for income/wealth outcomes, and the latter associated with less reliance on annual cropping (Jones et al. 1995; Vosti et al. 2002, respectively). The Vosti et al. (2002) study also found evidence that farmers' social contacts and migratory paths indicate the presence of ways for farmers to either ease on-farm resource constraints (through local trade in the absence of fully functioning markets), or lower their market access costs.[32]

Time-related factors – time since the lot was first settled and time since the current owner's arrival – also emerge as important for deforestation and land use patterns (Jones et al. 1995; Vosti et al. 2002; Alston et al. 1996).[33,34] But most studies have

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contingent on the decision to participate in markets at the frontier makes a difference for deforestation results.

[32]Having social organizations (churches, farmers' associations, labor unions) nearby or arriving in the project with a network of contacts was correlated with fewer annuals as a proportion of area. More migratory stops *before* settlement correlated with more perennial cropping (coffee prices were at an unusual peak), more stops *within* the settlement project meant fewer perennials and more land left forested (Vosti et al. 2002). According to Sydenstricker (1998), social organizations eased interactions with entities outside the project related to marketing and agricultural extension.

[33]In Jones et al. (1995), a longer time on lot was associated with a lower incidence of annual burning. Alston et al. (1996) found a link between time on lot and having a title, which, in turn, was associated with

observations on just one or two points in time per farm. And, the clumping of arrival times and lot openings doesn't allow disentangling of dynamics at the farm or household level from broader temporal (e.g., macroeconomic) trends.[35]

The studies in general support the view that farmers in these areas have had some success, but also voice concern about the extent of poverty. A child anthropometry survey in the Vosti et al. study revealed little evidence of malnutrition, but other survey data supported the contention that many settlers lived close to the per capita poverty line for Brazil, and a sizeable subset farther from market centers might be struggling to make ends meet. The studies' short timeframes and timing (at least a decade after the opening of the colonization projects) made it difficult to document the failure of farms, particularly those in the earliest stages of a colonization project.

Other studies have, however, started to take a closer look at on-farm dynamics – following farms through longer time periods. One study already mentioned discussed a pattern of early migrants falling into financial straits and selling land to a second wave of more affluent settlers (Leña 1991). In contrast, work by Campari (2005) showed no broad evidence of early settler turnover, or any difference in clearing behavior or farm size between early settlers who last and latecomers.[36] Other work is based on an

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increased investment on the lot related either to pasture or permanent cropping. In the Vosti et al. (2002) study, there was a tendency for ever less forest and ever more pasture on lots of increasing age. Controlling for other factors in a multivariate analysis, however, showed lots opening later had proportionally less forest and more pasture.

[34]Vance and Geoghegan (2002) improve on these treatments of time by applying survival analysis to the deforestation problem, that is, estimating a likelihood function in which probability of forest clearing is conditional on the length of time a piece of land has survived as forest.

[35]The econometric work by Thomas (2003) described above that limits the sample spatially and in terms of deforestation behavior to try to isolate areas where farms are predominantly in a land use steady state is an advance in this regard.

[36]Note that, for Campari, the early settlers were not necessarily the first to settle on their lots, but those present at the beginning of his survey period in 1981.

underlying conceptual investment model of a household trying to move out of subsistence annuals into cattle or perennial crops, and has: a) tracked behavior of farms within and across arrival cohorts (Brondizio et al. 2002); b) asked how household demographics through the household's life cycle could affect land use trajectories, and how the changing population structure of the settlement project over time fits in (McCracken et al. 2002), and c) used case studies to highlight factors that enhance successful farm investment into ranching or perennial cropping, or thwart it, prolonging the period of reliance on annuals (Browder 2002). Discussion returns to these findings at the end of the chapter, in a section on settlers' land use trajectories.

1.3.2.2 Analytical models: market imperfections and dynamics

As noted above, a basic household model adaptable to situations of incomplete markets (Singh et al. 1986b) exists and has been applied to tropical deforestation contexts (see, e.g., Angelsen 1999 and Shively and Pagiola 2004, both of which use a utility function that includes leisure in contexts where nonseparability of consumption and production decisions applies). A few analytical models of tropical deforestation incorporate dynamic optimization (Albers 1996; Tachibana et al. 2001; Vosti et al. 2002; Thomas 2003; Walker 2003; Shively and Pagiola 2004). The combination of nonseparability and dynamics is relatively rare (Walker 2003; Shively and Pagiola 2004).

In most of these models (except Thomas and Albers), labor allocation across activities in a labor-constrained environment plays a central role. For the Vosti et al. study set in the Brazilian Amazon, this includes on-farm production activities: annual

cropping, perennial cropping, and pasture generation/cattle management.[37] The linear programming (numerical simulation) household model is calibrated to conditions in the western Brazilian Amazon taken from survey and secondary data. The calibration incorporates yield declines with continued production as well as biomass gains from leaving land fallow (secondary forest regeneration).[38] It also includes detailed information about managing pastures and herds, and tracks herd growth. The model baseline shows pasture growing steadily at the expense of forest. Having more labor-using production activities like perennial cropping, however, slows the pace of forest to pasture conversion.

Constructed for policy analysis, the model was not tested for its steady state properties. Sensitivity tests on the optimal trajectory were run through specific simulations. It has several characteristics that limit its range of applicability. It: a) uses a finite time horizon (although tries to correct for deviations caused by model terminal conditions), so does not necessarily shed light on long run behavior or implications of model assumptions; b) employs an iterative dynamic optimization strategy that could yield a solution that deviates from the optimum (but does not appreciably within the parameter range explored); c) equates household utility with household profit maximization despite an imperfect market setting (so ignores how, e.g., the consumption of leisure might affect welfare and deforestation outcomes); and d) approximates all constraints using linear functions, limiting input substitutability in production methods as

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[37]For both the Tachibana et al. and the Shively and Pagiola studies, a setting that includes both uplands and lowlands, each having its own production activities, is central to the finding of lowland intensification drawing pressure off upland forests. The latter study includes longer, second order effects (e.g., income growth leading to intensification on upland farms).

[38]The model does not track specific plots of land, and accounts for soil fertility using a procedure that smoothes across plots cultivated in annuals and perennials. For pasture management, soil fertility consequences were part of the farmer's technology choice.

well as nonlinearities in biophysical feedback to farmer behavior.[39] While none of these assumptions or approximations may matter to the policy analysis for which the model was built, the steps taken to calibrate this model to a specific reality (many rotation systems and possible constraints) render it less useful for other combinations of parameter values, and thus to a broader theoretical understanding of model assumptions and outcomes.

Thomas (2003), also set in the Brazilian Amazon, abstracts away from this level of detail, relying on a diminishing-returns production function with land as the sole input, a land-clearing cost function with rising marginal costs, and a land constraint. All the dynamics are in land clearing – choice of land-clearing per period is the control variable; land cleared is the state variable. (This means there are no cross-year production effects due to, say, intensity of land use and its effect on soil fertility.) The author cites known labor market constraints in the region among other things as behind the production and cost function curvature assumptions. The setup allows him to show that there is a steady state land use at the land constraint or inside it (where the marginal cost of clearing in current value terms equals marginal revenue from production) that depends on parameters of the problem (the production and cost functions plus output price and discount rate). The curvature assumptions imply that deforestation is frontloaded: the farmer will deforest less and less over time, the model predicts, eventually petering out at the steady state. Under these assumptions, the deforestation process is self-braking.[40]

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[39]The model does offer some input flexibility in production via proliferation of choices of specific (linear) production systems. And, the model does allow for realistic herd growth using mortality and fertility rates, while tracking the demographic composition of the herd.

[40]So, the empirical finding discussed earlier that "mature" regions, inside frontiers, saw deforestation *rate* fall at higher deforestation *levels*, was in keeping with the theoretical model proposed.

A third analytical model set in the Brazilian Amazon, Walker (2003), is also more abstract (with far fewer parameters and variables) than the Vosti et al. model, but like that model, is calibrated using data from the area. The model includes three phases of land use: an initial phase of settler establishment and land clearing, followed by a shifting cultivation phase, followed in turn by a third phase ushered in by the arrival of markets. The key insight here is that farmers settling beyond full reach of markets often do so with the expectation that, at some point, the market frontier will "catch up" to their spatial location.

The first two phases, land clearing and rotational land use (shifting cultivation), occur under a utility-maximizing framework where available labor and preferences for leisure and subsistence goods enter into the simultaneous determination of consumption and production decisions. In the third phase, production decisions are separable, and maximization of utility – which now includes preferences for the market good – follows from the budget established by profit maximization. For the land clearing/shifting rotation phases, the farmer decides on the size of the plot to clear for farming each year and, for secondary forest, the age of the fallow when felled. Both the labor needed for clearing and the productivity of the farmland increase with the forest's age.[41] The timing of market arrival is considered exogenous to the farmer, but within the model, this timing is either definitive (a fixed parameter) or an expectation by the farmer about an amount of production (or land creation) needed for market participation (so could be part of the farmers' decisionmaking). There is also no possibility of a staggered market arrival – labor and output markets arrive at the same time.

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[41]Walker discusses how this and other shifting cultivation models are modifications of the Faustmann forestry model.

Using numerical simulation methods (and functions and parameters based on studies from the area), Walker finds the smaller the family labor endowment and the higher the local wage rate (in the third phase), the lower the deforestation levels. But these effects are dramatic only among farmers with the lowest discount rates (that is, who gave a relatively high value to future welfare). By design, deforestation occurs up front (in the land creation phase) preceding the arrival of markets. The study makes a further innovation by applying the numerical model to individual lots along a feeder road in order to simulate deforestation patterns over time for a local area.[42]

This study seeks to build on this existing work on small-scale settlers, a group that, as frequent first arrivers on a new frontier, plays a critical role in the frontier development process and forest-to-pasture local land use trajectory that accompanies it. Focusing on this group can highlight implications for land use and poverty of frontier development. It allows use of a theoretical household model that can be tailored to fit the situations of labor scarcity that settlers face, as well as extended in a straightforward manner to encompass some of the key dynamics to land use decision-making on farm – for this area, specifically pasture and herd growth dynamics. A solid farm-level dynamic model, in turn, can form the basis for starting to explore the simplest inter-agent interactions among small-scale settlers. This could lend some insights to how settlement areas develop, as well as suggest strategies for moving forward on more complicated inter-agent interactions along the frontier.

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[42]The simulation treats each lots deforestation event and age of fallow as random variables resulting from randomly varying household characteristics. There is a time lag built in to the sequential settling of lots away from the main artery, and a linear transport cost determines different farmgate output prices for each location.

1.4 An analytical household model for small-scale settlers: research approach

The research approach here is to develop a simple dynamic household utility optimization model to capture the key economic trade-offs colonists face. Scenarios explore how, under different assumptions about farm household characteristics and market conditions, the trade-offs lead to different land use sequences. The work highlights the situation for early settlers, when frontier farmers face labor- (and usually capital-) scarce environments. As a first step toward exploring effects of movement along the spectrum from autonomous to market-integrated households, the study then looks at a conjunct of two households with varying characteristics, and examines how bilateral trade in labor alters their optimal land use trajectories.

The work adds to a still-small number of analytical models on spatial processes of tropical deforestation and land use change amid quite a large number of empirical studies. The analytical model created here combines a number of features that appear singly in some other models. It:

- uses a utility maximization framework appropriate to the institutional context of early settlement – where less than full integration in markets means production (hence land use and deforestation) and consumption decisions are interdependent (Singh, Squire, and Strauss 1986b) – focusing on the scarce factor (labor), which has been found to be critical to land use decisions in both theoretical and empirical tropical deforestation studies in the Brazilian Amazon and beyond;

- takes into account farm-level dynamics, specifically of the pasture creation/herd growth investment decision, a central aspect often ignored in analytical studies;[43]

- focuses on the frontier as a dynamic environment, expected to evolve toward greater market integration over time;[44] and

- starts to explore landscape-level consequences of agent behavior by jointly modeling more than one household – in this instance, examining how opportunities for labor trading between neighboring households could affect local deforestation and land use trajectories.[45]

As such, it: a) like the Walker model, takes the non-separability of the market setting seriously and takes a step toward landscape implications of the household model, but via interaction among households rather than summing up household behaviors; and b) like the Thomas model, looks at deforestation within a simple analytical framework using a dynamic optimization model, but expands that framework to explicitly include the settlers' predominant land use – pasture – as an investment decision, while c) stylizing the detail found in the Vosti et al. linear programming model to permit a look at longer run implications of model assumptions (as seen in optimal trajectory and model steady state).

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[43]Vosti et al. (2002) looks at farm-level dynamics, but in a profit maximizing framework and with many constraints that may obscure underlying dynamics; Thomas (2003) also considers dynamics, but only of the deforestation decision, and also in a profit-maximizing framework.

[44]Walker (2003) does this, but limits the examination to exogenous market arrival. Vance and Geoghegan (2004) examine different market realities on the same landscape, but not how this spatial extent changes over time.

[45]Other studies (Evans et al. 2001; Walker 2003) look at landscapes by aggregating across non-interacting households with randomly assigned characteristics.

The paper contributes to an emerging literature that seeks to understand how behavior along the frontier – where farmers often do not fully participate in markets and thus make interdependent production and consumption decisions – influences landscape dynamics more broadly. This literature encompasses work that indicates the importance to land use outcomes of accounting for not only landscape variability in market participation (vs. assuming frontier farmers are fully market integrated) (Vance and Geoghegan 2004), but also the changes in market conditions over time at the frontier, with market participation arriving at some point after settlement (Walker 2003).

An investment model can provide a common theoretical structure underlying the two common farming stories told in the region – both the failure and success at improving livelihoods. A model using that structure could shed light on the conditions likely to lead to success or failure, or show how one story morphs into the other, with what implications for on-farm forest cover.

The model constructed here builds off the Vosti et al. model. It draws on that setting for initial parameter ranges within a less detailed, more abstracted, dynamic investment model aiming for a broader, theoretically based understanding of longer run land use outcomes and persistent poverty or successful moves out of poverty as byproducts of household decisionmaking. Model structure is informed by the findings of the Vosti et al. model and other work pointing to the importance of factor markets – especially for labor in this relatively land abundant area.

The remainder of the dissertation is organized as follows. Chapter 2 outlines a household land-use optimal control model and its first order conditions, highlighting the critical role played by marginal returns to labor (hereafter "returns to labor"), and

describes its steady state solution. Chapter 2 also analyzes the characteristics of an optimal trajectory, using pared down model versions to highlight the nature of labor trade-offs across different types of activity. Chapter 3 is devoted to understanding model dynamics more fully using a numerical version of the model (programmed in GAMS), again using simpler models to highlight key trade-offs. Chapter 4 presents results of sensitivity analysis to changed model parameters, discusses how the model can be used to inform policy, and gives a few policy-relevant simulations, including some scenarios of inter-household labor trade, as examples. Chapter 5 summarizes key research findings, lays out some shortcomings of the model and discusses some directions for further research.

Chapter 2 A Constrained Dynamic Utility (Early Settler) Household Model

A household model was constructed to explore reasons behind pasture's dominance in the study area, tracking household welfare over time as part of this. A simple dynamic (continuous time, infinite time horizon) model characterizes the household story described above: moving from reliance on annuals to investment in cattle in a labor-, and initially capital-, scarce environment against a backdrop of soil degradation.

This chapter outlines the dynamic model, describes its first-order conditions and how they relate to the household's time allocation decision, and explores characteristics of a steady state solution and optimal trajectory. The chapter ends by presenting parameters and steady state values for a baseline scenario representing an early settler in the Brazilian Amazon.

2.1 Model description

The model maximizes household utility (taking a Cobb-Douglas functional form) from leisure and an aggregate consumption good with a constant time preference. Biophysical realities such as poor soil quality plus economic ones such as limited supply of factor inputs, available production technologies, and prices jointly constrain household decisions. The model household produces an annual crop and cattle to be sold at (known) exogenous prices, or sells labor off the farm at a (known) market wage. Revenue from these sources supplemented by any exogenous income purchases the consumption good. The household has a fixed size (endowment of time) that it can allocate as it chooses between labor and leisure.

Annual cropping degrades soils at a (known) exponential and irreversible rate, matching the rapid yield drop-offs associated with the area's continuous cropping without external inputs. The farmer can mitigate yield declines by applying additional labor with diminishing returns, but, following trends in the area away from secondary forest re-growth, cannot recuperate land by fallowing or other means.[46] This assumption could usefully be relaxed to explore settings under which annual cropping could be viable in the long run (but would add considerable complexity to the model because of the need to track land use/soil fertility combinations on specific plots). Annual cropping is sometimes referred to in what follows as merely "cropping" or "farming."

The cattle herd grows according to a logistic (natural) growth function, matching a typical model of population growth within some carrying capacity bounds, modified by farmer choice of cattle sales or, should the farmer choose to build a herd faster than natural growth rates allow, purchases. Existing pasture degrades (its carrying capacity falls) at a known proportional rate. The farmer can use labor to reverse this carrying capacity decline by either recuperating degraded pasture or creating new pasture via deforestation.[47]

Making pasture carrying capacity recuperable in the model reflects trends from the first 30 years of cattle production in the study area, but glosses over the technological change that helped pasture continue to perform (so is not meant to imply that pastures can be maintained indefinitely with the same production system). An extreme assumption in

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[46]This assumption means the model cannot explore the shifting cultivation as traditionally practiced, as done in Walker (2003), or use of purchased inputs to halt degradation, and cannot permit annual cropping in the long run.

[47]As in annuals, the model does not allow for purchased inputs to substitute for labor in stopping pasture degradation.

the other direction would be that pasture cannot be recuperated at all, and that all new pasture must come from deforestation. Tuning the pasture degradation parameter via simulations can test the impact of these parameter extremes.[48] With the assumption that pasture can be recuperated, expanding carrying capacity requires deforestation, but maintaining carrying capacity does not. So, a "mature" (steady state) farm would be spatially stable.[49] Distinguishing between newly deforested areas and areas where pasture was being maintained vs. allowed to degrade (and between stocking rates on the two areas) would be another useful model extension that would add the complexity of needing to track specific plots of land on farm.

The model treats annuals and pasture as separate production activities, and does not make the link between these land uses on a specific plot of land (as in the land use trajectory in the previous chapter). Again, to do so would require spatial tracking land, and add considerably to the model's complexity. The aim here is to abstract away from those links to examine the principal trade-offs in the conversion of forest to pasture.[50] As formulated here, annuals thus don't represent an opportunistic use of land cleared for pasture. The distinction between opportunistic and 'stand-alone' annuals could be a useful one for empirical analyses of deforestation as part of a cycle of poverty or pathway towards capitalization.

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[48]With zero degradation, ever-expanding costs of transporting production back to the feeder road would erode profits until deforestation stopped at property boundaries or a zero profit distance. Pasture productivity would then sink below profitable levels.
[49]Whereas if pasture recuperation were not possible, maintaining a constant capacity would require continually expanding pasture into new areas.
[50]Again, this would be a useful model addition. Alternatively, the production function for "pasture" could possibly be tweaked to incorporate an initial phase of annuals cropping, or some additional summing-up constraints imposed on land in each category for each time step.

Other base model assumptions were chosen to approximate conditions for a newly arrived settler in the study area: the household relies on its own manpower (no hired labor), cattle provides its sole vehicle for storing wealth, and property rights are secure (farmers expect to trade land at value). The model can explore effects of a more fully functioning labor market by opening up the possibility of hiring in labor at the exogenous wage.[51] It can also look at consequences of insecure tenure by tweaking the terminal value function to some proportion of its true value.

Unlike its treatment labor, the model assumes integration in output markets. Access to and participation in output markets usually precedes those in labor markets. Still, in the earliest days of settlement isolation in this regard may be the rule (and local trading in production goods, along the lines modeled later in the dissertation for labor, may be important).[52] Another useful model extension would be to add these goods explicitly to the household's utility function, and endogenize the household participation in the outside market for these products, essentially broadening to output goods the approach applied to labor here. The simplified approach taken here avoids the complication of specifying preferences for these goods, while focusing on a critical factor input for which market participation often lags.

Equations 1 through 6 characterize the optimal control model with two state variables (herd size and carrying capacity) and two controls (cattle sales and labor used to increase pasture carrying capacity), followed by restrictions on parameters and variables. Variable and parameter descriptions appear in Table 1.

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[51]Note that with full participation in the labor market, the approach could switch from one of utility optimization to profit maximization.
[52]As noted above, fieldwork indicated the importance of local social contacts and found evidence of some trading among households, often involving labor, but sometimes in combination with other items.

$$Max_U \int_0^\infty U_t e^{-\rho t} dt \; ; \text{where} \; U = \alpha \ln(C_t) + \beta \ln(l_t) \qquad (1)$$

subject to:

$$\dot{N}_t = rN_t\left(1 - \frac{N_t}{K_t}\right) - H_t = F(N_t, K_t) - H_t \qquad (2)$$ — herd dynamics based on natural growth, stocking rate, and cattle sales

$$\dot{K}_t = -\delta K_t + D_t \qquad (3)$$ — pasture carrying capacity dynamics (biophysical degradation and pasture creation through labor)

$$Q_t = AS_0 e^{-dt} LF_t^{\ \psi} \qquad (4)$$ — crop farming production function

$$T = l_t + LF_t + \gamma D_t + LO_t \qquad (5)$$ — household time constraint

$$C_t = p_h H_t + p_a Q_t + wLO_t + Y \qquad (6)$$ — consumption based on income sources

Restrictions — *on parameters:* $\alpha + \beta = 1$, $0 < \psi < 1$, all parameters non-negative

— *on variable ranges:* $C > 0$, $l > 0$, $D \geq 0$, $LF \geq 0$, $LO \geq 0$, $K \geq \underline{K}$[53]

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[53]Where $\underline{K}$ is positive and arbitrarily small, set so that a steady state is well defined when pasture is an unattractive option. More generally, a positive K is needed so that the herd growth equation is defined.

Table 1. Model variables and parameters

VARIABLES	PARAMETERS	
UTILITY	*FARM/HH/TECH CHARACTERISTICS*	*HH RESOURCE ENDOWMENTS*
U = utility	ρ = hh time preference rate$^\sim$	
C = non-leisure consumption	α = hh relative pref. for cons'n$^\sim$	Y = income transfers
	p_c = farmgate price of cons'n good (=1)*	
l = leisure	β = hh relative preference for leisure$^\sim$	T = total hh labor
ANNUALS PROD'N		
Q = production of annual crops	S_0 = initial soil quality for annuals$^+$	
LF = annual crop labor	d = rate of soil degradation for annuals$^{+,\#}$	
	p_a = farmgate annuals output price*	
	A = technical efficiency of annual cropping$^\#$	
	ψ = diminishing returns to annuals labor coefficient$^\#$	
PASTURE/CATTLE PROD'N		
N = herd size (1st state var.)	r = natural herd growth rate$^{+,\#}$	N_0 = initial herd size
K = pasture carrying capacity (2nd state var.)	δ = rate of decline in pasture carrying capacity$^{+,\#}$	K_0 = initial cattle carrying capacity
H = cattle sales (controls N)	p_h = farmgate cattle output price*	
D = new carrying capacity (controls K)	γ = technical labor efficiency for new pasture capacity$^\#$	
λ = costate variable for $N^\wedge$		
φ = costate variable for $K^\wedge$		
HIRED OUT LABOR		
LO = hired out (family) labor	w = wage for hh labor hired out*	
$\mu_{LF,K,D,LO,l}$ = shadow values$^\wedge$ t subscripts time		

$^\wedge$- in Hamiltonian. Parameter types: $^\sim$ - preference, * - market, $^+$ - biophysical, $^\#$ - technical.

The model's general set-up mimics the common land use pattern on lots in the study area within Brazil's arc of deforestation of initial annual cropping giving way to pasture predominance. As already suggested, model parameters can be tuned to reflect various agronomic and economic conditions, as well as to set up limiting, extreme situations within which reality must lie. It can tell an investment story – of using annual cropping for early consumption needs and to invest in starting a herd – or show

protracted poverty – when conditions delay or prevent investment, or make it less attractive. Each household trades off present vs. future consumption, balancing the labor needed to generate that consumption against leisure.

Under some parameter combinations, cattle production is a viable long-run activity. With the study focus on pasture expansion, the model is geared to this situation. In these cases, providing the household has the means to start investing, lots should move along optimal trajectories from specific initial conditions (starting values for state variables plus household endowments of time and exogenous income transfers) towards a steady state in which cattle sales just match herd growth and pasture is recuperated just enough to balance its degradation. Higher wages should attract labor out of on-farm activities, either mitigating the inexorable drop-off in proceeds from annual crop sales or removing pressure to deforest for pasture expansion.

Household time is limited, and its allocation across activities – annual cropping, cattle-raising, off-farm work – and leisure lies at the crux of the household's optimization decision. The maximization problem's first order conditions (*FOCs*) illustrate this via an equality across marginal returns to household time (sometimes referred to as simply 'returns to labor' or 'returns to time' in what follows) in all activities undertaken at all t.

2.2 First order conditions and the labor allocation decision

The current value Hamiltonian for the dynamic optimal control problem (Equation 7) and its associated *FOCs* (Equations 8-16) below highlight how the household time constraint links the various activities via time allocation decisions.

The current value Hamiltonian is:

$$\hat{H}_t = \alpha \ln\left(p_h H_t + p_a A S_0 e^{-dt} LF_t^{\psi} + wLO_t + Y\right) + \beta \ln l_t + \lambda_t \left[rN_t \left(1 - \frac{N_t}{K_t}\right) - H_t \right]$$
$$+ \varphi_t \left(-\delta K_t + D_t\right) - \mu_{1t} \left(LF_t + \gamma D_t + LO_t + l_t - T\right) +$$
$$\mu_{LOt} LO_t + \mu_{Dt} D_t + \mu_{Nt} N_t + \mu_{Kt} \left(K_t - \underline{K}\right) \qquad (7)$$

The *FOCs* for 'dynamic' variables (*controls* – cattle sales and pasture capacity

expansion; *states* – herd size and pasture carrying capacity; and *costates* – marginal value

of an incremental increase in herd size and pasture capacity to lifetime utility in current

value terms), and 'non-dynamic' variables (leisure, annual cropping, and off-farm labor –

activities that derive their dynamics via a link through the time constraint to the model's

pasture dynamics) appear below.[54]

Dynamic variables

Controls:

$$\frac{\partial \hat{H}_t}{\partial H_t} = 0 \Rightarrow \frac{\alpha p_h}{C_t} = \lambda_t \qquad (8)$$

$$\frac{\partial \hat{H}_t}{\partial D_t} = 0 \Rightarrow \frac{\varphi_t + \mu_{Dt}}{\gamma} = \mu_{1t}, \ \mu_{Dt} > 0 \text{ when } D_t = 0 \qquad (9)$$

States and costates:

$$\dot{N}_t = rN_t \left(1 - \frac{N_t}{K_t}\right) - H_t = F(N_t, K_t) - H_t \qquad (10)$$

$$\dot{K}_t = -\delta K_t + D_t \qquad (11)$$

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[54]Crop labor's dynamic dimension, soil degradation, is exogenous to the decisionmaking in the model.

$$\dot{\lambda}_t = \lambda_t(\rho - F_N) - \mu_{Nt} = -\frac{\alpha p_h}{C_t^2}\dot{C}_t = -\frac{\alpha p_h}{C_t^2}\left(p_h\dot{H}_t + p_aAS_0e^{-dt}LF_t^{\psi}\left(\psi\frac{LF_t}{LF_t} - d\right) + w\,LO_t\right)$$ (12)

$$\dot{\varphi}_t = \varphi_t(\rho + \delta) - \lambda_t F_K - \mu_{Kt} = \gamma\dot{\mu}_{1t} - \dot{\mu}_{Dt}, \mu_{Kt} > 0 \text{ when } K_t = \underline{K}$$ (13)

'Non-dynamic' variables

Leisure:

$$\frac{\partial \hat{H}_t}{\partial l_t} = 0 \Rightarrow \frac{\beta}{l_t} = \mu_{1t}$$ (14)

Crop labor:

$$\frac{\partial \hat{H}_t}{\partial LF_t} = 0 \Rightarrow \frac{\alpha p_a AS_0 e^{-dt}\psi LF_t^{\psi-1}}{C_t} = \mu_{1t}$$ (15)

Wage labor:

$$\frac{\partial \hat{H}_t}{\partial LO_t} = 0 \Rightarrow \frac{\alpha w}{C_t} + \mu_{LOt} = \mu_{1t}, \mu_{LOt} > 0 \text{ when } LO_t = 0$$ (16)

Equations 14–16 and the second control *FOC* (Equation 9) establish the marginal utility for household time allocated to activities and leisure along the optimal path, namely:

$$\mu_{1t} = \frac{\varphi_t + \mu_{Dt}}{\gamma} = \frac{\beta}{l_t} = \frac{\alpha p_a AS_0 e^{-dt}\psi LF_t^{\psi-1}}{C_t} = \frac{\alpha w}{C_t} + \mu_{LOt}$$ (17)

Equation 17 ensures identical marginal returns across all activities undertaken, or, in the case of pasture, undertaken at a level above the arbitrarily small lower bound. Using the pasture costate *FOC* (Equation 13), the shadow value of new pasture carrying capacity can be related to problem parameters and choice variables.

$$\varphi_t(\rho + \delta) - \lambda_t F_K - \mu_{Kt} = \gamma\dot{\mu}_{1t} - \dot{\mu}_{Dt} \Rightarrow$$

$$\varphi_t = \frac{\gamma\mu_{1t} + \lambda_t F_K + \mu_{Kt} - \mu_{Dt}}{(\rho + \delta)} = \frac{-\gamma\frac{\beta}{l_t^2}\dot{l}_t + \frac{\alpha p_h}{C_t}r\left(\frac{N_t}{K_t}\right)^2 + \mu_{Kt} - \dot{\mu}_{Dt}}{(\rho + \delta)} \tag{18}$$

Substituting for φ_t in Equation 17 expresses the relationship among marginal returns to household time for pasture labor, leisure, farming, and wage labor (in that order) as:

$$\frac{-\gamma\frac{\beta}{l_t^2}\dot{l}_t + \frac{\alpha p_h}{C_t}r\left(\frac{N_t}{K_t}\right)^2 + \mu_{Kt} - \dot{\mu}_{Dt}}{\gamma(\rho + \delta)} + \frac{\mu_{Dt}}{\gamma} = \frac{\beta}{l_t} = \frac{\alpha p_a A S_0 e^{-dt}\psi L F_t^{\psi-1}}{C_t} = \frac{\alpha w}{C_t} + \mu_{LOt} \tag{19}$$

This critical expression indicates some characteristics of an optimal time allocation. The household always has some leisure time and engages in annual cropping along the trajectory (due to the Cobb-Douglas nature of the utility function and diminishing returns annuals production function). Manipulating Equation 19 yields the familiar Cobb-Douglas optimal result where the relative market shares of consumed goods (the consumption good and leisure) match the household's relative preferences for the respective goods.[55]

Low wages or unfavorable pasture conditions can drive returns to these activities below returns to leisure and cropping, so that the household does not engage in one or more of the other activities. The household can foregoe off-farm work ($\mu_{LOt}>0$ to maintain the equality), allow pasture to degrade ($\mu_{Dt}>0$), or maintain pasture at its lower bound ($\mu_{Kt}>0$).[56]

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[55]This is seen most clearly in the terms pertaining to cropping and wage work (recall that the consumption good is priced at unity): $\dfrac{\beta}{\alpha} = \dfrac{l_t}{C_t}p_a A S_0 e^{-dt}\psi L F_t^{\psi-1} = \dfrac{l_t}{C_t}\left(w + \mu_{LOt}\dfrac{C_t}{\alpha}\right)$.

[56]If pasture is degrading, μ_{Dt} is positive and declining over time. Once pasture degrades to $\underline{K}$, and $\mu_{Dt}=0$, $\mu_{Kt}>0$.

Pasture labor can appear concurrently with either off-farm work or annual cropping labor. These final two activities can, but need not, overlap, a fact reflected in the relationship between the respective coefficients of α/C_t in returns to annual and wage labor from Equation 19: $w \leq p_a A S_0 e^{-dt} \psi L F_t^{\psi-1}$. With an inequality, wages are too low for the household to work off farm. With the equality $w = p_a A S_0 e^{-dt} \psi L F_t^{\psi-1}$, however, both annual cropping and wage work are chosen. In that case, farming labor declines inexorably over time as a function of the wage and annual production parameters:

$$w = p_a A S_0 e^{-dt} \psi L F_t^{\psi-1} \Rightarrow L F_t = \left(\frac{w}{p_a A S_0 e^{-dt} \psi} \right)^{\frac{1}{\psi-1}} \tag{20}$$

Total household time less labor required for pasture maintenance $(l_t^{up} = T - \gamma \delta \underline{K})$ provides an upper bound for leisure. This limit can be approached but not reached in the trajectory towards the steady state, since a small amount of labor will be allocated to cropping. Conditions that crowd out labor and favor leisure are discussed later in the chapter. The next section outlines characteristics of a steady state, highlighting time allocation choices.

2.3 Steady state and returns to labor

The household dynamic optimization problem has an analytical steady state solution. In the steady state, annual cropping drops out (since the coefficient on $LF \to 0$ as $t \to \infty$), and $C^* = p_h H^* + w L O^* + Y$, $T = l^* + \gamma D^* + L O^*$. Any change in annuals parameters – crop output price, production efficiency, initial soil quality, and soil degradation rate – could affect the trajectory to the steady state but not the steady state itself.

The state and costate *FOCs* in the system at rest become:

$$\dot{N} = 0 \Rightarrow H^* = rN^*\left(1 - \frac{N^*}{K^*}\right) \tag{21}$$

$$\dot{K} = 0 \Rightarrow D^* = \delta K^* \tag{22}$$

$$\dot{\lambda} = 0 \Rightarrow \rho = r\left(1 - \frac{2N^*}{K^*}\right) \Rightarrow \frac{N^*}{K^*} = \frac{r - \rho}{2r} \Rightarrow N^* = \left(\frac{r - \rho}{2r}\right)K^* ^{57} \tag{23}$$

$$\dot{\varphi} = 0 \Rightarrow \varphi^* = \frac{\lambda r}{(\rho + \delta)}\left(\frac{N^*}{K^*}\right)^2 + \mu_K^* = \frac{\alpha p_h}{C^*}\frac{r}{(\rho + \delta)}\left(\frac{r - \rho}{2r}\right)^2 + \mu_K^* ^{58} \tag{24}$$

These equations reflect the steady state farmer using enough labor to offset pasture degradation, and selling the cattle generated by natural population growth. Equation 23 shows that herd growth rate r and personal discount rate ρ jointly determine an ideal proportion of carrying capacity to use (as long as $r > \rho$).[59] The equations determine the following steady state marginal value for household members' time:

$$\frac{\dfrac{\alpha p_h}{C^*}r\left(\dfrac{r - \rho}{2r}\right)^2 + \mu_K^*}{\gamma(\rho + \delta)} = \frac{\beta}{l^*} = \frac{\alpha w}{C^*} + \mu_{LO}^* \tag{25}$$

It follows from Equation 25 that higher steady state consumption means more steady state leisure.[60] The equation identifies different classes of steady state solution, depending on the status of μ_K^* and μ_{LO}^*. In this section, three steady state solution types

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[57]The multiplier on herd size (μ_N) is always zero in the steady state since there will be some cattle to accompany the always positive pasture capacity.

[58]Just the expression derived earlier for φ in Equation 18, substituting for proportion of steady state carrying capacity used and dropping the equation's dynamic term. D's shadow value does not appear because the pasture lower bound requires continual labor to maintain.

[59]Without this condition, cattle/pasture production is not viable and the problem loses its dynamic element. The optimal stocking rate is capped at 0.5 – an artifact of the functional forms used here.

[60]Except when leisure has reached its upper bound (when the household only works enough to maintain the pasture lower bound); conditions favoring a "life of leisure" are described later in the chapter.

are presented, then the effects of exogenous income and prices on the steady state solutions are explored.

2.3.1 Steady state solution types

The activity – pasture or wage labor – with the highest return per unit of labor in the steady state (highest coefficient of $1/C^*$ in Equation 25) will garner all available non-leisure household time. The farmer balances labor in the chosen activity against leisure. The solution is a corner, with the unfavored activity at its lower bound. If the coefficients of $1/C^*$ just match, there is indifference between allocating time to pasture and wage labor, and a (knife-edge) interior solution to the problem. This subsection describes the three steady state types and derives the solution for each type.

2.3.1.1 Conditions for solution types

If returns to pasture labor exceed those to off-farm labor in the steady state, the steady state is a corner solution with no off-farm labor ($LO^*=0$, $\mu_{LO}^*>0$) – a "pro-pasture" corner solution. If, on the other hand, labor yields higher returns off farm, the steady state is the other corner solution, with the household using just enough labor on farm to sustain pasture carrying capacity at its lower bound ($K^*=\underline{K}$, $\mu_K^*>0$) and taking advantage of the high wage – a "pro-wage labor" corner solution. If, finally, the returns to steady state labor for the two activities match, the steady state can be an interior solution, where both activities are undertaken at levels away from their bounding values.

