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The Valuation of Mortgage-Backed Securities

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Doctor of Philosophy in Management

by

Brett Radcliffe Dunn

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ABSTRACT OF THE DISSERTATION

The Valuation of Mortgage-Backed Securities

by

Brett Radcliffe Dunn
Doctor of Philosophy in Management
University of California, Los Angeles, 2020
Professor Mikhail Chernov, Co-Chair
Professor Francis A. Longstaff, Co-Chair

This dissertation focuses on a major challenge to asset pricing theory: the valuation of mortgage-backed securities.

In the first chapter of this dissertation (with Mikhail Chernov and Francis A. Longstaff), we develop a three-factor no-arbitrage model for valuing mortgage-backed securities in which we solve for the implied prepayment function from the cross-section of market prices. This model closely fits the cross-section of mortgage-backed security prices without needing to specify an econometric prepayment model. We find that implied prepayments are generally higher than actual prepayments, providing direct evidence of significant macroeconomic-driven prepayment risk premiums in mortgage-backed security prices. We also find evidence that mortgage-backed security prices were significantly affected by Fannie Mae credit risk and the Federal Reserve's quantitative easing programs.

In the second chapter, I study the relationship between funding liquidity and the valuation of mortgage-backed securities. Most of the financing for mortgage-backed securities occurs through a trade known as a dollar roll, the simultaneous sale and purchase of forward contracts on mortgage-backed securities that is analogous to a repurchase agreement. I develop a four-factor no-arbitrage model for valuing mortgage-backed securities that allows for the valuation of dollar rolls. Unlike previous models of the dollar roll, I allow for the possibility of a prepayment risk premium. I develop a new measure of mortgage specialness that is independent of prepayment risk premia and agency credit spreads. I find that specialness is related to measures of balance sheet constraints and primary dealer positions in mortgage-backed securities.

The dissertation of Brett Radcliffe Dunn is approved.

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University of California, Los Angeles

2020

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PUBLICATIONS

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Macroeconomic-Driven Prepayment Risk and the Valuation of Mortgage-Backed Securities, with Mikhail Chernov and Francis A. Longstaff (2018), Review of Financial Studies, 31(3), 1021–1056.

CHAPTER 1

Macroeconomic-Driven Prepayment Risk and the Valuation of Mortgage-Backed Securities

This chapter is a reprint of Mikhail Chernov, Brett R. Dunn, Francis A. Longstaff; Macroeconomic-Driven Prepayment Risk and the Valuation of Mortgage-Backed Securities, *The Review of Financial Studies*, Volume 31, Issue 3, 1 March 2018, Pages 1132–1183, published by Oxford University Press. Here, we develop a three-factor no-arbitrage model for valuing mortgage-backed securities in which we solve for the implied prepayment function from the cross-section of market prices. This model closely fits the cross-section of mortgage-backed security prices without needing to specify an econometric prepayment model. We find that implied prepayments are generally higher than actual prepayments, providing direct evidence of significant macroeconomic-driven prepayment risk premiums in mortgage-backed security prices. We also find evidence that mortgage-backed security prices were significantly affected by Fannie Mae credit risk and the Federal Reserve's quantitative easing programs.

Macroeconomic-Driven Prepayment Risk and the Valuation of Mortgage-Backed Securities

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We develop a three-factor no-arbitrage model for valuing mortgage-backed securities in which we solve for the implied prepayment function from the cross-section of market prices. This model closely fits the cross-section of mortgage-backed security prices without needing to specify an econometric prepayment model. We find that implied prepayments are generally higher than actual prepayments, providing direct evidence of significant macroeconomic-driven prepayment risk premiums in mortgage-backed security prices. We also find evidence that mortgage-backed security prices were significantly affected by Fannie Mae credit risk and the Federal Reserve's quantitative easing programs. (*JEL* G12, G13, G21)

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A mortgage-backed security is a securitized claim to the principal and interest payments generated by a pool of mortgage loans. Mortgage-backed securities have traditionally been issued either by agencies such as Fannie Mae, Freddie Mac, and Ginnie Mae, or by private issuers. As of the end of 2016, the total notional amount of agency mortgage-backed securities outstanding was $7.545

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trillion, making this market one of the largest sectors of the global fixed income markets.[1] Agency mortgage-backed securities have the attractive feature that the timely payment of principal and interest is backed by either an implicit or explicit government guarantee. Thus, the primary focus in agency mortgage-backed security valuation is on the timing of prepayments.

This paper advocates and implements a no-arbitrage approach to the valuation of mortgage-backed securities. Specifically, we propose a model that provides internally consistent valuation across the entire cross-section of mortgage-backed securities. Our strategy closely parallels that of standard affine term structure models which provide no-arbitrage valuation of bonds across all maturities. We solve for an implied risk-neutral prepayment function using the entire cross-section of mortgage-backed security prices. A key advantage of this approach is that by studying the implied prepayment function, we can identify the factors that the market views as important drivers of prepayment risk as well as the risk premiums associated with those factors. Thereby, we avoid modeling actual prepayment behavior via an econometric model, a daunting task by any measure. To account for the liquidity of mortgage-backed securities and perceived credit risk of the agency guaranteeing them, we allow for the possibility that mortgage cash flows may be discounted at a different rate than Treasuries. We apply our model to a broad cross-section and time series of actively traded mortgage-backed securities issued by Fannie Mae.

A number of important results emerge from the analysis. First, we find that our no-arbitrage model fits the cross-section of mortgage-backed security prices surprisingly well. The median root-mean-square error (RMSE) across the entire coupon stack is 25.7 cents per $100 notional, which is on the same order of magnitude as the bid-ask spread for mortgage-backed securities. This accuracy compares well to previous generations of valuation models for mortgage-backed securities. This is achieved using only a simple two-factor implied prepayment model instead of a formal econometric prepayment model, which often includes many explanatory variables. Our results indicate that the pricing of mortgage-backed securities in the market may be much more rational than is commonly believed among market practitioners.

Second, we find that implied risk-neutral prepayments behave very differently from actual prepayments. Furthermore, implied prepayment rates are not simply scaled versions of empirical prepayment rates. The average implied prepayment rate across all mortgage-backed securities in our sample is 25.13% per year. In contrast, the corresponding average empirical prepayment rate is 20.96%. The difference between the implied and empirical prepayment rates provides direct evidence that the market incorporates significant prepayment-related risk premiums into the prices of mortgage-backed securities.

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[1] See www.sifma.org/research/statistics.aspx.

Third, we find that implied prepayments are driven not only by interest rates, but also two additional macroeconomic risk factors—turnover and rate response. The turnover rate reflects prepayments occurring for exogenous reasons unrelated to interest rates, but possibly correlated with macroeconomic fluctuations. Examples include adverse income shocks or unemployment resulting in a move or a foreclosure, negative shocks to housing values resulting in underwater borrowers strategically defaulting on non-recourse loans, or homeowners with appreciated property taking cash-out mortgages to extract home equity. The rate response factor represents the time variation in the sensitivity of prepayments to mortgage refinancing incentives. For example, borrowers may be less able to refinance into a lower mortgage rate after declines in housing prices, during recessions in which borrowers' income or credit may have been impaired, or during periods in which mortgage lending standards are tightened. Intuitively, declines in the rate response factor can be viewed as a marketwide form of burnout (in contrast to the security-specific type of burnout often incorporated into econometric prepayment models).

Fourth, we study the determinants of the prepayment risk premium by decomposing it into the risk premiums associated with the turnover and rate response factors. We find that the turnover factor carries a significant positive premium throughout the entire sample period, consistent with the systematic nature of turnover risk. The risk premium for the rate response factor is also positive on average, but temporarily takes on negative values during the refi waves of 2001–2005. This result raises the possibility that a borrower's ability to refinance during the refi waves may have been influenced by housing values in addition to standard income and credit considerations.

Fifth, we find that cash flows from mortgage-backed securities are discounted at a rate 65.5 basis points (bps) higher on average than are cash flows from Treasuries. This spread varies significantly through time and is strongly correlated with the credit spread between Fannie Mae debt and Treasuries. Furthermore, the spread is significantly related to supply-related factors such as Federal Reserve purchases of mortgage-backed securities during its quantitative easing programs and the volume of mortgage settlement fails among primary dealers. These results provide direct evidence that agency credit/liquidity spreads influence the pricing of mortgage-backed securities.

Sixth and finally, we apply the fitted model to a number of interest-only/principal-only securities as an out-of-sample test of the framework. We find that the model closely matches the market prices of these securities.

1. Related Literature

Because agency mortgage-backed securities guarantee the timely payment of principal and interest, there is no direct borrower-related credit risk—a default is simply a prepayment from the investor's perspective. Instead, the primary sources of risk are interest rate changes, agency credit spreads, and the timing of prepayments. The valuation of mortgage-backed securities, however, is challenging because the reasons for terminating and prepaying a mortgage may depend on factors besides interest rates such as housing prices, employment status, or family size. For reviews of the literature, see Kau and Keenan (1995), Capone (2001), Hayre (2001), Wallace (2005), and Fabozzi (2016).

The first generation of pricing models was pioneered by Dunn and McConnell (1981a, 1981b) and extended by Brennan and Schwartz (1985). This framework approaches the valuation of mortgage-backed securities from the perspective of contingent claims theory. In particular, this approach models mortgage prepayments as the result of a borrower attempting to maximize the value of an implicit interest rate option. Dunn and Spatt (2005) and Stanton and Wallace (1998) extend the approach to model the prepayment decision as the result of minimizing lifetime mortgage costs in the presence of refinancing costs. The models in these papers imply an upper bound on mortgage prices that is often violated empirically, as demonstrated by Stanton (1995) and Boudoukh et al. (1997). Later papers add frictions to allow for higher mortgage prices and consider the value of the prepayment option jointly with the option to default. Important contributions are Titman and Torous (1989), Kau et al. (1992), Kau and Slawson (2002), Downing, Stanton, and Wallace (2005), Longstaff (2005), and many others. An important drawback of this modeling approach is that actual mortgage cash flows and mortgage-backed security prices often diverge significantly from those implied by these types of models.

The second generation of mortgage-backed security pricing models takes a more empirical approach. Typically, these models begin with a detailed econometric model of the historical behavior of prepayments, including elements such as geography, seasoning, burnout, seasonality, and other macroeconomic factors. Key examples of this approach include Schwartz and Torous (1989, 1992, 1993), Richard and Roll (1989), and Deng, Quigley, and Van Order (2000). In this framework, interest rate paths are simulated (under the risk-neutral probability measure) and the econometric prepayment model (estimated under the actual probability measure) is applied to specify the cash flows along each interest rate path. However, prepayments in these models are driven exclusively by interest rate changes, thus there is no scope for a separate prepayment risk premium.[2] In addition, market participants do not agree about which econometric prepayment model to use in projecting prepayments. Carlin, Longstaff, and Matoba (2014) show that there is major disagreement between dealers about forecasted prepayment rates. Forecasting actual prepayment rates is a difficult task that is fraught with many challenges and difficulties. Furthermore, these models often give prices that diverge

[2] That prepayment in these models is exclusively driven by interest rates does not imply that previous researchers were unaware that additional factors may be important. Rather, it illustrates the difficulty of incorporating additional macroeconomic factors into these types of second generation models. We are grateful to the referee for this insight.

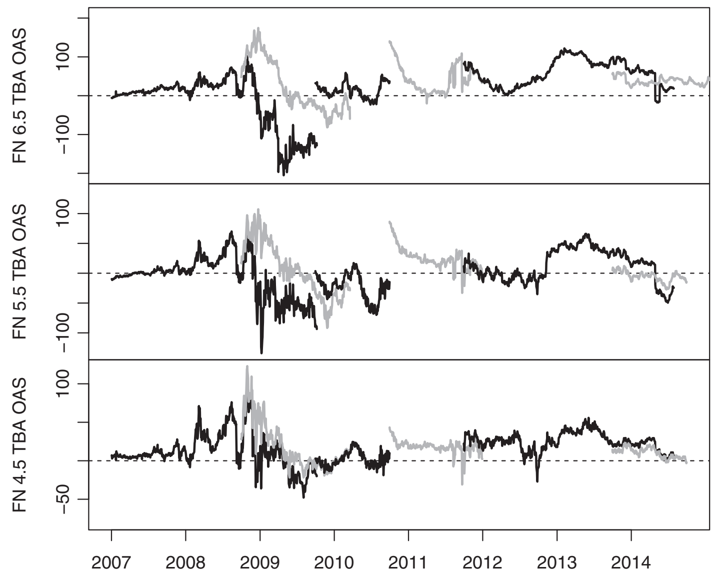

Figure 1
Effects of prepayment model changes on option-adjusted spreads
This figure shows the option-adjusted spread (OAS) in basis points for FNMA 4.50%, 5.50%, and 6.50% mortgage-backed securities implied by the series of prepayment models used by a specific major Wall Street dealer. Each line, alternating black and gray, represents a different version of the dealer's prepayment model. During the time period illustrated, the dealer used six different versions of its prepayment model. The option-adjusted spread is highly model dependent, and updates to the prepayment model can lead to large differences in the option-adjusted spread.

significantly from market prices, and can only be reconciled by introducing option-adjusted spreads into the framework.

Option-adjusted spreads are often more volatile than the underlying mortgage-backed security prices. This is shown in Figure 1, which plots the time series of option-adjusted spreads for FNMA 4.50%, 5.50%, and 6.50% mortgage-backed securities as given by the sequence of pricing models used by a major Wall Street mortgage dealer. As shown, the dealer changed its model frequently during the 2007–2015 period, primarily because the prior version of the model was failing to capture current market prices. The plot shows that changes in the model are often associated with large discontinuities in the time series of the option-adjusted spread that can be on the order of 50 bps or higher. This behavior in the option-adjusted spread, even when holding the dealer fixed, provides a motivation for basing empirical analysis on mortgage-backed security prices directly, rather than on option-adjusted spreads.

Several recent papers modify the basic econometric prepayment framework by allowing the model to depend on parameters implied from market prices. Specifically, a number of these papers allow the prepayment rate given by the econometric model to be scaled by a multiplier implied from the option-adjusted spreads of interest-only/principal-only securities. This approach is known as the implied-prepayment or break-even-prepayment model. Examples of this approach include Cheyette (1996), Chen (1996), Chan (1998), and Chaudhary (2006). A key advantage of this framework is that it allows for the possibility of a separate prepayment risk premium since the implied or risk-neutral prepayment rate need not equal the actual prepayment rate. This framework, however, has the drawback that a separate calibration is required for each pair of interest-only/principal-only securities—the implied multiplier is different for each pair of securities. Thus, this approach cannot provide consistent no-arbitrage pricing across the cross-section of mortgage-backed securities with varying coupon rates (the coupon stack). Furthermore, this framework is still tied to a specific econometric prepayment model. Levin and Davidson (2005) allow for two multipliers in scaling the components of their econometric prepayment model that they designate as turnover and refi risk. They also provide an example of how their model can be applied to the cross-section of mortgage coupon rates. Thus, their paper has some similarities to ours. Their approach is based on option-adjusted spreads, does not impose the no-arbitrage restriction, and depends on a specific econometric prepayment model.[3]

Although we primarily focus on developing a no-arbitrage valuation framework for mortgage-backed securities, some of our results have parallels in the recent literature on whether the expected returns of mortgage securities include prepayment risk premiums. For example, Gabaix, Krishnamurthy, and Vigneron (2007) study the interest-only strips market and document that their option-adjusted spreads covary with the moneyness of the market, consistent with a prepayment risk premium and the existence of specialized mortgage-backed security investors. An interesting paper by Boyarchenko, Fuster, and Lucca (2016) calibrates the break-even-prepayment model to the option-adjusted spreads of individual pairs of interest-only/principal-only strips. They find evidence of prepayment risk premiums in mortgage-backed securities. In particular, they find that prepayment risk premiums explain the cross-sectional smile in option-adjusted spreads and infer that the time variation in the implied option-adjusted spreads is due to a non-prepayment-related factor. Diep, Eisfeldt, and Richardson (2016) study Treasury-hedged mortgage-backed security returns and also find evidence of time-varying prepayment risk premiums. Furthermore, these prepayment risk premiums change signs over time in response to the relative supply of discount and premium mortgage-backed securities in the market.

Our paper provides a complementary perspective to this literature in several ways. First, Gabaix, Krishnamurthy, and Vigneron (2007), Boyarchenko, Fuster, and Lucca (2016), and Diep, Eisfeldt, and Richardson (2016) focus on the risk premiums in the expected returns of individual securities. In contrast, we focus on the risk premiums pertaining to the marketwide factors driving mortgage prepayments. We are able to measure these factor risk premiums by

[3] Other important contributions to this literature include Linetsky (2004), Goncharov (2006), Gorovoy and Linetsky (2007), Malkhozov et al. (2014), Hanson (2014), and Song and Zhu (2016).

comparing risk-neutral prepayment rates with empirical prepayment rates. By focusing on marketwide factor risk premiums, however, our approach does not allow us to study directly the cross-sectional structure of risk premiums in expected returns.[4] Second, Gabaix, Krishnamurthy, and Vigneron (2007), and Boyarchenko, Fuster, and Lucca (2016) study prepayment risk premiums through the lens of option-adjusted spreads. In contrast, our approach does not require the estimation of a formal econometric prepayment model. An implication of this, however, is that the implied prepayment model needs to be simple enough to be identified from the cross-section of TBAs in the market.[5] Fortunately, the results suggest that even a simple specification such as ours is able to capture the pricing of TBAs, IOs, and POs fairly accurately. We note, however, that our one-factor model of the U.S. Treasury term structure is limited in its ability to hedge the interest-rate risk of mortgage-backed securities. A multifactor model similar to that used by Boyarchenko, Fuster, and Lucca (2016) might be more appropriate for this purpose.

2. U.S. Agency Mortgage-Backed Securities

Agency mortgage-backed securities are issued by Fannie Mae (FNMA), Freddie Mac (FHLMC), or Ginnie Mae (GNMA).[6] Fannie Mae and Freddie Mac are government-sponsored enterprises (GSEs), whereas Ginnie Mae is a wholly owned government corporation. The U.S. agency mortgage-backed securities market is among the largest and most liquid bond markets worldwide. Furthermore, more than 70% of the $9.8 trillion U.S. home mortgage market serves as collateral for agency mortgage-backed securities. Immediately prior to the financial crisis of 2007–2008, private financial institutions accounted for more than 50% of U.S. mortgage-backed security issuance. Since the crisis, however, "private label" issuance has declined dramatically and now represents less than 4% of total mortgage-related issuance. In contrast, agency mortgage-backed security issuance has grown rapidly; the total notional size of the agency mortgage-backed security market increased 58% from 2006 to 2015.[7] In this section, we review the key features of agency mortgage-backed securities.

2.1 Credit quality

In exchange for monthly fees, the agencies guarantee the timely payment of mortgage interest and principal. The guarantee protects investors from defaults

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[4] To do so, for example, would require measuring the conditional default probability of Fannie Mae under both the actual and risk-neutral measures. We do not have sufficient observations to measure the actual conditional default probability of Fannie Mae.

[5] For example, we are limited in our ability to incorporate complex patterns of seasoning, burnout, and geographical concentration, etc., into the implied prepayment model. We are grateful to the referee for this observation.

[6] Fannie Mae, Freddie Mac, and Ginnie Mae refer to the Federal National Mortgage Association, the Federal Home Loan Mortgage Corporation, and the Government National Mortgage Association, respectively.

[7] See Federal Reserve Board Flow of Funds Table L.217 and www.sifma.org/resea rch/statistics.aspx.

on the underlying mortgages since delinquent mortgages must be purchased out of the trust at par by the issuer. This means that a default appears as a prepayment from an investor's perspective. Because GNMA securities carry the full faith and credit guarantee of the United States, their credit quality should be the same as that of U.S. Treasuries. FNMA and FHLMC securities carry a credit guarantee from the issuing GSE rather the United States. Historically, the GSE guarantee was viewed as an "implicit" government guarantee because investors believed that the government would back the agencies in times of stress. This view was validated in September 2008 when the government placed FNMA and FHLMC in conservatorship and provided them with unlimited access to collateralized funding. Both FNMA and FHLMC are supervised and regulated by the Federal Housing Finance Agency.[8]

2.2 Mortgage-backed security cash flows

In this paper, we focus on agency mortgage-backed securities backed by pools of fixed-rate mortgages. A fixed-rate mortgage is structured so that the borrower is obligated to make the same payment each month, consisting of interest and principal. In general, fixed-rate mortgages can be prepaid at any time without penalty. Each month, therefore, a pool of mortgages generates cash flows consisting of scheduled interest, scheduled principal, and possibly prepaid principal. A pass-through mortgage-backed security distributes to investors the principal and interest payments from the underlying mortgage loans, less guaranty and servicing fees. Because the guaranty and servicing fees are based on the outstanding balance, these fees decline over the life of the mortgage.[9]

Mortgage servicers collect and aggregate payments from the underlying mortgage loans and pass the payments to the mortgage-backed security trust. Mortgage payments are due on the first of the month (with a grace period determined by state law). Investors, however, receive the payments after a delay of 14, 19, or 24 days, depending on the mortgage-backed security program. If a loan becomes delinquent, servicers advance scheduled principal and interest until either the loan becomes current or is bought out of the trust at par. Servicers retain a monthly fee based on a percentage of the outstanding mortgage balance at the beginning of the month. This fee is often referred to as a "servicing strip" because the cash flows resemble an interest-only strip. In the FNMA, FHLMC, and GNMA II programs, mortgages with different gross coupons can be pooled together as long as the net coupon (gross coupon minus servicing and guaranty fees) is identical among all the loans in the mortgage pool. In the GNMA I program, the gross coupon is always 50 bps higher than the net coupon.

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[8] We note, however, that the current FNMA single family prospectus explicitly states that its certificates are not guaranteed by the United States and do not constitute a debt or obligation of the United States. Furthermore, the prospectus raises the possibility that if FNMA were to emerge from, and then later reenter, conservatorship, there is no assurance that the subsequent receiver or conservator would not repudiate the current guaranty.

[9] See www.fanniemae.com, www.freddiemac.com, and www.ginniemae.gov for more information about agency securitization programs.

2.3 Agency mortgage-backed security trading

Agency mortgage-backed securities trade on either a to-be-announced (TBA) basis or a specified-pool basis. The TBA market is a highly liquid forward market and accounts for 90% of all mortgage-backed security trading. From 2007 to 2014, the daily trading volume of U.S. agency mortgage-backed securities averaged $276 billion, which compares well with the $525 billion daily trading volume for U.S. Treasuries. Typically, pass-throughs are traded as specified pools if they command a premium over TBAs or if they are ineligible for TBA delivery.[10]

Similar to Treasury futures, a buyer of a TBA agrees to the trade without knowing the exact pools that will be delivered. Instead, the buyer and seller agree to six parameters: price, par amount, settlement date, agency program, mortgage type, and coupon. TBA trades generally settle to a monthly schedule set by the Securities Industry and Financial Markets Association (SIFMA). Nearly all TBA trades occur with settlement dates less than or equal to three months forward. Two days prior to the settlement date of the trade, the seller notifies the buyer of the exact pools that will be delivered (the 48-hour rule). The pools are then exchanged for the cash payment on the settlement date.

Market participants generally adhere to standards referred to as the "Good Delivery Guidelines" maintained by SIFMA. These guidelines specify the eligible collateral for a TBA trade and various operational guidelines such as the number of bonds per million dollars notional of a trade, the allowable variation in the delivery amount, and the costs of failing to deliver. TBA trades may also be executed with stipulations such as production year, weighted average maturity (WAM), weighted average loan age (WALA), FICO score, loan-to-value ratio, or geographic distribution. A stipulated TBA trade, however, would likely occur at a price higher than an unstipulated TBA (if the stipulations provide favorable prepayment characteristics).

2.4 Quantitative easing programs

Table 1 provides a listing of the major events in the agency mortgage-backed securities market during the study period. Among the most significant of these events are the Federal Reserve's quantitative easing programs, commonly known as QE I, QE II, and QE III. The first program, QE I, was announced on November 25, 2008 and directed the purchase of up to $500 billion of agency mortgage-backed securities and $100 billion of GSE debt. The stated goal of QE I was to reduce the cost and increase the availability of credit for the purchase of houses. QE I was expanded on March 18, 2009 to allow additional purchases of up to $750 billion of agency mortgage-backed securities and $100 billion of agency debt. The QE II program was announced on November 3, 2010 and

[10] Trading volume data comes from FINRA TRACE https://www.finra.org/indus try/trace/structure-product-activity-reports-and-tables. See Vickery and Wright (2013) for a discussion of the TBA market. Also see Carlin, Longstaff, and Matoba (2014). See Gao, Schultz, and Song (2017) for a discussion of the specified pool market.

Table 1
Major events in the agency mortgage-backed securities market

2002	Sep–Dec	High levels of refinancing activity after Federal Reserve lowers interest rates.
2003	Jan–Jun	Refinancing activity continues and reaches historically high levels.
2005	Jan–Jun	Mortgage delinquency rates reach historically low levels.
2007	Jun–Jul	Two Bear Stearns MBS funds suffer large losses and are liquidated. S&P places 612 subprime CDOs on creditwatch.
2008	Mar	Financially distressed Bear Stearns avoids bankruptcy by being acquired by JP Morgan.
	Jul	Federal Reserve Bank of New York is authorized to lend to FNMA and FHLMC if need arises.
	Sep	FNMA and FHLMC are placed into conservatorship, Lehman Brothers defaults.
	Nov	Federal Reserve announces QE I program to purchase up to $500 billion of agency MBS.
2009	Mar	Home Affordable Refinance Program and Stability Plan announced, making refinancing easier for high LTV loans.
	Mar	Federal Reserve expands QE I program to purchase up to an additional $750 billion of agency MBS.
	Dec	Treasury lifts all caps on the amount of FNMA and FHLMC preferred stock it may hold.
2010	Mar	QE I purchases of agency MBS ends.
	Aug	FOMC agrees to keep Fed holdings of securities at constant levels by reinvesting cash flows in Treasuries.
	Nov	Federal Reserve announces QE II program to purchase up to $600 billion of Treasuries.
2011	Jun	QE II purchases of Treasuries ends.
	Sep	Maturity Extension Program "Operation Twist" announced. Agency MBS cash flows to be reinvested in agency MBS.
2012	Sep	Federal Reserve announces QE III program, an open-ended program to purchase up to $40 billion of agency MBS per month.
2013	Jun	Ben Bernanke announces "tapering" of QE programs, Dow drops 659 points.
2014	Oct	QE III purchases of agency MBS and Treasuries ends.

Sources: https://www.stlouisfed.org/financial-crisis/full-timeline, https://research.stlouisfed.org/publications/review/13/01/Fawley.pdf, http://www.federalreserve.gov/releases/chargeoff/delallsa.htm.

authorized the purchase of up to $600 billion of longer-term Treasury securities. The QE III program was announced on September 13, 2012 and directed the purchase of up to $40 billion per month of agency mortgage-backed securities and $45 billion per month of Treasury securities. These programs had large effects on the supply of mortgage-backed securities in the market.[11]

3. Data

The primary data for the study consist of monthly prices (observed at the end of each month) from the TBA market for FNMA mortgage-backed securities with varying coupons. The sample period is January 1998 to September 2014. The data are obtained from a proprietary data set compiled by a major Wall Street mortgage-backed security dealer. However, we have cross validated the proprietary data with prices publicly available in the Bloomberg system and found the two sources to be very similar. To insure that we include only prices for actively traded mortgage-backed securities, we limit the data set to mortgage-backed securities with coupon rates that are within 300 bps of the current

[11] For a detailed discussion of the effects of the quantitative easing programs, see Gagnon, Raskin, Remache, and Sack (2011), Krishnamurthy and Vissing-Jorgensen (2011, 2013), Christensen and Rudebusch (2012), Thornton (2014), and Christensen and Gillan (2016).

Table 2
Summary statistics for FNMA mortgage-backed securities

Coupon	Average moneyness	Average CPR	Minimum price	Average price	Maximum price	N
2.50	−0.416	4.794	90.566	96.557	103.219	37
3.00	−0.146	2.449	89.258	98.382	105.555	49
3.50	0.057	6.637	92.250	100.048	107.250	70
4.00	0.470	9.355	87.688	102.255	107.758	74
4.50	0.055	10.244	90.609	99.804	108.313	137
5.00	0.493	15.543	93.484	101.862	111.047	145
5.50	0.345	14.703	86.500	100.816	111.969	184
6.00	0.656	18.082	89.813	102.026	113.031	185
6.50	0.830	21.787	92.531	102.496	113.219	160
7.00	1.208	29.586	94.875	103.632	113.906	150
7.50	1.499	33.953	97.094	103.975	109.563	132
8.00	1.881	34.790	99.188	104.705	108.250	120
8.50	2.104	35.830	101.031	104.974	108.688	91
9.00	2.231	41.759	102.500	105.229	107.563	58
9.50	2.374	22.478	103.000	105.478	107.188	35

This table reports summary statistics for FNMA mortgage-backed securities with the indicated coupon rates. Average moneyness denotes the average difference between the coupon rate and the current coupon mortgage rate. Average CPR denotes the average 3-month conditional prepayment rate. N denotes the number of observations. The sample consists of monthly observations for the period from January 1998 to September 2014.

coupon mortgage rate.[12] Furthermore, we only include prices for pools that trade as general collateral in the TBA market—we do not include prices for any mortgage-backed security that trades with a pay-up in the specified pools market.[13] The data set also includes 1-month and 3-month horizon conditional prepayment rate (CPR) information for each coupon.

Table 2 presents summary statistics for the data. As shown, the sample includes mortgage-backed securities with coupons ranging from 2.50% to 9.50%. Of course, not all coupons are actively traded throughout the entire sample period. The higher coupon mortgage-backed securities appear during the early part of the sample period when mortgage rates were considerably higher, and vice versa for the lower coupon mortgage-backed securities.

We also collect data for a wide variety of macroeconomic, mortgage market, and financial variables that will be used in the analysis throughout the paper. The appendix provides a description of each of these variables and the sources of the data. Finally, we collect historical data on Treasury constant maturity rates from the Federal Reserve H.15 release. We use a standard cubic spline approach to bootstrap the prices of zero-coupon bonds $D(t)$ for maturities ranging up to

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[12] Ideally, we would like to have a larger cross-section of coupon rates from which to estimate the model. We note, however, that the results are very similar when we use a more restrictive filter on the coupon rates included in the sample. See the discussion in Section 10.3.

[13] As discussed by Song and Zhu (2016), participants in the TBA market have incentives to deliver the cheapest collateral at settlement. This has little effect on our results, however, since we focus exclusively on the broad cohort of securities that are currently cheapest to deliver and do not carry a pay-up premium. Furthermore, a buyer in the TBA market can always stipulate delivery of the currently cheapest-to-deliver securities without having to pay a premium. A review of the quote sheets provided by a number of major dealers suggests that individual mortgage-backed securities can begin trading with a pay-up as small as 0.50 to 2.00 32nds. This places an upper bound on how much variation there can be in the values of the securities in the cohort of securities deliverable in the TBA market.

30 years for each month during the sample period (the methodology is described in the appendix).

4. Valuation Framework

In valuing mortgage-backed securities, we use a reduced-form framework in which an instantaneous prepayment process p_t plays the central role. Specifically, p_t is the fraction of the remaining notional balance of the underlying mortgage pool that is prepaid each instant. Thus, p_t can be viewed as a prepayment intensity or hazard rate. Our approach will be to solve for the implied value of p_t and its dynamics from the cross-section and time series of prices of mortgage-backed securities with different mortgage rates.

For expositional clarity, we assume for the present that mortgage cash flows are paid continuously and that the fixed mortgage rate m on the mortgages in the underlying pool is the same as the coupon rate on the mortgage-backed security. Let c denote the payment on a mortgage with an initial principal balance of one. Since the present value of the mortgage equals one at inception,

$$1 = c \int_0^T e^{-mt} \, dt, \tag{1}$$

$$= (c/m)(1 - e^{-mT}), \tag{2}$$

and the mortgage payment c is,

$$c = \frac{m}{1 - e^{-mT}}. \tag{3}$$

The mortgage payment c includes both interest and scheduled principal. Let I_t denote the principal balance of the mortgage at time t. The change in the principal balance is just the difference between the interest on the mortgage balance and the mortgage payment,

$$dI_t = (mI_t - c) \, dt. \tag{4}$$

Solving this first-order differential equation subject to the initial condition implies

$$I_t = \frac{1 - e^{-m(T-t)}}{1 - e^{-mT}}. \tag{5}$$

Now, consider a mortgage-backed security where the individual mortgages in the underlying pool are all T-year fixed-rate mortgages. Without loss of generality, we normalize the initial notional balance of the pool to be one. We denote the remaining notional balance of the underlying pool at time t as N_t, which, given the definition of p_t, can be expressed as

$$N_t = \exp\left(-\int_0^t p_s \, ds\right). \tag{6}$$

In turn, the remaining principal balance of the underlying pool is given by $N_t I_t$. It is important to distinguish between the remaining notional amount and the principal balance since mortgage payments are based on the original notional amount of the mortgages while prepayment cash flows are based on the remaining principal balance. The product $N_t I_t$ reflects both the effect of prepayments and, through Equation (4), the effect of scheduled principal payments on I_t.

Finally, let $F(m,T)$ denote the value of a mortgage-backed security where the underlying mortgages have a mortgage rate of m and maturity of T. The value of the mortgage-backed security at time zero is formally given by

$$F(m,T) = E^Q \left[\int_0^T \exp\left(-\int_0^t r_s + w_s \, ds \right) N_t \, (c + p_t \, I_t) \, dt \right], \tag{7}$$

where $E^Q[\,\cdot\,]$ denotes expectation under the risk neutral probability measure and r_t is the riskless interest rate. Following Duffie and Singleton (1997, 1999), Longstaff, Mithal, and Neis (2005), and many others, w_t plays the role of a credit/liquidity spread. The rationale for including w_t in the model is to allow for the possibility that cash flows from agency mortgages may be discounted a higher rate than Treasury cash flows, either because the credit of the agency may not be as strong, or because agency mortgages may be less liquid than Treasuries.

5. Prepayment Function

To complete the valuation framework for mortgage-backed securities, we need to specify the prepayment process p_t. Before doing this, however, it is useful to first consider some of the stylized facts about actual prepayment rates.

To illustrate the relation between prepayments and refinancing incentives, Figure 2 plots the prepayment rates for FNMA mortgage-backed securities as a function of the refinancing incentives for these securities. As shown, there is a strong relation between the prepayment rate and the refinancing incentive. When the coupon rate on the mortgage is lower than the current market rate, the borrower has no incentive to refinance. When the coupon rate is higher than the current market rate, the borrower may be able to reduce his mortgage costs by refinancing. Interestingly, the relation between prepayment rates and the refinancing incentive has the appearance of a piecewise linear function similar to that of a call option payoff.

In particular, when the prepayment option is out of the money, the relation is flat, although generally not zero. In fact, the prepayment rates for these mortgage-backed securities can be as high as 10% to 20%, because borrowers often prepay mortgages for reasons other than to reduce mortgage costs. For example, borrowers often prepay mortgages even when the market rate is higher than their mortgage rate for exogenous reasons such as a retirement or a career-related move. Also, borrowers may refinance into a higher mortgage rate to extract home equity after an increase in housing prices. During the recent financial crisis, a major source of exogenous prepayments has been the high rate

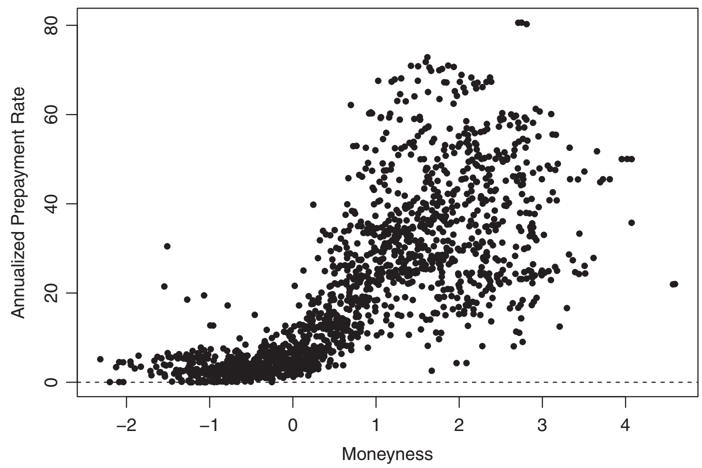

Figure 2
Prepayment rates for FNMA mortgage-backed securities
This figure plots the 3-month prepayment rates for FNMA mortgage-backed securities against the moneyness of the mortgage-backed securities. Moneyness is expressed in percentage points. The prepayment rates are expressed as annualized percentages of the outstanding principal balance of the mortgage-backed security. The data consist of monthly observations for all liquid coupons over the January 1998 to September 2014 sample period.

of foreclosures throughout the United States. A foreclosure results in the pass through of the entire remaining mortgage balance to the holders of an agency-guaranteed mortgage-backed security. Thus, foreclosures trigger prepayments for agency mortgage-backed securities.

When the prepayment option is in the money, the relation is generally increasing, but spreads out as the price increases. A closer inspection of the data, however, indicates that the relation is actually close to linear at a point in time, but that the slope of the relation varies over time. Thus, the unconditional relation appears spread out. To illustrate this, Figure 3 plots the prepayment rate and refinancing incentive relation for selected dates during the sample period. As shown, the prepayment functions display varying slopes over time.

Motivated by these stylized facts, we use a simple generic specification of the implied prepayment function that allows for both exogenous and rate-related prepayments. Specifically, we model the prepayment function as

$$p_t = x_t + y_t \max\big(0,\ m - a - b\, r_t(10)\big), \tag{8}$$

where $r_t(10)$ is the 10-year Treasury rate. In this specification, x_t denotes the exogenous hazard rate at which mortgages are prepaid in the absence of refinancing incentives. Intuitively, x_t captures all the non-interest-rate-related background factors that lead to prepayments. For example, when a borrower defaults and the mortgage is foreclosed, investors receive repayment of principal since agency mortgage-backed securities are guaranteed against default. Similarly, when a mortgage loan is put-back to its originators, investors

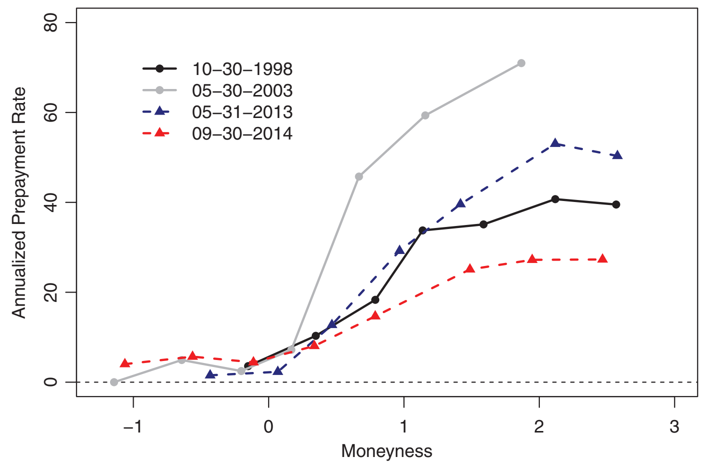

Figure 3
Prepayment rates for FNMA mortgage-backed securities for selected dates
This figure plots the 3-month prepayment rates for FNMA mortgage-backed securities against the moneyness of the mortgage-backed securities for the indicated dates. Moneyness is expressed in percentage points. The prepayment rates are expressed as annualized percentages of the outstanding principal balance of the mortgage-backed security.

receive their principal.[14] Thus, we will refer to x_t simply as the turnover rate. x_t is a hazard rate rather than a prepayment rate. The value of x_t, however, can be easily mapped into an annualized prepayment rate using the expression $1-e^{-x_t}$, and two values are generally close to each other. Thus, with little loss of generality, we can simply think of the units of x_t as being expressed in terms of a prepayment rate. The exact relationship is shown in the appendix.

The refinancing incentive is determined by the difference between the mortgage rate m and the implied rate at which mortgages can be refinanced. We allow this implied rate to be a general affine function $a+b\, r_t(10)$ of the 10-year Treasury rate $r_t(10)$, rather than constraining it to be a specific short-term or long-term rate. We use the 10-year Treasury rate since it is strongly correlated with mortgage rates—the correlation between the 10-year Treasury rate and the FNMA primary mortgage rate during the sample period is 0.9825. This suggests that representing the market mortgage rate as a linear function of the 10-year rate provides a realistic approximation. The values of a and b will be estimated from the data.[15]

The term y_t that multiplies the refinancing incentive term $\max(0,\, m - a - b\, r_t(10))$ in Equation (8) measures how sensitive borrowers are to refinancing incentives. For example, borrowers whose home values were less than their mortgage balances would typically have a very low propensity to refinance, or equivalently, a low value of y_t. After the introduction of the Home Affordable

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[14] We are grateful to the referee for this observation.