Equations 26-28 summarize the parameter relationships leading to each of the three cases (with subscripts p, w, and i on choice variables indicating the pro-pasture, pro-wage labor, and interior solutions, respectively). These expressions can be slightly

modified to accommodate the "life of leisure" scenarios discussed below by replacing the equalities with inequalities in all three cases.[61]

Pro-pasture corner:

$$\frac{p_h r\left(\dfrac{r-\rho}{2r}\right)^2}{\gamma(\rho+\delta)} > w \rightarrow \frac{\dfrac{\alpha p_h}{C_p^*} r\left(\dfrac{r-\rho}{2r}\right)^2}{\gamma(\rho+\delta)} = \frac{\beta}{l_p^*} > \frac{\alpha w}{C_p^*} \tag{26}$$

Pro-wage labor corner:

$$\frac{p_h r\left(\dfrac{r-\rho}{2r}\right)^2}{\gamma(\rho+\delta)} < w \rightarrow \frac{\dfrac{\alpha p_h}{C_w^*} r\left(\dfrac{r-\rho}{2r}\right)^2}{\gamma(\rho+\delta)} < \frac{\beta}{l_w^*} = \frac{\alpha w}{C_w^*} \tag{27}$$

Interior:

$$\frac{p_h r\left(\dfrac{r-\rho}{2r}\right)^2}{\gamma(\rho+\delta)} = w \rightarrow \frac{\dfrac{\alpha p_h}{C_i^*} r\left(\dfrac{r-\rho}{2r}\right)^2}{\gamma(\rho+\delta)} = \frac{\beta}{l_i^*} = \frac{\alpha w}{C_i^*} \tag{28}$$

The next three subsections derive steady state solutions for each problem type by combining Equations 26-28 (the equality of marginal returns to household time) and the time constraint.

2.3.1.2 Pro-pasture steady state

In the pro-pasture case:

$$LO_p^* = 0, \ l_p^* = \frac{\beta\gamma(\rho+\delta)}{\left(\dfrac{\alpha p_h}{p_h H_p^* + Y}\right) r\left(\dfrac{r-\rho}{2r}\right)^2} = \frac{\beta\gamma(\rho+\delta)\left(p_h H_p^* + Y\right)}{\alpha p_h r\left(\dfrac{r-\rho}{2r}\right)^2} \text{ and } l_p^* = T - \gamma D_p^* \Rightarrow$$

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[61]That is, $(=\dfrac{\beta}{l_i^*})$ with $(\leq\dfrac{\beta}{l_i^*})$.

$$\frac{\beta\gamma(\rho+\delta)\left(\dfrac{p_h D_p^*}{\delta}\left(\dfrac{r^2-\rho^2}{4r}\right)+Y\right)}{\alpha p_h r\left(\dfrac{r-\rho}{2r}\right)^2}=T-\gamma D_p^* \Rightarrow$$

$$\frac{\beta\gamma(\rho+\delta)p_h D_p^*}{\delta}\left(\frac{r^2-\rho^2}{4r}\right)+\alpha p_h r\left(\frac{r-\rho}{2r}\right)^2\gamma D_p^*=\alpha p_h r\left(\frac{r-\rho}{2r}\right)^2 T-\beta\gamma(\rho+\delta)Y \Rightarrow$$

$$D_p^*\gamma p_h\left(\frac{\beta(\rho+\delta)}{\delta}\left(\frac{r^2-\rho^2}{4r}\right)+\alpha r\left(\frac{r-\rho}{2r}\right)^2\right)=\alpha p_h r\left(\frac{r-\rho}{2r}\right)^2 T-\beta\gamma(\rho+\delta)Y \Rightarrow$$

$$D_p^*=\frac{\alpha p_h r\left(\dfrac{r-\rho}{2r}\right)^2 T-\beta\gamma(\rho+\delta)Y}{p_h\gamma\left[\dfrac{\beta(\rho+\delta)}{\delta}\left(\dfrac{r^2-\rho^2}{4r}\right)+\alpha r\left(\dfrac{r-\rho}{2r}\right)^2\right]} \Rightarrow$$

$$D_p^*=\frac{1}{\left(\dfrac{\beta(\rho+\delta)}{\delta}(r+\rho)+\alpha(r-\rho)\right)}\left\{\frac{\alpha T(r-\rho)}{\gamma}-\left(\frac{\beta(\rho+\delta)Y}{p_h\dfrac{(r-\rho)}{4r}}\right)\right\}.\text{[62]} \tag{29}$$

Relationships (already presented) between D_p^* and other steady state variables

$(l_p^*, H_p^*, C_p^*,$ and $U_p^*)$ fill out the pro-pasture corner solution. For leisure,

$$l_p^*=T-\gamma D_p^*=T-\frac{1}{\left(\dfrac{\beta(\rho+\delta)}{\delta}(r+\rho)+\alpha(r-\rho)\right)}\left\{\alpha T(r-\rho)-\left(\frac{\beta\gamma(\rho+\delta)Y}{p_h\dfrac{(r-\rho)}{4r}}\right)\right\}. \tag{30}$$

2.3.1.3 Pro-wage labor steady state

In the pro-wage labor case, the solution hinges on the lower bound for carrying capacity and the labor it fixes on the farm. Remaining household time is allocated efficiently between off-farm labor and leisure:

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[62]The first bracketed term represents a fraction of household time T; the second is exogenous income adjusted by the ratio of the leisure preference parameter to the money value return from pasture labor.

$$K_w^* = \underline{K}, \ D_w^* = \delta \underline{K}, \ N_w^* = \left(\frac{r-\rho}{2r}\right)\underline{K}, H_w^* = rN_w^*\left(1 - \frac{N_w^*}{\underline{K}}\right),$$

$$T = LO_w^* + l_w^* + \gamma\delta\underline{K}, \text{ and } C_w^* = p_h H_w^* + wLO_w^* + Y.$$

Since the marginal returns to time from off-farm wage work and household leisure match in the steady state, steady state leisure can be expressed in terms of steady state consumption components (i.e., cattle sales and off-farm labor) plus problem parameters: $\dfrac{\beta}{l_w^*} = \dfrac{\alpha w}{C_w^*} \Rightarrow l_w^* = \dfrac{\beta C_w^*}{\alpha w} = \dfrac{\beta\left(p_h H_w^* + wLO_w^* + Y\right)}{\alpha w}$. This steady state leisure expression substituted back into the time constraint provides an equation for steady state off-farm labor in terms of parameters:

$$T = LO_w^* + \frac{\beta\left(p_h H_w^* + wLO_w^* + Y\right)}{\alpha w} + \gamma\delta\underline{K} \Rightarrow \frac{LO_w^*}{\alpha} = T - \frac{\beta\left(p_h H_w^* + Y\right)}{\alpha w} - \gamma\delta\underline{K} \Rightarrow$$

$$LO_w^* = \alpha(T - \gamma\delta\underline{K}) - \frac{\beta}{w}\left[p_h\left(\frac{r^2 - \rho^2}{4r}\right)\underline{K} + Y\right].^{63} \tag{31}$$

Other steady state variables in terms of parameters are easily derived using relationships already presented. For l_w^*,

$$l_w^* = T - \gamma D_w^* - LO_w^* = T - \gamma\delta\underline{K} - \left(\alpha(T - \gamma\delta\underline{K}) - \frac{\beta}{w}\left[p_h\left(\frac{r^2 - \rho^2}{4r}\right)\underline{K} + Y\right]\right) =$$

$$T - \alpha T + \gamma\delta\underline{K}(\alpha - 1) + \frac{\beta}{w}\left[p_h\left(\frac{r^2 - \rho^2}{4r}\right)\underline{K} + Y\right] \Rightarrow$$

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[63]The expression's first term adjusts available time for allocation by the consumption preference parameter; the second involves, like the previous case, exogenous income adjusted by the leisure preference parameter over the money return to the labor activity in question.

$$l_w^* = \beta(T - \gamma\delta\underline{K}) + \frac{\beta}{w}\left[p_h\left(\frac{r^2 - \rho^2}{4r}\right)\underline{K} + Y \right].$$ (32)

2.3.1.4 Knife-edge case steady state

The marginal conditions regarding returns to household time and the time constraint jointly determine a unique steady state with variables expressed in terms of problem parameters for each corner solution. In the knife-edge instance of on-farm (from pasture labor) and off-farm (from wage labor) labor returns matching in the steady state, these conditions don't fully determine a solution, but define a steady state relationship between off-farm labor and pasture creation. The derivation of the solution follows the pro-wage case in Equation 32, but with D_i^* here substituting for $\delta\underline{K}$ there.

$$l_i^* = \frac{\beta\gamma(\rho + \delta)}{\left(\dfrac{\alpha p_h}{p_h H_i^* + wLO_i^* + Y}\right)r\left(\dfrac{r - \rho}{2r}\right)^2} = \frac{\beta\left(p_h H_i^* + wLO_i^* + Y\right)}{\alpha w} \text{ and } l_i^* = T - LO_i^* - \gamma D_i^* \Rightarrow$$

$$LO_i^* = \alpha(T - \gamma D_i^*) - \frac{\beta}{w}\left[p_h\left(\frac{r^2 - \rho^2}{4r}\right)\frac{D_i^*}{\delta} + Y \right], \text{ or, in terms of } l_i^*:$$

$$l_i^* = \beta(T - \gamma D_i^*) + \frac{\beta}{w}\left[p_h\left(\frac{r^2 - \rho^2}{4r}\right)\frac{D_i^*}{\delta} + Y \right].$$ (33)

Innumerable combinations of l_i^*, LO_i^* and D_i^* satisfy these requirements; in the knife-edge case, the marginal equality of returns to time and the time constraint do not suffice to identify a unique steady state candidate. Since this case is of no practical importance (such parameters will never match precisely), we leave it aside for the remainder of the discussion.

The next parts of this section discuss the roles of exogenous income and prices in labor allocation in the steady state.

2.3.2 Role of exogenous income in steady state labor allocation

Higher exogenous income means more leisure time in the steady state across all three problem types. High enough exogenous income makes leisure the most attractive steady state use of time, effectively crowding out all labor beyond what's needed to maintain a pasture lower bound.[64] In such instances, $l^*_{p,w,i} = T - \gamma\delta\underline{K}$, and $D^*_{p,w,i} = \delta\underline{K}$, replacing the solutions above. The threshold income level above which the household leads a steady state life of leisure derives directly from the steady state solutions above.[65]

$$\bar{Y}_{ss} = \max\left[\frac{\alpha p_h r\left(\frac{r-\rho}{2r}\right)^2}{\beta\gamma(\rho+\delta)}(T-\gamma\delta\underline{K}) - p_h\left(\frac{r^2-\rho^2}{4r}\right)\underline{K}, \alpha\frac{w}{\beta}(T-\gamma\delta\underline{K}) - p_h\left(\frac{r^2-\rho^2}{4r}\right)\underline{K}\right]. \quad (34)$$

2.3.3 Role of prices in steady state labor allocation

Higher product prices – namely p_h in the pro-pasture scenario and w in the pro-wage labor scenario (or both together in the knife-edge case) – drive the associated labor activity ever closer to an upper bound (given other parameters) by mitigating exogenous income's leisure-enhancing effect. In the pro-pasture case (Equations 29 and 30), higher p_h pushes D^*_p ever closer (but never equal) to T (leaving $l^*_p > 0$), with the equation's final term vanishing as $p_h \to \infty$. In the pro-wage labor case (Equations 31 and 32), ever higher wage moves time allocation towards a limit in which the household allocates its discretionary time (i.e., $T-\gamma\delta\underline{K}$) according to shares determined by consumption and leisure preference parameters: as $w \to \infty$, $LO^*_w \to \alpha(T - \gamma\delta\underline{K}); l^*_w \to \beta(T - \gamma\delta\underline{K})$.

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[64]Absent exogenous income, low enough returns to the most attractive labor activity could prompt a life of leisure, but only because of the pasture minimum $\underline{K}$. Unlike the case with exogenous income, lowering the arbitrarily small $\underline{K}$ can guarantee a labor optimum above the lower bound.

[65]The *max* function returns the first bracketed expression in the pro-pasture case, and the second in the pro-wage labor case; in the interior case the bracketed expressions are equal.

Cattle price influences time allocation in the pro-wage labor case due to the effect of the pasture minimum. A higher cattle price does not affect pasture labor in the pro-wage case, but induces a shift from wage labor toward leisure to compensate for the higher consumption (and utility) permitted by the same minimum pasture. Because wage labor's minimum is zero, the pro-pasture solution is not influenced by the specific wage rate.

2.4 Characteristics of an optimal trajectory

The optimal trajectory involves the farmer trading off labor and leisure over time so that (reprinting Equation 19 for easy reference):

$$\frac{-\gamma \frac{\beta}{l_t^2} \dot{l}_t + \frac{\alpha p_h}{C_t} r \left(\frac{N_t}{K_t} \right)^2 + \mu_{Kt} - \dot{\mu}_{Dt}}{\gamma(\rho + \delta)} + \frac{\mu_{Dt}}{\gamma} = \frac{\beta}{l_t} = \frac{\alpha p_a A S_0 e^{-dt} \psi L F_t^{\psi - 1}}{C_t} = \frac{\alpha w}{C_t} + \mu_{LOt}.$$

As in the steady state, the household will always take leisure time and consume something along the optimal trajectory, working in some activity above the minimum required unless there is adequate exogenous income to crowd out labor.

The dynamic time allocation trade-offs are complicated. To better understand these, we start with a simpler model – one without pasture investment (next subsection) – before proceeding to the full model. We discuss exogenous income's role at the end of each subsection.

2.4.1 Trajectory for model without investment

A simpler model, without pasture investment, consists of a sequence of static optimizations as the household allocates its time among leisure, crop labor, and wage labor. The farmer knows that each period brings a decline in soil productivity – this drives the changes in time allocation. Returns to time must be equated across activities,

from Equation 19 above (without the pasture term). It can be used to associate relationships among parameters with particular qualitative trajectories. We examine these next, first considering the case with no exogenous income, then adding exogenous income.

2.4.1.1 'No-investment' trajectory without exogenous income

Because returns to labor for annuals fall inexorably over time, whether or not these initially exceed returns to wage labor determines the nature of the no-investment model trajectory.

Let's suppose annuals initially garner higher returns than wage labor. Then, from Equation 19, $p_a A S_0 e^{-dt} \psi L F_t^{\psi-1} > w; \mu_{LO_t} > 0$, and the trajectory starts with crop labor only. The household relies solely on crop profits for income to buy the consumption good, making the marginal returns equation (substituting for C_t) $\dfrac{\beta}{l_t} = \dfrac{\alpha p_a A S_0 e^{-dt} \psi L F_t^{\psi-1}}{p_a A S_0 e^{-dt} L F_t^{\psi}}$,

or:

$$l_t = \frac{\beta}{\alpha} \frac{p_a A S_0 e^{-dt} L F_t^{\psi}}{p_a A S_0 e^{-dt} \psi L F_t^{\psi-1}} = \frac{\beta}{\alpha \psi} L F_t \qquad (35)$$

By substituting this expression for leisure in the time constraint $T = l_t + LF_t$, we can see how the household allocates its time in this part of the trajectory:

$$T = \frac{\beta}{\alpha \psi} LF_t + LF_t = \left(\frac{\beta}{\alpha \psi} + 1 \right) LF_t = \left(\frac{\beta + \alpha \psi}{\alpha \psi} \right) LF_t \Rightarrow$$

$$LF_t^* = \left(\frac{\alpha \psi}{\beta + \alpha \psi} \right) T \qquad (36)$$

and

$$l_t^* = T - \left(\frac{\alpha\psi}{\beta + \alpha\psi}\right)T = \left(1 - \frac{\alpha\psi}{\beta + \alpha\psi}\right)T = \left(\frac{\beta}{\beta + \alpha\psi}\right)T.^{66} \tag{37}$$

In other words, in the initial period where crop labor returns surpass wage labor

returns, the household allocates <u>fixed</u> proportions of its time to leisure and annuals labor

despite soil degradation's effects on yields.[67] Even as leisure holds steady, consumption

drops in this phase because it completely depends on profits from annual crops, which are

declining because of soil degradation.

The falling returns to annual cropping mean that, for a positive wage, there will

come a time (call it τ) when returns to annual cropping will equal returns to wage labor,

so that, from Equation 19:

$$w = p_a A S_0 e^{-dt} \psi L F_{t \geq \tau}^{\psi-1}, \mu_{LOt \geq \tau} = 0, LO_{t \geq \tau} > 0.$$

From this moment, consumption income comes from both cropping and wage

work. We can rewrite the marginal equality of returns for this phase by again substituting

for C_t, and now using the equality $w = p_a A S_0 e^{-dt} \psi L F_{t \geq \tau}^{\psi-1}$:

$$\frac{\beta}{l_{t \geq \tau}} = \frac{\alpha w}{p_a A S_0 e^{-dt} L F_{t \geq \tau}^{\psi} + w L O_{t \geq \tau}},$$

or

$$l_{t \geq \tau} = \frac{\beta}{\alpha} \frac{p_a A S_0 e^{-dt} L F_{t \geq \tau}^{\psi} + w L O_{t \geq \tau}}{p_a A S_0 e^{-dt} \psi L F_{t \geq \tau}^{\psi-1}} = \frac{\beta}{\alpha}\left(\frac{L F_{t \geq \tau}}{\psi} + L O_{t \geq \tau}\right). \tag{38}$$

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[66]This time allocation illustrates the effect of the diminishing returns annuals technology on the constant returns solution of $LF_t^* = \alpha T$, $l_t^* = \beta T$; time apportioned according to preference parameters for consumption and leisure in the Cobb-Douglas utility function.

[67]Soil degradation ends up being irrelevant for returns to annuals because consumption relies solely on annual cropping, so this term drops out of the expression. As we'll see below, this result no longer holds if exogenous income is included.

Taking this expression for leisure and putting it back in the time constraint

$T = l_{t \geq \tau} + LF_{t \geq \tau} + LO_{t \geq \tau}$ gives wage labor in terms of cropping labor:

$$T = \frac{\beta}{\alpha}\left(\frac{LF_{t \geq \tau}}{\psi} + LO_{t \geq \tau}\right) + LF_{t \geq \tau} + LO_{t \geq \tau} = \left(\frac{\beta + \alpha\psi}{\alpha\psi}\right)LF_{t \geq \tau} + \frac{LO_{t \geq \tau}}{\alpha} \Rightarrow$$

$$LO_{t \geq \tau} = \alpha T - \left(\frac{\beta + \alpha\psi}{\psi}\right)LF_{t \geq \tau}. \tag{39}$$

Substituting this for $LO_{t \geq \tau}$ in the expression above for leisure gives leisure in

terms of farming labor:

$$l_{t \geq \tau} = \frac{\beta}{\alpha}\left(\frac{LF_{t \geq \tau}}{\psi} + \alpha T - \left(\frac{\beta + \alpha\psi}{\psi}\right)LF_{t \geq \tau}\right) = \beta T + \frac{\beta}{\alpha\psi}LF_{t \geq \tau} - \beta\left(\frac{\beta + \alpha\psi}{\alpha\psi}\right)LF_{t \geq \tau} =$$

$$\beta\left[T + \frac{LF_{t \geq \tau}}{\alpha\psi} - \left(\frac{\beta + \alpha\psi}{\alpha\psi}\right)LF_{t \geq \tau}\right] = \beta\left[T + \left(\frac{1}{\alpha\psi} - \frac{\beta + \alpha\psi}{\alpha\psi}\right)LF_{t \geq \tau}\right] \Rightarrow$$

$$l_{t \geq \tau} = \beta\left[T + \left(\frac{1}{\psi} - 1\right)LF_{t \geq \tau}\right]. \tag{40}$$

Expressing farming labor $LF_{t \geq \tau}$ in terms of problem parameters, then substituting

this expression for $LF_{t \geq \tau}$ in equations for leisure and wage labor in terms of farming labor

gives the time allocation solution:

$$LF_{t \geq \tau}^{*} = \left(\frac{w}{p_a AS_0 e^{-dt}\psi}\right)^{\frac{1}{\psi - 1}} \tag{41}$$

$$l_{t \geq \tau}^{*} = \beta T + \beta\left(\frac{1 - \psi}{\psi}\right)\left(\frac{w}{p_a AS_0 e^{-dt}\psi}\right)^{\frac{1}{\psi - 1}} \tag{42}$$

$$LO_{t \geq \tau}^{*} = \alpha T - \left(\frac{\beta + \alpha\psi}{\psi}\right)\left(\frac{w}{p_a AS_0 e^{-dt}\psi}\right)^{\frac{1}{\psi - 1}}. \tag{43}$$

Unlike during the initial phase, once off-farm labor begins the household changes its time allocation between leisure and work over time. Leisure time starts to decline and working time to increase. Soil degradation drives the balance within working hours to shift towards wage work and away from annuals. Time allocation approaches a fixed allocation between leisure and wage work determined by relative preference for leisure and consumption, as annuals labor fades out (as $t \to \infty$, $l^* = \beta T$, $LO^* = \alpha T$, $LF^* = 0$).

Differentiating the relevant parts of Equation 19 with respect to time allows some additional conclusions to be drawn about rates of change during this phase.

$$-\frac{\beta \dot{l}_t}{l_t^2} = -\frac{\alpha p_a A S_0 e^{-dt} \psi L F_t^{\psi-1}}{C_t} \left(\frac{\dot{C}_t}{C_t} + d + \frac{(1-\psi) \dot{LF}_t}{LF_t} \right) = -\frac{\alpha w \dot{C}_t}{C_t^2} + \dot{\mu}_{LOt} \qquad (44)$$

During the off-farm work phase, the equality of returns to leisure time and off-farm labor[68] and the above expression together imply consumption and leisure must fall at the same proportional rate.[69] Since $w = p_a A S_0 e^{-dt} \psi L F_{t \geq \tau}^{\psi-1}$ in this phase, the above expression implies that farming labor falls at a constant proportional rate. That pace is determined by the rate of soil degradation and the degree of diminishing returns in annuals production.[70]

In sum, for $t < \tau$, the household fixes its allocation of time between leisure and cropping labor and its consumption falls. Starting at $t = \tau$, the household devotes ever

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[68] $\dfrac{\beta}{l_t^*} = \dfrac{\alpha w}{C_t^*}$.

[69] That is, $\dfrac{\dot{C}_t^*}{C_t^*} = \dfrac{\dot{l}_t^*}{l_t^*}$; with $\mu_{LOt} = 0$.

[70] More precisely, $d = -\dfrac{(1-\psi)\dot{LF}_t}{LF_t}$.

more time to off-farm work, taking time out of both annuals labor and leisure. By availing itself of wage work, the household slows the consumption declines due to soil degradation vis-à-vis the first, annuals only phase. If returns to annual cropping at $t = 0$ are not high enough, the $t \geq \tau$ solution holds for the entire trajectory.

2.4.1.2 Exogenous income in the 'no-investment' trajectory

Including exogenous income adds a term to solutions in both parts of the trajectory. This changes the qualitative nature of the no-investment model's solution. Exogenous income crowds out labor, similar to what was seen in the steady state analysis, but now cannot do so completely, since diminishing returns guarantee that the household always grows annuals. For $t < \tau$, the part of the trajectory with no wage labor and just annual cropping,

$$l_{t<\tau} = \frac{\beta}{\alpha} \left(\frac{LF_{t<\tau}}{\psi} + \frac{Y}{p_a A S_0 e^{-dt} \psi LF_{t<\tau}^{\psi-1}} \right) \Rightarrow$$

$$T = \left(\frac{\beta + \alpha\psi}{\alpha\psi} \right) LF_{t<\tau} + \frac{\beta Y}{\alpha p_a A S_0 e^{-dt} \psi LF_{t<\tau}^{\psi-1}}. \tag{45}$$

Higher exogenous income Y is associated with less cropping labor LF_t and more leisure l_t, but the effect is mitigated by a stronger relative household preference for consumption vis-à-vis leisure or by parameters more favorable to annual cropping (higher price, better technical efficiency, better soil fertility). A given Y, moreover, crowds out more labor as time passes because it offsets consumption declines that otherwise would grow ever larger because of soil degradation. The presence of exogenous income thus disrupts the earlier solution's fixed time allocation in the first phase. Note, though, that no matter how high exogenous income goes, cropping labor will persist at some, albeit tiny level because of the diminishing returns to labor in the activity.

In the second part of the trajectory, once off-farm labor begins, introducing exogenous income again shifts time toward leisure, but now takes time away from off-farm work. Cropping labor remains unaffected by exogenous income, tied as it is to the wage and other problem parameters. Soil degradation still prompts the household to shift time away from leisure and towards off-farm work. For $t \geq \tau$, then,

$$LF_{t \geq \tau}^* = \left(\frac{w}{p_a A S_0 e^{-dt} \psi} \right)^{\frac{1}{\psi - 1}}, \tag{46}$$

$$LO_{t \geq \tau}^* = \alpha T - \left(\frac{\beta + \alpha \psi}{\psi} \right) \left(\frac{w}{p_a A S_0 e^{-dt} \psi} \right)^{\frac{1}{\psi - 1}} - \frac{\beta Y}{\alpha w}, \tag{47}$$

$$l_{t \geq \tau}^* = \beta T + \beta \left(\frac{1 - \psi}{\psi} \right) \left(\frac{w}{p_a A S_0 e^{-dt} \psi} \right)^{\frac{1}{\psi - 1}} + \frac{\beta Y}{\alpha w}. \tag{48}$$

Setting the solution for off-farm work to zero and solving for exogenous income yields the threshold level at or above which exogenous income totally crowds out off-farm labor:

$$\bar{Y}_t = \frac{\alpha w}{\beta} \left[\alpha T - \left(\frac{\beta + \alpha \psi}{\psi} \right) \left(\frac{w}{p_a A S_0 e^{-dt} \psi} \right)^{\frac{1}{\psi - 1}} \right]. \tag{49}$$

The threshold shrinks over time because of the effect of soil degradation on labor allocation.

This discussion of a simple version of the model without investment shows the intricate nature of the household's optimal time trade-offs across work activities and leisure, even in the absence of model dynamics. Without investment, the household must devote ever more time to work and less to leisure as time passes. Once the off-farm work

opportunity with its steady wage becomes profitable, it allows the household to stave off a worsening situation caused by soil degradation.

Exogenous income can again bolster leisure time. Holding to its role of crowding out labor, exogenous income hits cropping labor in the trajectory's first phase with an effect that grows as soils degrade, and in the trajectory's second phase prompts a permanent transfer of time away from off-farm labor to leisure. The transfer to leisure associated with exogenous income is mitigated the greater the preference for the consumption good over leisure, and the more favorable the parameters toward the relevant work activity (annuals parameters in the first phase, and the wage in the second phase). In the second phase of the trajectory, the inexorable decline in cropping labor proceeds, driven by the rate of soil degradation.

2.4.2 A full model trajectory

With the pasture investment possibility added, labor allocation decisions tie in to model dynamics, altering the nature of the trajectory. We add to the above work opportunities that include an exogenously worsening situation (annual cropping with its soil degradation) and an exogenously stable option (off-farm work with its constant wage), a possibility of endogenously improving circumstances through investment (pasture with its associated herd growth dynamics).

A priori, we'd expect the investment opportunity, when profitable, to hasten the decline of annual cropping vis-à-vis the situation when wage labor is the only work alternative to annual cropping. Investment should also open up the possibility of gains in leisure as well as consumption over time. It could shorten or remove altogether the

attractiveness of an off-farm work phase (a phase which was inevitable in the previous model).

More generally, it could mean that for parameter ranges near the knife-edge steady state point,[71] the long-run 'losing' activity might start and die out along the trajectory. The following subsections examine some analytics characterizing the full model optimal trajectory, look at qualitative trajectories for parameter ranges characterized by the different steady state types, and revisit how exogenous income changes labor allocation along the optimal trajectory.

2.4.2.1 Full model trajectory: some analytical insights

In analytical terms, the dynamic link between leisure (its returns and how it changes over time) and pasture labor can be seen in the expression for returns to pasture labor:

$$\frac{\varphi_t + \mu_{Dt}}{\gamma} \equiv \frac{-\gamma \frac{\beta}{l_t}\frac{\dot{l_t}}{l_t} + \frac{\alpha p_h}{C_t} r\left(\frac{N_t}{K_t}\right)^2 + \mu_{Kt} - \dot{\mu}_{Dt}}{\gamma(\rho + \delta)} + \frac{\mu_{Dt}}{\gamma}. \tag{50}$$

When pasture is the long-run 'winner' and its capacity is expanding towards its steady state value ($\underline{K} < K < K^*$, $D>0$, $\mu_{Kt}=0$, $\mu_{Dt}=0$), the equality of marginal returns to time for pasture labor and leisure (as holds along the optimal path) simplifies to:

$$\frac{\alpha p_h}{\gamma C_t} r\left(\frac{N_t}{K_t}\right)^2 \left(\frac{\dot{l_t}}{l_t} + \rho + \delta\right)^{-1} = \frac{\beta}{l_t}. \tag{51}$$

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[71]That is, when $\dfrac{p_h r\left(\dfrac{r - \rho}{2r}\right)^2}{\gamma(\rho + \delta)} = w$.

This suggests that, in contrast with the no-investment model, leisure time can grow over the trajectory with pasture on the rise. Any proportional decline in leisure, moreover, must not exceed in magnitude the sum of the discount rate and pasture degradation rate.[72]

In the full model, the equation for differentiated marginal returns to time gains a new term for pasture returns, in which the stocking rate and its speed of adjustment[73] figure along with proportional change rates for leisure and consumption:

$$\frac{\dot{\varphi}_t + \mu_{Dt}}{\gamma} = -\frac{\beta \dot{l}_t}{l_t^2} = -\frac{\alpha p_a A S_0 e^{-dt} \psi L F_t^{\psi-1}}{C_t}\left(\frac{\dot{C}_t}{C_t} + d + \frac{(1-\psi)\dot{LF}_t}{LF_t}\right) = -\frac{\alpha w \dot{C}_t}{C_t^2} + \mu_{LOt} \qquad (52)$$

$$\frac{\dot{\varphi}_t + \mu_{Dt}}{\gamma} \equiv \frac{1}{\gamma(\rho+\delta)}\left[\frac{\gamma\beta \dot{l}_t}{l_t^2}\left(\frac{\dot{l}_t}{l_t} - \ddot{l}_t\right) + \frac{\alpha p_h r}{C_t}\frac{N_t}{K_t}\left(2\frac{\dot{N}_t}{K_t} + 2\frac{N_t}{K_t}\frac{\dot{K}_t}{K_t} - \frac{N_t}{K_t}\frac{\dot{C}_t}{C_t}\right) + \mu_{Kt} - \ddot{\mu}_{Dt}\right] + \frac{\dot{\mu}_{Dt}}{\gamma} \qquad (53)$$

As in the no-investment model, during any off-farm work phase, consumption and leisure move in the same direction at the same proportional rate of change, and annual cropping still declines at the same constant proportional rate.[74]

The investment opportunity, however, complicates the picture for a household engaged in annual cropping without wage work compared with the situation in the no-investment model. Consumption and leisure *can* move in opposite directions outside of

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[72] $\dfrac{\dot{l}_t}{l_t} > 0$, or $\dfrac{\dot{l}_t}{l_t} < 0$, $\left|\dfrac{\dot{l}_t}{l_t}\right| < \rho + \delta$.

[73] More precisely, $\dfrac{\dot{N}_t}{K_t}, \dfrac{N_t}{K_t}, \dfrac{\dot{K}_t}{K_t}$.

[74] $\dfrac{\dot{LF}_t}{LF_t} = \dfrac{d}{(1-\psi)}$.

the off-farm work phase. The result hinges on the sign of the expression within the time-differentiated returns to cropping labor,[75] so on the relative magnitudes of proportional change in consumption and crop labor (adjusted by the diminishing returns parameter) and the soil degradation rate. Mathematically, leisure and consumption's movements in opposite directions can be inferred from a series of conditions relating these entities.[76] Severe soil degradation can lead to rising leisure accompanied by falling consumption. Slow enough soil degradation (small d) can prompt more work and rising consumption alongside falling leisure.

We can use the type of steady state – pro-pasture labor, pro-wage labor, or the knife-edge case – to further characterize the optimal trajectory.

2.4.2.2 Full model trajectory and steady state type

This discussion takes the pro-wage labor steady state with extremely unfavorable pasture parameters as a starting point, then examines qualitative effects on the optimal trajectory as pasture parameters are boosted relative to the wage to the point of the knife-edge steady state, and beyond to the pro-pasture steady state.

When the full model's parameters indicate a pro-wage labor steady state and pasture is not profitable enough to use any labor beyond the minimum requirements, the

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[75] $\dfrac{\dot{C}_t}{C_t} + d + \dfrac{(1-\psi)\dot{LF}_t}{LF_t}$.

[76] $\dot{C} < 0, \dot{l} > 0$ if a) $\dot{LF} > 0, d > \left|\dfrac{\dot{C}}{C}\right| - (1-\psi)\dfrac{\dot{LF}}{LF}$, b) $\dot{LF} < 0, d > \left|(1-\psi)\dfrac{\dot{LF}}{LF} + \dfrac{\dot{C}}{C}\right|$;

$\dot{C} > 0, \dot{l} < 0$ if $d < (1-\psi)\left|\dfrac{\dot{LF}}{LF}\right| - \dfrac{\dot{C}}{C}$. This exercise, however, does not shed light on the other

parameter conditions that would prompt such configurations (e.g., conditions that would cause rising cropping labor over time despite soil degradation).

optimal trajectory would closely resemble that of the no-investment model, with a couple of changes caused by the full model's need for an initial pasture endowment and a minimum pasture level. The household would take advantage of any pasture endowment over the minimum requirement, as well as any herd endowment. Once the pasture had degraded to its minimum, the household would siphon off just enough labor needed to maintain this.

Because even the minimum pasture makes some contribution to consumption as long as it contains some cattle, a change in cattle price too small to alter pasture labor or herd size would still cause some other changes in the model. The additional revenue would play a role akin to exogenous income in the no-investment model. This means a shift towards leisure from the primary labor activity all along the trajectory and in the steady state. As in that no-investment model, the transfer from labor to leisure increases over time when annual cropping is the primary activity, but stays constant during the wage phase. As in the no-investment model, $p_a AS_0 e^{-dt} \psi L F_{t=0}^{\psi-1} > w$ is a necessary condition for an annuals phase without wage labor. The shift of labor from annual cropping to off-farm work would be swifter with more unfavorable annuals parameters (lower price, lower technical efficiency and more dramatically diminishing returns to labor, lower soil fertility, and higher soil degradation).

Boosting pasture parameters to prompt an active pasture phase complicates the picture considerably. If off-farm work is still more favorable in the long run (the model is still heading towards an off-farm work steady state), any pasture activity would be temporary. That pasture activity would presumably begin during the initial annuals

phase, prior to off-farm work's starting, in order to mitigate effects of soil degradation.[77] From that point, the household would manage the abandonment of its pasture and the sell-off of its herd while increasing wage work towards the steady state value. This temporary pasture phase could affect the timing of the wage phase's onset. By mitigating soil degradation, pasture would allow labor returns in a given period higher than would be the case if only annuals were undertaken, thus delaying the start of the off-farm work phase.

With yet more favorable pasture parameters so that the model switches to a pasture labor steady state, it is off-farm work that might enjoy a temporary phase, again as a tool to mitigate losses from soil degradation. Pasture labor would increase over time, with a different trend during annuals and off-farm labor phases because the pattern of labor re-allocation towards pasture would be differently influenced by exponential soil degradation and a constant wage, respectively.

Off-farm wage labor begins at τ as described in the no-pasture sequential static model. Because the advent of pasture should draw labor away from annuals cropping, τ should be earlier in the trajectory vis-à-vis a no-pasture model. With even more favorable pasture parameters, the labor drawn to pasture could approach steady state levels before τ, effectively choking off any period of wage labor.

As before, exogenous income provokes a transfer of time from the most favorable labor activity into leisure. The presence of some cropping labor, however minuscule, prevents an absolute 'life-of-leisure' threshold for exogenous income along the trajectory analogous to that seen for the steady state. Still, enough exogenous income can

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[77]This assumes that the household starts off with little pasture and no cattle. A higher endowment of pasture and cattle would delay any active pasture phase for a given set of parameters

completely choke off wage work or pasture activity above the minimum requirement, so that leisure absorbs all discretionary time outside the cropping sector (so $l_t = T - \gamma \delta \underline{K} - LF_t^*$ and $D_t = \delta \underline{K}$). In that sense, the steady state 'life-of leisure' threshold exogenous income $\bar{Y}_{ss}$ (Equation 34) is a special case, and endpoint, of a time-specific threshold $\bar{Y}_t$ above which leisure crowds out wage work and pasture in situations otherwise favorable to these. That suggests that the time-dependent threshold exogenous income depends in part on stocking rate,[78] as well as soil degradation (as seen in the wage phase of the no-investment model). So with a constant exogenous income, a household could move into or out of a relative 'life-of-leisure' over time.[79]

This qualitative description of the optimal trajectory draws on the general shapes of returns to labor in the various activities: annuals' declines due to exponential soil degradation, off-farm works' stability due to the constant wage, and pasture's increases due to the dynamics of herd growth. Details of these complicated trade-offs are affected, most importantly, by the specifics of herd growth and pasture degradation dynamics. The labor trade-offs imply, in turn, intricate relationships among the rates of change in components of household utility (leisure and consumption) as well as in proportion of pasture carrying capacity used. Chapter 3 revisits these aspects of the optimal trajectory using a numerical simulation model, after the following section presents baseline parameters for the model.

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[78] $\dfrac{N_t}{K_t}$, analogous to the role of the steady state stocking rate $\dfrac{r^2 - \rho^2}{4r}$ in the steady state exogenous income threshold.

[79] The actual exogenous income threshold would be complicated to derive, as it depends on a closed form solution for the optimal trajectory, but it would, as in the steady state, be a maximum function of income thresholds for wage labor and pasture activity above the minimum.

2.5 The model in context – baseline parameters and steady state

Baseline parameters were chosen with the study context in mind and to normalize steady state carrying capacity to close to one. The general structure of the model mimics the study area's reality of annuals giving way to pasture, with pasture becoming increasingly important in terms of area over time. Biophysical parameters reflect the gradual decline in carrying capacity of pasture under traditional (and still prevalent) management practices, as well as the tendency for smallholders to move toward lower shares of annual cropping in cleared area and contribution to income over time.[80] The household's situation captures that of smallholder migrants to forested regions: no outside income or meaningful off-farm labor opportunities, and no starting herd.

Table 2 shows parameter and analytical steady state values for the baseline model. Degradation rates reflect common usage of annuals areas for three years before a land-use change, and pasture area for seven years before recovery/replanting. The discount rate is similar in magnitude to that used in other studies in the Brazilian Amazon (Walker et al. 2000; Vosti et al. 2002). Relative preferences are set to favor consumption, in keeping with observed behavior in the area as households capitalize. Exogenous income is set to zero – a common situation for incoming small-scale migrants. The extent of annuals' profitability varies in the study region; the baseline parameters make annuals competitive with pasture in the short-run, and other scenarios are explored in subsequent sensitivity analyses.

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[80]The model's annuals represent a phase of reliance on these crops for current consumption and investment into pasture, not opportunistic use of deforested area.

Table 2. Baseline parameters and steady state values, plus selected reference values[81]

Parameter	Value	Parameter	Value	Variable	Steady State Value (SS)	Ref. Values
Y	0	S_o	1	l^*	0.234	$\bar{Y} = 0.517$
α	0.9	ρ	0.1	K^*	1.008	$\gamma D^* = 0.766$
β	0.1	r	0.5	D^*	0.121	$\varphi^*/\gamma = 0.427$
γ	6.33	δ	0.12	N^*	0.403	$\beta/l^* = 0.427$
p_h, p_a	1	d	0.04	H^*	0.121	$N^*/K^* = 0.400$
w	0.01	T	1	φ^*	2.70	
ψ	0.8	N_o, K_o	0, 0.05	C^*	0.121	
A	0.1	$\underline{K}$	10^{-5}	U^*	-2.046	

Herds grow naturally at rates greater than the discount rate $(r > \rho)$, making this production system potentially worthwhile. With no external income, the household never finds leisure crowding out all its non-cropping time $(Y = 0 < \bar{Y}_t \; \forall t)$. The relatively low wage ensures that returns to annual crop labor at $t=0$ exceed those to off-farm labor (since $p_a A S_0 e^{-dt} \psi L F_{t=0}^{\psi-1} > w$ even if all household time were allocated to cropping labor), so the optimal trajectory begins with annual cropping and no wage work.

The steady state shadow wage for pasture labor exceeds returns to wage labor (with baseline parameter values, as long as $w < 0.0574$),[82] so the trajectory ends at a steady state with pasture the only labor activity – a pro-pasture corner solution. Since capacity of pasture used at the outset is less than the steady state ideal, this ratio will increase over the course of the trajectory.

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[81]* indicates a steady state value, an underscore is a minimum value, and a line over a variable indicates an upper threshold.

[82]This follows from $\dfrac{p_h r \left(\dfrac{r - \rho}{2r} \right)^2}{\gamma(\rho + \delta)} > w$, or, using baseline parameter values, $0.0574 > 0.01$.

The next chapter uses the numerical model to present the baseline optimal trajectory with more detailed dynamics – when pasture labor begins, when annuals disappear, whether wage labor ever occurs during the optimal trajectory, and how time allocation shifts between leisure and labor. We then explore how different parameter values affect the optimal path.

Chapter 3 Numerical Simulation Model

Numerical simulation can readily generate close approximations of optimal timepaths for model variables.[83] The results can provide a foundation for identifying hypotheses about farm-level trajectories that could be tested empirically. Closed form steady state solutions and optimal path conditions already presented provide a useful check on the accuracy of the numerical model's approximations.

The numerical model uses discrete time, with a one year time step. This more accurately reflects the nature of seasonal agriculture than the continuous time of the theoretical model, in particular matching the production period for annual crops. The model described here, programmed in GAMS and using the nonlinear optimization solver CONOPT3,[84] runs from $t=0$ to $t=90$, after which a value function captures entry into an optimal steady state, truncating the infinite theoretical trajectory that approaches the steady state. In practice, the parameter values and numerical solver used limit how long the model simulation can run reliably in the lead-up to the steady state: $t=90$ was chosen because it handled the model run across a range of parameter values and yielded steady state results close to analytical values.

The numerical model also uses an approximation for the production function for annual crops in order to allow farming labor to zero out in finite time – a necessity for model convergence.[85] In addition, it includes a parameter to examine effects of tenure security. *LdMkt* sets the proportion of the value function the farmer expects to be able to receive for the farm (including the livestock assets) upon entering the steady state.

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[83]The knife-edge case is the exception. True parameter values point to an indifference between activities that the numerical model's approximations cannot capture.

[84]The solver finds local optima based on the generalized reduced gradient method (McCarl 2006).

[85]$Q_t = p_a A S_0 e^{-dt} \left[(LF_t + \varepsilon)^\psi - \varepsilon^\psi \right]$, with ε arbitrarily small, and $\varepsilon = 10^{-7}$ in the baseline run.

The program of the baseline model using parameters from Table 2, no property rights distortions (*LdMkt*=1), and a slight adjustment in the annuals production function ($\varepsilon=10^{-7}$) yielded numerical results going into the steady state (after *t*=90 in the program) matching analytical steady state values presented in Table 2 to at least two decimal points.

This chapter builds up to presentation and discussion of the baseline model simulation by exploring dynamic trajectories of similar but simpler models. The numerical results reflect the qualitative characteristics for optimal trajectories reached analytically in the previous chapter, plus details about the timing and nature of transitions. They provide a backdrop for the sensitivity analysis and policy simulations that follow in Chapter 4.

3.1 Understanding model dynamics via simpler models

The full baseline has wage labor, annual cropping, and cattle production as possible income sources. This section looks at simpler models with one or more of the income source components to shed light on model dynamics, especially how labor allocation changes over time. The approach is different than in the previous chapter, where discussion began with a no-investment model, then introduced pasture parameters, examining how, as they grew more favorable, the qualitative nature of the trajectory changed.

Here, instead, we build on the most basic *dynamic* model, with cattle as the only viable production activity. We then add wage labor, and examine trajectories at several different wages to get a feel for how this addition alters other model dynamics. In the next model, annual cropping is substituted for wage labor to illustrate how the

distinguishing aspects of this production activity – namely soil degradation and decreasing returns to labor – change the household's trajectory. The simpler models use the relevant baseline parameter values from Table 2 unless otherwise stated.

3.1.1 A model with pasture only

Using the baseline model structure with a zero crop price and zero wage generates a pasture only model. The simulation includes a small exogenous income (Y=0.01). It is required in this simulation in the first period to allow household consumption before a herd can be started (but is available in every period in the simulation). Figure 11 depicts household time allocation results.[86] At the outset, the household splits its time almost evenly between leisure and pasture labor. From there, leisure loses time to the only available alternative – pasture – with lower shifts per time step as the household approaches its steady state time allocation. Neither annuals nor off-farm work are viable options in this simulation, so their values stay at zero throughout the trajectory in this figure and those that follow.

Figure 11. Household time allocation – pasture only simulation

household time allocation
pasture only model Y=.01

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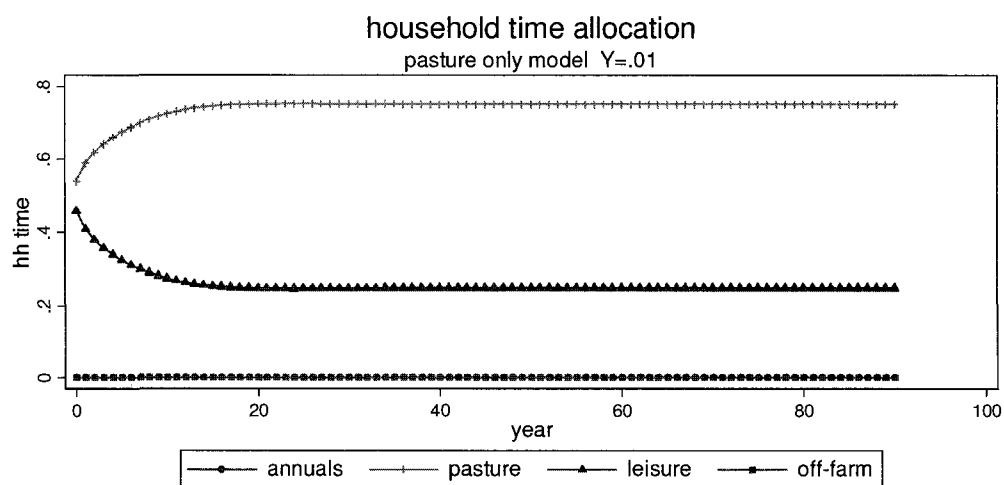
[86]Since overall household time is normalized to 1, this could be interpreted as shares of household time.

Leisure's high start despite the household's relative preference for consumption is a signal that the household is dynamically constrained in early days. Figure 12 confirms that the household's returns to labor increase along the trajectory as pasture investment gets underway.[87] The labor activity with the highest returns to time is the one the household undertakes. Returns to leisure match this amount – a graphical depiction of the equality of marginal utility of returns to time that occupied much of Chapter 2 and is the hallmark of optimizing behavior. Without opportunities for an immediate pay-off via annuals or wage labor, the household prefers leisure. As pasture investment gets underway, the household diverts time away from leisure.

Figure 12. Returns to household time – pasture only
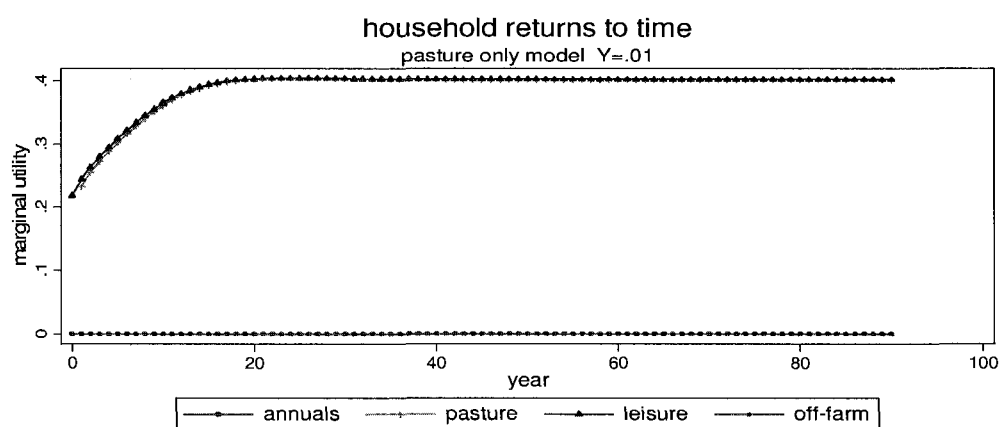
The household relies on herd growth for improving consumption beyond the small levels allowed by exogenous income. Figure 13 shows consumption trending upward as leisure falls off, in line with the increases in pasture labor.

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[87]The pasture curve starts at $t=1$, since in the discrete time model, returns to pasture labor are undefined at $t=0$. This is because, change in leisure time, undefined at $t=0$, is a component of pasture labor returns, as seen in the previous chapter.

Figure 13. Components of household utility (consumption and leisure) – pasture only[88]

household utility components
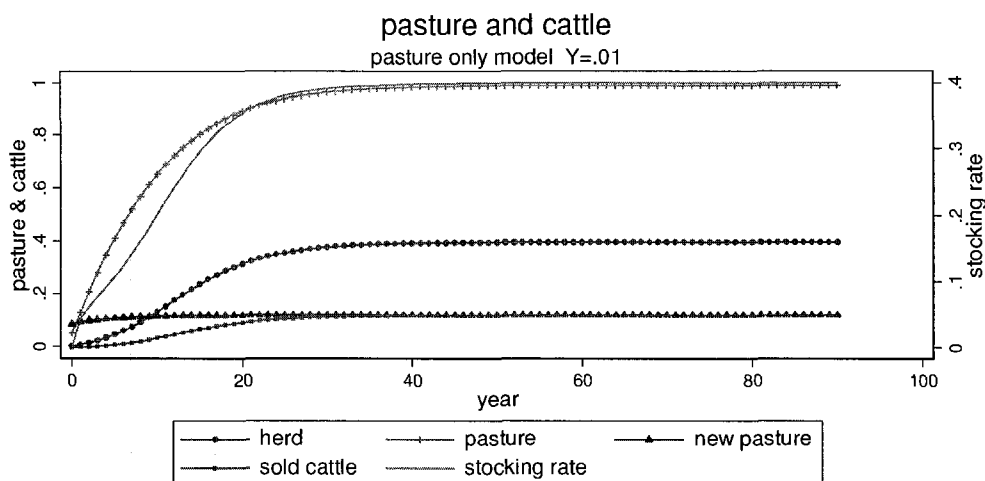pasture only model Y=.01
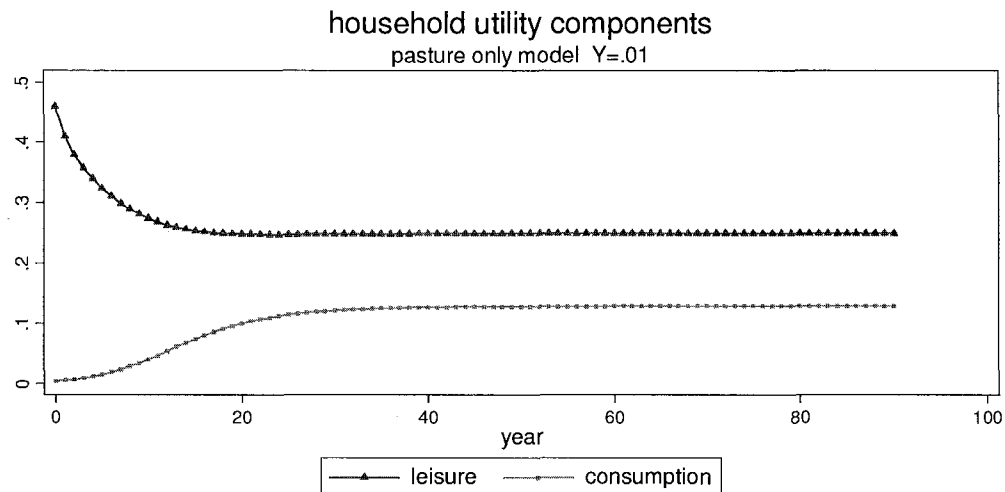
Exogenous income boosts leisure in the steady state, as in the baseline model (described in the previous chapter), but not throughout the entire trajectory. In the earliest phase, the household relies on exogenous income for its consumption and to provide capital to start a herd. The household starts to expand pasture right away.

Figure 14 (right-side axis) shows the cattle stocking rate (herd size divided by carrying capacity) rising swiftly, then tapering off toward its optimum as herd growth outpaces pasture expansion. The household buys cattle for its pasture endowment early on, then relies on population dynamics to grow the herd. It starts to sell cattle in year 4, increasing off-take as the herd expands. The pasture transition happens relatively quickly: it takes just over a decade for the household to hit 70% of pasture capacity, and just over 30 years to approach its fullest spatial extent.

[88]The way the model is scaled allows for easy viewing both utility components on the same graph; the y-axis here and in similar graphs that follow refers to time for leisure, and consumption level for consumption.

Figure 14. Household cattle and pasture management – pasture only[89]

pasture and cattle
pasture only model Y=.01

The next stripped-down model adds a constant off-farm wage to the pasture-only model to examine how an additional labor option affects the optimal trajectory.

3.1.2 A model with pasture and wage labor

This subsection examines how a market wage affects the optimal trajectory, keeping the zero crop price from the previous simulation and returning exogenous income to its zero baseline value. Having a second labor option changes labor allocation patterns, and the way that pasture is managed. Pro-pasture steady state and pro-wage labor steady state models, achieved by choosing wages on either side of the threshold wage that switches the baseline model's steady state (w=0.0574), are examined in turn.

3.1.2.1 Pasture/wage labor model with pro-pasture steady state

A range of wages makes off-farm work attractive in the short run only even though the model has a pro-pasture labor long-run steady state. The trajectory begins with off-farm labor and transitions to pasture labor. This qualitative trajectory is explored using two simulations, the baseline value w=0.01 and four times that, w=0.04,

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[89]Model scaling is such that both pasture and cattle can be easily viewed using the same y-axis scale.

both within the wage range ensuring a pasture steady state. We first describe the two scenarios, then discuss them in relation to the pasture-only model.

The opportunity for off-farm work drastically alters time allocation patterns from those seen in the pasture-only model (Figures 15 and 16). The third household time option – wage labor – eases the strict time trade-off previously seen between work in one activity (pasture) and leisure. The household uses wage labor in the early phase to provide consumption and capital to start off the herd. With a viable work option, leisure time just after settlement is sharply lower. The household gradually shifts its time allocation away from the constant wage and towards investment in pasture, and is still able to increase leisure. Eventually, wage labor will be zeroed out (since the model has a pro-pasture labor steady state). The higher the wage, the longer this takes (Figure 16 vs. Figure 15). A higher wage draws labor away from the pasture expansion, slowing the conversion of farm to pasture. A quadrupled wage (w=0.04 instead of w=0.01) prolongs the transition to all pasture labor by about 20 years – wage labor is phased out at t=29 rather than t=10.

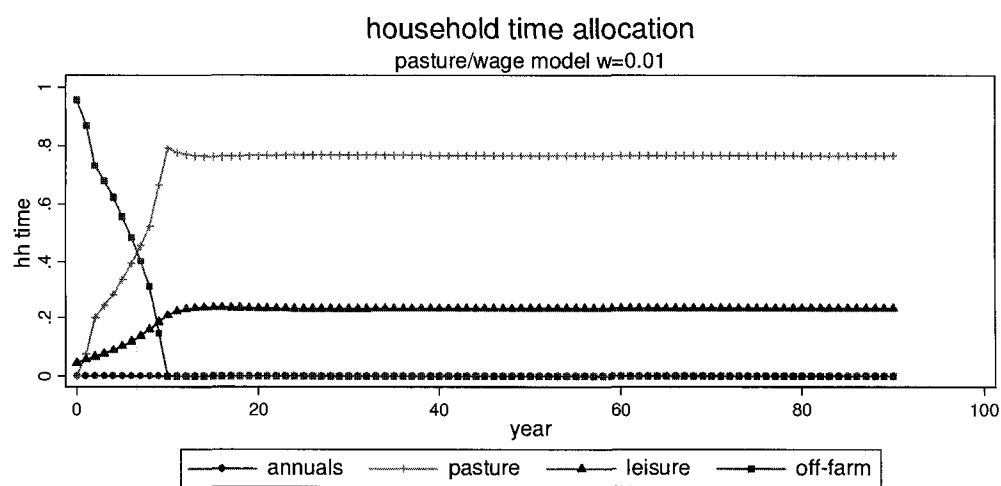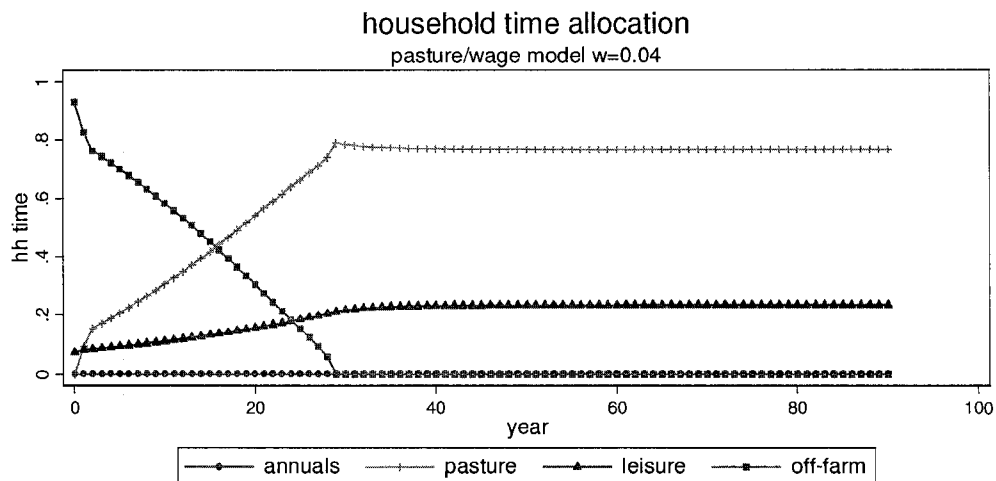Figure 15. Household time allocation – pasture and wage labor (w=0.01)

household time allocation
pasture/wage model w=0.01

Figure 16. Household time allocation – pasture and wage labor (w=0.04)

household time allocation
pasture/wage model w=0.04

Wage work is sacrificed to pasture as pasture investment first begins. The allocation of time to pasture development grows more steadily for the bulk of the wage phase. Over this period, pasture investment accelerates, but less so the higher the wage. The wage phase ends with an uptick in the allocation of labor to pasture, pushing this temporarily above its long term steady state value.[90]

Unlike the pasture-only model, the wage labor/pasture model, with its initial outside earning opportunity, sees household returns to labor fall over time (Figures 17 and 18), in keeping with leisure's upward trajectory. With the higher wage, pasture labor is deferred beyond the first year, as seen in its lower returns at the start of the trajectory. The higher the wage, the larger the initial marginal returns to labor.

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[90]The discrete time step, with its annual adjustment, is likely behind this uptick.

Figure 17. Returns to household time – pasture and wage labor (w=0.01)
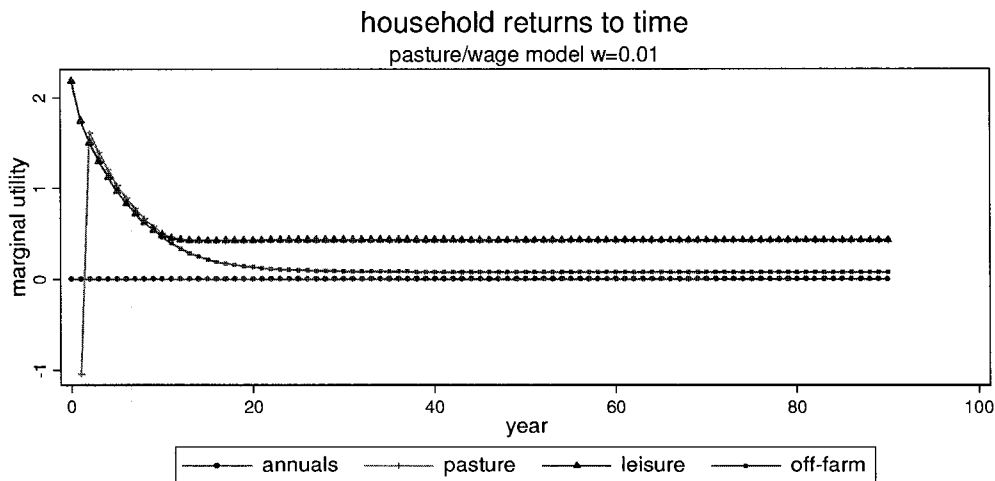
Figure 18. Returns to household time – pasture and wage labor (w=0.04)
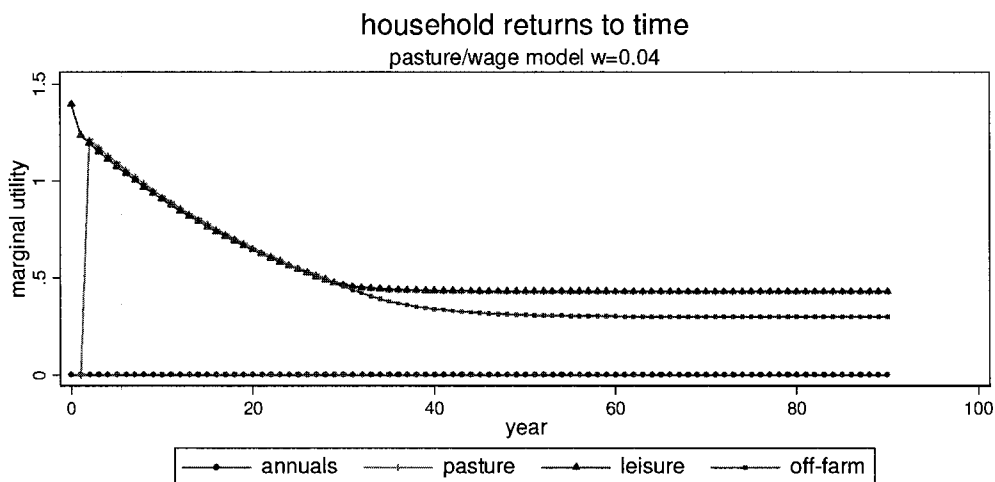
Figures 19 and 20 show consumption rising in step with the pasture transition. A lower wage means lower consumption early on, but the faster transition to pasture under these conditions means that consumption under the lower wage surpasses consumption for the higher wage within a generation (at t=17). In contrast with the pasture only simulation, where leisure must fall for consumption to rise, here leisure and consumption both rise throughout the trajectory. This matches the theoretical finding in the previous

chapter (Equation 44) that a wage phase ensures that leisure and consumption move in the same direction.

Figure 19. Household leisure and consumption – pasture and wage labor (w=0.01)

household utility components
pasture/wage model w=0.01
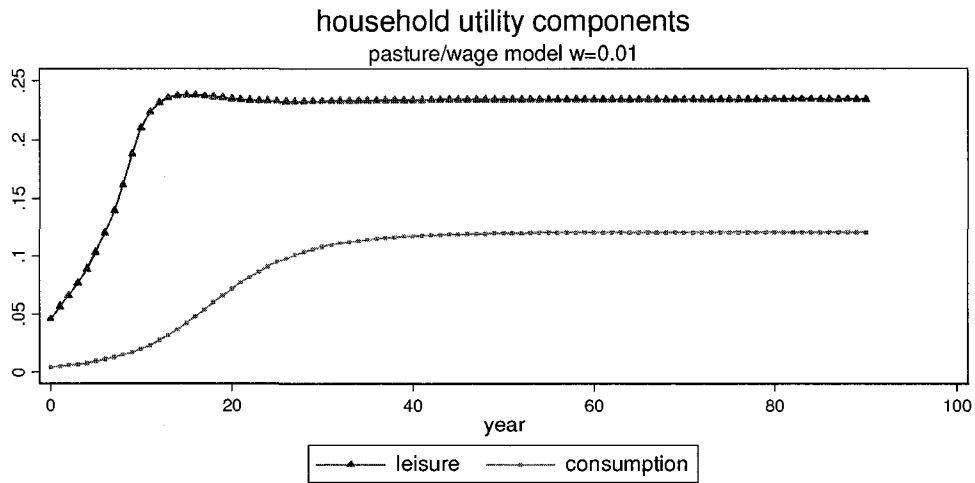
Figure 20. Household leisure and consumption – pasture and wage labor (w=0.04)

household utility components
pasture/wage model w=0.04
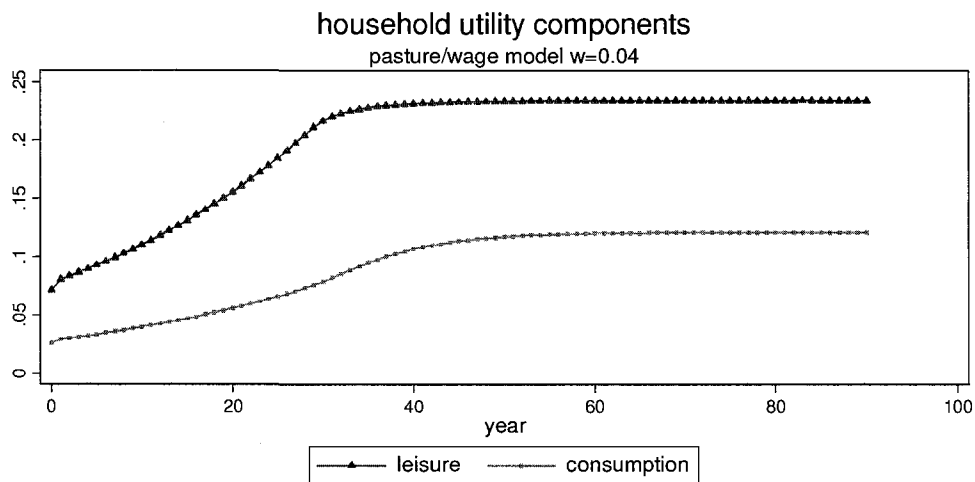
Figures 21 and 22 highlight steady proportional gains in leisure and consumption from one period to the next during the wage labor phase, identical because of the Cobb-Douglas utility function, and document the perturbations at the phase's start and finish. Notably, the wage labor phase's ending coincides with a short period of improving gains

in consumption starting with the uptick in pasture labor, before this reverses to taper toward zero change in the steady state.

Figure 21. Proportional change in utility components – pasture and wage labor (*w*=0.01)

change in leisure & consumption
pasture/wage model w=0.01
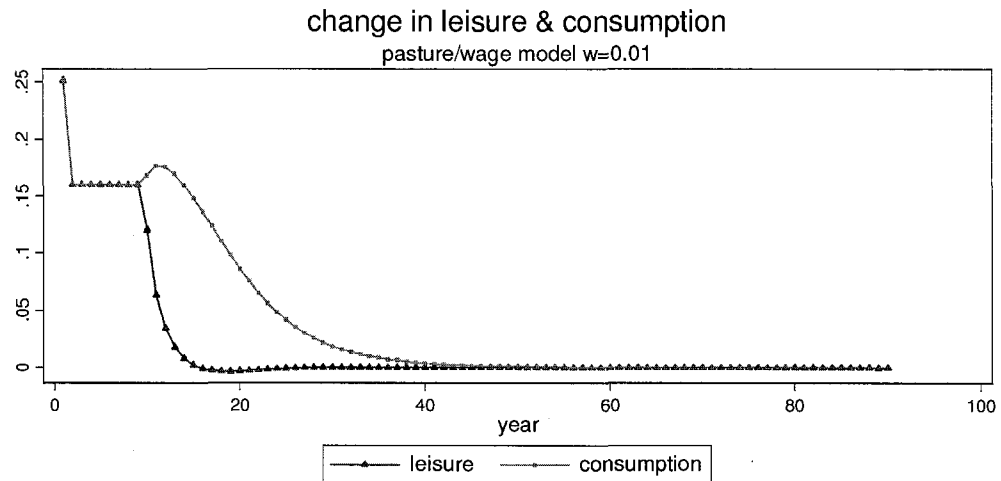
Figure 22. Proportional change in utility components – pasture and wage labor (*w*=0.04)

change in leisure & consumption
pasture/wage model w=0.04
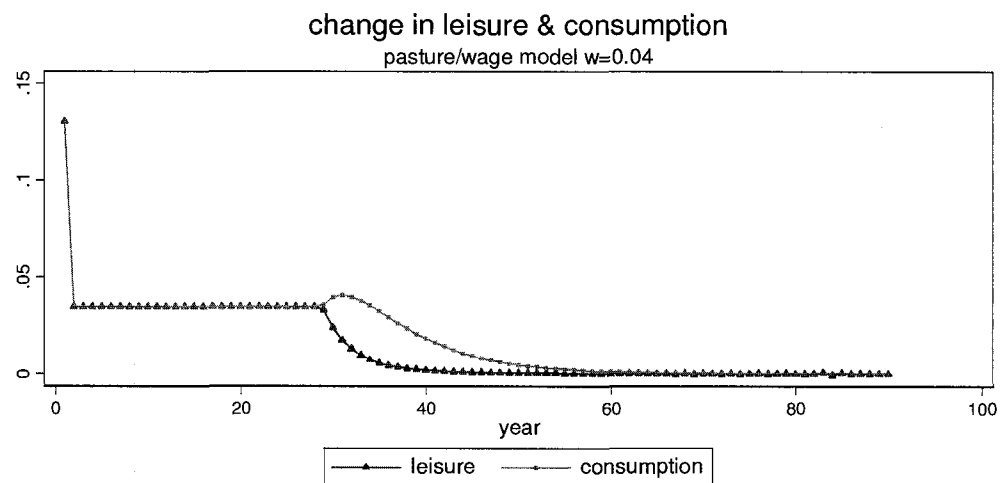
Figures 23 and 24 trace the slower expansion of pasture accompanying lower pasture labor levels under a higher wage scenario. With these parameters, the higher wage delays pasture's expansion to its full spatial extent by about a decade. The longer wage phase means more of the pasture's expansion is concurrent with off-farm work. By the high wage labor phase-out at *t*=29, pasture hits 70% of its optimum carrying capacity.

At the comparable point for the lower wage (t=10 wage labor phase-out), pasture capacity is only at about 40% of its optimum. After the cut-off, the lower wage scenario expands pasture much more quickly, so by t=29 it is close to its full capacity. A more drawn-out pasture expansion with the higher wage also allows the household to approach optimal stocking rate much sooner, during the wage phase. With the lower wage, faster pasture expansion with the same herd growth parameters means the household stays at a stocking rate considerably lower than the optimum during the wage phase.

Figure 23. Household herd and pasture management – pasture and wage labor (w=0.01)
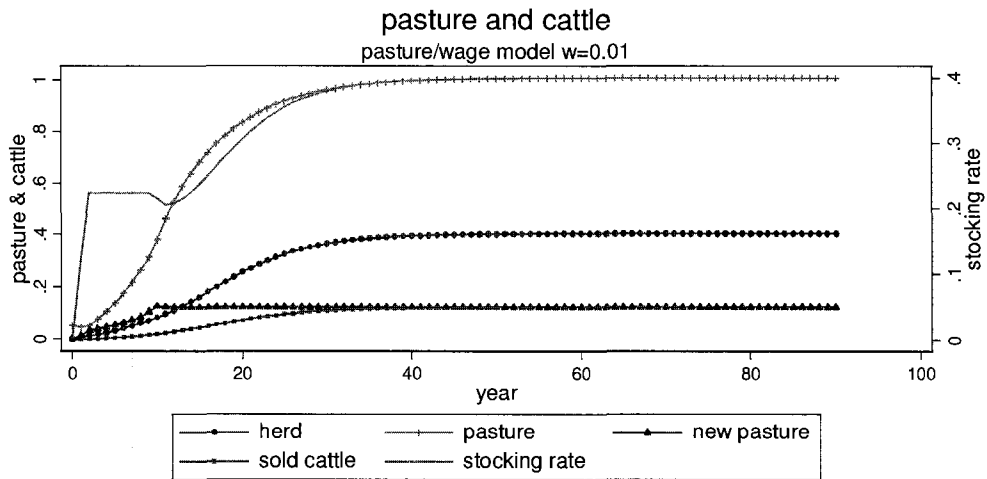
Figure 24. Herd and pasture management – pasture and wage labor (w=0.04)
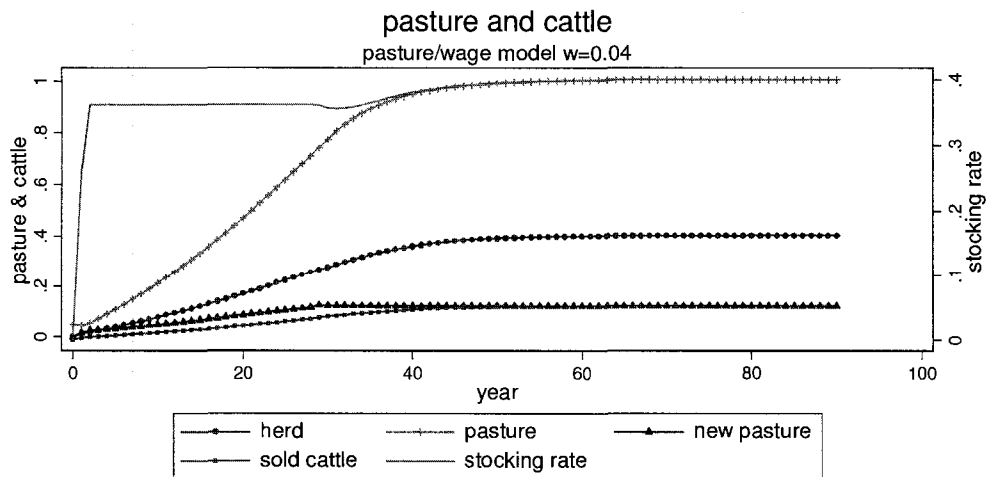
The wage scenarios also feature an oscillating pattern in herd growth vs. pasture capacity growth not seen in the pasture only model. Figures 25 and 26 depict the amount that herd growth rate exceeds pasture growth (if positive, stocking rate is rising), and show the oscillation coincides with the spurt in pasture labor at the wage labor phase-out (Figures 25 and 26). A higher wage, by making the decline in wage labor more linear, dampens the growth spurt in pasture at the phase-out, lessening and delaying the volatility of these stocking rate changes.