[15] m will typically be higher than $r_t(10)$ when a loan is originated, m remains fixed, whereas $r_t(10)$ varies over time. Because of this, the moneyness of the mortgage can become negative, and, therefore, the maximum operator is always relevant.

Refinancing Program (HARP) in 2009, however, this set of borrowers might have been much more likely to refinance given the same level of refinancing incentive. Thus, changes in home values might be one source of the time variation in the rate response factor. Similarly, the propensity to refinance could also vary with the required loan-to-value underwriting standards in the mortgage market. Given the role that y_t plays in the prepayment function, we denote it as the rate response factor. Since y_t is a multiplier for the refinancing incentive term, it is not expressed in any specific units. However, the product of y_t and the refinancing incentive represents a hazard rate, which, in turn, can be mapped into a prepayment rate as in the discussion above about the turnover rate.

Finally, it is important to acknowledge that our specification of the implied prepayment function in Equation (8) is among the simplest possible. In particular, our simple specification does not explicitly include many of the features that researchers and practitioners incorporate into formal econometric prepayment models such as seasoning, burnout, nonlinear dependence of refinancing activity on the refinancing incentive, housing values, macroeconomic conditions, etc. There are three reasons we have intentionally chosen one of the simplest possible specifications of the implied repayment function rather than mimicking state-of-the-art econometric prepayment models.

First, the implied prepayment function represents prepayments under the risk-neutral probability measure—not under the actual or econometric probability measure. If mortgage-backed security prices incorporate prepayment risk premiums, then the implied prepayment function could be very different from the actual or econometric prepayment function. For this reason, our approach will be to begin with the most basic risk-neutral specification, and then evaluate whether more complex features such as those used in state-of-the-art econometric prepayment models are necessary in modeling mortgage-backed security prices accurately.

Second, the reduced-form nature of the implied prepayment function allows for the possibility that the state variables x_t and y_t may play a similar role in modeling risk-neutral prepayments that features such as seasoning, burnout, etc. play in econometric modeling. In particular, time variation in the turnover factor may reflect changes in macroeconomic conditions. Similarly, the rate response factor can be viewed as a generalized form of burnout. For example, a decrease in the implied value of y_t may reflect a decline in the general willingness or ability of borrowers to refinance mortgages into lower rates in a way that parallels the usual security-specific notion of burnout. Note, however, that since y_t is a marketwide factor impacting all mortgage-backed securities, it clearly cannot capture seasoning and burnout in the usual cross-sectional sense.

Third, by choosing such a simple specification for the implied prepayment function, we are biasing the results against the model. If it turns out, however, that even with this simple implied prepayment specification, the model is able to capture the cross-section of mortgage-backed securities accurately, then this would provide strong support for the usefulness and viability of these types of implied prepayment models.

6. Estimation Methodology

In this framework, the value of a mortgage-backed security is a function of the three state variables: w_t, x_t, and y_t (in addition to the interest rate). To complete the specification of the model, we assume that the dynamics of the state variables are given by the following system of stochastic differential equations under the risk-neutral pricing probability measure,

$$dw = (\alpha_w - \beta_w\ w)\ dt\ +\ \sigma_w\ dZ_w, \tag{9}$$

$$dx = (\alpha_x - \beta_x\ x)\ dt\ +\ \sigma_x\ \sqrt{x}\ dZ_x, \tag{10}$$

$$dy = (\alpha_y - \beta_y\ y)\ dt\ +\ \sigma_y\ \sqrt{y}\ dZ_y. \tag{11}$$

The credit/liquidity spread w_t follows a mean-reverting process that can take on both positive and negative values. The spread parallels the specification used by Duffie and Singleton (1997, 1999), Longstaff, Mithal, and Neis (2005), and many others. The state variables x_t and y_t driving prepayments both follow mean-reverting square-root processes, ensuring that prepayment rates are always nonnegative. This specification of dynamics places this model within the familiar affine framework widely used throughout the financial literature.

To model the evolution of the riskless rate, we assume that r_t follows the single-factor Hull and White (1990) model

$$dr = (\alpha_r(t) - \beta_r\ r)\ dt\ +\ \sigma_r\ dZ_r, \tag{12}$$

where $\alpha_r(t)$ is a deterministic function of time, and β_r and σ_r are positive constants. The function $\alpha_r(t)$ allows for an exact fit to the Treasury term structure on a given date. The 10-year rate $r_t(10)$ that determines the refinancing incentive is an affine function of the short rate r_t. The interest rate model could easily be relaxed to allow for a more general multifactor specification.[16]

We allow for correlation between the state variables. Specifically, we assume that dZ_r is correlated with dZ_x and dZ_y, and that dZ_x and dZ_y are correlated with each other. We denote the correlation of dZ_r with dZ_x as $\rho_{r,x}dt$, the correlation of dZ_r with dZ_y as $\rho_{r,y}dt$, and the correlation of dZ_x with dZ_y as $\rho_{x,y}dt$.[17]

As discussed in the appendix, the parameters for the riskless rate are estimated separately from the mortgage model. For each date, we solve for β_r and σ_r parameters to minimize the relative pricing error over the swaption volatility

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[16] Clearly, the single-factor Hull White (1990) model has limitations relative to a more general multifactor model. For example, a multifactor model would likely perform better in terms of hedging the interest rate risk of mortgage-backed securities (see Gupta and Subrahmanyam 2005; Driessen, Klaassen, and Melenberg 2003). Some practitioners use multifactor models in their MBS valuation frameworks. We are grateful to the referees for these observations.

[17] As is common in the literature, we assume that dZ_w is uncorrelated with the other state variables. This standard assumption likely has little effect on the results. For example, see Duffie and Singleton (1997), Longstaff, Mithal, and Neis (2005), Pan and Singleton (2008), Longstaff et al. (2011), and Ang and Longstaff (2013).

Table 3
Estimates of model parameters

Parameter	Value	SE
a	0.01025	0.00142
b	0.86567	0.01485
α_w	0.00006	0.00081
α_x	0.00138	0.00098
α_y	0.03885	0.04279
β_w	0.00834	0.01216
β_x	0.00978	0.01104
β_y	0.00234	0.01057
σ_w	0.00020	0.02982
σ_x	0.02281	0.00793
σ_y	0.08945	0.03720
$\rho_{r,x}$	−0.15430	0.14225
$\rho_{r,y}$	0.12657	0.21620
$\rho_{x,y}$	−0.04890	0.11873

This table reports the estimate of the model parameters along with their asymptotic standard errors.

surface. Specifically, we fit these parameters to the 15 European swaptions with expirations of 1, 2, 3, 4, and 5 years, and tenors of 5, 7, and 10 years. Given the estimates of β_r and σ_r, the function $\alpha_r(t)$ is determined by fitting the model to the Treasury yield curve.

The estimation of the mortgage model can be viewed as consisting of three steps. First, we select an initial parameter vector θ, where $\theta = \{a, b, \alpha_w, \alpha_x, \alpha_y, \beta_w, \beta_x, \beta_y, \sigma_w, \sigma_x, \sigma_y\, \rho_{r,x}, \rho_{r,y}, \rho_{x,y}\}$. Second, conditional on θ and for each month t during the sample period, we solve for the values of w_t, x_t, and y_t that best fit the model to the prices of the cross-section of mortgage-backed securities with different coupon rates (the coupon stack) by minimizing the RMSE. w_t, x_t, and y_t are separately identifiable because each has different effect on mortgage-backed security prices. Specifically, the effect of an increase in w_t is to increase the discount rate on all mortgage-backed security cash flows, which has the effect of lowering the prices of all mortgage-backed securities.[18] In contrast, an increase in the turnover rate x_t has the effect of increasing the prepayment rate for all mortgage-backed securities. In turn, an increase in the prepayment rate increases the values of discount mortgage-backed securities while decreasing the values of premium mortgage-backed securities. Furthermore, an increase in y_t has the effect of increasing the prepayment rate for premium mortgage-backed securities, but has no impact on the prepayment rate or prices of discount mortgage-backed securities. Since the nonlinear structure of the prepayment function makes it difficult to express the price of mortgage-backed securities in closed-form, we use simulation to solve for the model-based mortgage-backed security values. Third, we iterate over alternative values of the parameter vector θ until we find the vector that results in the lowest global RMSE. Table 3 reports the parameter values obtained from the estimation along with their asymptotic

[18] Since an increase in w_t affects all mortgage-backed securities, its effect differs from that of an option-adjusted spread, which is security specific and constant through time.

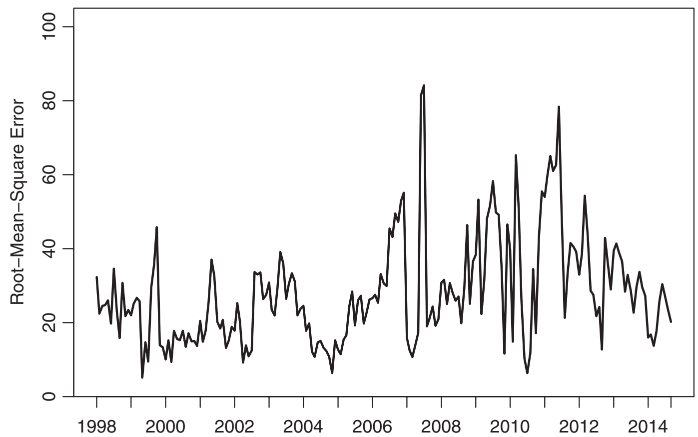

Figure 4
Root-mean-square errors from fitting the model
This figure plots the time series of root-mean-square errors from fitting the model to the cross-section of mortgage-backed security prices. The root-mean-square error is expressed as cents per $100 notional position.

standard errors. The outputs are the parameter values and the time series of state variables. The details of the mortgage model estimation process are described in the appendix.[19]

7. Implied Prepayment Factors

In this section, we discuss the empirical results and their implications. First, we examine how well the model is able to fit the market prices of mortgage-backed securities. We then study the properties of the three state variables of the model: the credit/liquidity spread w_t, the turnover rate x_t, and the rate response factor y_t.

7.1 Fitting mortgage-backed security prices

The coupon stack for each month in the sample period typically includes between 6 to 10 mortgage-backed securities with varying coupon rates at 50 bp increments. The estimation algorithm solves for the values of the three state variables w_t, x_t, and y_t that best fit the model to the coupon stack. Since there are more prices than state variables, it is clear that there will be residual differences between model values and market values. To quantify the magnitude of these differences, we compute the RMSE for each month in the sample period.

Figure 4 plots the time series of the RMSEs. As shown, the model fits the mortgage-backed security prices extremely well. For much of the sample

[19] The appendix also reports the results from a number of robustness checks including the inclusion of burnout and seasoning features in the model, the use of the swap curve as the discounting curve, the restriction of the set of TBAs used in the estimation to the five with coupons closest to the current coupon rate, and an analysis of the relation between fitting errors and TBA characteristics such as WALA.

Table 4
Summary statistics for the mortgage-backed security pricing factors

Statistic	Spread	Turnover	Rate response
Mean	65.534	8.233	11.492
Minimum	−40.968	0.100	0.592
Median	68.327	7.543	11.462
Maximum	208.157	26.508	32.383
SD	43.090	4.365	4.751
Serial correlation	0.842	0.688	0.704
Number	201	201	201

This table reports summary statistics for the agency credit/liquidity spread (spread), the turnover rate (turnover), and the rate response factor (rate response). The factors are estimated from the cross-section of mortgage-backed security prices. Spread is expressed in basis points. Turnover is expressed as a percentage. Rate response is expressed as a multiplier for the refinancing incentive. The sample consists of monthly observations for the period from January 1998 to September 2014.

period, the RMSEs range from about 5 to 30 cents for mortgage-backed security prices quoted in terms of a $100 notional position. This range compares well with the bid-ask spreads of actively traded mortgage-backed securities, which discussions with traders indicate are typically on the order of three to four ticks, or 32nds of a point. Once the financial crisis begins in 2008, however, the RMSEs tend to become larger in value. Intuitively, this may simply be the result of the massive shocks that the housing and mortgage markets experienced during the financial crisis, as well as a lack of liquidity and risk capital in the markets to arbitrage mispricing among mortgage-backed securities. The median RMSE for the pre-crisis period is 21.5 cents.[20] The median RMSE for the entire sample period is 25.7 cents.[21]

7.2 Mortgage-backed security pricing factors

The estimation algorithm solves for the implied values of the three factors driving mortgage-backed securities prices for each month during the sample period: the credit/liquidity spread, the turnover rate, and the rate response factor. Table 4 provides summary statistics for the implied values of these factors. These pricing factors are discussed individually below.

7.3 Credit/liquidity spread

Table 4 shows that the mean value of the credit/liquidity spread is about 65.5 bps with a standard deviation of 43.1 bps. This mean value is in relatively close agreement with the average spread on FNMA debt issues during the sample period. For example, the average spread of FNMA 10-year debt over Treasuries during the January 2000 to September 2014 period is 49.8 bps. We will study

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[20] To provide additional perspective, we also compute the RMSE under the static assumption that the future prepayment rate for each mortgage-backed security equals its current 3-month prepayment rate. In this estimation, however, we again solve for the implied credit/liquidity spread. The resultant RMSE is 242 cents. We are grateful to the referee for suggesting this comparison.

[21] The median RMSE for discount and premium mortgage-backed securities is 18.0 and 26.8 cents, respectively.

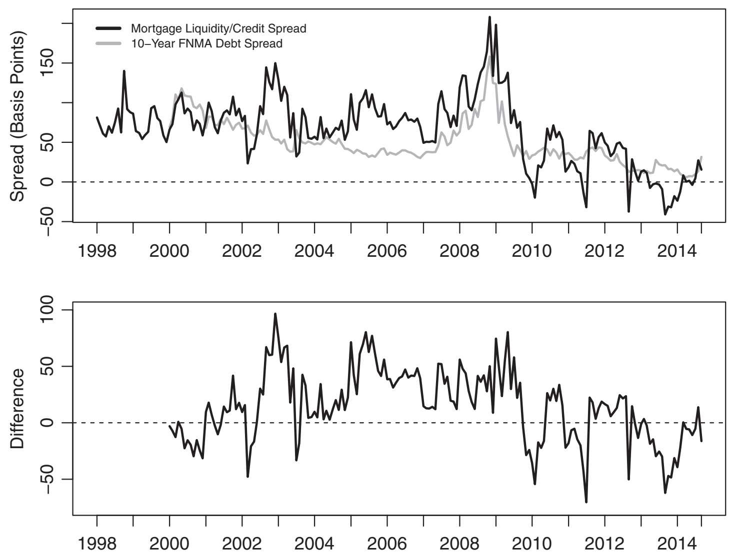

Figure 5
Implied credit/liquidity spread, the credit spread for FNMA agency debt, and the liquidity spread
The upper panel plots the time series of the implied credit/liquidity spread as well as the credit spread for 10-year FNMA agency debt over the 10-year Treasury rate. The lower panel plots the difference between the spreads, which is designated the liquidity spread. The spreads are expressed in bps.

the link between the implied spread and FNMA credit spreads in more depth shortly.

Figure 5 plots the time series of the implied credit/liquidity spread values over the sample period, along with the spread for FNMA agency debt. As shown, the majority of the implied spreads are positive. In particular, 184, or 91.5% of the 201 estimates are positive. That some of the implied spreads are negative, however, hints that the implied spreads may be reflecting more than the credit risk of FNMA bonds, particularly since FNMA credit spreads are uniformly positive throughout the 2000–2014 period.

This latter observation is reinforced by comparing the spread values shown in Figure 5 with the key events in the timeline given in Table 1. For example, the large decline in the spread beginning in April 2009 coincides with the large expansion of the QE I program to purchase an additional $750 billion of mortgage-backed securities. The large downward spike around September 2012 coincides with the announcement of the QE III program to purchase $40 billion of agency mortgage-backed securities per month. Thus, these observations hint that the massive purchases of mortgage-backed securities during QE I and QE III may have had an effect via new production and existing collateral being removed from the market. The potential effect is two-fold: a direct decrease in supply would increase prices and decrease spreads, an indirect effect on liquidity would increase spreads. It appears that the first effect dominates the second.

On the other hand, Figure 5 also shows that the implied spreads appear to be related to key events that may impact the credit risk of FNMA. For example, the spread attains its largest values during the Lehman crisis period of Fall 2008. However, after FNMA and FHLMC are placed into conservatorship and their credit risk is essentially defeased, the implied spread quickly returns to pre-crisis levels, and subsequently actually reaches historical lows.

To examine the properties of the implied spread in more detail, we regress monthly changes in the implied spread on a number of explanatory variables reflecting changes in the credit risk and liquidity of the mortgage-backed securities market. First, we include monthly changes in the yield spread between FNMA notes and Treasury notes with similar maturities. The intuition for including this spread is that if FNMA's cost of debt capital were to increase relative to that of the Treasury, then the value of the FNMA guarantee should decline, resulting in lower mortgage-backed security prices, or equivalently, higher implied spreads.

Second, we include three measures relating to the supply of mortgage-backed securities in the market. The first of these is the amount of mortgage-backed securities purchased by the Federal Reserve via its quantitative easing programs. The scale of these purchases represented a large fraction of the total available supply of mortgage-backed securities in the market and, therefore, could potentially have a sizable effect on the liquidity of these securities. The second is the total amount of settlement fails of mortgage-backed securities by primary dealers. Settlement fails occur when dealers face challenges in obtaining enough mortgage-backed security collateral to settle trades, and are a reflection of tight supply in the market.[22] The third is the net issuance of mortgage-backed securities. This measure reflects the change in the supply of mortgage-backed securities available in the financial markets.

Third, we include the change in primary dealers' holdings of mortgage-backed securities as reported by the Federal Reserve Bank of New York. The intuition for this measure is that when primary dealers increase their inventories, we would expect that the liquidity of the mortgage-backed securities market would improve, leading to a decline in the implied spread. Finally, we include the first two lagged values of the change in the credit/liquidity spread to control for the time series properties of this variable. Details of the variables used in this regression are provided in the appendix.

Table 5 presents the regression results. As illustrated, the change in the FNMA credit spread is strongly related to the change in the credit/liquidity spread implied from the prices of mortgage-backed securities. The regression coefficient is positive and highly significant with a t-statistic of 3.44. Although this result is very intuitive, to our knowledge, this is the first direct evidence that the credit risk of the agency guaranteeing the timely payment of principal and

[22] We are grateful to the referee for suggesting this explanatory variable.

Table 5
Results from the regression of monthly changes in the implied credit/liquidity spread on explanatory variables

Variable	Coefficient	t-statistic
Intercept	5.2450	1.90
First lagged change in implied spread	−0.3667	−5.68*
Second lagged change in implied spread	−0.1434	−2.17*
Change in FNMA spread	0.7371	3.44*
Fed MBS purchases	−0.1163	−2.08*
MBS settlement fails	−0.0193	−2.26*
Lagged MBS settlement fails	0.0093	0.74
Net MBS issuance	−0.2779	−1.93
Change in dealer inventories	−0.0176	−0.13
Adj. R^2		0.2052
N		176

This table reports the results from the regression of the monthly change in the implied credit/liquidity spread (measured in basis points) on its first two lagged values, the change in the FNMA credit spread (measured in basis points), Federal Reserve purchases of mortgage-backed securities (in $ billions), contemporaneous and lagged primary dealers' mortgage-backed security fails (in $ billions), net mortgage-backed security issuance (in $ billions), and the change in primary dealers' holdings of mortgage-backed securities (measured in $ billions). All changes are monthly. The t-statistics are based on the Newey-West (1987) estimator of the covariance matrix (with four lags). * denotes significance at the 5% level. The sample period is January 2000 to September 2014.

interest is related to the pricing of mortgage-backed securities. The regression coefficient of roughly 0.74 indicates that while the implied spread is closely related to the spread on FNMA debt, the relation is not one-to-one and that there are other drivers of the implied spread.

In particular, Table 5 shows that the supply-related variables have significant effects on the credit/liquidity spread, consistent with a liquidity interpretation of this variable. For example, the coefficient for Federal Reserve purchases is negative with a t-statistic of -2.08. Intuitively, this suggests that as the large purchases by the Federal Reserve crowded out other market participants, the resultant scarcity of mortgage-backed securities led to an increase in their prices. This effect is also consistent with the significant negative coefficient for settlement fails, which suggests that mortgage-backed securities increase in value when the supply of mortgage-backed security collateral is tight in the market.

Given the strong empirical relation between the credit/liquidity rate and the FNMA credit spread, a simple estimate of the size of the liquidity component in mortgage-backed securities can be obtained by subtracting the FNMA credit spread from the credit/liquidity spread. This difference or liquidity spread is also plotted in Figure 5. As shown, during the pre-crisis period, the liquidity spread is positive with an average value of around 23.2 bps. After the crisis of 2008, the liquidity spread declines to near zero with downward spikes coinciding with the initiation and extension of the QE I program. The initiation of the QE III program with its massive purchases of agency mortgage-backed securities coincides with the large negative spike in the liquidity spread. Discussions with industry sources suggest that as the Federal Reserve's purchases of agency mortgage-backed securities began to crowd other players out of the market, the difficulty of finding tradeable collateral made existing supplies of mortgage-backed securities trade at a premium. The liquidity estimates shown in Figure 5 are consistent with this view and with the regression results in Table 5. Furthermore, our results provide support for previous research that finds links between QE activity and mortgage-backed security prices including Krishnamurthy and Vissing-Jorgensen (2011, 2013), the Treasury Market Practices Group (2012), Kandrac (2013), Song and Zhu (2016), and Boyarchenko, Fuster, and Lucca (2016).

7.4 Turnover rate

Table 4 reports summary statistics for the implied turnover rate. The implied turnover rate is based on the risk-neutral probability measure since its value is inferred from the prices of mortgage-backed securities. Because prepayment rates are directly observable, however, the turnover rate under the actual or empirical probability measure can be directly estimated from the data. The details of the estimation procedure are given in the appendix.[23] As part of our analysis, we will contrast the properties of the empirical and implied turnover rates and examine their implications for risk premiums.

Figure 6 plots the time series of the implied turnover rate and the empirical turnover rate. As illustrated, virtually all of the implied turnover rates are higher than the realized turnover rates. Some of the largest values of the implied turnover rate occur during 2003 and 2005. Similarly, some of the largest spikes in realized turnover occur in 2003 and in 2004. Industry sources suggest that a sizable fraction of this turnover may have been motivated by borrowers attempting to "cash out" some of the equity in their homes resulting from the rapid increase in housing values. Thus, the increase in turnover rates during this period could partially reflect a shift towards consumption-related incentives for refinancing. Similarly, the spike in the implied turnover rate during the early stages of the financial crisis may reflect expectations of higher mortgage defaults and foreclosures.[24]

To explore this further, we regress quarterly changes in both the empirical and implied turnover rates on variables that reflect the state of the macroeconomy, risk premiums in the fixed income and other markets, and the level of distress in the mortgage markets. As macroeconomic measures, we include the lagged growth rate in U.S. personal consumption expenditures, the lagged change in the Conference Board Consumer Confidence Index, and the lagged change in the unemployment rate. As measures of risk premiums, we include the change

[23] We estimate the empirical turnover rate and rate response factor each month during the sample period from realized 1-month CPRs using a nonlinear regression framework. In this approach, we use the CPRs for the exact same set of mortgage-backed securities that we use in estimating the implied turnover and rate response factors.

[24] Although a number of the empirical turnover rates take values close to zero, only two of the empirical turnover rates are actually zero. These two zero values occur in months in which the CPRs for discount mortgage-backed securities are reported as identically zero.

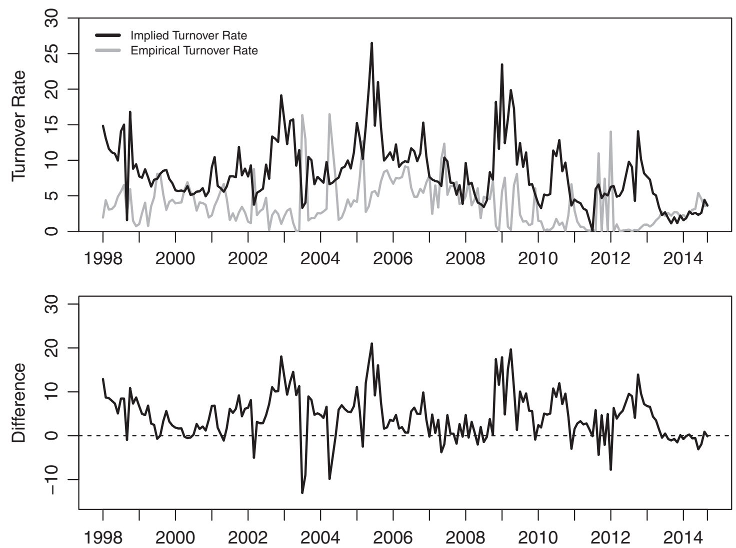

Figure 6
Implied and empirical turnover rates
The upper panel plots the time series of the implied and empirical turnover rates. The lower panel plots the difference between the implied and empirical turnover rates. The turnover rates are expressed as annualized percentages of the outstanding principal balance of the mortgage-backed security.

in the BBB corporate credit spread over Treasuries, the change in the Treasury two-year to 10-year term structure slope, and the change in the VIX index. Finally, to capture the level of distress in the mortgage markets, we include the lagged change in the mortgage foreclosure rate, and the doubly lagged change in the mortgage delinquency rate (both from the Mortgage Bankers Association National Delinquency Survey). We include these distress variables in these lagged forms since it is likely they would affect turnover with a delay. We also include the lagged changes in both the empirical and implied turnover rates as controls in the regression. The appendix provide details for each of the variables used in the regression.

Table 6 reports the results from the regressions. Focusing first on the regression for changes in the actual turnover rate, the results show that turnover is significantly positively related to consumption growth. A 1% increase in consumption maps into an increase in the turnover rate of 0.89%. This is consistent with anecdotal evidence that turnover increased during the mid-2000s as homeowners with increased equity in their homes used cash-out refinancings to fund high consumption.[25] On the other hand, the turnover rate

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25 The time series of consumption is measured with noise. Thus, our results should be viewed as suggestive rather than definitive. To examine the robustness of the results, we reestimated the regression in Table 6 separately for the first and second halves of the sample period. For the first half, the coefficient for consumption growth is 1.5293 with a t-statistic of 3.47. For the second half, the coefficient for consumption growth is 1.4690 with a t-statistic of 1.76. Thus, the coefficient estimates appear similar across the two halves of the sample period. We are grateful to the referee for raising this issue.

Table 6
Results from the regression of quarterly changes in the empirical and implied turnover rates on explanatory variables

	Actual turnover rate		Implied turnover rate	
Variable	Coefficient	t-statistic	Coefficient	t-statistic
Intercept	−0.8518	−2.19*	1.5006	2.25*
Lagged empirical turnover	−0.5179	−9.60*	0.1198	0.67
Lagged implied turnover	0.5779	0.07	−0.2888	−2.80*
Lagged change in consumption	0.8946	3.03*	−1.3014	−2.76*
Lagged change in consumer confidence	−0.0270	−1.24	−0.0353	−1.26
Lagged change in unemployment	4.0239	3.16*	−1.7677	−1.63
Change in credit spread	−0.0097	−3.02*	0.0070	3.21*
Change in term structure slope	−0.0218	−1.84	−0.0182	−1.49
Change in VIX	0.0721	0.87	0.1159	2.54*
Lagged delinquencies	1.9908	1.45	0.3445	0.30
Lagged foreclosures	−8.4475	−4.09*	1.2311	0.50
Adj. R^2		0.390		0.146
N		67		67

This table reports the results from regressions of the quarterly change in the turnover rate on the lagged change in the empirical turnover rate, the lagged change in the implied turnover rate, the lagged percentage change in personal consumption expenditures, the lagged change in The Conference Board Consumer Confidence Index, the lagged change in the unemployment rate, the change in 5-year BBB credit spreads over Treasuries (measured in basis points), the change in the Treasury 2- to 10-year slope (measured in basis points), the change in the VIX index, the doubly lagged change in the mortgage delinquency rate, and the lagged change in the foreclosure rate. The center panel presents the results for the regression in which the change in the empirical turnover rate is the dependent variable. The right panel presents the results for the regression in which the change in the implied turnover rate is the dependent variable. All variables are measured quarterly. The t-statistics are based on the Newey-West (1987) estimator of the covariance matrix (with three lags). * denotes significance at the 5% level. The sample period is January 1998 to September 2014.

is significantly positively related to changes in unemployment. In particular, a 1% increase in the unemployment rate maps into an increase in the turnover rate of 4.02%. This is consistent with the interpretation that involuntary turnover increases during economic downturns as borrowers face adverse shocks and distress-related prepayments increase (via foreclosures, employment-related moves, etc.). Lagged foreclosures are significantly negatively related to actual turnover. The reason for the negative sign of the relation is that foreclosures may actually have the effect of resolving uncertainty about future turnover. Thus, holding fixed delinquency rates, higher foreclosures during the current period reduce the overhang of distressed mortgages, resulting in lower future turnover rates. Finally, Table 6 shows that actual turnover is significantly related to the change in the BBB corporate credit spread, although the sign of the relation is negative.

Turning our attention now to the regression for changes in implied turnover, we see that implied turnover behaves very differently from actual turnover. In particular, the risk premium measures appear to be key drivers of the implied turnover rate. To see this, note that the most significant variable in the regression is the change in the BBB corporate credit spread. The positive sign for this coefficient indicates that increases in the credit spread are associated with higher implied turnover values. In addition, the coefficient for changes in the VIX index is also positive and significant, indicating that implied turnover tends to increase

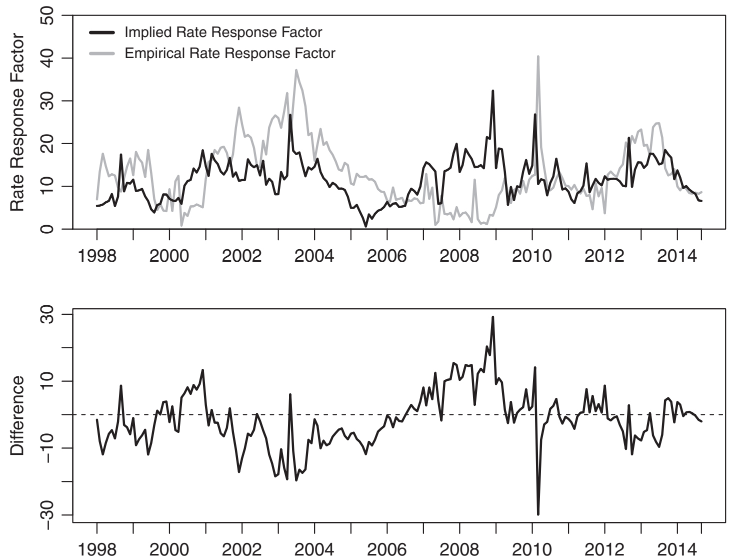

Figure 7
Implied and empirical rate response factors
The upper panel plots the time series of the implied and empirical rate response factors. The lower panel plots the difference between the implied and empirical rate response factors. The rate response factors are multipliers measuring the sensitivity of the prepayment hazard rate to the refinancing incentive.

with market volatility. In contrast, neither of the two mortgage market distress variables are significant. Of the macroeconomic measures, only consumption growth is significant. Finally, finding that the coefficients for consumption and the corporate credit spread have opposite signs than in the empirical turnover regression highlights that the behavior of implied prepayments can be very different from that of empirical prepayments.

In summary, the relation between actual turnover rates and macroeconomic factors such as consumption, unemployment, and foreclosures in the mortgage markets suggests that turnover risk may be very systematic in nature. If so, it would not be surprising if turnover risk were to carry a large risk premium. This possibility is strengthened by finding that changes in the implied turnover rate are more strongly correlated with financial market returns than with macroeconomic fundamentals. We will explore this issue in depth later in the paper.

7.5 Rate response factor

Table 4 also reports summary statistics for the implied rate response factor. As before, the empirical rate response factor is also estimated directly from observed prepayment data.

Figure 7 plots the time series of the implied and empirical rate response factors. As shown, the implied and empirical rate response factors display considerable time series variation and generally track each other closely. Some of the higher values of the empirical rate response factor occur during the

Table 7
Results from the regression of quarterly changes in the empirical and implied rate response factors on explanatory variables

	Actual rate response		Implied rate response	
Variable	Coefficient	t-statistic	Coefficient	t-statistic
Intercept	0.6771	0.46	−1.1468	−1.35
Lagged empirical rate response	−0.3750	−6.89*	−0.0271	−0.33
Lagged implied rate response	0.2501	1.15	−0.2491	−3.57*
Lagged change in consumption	−0.8365	−0.61	1.0449	1.58
Lagged change in consumer confidence	−0.0634	−0.83	0.0071	0.13
Lagged change in unemployment	−0.6027	−0.19	1.6600	1.41
Change in credit spread	−0.0029	0.66	0.0328	8.71*
Change in term structure slope	0.0322	0.08	0.0060	0.46
Change in VIX	−0.1971	0.20	−0.2234	−2.16*
Lagged change in LTV	−0.6099	0.26	0.9435	1.99*
Lagged change in credit availability	0.6785	0.60	−0.2598	−0.21
Adj. R^2		0.234		0.340
N		59		59

This table reports the results from regressions of the quarterly change in the rate response factor on the lagged change in the empirical rate response factor, the lagged change in the implied rate response factor, the lagged percentage change in personal consumption expenditures, the lagged change in the Conference Board Consumer Confidence Index, the lagged change in unemployment, the change in 5-year BBB credit spreads over Treasuries (measured in basis points), the change in the Treasury 2- to 10-year slope (measured in basis points), the change in the VIX index, the lagged change in the loan-to-value ratio for new FNMA mortgages, and the lagged change in the credit availability index. The center panel presents the results for the regression in which the change in the empirical rate response factor is the dependent variable. The right panel presents the results for the regression in which the change in the implied rate response factor is the dependent variable. All variables are measured quarterly. The t-statistics are based on the Newey-West (1987) estimator of the covariance matrix (with three lags). * denotes significance at the 5% level. The sample period is January 1998 to September 2014.

2001–2005 period when refinancings reached historically high levels. The implied rate response factor attains its highest values during the financial crisis. More recent increases in the rate response factor coincide with the rapid expansion of the Home Affordability Refinancing Program (HARP) in which investors with home values below their mortgage balance were allowed to refinance their homes.

As in the previous section, we regress quarterly changes in the empirical and implied rate response factors on a number of explanatory variables. In particular, we include the same set of macroeconomic variables and risk premium measures used in the previous regression discussed above. In addition, we include two measures that reflect the level of frictions that borrowers may face in obtaining mortgage credit in the market. The first measure is the average loan-to-value ratio for newly originated FNMA mortgages. Changes in this ratio reflect variation in loan underwriting standards. For example, a decrease in the loan-to-value ratio may indicate that lenders require higher downpayments in order for borrowers to obtain mortgage credit. The second measure is the housing credit availability index reported by the Housing Finance Policy Center. The appendix provides details for each of the variables used in the regression.

Table 7 reports the results from the regressions. As before, we begin with the results for the empirical rate response factor. As shown, only the lagged change in the empirical rate response is significant in the regression. In particular, none of the macroeconomic, risk premiums, or financial frictions variables are significant. This result suggests that changes in the empirical rate response factor may be driven more by idiosyncratic influences and are less systematic in nature than is the case for changes in turnover.

Focusing next on the implied rate response factor, we again find that it behaves differently from the empirical rate response factor. For example, the lagged change in the loan-to-value ratio is positive and significant in the regression. Thus, the implied rate response factor increases as mortgage underwriting guidelines are relaxed. Furthermore, the risk premium measures are again the most significant variables in the regression. In particular, the change in the BBB corporate credit spread is positive and highly statistically significant with a t-statistic of 8.71. In addition, the coefficient for the VIX is significantly negative.[26] Again, these results suggest that implied rate response factor incorporates a significant risk premium component. This issue is explored in the next section below.

8. Prepayment Risk Premium

In this section, we examine whether the market prices of mortgage-backed securities incorporate a risk premium for prepayment risk. Since we model prepayment risk as an explicit function of the turnover rate and the rate response factor, our framework also allows us to break down the total prepayment risk premium further into the components related to the turnover rate and the rate response factor. Mortgage-backed securities may also incorporate premiums for interest rate risk and agency credit risk. Rather than focusing on these well-known and extensively researched types of risk premiums, however, we exclusively focus on the prepayment risk premium. It is important to observe that our approach measures the marketwide risk premiums associated with the factors driving mortgage prepayments. This approach contrasts with that of recent papers such as Boyarchenko, Fuster, and Lucca (2016) and Diep, Eisfeldt, and Richardson (2016) that focus on the risk premiums incorporated into the expected returns of individual mortgage-backed securities. Thus, our paper provides a marketwide perspective on prepayment risk premiums that is complementary to the results of these other papers.

8.1 Is there a prepayment risk premium?

To address the issue of whether there is a prepayment risk premium, we follow the standard approach of comparing values estimated under the risk-neutral probability measure with those estimated under the actual or empirical probability measure. Because the implied prepayment function is estimated directly from the market prices of mortgage-backed securities, it represents the

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[26] We also estimate this regression including changes in the prepayment rate disagreement index of Carlin, Longstaff, and Matoba (2014). This variable was not significant.

prepayment function under the risk-neutral probability measure. In contrast, prepayments under the actual or empirical probability measure are directly observable. Our estimates of marketwide prepayment factor risk premiums are expressed in terms of hazard rates rather than in terms of the expected returns of individual securities.

It is important to observe that since the implied prepayment rate is based on the risk-neutral probability measure, the implied prepayment rate need not equal the empirical prepayment rate. This follows from Jarrow, Lando, and Yu (2005) who show that if hazard rates or intensities are sensitive to shocks that carry risk premiums (e.g., such as macroeconomic factors), then their values can differ between the risk-neutral and actual probability measures. This is analogous to what occurs in reduced-form credit models in which the risk-neutral default probability or hazard rate need not equal the actual default probability. A key difference, however, is that the actual probability of default is extremely difficult to measure given how rare default events are. Thus, it is very challenging to estimate the difference between risk-neutral and actual default probabilities.[27] In contrast, empirical prepayment rates are directly observed and differences between the prepayment rate under the risk-neutral and empirical probability measures are easily identified.

The implied prepayment rate for each mortgage-backed security is given by simply substituting its weighted average coupon rate into the fitted prepayment hazard rate function and solving for the corresponding prepayment rate. Observe that in doing this, we are solving for the instantaneous implied prepayment rate which can be directly compared to the 1-month realized CPR for the mortgage-backed security.[28]

The upper panel of Figure 8 plots the time series of the monthly averages for both the implied and empirical prepayment rates. These monthly averages are calculated as the simple average of the prepayment rates for all coupons for each month. The lower panel plots the time series of the prepayment risk premium, which is computed as the difference between the implied and realized prepayment rates. As shown in the upper panel, the implied and realized prepayment rates tracked each other closely up until the middle of 2006. Through most of the financial crisis, however, implied prepayments were much higher than empirical prepayments. This is particularly evident in the lower panel which shows that the prepayment risk premium attained large values during late 2008 and early 2009.

Table 8 presents summary statistics for the implied prepayment rates, the empirical prepayment rates, and the prepayment risk premium. The summary statistics in Table 8 are computed using the time series of the monthly averages. As shown, the average implied prepayment rate across the entire

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27 For example, see Huang and Huang (2012) and Giesecke et al. (2011).

28 To solve for the risk premium over longer horizons, we would need also need to solve for the parameters of the w_t, x_t, and y_t processes under the objective probability measure.

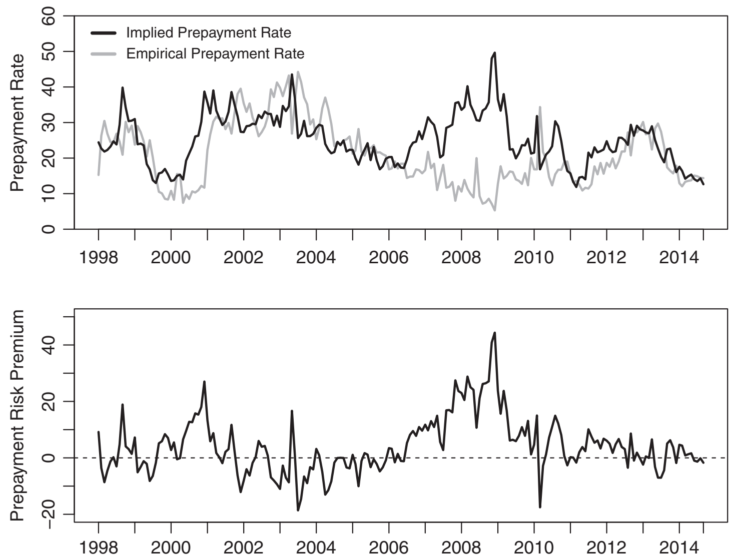

Figure 8
Implied and empirical prepayment rates and the prepayment risk premium
The upper panel plots the time series of the implied and empirical prepayment rates (both averaged across all coupon rates for each month). The lower panel plots the time series of the prepayment risk premium defined as the difference between the implied and empirical prepayment rates. The prepayment rates and the risk premium are expressed as annualized percentages of the outstanding principal balance of the mortgage-backed security.

Table 8
Summary statistics for prepayment rates and prepayment risk premiums

	Average	SD	Minimum	Median	Maximum	*N*
Implied prepayment rate	25.130	7.124	11.865	24.335	49.651	201
Empirical prepayment rate	20.956	8.717	5.289	19.056	44.250	201
Prepayment risk premium	4.174	9.839	−18.617	2.358	44.362	201
Implied turnover prepayment rate	7.378	3.641	0.100	6.890	23.023	201
Empirical turnover prepayment rate	3.575	2.774	0.000	3.005	13.795	201
Turnover risk premium	3.803	4.422	−10.353	3.884	17.905	201
Implied rate response prepayment rate	17.752	7.150	1.173	17.014	39.659	201
Empirical rate response prepayment rate	17.381	9.168	0.779	15.483	42.057	201
Rate response risk premium	0.371	10.554	−24.185	−0.986	34.370	201

This table reports summary statistics for the implied and empirical prepayment rates, the implied and empirical turnover prepayment rates, the implied and empirical rate response prepayment rates, and the corresponding risk premiums (defined as the difference between the implied and empirical prepayment rates). All variables are expressed as percentages. The sample consists of monthly observations for the period from January 1998 to September 2014.

sample of mortgage-backed securities is 25.130%. In contrast, the average empirical prepayment rate for the same sample of mortgage-backed securities is 20.956%. Thus, the implied prepayment rate is clearly different from the actual prepayment rate. The average difference between the implied and actual prepayment rates is 4.174%. The hypothesis that this difference is zero is strongly rejected by the data. These results provide direct confirmation that there is a substantial prepayment risk premium incorporated into mortgage-backed security prices. This direct evidence of prepayment risk premiums in the mortgage-backed securities market corroborates the evidence of prepayment risk premiums in the option-adjusted spreads of interest-only/principal-only securities reported by Gabaix, Krishnamurthy, and Vigneron (2007) and Boyarchenko, Fuster, and Lucca (2016).[29]

The result that the implied prepayment rate is substantially higher than the empirical prepayment rate has important implications for the pricing of mortgage-backed securities, particularly for the literature that relies on econometric models of prepayment. The prices of mortgages with coupon rates below the current market rate are increasing in the prepayment rate while the opposite is true for the prices of mortgages with coupon rates above the current market rate. Thus, the positive prepayment risk premium implies that discount mortgages will have higher values than implied by empirical prepayment functions, while the reverse will be the case for premium mortgages. These results are broadly consistent with the empirical evidence provided in Duarte, Longstaff, and Yu (2007). To provide more insight into the nature of the prepayment risk premium, it is useful to break it down into its components. In the following sections, we examine the turnover and rate response risk premiums separately.

8.2 Turnover risk premium

Similar to the previous section, can solve for the prepayment rate that is due exclusively to turnover by comparing the prepayment rate given by the hazard rate function to the prepayment rate obtained by setting $x_t = 0$ in the hazard rate function (details provided in the appendix). For clarity, we will designate this as the turnover prepayment rate to distinguish it from the turnover rate (which is a hazard rate rather than a prepayment rate). We can then identify the turnover risk premium by comparing the implied and empirical turnover prepayment rates.

Figure 9 plots the time series of the implied and empirical turnover-related prepayment rates along with the turnover risk premium, which is calculated as the difference. As shown, the turnover risk premium is generally positive throughout the sample period. The turnover risk premium, however, attains some of its largest values during the refinancing waves of the 2001–2005 period (total refinancing volume during this period was many times higher than the average during the prior ten years). The turnover risk premium also attains high values during the financial crisis. Recall that a positive turnover risk premium has the effect of increasing the values of discount mortgage-backed securities while decreasing the values of premium mortgage-backed securities.

[29] As an alternative way of corroborating the existence of prepayment risk premiums, we regress the monthly excess returns on the Bloomberg 30-Year MBS Return Index on the first two lagged values of the prepayment risk premium. The coefficient for the first lagged value is positive and has significant forecast power for excess returns (t-statistic of 1.94). We are grateful to the referee for suggesting this test.

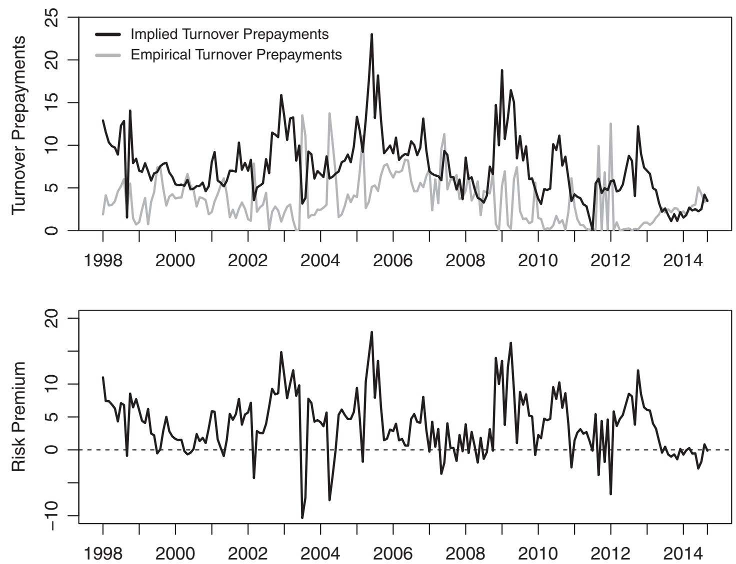

Figure 9
Implied and empirical turnover prepayment rates and the turnover risk premium
The upper panel plots the time series of the implied and empirical turnover prepayment rates. The lower panel plots the time series of the turnover risk premium defined as the difference between the implied and empirical turnover prepayment rates. The prepayment rates and the risk premium are expressed as annualized percentages of the outstanding principal balance of the mortgage-backed security.

Table 8 presents summary statistics for the implied and empirical turnover prepayment rates, and the turnover risk premium. The average implied turnover prepayment rate is roughly twice as large as its empirical counterpart. In particular, the average implied turnover prepayment rate is 7.378%, while the average empirical turnover prepayment rate is 3.575%. Thus, the average turnover risk premium is 3.803% for the sample period. This value is highly statistically significant.

Recall from the previous section that the average prepayment risk premium is 4.174% on average. Thus, the average turnover risk premium of 3.803% represents 91.11% of the entire average prepayment risk, making it the primary component. Given the earlier evidence that turnover risk is related to broad trends in the economy, these result suggest that much of the prepayment risk premium in mortgage-backed securities can be linked to the effects of non-interest-rate-related macroeconomic fluctuations on prepayment behavior.

8.3 Rate response risk premium

Following the approach in the discussion above, we identify the rate response prepayment risk premium as the difference between the prepayment rates due specifically to the implied and empirical rate response factors. The upper

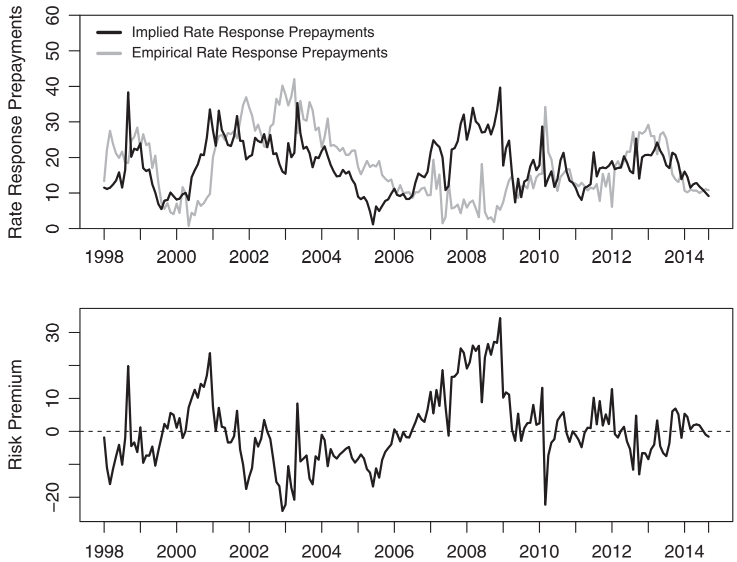

Figure 10
Implied and empirical rate response prepayment rates and the rate response risk premium
The upper panel plots the time series of the implied and empirical rate response prepayment rates. The lower panel plots the time series of the rate response risk premium defined as the difference between the implied and empirical rate response prepayment rates. The prepayment rates and the risk premium are expressed as annualized percentages of the outstanding principal balance of the mortgage-backed security.

panel in Figure 10 plots the time series of empirical and implied rate response prepayment rates; the lower panel plots the rate response risk premium.

As shown in the Figure 10, the empirical and implied rate response prepayment rates track each other closely during the 1998–2000 period, and both reach a level of about 30% by the end of 2000. Beginning in 2001, however, both the empirical and implied rate response prepayment rates start to decline, although the implied prepayment rate clearly declines more rapidly than the empirical prepayment rate. Because of this pattern, the rate response risk premium tends to be negative during the 2001–2005 period. With the arrival of the financial crisis in 2007–2008, the implied rate response prepayment rate increases rapidly and attains its highest levels. In contrast, the empirical rate response prepayment rate declines to very low levels similar to those during the 2000–2001 downturn. Thus, the rate response risk premium takes on very large positive values during the early stages of the financial crisis. In fact, during this period, the rate response risk premium is the dominant component of the total prepayment risk premium since the turnover risk premium is close to zero during the 2007–2008 period. With the inception of the HARP program in March 2009, the empirical and implied rate response prepayment rates quickly converge and track each other closely throughout the remainder of the sample period. This suggests that the HARP program and other similar interventions may have removed much of the systematic risk in the ability of borrowers to respond to refinancing incentives.

Table 8 also presents summary statistics for the implied and empirical rate response prepayment rates along with the risk premium. The average implied rate response prepayment rate of 17.752% is slightly higher than the average empirical rate response prepayment rate of 17.381%. The average rate response risk premium is positive with a value of 0.371%. A closer look at the data, however, indicates that the rate response risk premium is generally significantly positive, with the one exception of the 2001–2005 period. Excluding this period, the average rate response risk premium is 3.41% which closely compares with the overall average turnover risk premium of 3.80%.

There are good reasons to believe, however, that the 2001–2005 period may have been an unusual period during which the normal covariance between rate response and consumption may have changed signs. As one example, housing values increased dramatically during this 5-year period and many homeowners refinanced into higher balance loans (and even higher interest rate loans) in order to cash out equity and increase their consumption. For example, annual cash-out refis averaged $23.3 billion from 1993 to 2000, but increased more than 350% to $82.9 billion in 2001. Similarly, annual cash-out refis exceeded $100 billion from 2002 to 2005. During normal times, a borrower's credit and employment/income situation would be major determinants of their ability to refinance a mortgage. During this period of rapidly increasing housing values, however, borrowers were often able to refinance primarily on the strength of their home equity rather than the usual credit/income criteria. Thus, it is possible that the typical positive rate response risk premium reflects the covariance between consumption and the macroeconomic factors that affect borrowers' credit scores, employment, and household income. In contrast, the negative rate response risk premium during this period may represent compensation for a different set of risks (related to housing values) that temporarily drove refinancing activity during this period.

There are, however, other possible reasons this period may have been an unusual one for risk premiums. For example, Lustig and Van Nieuwerburgh (2005) argue that the ratio of housing collateral to human wealth is an important determinant of risk premiums. Their figure 6 shows that 2002 was associated with a 70-year high in the housing collateral ratio. Given the close link between housing values and the potential ability to refinance, it is possible that their results may help explain the decline in the rate response risk premium during this period. Similarly, the 2002 to 2005 period experienced a dramatic decline in the spread between BBB corporate yields and Treasury yields. This decline may also have been associated with a reduction in credit-related risk premiums, which in turn could have impacted the risk premium for the credit-availability-related rate response factor.

Numerous other examples of risk premiums change signs over time. For example, Fleckenstein, Longstaff, and Lustig (2017) show that the inflation risk premium changed signs during the 2010–2015 period. Vedolin (2013) shows that volatility risk premiums for individual stocks can be both positive and negative, and change signs frequently. The Federal Reserve's term structure model has implied negative term premiums throughout much of 2016.[30]

Finally, another important implication of these results is that rate-response-related refinancing activity is an important driver of mortgage-backed security pricing. On average, rate-response-related prepayments represent 79.27% of empirical prepayments and 69.45% of implied prepayments. This can easily be seen by comparing the empirical and implied turnover prepayment rates in Figure 9 with the empirical and implied rate response prepayment rates in Figure 10. In particular, the turnover prepayment rates seldom exceed 20% during the sample period, while the rate response prepayment rates often exceed 20% during the refi wave, financial crisis, and HARP periods. Thus, even though the rate response risk premium may be small on average, rate-response-related refinancing activity is the primary factor driving total prepayments. This means that rate-response-related refinancing activity is of first order importance both empirically as well as in the risk-neutral world in which mortgage-backed securities are priced.

9. Pricing IO/PO Securities

To test the robustness of our model, we value the interest-only (IO) and principal-only (PO) classes ("strips") of a selection of Fannie Mae stripped mortgage-backed securities (SMBS). An interest-only (IO) strip receives 100% of the interest and 0% of the principal from the pass-throughs backing the security, and a principal-only (PO) strip receives 0% of the interest and 100% of the principal. The market prices of IO and PO securities are highly sensitive to prepayment expectations, and these securities allow for a demanding out-of-sample test of our model. Traditional mortgage valuation models have difficulty pricing IO and PO strips, and our model performs significantly better, even though we make no adjustments for the specified nature of IO/PO collateral and the lower liquidity of the IO/PO market. In this section, we provide a brief overview of the IO/PO market, describe the data, and discuss the estimation and the results.

9.1 IO/PO markets

IOs and POs can be created as part of any collateralized mortgage obligation (CMO) deal. However, the most liquid sector of the IO/PO market are the IOs and POs created from SMBS deals. The reason SMBS are more liquid than CMOs is two fold. First, each SMBS deal has an exchange option. This option allows someone that holds both the IO and PO (i.e, the IO/PO "combo") to exchange the combo for a pass-through security for a small fee. The pass-through can then be sold in the specified pool market or the TBA market if the

[30] See https://www.bloomberg.com/news/articles/2016-05-19/not-much-worries-bond-traders-as-term-premium-falls-to-1962-lows .

collateral does not trade at premium to TBA. Second, each SMBS deal is very large, typically several billion dollars of notional.[31]

Even though SMBS is liquid compared to IOs and POs from CMOs, there are many reasons to believe that SMBS is still much less liquid than TBAs. First, the trading volume for the IO/PO market is tiny compared to that of the TBA market. TRACE data, obtained from a large MBS dealer, indicates that the daily trading volume of FNMA IO/POs was only 0.20% of the FNMA TBA volume from 2011 to 2017. Second, Chaudhary (2006) discusses how SMBS becomes less liquid as it seasons, and the value of exchange option may deteriorate as IO or PO strips get locked up in CMOs. Third, the funding markets of IO/POs are different than TBAs. The TBA market has a corresponding dollar roll market were MBS often trades "special," and similar to specialness in the Treasury repo market, this increases prices (see Song and Zhu (2016)). However, IOs and POs are financed in the MBS repo market where financing rates and haircuts are generally higher.

There is also reason to believe that even IOs and POs from the same SMBS are likely to have different liquidity. First, IOs have much greater price volatility (in percentage terms), and are subject to greater haircuts and holding costs. Second, POs have favorable accounting treatment for banks and they do not necessarily need to be marked-to-market. Finally, there is a greater supply of IOs in the market than POs. Each time an MBS pass-though is created, an IO strip, called a mortgage servicing right (MSR), is created, and a portion of the IO strip can be sold as an agency-guaranteed IO security.

Another dimension along which SMBS differ from TBAs is that each SMBS has unique collateral characteristics that often provide valuable prepayment behavior. Mortgage strip pricing reports from MBS dealers show that SMBS combo pay-ups can be as much as \$2 to \$3 per \$100 notional. This means that the collateral backing the IO/PO combo has superior prepayment behavior that commands a premium over TBA, even after accounting for the liquidity discount.

9.2 IO/PO data

We obtained end-of-day marks for IOs and POs from various SMBS trusts from two major Wall Street dealers. The sample begins in 2004 and we end our sample on December 31, 2009 because the quality of the marks deteriorate in later years. We found that the price of IO/PO combos were marked at constant spread to TBAs beginning in 2008 and by 2010 they were marked at constant spreads for months at a time. This leads us to question the quality of the marks. We focus on the 5.00%, 5.50%, and 6.00% coupons because Chaudhary (2006) indicates that these were the most liquid coupons in 2006 and they are traded throughout the sample period.

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[31] For an overview of the IO/PO market and the risks of IO and PO securities, see Hayre (2001), Chaudhary (2006), and Fabozzi (2016).

Table 9
Summary statistics for FNMA stripped mortgage-backed securities

Trust number	Trust size(bn)	Characteristics of the mortgage loans in each trust Vintage	WAC	ALOS	% LTV	% CA	1Q2010 3m CPR	Max pay-up	Dealer RMSE	N
5.0% pass-through coupon										
FNS 337	1.55	4/2003	5.64	155,558	71.46	24.70	14.97	0-152	0.52	72
FNS 340	2.24	6/2003	5.45	151,534	69.34	24.20	13.70	0-152	0.41	72
FNS 360	2.50	6/2005	5.69	158,768	70.15	19.10	15.73	0-100	0.31	54
FNS 377	3.78	12/2005	5.45	167,246	69.41	17.10	17.67	0-100	0.35	38
FNS 397	4.00	4/2009	5.49	198,319	76.02	22.30	14.10	0-000	2.86	4
5.5% pass-through coupon										
FNS 346	2.00	8/2003	5.98	158,320	70.54	29.20	16.43	0-164	0.38	72
FNS 354	2.90	9/2004	5.94	170,414	72.67	20.50	18.13	0-150	0.31	62
FNS 363	2.05	9/2005	5.93	180,474	71.59	16.30	15.57	0-110	0.37	48
FNS 379	4.45	2/2007	6.10	198,830	71.75	13.40	26.30	0-080	0.53	32
FNS 399	2.15	8/2008	5.99	187,733	73.49	20.40	24.83	0-004	–	2
6.0% pass-through coupon										
FNS 293	0.51	9/1993	6.70	93,785	95.00	14.80	12.53	1-035	–	72
FNS 344	2.20	3/2003	6.54	119,071	75.55	25.90	16.70	0-165	0.71	72
FNS 370	2.75	3/2006	6.43	165,217	72.51	13.90	23.03	0-125	0.48	43
FNS 372	3.00	5/2006	6.47	162,630	72.32	11.30	22.80	0-110	0.49	41

This table reports summary statistics for FNMA stripped mortgage-backed securities (SMBS). Each SMBS is identified by a trust number and has two classes: an IO class and a PO class. For the mortgage loans backing each SMBS, vintage denotes the weighted-average origination month, WAC denotes the weighted average coupon in percentage points, ALOS denotes the average loan size at origination in dollars, LTV denotes the loan-to-value ratio in percent, and % CA denotes the percentage backed by homes in California. 1Q210 3m CPR denotes the conditional prepayment rate for the 1st quarter of 2010. Max pay-up denotes the price difference between the IO price plus the PO price and the TBA price, where the price is expressed in points and ticks (32nds) (the last digit represents eights of a tick). N denotes the number of observations. Dealer RMSE denotes the root-mean-square error between two dealer's end-of-day marks for IO. The sample consists of end-of month observations for the period from January 2004 to December 2009.

Table 9 shows the summary statistics for the SMBS trusts. These deals tend to be very large—the average deal size is $2.6 billion notional. Each deal has different collateral characteristics. For example, each SMBS deal corresponds to a different vintage of mortgages and the underlying mortgage coupons, loan sizes, loan-to-value ratios, and geographic distributions are different for each SMBS. These characteristics translate into to different prepayment speeds and different prices for the IO/PO combo relative to TBA. Even though these are the most liquid SMBS securities, there is significant disagreement in the end-of-month dealer marks—the RMSE between the two dealer marks range from 0.31 to 2.86 dollars for the IO class of the SMBS deals. Surprisingly, FNS 397, which is the second largest deal and should be the most liquid, has the largest RMSE between marks.

9.3 Estimation and results

Our model is estimated from the most liquid sector of the MBS market—the TBA market. Using the fitted model, we calculate the prices of the interest-only and principal-only portions of each TBA coupon. We then compare the model's IO/PO prices for the 5.00%, 5.50%, and 6.00% coupon TBAs to 5.00%, 5.50%,

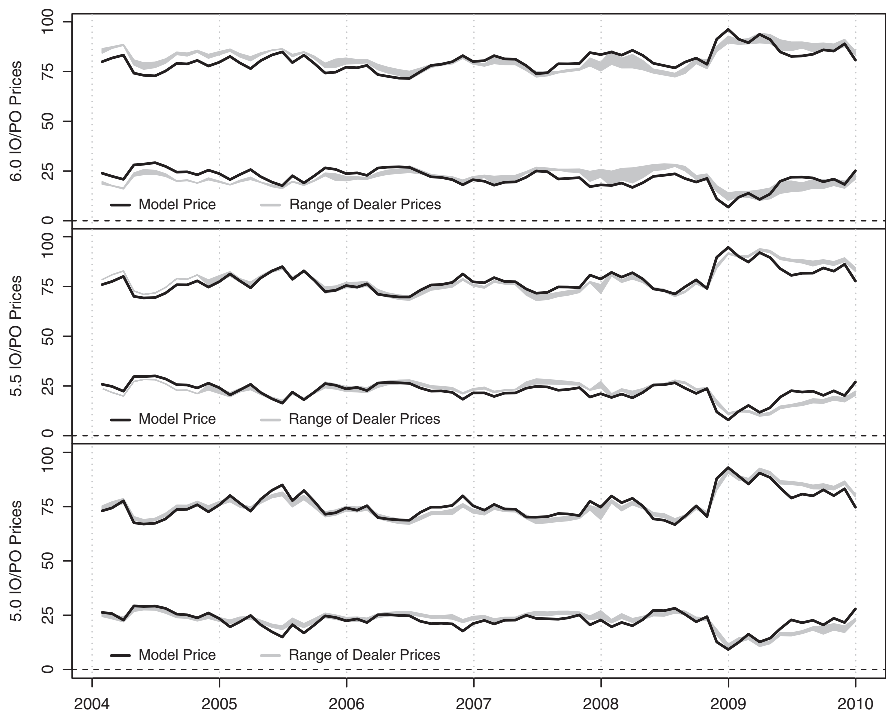

Figure 11
Model prices and dealer marks for IO/PO strips
This figure plots the IO and PO prices from the fitted model and compares them to dealer end-of-day marks for IO/PO strips. The range of dealer marks for the IO/PO strips is shown by the gray-shaded areas.

and 6.00% coupon IO and PO classes for the SMBS deals shown in Table 9. This means that our IO/PO prices incorporate the same credit/liquidity spread and prepayment assumptions as the TBA market. To accurately value SMBS IOs and POs, our model would need to be extended to account the different liquidity and prepayment characteristics of SMBS. It is not clear how our prices should compare to SMBS because it is unclear what the joint effect is of the different liquidity and prepayment characteristics.

Despite this, our model performs well in tracking the IO and PO prices for the SMBS pools. For example, the average correlation between our prices and the SMBS marks is 89.6%. Figure 11 plots the time series of market values and fitted values for the IOs and POs. As shown, the fitted values track the market values very closely. The RMSE between our IO prices and the closest SMBS IO strip is \$1.73, \$2.17, and \$2.56, for the 5.00%, 5.50%, and 6.00% coupons, respectively. This compares favorably to the RMSE between the dealer marks. The option-adjusted spreads from dealer prepayment models for both the IO and the PO classes of these SMBS ranged from −1,000 to 2,200 bps over the sample period, even though these models adjust the prepayment forecast given the collateral characteristics for each SMBS trust. Hence, our model drastically improves on the traditional approach.

10. Conclusion

We present a new three-factor no-arbitrage model for the valuation of mortgage-backed securities. Rather than using an empirical prepayment function, our approach solves for the implied prepayment function used by the market in pricing mortgage-backed securities. By studying the properties of the implied prepayment function, our goal is to shed light on the key drivers of prepayment risk as perceived by the market.

We show that this modeling framework is very successful in capturing the cross-sectional structure of mortgage-backed security prices. This result is important since it suggests that the standard approach of calibrating mortgage models to each mortgage-backed security separately using option-adjusted spreads may not be necessary.

We also find that implied prepayments can be very different from actual prepayments. This provides direct evidence that mortgage-backed securities incorporate significant prepayment risk premiums. Furthermore, the results indicate that macroeconomic factors play a large role in driving prepayment risk. In particular, we find that prepayment risk is driven not only by changes in interest rates, but also by changes in turnover and rate response factors. We find that these factors are related to macroeconomic fundamentals and are also associated with significant risk premiums.

Finally, we provide the first direct evidence that mortgage-backed security prices are also driven by changes in the credit risk of the agency guaranteeing the timely payment of principal and interest as well as by changes in the liquidity of these securities. These results are consistent with findings for other markets.

Our results suggest a number of possible directions for future research. Although the simple implied prepayment model we use performs well, it may be worthwhile to explore whether alternative specifications that include formal models of seasoning and burnout might enhance the performance further. In this paper, we have focused primarily on the pricing of TBAs. An interesting direction for future research might be the extension of the framework to the specified pools market or to broader categories of IOs, POs, and CMOs. Future work could also focus on identifying how much of the risk premium in the returns of mortgage-backed securities is due to agency credit and how much is due to actual prepayment risk. A framework such as ours that explicitly incorporates a credit/liquidity spread could provide a useful starting point in this analysis.

Appendix

A.1 Data Sources

Table A1 presents the definitions and data sources for the variables used in the study.

Table A1
Data definitions and sources

Data	Frequency	Description and source
FNMA MBS Prices	Monthly	Proprietary data set provided by a major Wall Street MBS dealer. Data cross-validated with Bloomberg data.
FNMA CPR Data	Monthly	One-month and three-month CPR prepayment rate data collected and provided by eMBS Inc.
Treasury CMT Data	Monthly	Constant maturity Treasury rates from Federal Reserve H.15 Selected Interest Rates Release.
Discount Function D(T)	Monthly	Discount function out to 30 years bootstrapped from Treasury CMT rates using standard cubic spline interpolation algorithm as described in Longstaff, Mithal, and Neis (2005).
Interest Rate Volatility	Monthly	Basis point volatility for 1-, 2-, 3-, 4-, and 5-year into 5-, 7-, and 10-year swaptions. Proprietary data set provided by major Wall Street MBS dealer.
FNMA Agency Credit Spread	Monthly	Ten-year FNMA cash flow spread (Z spread) to the Treasury curve. Proprietary data set provided by a major Wall Street MBS dealer.
Primary Dealers' MBS Holdings	Weekly	Federal Reserve Bank of New York: http://www.newyorkfed.org/markets/gsds/search.html.
Net MBS Issuance	Monthly	Net MBS issuance in $ millions provided by eMBS Inc.
Federal Reserve MBS Purchases	Weekly	Board of Governors of the Federal Reserve System, Mortgage-Backed Securities Held by the Federal Reserve: All Maturities [MBST], retrieved from FRED, Federal Reserve Bank of St Louis, https://research.stlouisfed.org/fred2/series/MBST. Weekly data aggregated to monthly and quarterly frequency.
Consumption Expenditures	Monthly	Seasonally adjusted at annual rates. US Bureau of Economic Analysis, Personal Consumption Expenditures [PCE], retrieved from FRED, Federal Reserve Bank of St. Louis, https://research.stlouisfed.org/fred2/series/PCE.
Unemployment Rate	Monthly	Seasonally adjusted unemployment rate provided by Bureau of Labor Statistics, http://data.bls.gov/timeseries/LNS14000000.
Consumer Confidence Index	Monthly	The Conference Board. Provided by Bloomberg (CONCCONF Index).
Delinquency Rate	Quarterly	Mortgage Bankers Association National Delinquency Survey, provided by Bloomberg (DLQTDLQT Index).
Foreclosure Rate	Quarterly	Mortgage Bankers Association National Delinquency Survey, provided by Bloomberg (DLQTFORE Index).
Credit Availability Index	Quarterly	Housing Finance Policy Center Index. Indicates the difficulty of getting a mortgage in the United States. The index calculates the percentage of owner-occupied purchase loans that are likely to default. http://www.urban.org/policy-centers/housing-finance-policy-center/projects/housing-credit-availability-index.
Primary Dealers' MBS Fails	Weekly	Total MBS settlement fails with primary dealers, retrieved from Federal Reserve Bank, https://www.newyorkfed.org/markets/gsds/search.html.
BBB Credit Spreads	Monthly	Five-year BBB fitted par spread to Treasuries assuming 40% recovery. Proprietary data provided by a major Wall Street MBS dealer.
2- to 10-Year Treasury Slope	Monthly	Federal Reserve Bank of St. Louis, 10-year Treasury constant maturity minus 2-year constant maturity [T10Y2Y], retrieved from FRED, https://fred.stlouisfed.org/series/T10Y2Y.
LTV of New FNMA MBS	Monthly	Balance-weighted average LTV of Fannie Mae fixed-rate 30-year MBS by origination month, Fannie Mae single-family loan performance data, http://www.fanniemae.com/portal/funding-the-market/portal/loan-performance-data.html.
VIX Volatility Index	Monthly	Chicago Board Options Exchange Volatility Index (VIX) provided by Bloomberg (VIX Index).
FNMA MBS OAS	Daily	Fannie Mae MBS option-adjusted spreads from various prepayment models. Data provided by a major Wall Street MBS dealer.

Table A2
Cash flow time line for a hypothetical 30-Year FNMA TBA trade

Date	Event	Time	Note
March 31	Trade date	0	Trade parameters: issuer, maturity, coupon, face value, price, settlement date.
April 6	Factor date		Pool factors are released by FNMA.
April 12	48-hour day		The buyer is notified of the pools the seller will deliver to settle the TBA trade.
April 14	Settlement date	t_s	The buyer wires the payment to the seller.
April 30	Record date	τ_1	Fedwire records the buyer as the new holder of record.
May 7	Factor date		Pool factors are released by FNMA, reflecting April prepayments.
May 25	Payment date	t_1	The buyer receives the first payment from the MBS.
⋮	⋮		⋮
May 31	Month end	τ_2	Payment at t_2 reflects prepayments over τ_1 to τ_2.
⋮	⋮		⋮
June 26	Payment date	t_2	The buyer receives the second payment from the MBS.
⋮	⋮		⋮
June 31	Month end	τ_3	Payment at t_3 reflects prepayments over τ_2 to τ_3.
⋮	⋮		⋮
July 25	Payment date	t_3	The buyer receives the third payment from the MBS.
⋮	⋮		⋮

This table shows the key events and cash flows surrounding a 30-year FNMA TBA trade executed on March 31.

A.2 Fannie Mae Mortgage-Backed Security Cash Flows

The pricing data are from the to-be-announced (TBA) market for 30-year Fannie Mae (FNMA) mortgage-backed securities (MBS). Before describing how we estimate the model, we consider the timing of cash flows generated by a FNMA TBA trade.

TBA trades settle in accordance with a monthly schedule set by the Securities Industry and Financial Markets Association (SIFMA). Thirty-year FNMA MBS falls into SIFMA's class A, which typically settles during the second week of the month. Because we select prices at the end of each month, the settlement date corresponding to these observations is around the second week of the following month (the exact settlement dates can be found on Bloomberg). On the notification date, 2 days prior to settlement, the buyer is notified of the exact pools to be delivered. On the settlement date, the buyer transfers a payment to the seller, which consists of the agreed on price (which we observe at month end) plus accrued interest on the face value of the pools identified on the notification date (the variance permitted on TBA trades is plus or minus 0.01% of the dollar amount of the transaction agreed to by the parties). On the record date, the last day of the month, Fedwire records the buyer as the new holder of the security. On the fifth or sixth business day of the next month, the pool factors (the ratio of the current balance to the original balance) are released. The pool factor determines the new face value of the mortgage after accounting for scheduled principal payments and prepayments from the previous month. Then, on the payment date later that month, the scheduled principal payments, interest payments, and prepayments, less servicing and guaranty fees are passed to the holder of the security. For FNMA MBS, the payment date is the 25th of the month. If the 25th day happens to fall on a bond market holiday or a weekend, the payment is made on the following business day. A time line for the timing of payments for a hypothetical TBA trade is shown in Table A2.

A.3 Adjustment for Fees

Fixed-rate mortgage pools consist entirely of fixed-rate loans, but the underlying loans may bear different fixed rates of interest. Interest on a fixed-rate pool is set on the issue date of the related certificates, and it is equal to the interest rates less the fee percentages for each loan in the pool. The fee percentage is the sum of the servicing fee and the guaranty fee for that loan. Fixed-rate loans in a single pool have interest rates that are within a 2% range (sometimes a wider range may be allowed). However, the pass-through rate of each loan in fixed rate pool is the same. Therefore, the pass-through rate will not change if prepayments occur.

Consider the cash flow generated by a pass-through of an individual fixed rate mortgage. Prior to either prepayment or default, the owner of the pass-through receives the constant cash flow c (consisting of both interest and scheduled principal) generated by the mortgage loan less the servicing and guaranty fees, which are a percentage of the principal balance I_t. Denote the servicing and guaranty fees by the constant g. Then the cash flow generated by the pass-through security, c_t^{PT}, in absence of prepayment, is

$$c_t^{PT} = c - gI_t. \tag{A1}$$

Therefore, the value of a FNMA MBS, after accounting for fees, is given by

$$F(m,T) = E^Q\left[\int_0^T \exp\left(-\int_0^t r_s + w_s \, ds\right) N_t \, (c + (p_t - g)\, I_t\,)\, dt\right], \tag{A2}$$

where

$$p_t = x_t + y_t \max(0,\ m - a - b\, r_t(10)). \tag{A3}$$

A.4 Valuation

Because of the independence of w_t from the other stochastic processes, we can write

$$F(m,T) = \int_0^T S(t)\, E^Q\left[\exp\left(-\int_0^t r_s \, ds\right) N_t \, (c + (p_t - g) I_t)\right] dt, \tag{A4}$$

where

$$S(t) = E^Q\left[\exp\left(-\int_0^t w_s \, ds\right)\right]. \tag{A5}$$

The expression for $S(t)$ is given by

$$S(t) = A(t)\exp(-w_0\, B(t)), \tag{A6}$$

where

$$\begin{aligned} A(t) = \exp\Bigg(&\left(\frac{\sigma_w^2}{2} - \frac{\alpha_w}{\beta_w}\right)t + \left(\frac{\alpha_w}{\beta_w^2} - \frac{\sigma_w^2}{\beta_w^3}\right)\big(1 - \exp(-\beta_w t)\big) \\ &+ \frac{\sigma_w^2}{4\beta^3}\big(1 - \exp(-\beta_w t)\big)\Bigg), \end{aligned} \tag{A7}$$

$$B(t) = \frac{1}{\beta_w}\big(1 - \exp(-\beta_w t)\big). \tag{A8}$$

We assume that the interest rate follows the Hull and White (1990) model,

$$dr = (\alpha_r(t) - \beta_r r)\, dt + \sigma_r \, dZ_r, \tag{A9}$$

where β_r and σ_r are positive constants and the deterministic function $\alpha_r(t)$ is chosen to match the Treasury term structure exactly. Given the market discount function $D(t)$ and values for β_r and σ_r,

$$\alpha_r(t) = -\frac{\partial^2 \ln D(t)}{\partial t^2} - \beta_r \frac{\partial \ln D(t)}{\partial t} + \frac{\sigma_r^2}{2\beta_r}\big(1 - \exp(-\beta_r t)\big). \tag{A10}$$

At time t (where t is measured in years), the 10-year interest rate $r_t(10)$ is an affine function of the short rate r_t,

$$r_t(10) = \frac{1}{10}(-A_r(t) + B_r\ r_t), \tag{A11}$$

where

$$A_r(t) = \ln \frac{D(t+10)}{D(t)} - B_r \frac{\partial \ln D(t)}{\partial t} - \frac{\sigma^2}{4a}\left(1 - \exp(-2\beta_r t)\right) B_r^2), \tag{A12}$$

$$B_r = \frac{1 - e^{-10\beta_r}}{\beta_r}. \tag{A13}$$

Therefore, we can rewrite the prepayment intensity in terms of r_t,

$$p_t = x_t + y_t \max(0, m - a - b(-A_r(t) + B_r\ r_t)/10). \tag{A14}$$

A.5 Adjustments For Discrete Cash Flows

In this section, we adjust the mortgage valuation formula to account for the actual cash flows from a TBA trade. Let

$$\tilde{T} \equiv \{0, t_s, t_1, \ldots, t_K\}, \tag{A15}$$

be the set of points in time related to the payments associated with a mortgage-backed security with K months until maturity. The valuation date, or trade date, is $t = 0$. The MBS settles at $t = t_s$, and the MBS investor receives payments on dates t_1 through t_K. Since the settlement dates are fixed by SIFMA, the amount of time from the valuation date $t = 0$ through the settlement date $t = t_s$ varies depending on the trade date. Also, because of holidays and weekends, each time step after settlement, that is, $\Delta t_i \equiv t_{i+1} - t_i$, $i = 1, \ldots, K-1$, may vary by a couple of days. Let CF_i be the cash flow received by the mortgage investor at time t_i. In our framework, the value of the mortgage is

$$F(m, K) = \frac{1}{S(t_s)} \sum_{i=1}^{K} S(t_i)\ E^Q[\ \exp(-\textstyle\int_{t_s}^{t_i}\ r_s\ ds)\ \text{CF}_i\ ]. \tag{A16}$$

To determine the cash flow CF_i at t_i, we can apply standard mortgage cash flow formulas (recall that in continuous time, the cash flow is $N_t[c + (p_t - g)I_t]$). Following Hayre (2001), for each dollar of a mortgage in month i,

$$\text{Monthly payment} = \text{PAY}_i = \frac{m/12}{1 - (1 + m/12)^{-K}}, \tag{A17}$$

$$\text{Balance (end of month)} = \text{BAL}_i = \frac{1 - (1 + m/12)^{-K+i}}{1 - (1 + m/12)^{-K}}, \tag{A18}$$

$$\text{Principal portion of payment} = \text{PRIN}_i = \text{PAY}_i \times (1 + m/12)^{-K-1+i}, \tag{A19}$$

$$\text{Interest portion of payment} = \text{INT}_i = \text{PAY}_i - \text{PRIN}_i. \tag{A20}$$

Let

$$\tilde{T}_{CF} \equiv \{\tau_0, \tau_1, \ldots, \tau_K\}, \tag{A21}$$

be the set of points in time relevant to determine the monthly cash flows of the MBS. This set corresponds to month-ends. For the example in Table A2, τ_0 is March 31st, the month end before the settlement date, and τ_1 is April 30th. It is possible that τ_0 is either before or after the trade date depending on whether the trade date and settlement date occur in the same month. However, because the data are observed at each month end, the elements of $\tilde{T}$ and $\tilde{T}_{CF}$ are ordered as

$$\tau_0 = 0 < t_s < \tau_1 < t_1 < \tau_2 < t_2 < \cdots < \tau_K < t_K, \tag{A22}$$

as shown in the example in Table A2.

Recall that N_t represents the fraction of the mortgage pool that has not yet prepaid (i.e., a survival factor). To keep track of monthly prepayments, we calculate the single monthly mortality (SMM), a common object in mortgage modeling. SMM is fraction of the pool's outstanding balance at the beginning of the month that is prepaid during the month. Hence,

$$\text{SMM}_i = \frac{N_{\tau_{i-1}} - N_{\tau_i}}{N_{\tau_{i-1}}}. \tag{A23}$$

Therefore, the prepayments in a given month i, PP_i, can be written as

$$\text{PP}_i = \left(\text{BAL}_{i-1} \times N_{\tau_{i-1}} - \text{PRIN}_i \times N_{\tau_{i-1}}\right) \times \text{SSM}_i. \tag{A24}$$

The cash flow CF_i received by the investor at t_i reflects the payments (scheduled and prepaid) at τ_i from the underlying mortgage loans, less servicing and guaranty fees g. Therefore,

$$\text{CF}_i = \text{PRIN}_i \times N_{\tau_{i-1}} + \text{PP}_i + \frac{m-g}{m} \times \text{INT}_i \times N_{\tau_{i-1}}. \tag{A25}$$

Given paths of r_t, x_t, and y_t, we calculate a path of p_t. After integration and exponentiation, we calculate N_t for the relevant time points. Then, the standard mortgage formulas provide the cash flows.