Figure 25. Difference in herd and pasture growth – pasture and wage labor (w=0.01)

difference in pasture & cattle growth rates
pasture/wage model w=0.01

————— proportional herd growth minus proportional pasture growth
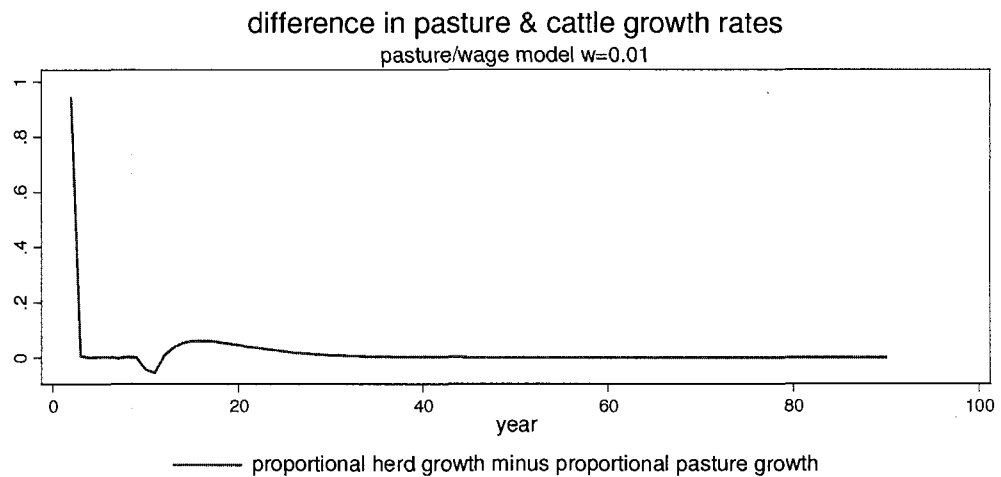
Figure 26. Difference in herd and pasture growth – pasture and wage labor (w=0.04)

difference in pasture & cattle growth rates
pasture/wage model w=0.04

————— proportional herd growth minus proportional pasture growth
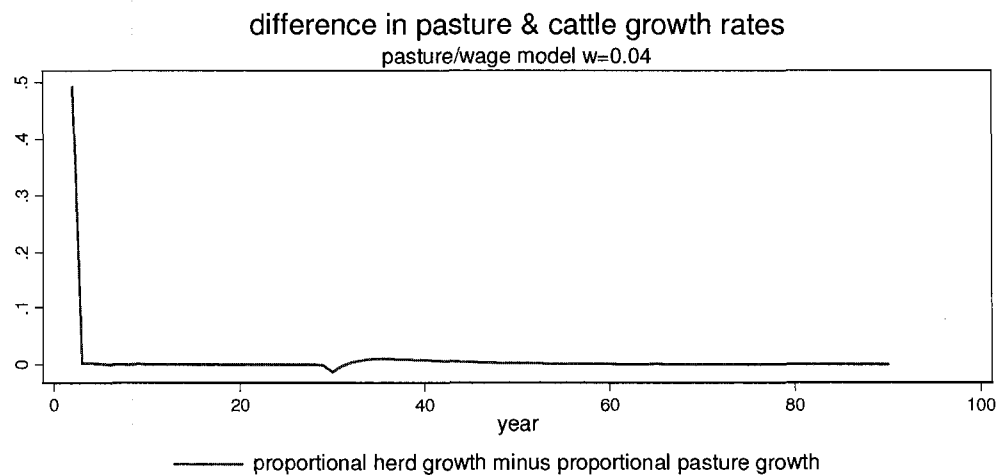
These model simulations demonstrate how the addition of a second labor alternative alongside pasture/cattle mitigates the consequences of a strict time trade-off between pasture and leisure. In the pasture-only model, consumption and pasture labor rise in tandem over time; leisure must therefore decline. Despite the model household's preference for consumption over leisure (α=0.9, β=0.1), leisure presents an attractive option early on because the small cattle/pasture operation absorbs so little manpower. As a by-product of this constrained beginning, the household's returns to time improve as pasture expands. The strict time allocation trade-off also results in a smooth approach to steady state pasture levels. As the trajectory moves closer to the steady state, the herd growth rate's advantage over the pasture growth rate narrows, the growth rates themselves slow, and pasture stocking rate approaches its long-term value smoothly from below.

With the possibility of wage work, a preferable (consumption-boosting) alternative to leisure exists early on. Even at the lowest wage examined here, off-farm labor dominates both leisure and pasture labor at the trajectory's start. In contrast to the pasture only model, returns to household time are highest at the outset, and fall over time.

The wage option relaxes the pasture only model's zero-sum time trade-off between pasture labor and leisure so the household can and does allocate more time to both along the trajectory as wage labor tapers off. Unlike exogenous income in the pasture only model, a higher wage does not bankroll a swifter transition to pasture, but rather prolongs the process: the wage attracts more labor off farm for longer.

The alternative labor activity also induces volatility in the approach to long-term cattle stocking rate. In both models, the household purchases a few animals to take

advantage of the slight pasture endowment, which causes a spurt in herd growth relative to pasture expansion. With the wage labor option, the household does not opt, as in the pasture only model, to continue to grow the herd faster than the pasture. It instead maintains the same stocking rate for the duration of the wage phase, and herd and pasture grow in lockstep. Proportional growth rates of leisure and consumption lock in to identical levels for this period. The higher the wage, the closer the stocking rate during this 'holding pattern' comes to the long-term ideal, and the slower is the proportional growth in leisure and consumption.

The household diverges from the holding pattern at the end of the wage phase. To zero out the wage category, it re-allocates relatively more labor to pasture production than under a single time-step during the holding pattern. So much labor, in fact, that pasture labor temporarily rises *above* its steady state value. With the extra labor, pasture grows faster (proportionally) than herd size – causing the see-saw growth pattern absent from the pasture only model.

3.1.2.2 Pasture/wage labor model with pro-wage labor steady state

With other baseline parameters, wages over the threshold level of $w=0.0574$ cause the household to trend towards a wage labor steady state with a minimum level of pasture ($\underline{K}$). At high wage levels, the household just lets the pasture degrade to its minimum. There are wage levels just above the threshold, however, where the household finds it worthwhile to take advantage of the pasture endowment by managing pasture's decline and applying some labor to counter pasture degradation. Figures 27 and 28 show the downward trending of pasture labor and related parameters in such a scenario. The time allocation graph does not display the off-farm work level – the residual category at close

to 90% of household time throughout – so that the minuscule amount of pasture labor (less than 2% of household time) is visible. The pasture endowment is decreased from the baseline value so that pasture labor zeroes out within the 90-year timeframe.

Figure 27. Household time allocation – pasture/wage model with wage just above threshold

household time allocation
pasture/wage model w=.057625
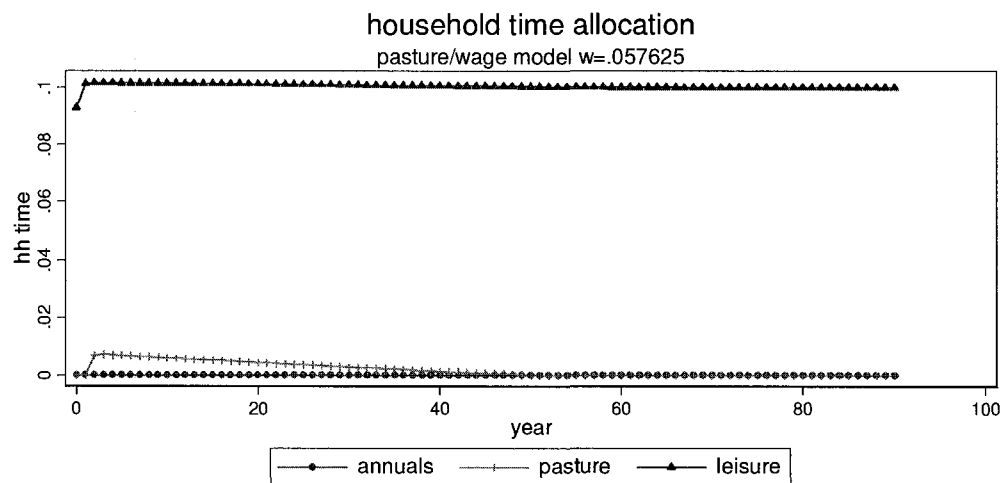
Figure 28. Pasture and cattle – pasture/wage model with wage just above threshold

pasture and cattle
pasture/wage model w=.057625
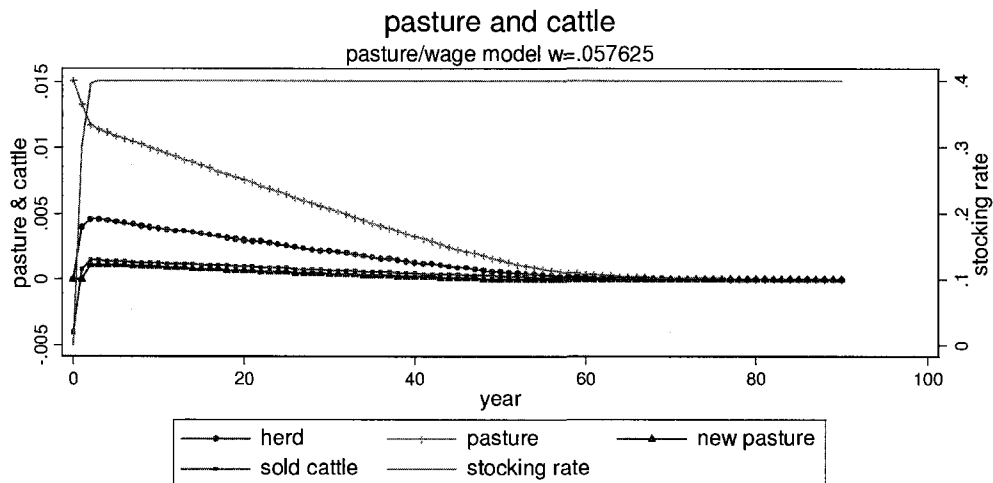
In the next simple model, annual cropping takes the place of wage labor as a second labor option alongside pasture.

3.1.3 A model with pasture and annuals

This subsection outlines the situation when the only labor option besides pasture is annual cropping. Absent soil degradation and decreasing returns technology, the transition to the steady state would mimic patterns seen in the wage and pasture model, with annuals playing the role of wage work.

As modeled here, soil degradation affects labor allocation just like an exponentially declining market wage would: however attractive annuals are at the outset, this alternative labor activity sees its returns to labor inexorably decline, and makes less sense to sustain as time passes. Because the functional form for annual cropping production has decreasing returns to labor, the farmer will always find some positive level of cropping labor attractive. In the simulation here, the slightly altered annuals production function already described allows crop production to go to zero. To get a model with annuals as the only labor alternative to pasture, the simulation also includes a positive crop price, a zero wage, and zero exogenous income.

The results show labor shifting out of annuals into pasture, with annuals largely gone by $t=40$. Actually, the baseline yields results identical to this one because the transition to pasture happens quickly enough so that the baseline wage rate $w=0.01$ is never attractive – that is, $p_a A S_o e^{-dt} \psi L F_t^{\psi-1} > w$ throughout the trajectory. We discuss the baseline results in the next section.

3.2 Baseline simulation

This section presents baseline results for time allocation, household utility, and pasture management along the optimal trajectory. Baseline results are then discussed in

the context of the simpler models presented above. Discussion focuses on the labor

transition to pasture to provide a benchmark for evaluating simulations to follow.

3.2.1 Household time allocation and returns to labor – baseline

Figure 29 shows household time allocation along the baseline trajectory. Figure

30 depicts changes in time allocated to each labor category along the first fifty years of

the trajectory. Both figures show the household cuts substantially into annuals time to

jumpstart the clearing for pasture in the model's second and third years (t=1,2). After

that, the household reallocates a smaller but ever increasing amount of labor from annuals

to pasture until t=22, after which the transfer decelerates. The phasing out of annuals

largely coincides with the transition to pasture (annuals' slow decline peters out at t=58).

As in the simpler models, pasture investment opens up the possibility of increased leisure

time, trending toward the steady state time allocation. The household settles close to its

optimum steady state time allocation once annuals are largely phased out. Because wage

work is never an attractive option, the baseline provides an example of a model with

pasture and crop labor as the two options for work.

Figure 29. Household time allocation – baseline simulation

household time allocation
baseline
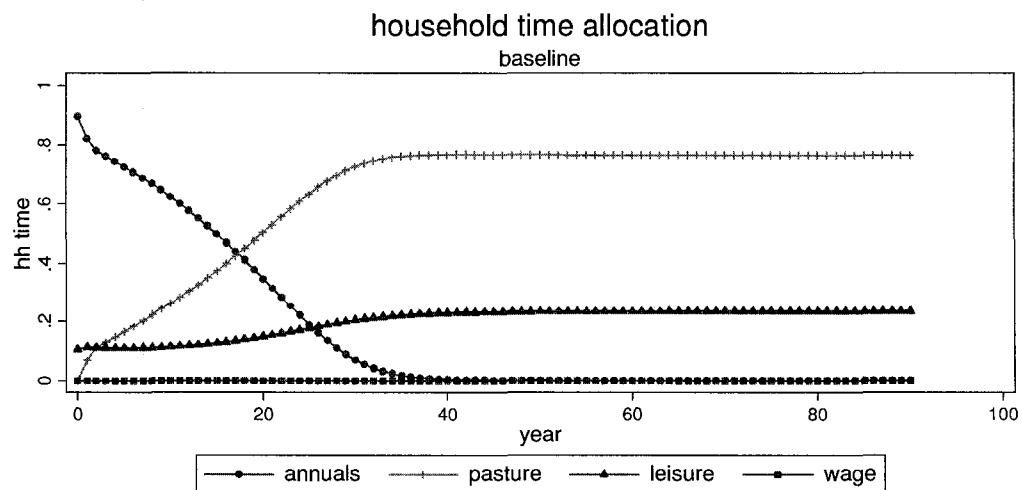
Figure 30. Household time allocation shifts from prior period, annuals phase – baseline

change in hh time allocation

baseline
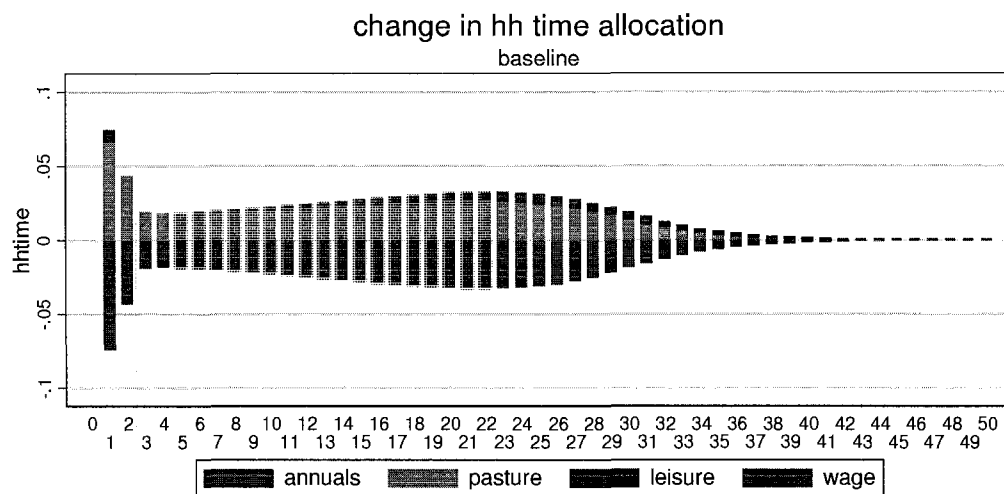
Figure 31 compares returns to household time to each alternative along the trajectory, and confirms that the other activities outcompete wage labor. As in the simpler models, the key marginal condition equality shows up here as returns to leisure matching returns to the activity undertaken at each point in the trajectory. In line with the brief period of decreasing leisure followed by an upturn, returns to time improve then decline, in contrast to their strict decline in the pasture/wage model.

Figure 31. Returns to household time – baseline

household returns to time
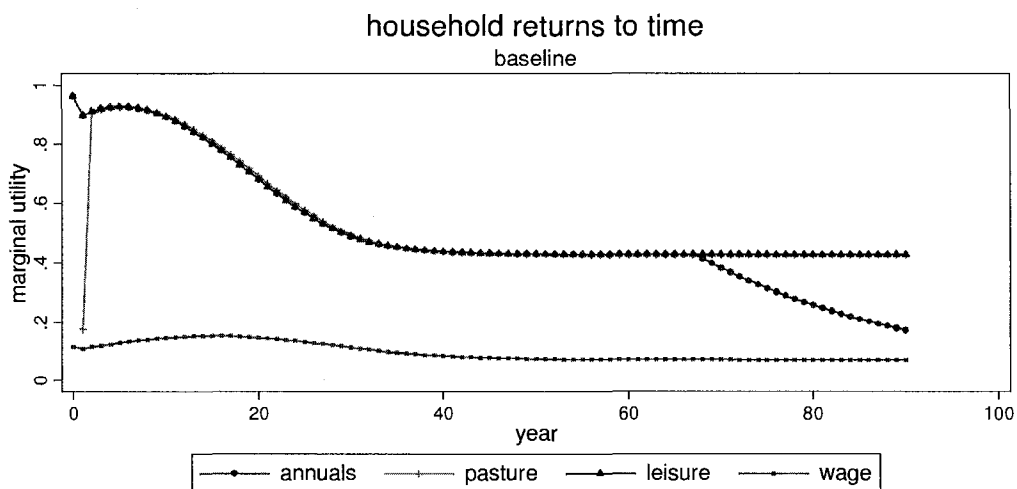
baseline

3.2.2 Household well-being – baseline

In the early years when the household seeks to expand pasture capacity and stave off a sharp decline in annuals production, it suffers declines in both leisure time and consumption, and (consequently) utility (Figure 32). The investment pays off in higher utility levels after pasture expansion gets underway. As pasture expands, first leisure, then consumption, reverses course and heads upwards.

Figure 32. Components of household utility (consumption and leisure) – baseline

household utility components
baseline
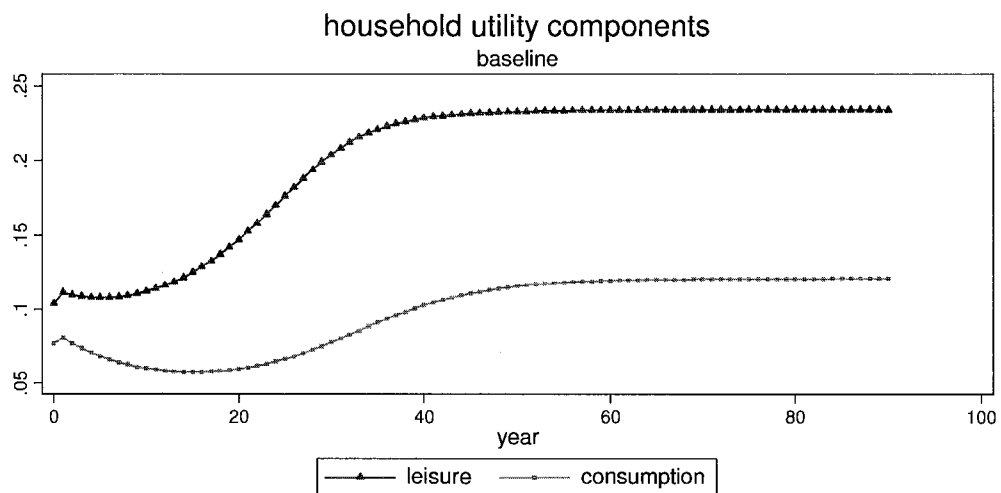
The period of declining consumption and increasing leisure (analytically shown to be feasible in Equation 52) is due to the lag between labor investment in pasture capacity and herd growth and increased cattle sales. Figure 33, showing proportional change in utility components from one year to the next, points up this period (roughly between $t=6$ and $t=15$). Soil degradation in annual cropping drives the gap between proportional change in leisure and consumption in the early phase (the gap disappeared in the pasture/wage simulations above, with no exogenous downward pressure on returns to the activity). The pasture endowment allows the household to boost leisure and consumption briefly before pasture expansion starts.

Figure 33. Proportional change in utility components – baseline

change in leisure & consumption
baseline
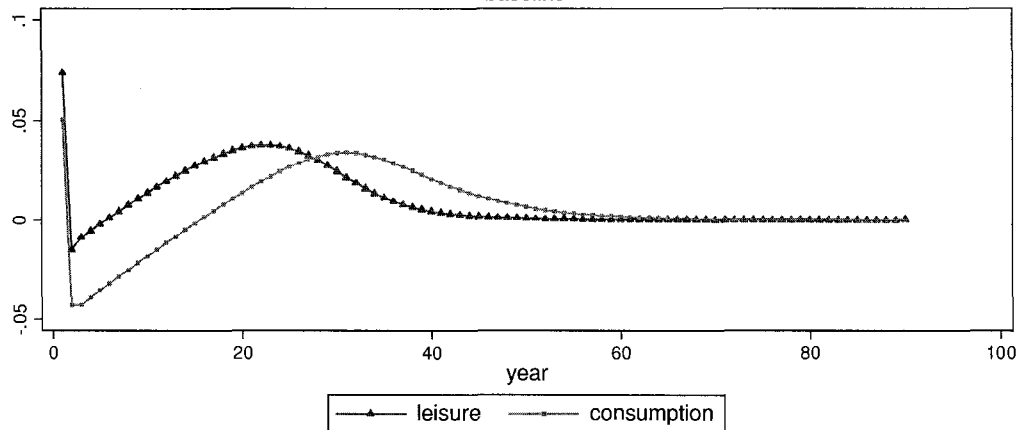
3.2.3 Herd and pasture management – baseline

Figure 34 depicts the other decisions about herd and pasture management that accompany labor's shift toward pasture. The farmer purchases cattle in the first two years to start the herd, then chooses to rely on natural growth adjusted via cattle sales to manage herd growth (rather than growing the herd through additional purchases). The result is a pasture stocked at close to 45% of capacity at $t=2$. Once the farmer starts to put labor into pasture, capacity expands faster than herd size, so stocking rate drops to close to 35% by $t=31$, as annuals production gets close to zero. At that point, pasture has hit roughly 75% of its steady state capacity.

Thereafter, herd growth only slightly outpaces pasture expansion, nudging stocking rate up towards its steady state level. The dynamics of herd growth allow an appreciable improvement in consumption levels in this last phase of pasture expansion with little additional labor shifted to pasture. With model assumptions about pasture maintenance and expansion, the farm grows close to its full spatial extent when pasture capacity nears its maximum, about 60 years after the start of the trajectory.

Figure 34. Household cattle and pasture management – baseline

Figure 34. Household cattle and pasture management – baseline
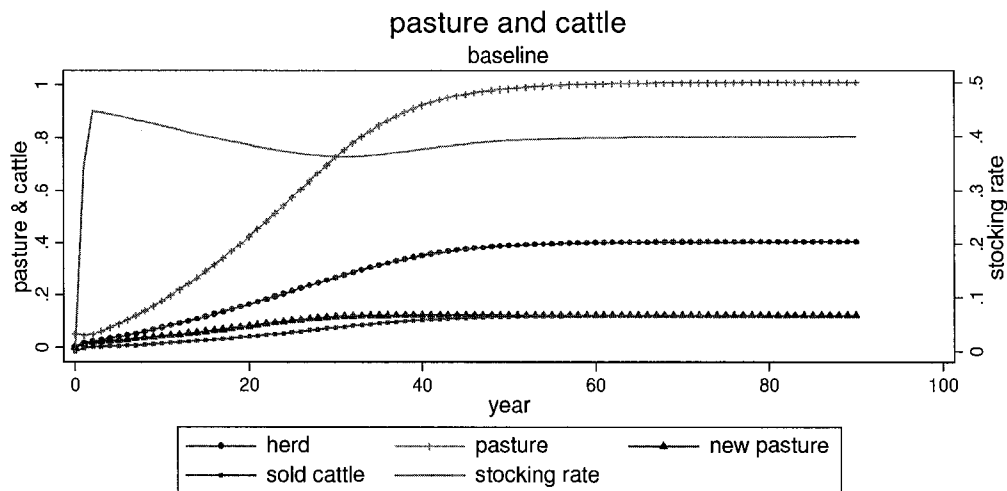
3.2.4 Farm shadow value, with and without the herd – baseline

Along with the pasture investment and herd growth, the value of the farm shifts

over the trajectory. Figure 35 depicts the trajectory of the farm's shadow value in utility

terms (the Hamiltonian from Equation 7). The figure splits out the contribution from

utility, and shows the shadow value of the farm without as well as with the herd.[91]

Figure 35. Shadow value of the farm, with and without the herd – baseline

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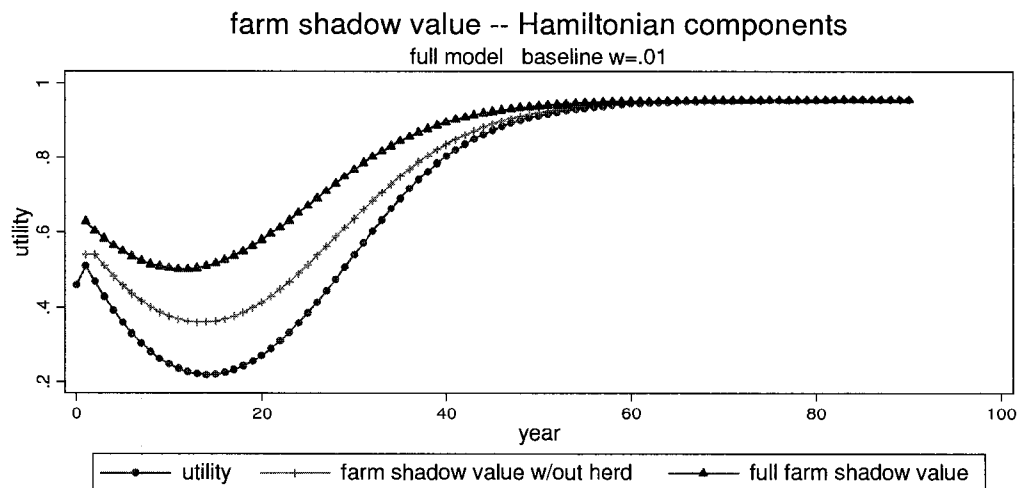
[91]The value without the herd is utility plus the change in pasture stock valued using that equation's costate φ_t; the value with the herd is the full Hamiltonian – the above value plus the change in herd size valued using the costate λ_t.

The trajectory shows a decline in value during the initial annuals phase when the soil is being degraded, mitigated by additions to the pasture and herd. Continued investment in pasture adds value to the farm, both directly and through herd growth and rising utility, climbing towards the steady state.

3.2.5 Baseline results in the context of simpler models

Baseline simulation dynamics mimic aspects of the pasture/wage labor simple model. The introduction of a second labor alternative opens the possibility for leisure to grow over time, and creates a labor transition during which pasture and another labor activity are undertaken. Returns to labor decline and overall utility improves in both models. For both models, the alternative labor phase involves distinct start-up and ramp-down periods on either side of a holding pattern period. Proportional rates of change in leisure and consumption as well as cattle stocking rates change during these sub-phases.

These broad similarities mask some differences in model behavior driven by the inexorable soil degradation in the baseline as against the constant wage of the pasture/wage labor model. In a bid to offset falling productivity, the baseline household allocates relatively more labor at first to the alternative – annuals – than in the pasture/wage model. The desire to stem soil degradation and build pasture translates into an early period of leisure declines, and generates the concave section in the returns to time graph, contrasted with the strictly convex returns to time graph for the pasture/wage labor model. This gives way to a push to abandon annuals more quickly, which in turn is offset by the effect of decreasing returns to labor in annuals production technology. As a result, instead of the gradual acceleration of labor shifting toward pasture seen in the pasture/wage labor model, in the baseline, as annuals decline the household first

accelerates the allocation of labor to pasture, then decelerates as the household moves closer to phasing out annuals altogether.

Baseline utility undergoes a reversal of fortune contrasting with the steady proportional increase in utility components in the pasture/wage labor. In the baseline, household leisure and consumption decline as the household attempts to compensate for degradation-induced productivity losses. In a second part of the annuals phase, the household increasingly abandons annuals, and leisure and consumption start to recoup their losses. In the baseline, proportional changes in leisure surpass those in consumption until the household nears its steady state pasture labor allocation, after gains in leisure have started to flag but gains in consumption have yet to peak. Instead, in the pasture/wage labor model, leisure and consumption undergo identical proportional changes until the end of the wage phase, when consumption's proportional gains rise while leisure's fall off.

Like a higher wage, more profitable annual cropping slows the transition to pasture while raising the cattle stocking rate early on. With sufficiently favorable annuals parameters, the early increase in stocking rate can actually overshoot the long-term ideal (as happens in the baseline).[92] In the baseline, stocking rate falls for much of the annuals phase (after the initial herd build up relative to pasture capacity), but in the wage/pasture model holds steady during the wage phase. For both models, stocking rate dips then resumes a rise towards its long run steady state once the labor alternatives to pasture largely disappear. In the baseline, since annuals decline smoothly towards zero, pasture

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[92]This overshoot cannot happen in the pasture/wage labor model because at the requisite wage the model shifts to a pro-wage labor steady state.

labor increases smoothly, contrasting with the surge in pasture labor that occurs at the wage labor phase-out in the pasture/wage labor model.

The utility and investment trends translate into a dip in the farm's shadow value in the baseline model accompanying the early decline of annuals, mitigated by the value of the herd and pasture, with the investment eventually reversing the trend and improving the farm's value. Wage work, by contrast, would slow the upward trend in farm value caused by pasture investment, but exhibit no such decline in farm shadow value.

The next chapter subjects the model to parameter changes and charts resulting changes in the optimal trajectory. It then presents a few policy simulations, extending lessons drawn from the sensitivity analysis.

Chapter 4 Sensitivity analysis and policy simulations

This chapter begins by presenting results of sensitivity analysis performed on the baseline model, separating out wage from non-wage parameter simulations. Wage parameter simulations receive special attention because of labor's importance in the model. A summing-up of the results of the sensitivity analysis argues for their policy relevance, emphasizing the importance of the model's dynamic nature in this regard.

The discussion of what model aspects make it amenable to explorations of policy issues continues in the ensuing section. In addition to dynamics, model strongsuits include flexibility – its structure can be fleshed out with more detail in several dimensions – and expandability – it can act as a building block for starting to model aggregations of households. This last approach holds promise as a contribution to the growing literature trying to bridge the gap between looking at frontier areas through the lens of a single household unable to access important markets, or of a completely market integrated unit. Examples of these model aspects are then presented in the form of policy simulations.

4.1 Sensitivity analysis

This section looks at how varying parameters in the full model affects the household trajectory. The sensitivity analysis summarizes how the trajectory shifts as parameters change in favor of one of the on-farm activities, or so as to alter household characteristics. It then focuses on how the wage rate affects the optimal trajectory, since this parameter targets in a direct and easily visible way the labor allocation decision at the heart of the investment decision.

4.1.1 Non-wage parameter simulations

A higher annuals price p_a, technical efficiency parameter A, or baseline soil fertility S_o affect the trajectory in the same way (since all enter the annuals production function in the same way). They stretch out the phase-out of annuals, delaying the onset of and prolonging the transition to pasture. The time allocation in the early phase mimics that seen in the no-investment model without exogenous income: the household maintains a set allocation between cropping and leisure. The trajectory starts with relatively high consumption, which then steadily falls due to soil degradation, even as leisure holds steady. A holding pattern in the pasture stocking rate also marks this period.[93] Once the pasture transition gets underway, leisure time is sacrificed early on, but then both leisure and consumption turn upwards (as in the baseline) as the investment's benefits begin to be felt. Using a doubling of the output price for the annual crop (p_a=2) as an example, Figures 36 to 39 illustrate these trends in time allocation, utility components and their proportional change, and pasture and cattle growth.

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[93]This is a byproduct of the initial endowment in pasture, which the household takes advantage of by buying cattle, and managing its growth/sell-off as the pasture degrades (overstocking the pasture relative to the long-run optimum).

Figure 36. Household time allocation, crop price doubled

household time allocation
full model pa=2
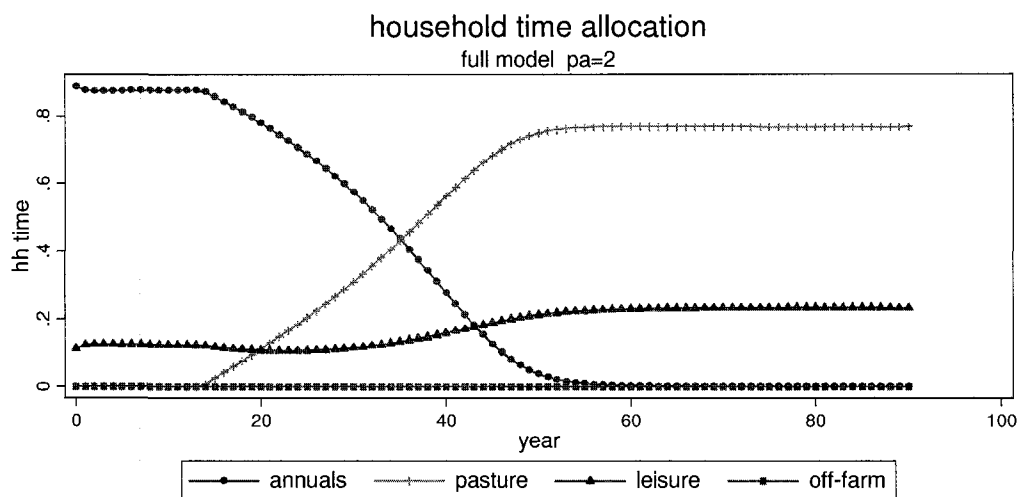
Figure 37. Household utility, crop price doubled

household utility components
full model pa=2
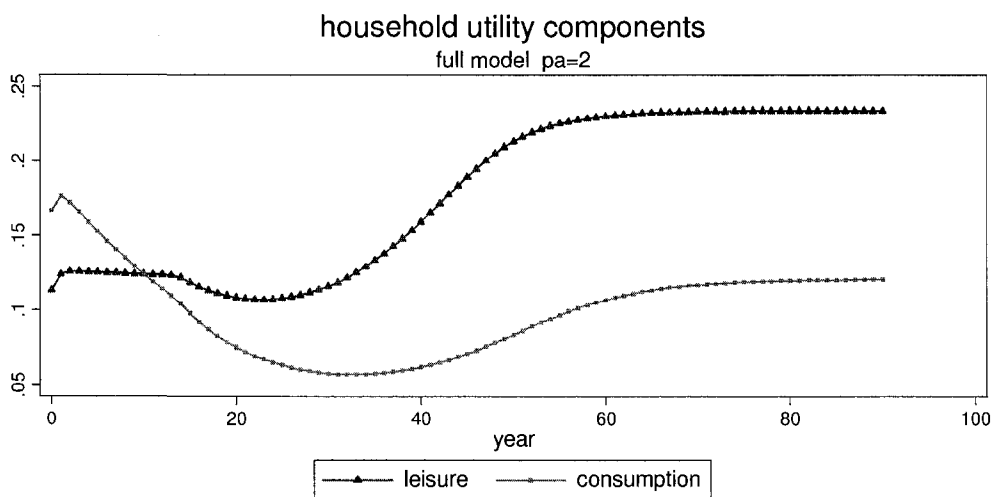
Figure 38. Household proportional change in utility, crop price doubled

proportional change in leisure & consumption
full model pa=2

leisure   consumption
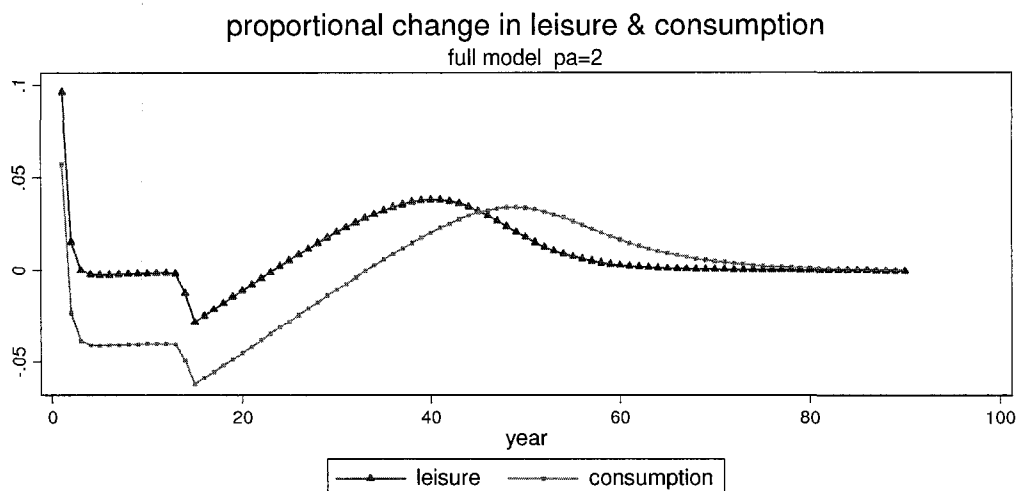
Figure 39. Pasture and cattle growth, crop price doubled

pasture and cattle
full model pa=2

herd   pasture   new pasture
sold cattle   stocking rate
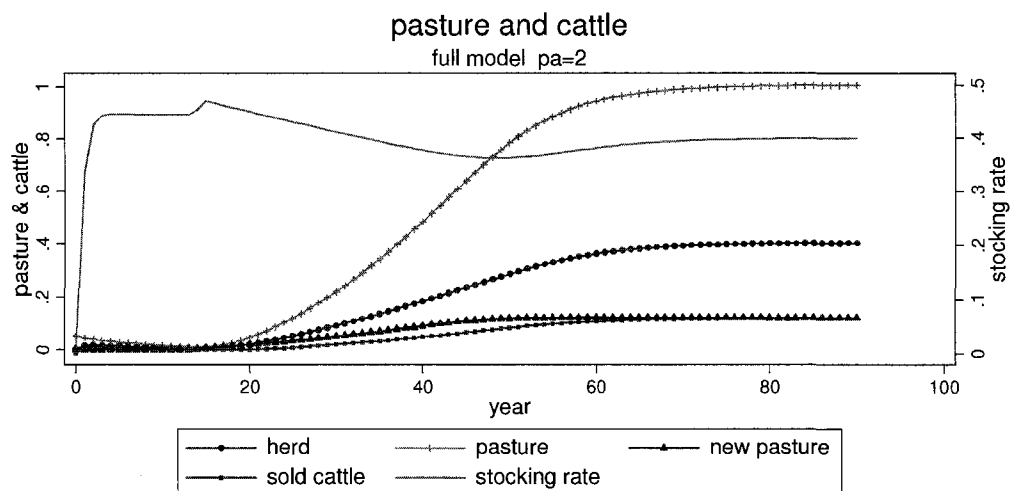
A lower soil degradation rate d likewise prolongs the transition to pasture, but by stretching it out rather than delaying its start. The closer annuals production technology gets to constant returns to scale (increasing ψ towards 1), the less crop labor's decline slows as it peters out. Decreasing returns mean less productivity early on, but mute the deleterious effect of soil degradation vis-à-vis the situation with constant returns.