A.6 Discount Function

We collect historical data on nominal-constant maturity Treasury rates from the Federal Reserve's H.15 statistical release. Then, we use a standard cubic spline to bootstrap the prices of zero-coupon bonds $D(t)$ for the relevant time points for up to 30 years. For a discussion of this methodology, see Longstaff, Mithal, and Neis (2005).

A.7 Estimation of Interest Rate Dynamics

As discussed above, in the dynamics for the riskless rate in the Hull and White (1990) model,

$$dr = (\alpha_r(t) - \beta_r\, r)\, dt + \sigma_r\, dZ_r, \tag{A26}$$

the deterministic function $\alpha_r(t)$ is chosen to match the Treasury term structure exactly. Therefore, estimation of the model involves finding the values of β_r and σ_r that best fit a set of market instruments on a given date.

Pass-through mortgage-backed securities are most sensitive to changes in intermediate-term yields (e.g., see Ho 1992 or Dunn et al. 2016). Moreover, the refinancing incentive in our model is a function of the 10-year Treasury yield. As such, we fit the interest rate model to the intermediate sector of the volatility surface. Because European swaptions are among the most liquid options on interest rates, we estimate the interest rate volatility using the swaption volatility surface. Specifically, we select the set of 1-, 2-, 3-, 4-, and 5-year into 5-, 7-, and 10-year at-the-money-forward European receivers swaptions, giving a total of 15 instruments. We then calculate the prices of at-the-money-forward receivers swaptions referencing the Treasury curve ($D(t_i)$ is from the Treasury curve). The price of a T into $t_n - T$ receivers swaption with payment dates $t_1, t_2, \ldots, t_n$ and normal volatility σ_N is

$$P = \sigma_N \sqrt{\frac{T}{2\pi}} \sum_{i=1}^{n} D(t_i). \tag{A27}$$

Corb (2012) provides an extensive discussion of the normal swaption model.

To calculate the prices of the swaptions in the Hull and White (1990) model, we apply the Jamshidian (1989) decomposition, allowing us to write the swaption price as a weighed sum of zero-coupon bond options, for which there are analytical formulas (see Brigo and Mercurio 2006 or Hull 2015 for a textbook treatment of this approach).

Finally, we solve for values of σ_r and β_r on a given date that minimize the sum of squared percentage price error:

$$\sum_{i=1}^{15} \left[\frac{P_{\text{market}}(i) - P_{\text{model}}(i)}{P_{\text{market}}(i)} \right]^2, \tag{A28}$$

of the 15 swaptions using the Levenberg-Marquardt algorithm. The use of the sum of squared percentage price errors as an objective function follows standard practice. The model fits the Black volatility surface of the 15 swaptions with a median RMSE of 38 bps. Over the sample period, the median Black volatility is 20.64%.

A.8 Estimation Methodology

The estimation of the model can be viewed as consisting of three steps.

1. We select an initial parameter vector θ, where $\theta = \{a, b, \alpha_w, \alpha_x, \alpha_y, \beta_w, \beta_x, \beta_y, \sigma_w, \sigma_x, \sigma_y, \rho_{r,x}, \rho_{r,y}, \rho_{x,y}\}$.
2. Conditional on θ and for each month t during the sample period, we solve for the values of w_t, x_t , and y_t that best fit the model to the prices for the cross-section of mortgage-backed securities with different coupon rates (the coupon stack) by minimizing the RMSE.

   Since the nonlinear structure of the prepayment function makes it difficult to express the price of mortgage-backed securities in closed-form, we use simulation to solve for the model-based mortgage-backed security values. The simulation step solves

   $$F(m, K) = \frac{1}{S(t_s)} \sum_{i=1}^{K} S(t_i)\, E^Q[\exp(-\textstyle\int_{t_s}^{t_i} r_s\, ds)\, CF_i], \tag{A29}$$

   for given values of w_t, x_t , and y_t. Since $S(t)$ has a closed-form solution, we can focus on the expectation in the equation above. We simulate paths of r_t, x_t, and y_t with monthly time steps, and then compute a path of p_t since

   $$p_t = x_t + y_t \max(0, m - a - b\, r_t(10)). \tag{A30}$$

   From p_t, we compute the survival factors $N(t)$ for each month, and then compute the mortgage cash flows CF_i. Along each path, we also compute the discount factor to apply to each cash flow. The average discounted cash flows over all paths provides the estimate of the expectation. Given the value of the expectation, we then solve for the mortgage price.

   The RMSE of the simulated prices and the market prices provides the objective function for the optimization. We use the controlled random search (CRS) algorithm of Kaelo and Ali (2006) to solve for the initial values of w_t, x_t, and y_t, which minimize the RMSE for each date t.
3. We apply the CRS algorithm to the parameter vector θ to find the vector that results in the lowest global RMSE. The outputs are the parameter values and the time series of state variables.

A.9 Identifying the Empirical Turnover and Rate Response Factors

In identifying the empirical turnover and rate response factors, we use the one-month conditional prepayment rates (CPRs) for the same set of mortgage-backed securities used to estimate the implied turnover and rate response factors. To illustrate how this is done, let CPR_{it} denote the 1-month realized CPR observed at time t for the ith mortgage-backed security, where $i = 1, 2, \ldots, n_t$, and where n_t is the number of individual mortgage-backed securities in the sample at time t. For a given t, we estimate the empirical turnover and rate response factors, denoted as $\hat{x}_t$ and $\hat{y}_t$, from the following cross-sectional nonlinear regression,

$$\text{CPR}_{it} = 1 - \exp(-\hat{x}_t - \hat{y}_t \max(0,\ m\ - a\ - b\ r_t(10))) + \epsilon_{it}, \tag{A31}$$

where ϵ_{it} denotes an i.i.d. normally distributed residual. The exponential term in this expression results from the mapping of the hazard rate function into the conditional prepayment rate. This nonlinear regression is estimated separately for each date t in the sample using the CPRs for the n_t individual mortgage-backed securities in the sample on date t. We solve for the best fitting values of $\hat{x}_t$ and $\hat{y}_t$ by minimizing the sum of squared residuals using a standard genetic algorithm numerical optimizer. We repeat this process using different sets of starting values to ensure that we achieve the global minimum. Given the relatively small values of n_t in the sample, we make a minor concession to the data and impose the lower bound constraints $\hat{x}_t \geq -\ln(1 - \min_i CPR_{it})$ and $\hat{y}_t \geq 0$ in the estimation to guard against the effects of outliers in the data. These lower bounds ensure that estimated empirical prepayment rate remains positive.

A.10 Decomposing the Prepayment Rate into Components

To make our approach to decomposing the prepayment rate into its turnover and rate response prepayment components more intuitive, we introduce the following notation:

$$\text{CPR} = 1 - \exp\big(-x_t - y_t \max(0,\ m\ - a\ - b\ r_t(10))\big), \tag{A32}$$

$$\text{CPR}_x = 1 - \exp\big(-x_t\big), \tag{A33}$$

$$\text{CPR}_y = 1 - \exp\big(-y_t \max(0,\ m\ - a\ - b\ r_t(10))\big), \tag{A34}$$

where CPR_x and CPR_y denote the CPR values resulting from setting y_t and x_t, respectively, equal to zero. From these expressions, it directly follows that,

$$1 - \text{CPR} = (1 - \text{CPR}_x)\ (1 - \text{CPR}_y), \tag{A35}$$

which implies

$$\text{CPR} = \text{CPR}_x + \text{CPR}_y - \text{CPR}_x\ \text{CPR}_y. \tag{A36}$$

The cross-product term in the above expression is typically very small. To decompose the CPR into turnover and rate response components, we simply distribute the cross-product term based on the values of CPR_x and CPR_y. Thus, the turnover prepayment rate is

$$\text{CPR}_x - \frac{\text{CPR}_x}{\text{CPR}_x + \text{CPR}_y}\ \text{CPR}_x\ \text{CPR}_y. \tag{A37}$$

Similarly, the rate response prepayment rate is

$$\text{CPR}_y - \frac{\text{CPR}_y}{\text{CPR}_x + \text{CPR}_y}\ \text{CPR}_x\ \text{CPR}_y. \tag{A38}$$

A.11 Burnout and Seasoning

In this section, we discuss how the implied prepayment model can be extended to allow for burnout and seasoning effects. Burnout refers to the fact that the longer a pool of mortgages is exposed to refinancing incentives, the less responsive the pool is to subsequent refinancing incentives. Burnout can be modeled as a cap h on the prepayment incentive so that

$$p_t = x_t + y_t \min(\max(0,\ m - a - b\, r_t(10)),\ h). \tag{A39}$$

Seasoning refers to the increase in prepayment speeds with the age of the pool. Mortgage pools can season with respect to both refinancing and turnover. Typically, seasoning is modeled as a ramp up to a steady-state level. For example, the Public Securities Association (PSA) standard prepayment model assumes a 30-month ramp-up period. In our model, we can incorporate burnout, turnover seasoning, and refinancing seasoning with the specification

$$p_t = \hat{x}_t + \hat{y}_t \min\big(\max(0,\ m - a - b\, r_t(10)), h\big), \tag{A40}$$

where

$$\hat{x}_t = \min\left(\frac{\text{WALA}_t}{30}, 1\right) x_t, \tag{A41}$$

$$\hat{y}_t = \min\left(\frac{\text{WALA}_t}{30}, 1\right) y_t, \tag{A42}$$

and WALA_t is the weighted average loan age in months for the mortgage pool at time t.

A.12 Additional Robustness Results

As discussed earlier, our results are based on what is perhaps the simplest possible specification of the implied prepayment function. As a robustness check, we examine how the results are affected when the implied prepayment function is modified to include seasoning and burnout effects. Recall that seasoning and burnout appear to be important features of empirical prepayment rates. Thus, it is possible that incorporating these features into the implied prepayment function may improve the ability of the model to explain mortgage-backed security prices. To capture seasoning effects, we use the standard prepayment model convention of the Public Securities Association (PSA) that prepayment rates increase or ramp up linearly over the first 30 months of the life of the mortgage loans underlying the mortgage-backed securities. Specifically, we replace the values of x_t and y_t in the implied prepayment function with the terms $\min(1, \text{WALA}/30)x_t$ and $\min(1, \text{WALA}/30)y_t$, where WALA denotes the weighted average life of the loans in months. To capture burnout effects, we replace the maximum operator in the implied prepayment specification in Equation (8) with the expression $\min(\max(0, m - a - b\, r_t(10)), 0.024)$. This specification implies that the refinancing incentive increases linearly from zero to 240 bps points in the money, but then becomes constant for mortgage-backed securities that are more than 240 bps in the money. Thus, this nonlinear specification implies that deep-in-the-money mortgage-backed securities exhibit burnout behavior and do not prepay at higher rates. The burnout threshold of 240 bps is determined by solving for the value that best matches the 3-month empirical prepayment rates for the mortgage-backed securities in the sample. We acknowledge, however, that since burnout is a function of the entire history of a mortgage-backed security, our approach of conditioning on the current refinancing incentive—necessitated by the limited cross-section of TBAs available in the sample—will likely not fully capture potential burnout effects.

We reestimate the model using this extended implied prepayment function. The model actually does worse when seasoning and burnout are incorporated into the implied prepayment function. Specifically, the median RMSE for the model increases from 25.7 cents to 26.5 cents when seasoning and burnout are included. The values of x_t and y_t we obtain using this implied prepayment specification are similar to those reported earlier, although slightly more volatile. In particular, the average values of x_t and y_t from this specification are 0.0972 and 11.334, respectively, which are close to those in Table 4.

As another robustness check, we also reestimate the model with the Hull and White (1990) model fitted to the swap curve rather than the Treasury curve. In doing this, we also recalibrate the

Hull and White model to match the same set of swaption volatilities using the same procedure as before.

The results from this exercise are very similar to those reported previously. The model fits the data slightly worse when the swap curve is used to discount cash flows—the median RMSE increases from 25.7 cents to 26.9 cents. The estimated values of x_t and y_t are virtually identical to those obtained previously using the Treasury curve to discount cash flows. Not surprisingly, the only discernible effect of using the swap curve is that the estimates of the credit/liquidity factor w_t are lower by the average swap spread during the sample period. In particular, the average value of w_t decreases from 65.5 bps to 24.4 bps.

To evaluate the effect on the empirical results, we reestimate the regression in Table 5 using the values of w_t obtained when the swap curve is used. The regression results are very similar to those before. In particular, the FNMA spread and settlement fails variables continue to be significant with the same sign and similar regression coefficients as before.

To evaluate the sensitivity of the results to the filters we used in creating the data set, we reestimated the model using only the five TBAs with coupons closest to the current coupon mortgage rate. The liquidity of TBAs is generally lower for the mortgage-backed securities with coupons that are farthest from the current coupon mortgage rate. Thus, this approach eliminates some of less liquid TBAs from the estimation since their prices are more likely to be measured with error. Despite the reduction in the number of TBAs used in the estimation, however, the estimates of the credit/liquidity spread, the turnover factor, and the rate response factor are very similar to those we obtain using the entire data set.

As a final robustness check on the model specification, we also regressed the pricing errors from the individual mortgage-backed securities on their price, price squared, WAC, and WALA. Although not shown, the results imply that there is little apparent relation between the pricing errors and these measures. The exception is the WALA of the TBA, which is significantly positively related to the pricing errors. This suggests that one possible direction for extending the simple implied prepayment specification used in this paper might be to incorporate the age of the loan into the model. Intuitively, this would parallel the seasoning and burnout features often incorporated into empirical prepayment models.

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CHAPTER 2

Funding Liquidity and the Valuation of Mortgage-Backed Securities

I study the relationship between funding liquidity and the valuation of mortgage-backed securities. Most of the financing for mortgage-backed securities occurs through a trade known as a dollar roll, the simultaneous sale and purchase of forward contracts on mortgage-backed securities that is analogous to a repurchase agreement. I develop a four-factor no-arbitrage model for valuing mortgage-backed securities that allows for the valuation of dollar rolls. Unlike previous models of the dollar roll, I allow for the possibility of a prepayment risk premium. I develop a new measure of mortgage specialness that is independent of prepayment risk premia and agency credit spreads. I find that specialness is related to measures of balance sheet constraints and primary dealer positions in mortgage-backed securities.

2.1 Introduction

The U.S. agency mortgage-backed securities (MBS) market is one of the largest and most important markets globally, with \$8.2 trillion notional outstanding as of June 2018.[1,2] On average, 94 percent of all MBS trading volume, around \$200 billion per day, occurs as forward contracts, known as to-be-announced (TBA) contracts.[3] In addition to being a highly liquid market to buy and sell MBS, the TBA market also serves as the most important funding market for MBS, through trades known as dollar rolls. A dollar roll is two simultaneous TBA trades for the same notional amount and type of MBS, but with different settlement dates. Economically, dollar rolls are similar to repurchase agreements, but their accounting treatment can be quite different.

This paper provides a theory of valuation of forward contracts on agency mortgage-backed securities in the presence of prepayment risk, issuer default risk and funding liquidity risk. The central contribution of the theory is that it provides the first measure of implied MBS financing rates that allows for prepayment risk. In addition, the model is the first to allow for the separate identification of an issuer credit spread and a funding liquidity spread. Intuitively, the funding liquidity spread is the channel through which factors such as capital constraints, liquidity, quantitative easing programs, and the cost or benefit of holding cash securities, impact the value of mortgage-backed securities. The funding liquidity spread determines if mortgages trade special, that is, if their funding cost is lower than other short-term rates.

My model identifies the funding liquidity spread through the assumption that holding cash mortgage-backed securities carries a storage cost, similar to commodities, but forward contracts do not. The cost of holding cash mortgage-backed securities can stem from frictions like balance sheet constraints and regulatory requirements. On the other hand, it is possible

[1] A mortgage-backed security (MBS) is a securitized claim to the principal and interest payments generated by a pool of fixed-rate mortgages. Agency MBS refers to MBS issued by Fannie Mae, Freddie Mac or Ginnie Mae.

[2] Source: https://www.sifma.org/resources/research/us-mortgage-related-issuance-and-outstanding/

[3] Source: https://www.sifma.org/resources/research/us-sf-trading-volume/

that there is a benefit to holding mortgages on balance sheet because the cash securities can become a scarce resource during times of Federal Reserve purchases or heavy deal flow of tranched securities. Specifically, I assume that an investor that holds a cash mortgage-backed security incurs a net cost v, measured as a percentage of market value, whereas an investor that holds of a forward contract does not incur this cost. Thus by trading in the forward market, and investor can avoid the net cost v while maintaining a long position in mortgage-backed securities.

The justification for a funding liquidity spread in the prices of TBA contracts stems from the accounting treatment of dollar rolls. Gao, Schultz, and Song (2017) find that dollar rolls account for about half of the volume in the TBA market. Dollar rolls are more likely to be treated as sales and purchases whereas repurchase agreements are treated as secured financing. This means that long positions in mortgage-backed securities funded with dollar rolls can be held off balance sheet. In contrast, cash mortgage-backed securities are held on balance sheet even if they are financed in the repo market. The difference in accounting treatment may be a channel through which balance sheet constraints and regulatory requirement impact the value of mortgage-backed securities.

Dollar rolls are a convenient way to measure financing costs because they are based on liquid market prices. Therefore, all market participants face approximately the same financing rates. In contrast, both the interest rate and haircut for repurchase agreements vary by counterparty, and the counterparty-level data are not readily available. Unlike the repo rate for a repurchase agreement, however, the financing cost for a dollar roll is unknown at inception due to uncertainty about future prepayments. Therefore, a model of prepayment behavior is necessary to estimate financing costs implied by dollar rolls.

The traditional approach to measuring MBS financing rates involves the use of a statistical prepayment model fitted to historical data. Given the model's forecast of future prepayments and the prices of TBA contracts across settlement months, one can solve for financing rates implied by dollar rolls. The details of this approach are discussed in section 2.4.2 and Appendix 2.13. This implied financing rate is often called the "break-even financing rate" because, given a prepayment assumption, this rate equates the value of "rolling"

MBS, i.e. financing MBS via a dollar roll, and holding MBS. To date, all studies of MBS funding costs follow this approach.

Despite the popularity of the traditional approach, recent research brings into question its underlying assumptions. First, using a statistical prepayment model calibrated to historical data rules out the possibility of a prepayment risk premium. Essentially, these models assume that "risk-neutral" and "real world" prepayment rates are identical. However, many papers including Gabaix, Krishnamurthy, and Vigneron (2007), Chernov, Dunn, and Longstaff (2017), Boyarchenko, Fuster, and Lucca (2019a), Diep, Eisfeldt, and Richardson (2019), and others, present evidence of a significant prepayment risk premium that impacts the value of MBS. Moreover, Chernov, Dunn, and Longstaff (2017) show that risk-neutral prepayments behave very differently than actual prepayments. Hence, industry-standard prepayment models are not aligned with the mounting evidence that MBS prices reflect prepayment risk. Without allowing for the possibility of prepayment risk premia, it is probable that break-even financing rates from the traditional approach are very different than the market's expectations.

Second, break-even financing rates are unique to each prepayment model (and pair of TBA contracts). Carlin, Longstaff, and Matoba (2014), however, show that there is substantial disagreement about prepayment forecasts among dealers. Therefore, switching from one dealer's model to another leads to different break-even financing rates. Also, as shown by Chernov, Dunn, and Longstaff (2017) dealers update their models frequently, leading to jumps in prepayment forecasts. Even switching between models of a particular dealer give different break-even financing rates. To deal with these discontinuities, Kitsul and Ochoa (2016), Song and Zhu (2019) and others use break-even financing rates that have been backfilled using a recent prepayment model calibrated to fit the historical prepayment experience over the sample period. As a result, break-even financing rates in these papers do not reflect contemporaneous prepayment expectations.

Besides the conceptual issues with break-even financing rates, even extracting meaningful information from them is problematic. Consider the problem of relating financing rates to expected returns. Following Gabaix, Krishnamurthy, and Vigneron (2007), Kitsul and Ochoa

(2016) and Song and Zhu (2016) use the model's option-adjusted spread (OAS) as a proxy for expected returns. MBS is valued via simulation methods, and the OAS is a constant value added to the short rate rate along all paths such the model price equals the market price. As shown by Young (2010) and it more detail in Appendix 2.14, the break-even implied financing rate equals the model's option-adjusted spread (OAS), to a first-order approximation. Therefore, the relationship between OAS and break-even financing rates is not of any economic significance; it is simply a mechanical relationship through the model.

Because break-even financing rates are model dependent, they are subject to model misspecification. To be sure, any implied financing rate is model-dependent and subject to misspecification. However, prepayment models are proprietary and very complicated. Therefore, from a researcher's prospective, a prepayment model is a "black box" and it is unclear how much or in what ways the model is misspecified. As discussed by Gabaix, Krishnamurthy, and Vigneron (2007), the explanation that results from OAS analysis are driven by model misspecification cannot be ruled out, and because break-even financing rates are OASs to a first order approximation, the same can be said about break-even financing rates.

My model has several advantages over the traditional approach. First, my model provides a new measure of MBS funding costs and a funding liquidity risk premium that is not confounded with prepayment and credit risk. Second, while the traditional approach provides a break-even financing rate for every possible dollar roll, my model includes an aggregate measure of the MBS funding rate, comparable to a general collateral MBS repo rate. The model is easily extended to allow for collateral-specific funding liquidity. Third, the model is a simple and transparent four-factor no-arbitrage model, and it is not subject to the "black-box" misspecification concerns of a traditional prepayment model.

I find that my no-arbitrage model fits the cross-section and term structure of mortgage TBA contracts extremely well. The median root-mean-square error (RMSE) across the entire coupon stack and three expiration dates is 15.9 cents per $100 notional, which is the same order of magnitude as the bid-ask spread for mortgage-backed securities. This level of accuracy is an improvement over Chernov, Dunn, and Longstaff (2017) and previous generations of valuation models for mortgage-backed securities. Over the sample period, I find that the mean credit spread is 48.7 basis points and the mean funding liquidity spread is 20.8 basis points.

Empirically, I find the agency MBS credit spread is strongly related to other measures of credit risk. The funding liquidity spread is strongly related to primary dealer positions in mortgage-backed securities, measures of balance sheet usage costs and proxies for debt overhang costs of the financial sector. Thus, my results provide evidence that intermediary balance sheet constraints impact the value of mortgage-backed securities, one of the largest and most important asset classes.

2.2 Related Literature

This paper studies the role of the funding market in the valuation of mortgage backed securities and it is related to several branches of literature.

My study is most closely related to Song and Zhu (2019) and Kitsul and Ochoa (2016). Both Song and Zhu (2019) and Kitsul and Ochoa (2016) study implied financing rates from JP Morgan's prepayment model. Song and Zhu (2019) focuses on the determinants of dollar roll specialness, the difference between implied financing rates and a short-term interest rate. Kitsul and Ochoa (2016) derive a measure of funding liquidity risk from implied financing rates and study its relationship to the model's option-adjusted spread. The main difference between my study and theirs is that I propose a new no-arbitrage framework to value the dollar roll, whereas those studies rely on the output of JP Morgan's prepayment model. There are several drawbacks to their approach. First, in order to have a continuous time series, they use a recent prepayment model that has been calibrated to historical data. This means that the implied financing rates are not what the contemporaneous model would have produced. Second, as discussed in section 2.4.2 and Appendix 2.14, the use of an option-adjusted spread model with the traditional dollar roll model implies that the implicit financing rate is approximately the short rate plus the option-adjusted spread, or equivalently, that specialness is approximately the option-adjusted spread. Therefore, the implied financing rate inherits all the problems of option-adjusted spreads. Also, the link between option-adjusted spreads and implied financing rates is not of economic significance – it is simply a by product of the model. Finally, these papers do not allow for a prepayment risk premium despite strong evidence that prepayment risk is a key factor in the valuation of mortgage-backed securities.

This paper also contributes to the literature on the financing and hedging of mortgage-backed securities. To the best of my knowledge, Duarte, Longstaff, and Yu (2007) was the first study to incorporate dollar rolls. They construct a fixed-income arbitrage trading strategy financed with a dollar roll and find that it produces high alpha for discount and par mortgages. More recently, Gao, Schultz, and Song (2017) focus the specified pool market and find that spikes in the trading volume of dollar rolls seem to reduce the trading costs for specified pools.

My four-factor no-arbitrage valuation model contributes the literature on the valuation of mortgage-backed securities. In this regard, my paper is most closely related to Chernov, Dunn, and Longstaff (2017), but the focus of my paper is quite different. I extend the framework of Chernov, Dunn, and Longstaff (2017) to allow for the valuation of dollar rolls and forward contracts of mortgage-backed securities. This involves separating the credit/liquidity spread of Chernov, Dunn, and Longstaff (2017) into distinct credit and liquidity factors identified through prices of forward contracts. Papers on the valuation of mortgage-backed securities include Dunn and McConnell (1981a), Dunn and McConnell (1981b), Brennan and Schwartz (1985), Richard and Roll (1989), Schwartz and Torous (1989, 1992, 1993), Kau, Keenan, Muller, and Epperson (1992), Stanton (1995), Boudoukh, Whitelaw, Richardson, and Stanton (1997), Stanton and Wallace (1998), Deng, Quigley, and Van Order (2000), Kau and Slawson (2002), Dunn and Spatt (2005), Downing, Stanton, and Wallace (2005), and Longstaff (2005) and others.

Two recent papers are similar to mine in that they explore the basis between derivative contracts and cash positions. I study the wedge between the implied funding rate of cash mortgage-backed securities compared to forward contracts on those securities. Du, Tepper, and Verdelhan (2018) shows that there are large and persistent violations of the covered interest rate parity relation in a number of major currencies. These violations can be interpreted as wedges between the implied funding rate of currency forward contracts and the spot interest rate in cash markets. Du, Tepper, and Verdelhan (2018) provide evidence that these violations are related to balance sheet factors in post-financial-crisis period. Fleckenstein and Longstaff (2020) complements Du, Tepper, and Verdelhan (2018) by showing that their results can be extended to periods much earlier than the post-crisis period in other large derivative markets. I develop a model of mortgage-backed securities that allows for different funding costs of forwards and cash positions, and through the estimation of the model I provide evidence that balance sheet constraints impact the valuation of mortgage-backed securities.

Like Fleckenstein and Longstaff (2020), this paper contributes to the pricing of derivatives by showing that there is a large funding-related basis between cash mortgage-backed securities and mortgage TBAs (forward contracts). Other studies of the bases between securities and their derivatives are Longstaff, Mithal, and Neis (2005), Duffie (2010), Bai and Collin-Dufresne (2019), Du, Tepper, and Verdelhan (2018), Fama and French (1987, 1988), Gabaix, Krishnamurthy, and Vigneron (2007), Duarte, Longstaff, and Yu (2007),Stanton and Wallace (2011) , Chernov, Dunn, and Longstaff (2017), Boyarchenko, Fuster, and Lucca (2019b), Song and Zhu (2019), Klingler and Sundaresan (2019), Brenner, Subrahmanyam, and Uno (1989), Constantinides, Jackwerth, and Savov (2013), Fleckenstein, Longstaff, and Lustig (2014), Haubrich, Pennacchi, and Ritchken (2012), Longstaff and Rajan (2008), Coval, Jurek, and Stafford (2009), Longstaff and Myers (2014), Ronn and Ronn (1989), Longstaff (1995), van Binsbergen, Diamond, and Grotteria (2019), and Pasquariello (2014, 2018).

I also contribute to the literature on the relation between asset pricing and the constraints of intermediaries. Examples include Nanda and Chowdhry (1998), Xiong (2001), Kyle and Xiong (2001), Gromb and Vayanos (2002), Krishnamurthy (2003, 2010), Brunnermeier and Pedersen (2009), Adrian and Shin (2010), Garleanu and Pedersen (2011), He and Krishnamurthy (2012, 2013), Adrian, Etula, and Muir (2014), Kondor and Vayanos (2019), Pasquariello (2014, 2018), Lewis, Longstaff, and Petrasek (2017), He, Kelly, and Manela (2017), Adrian, Boyarchenko, and Shachar (2017), Chen, Joslin, and Ni (2019), Andersen, Duffie, and Song (2019).

2.3 U.S. Agency Mortgage-Backed Securities

Agency mortgage-backed securities (MBS) are issued by either the government-sponsored enterprises (GSEs) Fannie Mae (FNMA) and Freddie Mac (FHLMC) or Ginnie Mae (GNMA), a wholly-owned government corporation.[4] Fannie Mae and Freddie Mac are supervised and regulated by the Federal Housing Finance Agency (FHFA) whereas Ginnie Mae is within the U.S. Department of Housing and Urban Development (HUD). The market for U.S. agency mortgage-backed securities is among the largest and most liquid markets globally. More than 70 percent of the $10.2 trillion home mortgages in the U.S. serve as collateral for agency mortgage-backed securities.

2.3.1 The Agency Guarantee

Perhaps the most attractive feature of agency mortgage-backed securities is their credit guarantee. In exchange for monthly fees, the agencies guarantee the timely payment of interest and principal from mortgage-backed securities. The guarantee protects investors from delinquencies and defaults on the underlying mortgages and from the failure of a direct servicer to remit borrower payments to the securitization trust. If a mortgage loan becomes delinquent, the agencies guarantee that investors continue to receive scheduled interest and principal until either the borrower cures the delinquency or the issuer buys the loan out of the trust at par plus interest. Therefore, defaults appear as prepayments from an investor's perspective.

The quality of the guarantee varies by issuer. Ginnie Mae securities carry the full faith and credit guarantee of the United States government. Thus, the credit quality of Ginnie Mae securities is the same as that of U.S. Treasuries. Fannie Mae and Freddie Mac securities, however, carry a credit guarantee from the issuing GSE rather the United States. Historically, investors have viewed a GSE guarantee as an "implicit" government guarantee, because they believed the government would back the agencies in times of stress. This view was

[4]Fannie Mae, Freddie Mac, and Ginnie Mae refer to the Federal National Mortgage Association, the Federal Home Loan Mortgage Corporation, and the Government National Mortgage Association, respectively.

validated in September 2008 when the government placed Fannie Mae and Freddie Mac in conservatorship and provided them with unlimited access to collateralized funding. Even though Fannie Mae and Freddie Mac are currently in conservatorship, they could default on their obligations in the future if their financial condition deteriorates or if they are placed into a new conservatorship or into receivership.

In the event that either Fannie Mae or Freddie Mac fail to make the payments required under the guarantee, investors would receive only the amounts paid on the underlying mortgage loans, which are generally limited to borrower payments and other recoveries on the loans. As a result, delinquencies and defaults on the underlying mortgage loans or a servicer's failure to remit borrower payments to the trust would adversely affect the amounts that investors receive each month.

2.3.2 Cash Flows of a Fixed-Rate Agency Pass-Through Security

In this paper, I focus on agency pass-through mortgage-backed securities backed by pools of fixed-rate mortgages. A fixed-rate mortgage is structured so that the borrower is obligated to make the same payment each month, consisting of interest and principal. In general, fixed-rate mortgages can be prepaid at any time without penalty. Each month, therefore, a pool of mortgages generates cash flows consisting of scheduled interest, scheduled principal, and possibly prepaid principal. A pass-through mortgage-backed security distributes to investors the payments from the underlying mortgage loans, less guaranty and servicing fees. Because the servicing and guaranty fees are based on the outstanding balance, these fees decline over the life of the mortgage. For some agency securitization programs, mortgages with different gross coupons can be pooled together as long as the net coupon (gross coupon minus servicing and guaranty fees) is identical among all the mortgages in the pool.[5]

Mortgage servicers collect and aggregate payments from the underlying mortgage loans and pass the payments to the mortgage-backed security trust. If a loan becomes delinquent,

[5]See www.fanniemae.com, www.freddiemac.com, and www.ginniemae.gov for more information about agency securitization programs.

servicers advance scheduled principal and interest until either the loan becomes current or is bought out of the trust. Although mortgage payments are due on the first of the month, with a grace period determined by state law, investors receive the payments after a delay depending on the mortgage-backed security program. The payment is sent to the certificate holder on record at one of the Federal Reserve Banks as of the close of business on the last day of the month immediately before the month in which the distribution date occurs.

For clarity, consider a certificate holder of a Fannie Mae mortgage-backed security. Fannie Mae distributes payments on the 25th day of each month, or the next business day if the 25th is a weekend or holiday. Each payment consists of the scheduled principal and interest due on the first of that month plus any prepayments received the previous month.[6] Therefore, if an investor is the holder of record on July 31st, they are entitled to the payment from the security on August 25th, and the payment consists of any prepayments (perhaps default-related) received during July, plus the scheduled interest and principal due on August 1st.

2.3.3 The Market for Agency Mortgage-Backed Securities

The U.S. agency mortgage-backed securities market is among the largest and most liquid bond markets worldwide. As of June 2018, the total notional value of agency mortgage-backed securities outstanding was $8.2 trillion. From 2007 to 2016, the average daily trading volume of U.S. agency mortgage-backed securities was $261 billion, compared to $520 billion for U.S. Treasuries and $22 billion for corporate debt.[7] Around 94 percent of mortgage pass-through securities are traded in a highly liquid forward market, known as the to-be-announced (TBA) market; the remainder trade as specified pools.[8]

Similar to Treasury futures, a buyer of a TBA agrees to the trade without knowing the

[6] Generally, servicers have chosen to treat prepayments in full received on the first business day of a month as if received on the last calendar day of the preceding month. As a result, such a prepayment will be passed through to certificate holders on the distribution date in the same month in which the prepayment actually was received.

[7] Outstanding notional value and trading volume are from SIFMA.

[8] The term "pools" often refers to the actual pass-through securities, even through the pools are technically the collateral backing the pass-through security.

exact pools that will be delivered. Instead, the buyer and seller agree to six parameters: price, par amount, settlement date, agency program, mortgage type, and coupon. At any point in time, a range of coupons (typically in 50 basis point increments) and three different settlement dates are actively traded. The seller notifies the buyer of the exact pools that will be delivered by 3:00pm two days prior to the settlement date of the trade (the 48-hour rule). The pools are then exchanged for the cash payment on the settlement date. In a specified pool trade, the buyer and seller agree on the exact pools when the trade is executed. Typically, pass-throughs are traded as specified pools if they command a premium over TBAs due to favorable prepayment characteristics or if they are ineligible for TBA delivery.

Market participants generally adhere to standards referred to as the "Good Delivery Guidelines" maintained by by the Securities Industry and Financial Markets Association (SIFMA).[9] These guidelines specify the eligible collateral for TBA trades and various operational guidelines such as the number of bonds per million dollars notional of a trade, the allowable variation in the delivery amount, and the costs of failing to deliver. TBA trades may also be executed with stipulations such as production year, weighted average maturity (WAM), weighted average loan age (WALA), FICO score, loan-to-value ratio, or geographic distribution. A stipulated TBA trade would likely occur at a price higher than an unstipulated TBA if the stipulations provide favorable prepayment characteristics.

TBA trades settle to a monthly schedule set by SIFMA.[10] Each agency and mortgage type is assigned a single settlement day each month. For example, 30-year Fannie Mae TBAs fall into SIFMA's Class A and settle around the 9th or 10th business day of the month. Therefore, the notification day (48-hour day) is around the 7th or 8th day of the month. Because TBAs are traded on the current face value rather than the original face value of the pass-through securities, it is necessary to know the amount of scheduled and prepaid principal during the month. This data is released on the 4th business day of the month through what is known as a pool factor. The pool factor is a multiplier on the

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[9] The good delivery guidelines can be found at http://www.sifma.org/uploadedfiles/services/standard_forms_and_documentation/ch08.pdf

[10] The settlement schedule can be found at www.sifma.org or on Bloomberg via the command TDAT ¡GO¿.

original face value that keeps track of principal payments. Multiplying the factor by the original face of the security gives the current face. Therefore, after the factor day, a trader that is short mortgage-backed securities through a TBA contract can figure out what pools they will deliver. It is important to keep in mind that neither the buyer nor the seller know the prepayments that will be experienced after settlement, limiting the amount of adverse selection in pool delivery.

Consider an investor who holds 30-year Fannie Mae mortgage-backed securities and sells a TBA for August 2017 settlement. Table 2.1 shows the timeline of this trade. Because a TBA is a forward contract, no cash is exchanged on the trade date t_0. On August 4th, the factor date, Fannie Mae releases the pool factors. The pool factors determine the face value of the securities as of August 1st as well as the final payment the seller will receive on August 25th. Knowing the current face value, the seller can determine how much of each pool they can deliver into the TBA. If necessary, another TBA trade could be executed to completely liquidate a position or cover a short. By August 10th at 3:00pm, the seller notifies the buyer as to the exact pools that will be delivered. On August 14th, the trade settles and the seller receives a cash payment. On August 31st, Fedwire, a settlement funds transfer system, records the buyer as the holder of record, which entitles the buyer to the September payment. On September 7th, the pool factors are released which determine the size of the buyer's first payment, received on September 25th. Thus, the size of the first payment is determined over three weeks after settlement.

2.4 The Dollar Roll

Dollar rolls are an important mechanism by which mortgage-backed securities are financed, hedged, and sold short. A dollar roll is two simultaneous TBA trades with different settlement dates but the same notional amount, agency program, mortgage type and coupon. For example, buying $100 million of 30-year Fannie Mae 3.5% coupon TBA for August settlement and selling $100 million of 30-year Fannie Mae 3.5% coupon TBA for September settlement is equivalent to buying $100 million of the 30-year Fannie Mae 3.5% coupon August/September dollar roll. An investor is said to "roll" their mortgage-backed securities if they simultaneously execute a sell order for a TBA for one settlement month and a buy order for the same TBA, in the same amount, for the following month. In this case, the investor is selling a dollar roll.[11] Essentially, the investor gives up ownership of mortgage-backed securities for one month in exchange for a one-month loan. In fact, by repeatedly selling dollar rolls, an investor can maintain a long position in mortgage-backed securities without ever taking delivery of physical securities.

Market participants trade dollar rolls for a variety of reasons. Traditional mortgage investors, such as asset managers, banks, endowments, hedge funds, REITs, insurance companies and pension funds, often sell dollar rolls to finance long positions in mortgage-backed securities. Mortgage servicers sell dollar rolls to hedge the negative duration of their portfolio of mortgage servicing rights because the hedge requires a long TBA position. Also, the Federal Reserve has sold dollar rolls to alleviate collateral shortages during its quantitative easing programs. Mortgage originators may sell dollar rolls if their loans are being funded faster than expected and they want to shift TBA settlements to an earlier month. On the other side, dealers often buy dollars roll to "roll in" mortgage-backed securities for delivery into a short TBA position or to acquire collateral for a collateralized mortgage obligation (CMO) deal. Also, investors and dealers buy dollar rolls to hedge less-liquid mortgage-backed securities, such as specified pools.

Table 2.2 provides an example of a money manager that uses dollar rolls for financing and hedging purposes. That table shows the balance sheets for five funds managed by Pacific Investment Management Company, LLC (PIMCO) as of March 31, 2018. The total fund assets exceed the net assets by a multiple ranging from 1.57 to 2.81. This leverage is achieved through a mix of liabilities and dollar rolls play and import role. Depending on the accounting treatment of the specific trade, dollar rolls can either be classified as sale-buyback transactions or appear as separate TBA investments on both asset and liability side

[11] Clearly, an investor would only sell their MBS position at the TBA price if their collateral was the cheapest-to-deliver. Otherwise, they would either swap the collateral for TBA or execute the sell leg of the dollar roll as a specified pool trade.

of the balance sheet. As shown, TBA investments range from 44.2% to 86.8% of the total liabilities and reverse repurchase agreements are comparable in size for only the long-term U.S. Government Fund.

Gao, Schultz, and Song (2017) highlights the importance of the dollar roll market. Using FINRA TRACE data, they find that dollar rolls tend to be very large trades and account for about half of the volume in the TBA market. They find that the majority of dollar roll trading occurs two to five days before the Class A settlement date, and that these spikes in volume seem to reduce the trading costs for specified pools. Figure 2.1 plots the daily trading volume for 30-year Fannie Mae TBA securities obtained from SIFMA. As shown, TBA trading volume varies significantly over the course of a month, and the trading volume four days prior to Class A settlement date is over 2.5 times the volume 10 to 23 days prior to Class A settlement. This variation seems to be due to dollar rolls. Gao, Schultz, and Song (2017) find that the mean size of a interdealer dollar roll trade is \$59.64 million while the mean size for trades with customers is over \$116.43 million. They estimate that the round-trip trading cost for these trades is less than \$0.01, reflecting the high liquidity of the TBA market.

2.4.1 Mechanics of a Dollar Roll

Consider a \$1 million 30-year Fannie Mae 3.5% coupon August/September dollar roll with a trade date of August 7th, 2017. Figure 2.2 shows a screenshot of the dollar roll analysis page on Bloomberg and provides the trade details. The August TBA has a price of 103-08[12] (103.25 in decimals) and settles on August 14, 2017. The September TBA has a price of 103-02 (103.0625 in decimals) and settles on September 13, 2017. Quoted prices do not include accrued interest. The prices that are actually paid are 103.3764[13] and 103.1792,[14] respectively. I denote the actual prices paid per one dollar of outstanding balance as P_1 and

[12] Prices are quoted in 32nds, so that 103-08 = 103 + 8/32 = 103.25.

[13] $103.25 + 13/360 \times 3.5 = 103.3764$

[14] $103.0625 + 12/360 \times 3.5 = 103.1792$

P_2, respectively. Thus, in this example, $P_1 = 1.033764$ and $P_2 = 1.031792$. I denote the trade balances in dollars as B_1 and B_2, respectively. For a \$1 million dollar roll, both B_1 and B_2 are \$1 million.

Table 2.3 shows the timeline of cash flows from settled Fannie Mae mortgage-backed securities, TBAs and dollar rolls. For simplicity, consider a pool of settled mortgage-backed securities with a outstanding balance B_0 in July. Each month, the outstanding balance declines due to scheduled and prepaid principal payments. The balances are B_1 in August and B_2 in September. The first column of cash flows in Table 2.3 shows the payments received from the settled mortgage-backed securities with balance B_0 in July. The payments $C_1, C_2, \ldots, C_N$ are received on the 25th day of each month (or the next business day). The cash flows from buying B_1^A outstanding balance of the August TBA are shown in the next column. On August 14th, the buyer of the TBA pays $P_1 B_1^A$ which entitles them to payments beginning on September 25th. Each payment represents the $B_1^A / B_X X$

The September TBA has a similar structure: the buyer pays $P_2 B_2$ on September 13th and begins receiving payments on October 25th.

If the investor were to sell the August/September roll, they would simultaneously sell the August TBA and buy the September TBA. On August 14th, the investor delivers the pools to the buyer and receives a cash payment of $P_1 B_1$. Because the investor was the holder of record on July 31st, however, they are still entitled to the payments from the pools passed through on August 25th, denoted by c_1 in Table 2.3, even though it is after the settlement date. The amount of this payment can be determined from the pool factors, which were released on August 4th. The new owner of the security becomes the holder of record on August 31st and will receive the September 25th payment, c_2, the magnitude of which is determined on the September factor day. Thus, the roll seller gives up the payment c_2. On September 13th, the roll seller pays $P_2 B_2$ and gets back the same cohort of mortgage-backed securities they sold for August settlement. The roll seller becomes the holder of record again on September 30th, and they receive the October 25 payment c_3. Thus, the roll seller gave up the payment c_2 and held a large cash position from August 14 through September 13.

The cash flows from the perspective of the roll buyer are similar to a secured loan. On August 14th, the roll buyer pays $P_1 B_1$ and received mortgage-backed securities. Then, on September 13th, the roll buyer receives $P_2 B_2$ and delivers back the mortgage-backed securities. Later, on September 25th the roll buyer receives the payment c_2.

In summary, a dollar roll consists of four cash flows. Two cash flows are determined at the time of the trade: (1) the payment to settle the front TBA and (2) the payment to settle the back TBA. In addition, there are two cash flows that are unknown: (1) the price and amount of back TBA purchased to true up the face value and (2) the amount of the payment c_2.

2.4.2 The Roll Analysis Model

The traditional approach to value dollar rolls involves a present value model that I refer to as the "roll analysis model," (RA model) after the roll analysis page on Bloomberg, shown in Figure 2.2. Appendix 2.14 provides a more detailed description of the RA model. The RA model is a simple present-value model that translates the "drop", the price difference between two consecutive TBA contracts, into two key metrics: an implied financing rate, called the "break-even financing rate," and an implied prepayment speed. Given the TBA prices, each prepayment speed implies a break-even financing rate, and vice-versa. Figure 2.3 shows that break-even financing rates are a function of prepayment rates.

The break-even financing rate is the interest rate q_{be} that equates the value of holding a mortgage-backed security to rolling it as of the back settlement date t_2^s. If the investor rolls their mortgages, they receive $P_1 B_1$ at time t_1^s and earn interest on the cash holdings at rate q_{be} from t_1^s to t_2^s. At time t_2^s, the value of this cash position is $[1 + q_{be}(t_2^s - t_1^s)]\, B_1 P_1$. Alternatively, if the investor holds their mortgages and sells for t_2^s settlement, they will receive $P_2 B_2$ at time t_2^s and they are entitled to the coupon payment c_2 at time t_2. The value of this payment at time t_2^s discounted at rate q_{be} is $c_2[1 + q_{be}(t_2 - t_2^s)]^{-1}$. The break-even financing rate q_{be} equates the value of the cash position from selling early after the interest earned to the value of the foregone interest payment and the proceeds from selling late as of the second settlement date. Thus, q_{be} solves

$$0 = \frac{c_2}{1 + q(t_2 - t_2^s)} + B_2 P_2 - \left[1 + q_{be}(t_2^s - t_1^s)\right] B_1 P_1. \tag{2.1}$$

The implied prepayment speed is the prepayment rate that gives the value B_2 such that Equation 2.1 holds for an arbitrary financing rate q.

In Appendix 2.14, I show that the break-even financing rate is approximately the option-adjusted spread plus the short-term interest rate:

$$q_{be} \approx \text{OAS} + r. \tag{2.2}$$

2.4.3 Shortcomings of the Roll Analysis Model

Break-even financing rates depend on the particular prepayment model used for valuation and represent a subjective option about the cost of funds for mortgage backed-securities. Break-even financing rates are subject to misspecification. Typically, market participants have their own proprietary prepayment model, and by looking at one specific model it is not possible to know what is implied by other prepayment models or the market as a whole. As discussed by Carlin, Longstaff, and Matoba (2014), prepayment forecasts are subject to significant disagreement, and this translates into disagreement about break-even financing rates. Furthermore, as noted by Chernov, Dunn, and Longstaff (2017), even the prepayment models of the same dealer can lead to different prepayment forecasts and break-even financing rates.

Table 2.4 shows the PSA forecasts provided by Bloomberg prepayment survey participants on October 15, 2010 for a 6% 30-year Fannie Mae TBA, obtained from Diep, Eisfeldt, and Richardson (2016). On that day, the front TBA price was \$108.1719 and the back TBA price was \$107.8906; the settlement dates were 11/10/2010 and 12/13/2010, respectively. The PSA forecasts shown are for the current term structure. Maturity denotes the weighted average maturity of the mortgages underlying the Fannie Mae mortgage-backed security. Coupon denotes the pass-though rate for the Fannie Mae mortgage-backed security.

WAC denotes the weighted-average coupon or mortgage rate of the mortgages underlying the Fannie Mae mortgage-backed security. PSA is the percentage of the Public Securities Association prepayment model. This model assumes an annualized prepayment rate of 0.2% in month one, a rate increase by 0.2% each month until reaching 6% in month 30. From the 30th month onward, the model assumes an annualized prepayment rate of 6% of the remaining balance. CPR denotes the conditional prepayment rate implied by the PSA speed and the maturity plus one month. The financing rate is the break-even implied financing rate in basis points from the roll analysis model. The dealer CPR forecasts range from 7.8% to 44.5% leading to financing rates ranging from -127 bp to 207 bp, a 334 bp range. With such a wide range, it is impossible to know if mortgages are trading special or not.

Figure 2.4 illustrates the model-dependence of break-even financing rates through a sequence of models from a major Wall Street dealer. Prepayment models are fit to historical data, and as new prepayment speed as realized, the models are updated to take into account current market conditions. As shown in Figure 2.4, when the dealer updates the prepayment model, the break-even financing rate changes, sometimes as much as five percent. Thus, even the internal models of the same dealer can disagree about the break-even financing rate by up to 5%.

Another drawback of working with break-even financing rates is the fact that they are mechanically related to the option-adjusted spread. I show that the difference between break-even financing rates and short-term interest rates is approximately the option-adjusted spread. Therefore, dollar roll specialness, is approximately OAS. Hence, a main result of Song and Zhu (2016), that specialness is related to mortgage-backed security returns, is not valid; it is simply a byproduct of the mechanical interaction between two model-dependent measures.

2.4.4 Dollar Rolls and Reverse Repurchase Agreements

Most financing in the MBS market occurs through dollar rolls (Young (2010)). In many respects, a dollar roll is similar to a reverse repurchase agreement. A reverse repurchase agreement (reverse repo) is the purchase of securities in exchange for cash with the simultaneous agreement to resell the exact same securities at a specific price at a future date. The difference between the two prices determines the repo rate, the interest earned from lending cash. For a dollar roll, the effective interest rate also depends on the price difference between TBAs and the future path of TBA prices and realized prepayments.

Despite the similarity between a dollar roll and a reverse repo, there are important differences. First, the financing rate for a dollar roll is not known at the inception of the trade because it depends on the realized prepayment speed. Second, in a repo contract, any payments generated by the collateral are forwarded to the investor, whereas the dollar roll buyer is the owner of the MBS and receives one month's payment. Third, repo contracts are subject to haircuts, which can be substantial during times of market stress, as noted by Copeland, Martin, and Walker (2014). Fourth, repo financing can dry up completely once a counterparty becomes too risky (e.g. Lehman Brothers during the financial crisis) as discussed by Gorton and Metrick (2012). Fifth, discussions with Wall Street traders have indicated that pricing on repo contracts depend not on only a counterparty's credit but also its relationship with the firm: counterparties with a good relationship have lower repo rates than others. In contrast, dollar rolls have essentially the same contract terms no matter the counterparty. As long as an investor has access to the TBA market, they can obtain financing through dollar rolls and the implied interest rate is going to be the same rate for everyone.

For accounting purposes, selling dollar rolls can be treated as a sale and purchase transaction rather than a financing transaction, because the MBS returned does not meet the criteria of being "substantially similar" to the original MBS. Thus, MBS financed with dollar rolls can be held off balance sheet whereas MBS financed with repurchase agreements is held on balance sheet.[15]

Dollar rolls facilitate the shorting of mortgage-backed securities in the same way that

[15] Financial Accounting Standards Board (FASB) Accounting Standards Codification (ASC) 860 sets the conditions under which a security borrower returns "substantially the same" security, which gives the security lender "effective control" over the security. If the lender maintains "effective control" over the security, the agreement is considered a financing transaction rather than a sale/purchase.

reverse repurchase agreements do for U.S Treasuries. To short a security, a trader needs to borrow the security, sell it, and then repurchase the *exact same* security that they borrowed. This is possible to do in the U.S. Treasury market because there are a relatively small number of issues outstanding. For example, as of June 2017, there were 237 U.S Treasury Notes outstanding and the average issue size was $37 billion.[16] Thus, if a dealer acquires a U.S. Treasury through a reverse repurchase agreement, they can be confident that they can sell the security and find it later to cover the short. Each trade is only a small portion of the entire issue size and dealers maintain an inventory of Treasury issues, especially the on-the-runs. In contrast, as of June 2017, there were 529,572 Fannie Mae MBS pools and the average current face value was $7.65 million.[17] If an MBS is sold, it can easily be locked away in an available for sale or held to maturity account and it can be impossible to find the security to cover the short. Dollar rolls solve this problem because the exact same security does not need to be delivered. Instead, a trader only needs to deliver TBA collateral, and there have been only 16 liquid coupons traded since 1998.

2.5 The Data

The main dataset for this study consists of end-of-day prices from the TBA market for Fannie Mae mortgage-backed securities with varying coupons. The TBA prices are observed at a daily frequency from January 1, 1998 through August 30, 2017. The data are obtained from a proprietary dataset complied by a major Wall Street mortgage-backed security dealer. But, the prices available from the Bloomberg system are similar. At any given time, the dataset includes up to sixteen different coupons, not all of which are liquid. The dataset also includes the current outstanding balance and gross issuance for each coupon on a monthly frequency, as well as the weighted average coupon (WAC) and the weighted average maturity (WAM) for each cohort. The weighted average loan age (WALA) for each cohort is the maturity at origination (360 months) less the current weighted average maturity (WAM).

[16]Source: Monthly Statement of the Public Debt of the United States, June 30, 2017, U.S. Treasury.

[17]Source: Fannie Mae PoolTalk®

To ensure that I only include prices for actively traded mortgage-backed securities, I apply a series of filters. First, I include the six coupons with the largest outstanding balance in a given month. Then, I add the four coupons with the largest gross issuance in a given month (if they are not already in the dataset). The logic behind using gross issuance as a filter is that after a large move in interest rates, the mortgages close to the current coupon may not have a large outstanding balance because of the new interest rate environment. Nevertheless, these coupons are still actively traded which is reflected through high gross issuance.

Table 2.5 presents the summary statistics for the main dataset. As shown, the sample includes mortgage-backed securities with coupons ranging from 2.00 percent to 9.50 percent. Of course, not all coupons are actively traded throughout the entire sample period. The higher coupon mortgage-backed securities appear during the early part of the sample period when mortgage rates were considerably higher, and the lower coupon mortgage-backed securities appear later in the sample period.

To calculate the time that each mortgage was in the money, I compared the primary mortgage rate from Freddie Mac's Primary Mortgage Market Survey® (PMMS) to the weighted average coupon (WAC) of each mortgage-backed security. First, given each mortgage-backed securities weighted average maturity (WAM) and the current date, I calculated the origination date for each mortgage-backed security. For each date, I calculated the number of observations where the PMMS rate was less than the WAC from the origination date through the current date. Then, I divided this count by the total number of data points in the PMMS survey from origination through the current date. This gives the fraction of the time since origination that the mortgage is in the money. I then multiplied this quantity by the WALA / 12 to get the time in years the mortgage was in the money.

I also collect data from Bloomberg. I collect primary dealer net MBS holdings (PDPPMOPRT Index and PDPPMOR2 Index), the level of the VIX index (VIX Index), the Treasury-Eurodollar spread (.TED G Index), and the BBB-rated ten-year credit spread to Treasuries (CSI BBB Index). Following Fleckenstein and Longstaff (2020), I calculate the turn-of-year premium in Eurodollar futures from the Eurodollar futures prices for various maturities obtained from Bloomberg. The time series of the turn-of-year premium is shown in Figure 2.11.

2.6 Valuation Framework

To value agency mortgage-backed securities, I propose a reduced-form framework with three key processes: a prepayment process p_t, an agency credit spread process w_t, and a funding liquidity process v_t. As in Chernov, Dunn, and Longstaff (2017), the prepayment process p_t is the fraction of the remaining notional balance of the underlying mortgage pool that is prepaid each instant. The agency credit spread w_t accounts for the credit risk associated with the agency that guarantees the mortgage-backed security cash flows. Finally, the funding liquidity process v_t accounts for the holding cost of mortgage-backed securities and is the market-wide fractional carrying cost of mortgage-backed securities.

Agency mortgage-backed securities are subject to two types of default risk: defaults on the underlying mortgage loans and a default by the issuing agency. Due to the guarantee of the issuing agency, any defaults on the underlying loans are paid at par. Thus, defaults appear as prepayments from an investor's perspective, provided the agency itself does not default. If the issuing agency were to default on its guarantee, the mortgage-backed security essentially becomes a non-agency, or "private-label," security without any credit protection. In this case, the investor continues to receive payments from the security, but the payments may be delayed due to either delinquencies on the underlying mortgages or the failure of servicers to remit borrower payments to the securitization trust. Also, the payments may be lower due to mortgage modifications or losses from foreclosures. Therefore, if the issuing agency defaults, the mortgage-backed security experiences some fractional loss in market value.

As discussed in detail in Appendix 2.13, I apply the framework of Duffie and Singleton (1997, 1999) to value mortgage-backed securities with a fractional loss of market value at default as well as a liquidity factor. I value mortgage backed securities by finding the risk-neutral expected value of the promised cash flows discounting by the "default and liquidity-adjusted" short rate process $r_t + w_t + v_t$.

The liquidity process v_t captures the capital constraints of mortgage investors. Formally, the owner of a mortgage-backed security with value V_t must pay a flow of $v_t V_t$ to hold the mortgage. When the spread v_t is positive, investors are discounting mortgages at a higher rate than Treasuries, reflecting a liquidity discount or a higher cost of funds for a specialized mortgage investor. If balance sheets are full, there is an opportunity cost to hold mortgage-backed securities and this is reflected as a positive value of v_t. On the other hand, when the spread is negative, mortgage-backed securities are trading "special," that is they are discounted at a lower rate than Treasuries. Holding mortgages-backed securities may be valuable due to Federal Reserve purchases, demand from dealers to cover short positions, among other factors, and they may command a convenience yield. I model the net impact of these market dynamics as a spread v_t.

In this section, I provide an overview of the valuation framework; more details can be found in Appendix 2.13.

2.6.1 Mortgage Cash Flows

Consider a pass-through mortgage-backed security with N months to maturity and a coupon rate m. The mortgage loans backing the security have a weighted-average coupon WAC. The WAC is greater than the coupon rate m due to servicing and guaranty fees. The holder of an mortgage-backed security is promised monthly payments $\{C_{t_i}\}$, $i = 1, \ldots, N$, that are paid at times $t_1, \ldots, t_N$. Each payment C_{t_i} consists of interest, scheduled principal and any prepaid principal.

In the absence of prepayments, the interest and principal payments are known on the valuation date. Let PRIN_i, INT_i, and BAL_i be the scheduled principal payment, the interest payment, and the ending outstanding principal balance, respectively, for month i, $i = 1, \ldots, N$, under the assumption that there are no prepayments on the underlying loans. These quantities are given by standard mortgage formulas provided in Appendix 2.13.

The path of prepayment hazard rate p_t determines the amount of prepayments each month, and thus drives the variability in the payments C_{t_i}. To keep track of prepayments, I calculate the fraction of the mortgage pool that has not yet prepaid as of the end of month i, which I denote as Q_i. The survival factor Q_i links mortgage cash flows with and without prepayments. Given the definition of p_t, the fraction of the pool that has not yet prepaid at time t_i can be expressed as

$$Q_i = \exp\left(-\int_0^{t_i} p_s ds\right),\ i = 0, 1, \ldots, N. \tag{2.3}$$

Then, given the previous month's survival factor Q_{i-1}, the actual mortgage payments and ending balances are given by

$$\begin{aligned}
\mathrm{PAY}'_i &= \text{Scheduled monthly payment} = \mathrm{PAY}_i \times Q_{i-1}, && (2.4)\\
\mathrm{PRIN}'_i &= \text{Scheduled principal payment} = \mathrm{PRIN}_i \times Q_{i-1}, && (2.5)\\
\mathrm{INT}'_i &= \text{Scheduled interest payment} = \mathrm{INT}_i \times Q_{i-1}, && (2.6)\\
\mathrm{PP}_i &= \text{Prepaid Principal} = \left(\mathrm{BAL}'_{i-1} - \mathrm{PRIN}'_i\right) \times \frac{Q_{i-1} - Q_i}{Q_{i-1}}, && (2.7)\\
\mathrm{BAL}'_i &= \mathrm{BAL}'_{i-1} - \mathrm{PRIN}'_i - \mathrm{PP}_i = \mathrm{BAL}_i \times Q_i. && (2.8)
\end{aligned}$$

Thus, each payment is given by

$$C_{t_i} = \mathrm{PRIN}'_i + \mathrm{PP}_i + \frac{m}{\mathrm{WAC}} \times \mathrm{INT}'_i. \tag{2.9}$$

Note that the pass-though holder receives the entire principal payment from the underlying loans, but interest in paid out at rate m rather than rate WAC.

2.6.2 Spot Value of Mortgage-Backed Securities

As shown in the appendix, the value of the mortgage is given by discounting the promised cash flows by the "default and liquidity-adjusted" short rate process $r_t + w_t + v_t$. Thus, the pre-default value V_t of the mortgage-backed security at time t is given by

$$V_t = \sum_{i=1}^{N} E_t^{\mathbb{Q}} \left[\exp\left(-\int_t^{t_i} (r_u + w_u + v_u)\, du \right) C_{t_i} \right], \tag{2.10}$$

where $E_t^{\mathbb{Q}}[\cdot]$ denotes the expectation under the risk-neutral probability measure and C_{t_i} is given by Equation 2.9.

2.6.3 Forward Contracts on Mortgage-Backed Securities (TBA Contracts)

To-be-announced (TBA) contracts on mortgage-backed securities are forward contracts. In a TBA contract, the seller agrees to deliver a given principal balance of mortgage-backed securities at settlement. The principal balance for the trade is measured at settlement, not at the time of the trade.

I assume that the forward contract itself is not subject to its own liquidity spread. Thus, the liquidity spread v_t measures the relative liquidity of cash mortgages to TBAs. That is, the liquidity spread v is a cost incurred by an investor in mortgage-backed securities but by investors in forward contracts on mortgage-backed securities. Standard arguments, provided in the appendix, imply that the price $F^M(t_0, t_s)$ at time t_0 of a TBA (forward) contract M months forward for settlement at t_s is given by

$$F^M(t_0, t_s) = \frac{100}{\mathrm{BAL}_{t_s}} \frac{E_{t_0}^{\mathbb{Q}}\left[e^{\int_{t_0}^{t_s} v_u du} Q_{t_M}^{-1} \right]}{E_{t_0}^{\mathbb{Q}}\left[e^{-\int_{t_0}^{t_s} r_u du} \right]} V_{t_0} - \frac{100}{\mathrm{BAL}_{t_s}} \sum_{i=1}^{M} \frac{E_{t_0}^{\mathbb{Q}}\left[e^{-\int_{t_0}^{t_i} (r_u + w_u) du + \int_{t_i}^{t_s} v_u du} Q_{t_M}^{-1} C_{t_i} \right]}{E_{t_0}^{\mathbb{Q}}\left[e^{-\int_{t_0}^{t_s} r_u du} \right]}. \tag{2.11}$$

In Equation 2.11, $\mathrm{BAL}_{t_s} \times Q_{t_M}$ is the outstanding principal balance (after prepayments) at the time of settlement. Thus, the quantity $100 \times BAL_{t_s}^{-1} \times Q_{t_M}^{-1}$ has the effect of scaling the spot value to account for the fact that a TBA price is for \$100.00 of MBS at the time of settlement. With principal payments between the time of trade and settlement, the forward contract represents a larger ownership in the cohort of mortgages compared to the spot contract. The quantity $\exp\left(\int_{t_0}^{t_s} v_u du \right)$ reflects the liquidity cost of holding cash mortgage-backed securities between the valuation date t_0 and the settlement date t_s. Dividing by the zero coupon bond price $E_{t_0}^{\mathbb{Q}}\left[e^{-\int_{t_0}^{t_s} r_u du}\right]$ accounts for the interest that can be earned on the cash from selling cash mortgage-backed securities and buying TBAs. Finally, the forgone payments C_{t_i} reduce the value of a TBA contract compared to cash mortgage-backed securities.

The key difficulty in comparing different values and prices accross settlement dates is the fact that prepayments are unknown. Therefore, for two TBA trades with different settlement dates but the same face value, the later settlement date represents a larger ownership interest in a cohort of mortgages and the difference in the ownership interest is unknown due to prepayments. Because the price of a TBA contract is essentially the value of a trade with \$100 face value, the same difficulty carries over to prices.

2.7 The Prepayment Function

To complete the valuation framework for mortgage-backed securities, I specify a prepayment process p. Following Chernov, Dunn, and Longstaff (2017), I use a simple generic specification of the implied prepayment function that allows for both exogenous and rate-related prepayments. I extend the process presented in Chernov, Dunn, and Longstaff (2017) to allow for burnout. Burnout refers to the slowing of prepayment rates as mortgages stay in-the-money. Typically, after interest rates fall, prepayment speeds increase as borrowers refinance into lower rate mortgages. Some borrowers are unable to refinance despite an incentive to do so for various factors like poor credit quality or a lack of home equity. Over time, the borrows that are not able to refinance become a larger percentage of the cohort of mortgages that are in the money, leading to lower prepayment speeds for a particular refinancing incentive.

The key stylized facts about mortgage prepayments are illustrated in Figure 2.5. The top panel plots individual one-month prepayment rates for Fannie Mae 30-year mortgages as a function of the moneyness. Because the data are noisy, I show the Nadaraya–Watson kernel regression estimate (Nadaraya (1964); Watson (1964)) with Scott (1992) bandwidths. As shown, there is a clear relationship between prepayment rates and the refinancing incentives. When mortgages are out of the money, prepayments are generally low with 90 percent of the observations falling between 0.20 percent and 7.32 percent CPR. When the mortgage rate is lower than the market rate, the borrow has no incentive to refinance for the purpose of reducing mortgage costs. For in-the-money mortgages, prepayments generally increase linearly for a refinance incentive between 0.25 percent and 1.25 percent. Above a 1.25 percent refinance inventive, prepayment rates rise at slower rate due to burnout.

The bottom panel illustrates burnout in prepayment speeds. This chart plots prepayment rates for in-the-money mortgages against the amount of time those mortgages had been in the money. Again, due to the amount of noise in the data, I show the kernel regression estimates. As shown, mortgages that have been in the money for two years pay close to 40 CPR in general, but after eight years in the money, prepayment rates are generally around 20 percent.

I assume that the the prepayment function p_t is given by

$$p_t = x_t + y_t e^{-\beta\tau} \max\left(0, m - a - br_t(10)\right) \tag{2.12}$$

where $r_t(10)$ is the 10-year Treasury rate. The turnover factor x_t is the exogenous hazard rate at which mortgages are prepaid in the absence of refinancing incentives. The factor x_t captures non-interest rate related prepayments like mobility, default, and cash-out refinancing. The refinancing incentive is determined by the difference between the mortgage rate m and the implied rate at which mortgages can be refinanced. I allow the implied rate to be an affine function $a + br_t(10)$ of the 10-year Treasury rate $r_t(10)$, rather than constraining it to be a specific rate. The 10-year Treasury rate is strongly correlated with mortgage rates, and representing the mortgage rate as a linear function of the 10-year Treasury rate is a realistic assumption. The values of the constants a and b are estimated from the data.

The term $y_t e^{-\beta\tau}$ that multiplies the refinancing incentive term $\max\left(0, m - a - br_t(10)\right)$ in 2.12 measures how sensitive borrowers are to refinancing incentives. As discussed in Chernov, Dunn, and Longstaff (2017), the rate response factor y_t captures how home values, government programs like the Home Affordable Refinancing Program (HARP) and underwriting standards impact prepayment rates. In addition, loan officer capacity constraints and warehouse financing constraints could be reflected in y_t. I extend the model of Chernov, Dunn, and Longstaff (2017) to allow for burnout, the slowing of prepayment rates as mortgages stay in-the-money. Here, β is the burnout parameter and τ is the amount of time in years that the mortgage is in the money. Thus, as mortgages stay in the money for a longer time, the factor $e^{-\beta\tau}$ becomes smaller, slowing prepayment speeds for a given level of y_t and refi incentive. I find that adding the burnout parameter provides a better fit to the data. Chernov, Dunn, and Longstaff (2017) provides a detailed discussion of the advantages of choosing a simple, transparent parameterization of the prepayment function.

2.8 Model Identification

My model is identified through the assumption that mortgage to-be-announced contracts and cash mortgage-backed securities have different liquidity characteristics. Specifically, I assume that an investor in a mortgage-backed security incurs a net cost v, measured as a fraction of market value, whereas an investor in a forward contract on mortgage-backed securities does not incur this cost. The cost v can be thought of as the relative balance sheet costs associated with holding cash mortgage-backed securities compared to to-be-announced (forward) contracts.

The accounting treatment for dollar rolls justifies a basis in the funding cost for cash mortgage-backed-securities compared to mortgage to-be-announced contracts. As discussed in Section 2.4.4 and Song and Zhu (2019), dollar rolls are more likely to be treated as sales and purchases whereas repurchase agreements are treated as secured financing. This means that long positions in mortgage-backed securities funded with dollar rolls can be held off balance sheet. In contrast, cash mortgage-backed securities are held on balance sheet even if they are financed in the repo market. The difference in accounting treatment may be a channel through which balance sheet constraints and regulatory requirement impact the value of mortgage-backed securities.

Beyond differences accounting, there could be other factors driving the funding cost basis. It is possible that there is a benefit to holding mortgages on balance sheet because the cash securities can become a scare resource during times of Federal Reserve purchases or heavy deal flow of tranched securities. In additional, many studies have documented differences between the implicit funding rates for derivatives and the actual funding rates in cash markets. Examples include Brenner and Galai (1986), Ronn and Ronn (1989), Longstaff (1995), Longstaff, Mithal, and Neis (2005), Duffie (2010), Du, Tepper, and Verdelhan (2018), Bai and Collin-Dufresne (2019), and Song and Zhu (2019).

To illustrate what this assumption achieves, I demonstrate that each factor, v_0, w_0, x_0,and y_0, has a different effect on mortgage-backed security prices. Thus, the minimum root-mean-square error corresponds to a unique parameter vector. Figure 2.6 provides the intuition for model identification and shows how each factor impacts the coupon stack and the term structure of forward contracts in different ways. Each line connects the TBA prices for front month settlement and each crosshair indicates a price for back month settlement. The vertical axis is the mortgage price, and the horizontal axis is the coupon rate. The vertical distance between each crosshair and the solid line indicates the drop between front month and back month prices.

The top left panel shows the impact of the rate response factor y_0 on TBA prices. Recall that y_0 multiplies the refinancing incentive. Thus, changes in y_0 have little impact on mortgages without an incentive to refinance. The only way y_0 impacts these prices is through possible future states of the world with lower mortgage rates. But, for mortgages that are in-the-money, y_0 has the effect of increasing the prepayment rate and lowering the price. Increasing y_0 makes the plot of prices along the coupon stack more concave.

The upper right panel shows that increasing the turnover factor x_0 tends to increase the prices for out-of-the-money money mortgage-backed securities and decrease the prices for in-the-money mortgage-backed securities. An increase in the turnover rate x_0 has the effect of increasing the prepayment rate for all mortgage-backed securities, independent of the refinancing incentive. In turn, an increase in the prepayment rate increases the values of discount mortgage-backed securities while decreasing the values of premium mortgage-backed securities. Thus, changes in x_0 change the slope of the coupon stack, but not so much the level of curvature.

The bottom left panel shows the impact of the credit spread w_0 on TBA prices. Because w_0 is the spread at which mortgage cash flows are discounted over the risk-free rate (ignoring v_0), increasing w_0 decreases mortgage-backed securities prices by roughly the a similar amount across the coupon stack. Thus, changes in w_0 can be thought of as determining the overall level of prices.

Finally, the lower right panel shows the impact of the funding liquidity spread v_0. My model assumes that a holder of a mortgage-backed security incurs a net cost v, measured as a percentage of market value, whereas the holder of a forward contract does not incur this cost. By trading in the forward market, and investor can remove the mortgage-backed securities from their balance sheet while maintaining a long position. Thus, the credit spread w and the funding liquidity spread v affect the relative value of TBA prices in different ways. The lower right panel demonstrates that increasing w_0 while decreasing v_0 can give similar front TBA prices but lower back TBA prices. Thus, the values of w_0 and v_0 are determined jointly from the overall level of prices as well as the relative prices of TBA contracts with different settlement dates.

2.9 Estimation Methodology

In the Chernov, Dunn, and Longstaff (2017) framework, the value of a mortgage-backed security is a function of the three state variables: w_t, x_t, and y_t (in addition to the interest rate). I add a fourth state variable v_t that determines the relative pricing of mortgage-backed securities across settlement dates. To complete the specification of the model, I assume that the dynamics of the state variables are given by the following system of stochastic differential equations under the risk-neutral pricing measure,

$$
\begin{aligned}
dv &= (\alpha_v - \beta_v v)dt + \sigma_v dZ_v \\
dw &= (\alpha_w - \beta_w w)dt + \sigma_w \sqrt{w} dZ_w \\
dx &= (\alpha_x - \beta_x x)dt + \sigma_x \sqrt{x} dZ_x \\
dy &= (\alpha_y - \beta_y y)dt + \sigma_y \sqrt{y} dZ_y.
\end{aligned}
$$

The funding spread v_t follows a mean-reverting processes that can take on both positive and negative values. The credit w_t and the state variables x_t and y_t driving prepayments follow mean-reverting square-root processes, ensuring that prepayment rates and the credit spread are always nonnegative. Note that this specification of dynamics places this model within the familiar affine framework widely used throughout the financial literature.

To model the evolution of the riskless rate, I assume that r_t follows the single-factor Hull and White (1990) process

$$dr = (\alpha_r(t) - \beta_r r)dt + \sigma_r dZ_r,$$

where $\alpha_r(t)$ is a deterministic function of time and β_r and σ_r are positive constants. The function $\alpha_r(t)$ allows for an exact fit to the Treasury term structure on a given date. The ten-year rate $r_t(10)$ that determines the refinancing incentive is an affine function of the short rate r_t. The interest rate model can easily be relaxed to allow for a more general multi-factor specification.

The parameters for the riskless rate are estimated separately from the mortgage model. For each date, I solve for β_r and σ_r to minimize the relative pricing error over the volatility surface. To match the risk characteristics of mortgages, I consider options on 5-, 7-, and 10-year Treasuries with 1-, 2-, 3-, 4-, 5-year expiries. Given the estimates of β_r and σ_r, the function $\alpha_r(t)$ is determined by the Treasury yield curve. The details of this methodology can be found in the Appendix of Chernov, Dunn, and Longstaff (2017).

I allow for correlation between the state variables. Specifically, I assume that Z_r is correlated with Z_x and Z_y, Z_x and Z_y are correlated with each other, and both Z_w and Z_v

are independent of all other Brownian motions, such that

$$dZ_r dZ_x = \rho_{rx} dt$$
$$dZ_r dZ_y = \rho_{ry} dt$$
$$dZ_x dZ_y = \rho_{xy} dt.$$

The estimation of the mortgage model can be viewed as consisting of three steps. First, I select an initial parameter vector

$$\theta = \{a, b, \beta, \alpha_v, \alpha_w, \alpha_x, \alpha_y, \beta_v, \beta_w, \beta_x, \beta_y, \sigma_v, \sigma_w, \sigma_x, \sigma_y, \rho_{rx}, \rho_{ry}, \rho_{xy}\}.$$

Second, conditional on and for each day t during the sample period, I solve for the values of v_t, w_t, x_t, and y_t that best fit the model to the prices for the coupon stack (the cross section of mortgage-backed securities with different coupon rates) by minimizing the root mean squared error (RMSE). Since the nonlinear structure of the prepayment function makes it difficult to express the price of mortgage-backed securities in closed-form, I use simulation to solve for the model-based mortgage-backed security values. Third, I iterate over alternative values of the parameter vector until I find the vector that results in the lowest global root mean square error (RMSE). The outputs of the estimation are the parameter values and the time series of state variables. Table 2.7 reports the parameter values obtained from the estimation along with their asymptotic standard errors.

2.10 Implied Prepayment Factors

In this section, I review the model's fit to the market prices of mortgage to-be-announced contracts. Then, I discuss the empirical results for the implied turnover rate x, the implied rate response factor y, and the agency MBS credit spread w. Because the turnover factor and rate response factor were discussed extensively in Chernov, Dunn, and Longstaff (2017), I only discuss those factors briefly. I discuss the funding liquidity spread v in the next section.

2.10.1 Fitting To-Be-Announced Mortgage-Backed Security Prices

The coupon stack each day typically includes six to ten TBAs with varying coupon rates at 50 basis point increments across three different expiration months. This gives a total of between 18 and 30 individual TBA contracts. The estimation algorithm solves for the values of the four state variables v_t, w_t, x_t, y_t that best fit the model to markets prices. Because there are many more TBA contracts that state variables, there are residual differences between model values and market values. To quantify this difference, I compute the RMSE for each day in the sample period.

Figure 2.7 plots the time series of the RMSEs. As shown, the model fits TBA prices extremely well and improves upon the model of Chernov, Dunn, and Longstaff (2017), especially during the financial crisis of 2008. Much of the improvement is the result of adding the burnout factor. Over the sample period, the RMSEs range from 3 cents to 50 cents, with 90 percent of the results falling between 5 cents and 39 cents. This range compares well with bid-ask spreads for mortgage TBAs. The median RMSE for the entire sample period is 15.9 cents.

2.10.2 Turnover and Rate Response

Both the turnover and rate response factors are similar to the results of Chernov, Dunn, and Longstaff (2017). I find that mean turnover rate is 8.813 and the mean rate response factor is 12.973 compared to 8.233 and 11.492, respectively. The correlations between my factors and the factors of Chernov, Dunn, and Longstaff (2017) are 0.74 and 0.60, respectively. It makes sense that the rate response factor has a lower correlation due to the addition of the burnout factor. Standard deviations are similar, and I find higher serial correlation due to the fact that my model is estimated daily.

2.10.3 Credit spread

My model decomposes the credit/liquidity spread of Chernov, Dunn, and Longstaff (2017) into a credit spread w and a liquidity spread v. As shown in Table 2.7, the mean credit spread w is 48.7 basis points over the sample period, and it ranges from 1.9 basis points to 361.5 basis points. The mean credit spread is comparable to the mean level of agency debt spreads, as is expected given that agency mortgages and agency debt have similar levels of credit risk. For example, the mean level of the Credit Suisse Liquid Agency 10+ year spread (AGIN10BS Index on Bloomberg) was 43.1 basis points from January 2000 to August 2017. Over the same period, the mean level of the implied credit spread w was 48.9 basis points. The middle panel of Figure 2.8 plots the time series of the implied MBS credit spread w compared to the Bloomberg Barclays US agency spread. The implied spread tracks the agency spread closely, but the implied credit spread is more volatile, likely due to estimation error.

In my model, the implied credit spread should be independent of liquidity factors. To explore this, I regress the monthly changes in the implied credit spread on changes in the U.S. composite BBB-rated 10 year yield minus the US Treasury 10 year yield (CSI BBB Index on Bloomberg), changes in the Treasury-Eurodollar (TED) Spread, and the lagged changes in both the TED spread and the implied credit spread. The Treasury-Eurodollar spread is the yield on three-month LIBOR minus the yield on three-month Treasury Bills. Thus, the TED spread is a measure of the borrowing rate for financial institutions less the borrowing rate of the U.S. Government.

I find that the regression fits the data well, with an adjusted R^2 of 0.422 and all variables are significant and the 5% level. A 100 basis point increase in the BBB credit spread implies a 34.97 increase in the implied credit spread. Recall that the BBB credit spread, like any yield spread also has a liquidity component whereas the implied credit spread w should be independent of liquidity. In the regression, I include a more credit-sensitive index, the change in the BBB-Treasury spread and a less credit-sensitive index, the TED spread. Because the coefficient on the BBB - Treasury spread is positive but the coefficients on the change in the

TED spread and the lagged change in the TED spread are negative, I interpret these results as evidence that the implied credit spread w appears to be a pure credit factor.

2.11 Properties of the Funding Liquidity Spread

This section discussed the properties of the funding liquidity spread and the empirical results.

2.11.1 The Size of the Funding Liquidity Spread

Over the entire 1998-2017 sample period, the average value of the funding liquidity spread is 20.80 basis points. The funding liquidity spread averaged 28.63 basis points in the 1998-2007 pre-crisis period and 12.71 in the post-crisis period. On average, mortgages trade at a discount to Treasuries, that is the funding liquidity spread positive on average, but there are periods of time when the the funding liquidity spread can be negative, up to -127 basis points.

2.11.2 Balance Sheet Usage

In this section, I analyze the relationship between the funding liquidity spread and balance sheet usage costs for financial intermediaries. I discuss a number of variables that measure the balance sheet pressures in the mortgage market. I regress the monthly changes in the implied funding liquidity spread v on its lagged value, the percentage change in net primary dealer positions in mortgage backed securities, the percentage change in gross issuance of mortgage backed securities, the change in the Treasury-Eurodollar (TED) spread, the lagged change in the Treasury-Eurodollar (TED) spread, the lagged change in the turn-of-year premium, and the change in the VIX index. The results of the regression are shown in Table 2.9. The adjusted R^2 statistic is 0.386.

Holdings and issuance of mortgage-backed securities Holdings of mortgage-backed securities could impact their funding liquidity. When primary dealers have large holdings of mortgage-backed securities, they may be unlikely to add more to their balance sheet or provide repo financing. Thus, the funding liquidity spread v should be wider in periods of time when net holdings of mortgage backed securities are high. This corresponds to periods of time where there is greater balance sheet pressures on primary dealers and likely other market participants. I find that primary dealer holdings are highly significant with a t-statistic of 22.63. Thus, primary dealer net holdings of mortgage-backed securities does seem to be related to the implied funding rates of mortgage-backed securities.

While only significant at the 15 percent level, I find that gross MBS issuance also tends to correspond with wider liquidity spreads. Intuitively, when gross issuance is high it puts more balance sheet pressure on warehouse lines and intermediary balance sheets.

Balance sheet usage costs As a measure of balance sheet usage costs, Du, Tepper, and Verdelhan (2018) use the spread between the interest on excess reserves rate (IOER) and Libor. Because this spread is only available in the post-crisis period, I follow Fleckenstein and Longstaff (2020) and use the turn-of-year premium in Eurodollar futures prices as a proxy for the cost of balance sheet usage.