Because annuals don't persist to the steady state, none of these changes affects steady state values.

Changing cattle/pasture-related parameters (higher price p_h, lower labor requirements to convert land into pasture γ, slower pasture degradation δ, improved herd growth rates r) unsurprisingly speeds the transition to pasture. This means consumption is boosted sooner (and overall relative to leisure) vis-à-vis the baseline and enough labor comes out of annuals to increase time in both pasture production and leisure earlier in the trajectory. Fast-growing pasture means the dip in stocking rate is more pronounced than in the baseline. Taking the case of doubling the output price for cattle (p_h=2) as an example, Figures 40 to 43 illustrate these trends in time allocation, utility components and their proportional change, and pasture and cattle growth.

Figure 40. Household time allocation, cattle price doubled
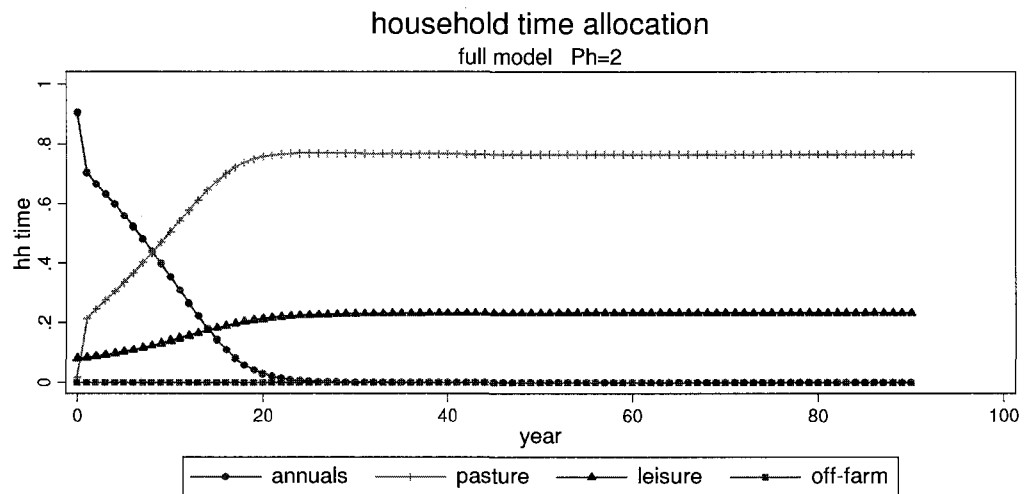
Figure 41. Household utility, cattle price doubled

household utility components
full model Ph=2
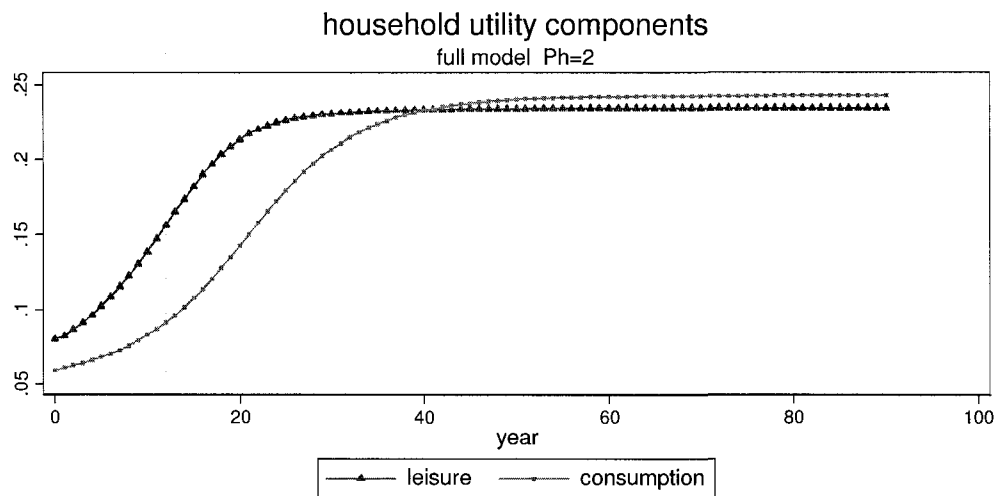
Figure 42. Household proportional change in leisure and consumption, cattle price doubled

proportional change in leisure & consumption
full model Ph=2
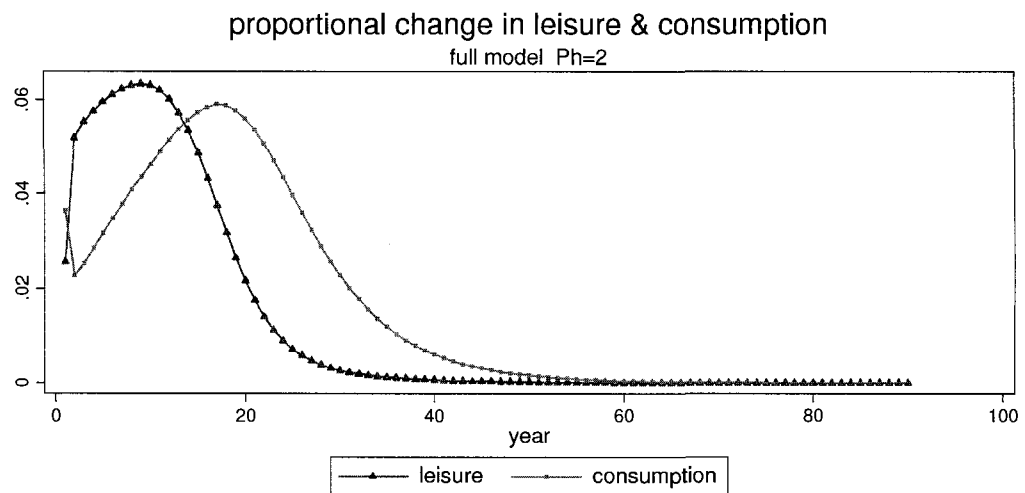
Figure 43. Pasture and cattle growth, cattle price doubled
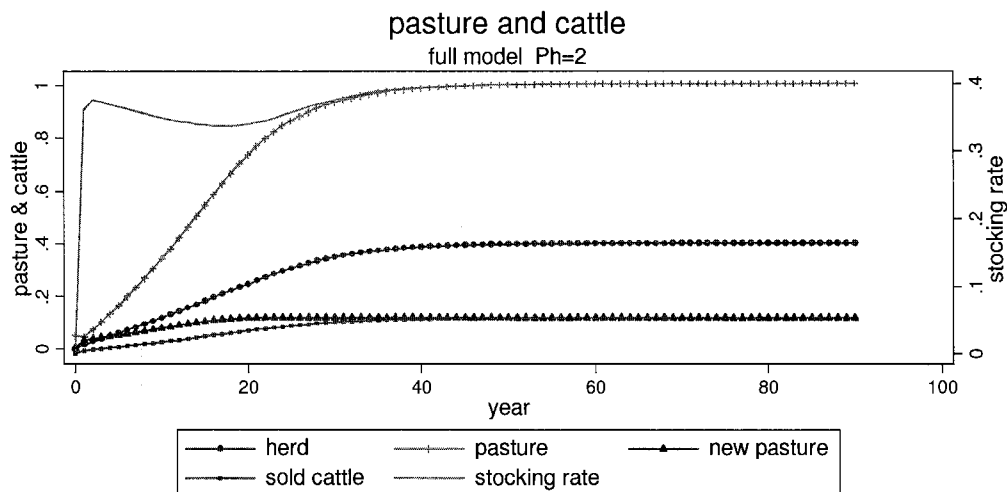
Two parameter changes – a higher herd growth parameter r and lower degradation rate δ – spark the biggest initial transfer of labor from annuals to jumpstart pasture because their effect amplifies over time. As shown in Chapter 2, neither output price nor efficiency of labor in converting land to pasture affects steady state labor allocation, except through exogenous income, if this is present. Of course, more efficient pasture conversion (a lower γ) means a given amount of labor results in higher carrying capacity. Slower pasture degradation results in more pasture but less pasture labor in the steady state. Higher herd growth rates prompt increases in both pasture labor and carrying capacity in the steady state, as well as a higher stocking rate.[94]

Adjusting household characteristics and preference parameters also had expected results. More exogenous income Y along the trajectory increases leisure time throughout but especially at the outset, and leads to a lower pasture carrying capacity in the steady state. It does not significantly alter the timeline of the transition, though. Increasing the

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[94]The steady state shifts in these simulations jibe with what the pro-pasture steady state labor solution showed in Chapter 2.

household's time endowment T also only slightly changes the timing of events. With the extra labor, the household starts pasture in the first year rather than the second. The lion's share of this time comes from annuals, but both annuals and leisure take up slightly less of household time in proportional terms than in the baseline. With more time allocated to all categories, this scenario leads to greater amounts of forest conversion and pasture carrying capacity. A lower household discount rate ρ hastens the transition to pasture, and leads to more carrying capacity in the steady state, and a higher stocking rate. Conversely, a higher discount rate delays the start of pasture and leads to lower carrying capacity and lower stocking rates in the steady state. A greater relative preference for leisure (higher β, lower α) unsurprisingly delays the transition to pasture and makes it a smaller event when it does happen. The household still draws down leisure time in the critical first years of pasture formation, though. Tenure insecurity, modeled as the farmer's expected value for sale of the farm being less than its full value ($LdMkt<1$), causes some disinvestment in pasture, starting slowly and accelerating in the final years of the trajectory. Less secure tenure speeds up the disinvestment process, so that the household may stop pasture conversion altogether before the sale. Insecure tenure, however, does not affect the pasture build-up.

4.1.2 Discussion of non-wage parameter simulations

The simulations underline the basic points that, relying on annual cropping only, the household is destined to see consumption levels fall; in order to improve consumption levels, the household must invest in pasture, and the timing of the upturn in consumption is closely tied to the timing of pasture expansion/herd growth. Which scenario is optimal for the household – greater consumption today or tomorrow – depends on which labor

activity is more productive, but also on the discount rate and how much the household would prefer to consume goods vs. leisure time.

More specifically, a strong household preference for leisure, relatively favorable production or price parameters for annuals vis-à-vis pasture, and a high discount rate all yielded a similar time allocation pattern: an initial phase where the time allocation to annual cropping vs. leisure stays the same each year, despite soil degradation. Sometimes touted as reflective of 'satisficing' or 'subsistence' behavior (basically an inherent preference for leisure over consumption beyond basic needs level), such a trajectory is also a plausible path for a poor household, with a strong time preference for the present or whose conditions mean slower herd growth or more rapid pasture degradation.

Even with a strong relative preference for consumption, taking more leisure time is shown to be the optimal course early in the baseline's trajectory: at this point there are no more attractive labor-using alternatives – soil degradation makes annual cropping less attractive, and pasture expansion is tied to dynamics of herd growth, the discount rate, and pasture degradation rates. That increasing exogenous income Y results in the household taking more leisure time, rather than investing more in pasture, underscores the point: without more labor (that is, the ability to hire labor) to generate the pasture, any hastened investment would go for nought.

The patterns of pasture/herd growth are also of interest. Households make the transition by first expanding pasture faster than natural herd growth, then letting cattle numbers catch up as the herd grows larger. Where parameters delay the start of this transition (e.g., the pro-annuals scenarios), the household gets enough cattle to take

advantage of its pasture endowment, then maintains a near constant stocking rate as the pasture degrades (by selling off the requisite herd growth) until the time comes for the pasture transition. In both the baseline and the doubled annual output price ($p_a=2$) scenario, this leads to a stocking rate early on that overshoots the long-run ideal. With a longer annuals period, moreover, this relatively high stocking rate is sustained for some time.

4.1.3 Wage simulations

The nature of the annuals/wage trade-off and the pasture-to-wage steady state threshold delimits a range of exogenous wages that, given other baseline parameters, yields three phases, possibly overlapping, during the trajectory – a wage phase, a pasture phase, and an annuals phase – a three-activity wage range.

4.1.3.1 Wage ranges described

The lowest viable wage to produce a wage phase, the highest wage to allow a pasture phase, and the wage to choke off annuals production altogether are difficult to pinpoint *a priori* (that is, without solving the dynamic optimization problem), since the competing activities' returns to time all include model variables. The baseline scenario numerical results, however, can provide insight here.[95]

Figure 44 reprises the returns to time information for the baseline, but also depicts the shadow wage, or "wage equivalent," for each time period – the wage at which returns

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[95]This section uses a higher level of precision in reporting results to highlight switching points in the numerical model.

to off-farm labor would match returns to leisure for that time period.[96] As long as

leisure's proportional change outpaces consumption's (until t=27 in the baseline), wage

equivalence falls.[97] Thereafter, these relative positions switch, and the shadow wage

rises. At this point the household starts to really reap the benefit of its pasture

investment. The dip, then rise in wage equivalence highlights the potential for a constant

wage to improve the household's situation. It provides an opportunity to remove the dip

in returns to time, as long as the cost in pasture investment foregone is not too great.

Figure 44. Household wage equivalence – baseline

household wage equivalence & returns to time
full model baseline w=.01
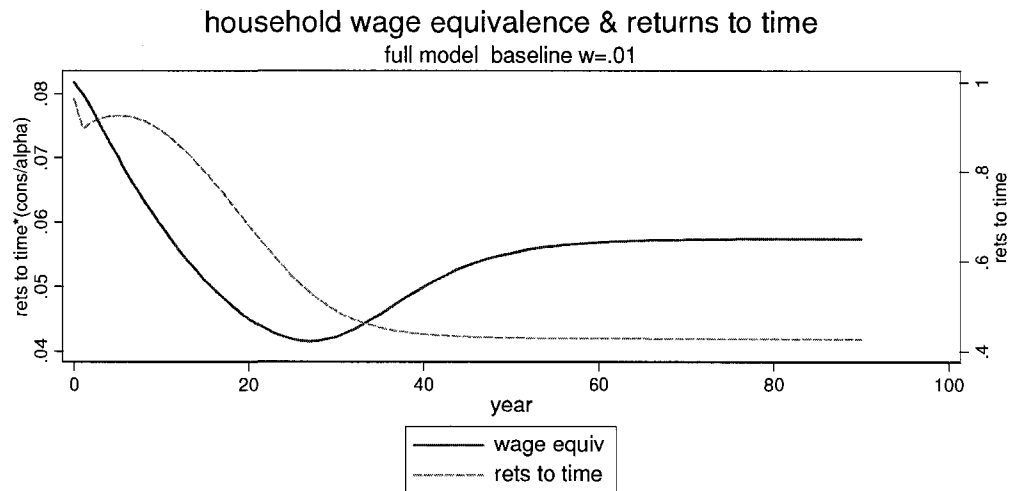
The lowest wage equivalent during the baseline, w_t=0.04152 (coming at t=27),

provides a lower bound for a wage phase on the way to a pro-pasture labor steady state.

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[96]More precisely, this is the shadow wage resulting from the equality of marginal returns to labor,

$$w_t + \mu_{LOt} \frac{C_t}{\alpha} = \frac{\beta C_t}{\alpha l_t}.$$

[97]To see why, consider the discrete time situation, where the change in wage equivalence

$$\Delta we_t = \frac{\beta}{\alpha}\left[\frac{C_t + \Delta C_t}{l_t + \Delta l_t} - \frac{C_t}{l_t}\right].$$ The sign of the bracketed expression determines the direction of the

shadow wage change. It is negative when $\frac{\Delta C_t}{C_t} < \frac{\Delta l_t}{l_t}$, or when the proportional change in consumption

falls shy of that in leisure.

The wage equivalent for the first year that labor goes into pasture expansion, $w_t=0.08001$ at $t=1$, provides an upper bound for a wage that would allow a pasture phase on the way to a pro-wage labor steady state. Because the baseline includes annuals in the model's first year at a higher wage equivalent, this implies that any wage that allows a pasture phase should also allow an annuals phase – establishing a 'three-activity' wage range.

Numerical simulations show the lowest viable wage falls in the expected region: imposing an exogenous wage of $w=0.415$, lower than the lowest baseline wage equivalent of 0.412, does not result in hired labor, but with $w=0.416$, above the baseline wage equivalent threshold, the household chooses one year of off-farm work, at $t=27$. The highest wage to yield a pasture phase falls between $w=0.073$ and $w=0.074$. The former yields a result with one year of pasture labor in $t=3$, and the latter no pasture phase. At higher wages, simulations reflect the sequential static case.[98]

4.1.3.2 Simulations in the 'three-activity' wage range

This section presents three wage scenario variations on the baseline within the wage range that prompts phases of annuals, wage labor, and pasture labor in order to give a qualitative sense for how the optimal trajectory changes as the wage rises. The first two wages fall in the range that lead to a pro-pasture labor steady state ($w=0.045$ and $w=0.05$, both fitting the pro-pasture labor steady state requirement of $w<0.0574$). The third wage ($w=0.06$; i.e., $w>0.0574$, but not by much) allows a pasture phase along the trajectory to a pro-wage labor steady state.

Figure 45, depicting wage equivalence paths for all wage simulations, shows the simulations differ only slightly in wage equivalence early in the trajectory. Then off-

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[98]Diminishing returns to annuals technology means it takes a much higher wage to choke off annuals altogether. At $w=0.21$ annuals labor starts out below 0.01 of household time.

farm work starts, forestalling a further drop in wage equivalence. Flat wage equivalence marks the period of off-farm work. The higher the wage, the longer is this period and the greater is the delay in the subsequent rebound of wage equivalence.[99]

Figure 45. Household wage equivalence as exogenous wage increases

household wage equivalence
wage simulations compared
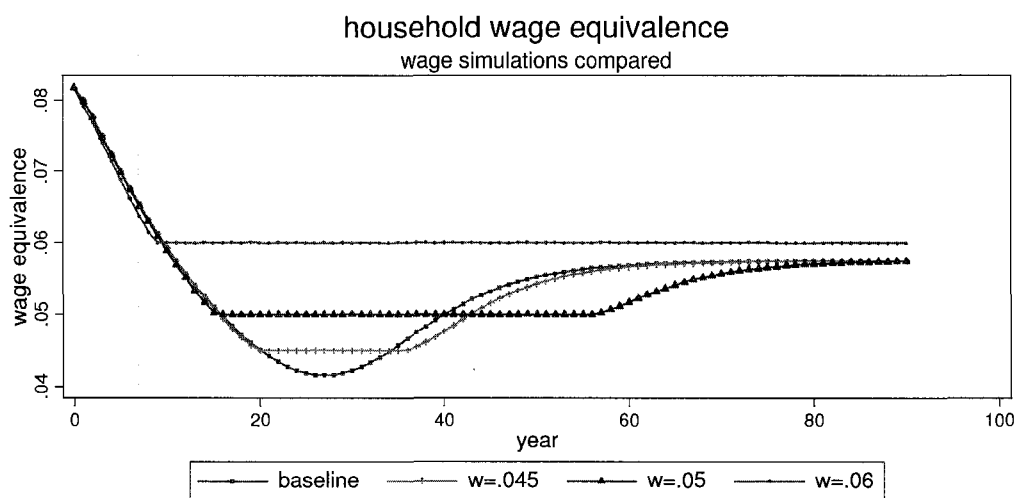
The time allocation graphs in Figures 46 to 48 tell the main story: a higher wage hastens the annuals phase-out and delays the transition to pasture. The household both prepares for and comes out of the wage phase with a one year upturn in pasture labor allocation.[100] The proportional decline in annuals during the wage phase is constant and matches the theoretical result from the previous chapter.[101]

[99]The rebound is the hallmark of pasture expansion and its accompanying rise in consumption and leisure.
[100]These resemble the abrupt adjustments seen in the pasture/wage model (without annuals), and again are likely due to the discrete time nature of the model.

[101] That is, $\dfrac{\dot{LF}_t}{LF_t} = \dfrac{d}{(1-\psi)}$.

Figure 46. Household time allocation, simulation with w=.045

household time allocation
full model   w=.045
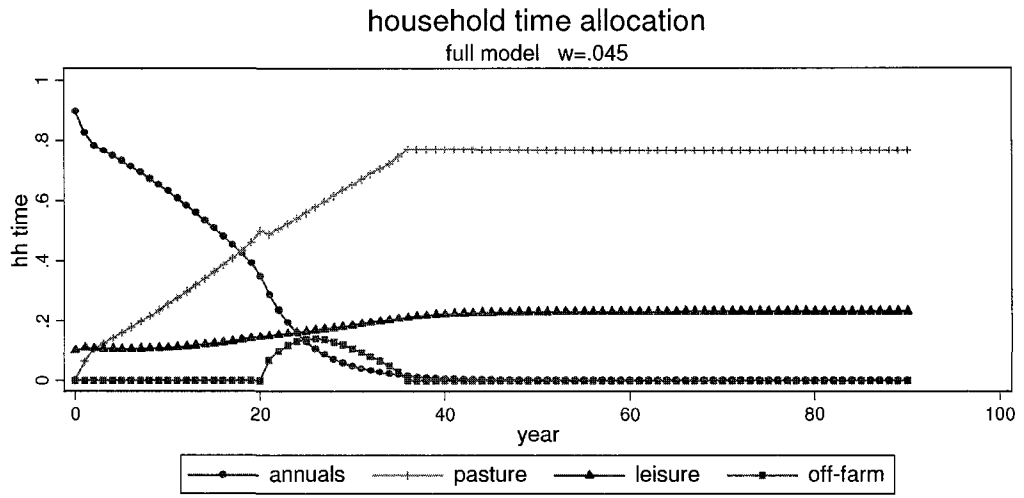
Figure 47. Household time allocation, simulation with w=.05

household time allocation
full model   w=.05
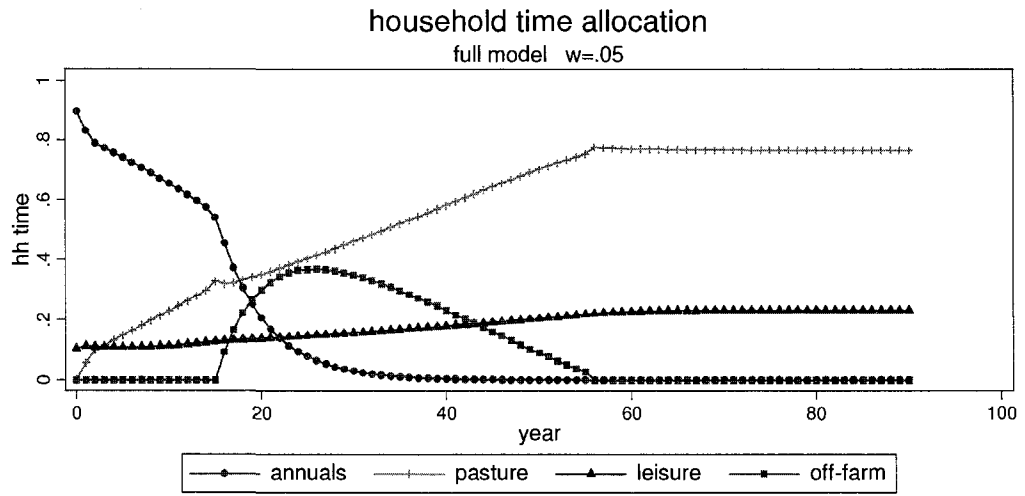
Figure 48. Household time allocation, simulation with w=.06

household time allocation
full model w=.06
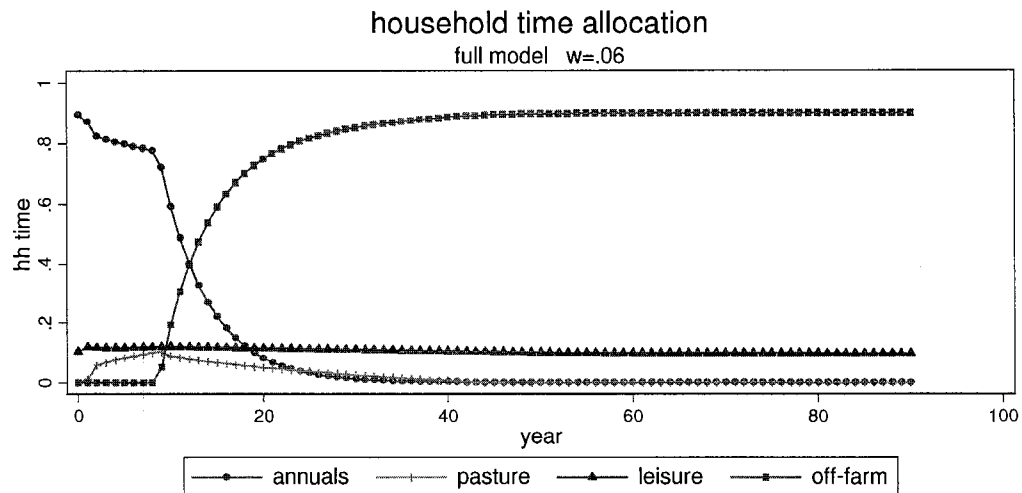
Figures 49 to 51 show pasture expanding more gradually as the wage rises, and expanding slightly before shrinking at the wage just beyond the pro-wage labor steady state threshold. The wage phase smoothes out pasture growth, making it nearly linear during this phase, and with a lower slope the higher the wage. Like the baseline model, all these wage simulations are characterized by an early period in which the stocking rate overshoots its long-term ideal, then starts to fall, as pasture expansions outpaces herd growth. But the decline of the stocking rate is cut short by the advent of off-farm work; instead, constant stocking rate remains constant during the wage labor phase. The higher the wage, the closer this constant stocking rate comes to the long-term ideal.

Figure 49. Pasture and cattle, simulation with *w*=.045

pasture and cattle
full model w=.045
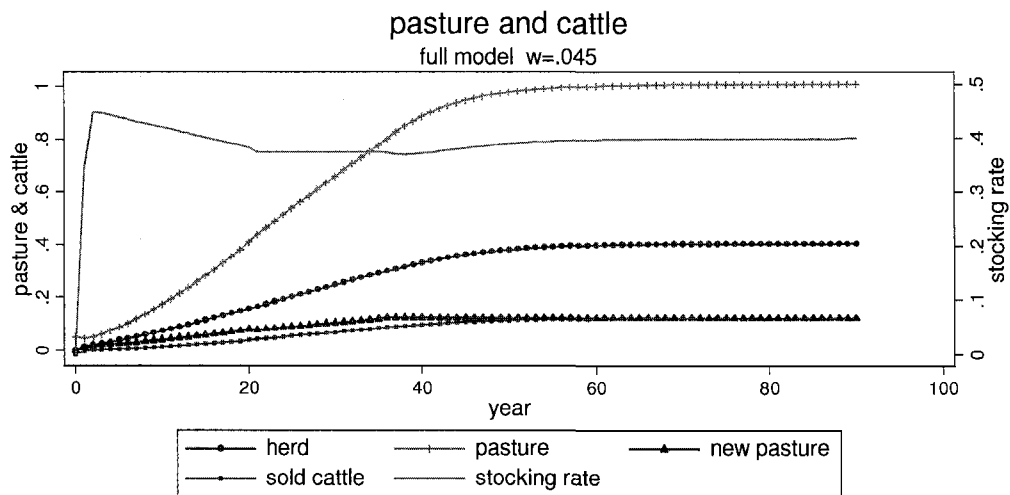
Figure 50. Pasture and cattle, simulation with *w*=.05

pasture and cattle
full model w=.05
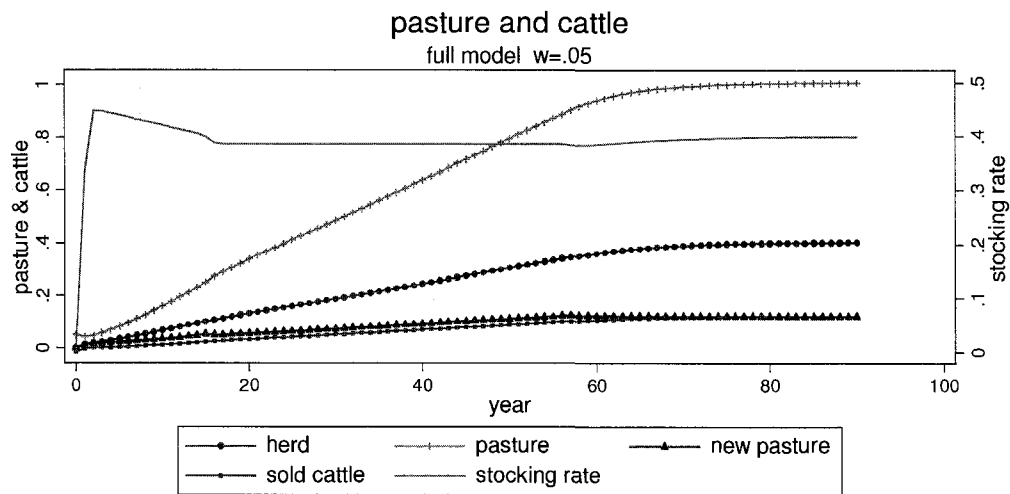
Figure 51. Pasture and cattle, simulation with *w*=.06
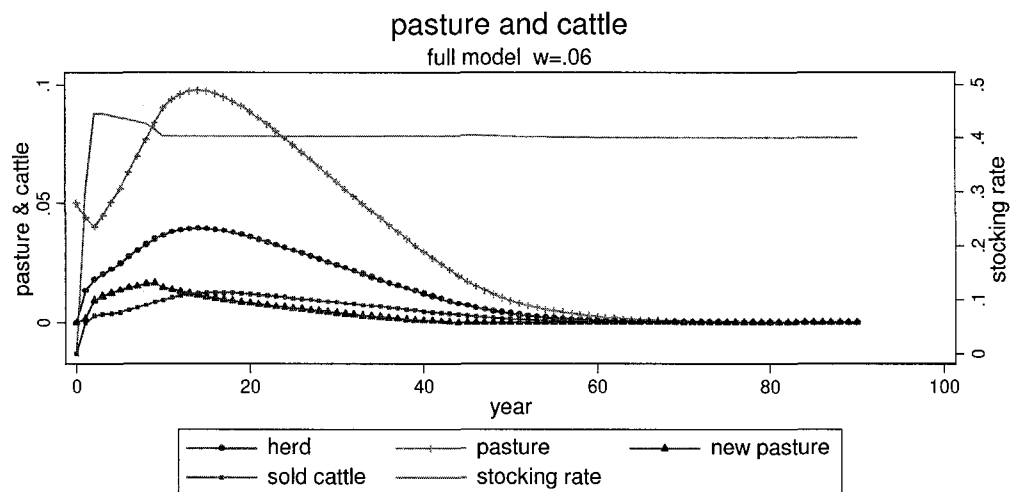
The period of equal proportional change in leisure and consumption, discussed in

Chapter 3 and seen in the pasture/wage model simulation, reappears here during the wage

phase (as Figure 52 illustrates for *w*=0.045). It essentially stretches out the crossover of

proportional change in consumption overtaking that of leisure seen in the baseline.

Figure 52. Proportional change in utility components, simulation with *w*=.045
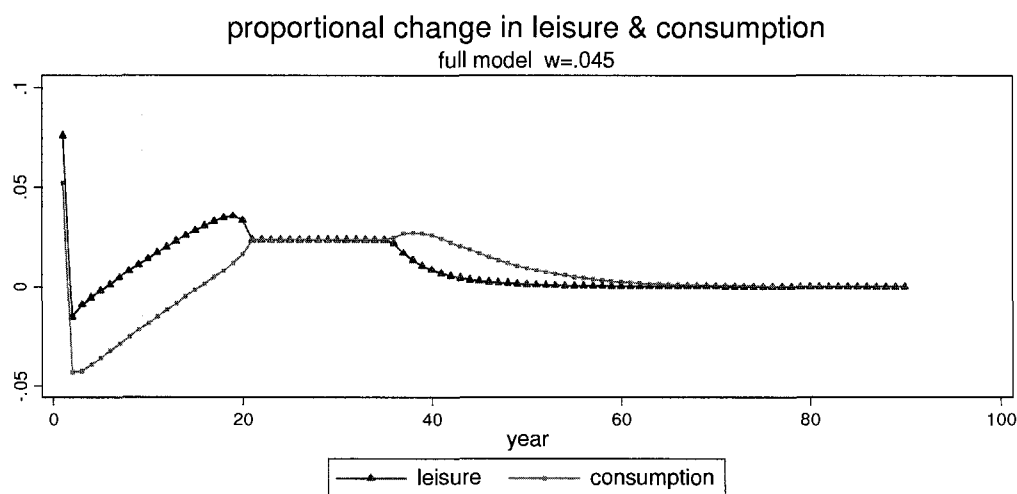
This holding pattern is due to slower investment in pasture, which dampens the

consequent improvement in consumption and leisure over time (Figures 53 to 55). At the

highest wage used here in the pro-wage labor steady state range, leisure and consumption

fail to turn upward altogether. Clearly, overall utility gains from choosing off-farm work

come from benefits early in the trajectory.