As discussed in Fleckenstein and Longstaff (2020), Eurodollar futures settle according to a quarterly schedule and represent the value of the three-month Libor rate at expiration. Thus, the December contracts represent loans that remain on balance sheet over year end, while the March, June and September contracts do not. Musto (1997), Griffiths and Winters (2005), Fleckenstein and Longstaff (2020), and others show that financing rates tend to spike near the end of a year as financial institutions face additional balance-sheet-related pressure to hold cash. Thus, the expected size in the spike in year-end Libor provides a measure of the balance sheet usage costs financial institutions face.

Like Fleckenstein and Longstaff (2020), I calculate the turn-of-the-year premium as the difference between the futures price for a December contract and the average of the futures prices for the contracts expiring 3 months earlier and later (September and March). The turn-of-the-year premium provides a continuous market-based measure of the difference between the expected value of Libor in December and the average expected value of Libor in the months bracketing December. Thus, the turn-of-the-year premium represents the incremental cost of balance sheet usage at year-end relative to other months. As discussed in Fleckenstein and Longstaff (2020), changes in the turn-of-year premium can be viewed as providing a simple "difference-in-differences" instrument for the time variation in the cost of balance sheet usage. This measure of the cost of balance sheet usage is available over the entire sample period, unlike the spread between the interest on excess reserves rate (IOER) and Libor. Figure 2.11 plots the turn-of-the-year premium.

I find the the funding liquidity spread is linked to the cost of balance sheet usage throughout the sample period. The t-statistic is 1.92 and is significant at the 5.5 percent level. Based on the regression, a 1 basis point increase in the turn-of-year premium corresponds to a 5.4 basis point increase in the liquidity spread.

Debt overhang costs As discussed in Myers (1977), Andersen, Duffie, and Song (2019), Fleckenstein and Longstaff (2020) and others, debt overhang costs may represent a large component of the costs faced by financial intermediaries in deploying their balance sheet. I use several proxies for the riskiness of financial sector debt to reflect the magnitude of the debt overhang costs of intermediaries. First, I include in the regression the change in the Treasury-Eurodollar (TED) spread and its lagged value. The Treasury-Eurodollar spread tracks the 3-month borrowing cost at banks over 3-month Treasury Bills. Thus, the TED spreads tends to increase as financial conditions worsen. I also include changes in the VIX index because the risk of corporate debt is inversely related to stock prices as implied by structual credit models and discussed in Fleckenstein and Longstaff (2020).

I find a stong relationship between the funding basis and debt overhang proxies. The t-statistics for the change in the TED spread and the lagged change in the TED spread are 3.17 and 6.21, respectively. The t-statistic of the change in the VIX indes is 4.26. Thus, when the proxies for debt overhang costs increase, the implied funding-liquidity spread also tends to increase.

Taken together, these regression results provide compelling evidence that balance sheet pressures such as debt over hang costs and balance sheet usage costs appear to be reflected in the prices of mortgage-backed securities through the funding liquidity spread v.

2.12 Conclusion

I present a new four-factor no-arbitrage model for the valuation of forward contracts on mortgage-backed securities. Building upon Chernov, Dunn, and Longstaff (2017), my approach solves for the implied prepayment function, the implied credit spread, and the implied funding liquidity spread. I show that this modeling framework is successful in capturing the cross-section of mortgage-backed security prices, and that implied prepayment models provide an important alternative to traditional approaches. I find that the funding liquidity spread is related to measures of balance sheet usage costs, debt overhang costs, and primary dealer positions in mortgage-backed securities.

2.13 Appendix: Details of the Model

2.13.1 Cash Flows of Mortgage-Backed Securities

The cash flow from a fixed-rate mortgage-backed security (MBS) depends on the realized prepayment rate for the pool of loans that back the security. Each month until maturity, the holder of an MBS receives a monthly payment, consisting of interest, scheduled principal and any prepaid principal. The monthly payment is determined recursively: the current principal balance determines the interest and scheduled principal payments for the next month, and next month's prepayments and scheduled principal determine next month's principal balance.

2.13.1.1 Cash Flows without Prepayments

Consider a pool of mortgage loans with a weighted-average coupon WAC and a remaining maturity of N months. Following Hayre (2001), for each dollar of mortgage outstanding balance in month i, the scheduled payment is

$$\text{Monthly payment} = \text{PAY}_i = \frac{\text{WAC}/12}{1-(1+\text{WAC}/12)^{-N}}. \tag{2.13}$$

The principal and interest portions of the payment are given by

$$\text{Principal portion of payment} = \text{PRIN}_i = \text{PAY}_i \times (1+\text{WAC}/12)^{-N-1+i}, \tag{2.14}$$

$$\text{Interest portion of payment} = \text{INT}_i = \text{PAY}_i - \text{PRIN}_i. \tag{2.15}$$

As the mortgages season, the monthly payment is constant while the principal payment increases and the interest payment decreases. In the absence of prepayments, the remaining balance at the end of the month is given by

$$\text{Remaining balance (end of month)} = \text{BAL}_i = \frac{1-(1+\text{WAC}/12)^{-N+i}}{1-(1+\text{WAC}/12)^{-N}}. \tag{2.16}$$

2.13.1.2 Cash Flows with Prepayments

To keep track of prepayments, I calculate the fraction of the mortgage pool that has not yet prepaid as of the end of month i, which I denote as Q_i. As discussed in Hayre (2001), the survival factor Q_i links MBS cash flows with and without prepayments. With PAY_i, PRIN_i, INT_i and BAL_i defined in the previous section, let PAY'_i, PRIN'_i, INT'_i and BAL'_i be the corresponding values with prepayments. Let PP_i be the prepaid principal in month i. Then, given the previous month's survival factor Q_{i-1},

$$\text{PAY}'_i = \text{Scheduled monthly payment} = \text{PAY}_i \times Q_{i-1}, \tag{2.17}$$

$$\text{PRIN}'_i = \text{Scheduled principal payment} = \text{PRIN}_i \times Q_{i-1}, \tag{2.18}$$

$$\text{INT}'_i = \text{Scheduled interest payment} = \text{INT}_i \times Q_{i-1}, \tag{2.19}$$

$$\text{PP}_i = \text{Prepaid Principal} = \left(\text{BAL}'_{i-1} - \text{PRIN}'_i\right) \times \frac{Q_{i-1}-Q_i}{Q_{i-1}}, \tag{2.20}$$

$$\text{BAL}'_i = \text{BAL}'_{i-1} - \text{PRIN}'_i - \text{PP}_i = \text{BAL}_i \times Q_i. \tag{2.21}$$

The single monthly mortality (SMM) rate in month i is defined to be

$$\text{SMM}_i = \frac{Q_{i-1} - Q_i}{Q_{i-1}}. \tag{2.22}$$

Given a SMM rate, the conditional prepayment rate (CPR) is given by

$$\text{CPR} = 1 - (1 - \text{SMM})^{12}. \tag{2.23}$$

Thus, Equation 2.20 can be written as

$$\text{PP}_i = \text{Prepaid Principal} = \left(\text{BAL}'_{i-1} - \text{PRIN}'_i\right) \times \text{SMM}_i. \tag{2.24}$$

2.13.1.3 Servicing and Guaranty Fees

The holder of a mortgage-backed security does not receive the entire interest payment from the underlying mortgage loans, because mortgage servicers hold back servicing and guaranty fees. These fees are a fixed percentage of the outstanding balance of the loans, and the holder of the mortgage-backed security receives a lower coupon rate on the mortgage-backed security compared to the underlying loans. Let m be the pass-through coupon rate on the mortgage-backed security. Then, the total cash flow C_{t_i} to the pass-through holder for month n, at time t_i, is

$$C_{t_i} = \text{PRIN}'_i + \text{PP}_i + \frac{m}{\text{WAC}} \times \text{INT}'_i. \tag{2.25}$$

The pass-though holder receives the entire principal payment from the underlying loans, but interest in paid out at rate m rather than the weighted-average coupon (WAC) rate on the underlying loans. All cash flow variation is reflected in variation in the survival factor Q_i. Because this paper studies agency mortgage-backed securities, any defaults on the underlying mortgage loans are reflected as a higher prepayment rate.

2.13.1.4 Timing of Cash Flows

Investors in mortgage-backed securities receive payments from the underlying mortgage loans after a delay. Mortgage payments are due on the first of the month, with a grace period determined by state law, but they are passed to investors sometime later that month. The exact payment day is specified in the prospectus for the particular mortgage-backed security program. For example, investors in Fannie Mae MBS receive payments on the 25th day of the month, or the next business day if the 25th is a weekend or holiday. Each monthly payment consists of the scheduled principal and interest due on the first of the month plus any prepayments over the previous month. Typically, prepayments received on the first of the month are paid the same month, but this too depends on the particular MBS program and is given in the prospectus. An investor is entitled to the monthly cash flow provided they are the holder of record on the last day of the previous month.

To model the timing of cash flows, I define three sets of points in time. The first,

$$\mathcal{T}_{\mathrm{CF}} \equiv \{t_0^c, t_1^c, \ldots, t_N^c\} \tag{2.26}$$

is the set of $N+1$ times relevant to determine the monthly prepayments of the MBS. Two consecutive points define the starting and ending times over which prepayments are aggregated. For Fannie Mae MBS, each t_i^c is the end of a first day of the month. The second set,

$$\mathcal{T}_{\mathrm{PMT}} \equiv \{t_1, t_2, \ldots, t_N\}, \tag{2.27}$$

is the set of N times the mortgage-backed security holder receives the N mortgage payments. For Fannie Mae mortgage-backed securities, each t_i is the end of the 25th day of the month, or the next business day if the 25th is a weekend of holiday. Finally, the third set,

$$\mathcal{T}_{\mathrm{RD}} \equiv \{t_1^r, t_2^r, \ldots, t_N^r\}, \tag{2.28}$$

is the set of record dates which are month ends.

As an example, consider the first payment from a MBS, paid at time t_1. This payment consists of the scheduled principal and interest due on the 1st of the month (time t_1^c) plus any prepayments over the previous month (after t_0^c through t_1^c). An investor is entitled to this payment if they hold the MBS at t_1^r. If the investor purchases the MBS after t_1^r, the investor does not receive the cash flow paid at time t_1.

2.13.1.5 Prepayment Process

I assume that prepayments are driven by an instantaneous prepayment process p_t, where p_t is the fraction of the remaining notional balance of the underlying pool that is prepaid each instant. Thus, p_t can be viewed as a prepayment intensity or hazard rate and Q_i as a discretely-measured survival factor. Given the definition of p_t, the fraction of the pool that has not yet prepaid can be expressed as

$$Q_i = \exp\left(-\int_{t_0^c}^{t_i^c} p_s ds\right),\ i = 0, 1, \ldots, N. \tag{2.29}$$

The continuous time analogue of Equation 2.29 is

$$Q_t = \exp\left(-\sum_{i=1}^{N} \mathbf{1}_{\left[t \geq t_i^c\right]} \int_{t_{i-1}^c}^{t_i^c} p_s ds\right),\ t \in \left[t_0^c, t_N^c\right]. \tag{2.30}$$

2.13.1.6 Agency Credit Risk

Mortgage-backed securities issued by Fannie Mae and Freddie Mac are guaranteed by the issuing agency, rather than the U.S. Government. If the issuing agency were to default on their guarantee, the mortgage-backed security essentially becomes a non-agency, or "private-label," mortgage-backed security without any credit protection. In the case of default, the investor continues to receive payments from the security, but the payments may be delayed due to delinquencies on the underlying mortgages or the failure of servicers to remit borrower payments to the securitization trust. Also, the payments may ultimately be lower due to mortgage modifications or losses from foreclosures. Therefore, if the issuing agency defaults, the mortgage-backed security experiences some kind of loss in market value. Formally, after default, the cash flow process switches to a process $C^*_{t_i}$, reflecting delinquencies and losses on the underlying mortgages.

2.13.2 Valuation Framework

I take as given an arbitrage-free setting where the value of a mortgage-backed security is given by

$$V_t = \sum_{i=1}^{N} E_t^{\mathbb{Q}} \left[\exp\left(-\int_t^{t_i} r_u du \right) \left(C_{t_i} \mathbf{1}_{[\tau > t_i]} + C^*_{t_i} \mathbf{1}_{[\tau \leq t_i]} \right) \right], \tag{2.31}$$

where $E_t^{\mathbb{Q}}[\cdot]$ denotes the expectation under the risk-neutral probability measure, τ is the time of default of the issuing agency, and r_t is the default-free short rate.[18] In Equation 2.31 it is understood that a record date must pass before the first payment is received. In other words, on each record date V_t drops by the expected value of the first payment. I rewrite Equation 2.31 as

$$V_t = \sum_{i=1}^{N} E_t^{\mathbb{Q}} \left[\exp\left(-\int_t^{t_i} r_u du \right) C_{t_i} \mathbf{1}_{[\tau > t_i]} \right] + E_t^{\mathbb{Q}} \left[\mathbf{1}_{[\tau \leq t_N]} \exp\left(-\int_t^{\tau} r_u du \right) (1 - L_\tau) V_{\tau-} \right] \tag{2.32}$$

where V_{t-} is the value V_t just prior to default and L_t,[19] the fraction loss of market value at default, is given by

$$L_t = 1 - \frac{1}{V_{t-}} \sum_{i=1}^{N} E_t^{\mathbb{Q}} \left[\exp\left(-\int_t^{t_i} r_u du \right) C^*_{t_i} \right]. \tag{2.33}$$

Duffie and Singleton (1999) show that equation 2.32 is equivalent to

$$V_t = \sum_{i=1}^{N} E_t^{\mathbb{Q}} \left[\exp\left(-\int_t^{t_i} (r_u + w_u)\, du \right) C_{t_i} \right], \tag{2.34}$$

[18] For a formal treatment see Bielecki and Rutkowski (2004).

[19] I assume that L_t is predictable, which is satisfied by continuity and the fact that that V_t satisfies the no-jump condition. See Duffie and Singleton (1999) for more details.

where $w_t = h_t L_t$, and h_t is the hazard process for the stopping time τ representing default. For simplicity, I model w_t directly, rather than h_t and L_t individually.

My model deviates from the Duffie and Singleton (1999) framework in that the investor never actually receives the recovery payment $(1 - L_\tau)V_{\tau-}$ at default. The value $(1 - L_\tau)V_{\tau-}$ represents the value at time τ of the future (less valuable) payments the investor receives from the mortgage-backed security.

2.13.3 The Funding Liquidity Spread

Mortgage-backed securities may be discounted at a rate higher or lower than Treasuries, the risk-free benchmark. A discount rate higher that Treasuries can be interpreted as a liquidity discount or a higher cost of funds for a specialized mortgage investor. A discount rate lower that Treasuries can arise through a convenience yield from holding mortgages-backed securities. For example, Federal Reserve purchases, dealers covering short positions (possibly from securitization deals), quarter-end window dressing and other factors can make holding physical securities valuable. It is also possible the TBA-eligible mortgage-backed securities are more liquid than off-the-run Treasuries.

Following Duffie and Singleton (1997, 1999), Longstaff, Mithal, and Neis (2005), Chernov, Dunn, and Longstaff (2017) and others, I model the net impact of these market dynamics as a spread v_t. Formally, the owner of a mortgage-backed security must pay a flow of $v_t V_t$ to hold the mortgage.[20] If the spread is negative, mortgage-backed securities are trading "special," that is they are discounted at a lower rate than Treasuries. Duffie and Singleton (1999) show that the pre-default value V_t of the mortgage-backed security at time t is given by

$$V_t = \sum_{i=1}^{N} E_t^{\mathbb{Q}} \left[\exp\left(-\int_t^{t_i} (r_u + w_u + v_u)\, du \right) C_{t_i} \right], \tag{2.36}$$

[20] I assume that the flow v_t is also paid by investors after any default by the issuing agency as well. Therefore,

$$L_t = 1 - \frac{1}{V_{t-}} \sum_{i=1}^{N} E_t^{\mathbb{Q}} \left[\exp\left(-\int_t^{t_i} (r_u + v_u)\, du \right) C_{t_i}^{*} \right]. \tag{2.35}$$

because the discounted gain process is a $\mathbb{Q}$-martingale. Equation 2.36 corresponds to the model of Chernov, Dunn, and Longstaff (2017) if the sum of w_t and v_t is replaced with a single credit/liquidity spread. In this paper, I model the credit spread w_t and the liquidity spread v_t individually.

2.13.4 Forward Contracts on Mortgage-Backed Securities (TBA Contracts)

The most liquid sector of the mortgage market is the to-be-announced (TBA) market. TBA contracts are forward contracts on mortgage-backed securities. In a TBA contract, the seller agrees to deliver a given principal balance of mortgage-backed securities at settlement. The principal balance for the trade is measured at settlement, not at the time of the trade.

As shown by Cox, Ingersoll, and Ross (1981), the forward price $F(t_0, t_s)$ of a security at time t_0 for settlement at time t_s is given by

$$F(t_0, t_s) = \frac{E^{\mathbb{Q}}_{t_0}\left[\exp\left(-\int_{t_0}^{t_s} r_u du\right) X_{t_s}\right]}{E^{\mathbb{Q}}_{t_0}\left[\exp\left(-\int_{t_0}^{t_s} r_u du\right)\right]}, \tag{2.37}$$

where X_{t_s} is the value of the security at time t_s. For a mortgage-backed security,

$$X_t = \mathbf{1}_{[\tau > t]} V_t + \mathbf{1}_{[\tau \leq t]} V_t^*, \tag{2.38}$$

where V_t^* is the post-default value given by

$$V_t^* = \sum_{i=1}^{N} E^{\mathbb{Q}}_t \left[\exp\left(-\int_t^{t_i} (r_u + v_u)\, du\right) C^*_{ti}\right] = (1 - L_t)\, V_t. \tag{2.39}$$

TBA prices are quoted for \$100.00 principal delivered at time t_s. To match the market convention, I normalize the delivery balance to 100 by changing Equation 2.37 to

$$F(t_0, t_s) = \frac{E^{\mathbb{Q}}_{t_0}\left[\exp\left(-\int_{t_0}^{t_s} r_u du\right) X_{t_s} \times 100 \times \mathrm{BAL}_{t_s}^{-1} Q_{t_s}^{-1}\right]}{E^{\mathbb{Q}}_{t_0}\left[\exp\left(-\int_{t_0}^{t_s} r_u du\right)\right]}. \tag{2.40}$$

BAL_t and Q_t in Equation 2.40 should be interpreted as BAL_i and Q_i such that $t \in [t_i, t_{i+1})$.

Because BAL_t is not random and known at time t_0, Equation 2.40 can be rewriten as

$$F(t_0, t_s) = \frac{100}{\mathrm{BAL}_{t_s}} \frac{E_{t_0}^{\mathbb{Q}}\left[\exp\left(-\int_{t_0}^{t_s} r_u du\right) X_{t_s} Q_{t_s}^{-1}\right]}{E_{t_0}^{\mathbb{Q}}\left[\exp\left(-\int_{t_0}^{t_s} r_u du\right)\right]}. \tag{2.41}$$

2.13.4.1 Dynamics of X_t

Following Duffie and Singleton (1999), I assume that the default time τ has a risk-neutral default hazard rate process h. Then, the default indicator process H, which is zero before default and 1 afterward (that is, $H_t = \mathbf{1}_{[\tau \leq t]}$), can be written in the form [21]

$$dH_t = (1 - H_t)\, h_t dt + dn_t, \tag{2.42}$$

where n_t is a martingale under $\mathbb{Q}$. Equation 2.38 can then be rewritten as

$$X_t = V_t + H_t\left(V_t^* - V_t\right). \tag{2.43}$$

Integration by parts for semimartingales implies that

$$dX_t = dV_t + dH_t V_{t-}^* + H_{t-} dV_t^* + dH_t \Delta V_\tau^* - dH_t V_{t-} - H_{t-} dV_t - dH_t \Delta V_\tau. \tag{2.44}$$

Following Duffie and Singleton (1999), I assume that V and V^* jump at default with zero probability so that $\Delta V_\tau = 0$ and $\Delta V_\tau^* = 0$ where the equality is meant in the almost sure sense. This means that the probability of a default occurring at the time of a coupon payment is zero. With this assumption, Equation 2.44 becomes

$$dX_t = dV_t + H_t(dV_t^* - dV_t) + dH_t(V_t^* - V_t). \tag{2.45}$$

[21] This follows from the Doob-Meyer decomposition of a counting process.

Equation 2.42 implies that

$$dX_t = dV_t + H_t(dV_t^* - dV_t) + (V_t^* - V_t)\,(1 - H_t)\,h_t dt + dn_t^1, \tag{2.46}$$

where $dn_t^1 = (V_t^* - V_t)dn_t$ is a martingale.[22]

Let $\widehat{C}_t$ and $\widehat{C}_t^*$ be the *cumulative* cash flow paid by an agency and the otherwise equivalent non-agency mortgage-backed security, respectively. Formally,

$$\widehat{C}_t = \sum_{i=1}^{N} C_{t_i} \mathbf{1}_{[t \geq t_i]} \tag{2.47}$$

and

$$\widehat{C}_t^* = \sum_{i=1}^{N} C_{t_i}^* \mathbf{1}_{[t \geq t_i]}. \tag{2.48}$$

Cox and Ross (1976) and Duffie, Schroder, and Skiadas (1996) show that the risk-neutral dynamics of the pre-default value are given by

$$dV_t = (r_t + h_t L_t + v_t)\,V_t dt - d\widehat{C}_t + dm_t. \tag{2.49}$$

and the post-default value by

$$dV_t^* = (r_t + v_t)\,V_t^* dt - d\widehat{C}_t^* + dm_t^*, \tag{2.50}$$

where m and m^* are martingales under $\mathbb{Q}$. Because $V_t^* = (1 - L_t)V_t$, it follows that

$$dV_t^* - dV_t = -\,(r_t + v_t + h_t)\,L_t V_t dt + d\widehat{C}_t - d\widehat{C}_t^* + dm_t^1, \tag{2.51}$$

where $m_t^1 = m_t^* - m_t$ is a martingale. Substituting Equations 2.49 and 2.51 in Equation 2.45

[22]The fact this is a martingale is given by Lemma 2 of Duffie, Schroder, and Skiadas (1996).

gives

$$dX_t = dV_t + H_t(dV_t^* - dV_t) + (V_t^* - V_t)(1 - H_t) h_t dt + dn_t^1 \tag{2.52}$$

$$\begin{aligned} &= (r_t + h_t L_t + v_t) V_t dt - d\widehat{C}_t + dm_t \\ &\quad - (r_t + v_t + h_t) H_t L_t V_t dt + H_t \left(d\widehat{C}_t - d\widehat{C}_t^* \right) + H_t dm_t^1 \\ &\quad + (V_t^* - V_t)(1 - H_t) h_t dt + dn_t^1. \end{aligned} \tag{2.53}$$

Because

$$V_t^* - V_t = (1 - L_t)V_t - V_t = -L_t V_t, \tag{2.54}$$

Equation 2.52 becomes

$$\begin{aligned} dX_t &= (r_t + h_t L_t + v_t) V_t dt - d\widehat{C}_t \\ &\quad - (r_t + v_t + h_t) H_t L_t V_t dt + H_t \left(d\widehat{C}_t - d\widehat{C}_t^* \right) \\ &\quad - L_t V_t (1 - H_t) h_t dt + d\hat{m}_t. \end{aligned} \tag{2.55}$$

where $\hat{m}_t = m_t + H_t m_t^1 + n_t^1$ is a martingale. After canceling terms, Equation 2.55 is

$$dX_t = (r_t + v_t) [V_t + H_t (V_t^* - V_t)] dt - d\widehat{C}_t + H_t \left(d\widehat{C}_t - d\widehat{C}_t^* \right) + dM_t. \tag{2.56}$$

Equations 2.56 and 2.43 imply that the dynamics of X_t are given by

$$dX_t = (r_t + v_t) X_t dt - d\widehat{C}_t + H_t \left(d\widehat{C}_t - d\widehat{C}_t^* \right) + d\hat{m}_t. \tag{2.57}$$

2.13.4.2 Dynamics of $X_t Q_t^{-1}$

Both the processes X_t and Q_t jump after each record date passes because the MBS trades ex-coupon. Itô's formula implies that

$$d\left(X_t Q_t^{-1}\right) = Q_{t-}^{-1} dX_t^{(c)} + X_{t-} dQ_t^{(c)} + dX_t^{(c)} dQ_t^{(c)} + \left[X_t Q_t^{-1} - X_{t-} Q_{t-}^{-1} \right] \mathbf{1}_{[t \in \mathcal{T}_{\mathrm{RD}}]}, \tag{2.58}$$

where $X_t^{(c)}$ is the continuous part of X_t and $\mathcal{T}_{\mathrm{RD}}$ is the set of record dates given in Equation 2.28. Given the definition of Q_t in Equation 2.30, Q_t is constant between record dates and $dQ_t^{(c)} = 0$. Therefore, after adding and subtracting $X_t Q_{t-}^{-1} \mathbf{1}_{[t \in \mathcal{T}_{\mathrm{RD}}]}$, Equation 2.58 can be rewritten as

$$d\left(X_t Q_t^{-1}\right) = Q_{t-}^{-1} dX_t^{(c)} + \left[X_t\left(Q_t^{-1} - Q_{t-}^{-1}\right) + Q_{t-}^{-1}\left(X_t - X_{t-}\right)\right] \mathbf{1}_{[t \in \mathcal{T}_{\mathrm{RD}}]}. \tag{2.59}$$

Therefore,

$$d\left(X_t Q_t^{-1}\right) = Q_{t-}^{-1} dX_t + X_t\left(Q_t^{-1} - Q_{t-}^{-1}\right) \mathbf{1}_{[t \in \mathcal{T}_{\mathrm{RD}}]}. \tag{2.60}$$

Equations 2.57 and 2.60 imply that

$$\begin{aligned} d\left(X_t Q_t^{-1}\right) = {} & \left(r_t + v_t\right) Q_{t-}^{-1} X_t dt + Q_{t-}^{-1}\left[-d\widehat{C}_t + H_t\left(d\widehat{C}_t - d\widehat{C}_t^*\right)\right] \\ & + X_t\left(Q_t^{-1} - Q_{t-}^{-1}\right) \mathbf{1}_{[t \in \mathcal{T}_{\mathrm{RD}}]} + d\hat{\hat{m}}_t, \end{aligned} \tag{2.61}$$

where $\hat{\hat{m}}_t$ is a martingale.

2.13.4.3 Discounted Process

Consider the discount factor

$$A_t = \exp\left(-\int_0^t \left(r_u + v_u\right) du\right). \tag{2.62}$$

Integration by parts implies that the differential of the normalized security price $X_t Q_t^{-1}$ discounted by A_t is given by

$$d\left(A_t X_t Q_t^{-1}\right) = A_t Q_{t-}^{-1}\left[-d\widehat{C}_t + H_t\left(d\widehat{C}_t - d\widehat{C}_t^*\right)\right] + A_t X_t\left(Q_t^{-1} - Q_{t-}^{-1}\right) \mathbf{1}_{[t \in \mathcal{T}_{\mathrm{RD}}]} + dM_t, \tag{2.63}$$

where $M_t = A_t\hat{\hat{m}}_t$ is a martingale. Integration from the time of trade, t_0, to the time of settlement, t_s, gives

$$
\begin{aligned}
A_{t_s}X_{t_s}Q_{t_s}^{-1} = A_{t_0}X_{t_0}Q_{t_0}^{-1} &- \sum_{t_i\in\mathcal{T}} A_{t_i}Q_{t_{i-1}}^{-1}\left[C_{t_i} - H_{t_i}\left(C_{t_i} - C_{t_i}^{*}\right)\right] \\
&+ \sum_{t_i\in\mathcal{T}} A_{t_i}X_{t_i}\left(Q_{t_i}^{-1} - Q_{t_{i-1}}^{-1}\right) + M_{t_s} - M_{t_0}.
\end{aligned} \tag{2.64}
$$

where $\mathcal{T}$ is the set of payment times between t_0 and t_s. Using the facts that $X_{t_0} = V_{t_0}$, the pre-default value of the mortgage-backed security, and Q_{t_0} is normalized to one, Equation 2.64 is equivalent to

$$
\begin{aligned}
A_{t_0}^{-1}A_{t_s}X_{t_s}Q_{t_s}^{-1} = V_{t_0} &- \sum_{t_i\in\mathcal{T}} A_{t_0}^{-1}A_{t_i}Q_{t_{i-1}}^{-1}\left[C_{t_i} - H_{t_i}\left(C_{t_i} - C_{t_i}^{*}\right)\right] \\
&+ \sum_{t_i\in\mathcal{T}} A_{t_0}^{-1}A_{t_i}X_{t_i}\left(Q_{t_i}^{-1} - Q_{t_{i-1}}^{-1}\right) + A_{t_0}^{-1}\left(M_{t_s} - M_{t_0}\right).
\end{aligned} \tag{2.65}
$$

Multiplying both sides of Equation 2.65 by $\exp\left(\int_{t_0}^{t_s} v_u du\right)$ and taking the expectation at time t_0 gives

$$
\begin{aligned}
E_{t_0}^{\mathbb{Q}}\left[e^{-\int_{t_0}^{t_s} r_u du}X_{t_s}Q_{t_s}^{-1}\right] = E_{t_0}^{\mathbb{Q}}\left[e^{\int_{t_0}^{t_s} v_u du}\right]V_{t_0} \\
&- \sum_{t_i\in\mathcal{T}} E_{t_0}^{\mathbb{Q}}\left[e^{-\int_{t_0}^{t_i} r_u du + \int_{t_i}^{t_s} v_u du}Q_{t_{i-1}}^{-1}\left[C_{t_i} - H_{t_i}\left(C_{t_i} - C_{t_i}^{*}\right)\right]\right] \\
&+ \sum_{t_i\in\mathcal{T}} E_{t_0}^{\mathbb{Q}}\left[e^{-\int_{t_0}^{t_i} r_u du + \int_{t_i}^{t_s} v_u du}X_{t_i}\left(Q_{t_i}^{-1} - Q_{t_{i-1}}^{-1}\right)\right].
\end{aligned} \tag{2.66}
$$

2.13.4.4 Forward Price

Substituting Equation 2.66 into Equation 2.41 implies that the forward price $F(t_0, t_s)$ at time t_0 for settlement at t_s is given by

$$
\begin{aligned}
F(t_0, t_s) = {} & \frac{100}{\mathrm{BAL}_{t_s}} \frac{E_{t_0}^{\mathbb{Q}}\left[e^{\int_{t_0}^{t_s} v_u du}\right]}{E_{t_0}^{\mathbb{Q}}\left[e^{-\int_{t_0}^{t_s} r_u du}\right]} V_{t_0} \\
& - \frac{100}{\mathrm{BAL}_{t_s}} \sum_{t_i \in \mathcal{T}} \frac{E_{t_0}^{\mathbb{Q}}\left[e^{-\int_{t_0}^{t_i} r_u du + \int_{t_i}^{t_s} v_u du} Q_{t_{i-1}}^{-1}\left[C_{t_i} - \mathbf{1}_{[\tau \le t_i]}\left(C_{t_i} - C_{t_i}^*\right)\right]\right]}{E_{t_0}^{\mathbb{Q}}\left[e^{-\int_{t_0}^{t_s} r_u du}\right]} \\
& + \frac{100}{\mathrm{BAL}_{t_s}} \sum_{t_i \in \mathcal{T}} \frac{E_{t_0}^{\mathbb{Q}}\left[e^{-\int_{t_0}^{t_i} r_u du + \int_{t_i}^{t_s} v_u du} X_{t_i}\left(Q_{t_i}^{-1} - Q_{t_{i-1}}^{-1}\right)\right]}{E_{t_0}^{\mathbb{Q}}\left[e^{-\int_{t_0}^{t_s} r_u du}\right]}.
\end{aligned} \tag{2.67}
$$

Typically, there are three liquid TBA settlement months. With mortgage payments at a monthly frequency, $\mathcal{T}$ is smaller than three payments, i.e. smaller than $\{t_1, t_2, t_3\}$. Below, I will consider four different cases for where the settlement date is relative to the trade date.

Case 1: t_0 and t_s are in the same month. In the case that t_0 and t_s are in the same month, there are no payments between the time of trade and settlement. Thus,

$$
F(t_0, t_s) = \frac{100}{\mathrm{BAL}_{t_s}} \frac{E_{t_0}^{\mathbb{Q}}\left[e^{\int_{t_0}^{t_s} v_u du}\right]}{E_{t_0}^{\mathbb{Q}}\left[e^{-\int_{t_0}^{t_s} r_u du}\right]} V_{t_0}. \tag{2.68}
$$

Case 2: The settlement date t_s is in the next month. In the case that t_0 is in month 1 and t_s is in the next month, one payment is foregone between the trade date and the settle date. Therefore, Equation 2.67 becomes

$$
\begin{aligned}
F(t_0, t_s) = \frac{100}{\mathrm{BAL}_{t_s}} \frac{E^{\mathbb{Q}}_{t_0}\left[e^{\int_{t_0}^{t_s} v_u du}\right]}{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_s} r_u du}\right]} V_{t_0} \\
- \frac{100}{\mathrm{BAL}_{t_s}} \frac{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_1} r_u du + \int_{t_1}^{t_s} v_u du} Q_{t_0}^{-1}\left[C_{t_1} - \mathbf{1}_{[\tau \le t_1]}\left(C_{t_1} - C^*_{t_1}\right)\right]\right]}{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_s} r_u du}\right]} \\
+ \frac{100}{\mathrm{BAL}_{t_s}} \frac{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_1} r_u du + \int_{t_1}^{t_s} v_u du} X_{t_1}\left(Q_{t_1}^{-1} - Q_{t_0}^{-1}\right)\right]}{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_s} r_u du}\right]}.
\end{aligned}
\tag{2.69}
$$

Consider the last quantity, integration from t_0 to t_1 of Equation 2.63 implies that

$$
\begin{aligned}
A_{t_1} X_{t_1} Q_{t_1}^{-1} = A_{t_0} X_{t_0} Q_{t_0}^{-1} - A_{t_1} Q_{t_0}^{-1}\left[C_{t_1} - H_{t_1}\left(C_{t_1} - C^*_{t_1}\right)\right] \\
+ A_{t_1} X_{t_1}\left(Q_{t_1}^{-1} - Q_{t_0}^{-1}\right) + M_{t_1} - M_{t_0}.
\end{aligned}
\tag{2.70}
$$

Multiplying Equation 2.70 by $Q_{t_0} A_{t_0}^{-1}$, subtracting $A_{t_1} X_{t_1} Q_{t_1}^{-1}$ from both sides, and using the fact that $X_{t_0} = V_{t_0}$, and implies

$$
A_{t_0}^{-1} A_{t_1} X_{t_1} = V_{t_0} - A_{t_0}^{-1} A_{t_1}\left[C_{t_1} - H_{t_1}\left(C_{t_1} - C^*_{t_1}\right)\right] + A_{t_0}^{-1} Q_{t_0}\left(M_{t_1} - M_{t_0}\right). \tag{2.71}
$$

Then, multiplying by $\exp\left(\int_{t_0}^{t_s} v_u du\right)$ gives

$$
\begin{aligned}
e^{-\int_{t_0}^{t_1} r_u du + \int_{t_1}^{t_s} v_u du} X_{t_1} = e^{\int_{t_0}^{t_s} v_u du} V_{t_0} - e^{-\int_{t_0}^{t_1} r_u du + \int_{t_1}^{t_s} v_u du}\left[C_{t_1} - H_{t_1}\left(C_{t_1} - C^*_{t_1}\right)\right] \\
+ e^{\int_{t_0}^{t_s} v_u du} A_{t_0}^{-1} Q_{t_0}\left(M_{t_1} - M_{t_0}\right).
\end{aligned}
\tag{2.72}
$$

Substituting Equation 2.72 into Equation 2.69 implies

$$
\begin{aligned}
F(t_0, t_s) = {} & \frac{100}{\mathrm{BAL}_{t_s}} \frac{E_{t_0}^{\mathbb{Q}}\left[e^{\int_{t_0}^{t_s} v_u du}\right]}{E_{t_0}^{\mathbb{Q}}\left[e^{-\int_{t_0}^{t_s} r_u du}\right]} V_{t_0} \\
& - \frac{100}{\mathrm{BAL}_{t_s}} \frac{E_{t_0}^{\mathbb{Q}}\left[e^{-\int_{t_0}^{t_1} r_u du + \int_{t_1}^{t_s} v_u du} Q_{t_0}^{-1}\left[C_{t_1} - \mathbf{1}_{[\tau \le t_1]}\left(C_{t_1} - C_{t_1}^*\right)\right]\right]}{E_{t_0}^{\mathbb{Q}}\left[e^{-\int_{t_0}^{t_s} r_u du}\right]} \\
& - \frac{100}{\mathrm{BAL}_{t_s}} \frac{E_{t_0}^{\mathbb{Q}}\left[e^{-\int_{t_0}^{t_1} r_u du + \int_{t_1}^{t_s} v_u du}\left(Q_{t_1}^{-1} - Q_{t_0}^{-1}\right)\left[C_{t_1} - \mathbf{1}_{[\tau \le t_1]}\left(C_{t_1} - C_{t_1}^*\right)\right]\right]}{E_{t_0}^{\mathbb{Q}}\left[e^{-\int_{t_0}^{t_s} r_u du}\right]} \\
& + \frac{100}{\mathrm{BAL}_{t_s}} \frac{E_{t_0}^{\mathbb{Q}}\left[e^{\int_{t_0}^{t_s} v_u du}\left(Q_{t_1}^{-1} - Q_{t_0}^{-1}\right)\right]}{E_{t_0}^{\mathbb{Q}}\left[e^{-\int_{t_0}^{t_s} r_u du}\right]} V_{t_0}.
\end{aligned} \tag{2.73}
$$

which is equivalent to (since $Q_{t_0} = 1$)

$$
\begin{aligned}
F(t_0, t_s) = {} & \frac{100}{\mathrm{BAL}_{t_s}} \frac{E_{t_0}^{\mathbb{Q}}\left[e^{\int_{t_0}^{t_s} v_u du} Q_{t_1}^{-1}\right]}{E_{t_0}^{\mathbb{Q}}\left[e^{-\int_{t_0}^{t_s} r_u du}\right]} V_{t_0} \\
& - \frac{100}{\mathrm{BAL}_{t_s}} \frac{E_{t_0}^{\mathbb{Q}}\left[e^{-\int_{t_0}^{t_1} r_u du + \int_{t_1}^{t_s} v_u du} Q_{t_1}^{-1}\left[C_{t_1} - \mathbf{1}_{[\tau \le t_1]}\left(C_{t_1} - C_{t_1}^*\right)\right]\right]}{E_{t_0}^{\mathbb{Q}}\left[e^{-\int_{t_0}^{t_s} r_u du}\right]}.
\end{aligned} \tag{2.74}
$$

The formula for value of the forward contract given in Equation 2.81 involves the default time τ and the post-default cash flows $\{C_{t_i}^*\}$. I assume that the value of the post-default cash flows can be determined by discounting the promised cash flows at the risk-adjusted discount rate $r_t + w_t$.[23] The forward price is then

$$F(t_0, t_s) = \frac{100}{\mathrm{BAL}_{t_s}} \frac{E^{\mathbb{Q}}_{t_0}\left[e^{\int_{t_0}^{t_s} v_u du} Q^{-1}_{t_1}\right]}{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_s} r_u du}\right]} V_{t_0} - \frac{100}{\mathrm{BAL}_{t_s}} \frac{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_1}(r_u+w_u)du + \int_{t_1}^{t_s} v_u du} Q^{-1}_{t_1} C_{t_1}\right]}{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_s} r_u du}\right]}. \tag{2.75}$$

Note that

$$C_{t_1} = \mathrm{PRIN}'_1 + \mathrm{PP}_1 + \frac{m}{\mathrm{WAC}} \times \mathrm{INT}'_1 \tag{2.76}$$

$$= \mathrm{PRIN}_1 \times Q_0 + (\mathrm{BAL}_0 - \mathrm{PRIN}_1) \times Q_0 \times \mathrm{SMM}_1 + \frac{m}{\mathrm{WAC}} \times \mathrm{INT}_1 \times Q_0 \tag{2.77}$$

$$= \mathrm{PRIN}_1 \times Q_0 + (\mathrm{BAL}_0 - \mathrm{PRIN}_1) \times (Q_0 - Q_1) + \frac{m}{\mathrm{WAC}} \times \mathrm{INT}_1 \times Q_0 \tag{2.78}$$

$$= \mathrm{PRIN}_1 + (\mathrm{BAL}_0 - \mathrm{PRIN}_1) \times (1 - Q_1) + \frac{m}{\mathrm{WAC}} \times \mathrm{INT}_1 \tag{2.79}$$

Because PRIN_1, BAL_1, and INT_1 are known at t_0, all the uncertainty is through the pre-

[23]The cash flows over the life of the forward contract make up a small portion of the value of the mortgage-backed security. Also, given the low probability of agency default, a simple assumption about the value of those cash flows has little impact on the value of the forward contract.

For the 30-year mortgage-backed securities considered in this study, the average coupon was 6.0 percent. For a 30-year mortgage with 6.0% pass through coupon and 6.5% coupon on the underlying mortgages, each scheduled payment (interest plus principal) is \$0.63 on a face value of \$100.00. Over the sample period the average conditional prepayment rate (CPR) is 21 percent. The single monthly mortality (SMM) rate[24] is 1.94%. The SMM multiplied by the principal balance after scheduled principal payments gives the amount of prepayments in a given month. Therefore, the prepayments are \$1.94 in the first month (the scheduled principal is only \$0.09) and the total payment is \$2.58. The each initial payment then is typically less than 2.5% of the value of the mortgage-backed security.

Since January 1st, 2000, the beginning of the available time series, 2-year agency spreads have been under 200 basis points. With an optimistic 60% recovery rate and a spread of 200 basis points, the default hazard rate is 500 basis points. Using this as an upper bound on the default hazard rate for agency mortgage-backed securities (even though credit spreads include liquidity factors), the risk-neutral probability of default over three months is $1 - \exp(-0.05/4) = 1.2\%$. Therefore, the value of each monthly cash flow in the case of default is less than $2.5\% \times 1.2\%$, or 3 basis points, within the bid-ask spread for mortgage-backed securities.

Using the rate $r + w$ to discount cash flows assumes that C_{t_i} is greater than zero because $w_t = h_t L_t$. Due to the factor L, w is lower than the default hazard rate but with zero recovery promised cash flows are discounted at the hazard rate. Most borrowers continue to pay their mortgages even if the agency defaults. Therefore, this assumption should put the error well within the worst case 3 basis points.

payment speed. Thus,

$$F(t_0,t_s) = \frac{100}{\mathrm{BAL}_{t_s}} \frac{E^{\mathbb{Q}}_{t_0}\left[e^{\int_{t_0}^{t_s} v_u du} Q^{-1}_{t_1}\right]}{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_s} r_u du}\right]} V_{t_0}$$
$$- \frac{100}{\mathrm{BAL}_{t_s}} \frac{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_1}(r_u+w_u)du+\int_{t_1}^{t_s} v_u du} Q^{-1}_{t_1}\right]\left(\mathrm{PRIN}_1 + \frac{m}{\mathrm{WAC}} \times \mathrm{INT}_1\right)}{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_s} r_u du}\right]}$$
$$- \frac{100}{\mathrm{BAL}_{t_s}} \frac{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_1}(r_u+w_u)du+\int_{t_1}^{t_s} v_u du}\left(Q^{-1}_{t_1} - 1\right)\right](\mathrm{BAL}_0 - \mathrm{PRIN}_1)}{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_s} r_u du}\right]}. \tag{2.80}$$

Case 3: The settlement date t_s is two months forward. In the case that t_0 is two months forward (two record dates pass over the life of the contract), two payments are foregone between the trade date and the settle date. Therefore, Equation 2.67 implies

$$F(t_0,t_s) = \frac{100}{\mathrm{BAL}_{t_s}} \frac{E^{\mathbb{Q}}_{t_0}\left[e^{\int_{t_0}^{t_s} v_u du}\right]}{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_s} r_u du}\right]} V_{t_0}$$
$$- \frac{100}{\mathrm{BAL}_{t_s}} \frac{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_1} r_u du+\int_{t_1}^{t_s} v_u du} Q^{-1}_{t_0}\left[C_{t_1} - \mathbf{1}_{[\tau\le t_1]}\left(C_{t_1} - C^*_{t_1}\right)\right]\right]}{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_s} r_u du}\right]}$$
$$- \frac{100}{\mathrm{BAL}_{t_s}} \frac{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_2} r_u du+\int_{t_2}^{t_s} v_u du} Q^{-1}_{t_1}\left[C_{t_2} - \mathbf{1}_{[\tau\le t_2]}\left(C_{t_2} - C^*_{t_2}\right)\right]\right]}{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_s} r_u du}\right]}$$
$$+ \frac{100}{\mathrm{BAL}_{t_s}} \frac{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_1} r_u du+\int_{t_1}^{t_s} v_u du} X_{t_1}\left(Q^{-1}_{t_1} - Q^{-1}_{t_0}\right)\right]}{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_s} r_u du}\right]}$$
$$+ \frac{100}{\mathrm{BAL}_{t_s}} \frac{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_2} r_u du+\int_{t_2}^{t_s} v_u du} X_{t_2}\left(Q^{-1}_{t_2} - Q^{-1}_{t_1}\right)\right]}{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_s} r_u du}\right]}. \tag{2.81}$$

Equation 2.74 implies

$$\begin{aligned}
F(t_0,t_s) = {} & \frac{100}{\mathrm{BAL}_{t_s}} \frac{E^{\mathbb{Q}}_{t_0}\left[e^{\int_{t_0}^{t_s} v_u du} Q^{-1}_{t_1}\right]}{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_s} r_u du}\right]} V_{t_0} \\
& - \frac{100}{\mathrm{BAL}_{t_s}} \frac{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_1} r_u du + \int_{t_1}^{t_s} v_u du} Q^{-1}_{t_1}\left[C_{t_1} - \mathbf{1}_{[\tau \le t_1]}\left(C_{t_1} - C^*_{t_1}\right)\right]\right]}{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_s} r_u du}\right]}. \\
& - \frac{100}{\mathrm{BAL}_{t_s}} \frac{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_2} r_u du + \int_{t_2}^{t_s} v_u du} Q^{-1}_{t_1}\left[C_{t_2} - \mathbf{1}_{[\tau \le t_2]}\left(C_{t_2} - C^*_{t_2}\right)\right]\right]}{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_s} r_u du}\right]} \\
& + \frac{100}{\mathrm{BAL}_{t_s}} \frac{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_2} r_u du + \int_{t_2}^{t_s} v_u du} X_{t_2}\left(Q^{-1}_{t_2} - Q^{-1}_{t_1}\right)\right]}{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_s} r_u du}\right]}.
\end{aligned} \tag{2.82}$$

Consider the last quantity, integration from t_1 to t_2 of Equation 2.63 implies that

$$\begin{aligned}
A_{t_2} X_{t_2} Q^{-1}_{t_2} = {} & A_{t_1} X_{t_1} Q^{-1}_{t_1} - A_{t_2} Q^{-1}_{t_1}\left[C_{t_2} - H_{t_2}\left(C_{t_2} - C^*_{t_2}\right)\right] \\
& + A_{t_2} X_{t_2}\left(Q^{-1}_{t_2} - Q^{-1}_{t_1}\right) + M_{t_2} - M_{t_1}.
\end{aligned} \tag{2.83}$$

Subtracting $A_{t_2} X_{t_2} Q^{-1}_{t_2}$ from both sides of Equation 2.84 and multiplying by $Q_{t_1} A^{-1}_{t_0}$ implies

$$\begin{aligned}
A^{-1}_{t_0} A_{t_2} X_{t_2} = {} & A^{-1}_{t_0} A_{t_1} X_{t_1} - A^{-1}_{t_0} A_{t_2}\left[C_{t_2} - H_{t_2}\left(C_{t_2} - C^*_{t_2}\right)\right] \\
& + A^{-1}_{t_0} Q_{t_1}\left(M_{t_2} - M_{t_1}\right).
\end{aligned} \tag{2.84}$$

Substituting 2.71 into Equation 2.84 implies

$$\begin{aligned}
A^{-1}_{t_0} A_{t_2} X_{t_2} = {} & V_{t_0} - A^{-1}_{t_0} A_{t_1}\left[C_{t_1} - H_{t_1}\left(C_{t_1} - C^*_{t_1}\right)\right] - A^{-1}_{t_0} A_{t_2}\left[C_{t_2} - H_{t_2}\left(C_{t_2} - C^*_{t_2}\right)\right] \\
& + A^{-1}_{t_0} Q_{t_1}\left(M_{t_2} - M_{t_1}\right) + A^{-1}_{t_0} Q_{t_0}\left(M_{t_1} - M_{t_0}\right).
\end{aligned} \tag{2.85}$$

Then, multiplying by $\exp\left(\int_{t_0}^{t_s} v_u du\right)$ gives

$$
\begin{aligned}
e^{-\int_{t_0}^{t_2} r_u du+\int_{t_2}^{t_s} v_u du} X_{t_2} = e^{\int_{t_0}^{t_s} v_u du} V_{t_0} &- e^{-\int_{t_0}^{t_1} r_u du+\int_{t_1}^{t_s} v_u du}\left[C_{t_1} - H_{t_1}\left(C_{t_1} - C^*_{t_1}\right)\right] \\
&- e^{-\int_{t_0}^{t_2} r_u du+\int_{t_2}^{t_s} v_u du}\left[C_{t_2} - H_{t_2}\left(C_{t_2} - C^*_{t_2}\right)\right] \\
&+ e^{\int_{t_0}^{t_s} v_u du} A_{t_0}^{-1} Q_{t_1}\left(M_{t_2} - M_{t_1}\right) + e^{\int_{t_0}^{t_s} v_u du} A_{t_0}^{-1} Q_{t_0}\left(M_{t_1} - M_{t_0}\right).
\end{aligned}
\tag{2.86}
$$

Substituting Equation 2.86 into Equation 2.82 implies

$$
\begin{aligned}
F(t_0, t_s) = \frac{100}{\mathrm{BAL}_{t_s}} & \frac{E^{\mathbb{Q}}_{t_0}\left[e^{\int_{t_0}^{t_s} v_u du} Q_{t_1}^{-1}\right]}{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_s} r_u du}\right]} V_{t_0} \\
&- \frac{100}{\mathrm{BAL}_{t_s}} \frac{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_1} r_u du+\int_{t_1}^{t_s} v_u du} Q_{t_1}^{-1}\left[C_{t_1} - \mathbf{1}_{[\tau \le t_1]}\left(C_{t_1} - C^*_{t_1}\right)\right]\right]}{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_s} r_u du}\right]}. \\
&- \frac{100}{\mathrm{BAL}_{t_s}} \frac{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_2} r_u du+\int_{t_2}^{t_s} v_u du} Q_{t_1}^{-1}\left[C_{t_2} - \mathbf{1}_{[\tau \le t_2]}\left(C_{t_2} - C^*_{t_2}\right)\right]\right]}{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_s} r_u du}\right]} \\
&+ \frac{100}{\mathrm{BAL}_{t_s}} \frac{E^{\mathbb{Q}}_{t_0}\left[e^{\int_{t_0}^{t_s} v_u du}\left(Q_{t_2}^{-1} - Q_{t_1}^{-1}\right)\right]}{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_s} r_u du}\right]} V_{t_0} \\
&- \frac{100}{\mathrm{BAL}_{t_s}} \frac{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_1} r_u du+\int_{t_1}^{t_s} v_u du}\left[C_{t_1} - H_{t_1}\left(C_{t_1} - C^*_{t_1}\right)\right]\left(Q_{t_2}^{-1} - Q_{t_1}^{-1}\right)\right]}{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_s} r_u du}\right]} \\
&- \frac{100}{\mathrm{BAL}_{t_s}} \frac{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_2} r_u du+\int_{t_2}^{t_s} v_u du}\left[C_{t_2} - H_{t_2}\left(C_{t_2} - C^*_{t_2}\right)\right]\left(Q_{t_2}^{-1} - Q_{t_1}^{-1}\right)\right]}{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_s} r_u du}\right]}
\end{aligned}
\tag{2.87}
$$

which is equivalent to

$$F(t_0,t_s) = \frac{100}{\mathrm{BAL}_{t_s}} \frac{E^{\mathbb{Q}}_{t_0}\left[e^{\int_{t_0}^{t_s} v_u du} Q_{t_2}^{-1}\right]}{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_s} r_u du}\right]} V_{t_0}$$
$$- \frac{100}{\mathrm{BAL}_{t_s}} \frac{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_1} r_u du + \int_{t_1}^{t_s} v_u du} Q_{t_2}^{-1}\left[C_{t_1} - \mathbf{1}_{[\tau \le t_1]}\left(C_{t_1} - C^*_{t_1}\right)\right]\right]}{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_s} r_u du}\right]}.$$
$$- \frac{100}{\mathrm{BAL}_{t_s}} \frac{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_2} r_u du + \int_{t_2}^{t_s} v_u du} Q_{t_2}^{-1}\left[C_{t_2} - \mathbf{1}_{[\tau \le t_2]}\left(C_{t_2} - C^*_{t_2}\right)\right]\right]}{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_s} r_u du}\right]}. \tag{2.88}$$

The forward price is then

$$F(t_0,t_s) = \frac{100}{\mathrm{BAL}_{t_s}} \frac{E^{\mathbb{Q}}_{t_0}\left[e^{\int_{t_0}^{t_s} v_u du} Q_{t_2}^{-1}\right]}{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_s} r_u du}\right]} V_{t_0} - \frac{100}{\mathrm{BAL}_{t_s}} \frac{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_1} (r_u + w_u) du + \int_{t_1}^{t_s} v_u du} Q_{t_2}^{-1} C_{t_1}\right]}{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_s} r_u du}\right]}$$
$$- \frac{100}{\mathrm{BAL}_{t_s}} \frac{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_2} (r_u + w_u) du + \int_{t_2}^{t_s} v_u du} Q_{t_2}^{-1} C_{t_2}\right]}{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_s} r_u du}\right]}. \tag{2.89}$$

Note that

$$\begin{aligned}
C_{t_1} &= \mathrm{PRIN}'_1 + \mathrm{PP}_1 + \frac{m}{\mathrm{WAC}} \times \mathrm{INT}'_1 && (2.90)\\
&= \mathrm{PRIN}_1 \times Q_0 + (\mathrm{BAL}_0 - \mathrm{PRIN}_1) \times Q_0 \times \mathrm{SMM}_1 + \frac{m}{\mathrm{WAC}} \times \mathrm{INT}_1 \times Q_0 && (2.91)\\
&= \mathrm{PRIN}_1 \times Q_0 + (\mathrm{BAL}_0 - \mathrm{PRIN}_1) \times (Q_0 - Q_1) + \frac{m}{\mathrm{WAC}} \times \mathrm{INT}_1 \times Q_0 && (2.92)\\
&= \mathrm{PRIN}_1 + (\mathrm{BAL}_0 - \mathrm{PRIN}_1) \times (1 - Q_1) + \frac{m}{\mathrm{WAC}} \times \mathrm{INT}_1 && (2.93)
\end{aligned}$$

$$\begin{aligned}
C_{t_2} &= \mathrm{PRIN}'_2 + \mathrm{PP}_2 + \frac{m}{\mathrm{WAC}} \times \mathrm{INT}'_2 && (2.94)\\
&= \mathrm{PRIN}_2 \times Q_1 + (\mathrm{BAL}_1 - \mathrm{PRIN}_2) \times Q_1 \times \mathrm{SMM}_2 + \frac{m}{\mathrm{WAC}} \times \mathrm{INT}_2 \times Q_1 && (2.95)\\
&= \mathrm{PRIN}_2 \times Q_1 + (\mathrm{BAL}_1 - \mathrm{PRIN}_2) \times (Q_1 - Q_2) + \frac{m}{\mathrm{WAC}} \times \mathrm{INT}_2 \times Q_1 && (2.96)\\
& && (2.97)
\end{aligned}$$

Thus,

$$
\begin{aligned}
F(t_0,t_s) = {} & \frac{100}{\mathrm{BAL}_{t_s}} \frac{E^{\mathbb{Q}}_{t_0}\left[e^{\int_{t_0}^{t_s} v_u du} Q_{t_2}^{-1}\right]}{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_s} r_u du}\right]} V_{t_0} \\
& - \frac{100}{\mathrm{BAL}_{t_s}} \frac{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_1}(r_u+w_u)du+\int_{t_1}^{t_s} v_u du} Q_{t_2}^{-1}\right]\left(\mathrm{PRIN}_1 + \frac{m}{\mathrm{WAC}} \times \mathrm{INT}_1\right)}{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_s} r_u du}\right]} \\
& - \frac{100}{\mathrm{BAL}_{t_s}} \frac{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_1}(r_u+w_u)du+\int_{t_1}^{t_s} v_u du} Q_{t_2}^{-1}\left(1-Q_{t_1}\right)\right]\left(\mathrm{BAL}_0 - \mathrm{PRIN}_1\right)}{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_s} r_u du}\right]} \\
& - \frac{100}{\mathrm{BAL}_{t_s}} \frac{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_2}(r_u+w_u)du+\int_{t_2}^{t_s} v_u du} Q_{t_2}^{-1} Q_1\right]\left(\mathrm{PRIN}_2 + \frac{m}{\mathrm{WAC}} \times \mathrm{INT}_2\right)}{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_s} r_u du}\right]}. \\
& - \frac{100}{\mathrm{BAL}_{t_s}} \frac{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_2}(r_u+w_u)du+\int_{t_2}^{t_s} v_u du} Q_{t_2}^{-1}\left(Q_1 - Q_2\right)\right]\left(\mathrm{BAL}_1 - \mathrm{PRIN}_2\right)}{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_s} r_u du}\right]}.
\end{aligned} \tag{2.98}
$$

General Case: The settlement date t_s is M months forward. Continued application of the derivations above imply that the price $F^M(t_0,t_s)$ a TBA (forward) contract M months forward is given by

$$
\begin{aligned}
F^M(t_0,t_s) = {} & \frac{100}{\mathrm{BAL}_{t_s}} \frac{E^{\mathbb{Q}}_{t_0}\left[e^{\int_{t_0}^{t_s} v_u du} Q_{t_M}^{-1}\right]}{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_s} r_u du}\right]} V_{t_0} \\
& - \frac{100}{\mathrm{BAL}_{t_s}} \sum_{i=1}^{M} \frac{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_i}(r_u+w_u)du+\int_{t_i}^{t_s} v_u du} Q_{t_M}^{-1} C_{t_i}\right]}{E^{\mathbb{Q}}_{t_0}\left[e^{-\int_{t_0}^{t_s} r_u du}\right]}.
\end{aligned} \tag{2.99}
$$

2.14 Appendix: The Traditional Approach to Dollar Roll Valuation

In this section, I present the industry-standard model for the valuation of dollar rolls. I refer to this model as the "roll analysis model," after the roll analysis (RA) page on Bloomberg, shown in Figure 2.2. The RA model is a simple present-value model that translates the "drop", the term for the price difference between two consecutive TBA contracts, into two key metrics: an implied financing rate, called the "break-even financing rate," and an implied prepayment speed. Given the TBA contracts' collateral characteristics and the drop, each prepayment speed maps to a specific break-even financing rate, and vice-versa. Because prepayment speeds are derived from a prepayment model, break-even financing rates depend on the particular prepayment model used to value the TBA contracts. I show that specialness, the difference between break-even financing rates and short-term interest rates, is approximately the option-adjusted spread.

2.14.1 The Roll Analysis Model

The roll analysis (RA) model involves three main assumptions: (1) the same mortgage-backed security is delivered into each TBA, (2) a constant cost of funds discounts cash flows, and (3) future prepayments are known for certain today. Consider a dollar roll formed by two TBA trades, both executed simultaneously at $t = 0$. The time line of events is shown in Table 2.3. The first trade settles in the front month, at time t_1^s (August 14th in Table 2.3), and is for the purchase of B_1 dollars of outstanding principal at time t_1^s. The second trade settles in the back month, at time t_2^s (September 13th in Table 2.3), and is for the sale of B_2 dollars of outstanding principal at time t_2^s. The RA model assumes that trades involve the exact same mortgage-backed security. Without loss of generality, consider a mortgage-backed security with N months remaining maturity that generates a cash flow C_n from month $n = 1$ to month $n = N$. These N cash flows are passed through to the certificateholder at times $t_1, t_2, \ldots, t_N$, after the payment delay for the MBS program. The dirty prices (clean price plus accrued interest) of these TBAs are F_1 and F_2, respectively. The prices are given as a percentage of the current outstanding balance.

Now, consider the net cash flows from the dollar roll. At t_1^s, the roll buyer pays B_1F_1 for the mortgage backed-security. At the end of the month, the roll buyer is recorded as the certificate holder, which entitles them to the cash flow stream starting at t_2 the following month. However, prior to receiving the cash flow C_2 the following month, the second TBA trade settles at time t_2^s, at which time the roll buyer receives B_2F_2. The RA model assumes that prepayments for the first month are known at $t = 0$, which is not the case. In reality, the current balance at time t_2^s, which is determined by the first month's prepayments, is known at time t_2^f, the factor day. With this assumption, the RA model sets B_2 equal to B_1 less the scheduled principal and prepayments, and the cash flows for the long and short positions offset each other after the first month. As a result, the entire position in the mortgage-backed security is sold for settlement at time t_2^s. To summarize, the RA model assumes a dollar roll consists of three cash flows: (1) $-B_1F_1$ at t_1^s, (2) $+B_2F_2$ at t_2^s, and (3) $+C_2$ at t_2.

The RA model also assumes that a constant cost of funds q discounts the cash flows of the dollar roll. The cost of funds q is quoted as an annual rate with the Actual/360 (money market) day count convention, and all time differences assume an Actual/360 day count. The RA model calculates the value $V(t_2^s)$ of the dollar roll at time t_2^s, which is given by

$$V(t_2^s) = \frac{C_2}{1 + q(t_2 - t_2^s)} + B_2F_2 - \left[1 + q(t_2^s - t_1^s)\right] B_1F_1. \tag{2.100}$$

The break-even financing is the rate q_{be} that sets $V(t_2^s)$ equal to zero. Therefore, q_{be} solves

$$\begin{aligned}
0 &= \frac{C_2}{1 + q_{be}(t_2 - t_2^s)} + B_2F_2 - \left[1 + q_{be}(t_2^s - t_1^s)\right] B_1F_1 \\
&= C_2 + \left[1 + q_{be}(t_2 - t_2^s)\right] B_2F_2 - \left[1 + q_{be}(t_2 - t_2^s)\right]\left[1 + q_{be}(t_2^s - t_1^s)\right] B_1F_1 \\
&= C_2 + B_2F_2 - B_1F_1 + q_{be}\left[(t_2 - t_2^s)B_2F_2 - (t_2 - t_1^s)B_1F_1\right] \\
&\quad - q_{be}^2(t_2 - t_2^s)(t_2^s - t_1^s)B_1F_1
\end{aligned} \tag{2.101}$$

Let

$$\phi = C_2 + B_2F_2 - B_1F_1 \tag{2.102}$$

$$\psi = (t_2 - t_2^s)B_2F_2 - (t_2 - t_1^s)B_1F_1 \tag{2.103}$$

$$\omega = -(t_2 - t_2^s)(t_2^s - t_1^s)B_1F_1 \tag{2.104}$$

so that q_{be} solves

$$\omega q_{be}^2 + \psi q_{be} + \phi = 0. \tag{2.105}$$

Then, after choosing the correct root, the quadratic formula implies

$$q_{be} = \frac{-\psi - \sqrt{\psi^2 - 4\phi\omega}}{2\omega}. \tag{2.106}$$

Because ω and q_{be}^2 are relatively small, I can approximate q_{be} by

$$\hat{q}_{be} = -\frac{\phi}{\psi}, \tag{2.107}$$

which results from setting the last term in Equation 2.101 equal to zero. The financing rate $\hat{q}_{be}$ sets $V(t_2) = 0$, compared to q_{be} which sets $V(t_2^s) = 0$.

As a numerical example, consider the RA model shown in Figure 2.2. There,

$$\begin{aligned}
B_1 F_1 &= 1{,}032{,}500.00 + 1{,}263.89 = 1{,}033{,}763.89, \\
B_2 F_2 &= 1{,}026{,}140.34 + 1{,}161.59 = 1{,}027{,}301.93, \\
C_2 &= 7{,}268.07, \\
t_2^s - t_1^s &= \frac{30}{360}, \\
t_2 - t_2^s &= \frac{12}{360}, \\
q_{be} &= 0.009331 = 93.31 \text{ bp}, \\
\hat{q}_{be} &= 0.009334 = 93.34 \text{ bp}.
\end{aligned}$$

2.14.2 Connection between OAS and Dollar Rolls

Consider a dollar roll involving two TBA trades: one for front month settlement and one for back month settlement. The dollar roll is valued using the RA model. Therefore, I assume that the mortgage pools are identical for each settlement month; the only difference is that the pools delivered for back month settlement are seasoned by one additional month compared to the pools delivered in the front month. The front month TBA has a remaining maturity of N months and the back month TBA has a remaining maturity $N-1$ months. Therefore, if the balance delivered in the back month is appropriately scaled, both TBA trades generate the same cash flows after the second month. To be sure, in reality, it is impossible to know exactly how to scale the balance, because the future prepayment rate is unknown. Let $t = 0$ be the time at which the dollar roll trade is executed. The front-month TBA settles at t_f and the back-month TBA settles at $t_b > t_f$. The first payment from the mortgage-backed securities delivered in the front-month TBA trade is paid at $t_1 > t_b$, and the remaining payments are made at $t_2, t_3, \ldots, t_N$.

The TBA trades are valued via a M-draw Monte Carlo simulation. For each draw i, the simulation generates a short rate path $\{r_i(t), t \in [0, t_N]\}$. For clarity, I assume that the path is continuous. Then, given a path of the short rate, and possibly other factors, the simulation generates a vector of cash flows $(c_i(t_1), c_i(t_2), \ldots, c_i(t_N))$, where each cash flow $c_i(t_j)$ corresponds to a payment time t_j. Note that a cash flow $c_i(t_j)$ includes scheduled principal and interest plus any prepayments. Let $B(t_f)$ and $B(t_b)$ be the outstanding principal balances of the pools at t_f and t_b, respectively. Due to the time it takes to close on a refinancing application, the simulation gives the same cash flow $c(t_1)$ in each draw. Similarly, the RA model assumes that the first cash flow $c(t_1)$ is known with certainty at $t = 0$. Knowing the first cash flow implies that future balance $B(t_b)$ is known at t_0 as well (again, in reality, this balance is random). For each TBA trade, the option-adjusted spread (OAS) is found by solving for the spread over the interest rate such that the model price equals the the market price. Let OAS_f and OAS_b be the option-adjusted spreads for the front month and back month, respectively. Therefore, the price of the front month TBA as a percentage of the outstanding balance is given by

$$P_f = \frac{1}{M} \sum_{i=1}^{M} \sum_{j=1}^{N} \frac{c_i(t_j)}{B(t_f)} e^{-\int_{t_f}^{t_j} \left(r_i(s) + \mathrm{OAS}_f\right) ds}, \tag{2.108}$$

and the price of the back month TBA is given by

$$P_b = \frac{1}{M} \sum_{i=1}^{M} \sum_{j=2}^{N} \frac{c_i(t_j)}{B(t_b)} e^{-\int_{t_b}^{t_j} (r_i(s) + \mathrm{OAS}_b) ds} \ , \tag{2.109}$$

Given the time it takes to refinance and the fact that $c(t_1)$ is non random in the simulation, I can rewrite equation 2.108 as

$$P_f = \frac{c(t_1)}{B(t_f)} e^{-\left(F(t_f,t_1)+\mathrm{OAS}_f\right)(t_1-t_f)} + \frac{1}{M}\sum_{i=1}^{M}\sum_{j=2}^{N} e^{-\int_{t_f}^{t_j}\left(r_i(s)+\mathrm{OAS}_f\right)ds}\frac{c_i(t_j)}{B(t_f)}, \tag{2.110}$$

where $F(s,t)$ is the market forward rate between s and $t > s$. Note that

$$e^{-F(t_1,t_2)(t_2-t_1)} = \lim_{M\to\infty}\frac{1}{M}\sum_{i=1}^{M} e^{-\int_{t_1}^{t_2} r_i(s)ds} \tag{2.111}$$

for interest rate models that fit the yield curve exactly, as is standard for prepayment models. Focus now on the second term of equation 2.110, which I denote as A:

$$\begin{aligned}
A &= \frac{1}{M}\sum_{i=1}^{M}\sum_{j=2}^{N} e^{-\int_{t_f}^{t_j}\left(r_i(s)+\mathrm{OAS}_f\right)ds}\frac{c_i(t_j)}{B(t_f)} \\
&= \frac{B(t_b)}{B(t_f)}\frac{1}{M}\sum_{i=1}^{M} e^{-\int_{t_f}^{t_b}\left(r_i(s)+\mathrm{OAS}_f\right)ds}\sum_{j=2}^{N} e^{-\int_{t_b}^{t_j}\left(r_i(s)+\mathrm{OAS}_f\right)ds}\frac{c_i(t_j)}{B(t_b)} \\
&= \frac{B(t_b)}{B(t_f)} e^{-\mathrm{OAS}_f(t_b-t_f)}\frac{1}{M}\sum_{i=1}^{M} e^{-\int_{t_f}^{t_b} r_i(s)ds}\sum_{j=2}^{N} e^{-\int_{t_b}^{t_j}\left(r_i(s)+\mathrm{OAS}_f\right)ds}\frac{c_i(t_j)}{B(t_b)} \\
&= \frac{B(t_b)}{B(t_f)} e^{-\left(F(t_f,t_b)+\mathrm{OAS}_f\right)(t_b-t_f)}\frac{1}{M}\sum_{i=1}^{M}\sum_{j=2}^{N} e^{-\int_{t_b}^{t_j}\left(r_i(s)+\mathrm{OAS}_f\right)ds}\frac{c_i(t_j)}{B(t_b)}.
\end{aligned}$$

Note that

$$\begin{aligned}
&\frac{1}{M}\sum_{i=1}^{M}\sum_{j=2}^{N} e^{-\int_{t_b}^{t_j}\left(r_i(s)+\mathrm{OAS}_f\right)ds}\frac{c_i(t_j)}{B(t_b)} \\
&\qquad = \frac{1}{M}\sum_{i=1}^{M}\sum_{j=2}^{N} e^{-\int_{t_b}^{t_j}(r_i(s)+\mathrm{OAS}_b)ds} e^{-\int_{t_b}^{t_j}\left(\mathrm{OAS}_f-\mathrm{OAS}_b\right)ds}\frac{c_i(t_j)}{B(t_b)} \qquad .
\end{aligned}$$

Empirically, $\mathrm{OAS}_f - \mathrm{OAS}_b$ is very small. For example, Figure 2.12 shows that OAS_f and OAS_b track each other very closely. In changes, the time series are 90% correlated and 99.6% correlated in levels. The mean absolute OAS for the front month is 19.995 bp and the mean absolute OAS for the back month is 19.971 bp, and the mean absolute difference is just 0.024

bp. Therefore,

$$e^{-\int_{t_b}^{t_j}\left(\mathrm{OAS}_f-\mathrm{OAS}_b\right)ds}$$

is very close to one, and I set

$$\begin{aligned}
A &= e^{-\left(F(t_f,t_b)+\mathrm{OAS}_f\right)(t_b-t_f)}\frac{B(t_b)}{B(t_f)}\frac{1}{M}\sum_{i=1}^{M}\sum_{j=2}^{N}e^{-\int_{t_b}^{t_j}(r_i(s)+\mathrm{OAS}_b)ds}\frac{c_i(t_j)}{B(t_b)}+\varepsilon \\
&= e^{-\left(F(t_f,t_b)+\mathrm{OAS}_f\right)(t_b-t_f)}\frac{B(t_b)}{B(t_f)}P_b+\varepsilon.
\end{aligned}$$

Hence, equation 2.110 becomes

$$P_f=\frac{c(t_1)}{B(t_f)}e^{-\left(F(t_f,t_1)+\mathrm{OAS}_f\right)(t_1-t_f)}+\frac{B(t_b)}{B(t_f)}P_b e^{-\left(F(t_f,t_b)+\mathrm{OAS}_f\right)(t_b-t_f)}+\varepsilon. \tag{2.112}$$

Now, I solve for the break-even financing rate with equation 2.107. Equation 2.102 implies

$$\begin{aligned}
C &= c_1+B_bP_b-B_fP_f \\
&= c_1+B_bP_b-c(t_1)e^{-\left(F(t_f,t_1)+\mathrm{OAS}_f\right)(t_1-t_f)}-B(t_b)P_b e^{-\left(F(t_f,t_b)+\mathrm{OAS}_f\right)(t_b-t_f)} \\
&= c_1\left(1-e^{-\left(F(t_f,t_1)+\mathrm{OAS}_f\right)(t_1-t_f)}\right)+B_bP_b\left(1-e^{-\left(F(t_f,t_b)+\mathrm{OAS}_f\right)(t_b-t_f)}\right) \\
&\approx c_1\left(F(t_f,t_1)+\mathrm{OAS}_f\right)(t_1-t_f)+B_bP_b\left(F(t_f,t_b)+\mathrm{OAS}_f\right)(t_b-t_f) \\
&\approx \left(F(t_f,t_b)+\mathrm{OAS}_f\right)\left[(t_1-t_f)c_1+(t_b-t_f)B_bP_b\right], \qquad (2.113)
\end{aligned}$$

and equation 2.103 becomes

$$\begin{aligned}
D &= (t_1-t_b)B_bP_b-(t_1-t_f)B_fP_f \\
&\approx (t_1-t_b)B_bP_b-(t_1-t_f)c_1\left[1-\left(F(t_f,t_1)+\mathrm{OAS}_f\right)(t_1-t_f)\right] \\
&\qquad -(t_1-t_f)B_bP_b\left[1-\left(F(t_f,t_b)+\mathrm{OAS}_f\right)(t_b-t_f)\right] \\
&\approx (t_1-t_b)B_bP_b-(t_1-t_f)\left(c_1+B_bP_b\right) \\
&= -\left[(t_1-t_f)c_1+(t_b-t_f)B_bP_b\right] \qquad (2.114)
\end{aligned}$$

Therefore, the break-even financing rate is

$$\begin{aligned}
\hat{q}_{be} &= -\frac{C}{D} \\
&\approx \frac{\left(F(t_f, t_b) + \mathrm{OAS}_f\right)\left[(t_1 - t_f)c_1 + (t_b - t_f)B_b P_b\right]}{(t_1 - t_f)c_1 + (t_b - t_f)B_b P_b} \\
&= F(t_f, t_b) + \mathrm{OAS}_f
\end{aligned} \tag{2.115}$$

Figure 2.13 plots the break-even financing rate versus the option-adjusted spread plus the short rate. The break-even financing rates are close to the 45-degree line and show that this relationship hold up empirically. Because the break-even financing rate is a function of the option-adjusted spread, it is not possible to attach any meaning to the relationship between the two. Both Kitsul and Ochoa (2016) and Song and Zhu (2016) try to link break-even financing rates to returns via OAS. This analysis shows that doing so is flawed.

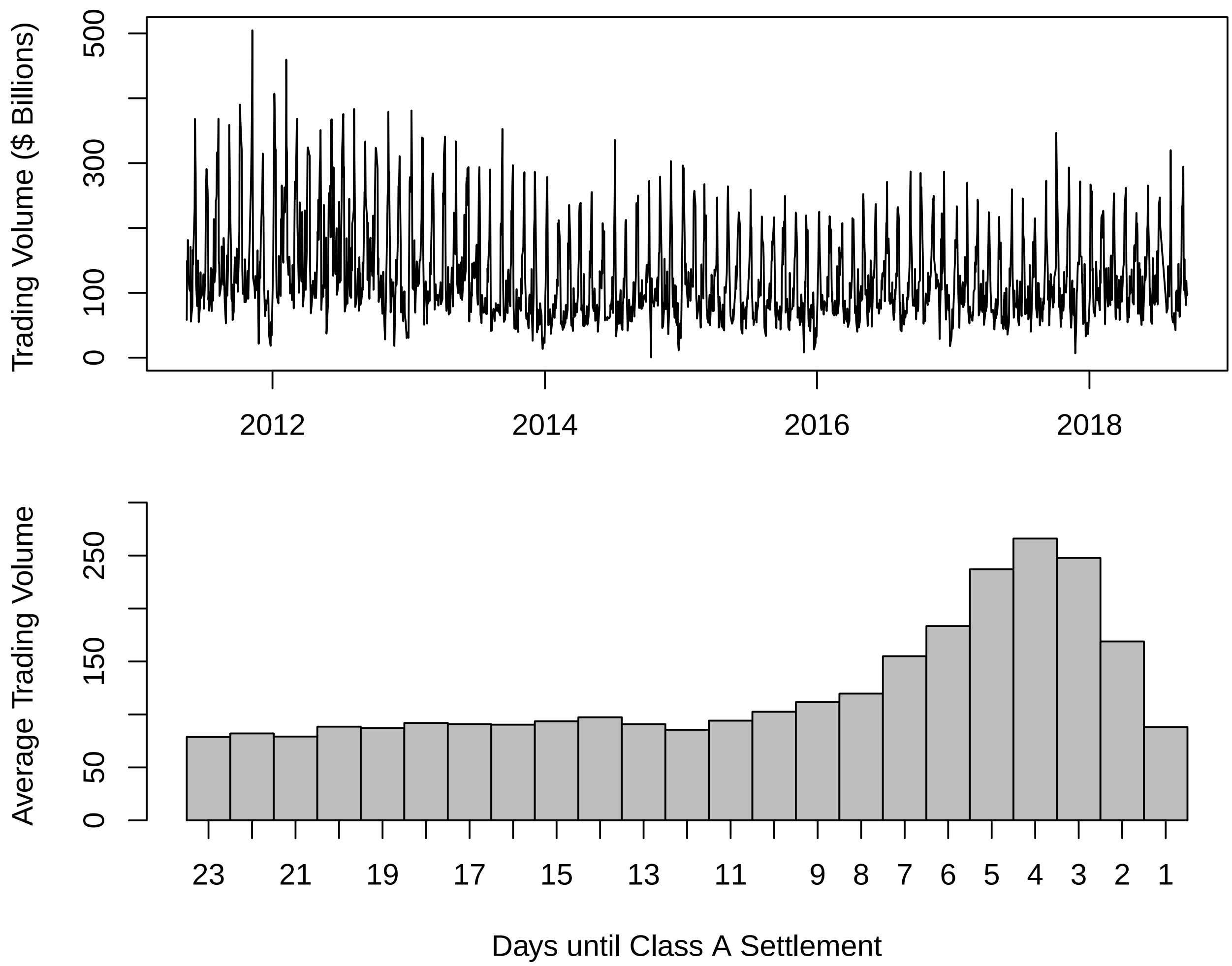

Figure 2.1. Fannie Mae 30-year TBA trading volume. The upper panel plots the daily trading volume for 30-year Fannie Mae TBAs, including both outright TBA trades and dollar rolls. Trading volume is highly volatile and displays a strong intramonth pattern. The lower panel plots the average daily trading volume of Fannie Mae 30-year TBAs against the number of days until settlement. On average, the factor day occurs 4 days prior to settlement. The notification day is two days prior to settlement. Thus, most of the TBA trading over the month occurs in the first part of the month prior to notification.

DOLLAR ROLL ANALYSIS

PAGE 1 OF 2

FNCL 3½

MORTGAGE DATA		DEFAULT PRICING		SETTLEMENT DATE	EVALUATION OF ARBITRAGE	
NET 3 ½	GROSS 4.13	IMM. PRICE	103-8	8/14/17	B/E FIN RATE	0.933
ORIGINAL TERM	30y 0m	FORW DROP	- 6.00 $\frac{1}{32}$s		(BREAKEVEN ACT/360)	
REMAINING TERM	29y 5m	FORW PRICE	103-2	9/13/17	SOLVE FOR	B
BALANCE	1,000,000.00					
STATED DELAY	54	REINV RATE	0.93	(ACT/360)	ARB $/MM	-0.00
GPM PLAN	N/A				32NDS	-0.00
		PREPAYMENT	214.00 PSA	PPL CPR PSA SMM	BP	0
			B.Median			

ANALYSIS OF ALTERNATIVES

MORTGAGE ROLL		CONTINUED HOLDING	
COST AT 103-8	1,032,500.00	FUTURE VALUE OF PAYMENTS	7,265.80
ACCRUED 13 DAYS	1,263.89	REMAINING PRINCIPAL:	
AMOUNT INVESTED	1,033,763.89	995,648.60 AT 103-2	1,026,140.34
30 DAYS INTEREST	803.84	ACCRUED 12 DAYS	1,161.59
TOTAL FUTURE VALUE	1,034,567.73	TOTAL FUTURE VALUE	1,034,567.73

CONTINUED HOLDING OF SECURITY

PAYMENT DATE	PRINCIPAL BALANCE	INTEREST	PRINCIPAL PAYMENT	NET CASH FLOW	VALUE AS OF 9/13/17
9/25/17	995,648.60	2,916.67	4,351.40	7,268.07	7,265.80

Figure 2.2. Dollar roll analysis on Bloomberg. This figure shows the calculation of the breakeven financing rate for the 30-year Fannie Mae 3.5% dollar roll on August 7th, 2017. The breakeven financing rate is found by equating the value to rolling the mortgage-backed securities to holding the mortgage-backed securities. The 0.93% financing rate is dependent on the assumption of a 214 PSA prepayment rate. To find this screen on a Bloomberg terminal, type `FNCL 3.5 8/17 <MTGE> RA <GO>`.