Figure 53. Household utility components, simulation with w=.045

household utility components
full model w=.045
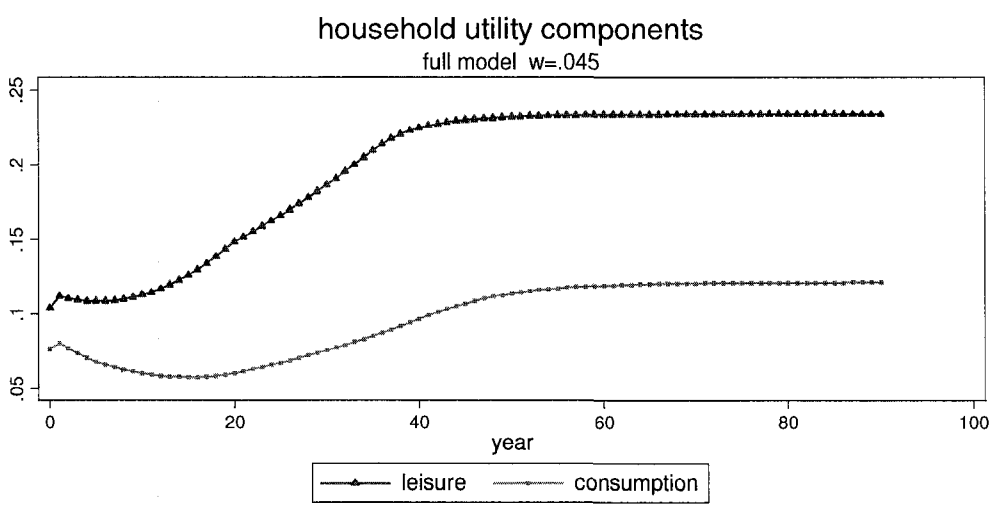
Figure 54. Household utility components, simulation with w=.05

household utility components
full model w=.05
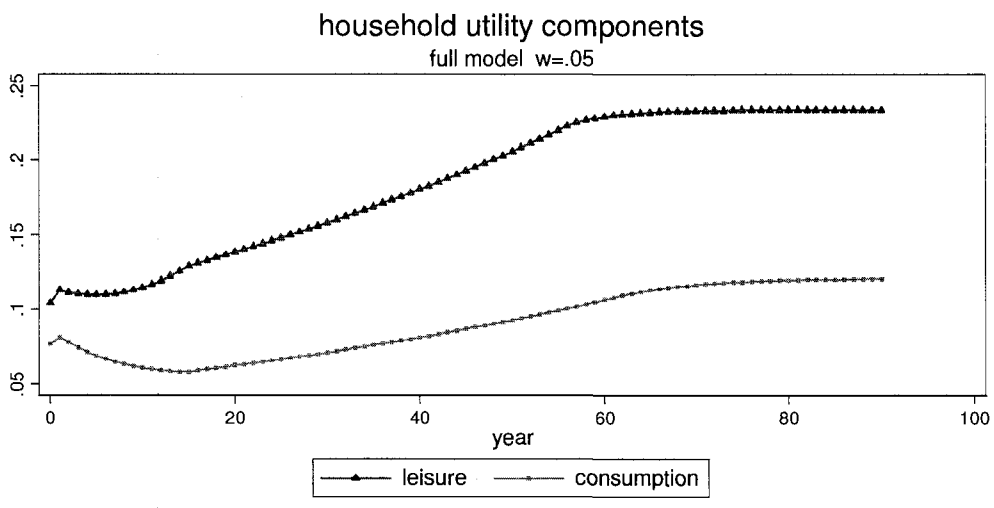
Figure 55. Household utility components, simulation with w=.06

household utility components
full model w=.06
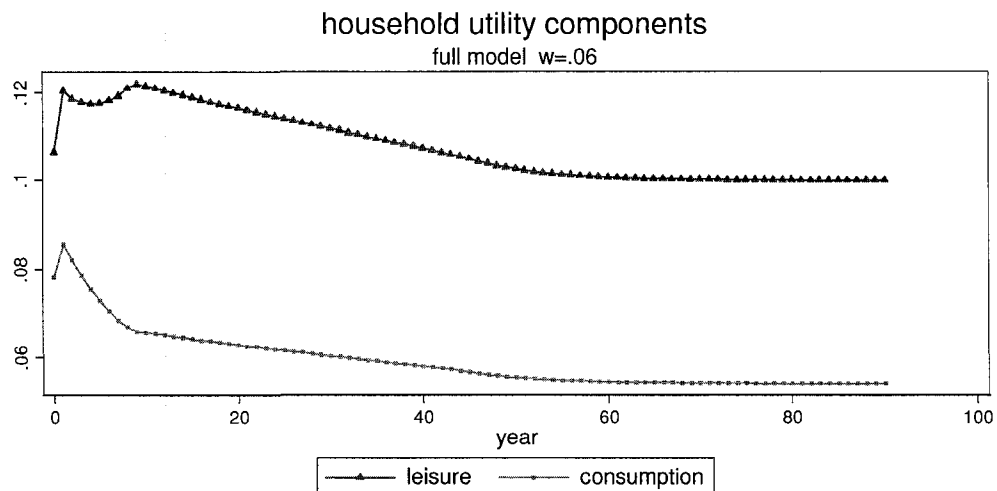
4.1.4 Discussion of wage simulations

Altering the wage affects the timing of the off-farm work phase, and significantly influences the trajectory of pasture expansion after off-farm work begins. But changing the wage also influences the initial part of the trajectory. The magnitude of effects from the off-farm work phase is more clearly seen by overlaying trajectories for different wage scenarios.

Figures 56 and 57 depict the components of utility – consumption and leisure – across the wage simulations. A higher wage translates into more consumption and leisure in the early part of the trajectory, even before the onset of the off-farm work phase.

Figure 56. Household consumption across wage simulations

household consumption
wage simulations compared
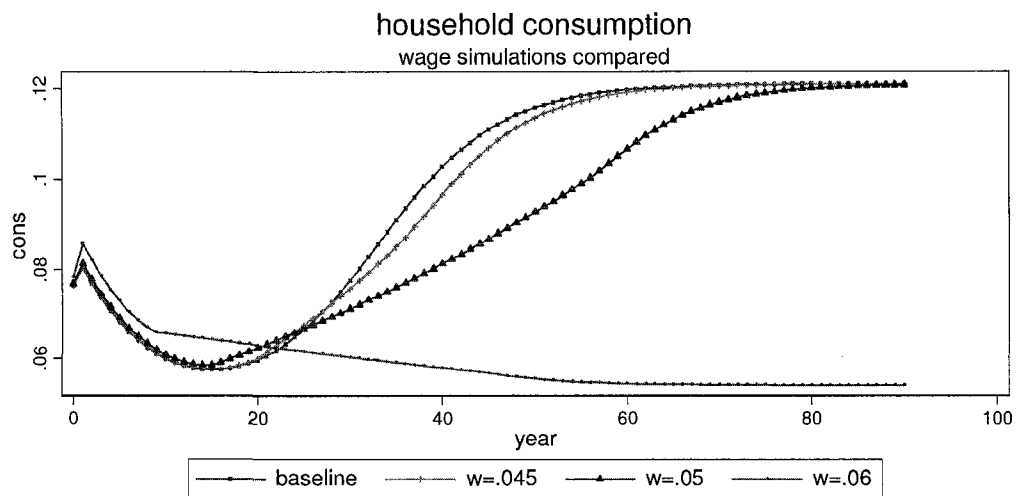
Figure 57. Household leisure across wage simulations

household leisure
wage simulations compared
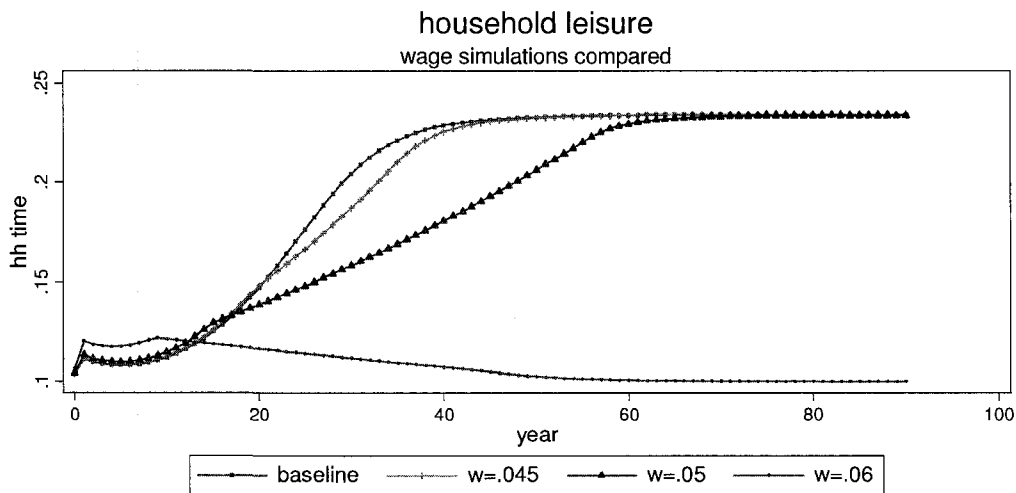
Comparing the rest of the household's time allocation across simulations (Figures 58 to 60 for crop labor, pasture labor, and off-farm work respectively) shows that the household's early consumption gains with a higher wage come from increasing its cropping labor at the expense of pasture before the off-farm work phase starts.

Figure 58. Household cropping labor across wage simulations

household ag labor
wage simulations compared
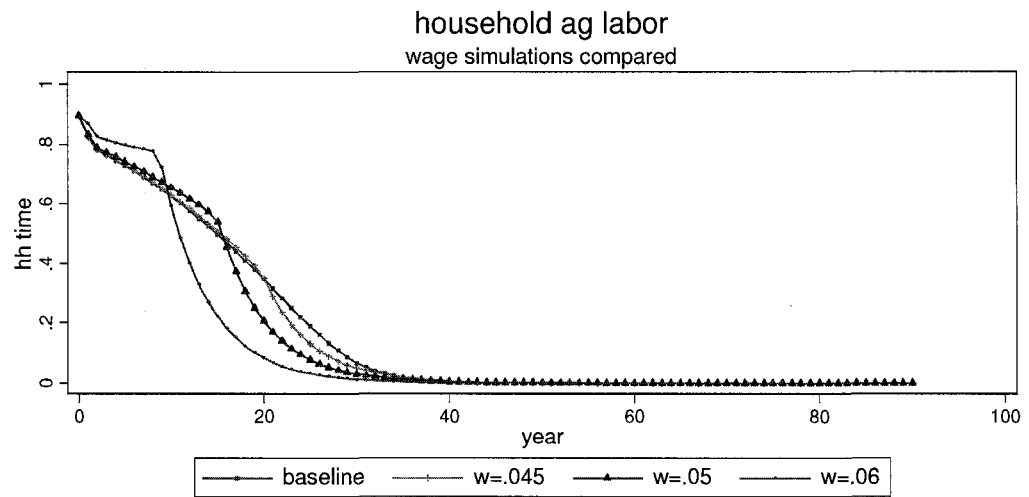
Figure 59. Household pasture labor across wage simulations

household pasture labor
wage simulations compared
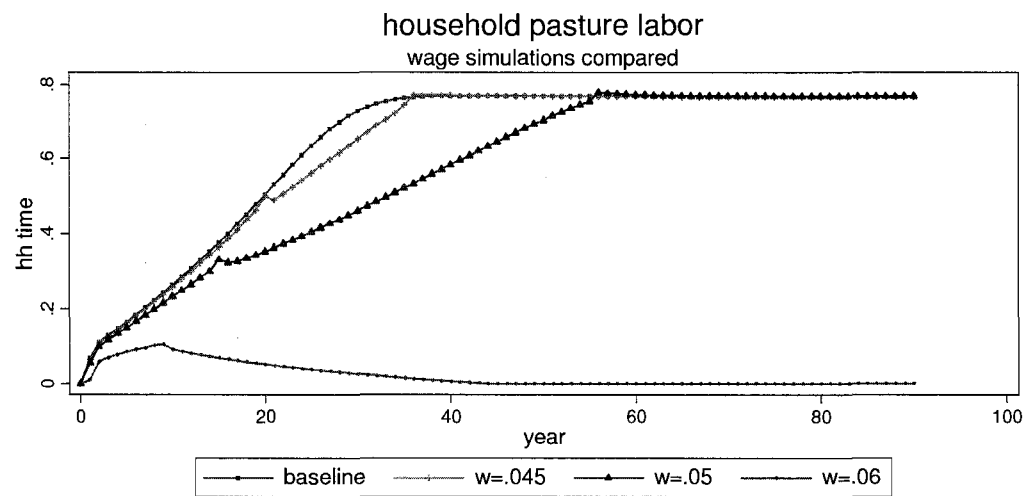
Figure 60. Household off-farm labor across wage simulations
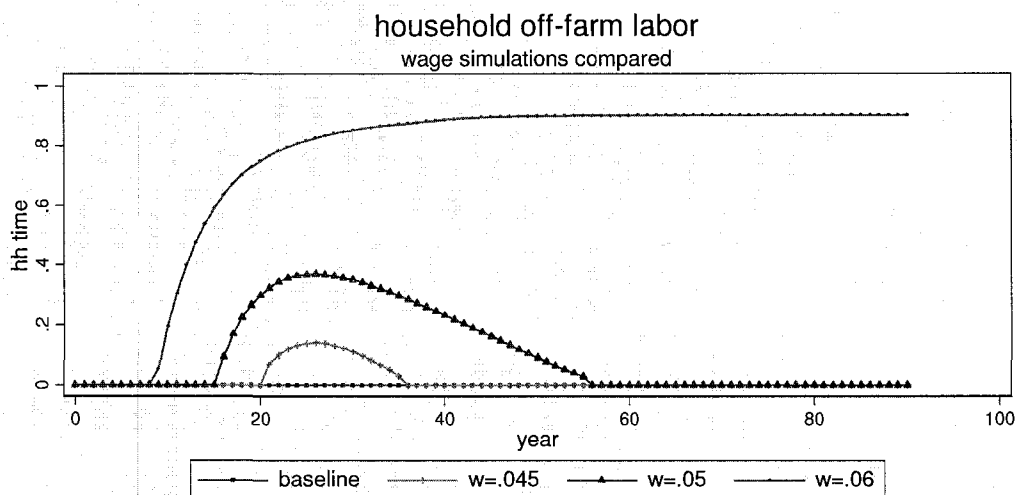
In sum, as in the pro-annuals simulations above, raising the wage shifts the trajectory towards more consumption and leisure earlier in the trajectory to offset the slower gains in consumption and leisure later due to the more gradual transition to pasture. Relatively small wage increases show ever larger deviations from the baseline's consumption and leisure trajectory. This is especially true after off-farm work begins as the cumulative effects of foregone investment make themselves felt. Even before off-farm work begins, the household adjusts to the off-farm work period by re-allocating labor away from pasture and towards both annuals and leisure, to boost nominal utility in early years when the discount rate's effect is weaker.

4.1.5 Lessons from sensitivity analysis

The sensitivity analysis showed how changing particular parameters shifts the balance among the three labor activities and leisure time. This section sums up some results relevant for policy questions, underscoring the importance of the model's dynamic nature in this regard.

The model's three labor alternatives illustrate more general categories of productive activity. Off-farm work at a constant wage gives a steady return to household labor. As an investment, pasture and herd expansion show increasing returns to labor over time. Because of soil degradation, returns to household labor tend to fall under annual cropping.

The sensitivity analysis results emphasize the basic point that the hope for rising consumption lies solely with the investment activity. A trajectory dominated by annuals is one whose consumption is dropping. Wage labor heralds a period of steady consumption. Deferring pasture investment postpones the point at which the household ceases to consume less year after year, and starts to consume more.

A high household time preference rate pushes the optimum towards a trajectory that frontloads consumption, investing more heavily in annuals despite soil degradation. This defers investment in pasture, and ensures early consumption gains are shortlived. Eventually, pasture investment leads again to rising consumption. Other parameters, such as favorable annuals parameters and high wages, have similar results.

Because the household must rely on family labor alone in the absence of a functioning labor market, leisure preferences also enter the household's time allocation calculus along its trajectory. Even with a strong preference for consumption, the household increases leisure time when no attractive labor-using activity is available. The possibility of multiple labor-using activities, moreover, allows for consumption to improve along with leisure. This happens when the trajectory involves a viable wage on the way to a pasture steady state. What was early pasture labor in the baseline gets allocated to annuals and leisure prior to the wage phase in the wage scenarios, resulting in

higher upfront consumption and leisure. Similarly, the pasture investment itself allows for both utility components to increase in tandem. With parameters favorable to annual cropping, however, consumption levels trend downward as leisure time holds steady in the pre-pasture phase.

Even the rudimentary sensitivity analysis simulations presented here have demonstrated the model's potential for placing observable land-use patterns of policy interest in the context of an economically rational household dynamic optimization trajectory. For example, a lengthened, stable annuals phase with overstocked pasture (vis-à-vis long-term ideals, not carrying capacity) appear with a high discount rate or parameters relatively favorable to annuals, plausible conditions for poor farmers unable to break into pasture investment.

Pasture expansion exhibits different patterns depending on what other labor activities are attractive. Under annuals, the household first accelerates then decelerates the pace of labor re-allocation toward pasture. Under wage labor, the labor re-allocation to pasture linearizes and slows. Farmers prepare for a period of lower pasture growth with, ironically, a brief upsurge in pasture labor. Parameters such as fast herd growth, low discount rate, improved labor efficiency for pasture transition, reduced pasture degradation rates, or higher cattle prices – all possible hallmarks of successful ranchers – speed the transition to pasture. The first two of these (herd growth r and discount rate ρ) also raise the wage needed to draw labor into permanent off-farm work.

Changes in stocking rate can indicate something about a household's overall trajectory. A rising stocking rate can suggest the household is nearing its steady state pasture capacity, whereas a declining stocking rate could signal the household is in its

fastest pasture expansion phase, and a stocking rate holding steady with changing pasture expansion points to labor being siphoned off to another use.

The results showcase the model's dynamic nature as a strength for looking at policy issues. It opens up a more nuanced exploration of specific land use behaviors as part of a longer term investment plan, quite different from what would emerge assuming a myopic household that optimizes period by period. The next section discusses this aspect of the model as well as others that make it of potential use for investigating policy issues, and presents some policy simulations to illustrate these aspects.

4.2 Policy simulations

The model's potential for shedding light on policy questions stems from its relatively simple structure. A simple structure makes time trade-offs and their implications more transparent. It leaves plenty of room for fleshing out detail in the form of additional activities (or technology levels within the activities), finer time-steps, or additional constraints, and provides a context for interpreting more complicated trade-offs as features are added. At the same time, it has the potential to be expanded in modular fashion to examine questions of inter-household interaction, again supplying a context for understanding how this dimension alters on-farm timepaths.

This section provides some discussion and presents examples for each of these model features: dynamics, flexibility on the detail with which the model characterizes the household's situation, and expandability beyond the single household. Each has its own subsection. Taken together, they suggest how the model might start to explore some dynamic aggregation issues – the complicated feedbacks between a dynamically optimizing farm frontier household and its often rapidly changing context.

4.2.1 Dynamics – discussion and examples

The sensitivity analysis touched on the model's primary strength for looking at policy questions, namely its dynamics. More concretely, by incorporating the region's predominant land use – cattle production – as an investment choice, the model sheds light on not only the decision to expand pasture, but also how other production activities influence the pasture transition through the scarce factor labor. As already seen, this perspective could be useful for interpreting empirical patterns on the ground, lending structure (and more specific hypotheses to test) to, for example, econometric analyses that subsume all dynamics under one or a handful of 'time counter' variables.

In addition, the model can easily incorporate non-autonomous timepaths for its parameters, and thus show how the farmer would act with expectations of changing parameters (which could be compared to, e.g., a myopic scenario where a change catches the farmers by surprise).

First, we look at a few scenarios that directly relate to the local labor market, in keeping with our focus on this scarce factor.[102] We examine the effects of a discrete wage hike during the trajectory (that might be associated with, e.g., a known infrastructure improvement that opens up additional off-farm opportunities), a gradual, continuous improvement in the wage rate (as might be anticipated with steady development of the frontier), and household demographics that change over time (to model what could happen with an aging rural sector).

Finally, we explore how changing relative output prices over time alter optimal timepaths. Scenarios are chosen to shed some light on how the expected development of

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[102]As is the case for all policy simulations presented, these scenarios are meant to be illustrative, not exhaustive.

these markets from settlement toward a steady state in relative prices accompanying maturation of the area could affect land use and deforestation. This provides a somewhat crude proxy for an approach that would fully endogenize early settler participation in output as well as input markets, and thus capture additional aspects of the critical establishment phase.

4.2.1.1 Discrete increase in the wage rate (w=0.05 at t=25)

In this simulation, the farmer can anticipate a rise in wage starting at a discrete point in time. This reflects one aspect of, for example, a planned infrastructure improvement.[103] The wage changes from its baseline level of w=.01 in t=0 to w=.05 in t=25. The baseline wage equivalence graph from the previous chapter (Figure 45) again indicates the minimum wage needed to affect the baseline run, but now the timepath matters, not just the trajectory's overall minimum. Wages above the overall minimum have no effect if they come after that minimum and too late in the trajectory to surpass the baseline's (then rising) wage equivalent. If the discrete change occurs early enough, it will produce the same result as a constant wage scenario.[104]

Figure 61 shows time allocation for this scenario. Given that the discrete change comes several years after the household would start to work under a constant w=0.05 wage, it's no surprise that the household takes advantage of the new wage the first year that it is available. As before, the household prepares for its off-farm work stint by devoting more labor to pasture expansion right before the wage change. Perhaps less

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[103]We leave aside for now the other price changes that would ensue, so assume the exogenous wage-changing event leaves unaffected the relative price of the consumer good, cattle, and the annual crop. We take up the question of changing relative output prices again later in this section.

[104]For example, with a constant wage of w=.05, off-farm work begins at t=16, so a discrete change before that time will yield the same trajectory as the constant wage.

intuitive is the way the household surges into the new activity, using more than half of its time working off farm in the first year. Commensurate with this is a larger surge in pasture labor before, and a bigger dip in pasture labor simultaneous with, this early foray into the new activity.

Figure 61. Household time allocation – discrete wage change to *w*=.05 in *t*=25

household time allocation
full model   w=.01 @ t0 to w=.05 @ t25
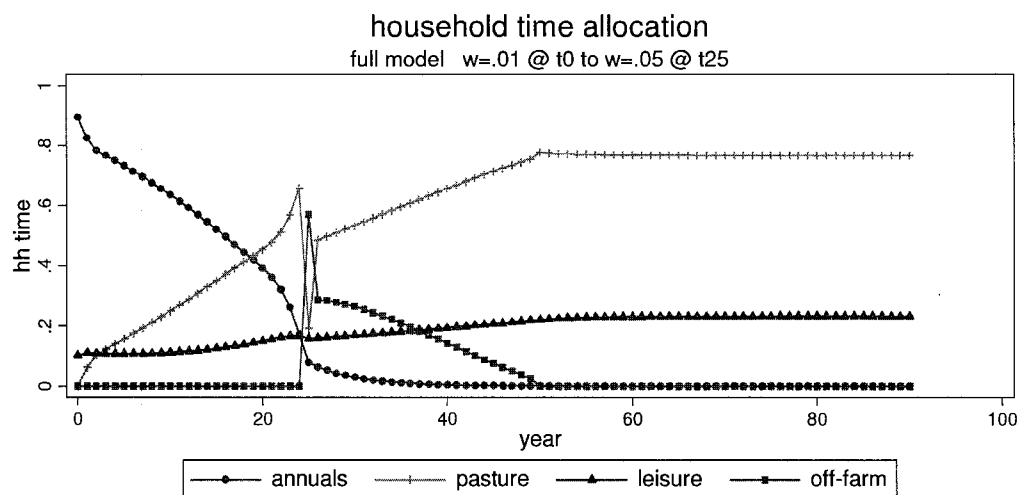
At the same time as pasture surges, the household accelerates its exit from annual cropping, and takes a bit more leisure time. Thereafter, pasture labor rises more steadily and off-farm work quickly peaks then starts to decline, at levels considerably lower and with an earlier phase out than in the constant *w*=.05 wage simulation.

Less off-farm work follows from greater investment in pasture before the wage change than in the constant wage scenario. Overall, off-farm work still slows the transition to pasture compared to the baseline, but less severely than under the constant *w*=.05 wage. The household allocates an amount of pasture labor early on in between that seen in the baseline and the constant wage scenario. Accordingly, this means more agricultural labor and leisure early on than in the baseline, but less than in the constant wage scenario. Agricultural labor and pasture labor are considerably higher just before

year 20 than in the constant wage scenario. This is due to the constant wage scenario's

earlier off-farm work start, lower initial pasture investment and hastened annuals phase-

out.

Deferring the deferred onset of off-farm work shows up as a pronounced dip in

the stocking rate resulting from the one-year pasture expansion surge (Figure 62)

combined with lower cattle sales.

Figure 62. Pasture and cattle – discrete wage change to *w*=.05 in *t*=25
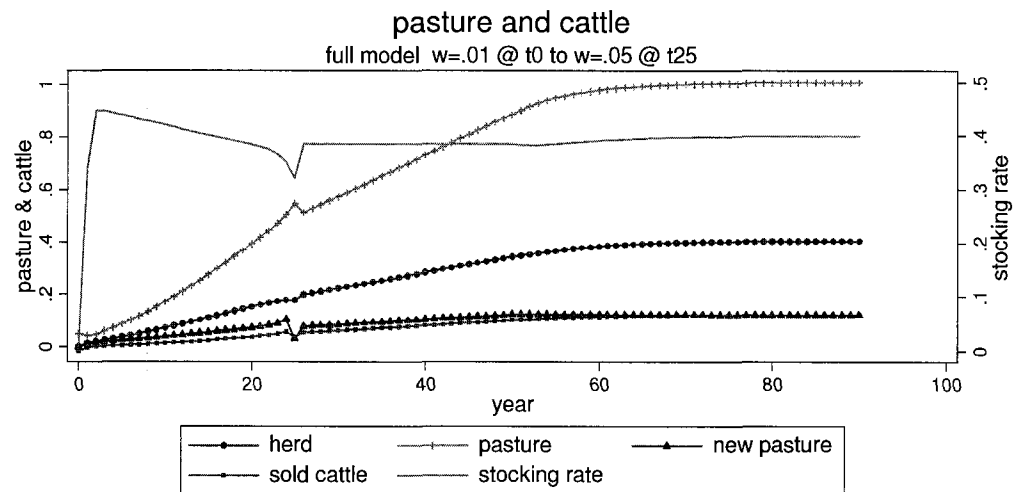
The transition introduces a blip in consumption and leisure trends as well,

allowing consumption to rise at a faster rate than leisure briefly heading into the off-farm

work phase (Figures 63 and 64).[105]

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[105]The requirement that proportional change in leisure and consumption must match during an off-farm work phase holds here – they are different in the first year of off-farm work, but jibe starting in *t*=26, the first year that proportional change under the off-farm work phase is calculated.

Figure 63. Household utility components – discrete wage change to w=.05 in t=25

household utility components
full model w=.01 @ t0 to w=.05 @ t25
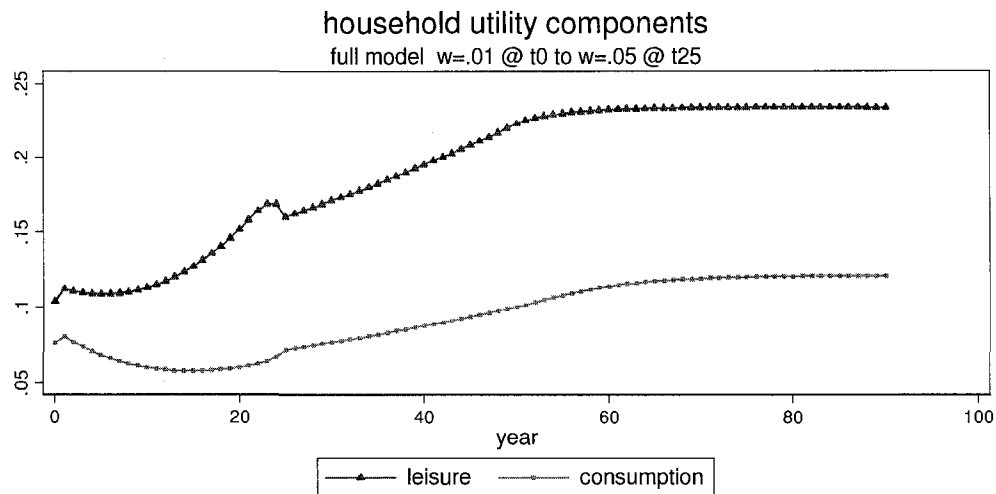
Figure 64. Household proportional change in utility – discrete wage change to w=.05 in t=25

proportional change in leisure & consumption
full model w=.01 @ t0 to w=.05 @ t25
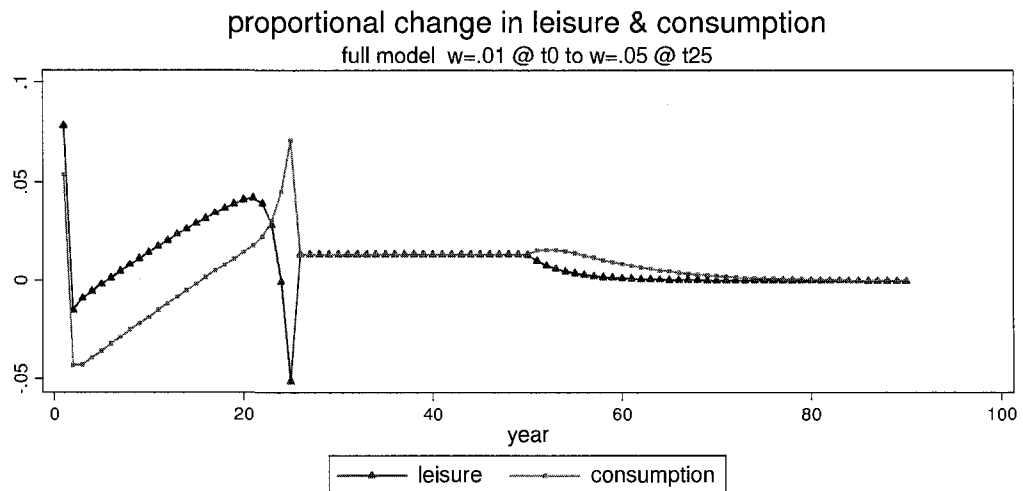
4.2.1.2 Gradually rising wage rate (2.5% per annum)

This scenario simulates a steadily growing exogenous wage from w=0.01 in t=0 to w=.092 in t=90.[106] As Figure 65 shows, the rising wage is eventually (at t=67, with w=.0523) sufficient to draw labor off-farm and start the drawdown of pasture. The

[106]So that the model will converge, the t=90 wage holds into the steady state.

wage's rise in effect 'chases' and finally surpasses the wage equivalence gain from

pasture investment.

Figure 65. Household time allocation – wage growth at 2.5%/yr

household time allocation
full model   w=.01 @ t0 + 2.5%/yr
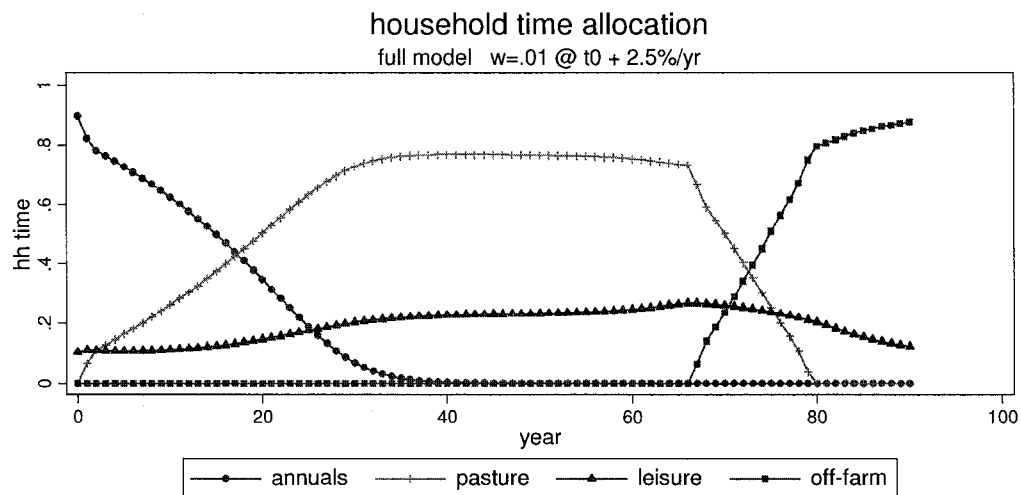
Pasture labor hits a peak in *t*=41; thereafter, the farmer anticipates the coming

wage by tapering off pasture labor slowly, then more quickly when off-farm work begins.

The farmer allows pasture to degrade more quickly than the herd is shrunk, so that

stocking rate rises during this period (Figure 66).[107]

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[107]Depending on conditions, lowered capacity could mean abandonment of some pasture, or a lowering of productivity within a given area of pasture.

Figure 66. Pasture and cattle – wage growth at 2.5%/yr

pasture and cattle
full model w=.01 @ t0 + 2.5%/yr
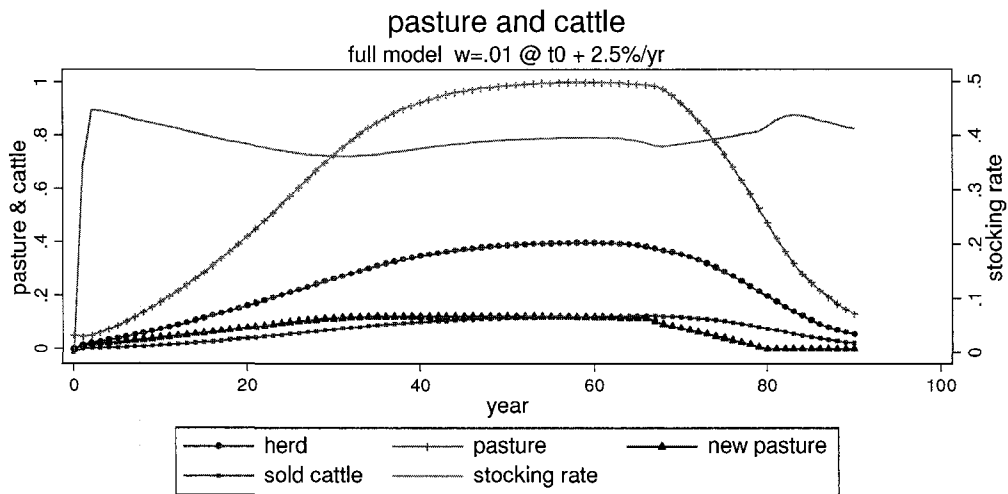
Dropping leisure time and consumption levels mark the pasture phase-out period (Figure 67). Proportional drops in leisure outpace those in consumption (Figure 68).

Figure 67. Household utility components – wage growth at 2.5%/yr

household utility components
full model w=.01 @ t0 + 2.5%/yr
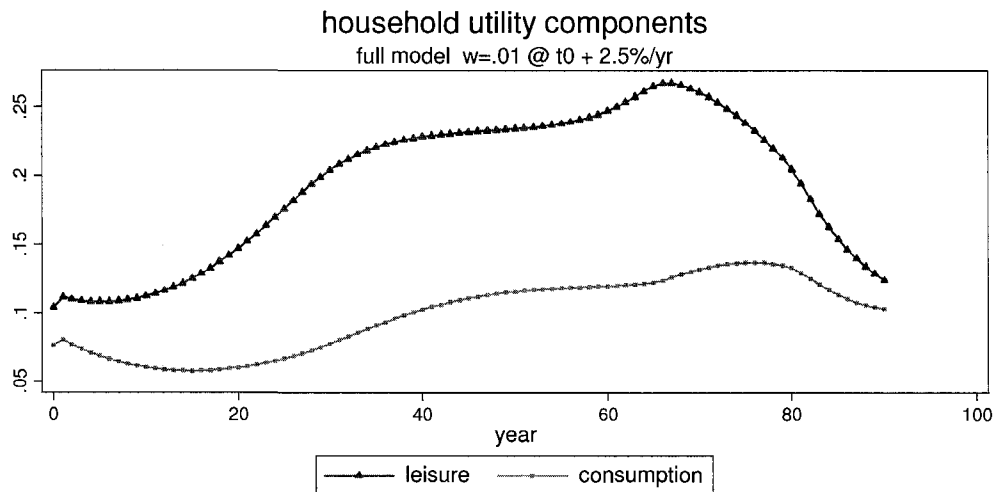
Figure 68. Proportional change in utility components – wage growth at 2.5%/yr

proportional change in leisure & consumption
full model w=.01 @ t0 + 2.5%/yr
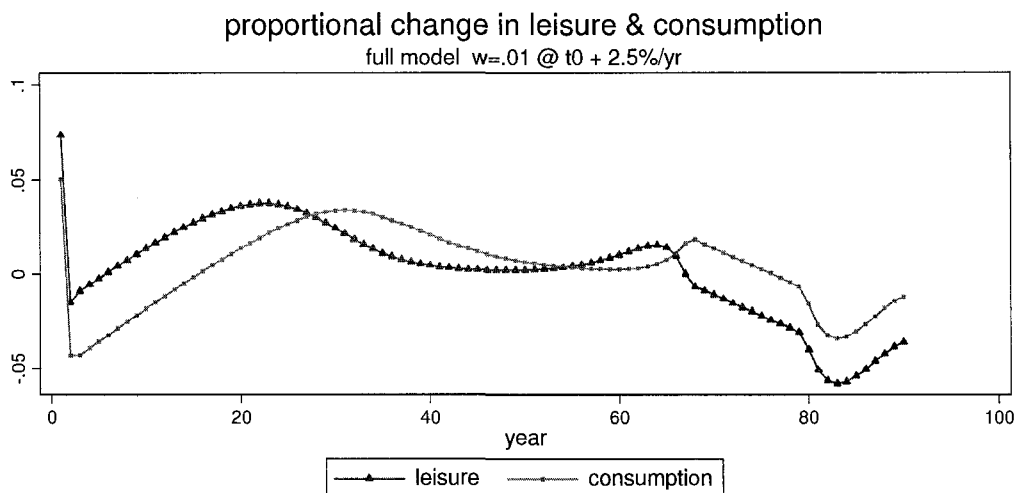
4.2.1.3 Changing output prices

Output prices would also be expected to change over time, particularly over the course of settlement maturation, with effects on individual farms' optimal paths.[108]

In the early days of settlement, pasture investment has yet to begin or is in its earliest stages, and the household has limited outside resources to meet consumption needs. Reliance on annual cropping is strong. As investment starts to loosen this tightly constrained system, annuals prices fall off relative to cattle prices over time towards a steady state of relative prices.

Modeling this drop-off as an exponential decrease in the annuals price would be equivalent in modeling terms to raising the soil degradation rate. The effect, as seen already, would be a hastened exit from annuals and transition to pasture. The steeper drop-off could make the difference in the attractiveness of off-farm work during pasture

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[108]As noted earlier, this approach approximates a more accurate depiction of frontier life that endogenizes participation in the output markets.

investment – either sparking an wage labor phase where the baseline had none, or causing the wage labor phase to start earlier.

It is also likely that a sudden change that improves access to outside markets, such as in the discrete jump in wages in the simulation above, would also spark an increase in cattle prices, given the rising regional prices for cattle driven by growing demand outstripping supply. A jump in cattle prices concurrent with the wage rise would tend to erode the effects seen with the wage increase. These include impacts seen early in the trajectory, before the price change occurs – an expectation of a rise in cattle prices would favor pasture investment to position the farmer to take advantage of the rise when it occurs, counterbalancing the annuals-enhancing outcome of the expectation of higher wages. The magnitude of the relative price change would determine which effect was stronger (and whether an off-farm work phase still would occur).[109]

4.2.1.4 Aging of the farm household

In this scenario, the household's time in each period changes along a timepath to roughly approximate the household's lifecycle. Household demographics have been posited to be critical for frontier settlers' land use patterns, with the idea that changing availability of the scarce factor labor as the household ages will affect farmers' capacity to implement certain technologies (Brondizio et al. 2002; McCracken et al. 2002; Walker et al. 2002). Early on in the settlement process, the concern is that the household with more young children will be more vulnerable to poverty, affecting the household's ability

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[109]In the study area in western Brazilian Amazon, infrastructure improvements can be not just in the direction of established markets in Brazil, but open up access *to* Brazil from neighboring Bolivia. Such a scenario could draw in migrant labor from Bolivia, opening up the possibility of a lower local wage and more hiring in on farm. With labor constraints removed, a significant brake on pasture development would be lifted (as seen in other model, e.g., Vosti et al. 2002). Because this model focuses on labor scarcity, it would need to be significantly modified (including more detail on pasture and the local cattle sector, beyond the dynamics of herd growth) to simulate such a scenario.

to make successful investments. More broadly, there is a concern that, over time, an aging rural population as younger workers are drawn toward urban sectors will only reinforce the tendency to rely on cattle production.

This scenario draws on the study by Vosti et al. (2002) for the western Brazilian Amazon, plus secondary data about age of household head when household size peaks for this cohort in Brazil (Barros et al. 2000),[110] to come up with an assumption of the household growing 2.2% per year from settlement for the first five years (reflecting in-migration of family members as well as expansion of the family), then declining 2.8% annually for the finite time horizon of the model.[111] As before, in each time period the household chooses how to allocate its available time across activities and leisure. The figures are admittedly not rigorous, but should suffice to provide the gist of how household aging might influence the household's trajectory.[112]

As Figure 69 shows, the household uses its growth phase to slow the annuals phase-out as well as speed up initial investment in pasture. Time allocated to pasture expansion and maintenance continues to grow at a slower pace in the aging phase, then peaks and starts to decline as the household time continues to shrink (Figure 70).

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[110]More specifically, household heads averaged 31 years old upon arrival, and were 46 years old at the time of the study. Dependency ratios at the outset reflected the usual practice of households migrating in stages, with adult male labor predominating at settlement. The rest of the family moves in within a few years. For this cohort across Brazil, household size peaked at about 40 years of age. But assigning children a proportion of adult 'time' (to get at the differences in labor and consumption of non-leisure goods, here an admittedly *ad hoc* 25%) means household labor can peak before household size. These assumptions, plus the dependency ratio at the time of the study, yielded the growth rates used.

[111]The end to household size shrinkage is also somewhat arbitrary, but fits with the idea that some rural population will remain.

[112]Indeed, the simulation might best be interpreted at a broader regional level, exploring the situation if the agricultural sector were to lose its working age population. This avoids issues about arrival of labor (and land) markets along the trajectory, as well as the transfer of farm ownership across generations. These situations could be incorporated in separate simulations.

Figure 69. Household time allocation – aging household simulation

household time allocation
aging household simulation
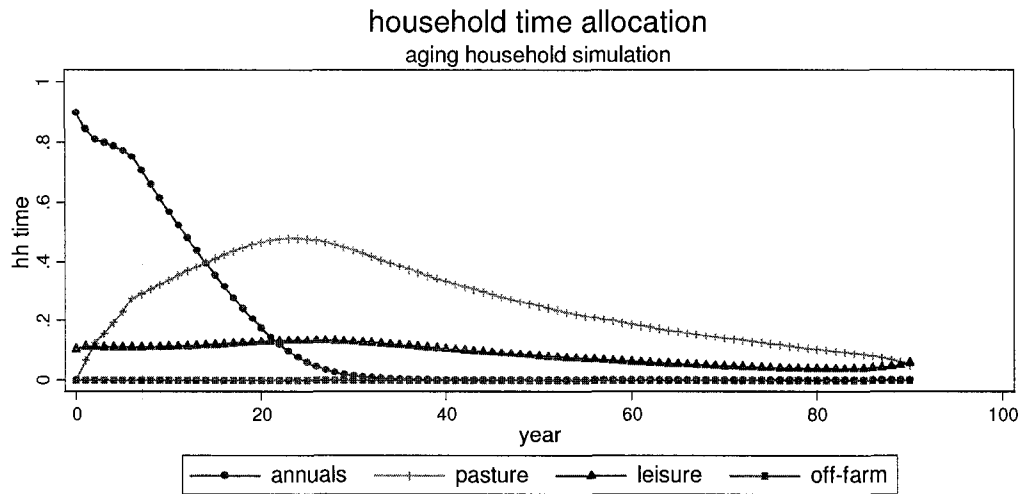
Figure 70. Change in time allocation from prior period – aging household simulation

change in hh time allocation
aging household simulation
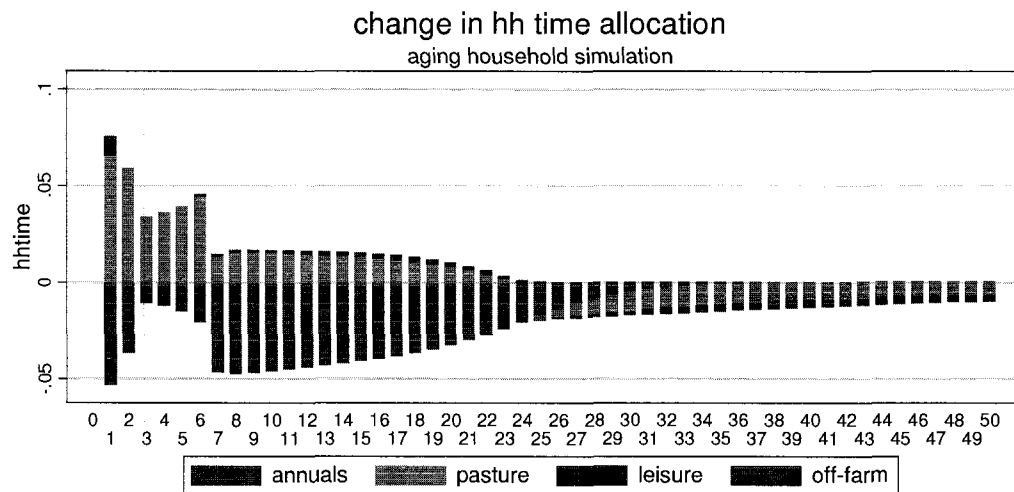
Proportion of time allocated to each category, however, is similar to the baseline for most of the trajectory (Figure 71). The aging household allocates a slightly higher proportion of time to pasture during the build-up phase, and an increasingly more of its time to leisure as aging shrinks household size.

Figure 71. Household proportional time allocation – aging household simulation

household time allocation - proportion of hhsize
aging household simulation
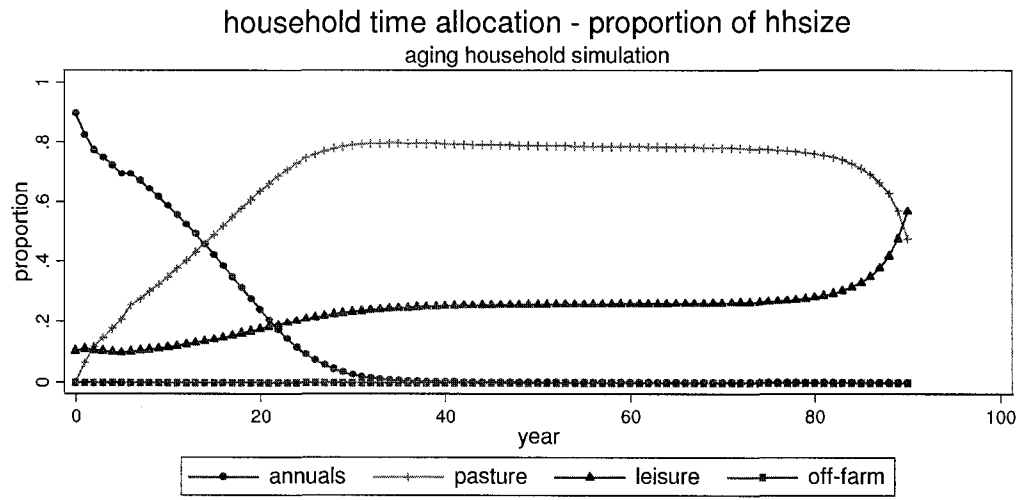
Figures 72 and 73 show that the household's aging erodes much of the consumption growth built on the pasture investment. With these growth rates and growth phases, pasture investment never allows household consumption to rise above its starting levels. Still, pasture investment does usher in a period of consumption growth after the household is in its aging phase (t=17 to t=34). On a per capita basis, moreover, household consumption starts to rise soon after the aging phase begins, as gains due to pasture start to kick in. The per capita gains continue through the period where pasture capacity is falling (so the aging outpaces the decline in pasture productivity).

Figure 72. Household utility – aging household simulation
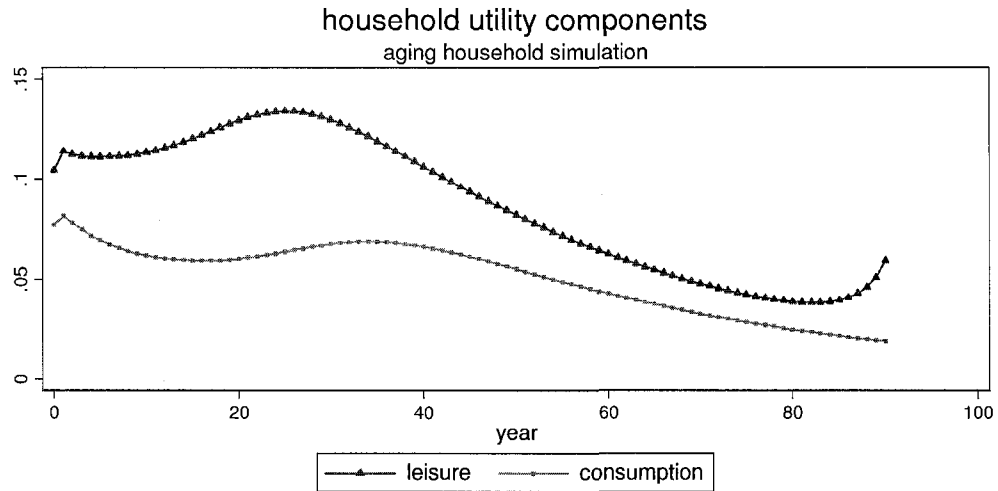
Figure 73. Proportional change in utility components – aging household simulation
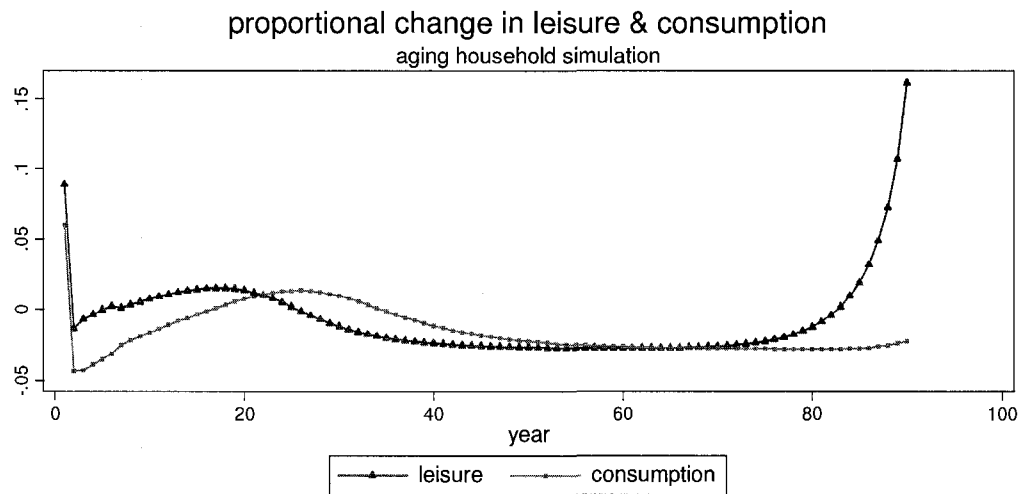
The household's aging makes the transition to the optimal stocking rate more gradual. The dip in stocking rate accompanying the growth in pasture stretches out into the phase where the pasture is declining (Figure 74).

Figure 74. Pasture and cattle management – aging household simulation

pasture and cattle
aging household simulation
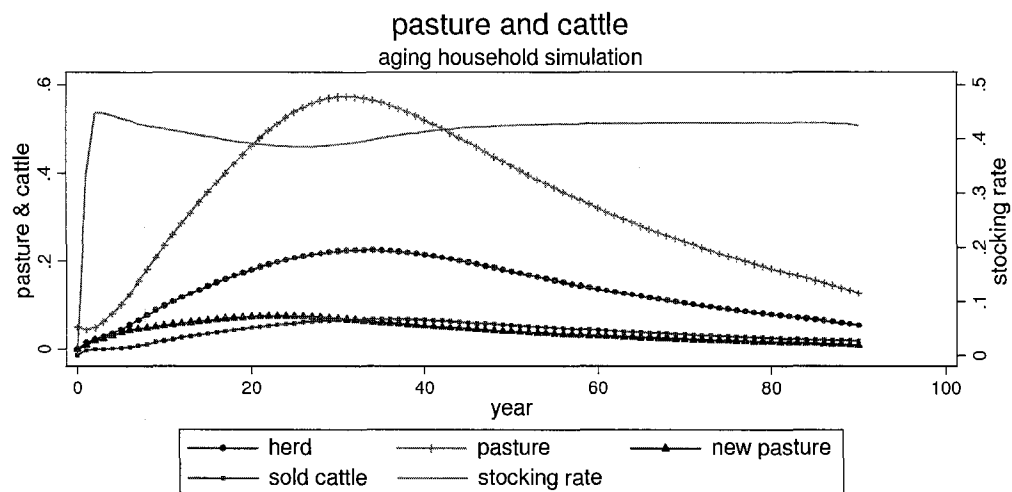
4.2.2 Flexibility to add detail – discussion and example

The model's aim is to capture the broadest brushstrokes of the frontier farmer's economic trade-offs. It focuses on the forest-to-pasture investment decision so predominant in the western Brazilian Amazon. It also tries to characterize the general economic and biophysical environment confronting the small farmer-migrant – namely labor's scarcity and the challenges posed by soil degradation.

In its simplicity, the model glosses over many aspects of the local farming situation. For instance, farmers have a much broader range of technologies and activities available to them, there are seasonal gluts and gaps in labor use that influence the time-dependent shadow value of time, and local conditions – regulations and infrastructure – impose additional constraints on household decisions and involve several categories of risk.[113]

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[113]See Vosti et al. (2002) for more on these details – such as production quotas on milk, and regulations on forest clearance – in the western Brazilian Amazon.

Still, the model's simplicity should make time trade-offs involved more transparent as the modeled scenario grows more complex, expanding farmers' choice set, increasing the granularity of the decision time-step, or imposing additional constraints.[114]

For example, as demonstrated in the sensitivity analysis, categorizing activities by their patterns of returns to labor over time can yield insights prior to modeling about the nature of time allocation transitions during the trajectory. In the simplest case, activities whose return does not depend on the amount of labor allocated (such as wage labor or constant returns production technology) will compete head-to-head in each time period for the household's labor (that is, the two activities will never be undertaken simultaneously).

The policy simulation example here adds an activity to the mix with a slightly different returns-to-labor profile than that seen up to now. We look at harvest of forest products, and assume that their production falls as pasture capacity expands, due to the fact that the forest itself is both receding (so harder to access) and shrinking in size. In essence, this yields a production function that degrades over space, where that space is explicitly linked to land in pasture. More specifically,

$$G_t = Be^{-\Delta K_t} LG_t \tag{54}$$

where G_t is production of the forest product, B is a labor efficiency coefficient, Δ is the space degradation rate, and LG_t is forest product labor (K_t, as before, is pasture carrying capacity). The amount of forest product collected is then sold; its value is added to the

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[114]Some changes, of course, require more drastic re-working of the model structure to incorporate than others.

consumption equation. For the policy simulation, the forest product price is set to 1, $B=.05$ (or half the labor efficiency of annuals production A), and $\Delta=.2$.

The simulation results show forest production playing a role similar to that of off-farm work in bridging the transition between annuals and pasture. This is visible in the household's time allocation trajectory (Figure 75): agricultural labor is slightly higher early in the trajectory than in the baseline and pasture is lower, with a brief surge prior to the start of the forest product's collection.

Figure 75. Household time allocation – forest product simulation
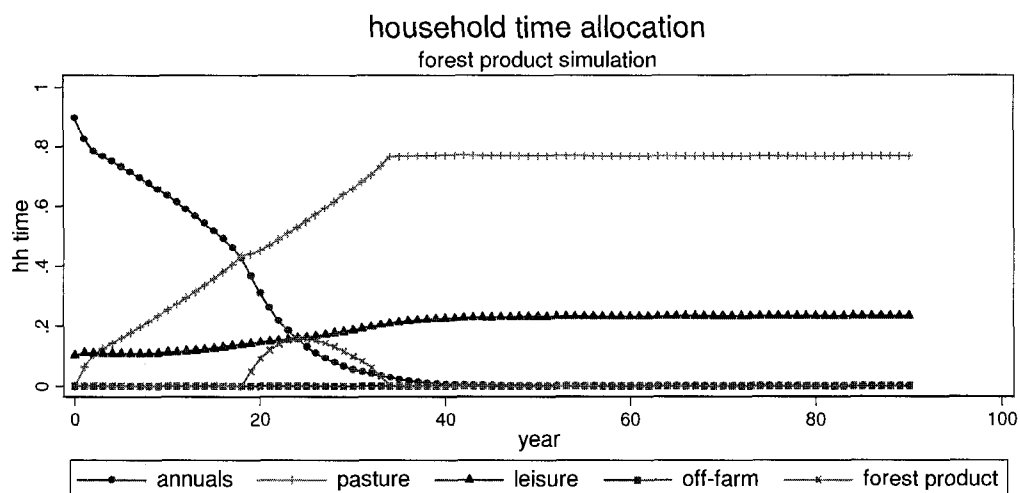
Now, the third activity introduces a period where the gap between proportional change in leisure and proportional change in consumption is narrowed (but not closed at a constant rate as with off-farm work), as seen in Figure 76.

Figure 76. Proportional change in utility components – forest product simulation

proportional change in leisure & consumption
forest product simulation
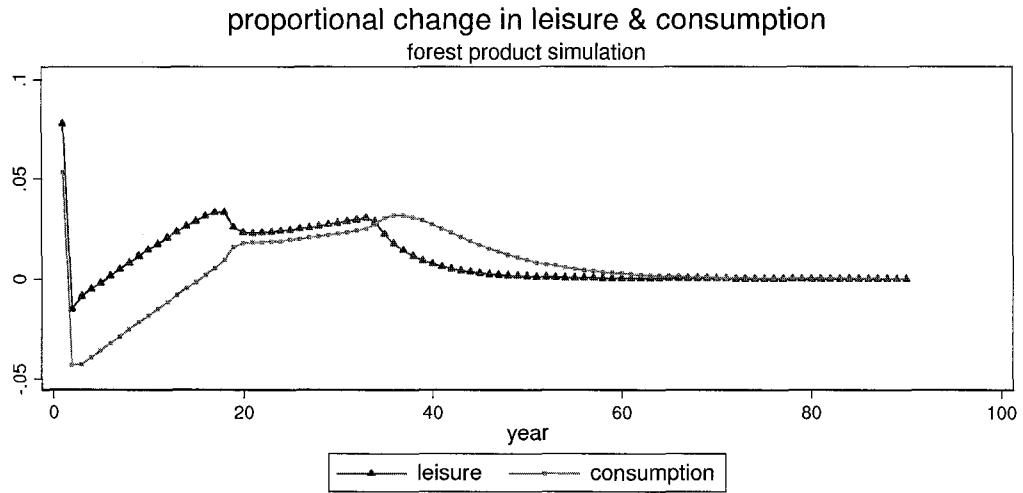
In terms of pasture growth, there's a slight delay in the transition owing to the slower start, despite a catch-up period in pasture labor after the forest-product phase. The result is a timepath for the stocking rate that resembles that in the off-farm work simulations, except that stocking rate declines slightly rather than holds constant during the third-activity phase (Figure 77).

Figure 77. Pasture and cattle – forest product simulation

pasture and cattle
forest product simulation
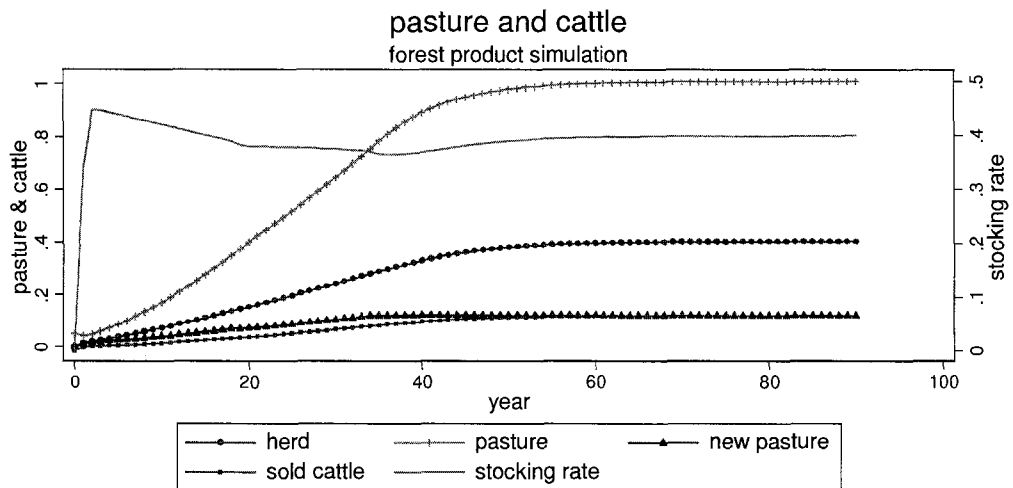
This simulation sets up an explicit trade-off between pasture and forest production. With less efficient labor productivity than annuals, though, forest production is not attractive until soil degradation has taken its toll on annuals production. At the same time, forest production becomes less attractive the more pasture is expanded. This leaves the window of opportunity for forest products as a bridge between the two other labor activities, with these parameters.[115]

4.2.3 Expandability beyond the household through labor trades – discussion and examples

The model's relative simplicity makes it a promising foundation for exploring aggregation beyond a single household – how groups of households with the potential to interact may behave in a way that's different from a simple summing up of their individual behaviors. In keeping with this paper's focus on labor, this section examines scenarios involving households trading labor. For these simulations, households participate in output markets (so don't trade these commodities) as before.

4.2.3.1 Households with different labor endowments

Otherwise identical households with different household sizes – labor endowments – would have differing returns to time, setting up the possibility of gains from trade. In this example, a baseline household (household A) has a neighbor (household B) with twice its labor endowment (household A has $T=1$, and household B has $T=2$). Figure 78 traces the shadow wage, or wage equivalence, for each of the two

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[115]Obviously, other parameters change the dynamic. Having forest production more competitive with annuals production at the outset but still not as favorable as pasture in the long run accelerates the transition out of annuals and into pasture. Currently, forest products are not so competitive; Vosti et al. (2002) suggest their collection and sale exploits seasonality in the labor cycle and intra-household differences in the shadow value of labor (so those with lower labor value – children – do the collecting).

households over the optimal trajectory in the absence of trade, as well as their common wage equivalence under trade.[116]

Figure 78. Wage equivalence – two households of different sizes with and without trade

hh wage equivalence -- hhA T=1 hhB T=2
no trade vs. trade
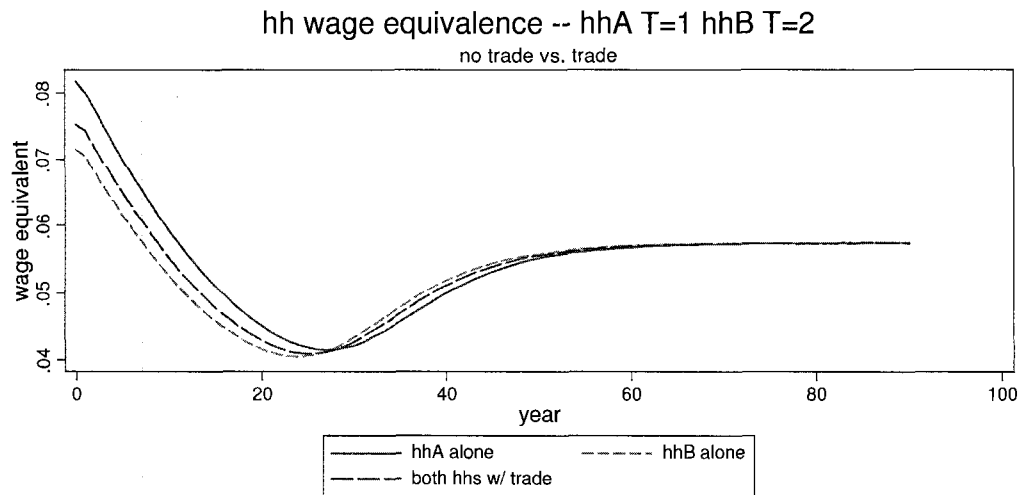
The smaller household has a higher wage equivalence early in the trajectory, when annuals activity is predominant and pasture is expanding. The larger household's wage equivalence nadir is lower and turns around sooner, so once the annuals phase-out is nearing its completion and pasture is approaching its steady state capacity, the larger household starts to have a slightly higher wage equivalence which lasts until the households are both extremely close to steady state time allocation.[117] Under trade, the wage equivalence falls between these two.

When the two households are allowed to trade labor, the optimal result involves the larger household trading an amount of labor to the smaller household to equalize the labor endowments across farms (household B trades half a unit of labor to household A in

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[116]The figures for the baseline run appear Section 3.2 in the previous chapter; a description of the expanded household size simulation appears in the sensitivity analysis in Section 4.1.1 earlier in this chapter.

[117]Starting in the cross-over period, the smaller household starts to have a greater absolute quantity of annuals than the larger household – its phase-out of annuals is slightly slower than the larger household's.

each period, leaving each household with 1.5 units of labor). Figure 79 depicts the time

allocation for each household after the trade (and includes the 0.5 traded from household

B to household A).

Figure 79. Time allocation of each household after trade – differing labor endowments

time allocation in each hh -- hhA T=1 hhB T=2
baseline with trade
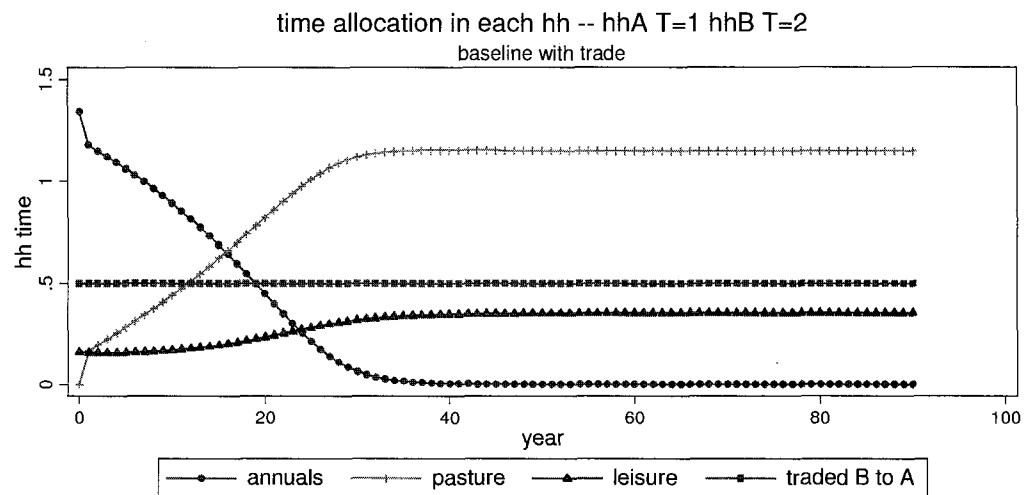
Overall, the time allocation for the two household system is extremely close with

or without trade. Still, introducing trade encourages slightly more annuals labor and

slightly less pasture labor in the transition to pasture, as seen in Figure 80 (which depicts

the difference in the two household systems – with and without trade). Steady state

values don't change because of trade.[118]

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[118]This follows from the fact that any spatial advantage to be garnered in this scenario comes from location
of annual cropping (not available in the steady state) or herd growth (not applicable in the steady state).

Figure 80. Trade effect on two-household time allocation – differing labor endowments

time allocation difference -- 2hh system
w/ trade - w/out trade hhA T=1 hhB T=2

legend: annuals, pasture, leisure
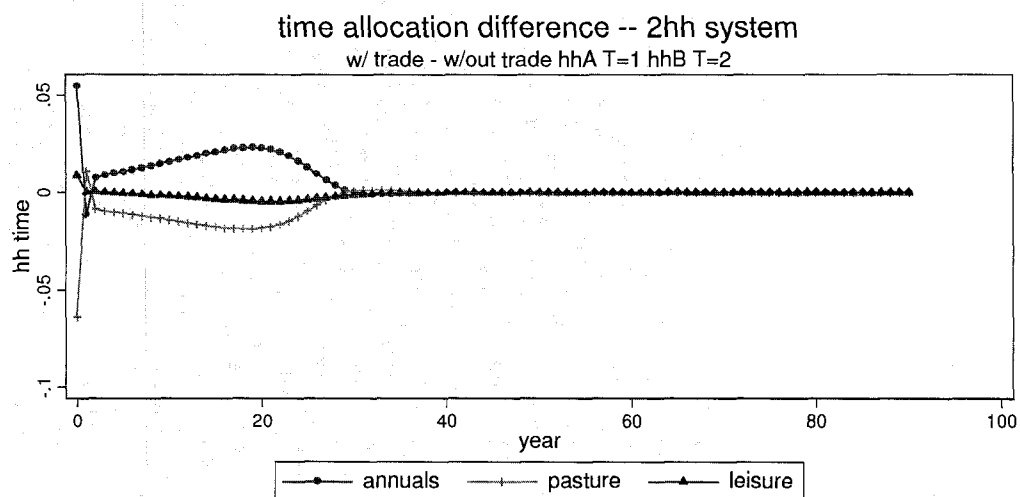
As a result, there is slightly less cattle and pasture overall along the trajectory with trade than without, and a slightly higher stocking rate (Figure 81).

Figure 81. Trade effect on two-household cattle and pasture – differing labor endowments[119]

cattle & pasture difference -- 2hh system
w/ trade - w/out trade hhA T=1 hhB T=2

legend: cattle, pasture, stocking rate
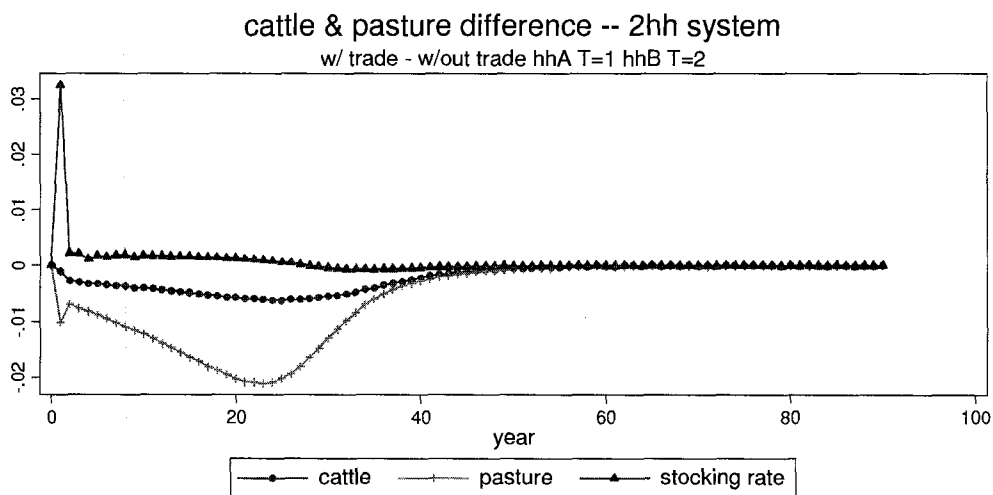
The present discounted benefits of the system (taken as the sum of the households' present discounted benefits), as they must, improve under trade. With the parameters used here, the gain amounts to approximately a 2% improvement in overall

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[119]Again, scaling allows the differences in cattle, pasture capacity, and stocking rate to be viewed on the same y-axis; units are for herd numbers, pasture capacity, and their ratio, respectively.

system present discounted benefit. Since the smaller household improves over its pre-trade position while the larger household suffers, the trade must involve a transfer from household A to household B to ensure both households gain (and the transaction costs of setting up the trade system cannot swamp the gains from trade).

4.2.3.2 "Small farmer-rancher" trade

This example examines the case where two otherwise identical households differ in their time preference parameter. The parameter is here best interpreted as an opportunity cost of capital rather than an innate preference, since it strives to capture differential access to market opportunities upon settlement. The econometric work in the Vosti et al. (2002) field study shows that settlers' social contacts, both within and outside settlement projects, have an effect on their investment (land use) patterns. The time preference parameter provides a convenient (albeit crude) proxy for the raft of factors that could be working to ease one settler's access to production or marketing related resources vis-à-vis a neighbor.[120] As discussed in the sensitivity analysis, a shift in this parameter alters household returns to time, again setting up an opportunity for gains from trade. Figure 82 shows the wage equivalence for the two households' timepaths when acting autonomously, and their timepath when allowed to trade labor.[121]

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[120]The difference is hypothesized to be related to different access to resources, especially early on in the land use trajectory. In the absence of hard evidence regarding the importance of particular factors in this phenomenon, the time preference should give a ballpark idea of possible effects.

[121]As will be discussed below, this scenario, while converging within limits imposed by the numerical simulation program, cannot completely converge. The traded labor scenario timepath is closely approximated here by averaging the wage equivalence for the two trading partners with the converged numerical program. As we shall see, with tighter convergence limits or longer time horizons, the two traders' timepaths move closer, but with the small farmer doing a disproportionate amount of the moving.

Figure 82. Wage equivalence – small-scale farmer and rancher, no trade vs. trade

hh wage equivalence -- hhA rho=.10 hhB rho=.07
no trade vs. trade
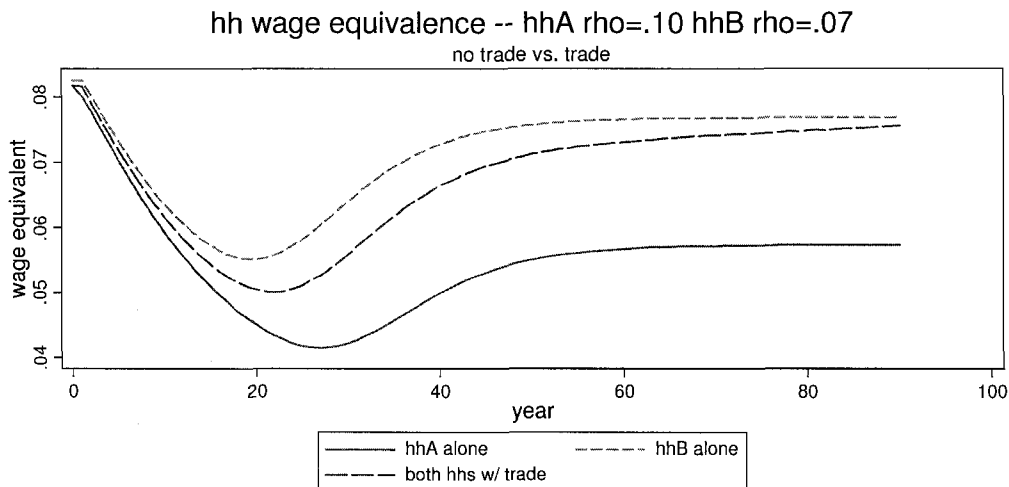
As a result of trade, the 'small-scale farmer' (with the higher discount rate) trades labor to the 'rancher' (with the lower discount rate) neighbor, with the pasture labor ending up consolidated on the rancher's farm (Figure 83). The small-scale farmer still finds it profitable, however, to invest in expanding pasture during the early phase, even trading ever less labor to the rancher, before embarking on a path of increasing labor trades and drawdown of own-pasture labor (and pasture itself). The pattern continues as the model hits the 90-year mark.[122]

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[122]This indicates that the model has not yet closed in on a steady state at this point. Since the model locks in time allocation at this point, the overall value function will underestimate the optimal. More on this below.

Figure 83. Household time allocation – small farmer-rancher trade

hh time allocation -- hhA rho=.10 hhB rho=.07
small scale farmer - rancher trade

Graphs by hh id
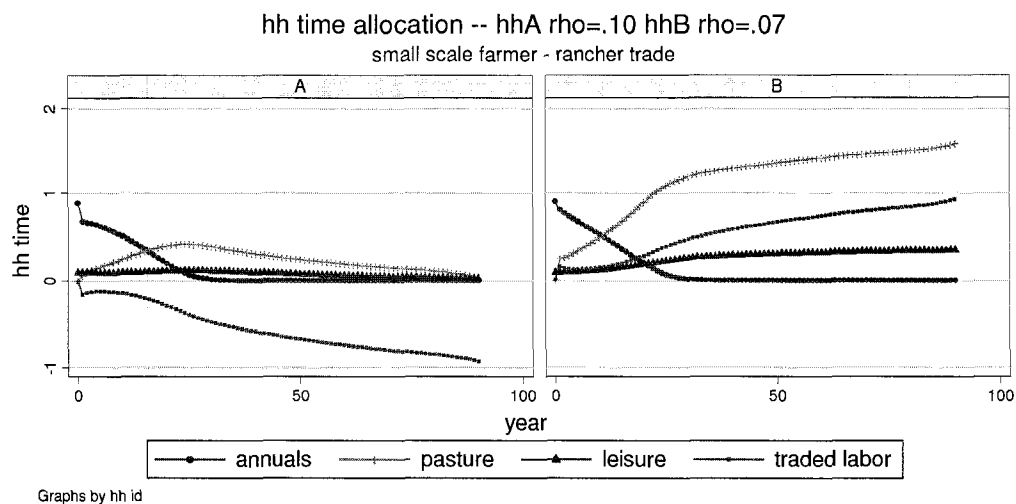
As a result of the trade, the two household system's (both households combined) time allocation shows a slight increase in pasture vis-à-vis the no-trade scenario, particularly in the primary build-up phase (Figure 84). Most of this is labor is taken away from annuals, with a slight amount coming from leisure. Once annuals are out of the picture, however, time continues to be transferred in the system from leisure to pasture (the small-scale farmer transfers pasture labor to the rancher, but also eats into leisure time to trade additional labor). The transfer continues after the no-trade system settles down (so the gap between pasture and leisure in the trade vs. no trade system continues to grow within the 90-year time horizon).

Figure 84. Trade effect on two-household time allocation – small farmer-rancher trade
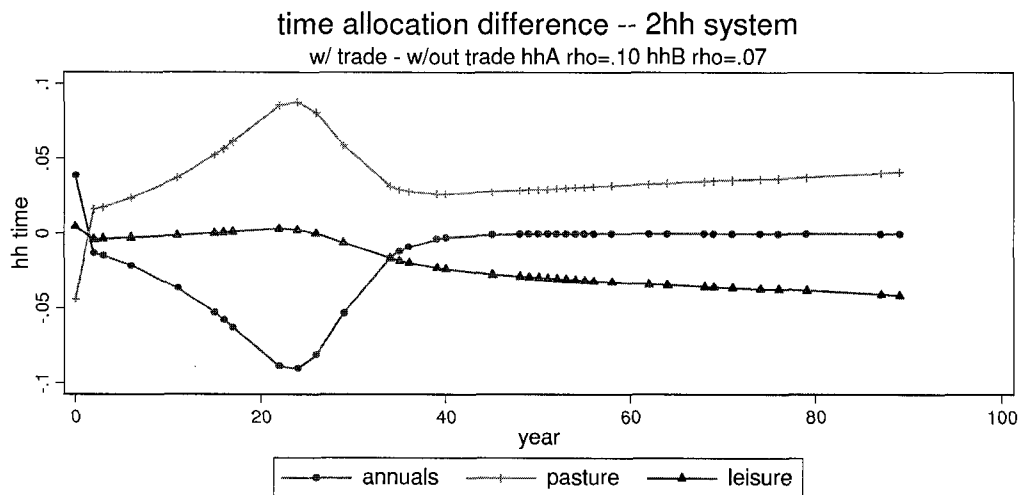
Correspondingly, pasture and herd grow slightly beyond what they would without

trade (Figure 85), increasingly so over time.

Figure 85. Trade effect on two-household cattle and pasture – small farmer-rancher trade
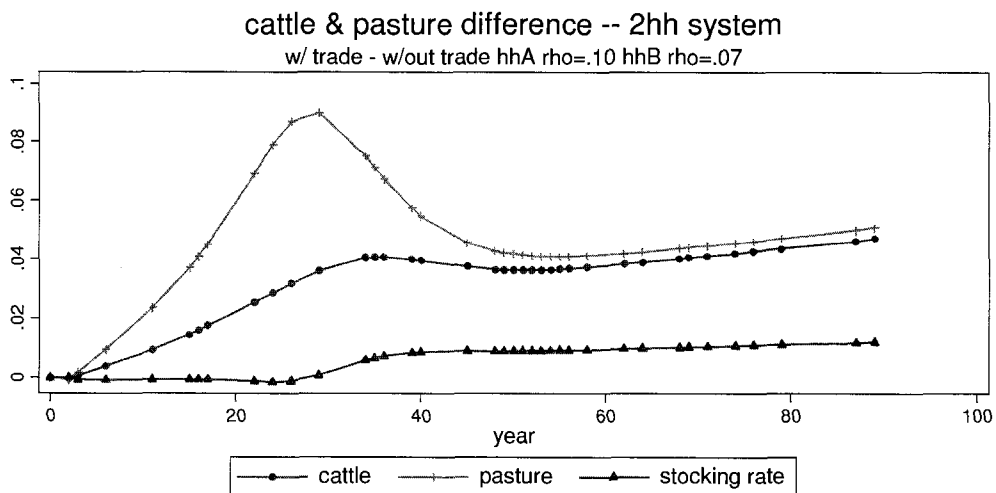
The overall improvement in system present discounted benefits, with the

parameters used here, amounts to just under 1% (and any additional gains from spinning

out the trajectory further would be minimal). To happen, the deal must involve a re-

distribution of the gains from trade from household B to household A to ensure both

households improve upon their no-trade situation. Without this distribution of benefits, the 'rancher' is better off, and the 'small-scale farmer' worse off, with trade than without it.

Given the trends at year 90, a few words about the system's steady state are in order. With trade, it's the system as a whole, and not each household, that has an upper limit of allocation to work and leisure matching preference parameters. Already at year 90, the small-scale farmer is well below the 10% leisure lower threshold (and the rancher is well above it). As is the case for the pro-wage labor model, it is the lower bound on pasture capacity in the small-scale farmer-rancher trade model that fixes the steady state solution. Trade continues to expand until the small-scale farmer uses the minimum amount to maintain pasture at its lower bound. This locks in an amount of leisure for the small-scale farmer needed to equate returns to time across the system. In terms of Figure 82, the already apparent trend of the system's wage equivalent approaching that of the stand-alone rancher continues. One way to think of why this occurs is to recall the critical role of the time preference parameter in the steady state solution. With trade, the system's time preference essentially falls between the trading partners' discount rates, weighted by the size of the trade. Since trade changes over time, so does this system parameter: more of the system's labor and production moving over to the farm with the lower discount rate means the system's time preference also moves toward the lower discount rate. Only the pasture lower bound anchors the discount rate at a particular point, permitting the steady state to exist.

4.2.4 Lessons from policy simulations

The policy simulations highlighting household dynamics focused on how timepaths for exogenous parameters influence the optimal trajectory. They showed that the expectation of a future wage increase (that is, with the farmer planning for the wage hike) high enough to draw labor off farm dampens pasture development from the get-go. The timing of a wage increase makes a difference. If the wage rises after the household would ideally start to find such a wage attractive, the household responds to the delay by investing more in pasture and less in agriculture and leisure in the lead-up to the off-farm work phase than under a constant wage scenario. The household then protects its earlier investment against degradation by allocating more time to pasture during the wage phase. Even with foresight, moreover, it is rational for the household to show volatile time allocation patterns around the time of a discrete wage change. Should the discrete exogenous change (a new road, for example) also cause a jump in pasture prices, this would attenuate the above effect, and potentially remove the rationale for working off farm. More generally, the relative decline in annual price relative to pasture price expected to accompany settlement development would prompt the farmer to invest more in pasture earlier.

Where changed conditions remove pasture as a long-run option (gradual wage increase, aging household), the household still invests in pasture and grows a herd early on, then uses it as a store of wealth to draw on and draw down later. The pasture phase-out mirrors its build-up in that leisure leads consumption in the change (now down rather than up), and stocking rate again rises, but now because the household sells off its herd at a slower rate than it allows its pasture to degrade. When the phase-out is due to a rising

wage, the household works more in order to consume less. When the pasture phase-out happens because of a shrinking household size, each household member gains in both leisure time and amount consumed.

The policy simulations also demonstrated the flexibility of the model to incorporate additional activities through an example using collection of forest products as an alternative for labor. The innovation here was to incorporate space into the production function in a form explicitly linked to another on-farm activity: the overall productivity of the forest shrank as pasture capacity expanded. With the parameters used, collection and sale of forest products provide a useful bridge between annual cropping and pasture, much the way that off farm labor had in the constant wage scenario. Since pasture remains favorable in the long term, though, it continues to expand, and chokes off forest production not only by drawing away labor (as in the constant wage case) but also by reducing the returns to forest production per unit of labor.

The trade-scenario policy simulations provided a first look beyond a single household in this dynamic frontier environment. The first simulation examined labor trade between two neighboring households with different labor endowments. The second simulation allowed labor trade between a 'small-scale farmer' and a 'rancher' (households with different opportunity costs of capital, proxied by different discount rates on their utility functions). Neither scenario, with the parameters used, had a large effect on either overall system benefits or overall land use. Nonetheless, there were changes in each case that could scale up to bigger changes across a landscape of non-market-integrated, labor-trading households.

The trading scenario between otherwise identical households with different labor endowments shows how households can leverage across space (the two farms) to best play off the dual diminishing-returns to annuals cropping against the dual increasing returns accompanying pasture growth due to herd dynamics. Results show the households gain more by reallocating more labor to annuals than in the absence of trade, dampening overall pasture growth. Because the households share all other parameters, the system can always improve by shifting time until each household has the same time allocation between work and leisure, so the two farms share identical land use patterns after trade. This is most easily seen considering returns to leisure,[123] which must be equated across households in an interior solution. This can only occur if households enjoy identical amounts of leisure time.

When households with different time preferences trade labor, on the other hand, the result is to boost pasture relative to the situation without trade. Still, not as much as you might expect, since the small-scale farmer does not act purely as an employee to the rancher. Rather, it is still best for the small-scale farmer to build up a stock of pasture and cattle early on, then manage its drawdown as ever more labor is traded to the rancher.

Together, the policy simulation examples illustrate that the model can demonstrate sometimes surprising changes in household timepaths for land use, and more importantly, illuminate the economic trade-offs involved in the result.

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[123] $\dfrac{\beta}{l_{ti}}$, where i subscripts the household (i=A,B).

Chapter 5 Conclusions

Deforestation has garnered a great deal of international attention principally because of its spillover effects on the larger common good, such as loss of an effective carbon sink and biodiversity. Many have pointed to neglect for such externalities in the behavior of those who deforest, driving them to clear beyond what would be socially optimal.

Conventional wisdom once held that settlements in the Brazilian Amazon were filled with subsistence farmers perpetually driven to clear new forest for annuals in order to survive, with no path out of poverty. Increasingly, evidence tips toward the view that cattle profits lie behind farm-level decisions to invest in land clearing primarily for pasture expansion, with annuals production largely a by-product of the clearing process. The two perspectives are often characterized as competing visions.

This dissertation has sought to shed light on local deforestation patterns of pioneer settlers in the western Brazilian Amazon using a dynamic optimization model for a small-scale household. In keeping with the idea that private incentives drive behavior, economic actors in the model choose to use land based on its productive value (that is, not accounting for harm to the public good caused by deforestation) and their own preferences for leisure and consumption. The research examines deforestation as part of a larger investment decision by the household, and focuses on the decision to expand pasture for cattle production – the area's predominant use for cleared land – as well as the scarce factor, labor. Such an investment model can suggest when the competing visions of small-scale households result from different snapshots in time of a single farm, and/or relevant visions at different times of a settlement's history.

This concluding chapter recaps principal findings from each chapter, draws some overall lessons from the findings (looking at how model results compare with some other theoretical and empirical studies in the process), and discusses directions for further research (examining some model shortcomings in the process).

5.1 Summary

Chapter 1 laid out some facts about deforestation and its recent history in the Brazilian Amazon, and established how the study fits in with the growing literature on the topic. It argued for a focus on the frontier as the leading-edge of settlement which empirical studies have shown to have not only immediate deforestation effects, but also long-lasting consequences for subsequent deforestation patterns. It also argued for a focus on the small-scale settler as a critical actor in frontier settlement – because of their sheer numbers, their precarious economic situation, and their role in various explanations of the deforestation process. It maintained that the current work could help fill important gaps in our understanding of small-scale frontier farmers' deforestation patterns by:

- taking an investment perspective, and exploring how pasture/herd dynamics affect the household's optimal trajectory;

- modeling consumption and production decisions jointly to capture the lack of complete market participation by frontier households, with an initial reliance on family labor;

- exploring the consequences of labor market *integration* by examining the relationship between the external wage, household time allocation, and land use patterns; and

- taking a first step towards understanding how frontier labor market *deepening* – the beginnings of an endogenous local labor market – could alter landscape-level land use by examining bilateral labor trade among neighbors and its affects on broader land use patterns.

Chapter 2 laid out the theoretical model. It also introduced the critical equation that captures the household's optimization of labor use by allocating across activities and leisure until marginal returns to its time match (or are prevented from doing so by a constraint). This led to the derivation of a threshold wage, related to cattle production parameters and the discount rate, beyond which a household will engage in off-farm work in the long-run, and below which pasture will be the favored long-run activity. The all-or-nothing nature of the competition between activities in the steady state follows from how labor enters the relevant production functions.[124] Exogenous income favors leisure and crowds out labor in the steady state; this effect is mitigated by higher prices for the steady state production activities.

Chapter 2 also explored the nature of the model's optimal trajectory through analytics, looking first at a simpler, 'no-investment' version of the model. In the simpler model, household time allocation holds steady throughout the annuals only phase despite soil degradation. Eventually soil degradation makes off-farm work attractive. Once this happens, consumption and leisure decline at the same proportional rate. At the same time, annual cropping falls at a constant proportional rate determined by the degradation rate and the extent of diminishing returns to labor in annuals production. These findings from the off-farm work phase hold for the full investment model. Outside the off-farm

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[124]Since pasture expansion depends linearly on deforestation labor, and wage does not change with volume of labor, these terms drop out in the equation for marginal returns to time across categories.

work phase, however, consumption and leisure can move in opposite directions in the full model. Unlike in the no-investment model, moreover, leisure can rise, but its pace is limited by the discount rate and pasture degradation rate. Chapter 2 ended by presenting parameters and steady state values for a baseline model.

Chapter 3 looked at model dynamics more closely using numerical simulations, and built up from a simple dynamic model with only pasture and labor, to one that included wage labor, to a full model that also included annual cropping. Results showed that the labor-leisure trade-off in the pasture only model – where pasture labor grows over time at the expense of leisure – gave way to more complicated time trade-offs once wage labor was added. With the extra labor alternative, the household delays the transition to pasture, the returns to its time fall rather than rise over time, and consumption and leisure can both rise. Beyond the delay in the pasture transition, the household responds to the extra labor phase by changing how it manages relative growth of pasture and the herd: the lower the returns to the alternative activity, the more time the household allocates to pasture expansion during the extra labor phase, and the lower the herd stocking rate during the phase.

The nature of returns to time in the alternative labor activity also affects the optimal trajectory. The transition out of (constant returns) wage labor is swift and relatively linear compared to the phasing out of (decreasing returns) annuals labor. Leisure and consumption both rise throughout the pasture-wage labor model, with identical proportional changes during the off-farm work phase (due to the Cobb-Douglas form of the utility function). In the pasture-annuals labor model, leisure and consumption decline early on while the household is coping with soil degradation and

trying to start its herd. Pasture investment allows the trend to reverse. At first, leisure rises faster than consumption (in proportional terms), but by a narrowing margin until eventually consumption takes the lead. This occurs when the shadow wage reverses its early downward trend and starts to rise (approximately when leisure time first surpasses annual cropping labor). Following the household's utility timepath (but mitigated by the growth of pasture and herd assets, the farm's shadow value shows an initial decline, followed by a turnaround and rise to the steady state.

Chapter 4 began by summarizing how varying the model parameters affects the optimal trajectory. Findings included that relatively favorable economic returns for annuals, by delaying investment in pasture, mimic a 'no-investment' model in the early part of the trajectory: a period of no change in time allocation, falling consumption due to soil degradation, and overstocking (relative to long-run ideals) of any existing pasture. Relatively favorable returns for pasture, in contrast, hasten the drop-off in annuals and makes for a faster pasture build-up, generating sharper concurrent proportional gains in leisure and consumption that occur earlier. Because the discount rate weights how the household values future vs. present earnings, changing it switches the balance between the immediate pay-off (annuals) and deferred pay-off (pasture) activities. Exogenous income is not used to speed up herd growth, but rather boosts household leisure.

With Chapter 4's wage sensitivity simulations, the dissertation looked more directly at how dynamics affect household market participation. The section used the wage equivalent – the shadow wage (returns to household labor expressed in money terms) – to highlight the threshold external wage for the household's participation in the labor market. Results showed that, with a constant external wage, the dynamic

endogenous wage equivalent generated by household optimization means that the household can move into a situation where market participation becomes viable because of its investment in pasture. In other words, participation in the external labor market may be induced not by an exogenous change in an external market, but by a change in the household's value of labor due to pasture investment. In addition, the simulations demonstrated that attractive options for labor in the short-run slow down the pasture expansion process.

The particular features of the returns to time in each "bridging" activity for labor determine the order in which, how quickly, and with what degree of overlap the household transitions through the available activities. Technologies with constant returns to labor (like wage work) lead to a more abrupt start-up and phase-out. This leads to volatile time allocation behavior around those periods. Finally, the household prepares for the bridging phase through small changes early on in the trajectory, since to compensate for the lost investment it must realize more gains before discounting takes its toll. The expectation of an off-farm work phase thus encourages more annual cropping and leisure early in the trajectory, but then a faster drop-off in annual cropping and slower rise in leisure as well as pasture expansion during the off-farm work phase.

Chapter 4's policy simulations aimed to showcase the model's strengths for looking at policy issues. The first set of policy simulations showed how model structure could incorporate dynamics in parameters both on and beyond the farm. The changing wage scenarios showed how a (suddenly or gradually) rising exogenous wage would affect on-farm dynamic shadow wages and therefore land use trajectories. This added some volatility in time allocation and pasture management behavior, even though the

change was foreseen by the farmer. Other prices could realistically change in tandem with the wage, mitigating or exacerbating these effects depending on the net change in relative prices. Expected changes in relative output prices favoring pasture over time as the settlers move away from their early strong reliance on annuals would hasten the transition to pasture. Two scenarios (the gradual wage change and the aging household) resulted in the household building up, then drawing down pasture as an activity. They provided a look at how the household manages that drawdown by allowing pasture to degrade faster than it sells off the herd (so increasing stocking rates), essentially feeding off its earlier investment (keeping consumption levels relatively high) during the drawdown.

A simulation adding an activity (forest product collection) to the household's labor option demonstrated the model's flexibility to additional detail. The choice of activity for this simulation was informed by the debate around saving forests by adding (commercial value) to them.[125] In addition, the simulation demonstrates that space, which enters the baseline model implicitly but not explicitly, might be incorporated short of the computationally expensive step of tracking bits of land over time, by modeling links between land uses within relevant production functions. For the simulation, the forest product was given a price that allowed it to act as an alternative "bridging" activity towards long-run pasture. That pasture expansion itself directly lowered productivity of forest product collection resulted in a bridging activity that, like wage work, slowed the pasture build-up, and increased labor in annual cropping and leisure before the forest

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[125]Note, though, that the forest production function used in this simulation did not explicitly model possible overexploitation of the forest resource (harming future production or other public goods related to the forest).

product phase. Unlike under wage work, though, the forest product phase did not see identical proportional gains in leisure and consumption, but a narrower gap of leisure's growth over consumption than in the annuals phase. Thus, the bridging activity slowed pasture growth compared to the baseline, but not as much as a wage work phase, whose returns are not penalized as pasture expands.

Chapter 4's final set of policy simulations introduced bilateral trade in labor between two neighboring households differing in one parameter (to create the opportunity for gains from trade). In these scenarios, trade induces a 'landscape-level' change (where the landscape is narrowly drawn to encompass the two households) in each household's returns to time compared to the no-trade scenario. This is a rudimentary example of 'dual-level' dynamics involving changes in the local labor market as well as on-farm dynamics. Each dynamically optimizing household responds to a dynamic external environment, *and* that environment is shaped by their own actions. Critical to maximizing the gains from trade is the two households' capacity to leverage production activity across space, taking advantage of non-linearities in the relevant production functions (here, annuals and cattle) which apply on each farm.

With the parameters chosen, neither scenario generated substantial, but both generated some, gains from trade and deviations in time allocation (and thus land use) for the two-household system (taken to be the sum across households of time in each category). More specifically, trade between households of varying sizes (available time) led to trade that equalized the situations on both farms. The result was a trajectory that favored annuals – slightly more annuals early on slowed the transition to pasture for the system as a whole. In contrast, trading partners with different discount rates favored the

growth of pasture in the two-household system vis-à-vis a no-trade system. The trade scenario includes a faster initial build-up, and a second, more gradual phase of pasture expansion once annuals are out of the picture. In the initial build-up, the small-scale farmer still finds it attractive to expand pasture and have a herd. This then gets drawn down in the second phase, as the "small-scale farmer" (with the higher discount rate) trades increasing amounts of labor to the "rancher" (with the low discount rate). The trend proceeds until only the labor needed to maintain the lower threshold amount of pasture on the "small farm." The trading system allows households to enjoy less free time than would be possible in a stand-alone system, where their leisure preference sets a floor on the optimum.

5.2 Research conclusions

The story told by this dissertation and this model is one of deforestation as an investment decision amid labor scarcity, with results following from the household's drive to equalize returns to time across its activities. The overall pattern of successful investment tied to larger pasture capacity and increased forest loss – the greater the more land-extensive is the pasture operation – is familiar from other studies. This section highlights some principal conclusions from the research, looking at several broad categories – long-run behavior, behavior along the trajectory, and how relaxing constraints affects outcomes.

• *Determinants of long-run land use (wage vs. pasture and the discount rate)*

The household's trajectory tends toward one of two steady state outcomes. Either the household devotes all its discretionary time to working off farm for a constant wage, or it works completely on farm to maintain a given pasture level against losses from

degradation, living off sales from herd growth. This fate is determined by whether the long-run shadow wage for pasture surpasses the exogenous wage. The steady state pasture shadow wage is, in turn, determined by exogenous parameters such as the price of cattle, along with pasture technology parameters such as herd growth rate, degradation rate, and the efficiency of labor in converting land to, or maintaining existing, pasture. The only household characteristic relevant to this trade off is the discount rate. The all-or-nothing nature of the steady state stems from the fact that, for both activities, steady state returns to labor do not depend on the amount of labor employed.

• Determinants of long-run pasture capacity (price plays a secondary role)

While price plays a central role in determining whether the household chooses pasture as its long run activity, it has a more peripheral role in shaping what that activity looks like. Long-run pasture capacity depends on a number of characteristics of the production technology (namely pace of pasture degradation, the efficiency of labor in converting forest to pasture or in fighting pasture degradation, and the speed of herd growth) and the household (namely household size and relative preferences for consumption and leisure). A higher cattle price can affect long-run capacity only in the presence of exogenous income, boosting pasture capacity by mitigating its leisure-enhancing effect. The optimal long-run stocking rate, on the other hand, follows from the household balancing herd growth against the discount rate, and depends only on these two parameters.

• Determinants of on-farm deforestation trajectory (critical role of labor alternatives)

Other models and studies have uncovered various patterns of on-farm deforestation over time. Deforestation has been found to either: a) be strongly

frontloaded near the time of settlement (Walker 2003 by design; Fearnside 1984); b) be frontloaded, but taper off more gradually over time as the farm moves toward a steady state in land use (Thomas 2003); c) proceed fairly steadily after an initial establishment phase (Vosti et al. 2002); d) or to exhibit a bimodal pattern (two pulses of increased deforestation) (Brondizio et al. 2002). These last two studies also include some empirical evidence that settlers arriving later tend to deforest more. In Brondizio et al. (2002), the two-pulse pattern gives way to a larger single upward thrust of deforestation yet to resolve into a single, or multiple pulse), but Campari (2005) finds no such difference.

In our model, since the speed of the transition to pasture, and thus deforestation, is intimately linked to available labor alternatives, various patterns emerge depending on parameters. The pasture-only model mimics the pattern in Thomas (2003). In this model, however, in order to expand pasture the farmer must apply labor to both maintain existing pasture as well as expand into new areas. The presence of an alternative for labor changes the story, and slows the transition to pasture.

When that labor alternative is annuals as in the baseline, the household has a faster initial pace of pasture expansion. Once the household spends more time in leisure than in annual cropping (a sign of investment payoff), pasture expansion's pace slows. If pasture looks less attractive in the short term (or is less accessible), the model shows an initial holding pattern in annuals, and the pasture build-up occurs later, when soil degradation erodes annuals' productivity. The model reproduced this basic pattern under a variety of conditions, including better soils and a high discount rate. It would be

consistent with a double-pulse of land clearing such as Brondizio et al. (2002) describe.[126]

When the labor alternative is off-farm work, as in other studies, off-farm work takes pressure off the forest, in that it prolongs the transition to pasture. As formulated here, though, it will not affect long run pasture capacity unless the wage is high enough to wholly replace pasture as a long-run activity. Even in this case, pasture can appear as an investment in the shorter run. When off-farm work appears as a phase along the way to a pro-pasture labor steady state, pasture's gains are steadier, similar to those in the Vosti et al. (2002) study. Indeed, off-farm work heralds a phase of steady increase in leisure and consumption as well. Moreover, the household's changing involvement in off-farm work in the face of a constant wage underlines the point that the household's changing conditions – improving, in the case of pasture investment – at some point open up a previously closed off opportunity in off-farm work. "Market arrival" in this case occurs with no change in the exogenous market.

With multiple labor activities available, more complicated transition patterns appear, as alternative labor activities may overlap (e.g., the three-activity phase of the trajectory of the wage simulations as well as the forest product policy simulation). Land use patterns in the various phases are determined by the various alternative returns to labor over time – annuals' decline because of soil degradation, pasture's rise due to investment, the steady return from wage work, and forest product's decline over space linked to the rise of pasture.

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[126]The more short-lived the annuals phase, the closer the pulses. In the extreme this moves closer to the frontloaded 'single pulse' of the pasture-only model.

The introduction of a new phase, or a change in parameters that alters the timing of an existing phase along the trajectory to steady state pasture causes marked changes in the trajectory subsequent to the start-up point. This is not surprising because the diversion of labor disrupts pasture investment, with an effect that amplifies during the alternative labor phase (because of the nature of investment). More subtle are the trajectory changes prior to the start-up point, as the farmer adjusts land use behavior even in the much-constrained early period to take account of the changed trajectory.

• *Volatility of behavior along trajectory (byproduct of rational behavior)*

Dynamic optimization leads to rational shifting between land uses over time that can include short-term volatility in time allocation. This happens when the household prepares for its off-farm work phase by increasing pasture labor in the previous year. The volatility is the more extreme the longer access to the wage is delayed beyond the point at which the household would ideally choose to avail itself of the off-farm work option. This holds true even though exogenous parameter timepaths are completely foreseen.

It can also be rational for the farmer to temporarily overshoot optimal stocking rates. In the face of favorable conditions for annuals, the household responds by overstocking its pasture relative to the long-run optimum, then understocking during pasture expansion.

From the broadest perspective, the activity phases themselves can look like volatile behavior. This is most striking in the case of pasture, where various conditions (a long-run wage that only slightly beats out pasture or is expected to in the future, a household expecting to see available labor dry up, a household trading its labor with a

neighbor who has a lower discount rate) lead to investment in, then disinvestment of, pasture and cattle production.

• *Nature of constraints upon arrival (few options for the early settler)*

The finding that, upon arrival, the settler has few options echoes similar ones in other studies. In our model, the lack of alternatives for labor in the early phase severely constrains what the household does. In the pasture-only model, the household begins with a near equal division of labor and leisure despite a market preference for consumption over leisure precisely because returns to labor are so low before the pasture investment gets going. The baseline has an alternative to pasture that pays off in the short-run: annuals. Proportional allocation between work and leisure is initially close to matching relative preferences for consumption vs. leisure, with work garnering the lion's share of the household's time. The household chooses to sacrifice leisure during the critical period where it needs labor both to build up pasture and stave off losses in annuals from soil degradation.

Because the pay-off to pasture takes time, the initial time allocation is not terribly sensitive to even substantial changes in production or price parameters. Even under conditions relatively favorable for pasture, annuals figure largely in the 'work' category early on.[127] If the household prefers consumption, the household uses most of its time to cultivate annuals. While not explicit in this model, this would imply an initial burst of deforestation to clear land for annuals.

In the model, the farm's shadow value falls during the initial part of the annuals phase, with its dropping utility and still meager assets due to pasture and herd growth.

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[127]When the favorable conditions are due to a higher cattle price, higher upfront cost to start a herd will also constrain the household's early options.

The turnaround to rising consumption and leisure, with faster asset growth, heralds a rising farm value.

• *Relaxed household labor and capital constraints (varied trajectory effects)*

Relaxing factor constraints does make some difference. With a larger time endowment, households hasten the start-up and build-up of pasture to a higher level of capacity, carving out time (in proportional terms) mostly from annuals, and a little from leisure. Other studies have found that a larger household size does indeed increase levels of deforestation overall. In addition, though, where more labor-intensive investment alternatives (e.g., perennials) exist, having more manpower benefits the labor-intensive land use as a proportion of cleared land, while leaving pasture, where viable, as the predominant use. If a more labor-intensive investment option were included in this model, its dynamic labor returns, alongside the other alternatives, would determine whether or not it is adopted.

With some exogenous income, the household boosts its leisure time – with no labor market, it cannot use capital to relieve its labor constraint. This provides a possible explanation for a finding by Vosti et al. (2002) that puzzled the authors – that more resources on arrival translated into larger proportions of forest left on farm.

• *Effects of labor trading across households (an endogenous way to relax constraints)*

Trading labor does just a little to improve overall system benefits. The critical labor constraints still exist, and, in these simulations where trading households arrive simultaneously, both trading partners face the most severe labor constraints at the same time. Still, trade in labor opens up the possibility of leveraging across space, making use of identical production functions in play in two locations, to improve overall returns to

time. While increasing one household's labor endowment slightly favors pasture (as seen in the sensitivity analysis), allowing labor to flow between otherwise identical neighbors with different labor endowments slightly favors annuals and slows the transition to pasture. It erases heterogeneity between farms; after trade, both households exhibit identical land use and consumption patterns. Having neighbors with different time preferences, on the other hand, opens up trade that effectively raises the time preference of the system, and consequently favors pasture growth.

The simulation results depend on transaction costs to set up the terms of trade being less overall gains from trade; with the small number of traders here (two), a Coasian deal should be possible.

5.3 Directions for future research

This model has shortcomings, some already noted. The broad spectrum of alternatives for labor is only partially captured in the production choices included here. Modeling time to market production, for example, could lead to interesting shifts in patterns seen here. Although labor available to use cleared land drives decisions to deforest, seasonality in labor use has also been shown to matter to land use, and is glossed over here. In addition, the model's findings hinge critically on the forms of the production functions used for the alternative activities, which considerably (and unrealistically) limit the flexibility with which farmers respond to changing circumstances. The characterization also omits determinants of farm size beyond the key components examined here – generation of pasture capacity and herd growth – such as broader economies of scale and scope associated with production or marketing. Indeed,

it bypasses any spatially explicit link between production and land (a feature explored further below).

Also potentially significant to outcomes, the model simplifies the situation faced by the migrant settling a 'pristine' (completely forested) lot by focusing on ties (or lack thereof) to the labor market, sidestepping the absence or precariousness of early links to output markets. Relatedly, it presupposes certainty for the farmer, ignoring the uncertain, risk-ridden environment faced by settlers that so many studies have highlighted. With regard to trade, it ignores how strategic considerations might affect farmers' behavior.

These factors would be expected to be more important for a newly opened area earlier in the life of a colonization project, and for a given farm earlier in the settlement process. The model results highlight that this period could indeed be critical, since the shadow value of the farm is falling initially.[128] Farm-level shocks could prevent a smooth transition to investment, leading to a potential for distress sales. Broader shocks affecting conjuncts of farms could steer settlement and accompanying development away from afflicted areas.

The simplifications serve a purpose. A model under certainty is a precursor to one under uncertainty or one that applies a game theoretic approach, and reveals how much of observable patterns can be captured without introducing risk or more complicated assumptions about relationships between trading partners. Similarly, limiting the number of market limitations, especially as regards interhousehold trades, constitutes a logical first step from which to build a more nuanced model of market participation and/or emergence of local trading area. At the same time, since functioning

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[128]The model's treatment of deforestation may mean its results overstate this case, however, as is explained below.

factor markets typically lag those of outputs, the focus on labor provides some useful information for an intermediate situation.

Each of these shortcomings provides a direction for further model development. Despite these shortcomings, this relatively simple model generates broad results in line with the observable conversion of forest to pasture, and patterns within that trajectory that may be at first glance surprising, including holding pattern phases and volatility highlighted in the previous subsection.

One model shortcoming that could point the way towards valuable future research lies in its simple treatment of space, both on and off the farm. The model accounts for space on farm solely through its treatment of pasture. With expanding pasture capacity comes forest loss. In pasture capacity, the model employs a concept cut loose from pasture size. Therefore, the link to amount of forest felled must rest on additional assumptions, including the extent to which maintenance of pasture in the face of degradation involves clearing additional land. Because deforestation is labor intensive, a modified model could usefully split out the expansion of pasture capacity from its maintenance. This would distinguish between labor requirements for each, and establish a clearer tie between the expansion of capacity and loss of forest. It would also increase the model's flexibility to fit different circumstances regarding pasture maintenance.

The 'deforest to increase pasture capacity' approach also abstracts away from the land use trajectory commonly seen in the area, where a plot of land is deforested and used for annuals before being planted to pasture. This means that annuals cropping can take place without deforestation (though again some assumptions about how to interpret this situation could remedy this problem), but also misses the fact that annuals and pasture

can piggyback on the same forest clearing. Modeling this more accurately could make a difference in the severely time-constrained early days of the model household, perhaps lending more insight into factors that lead one household to a successful investment, while another fails to make this transition. Making this change, for instance, would push some of the investment value of generated pasture forward in the model, easing the drop in farm shadow value associated with the early period.

Such alterations would make the model more amenable to certain policy questions, including how the lot size distributed by the government compares to that desired by the farmer, and what factors affect land trades (and when do they indicate distress sales).

Off the farm, the model could incorporate space more concretely in the way it links the household to existing markets, as well as to other households. For example, a landscape of identical isolated households could face differing effective wages despite a common exogenous wage due to transaction costs related to distance. Modeling that distance cost would trigger an interesting spatial pattern of 'market arrival' occasioned by dynamic, household-level wage equivalents hitting the external effective wage at different points in time. Similarly, the two-household trading conjunct could be modeled in the presence of an exogenous wage to see how that affects trades and overall land use patterns.

More locally, transaction costs involved in labor trades could be more explicitly related to distance to establish regions of viable labor trading around a given farm, and try to build on insights here related to market deepening. Critical would be what happens to transaction costs as potential trading partners are added. This is similar to the

approach used by Berger (2001), who uses Monte Carlo simulations of trades (with different orderings of bilateral trade pairings) to generate landscape-level outcomes. Another direction for the labor trades would involve bringing in a game theoretic approach, perhaps including arrival time. These approaches to the labor trades could be extended to output products, perhaps shedding light on what types of local trades would best alleviate constraints in the early, pre-market period.

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