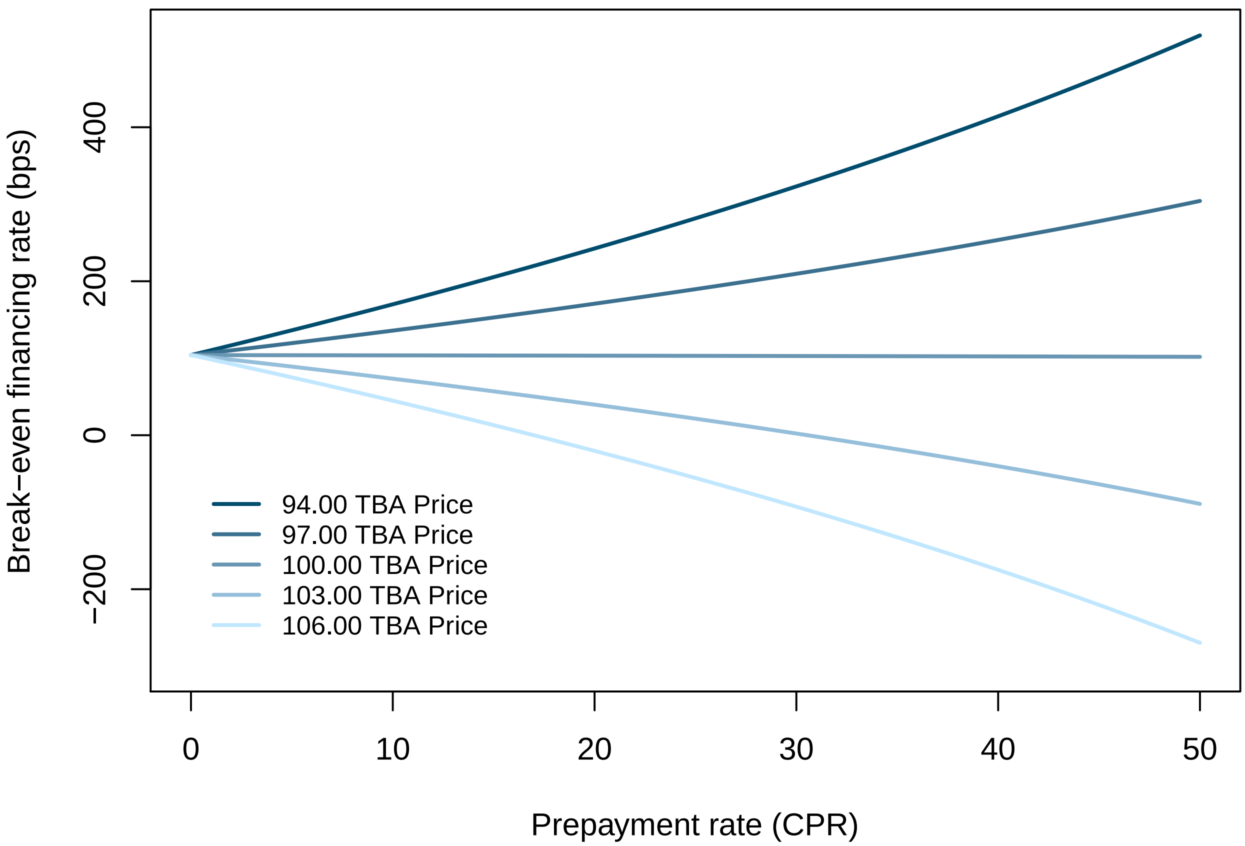

Figure 2.3. Break-even financing rates versus prepayment rates for various MBS prices. This figure shows the relationship between q_{be} and prepayment speeds. Higher prepayments lower the breakeven financing rate for premium MBS and increase the financing rate for discount MBS.

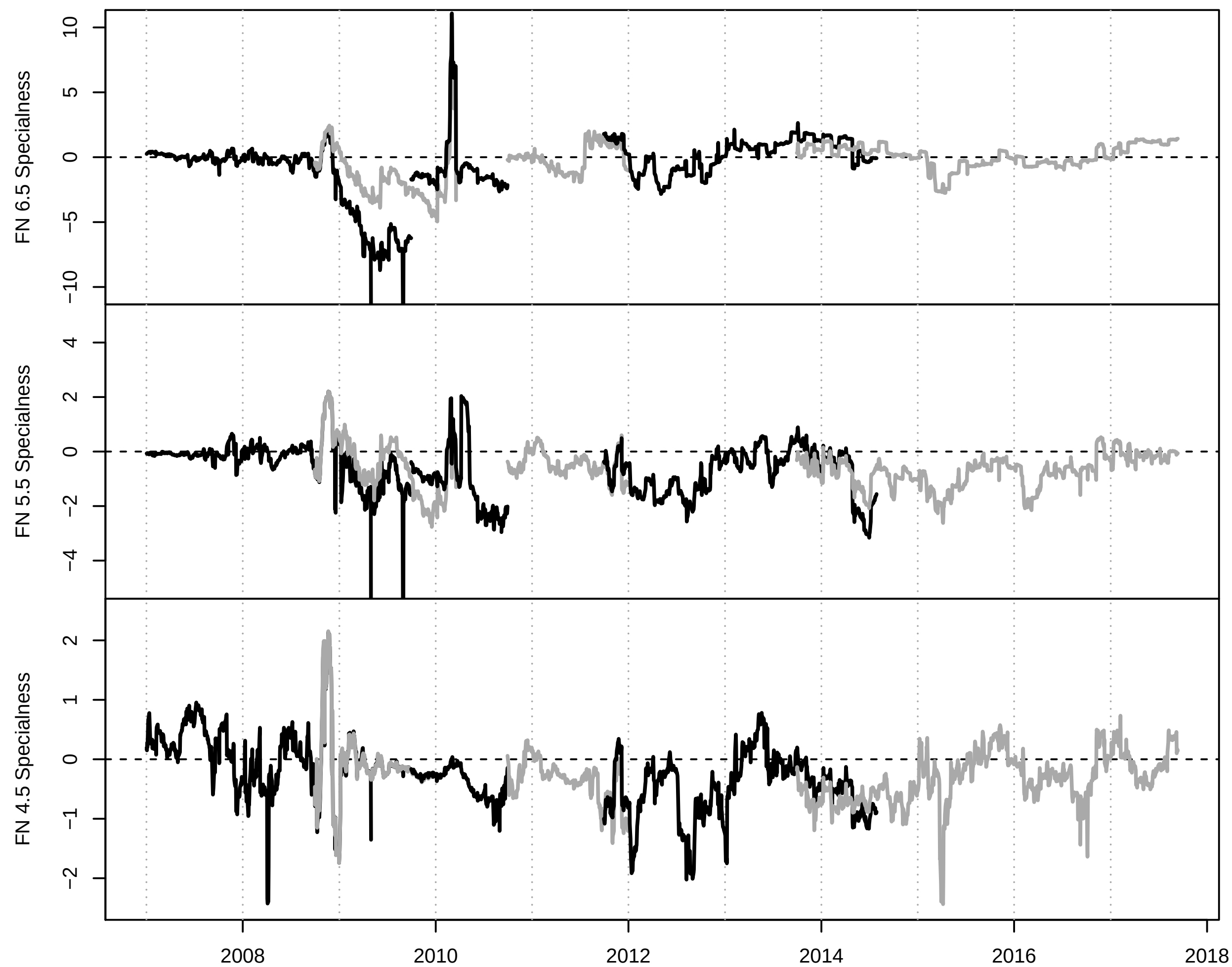

Figure 2.4. Effects of prepayment model changes on specialness. This figure shows the specialness in percentage points for FNMA 4.50, 5.50, and 6.50 percent mortgage-backed securities implied by the series of prepayment models used by a specific major Wall Street dealer. Specialness is a model's implied financing rate minus 1 month Libor. Each line, alternating black and gray, represents a different version of the dealer's prepayment model. During the time period illustrated, the dealer used six different versions of its prepayment model. Specialness is highly model dependent, and updates to the prepayment model can lead to large differences up to several hundred basis points.

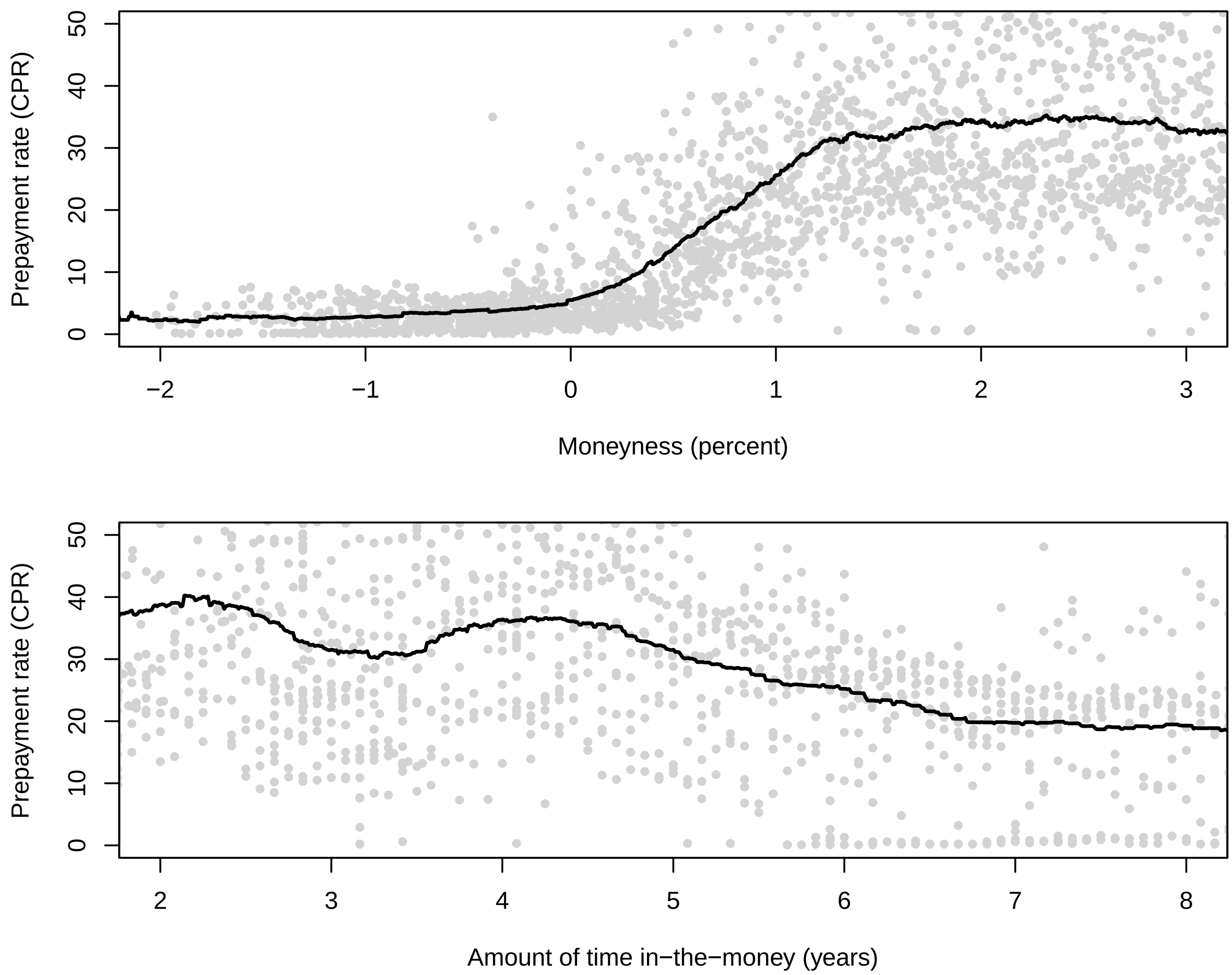

Figure 2.5. Prepayment rates for Fannie Mae mortgage-backed securities. The top panel plots the one-month prepayment rates for Fannie Mae mortgage-backed securities against the moneyness of the mortgage-backed securities. The bottom panel plots one-month prepayment rates for in-the-money Fannie Mae mortgage-backed securities against the amount of time the securities they have been in-the-money. Moneyness is expressed in percentage points. The prepayment rates are expressed as annualized percentages of the outstanding principal balance of the mortgage-backed security. The black line in each panel is the Nadaraya–Watson kernel regression estimate (Nadaraya (1964); Watson (1964)) with Scott (1992) bandwidths.

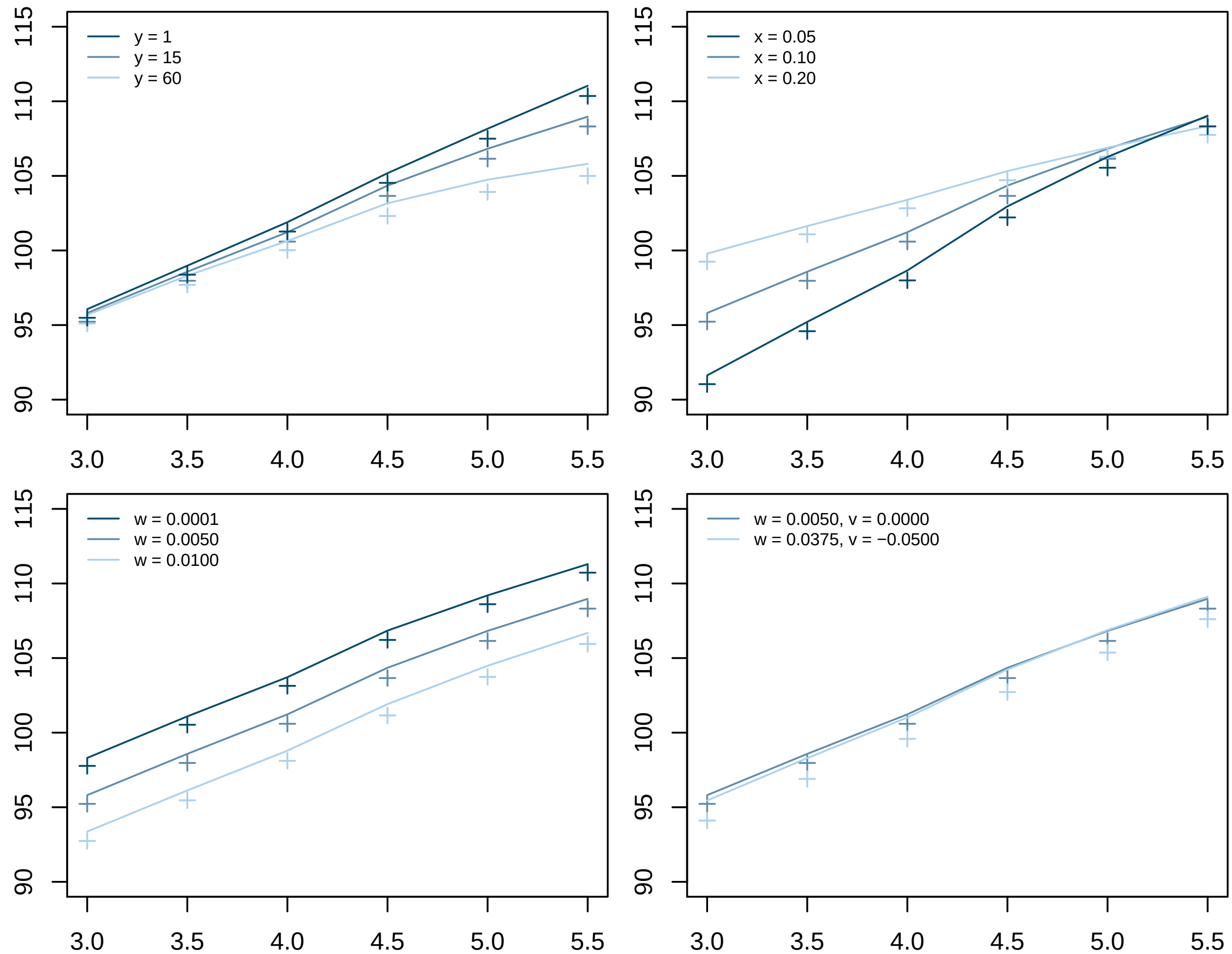

Figure 2.6. The effects of model parameters on model prices. This figure shows the effects of changes in model parameters on the prices of mortgage-backed securities (measured in dollars) with coupons ranging from 3.0% to 5.5% on 12/30/2013. The primary mortgage rate on 12/30/2013 was 4.5%. Each line connects the prices for front month settlement on 1/13/2014. Each crosshair indicates a price for back month settlement on 2/13/2014. The vertical axis is the mortgage price, and the horizontal axis is the coupon rate. In each panel, the grey line and crosshairs indicate model prices with $v_0 = 0.0$, $w_0 = 0.005$, $x_0 = 0.1$, and $y_0 = 15$ and the blue and black lines represent the prices for a shock to the initial values. The upper left panel shows that decreasing y_0 increases the prices for in-the-money mortgage-backed securities. The upper right panel shows that increasing x_0 tends to increase the prices for out-of-the-money money mortgage-backed securities and decrease the prices for in-the-money mortgage-backed securities. Increasing w_0 decreases mortgage-backed securities prices by about the same amount across the coupon stack. The lower right panel shows that increasing w_0 while decreasing v_0 can give similar front prices but lower back prices. Thus, y_0, x_0 and the sum $w_0 + v_0$ are estimates from the prices along the coupon stack whereas the split between w_0 and v_0 is estimated from the difference in prices along settlement dates.

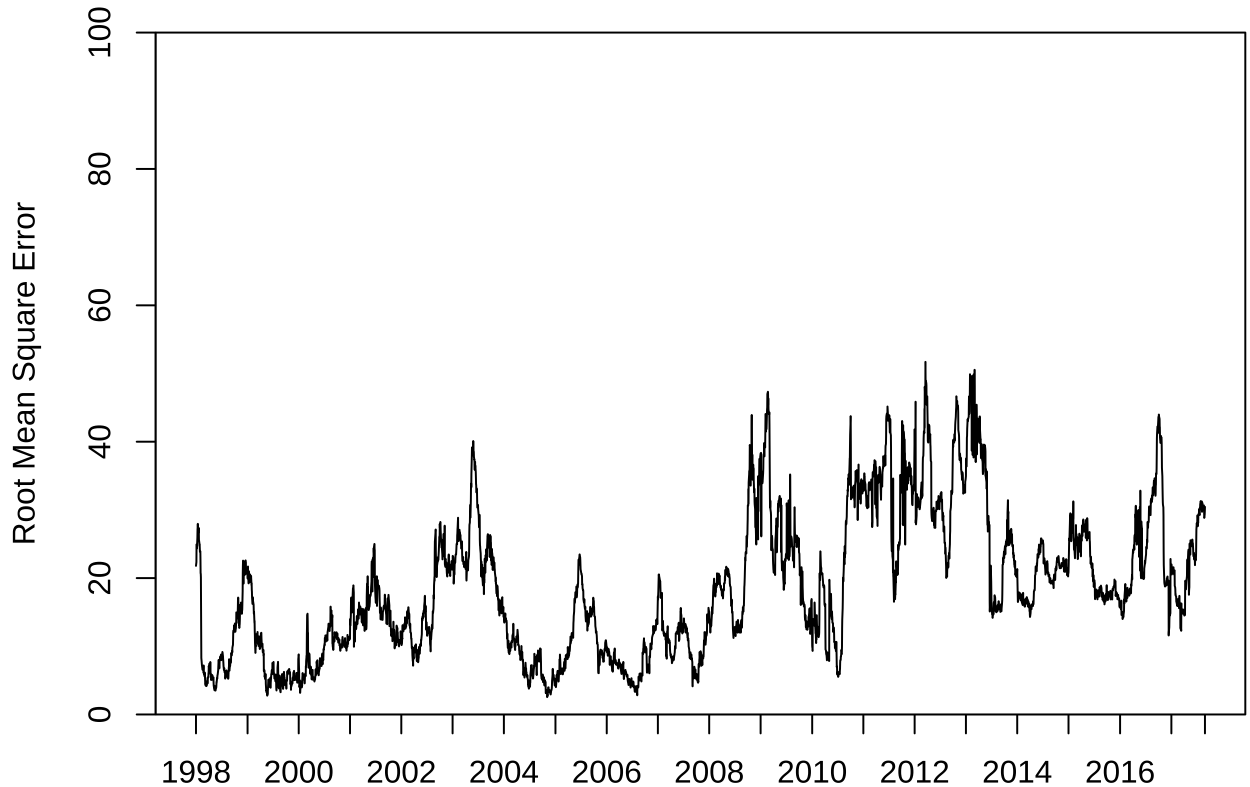

Figure 2.7. Root-mean-square errors from fitting the model. This figure plots the time series of root-mean-square errors from fitting the model to the cross-section of mortgage backed security prices. The root-mean-square error is expressed as cents per $100 notional position.

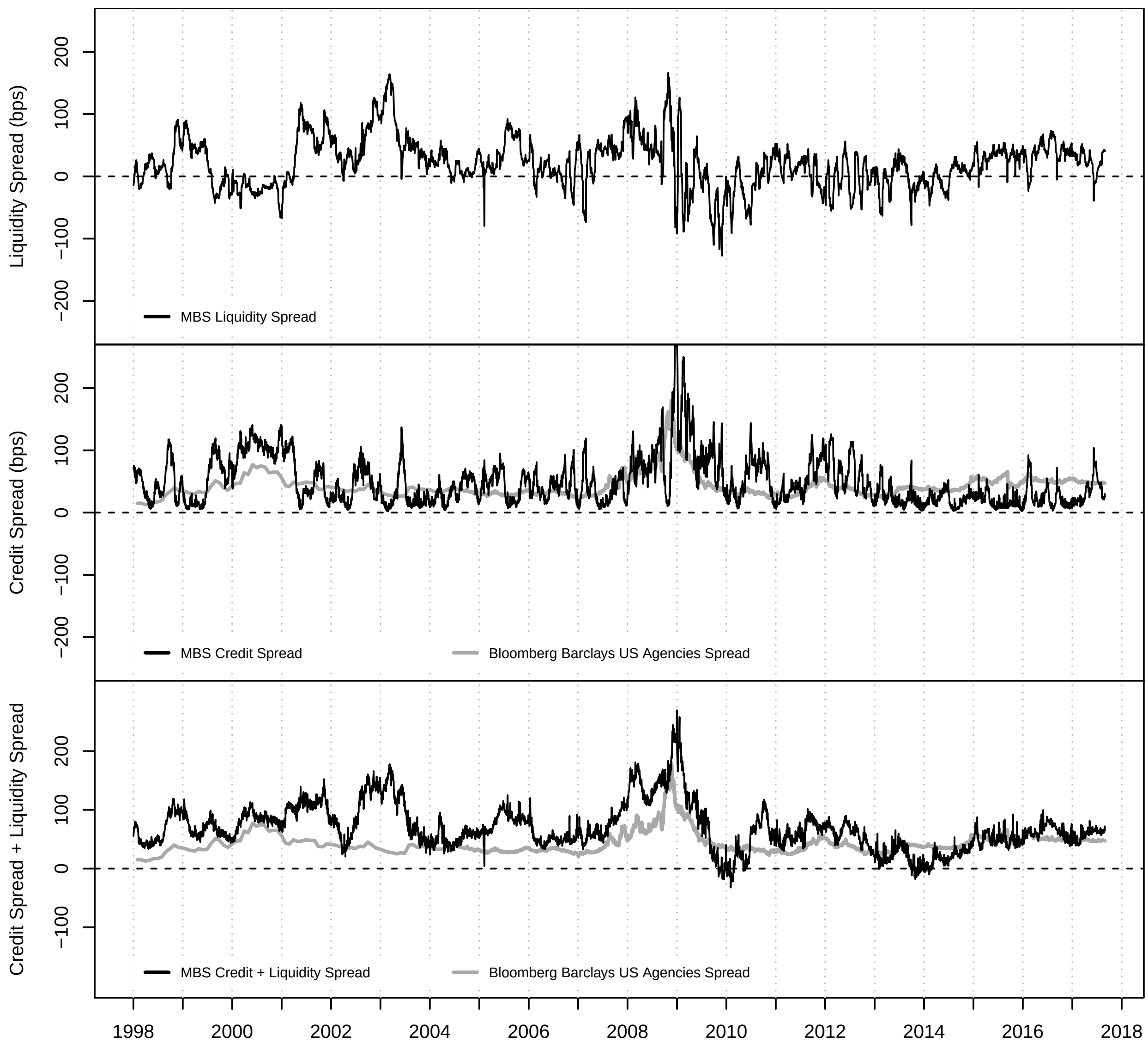

Figure 2.8. Implied MBS liquidity spread, implied MBS credit spread, combined MBS credit/liquidity spread and credit spread for agency debt. The upper panel plots the time series of the implied liquidity spread for Fannie Mae mortgage-backed securities. The middle panel plots the implied credit spread for Fannie Mae mortgage-backed securities. The bottom panel plots the sum of the credit and liquidity spread and compares it to the spread of the Bloomberg Barclays US Agencies Index to Treasuries. The spreads are expressed in basis points.

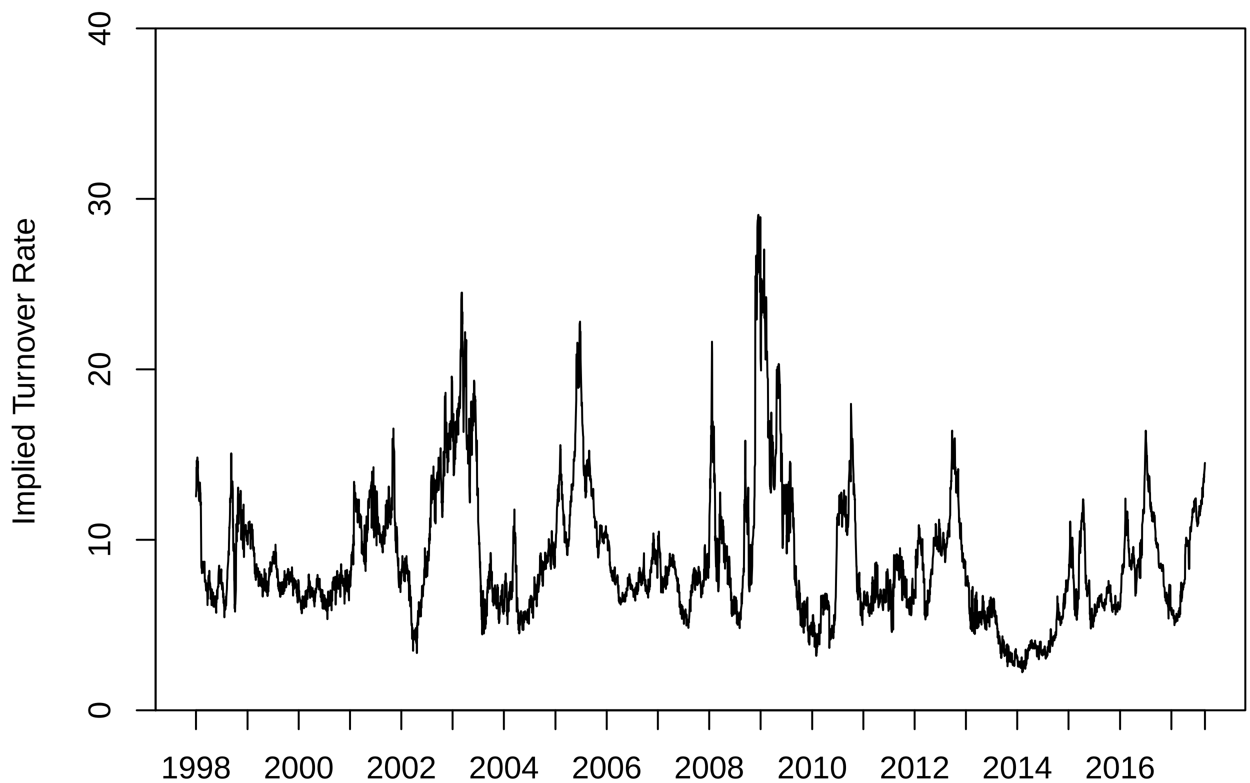

Figure 2.9. Implied turnover rate. This figure plots the time series of the implied turnover rate. The turnover rate is expressed as an annualized percentage of the outstanding principal balance of the mortgage-backed security.

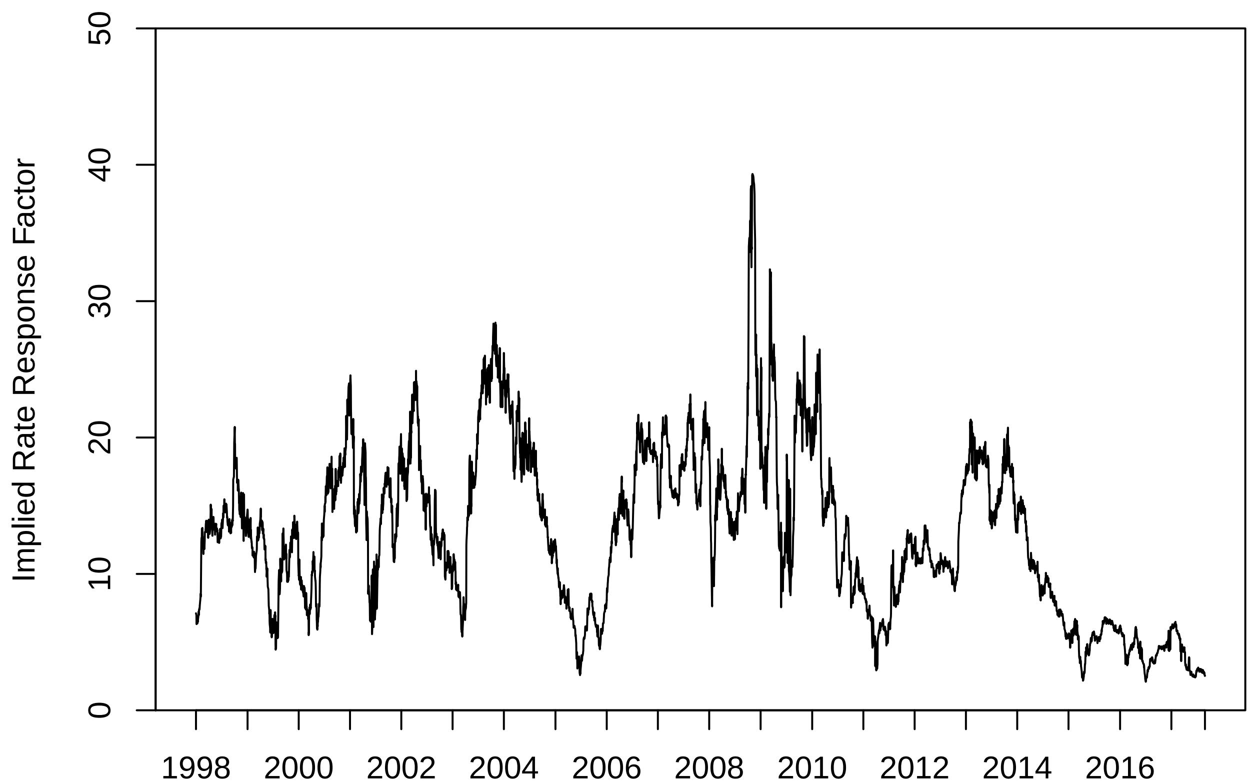

Figure 2.10. Implied rate response factor. This figure plots the time series of the implied rate response factor. The rate response factor is a multiplier measuring the sensitivity of the prepayment hazard rate to the refinancing incentive.

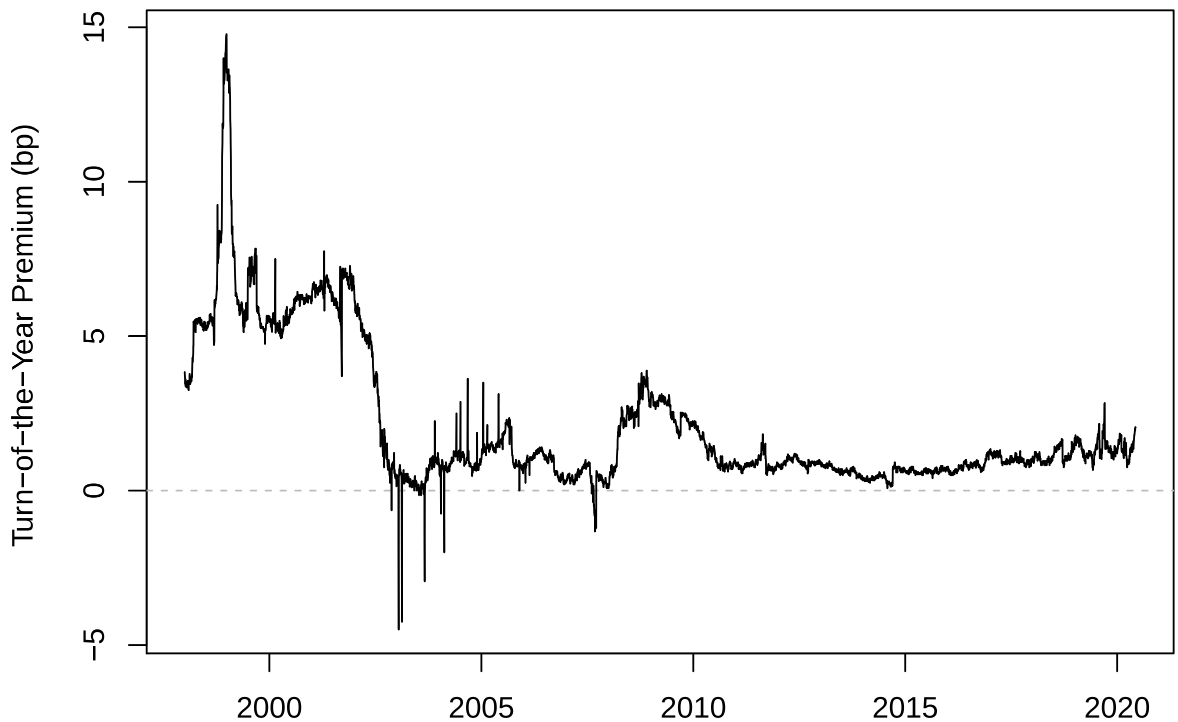

Figure 2.11. The turn-of-the-year premium. This figure plots the time series of the index of the turn-of-the-year premium in Eurodollar futures prices.

Figure 2.12. Option-adjusted spreads for rolling front month and rolling back month Fannie 5.0 TBAs. The top panel plots the time series of option-adjusted spreads for the rolling front month and back month Fannie Mae 5.0 percent TBA contracts. The bottom panel shows the difference in option-adjusted spreads for the two contracts. The difference is trimmed at the 99% level to remove outliers.

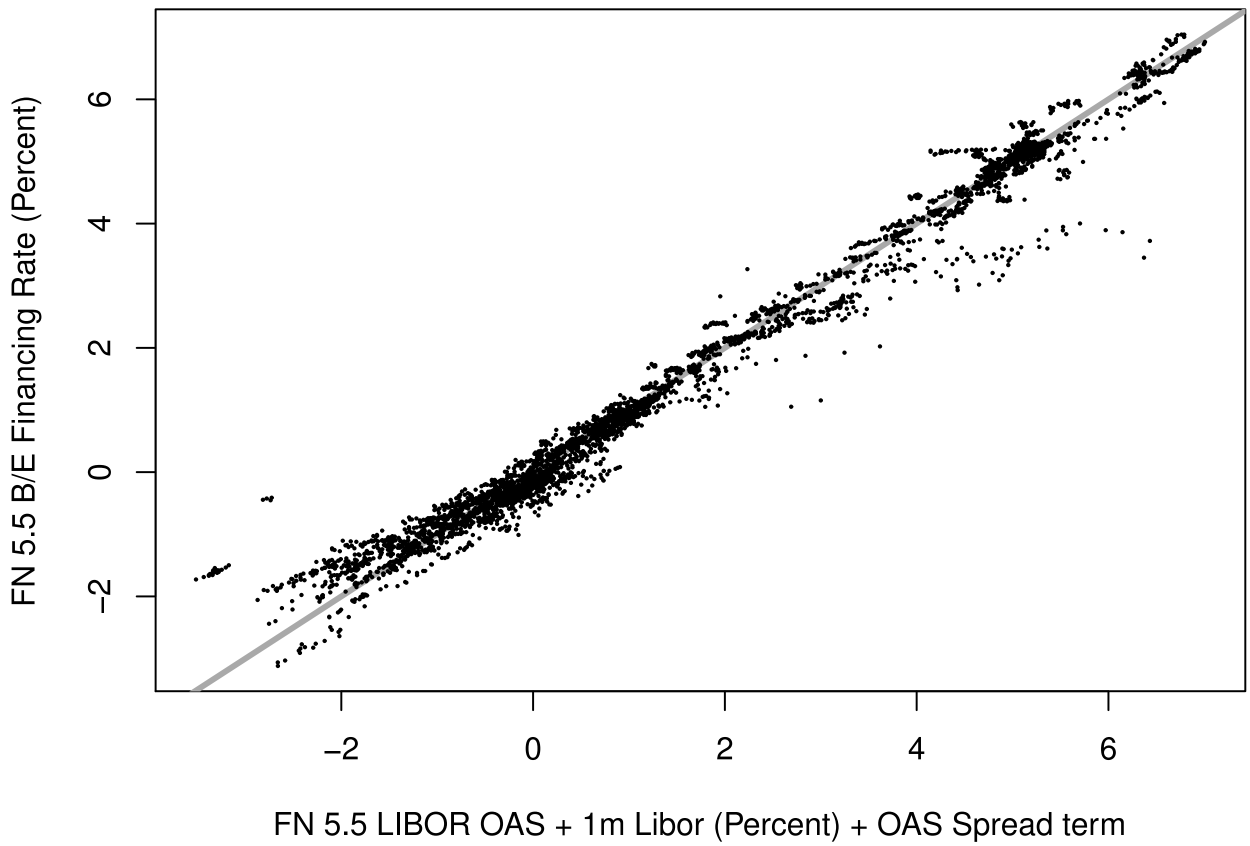

Figure 2.13. Break-even Financing Rates are a function of Option-Adjusted Spreads. This figure plots the breakeven financing rate versus Fannie Mae Libor OAS plus 1 month Libor plus the spread correction term. The grey line is the 45-degree line. Empirically, the breakeven financing rate q_{be} is approximately equal to the short rate r plus the option-adjusted spread over Libor.

Table 2.1

Cash flow timeline for a hypothetical 30-year Fannie Mae TBA trade. This table shows the key events and cash flows surrounding a 30-year Fannie Mae TBA trade for August settlement. The trade date may either be before or after the factor date.

Date	Event	Time	Note
	Trade Date	t_0	Trade parameters: issuer, maturity, coupon, face value, price, settlement date.
August 4	Factor Date		Pool factors are released by FNMA, reflecting prepayments from Jul 2 to Aug 1.
August 10	48-Hour Day		The buyer is notified of the pools the seller will deliver to settle the TBA trade.
August 14	Settlement Date	t_s	The buyer wires the payment to the seller.
August 31	Record Date		Fedwire records the buyer as the new holder of record.
September 7	Factor Date		Pool factors are released by FNMA, reflecting August prepayments.
September 25	Payment Date	t_1	The buyer receives the first payment from the MBS.
⋮	⋮		⋮
October 25	Payment Date	t_2	The buyer receives the second payment from the MBS.
⋮	⋮		⋮
November 27	Payment Date	t_3	The buyer receives the third payment from the MBS.
⋮	⋮		⋮
December 26	Payment Date	t_4	The buyer receives the third payment from the MBS.
⋮	⋮		⋮

Table 2.2

An example of a money manager that uses dollar rolls. This figure shows the balance sheets for various funds managed by Pacific Investment Management Company, LLC (PIMCO). Each fund uses some degree of leverage. Leverage is calculated as the ratio of total assets to net assets. Leverage is achieved through various financing transactions such as reverse repurchase agreements and dollar rolls. Dollar rolls account for a large portion of fund leverage, with TBA purchases ranging from 44.2% to 89.5% of total liabilities. Source: PIMCO Funds Annual Report dated March 31, 2018.

(Amounts in thousands of dollars)	Total Return Fund IV	Long-Term U.S. Government Fund	GNMA Fund	Mortgage-Backed Securities Fund	Mortgage Opportunities
Assets:					
Investments, at value	2,148,871	1,888,786	1,198,347	295,469	7,475,024
Financial derivative instruments	6,423	9,286	715	169	17,552
Cash, deposits, and currency	17,996	749	2,182	478	5,433
Receivable for investments sold	71	5,396	12,516	3,702	82,121
Receivable for TBA investments sold	413,735	313,564	397,941	184,430	3,432,278
Receivable for Fund shares sold	1,422	151	211	40	6,036
Interest and/or dividends receivable	8,843	12,542	2,352	663	21,050
Other assets	0	1	0	0	0
Total Assets	2,597,361	2,230,475	1,614,264	484,951	11,039,494
Liabilities:					
Payable for reverse repurchase agreements	0	334,991	12,211	0	739,501
Payable for sale-buyback transactions	72,245	16,770	1,098	0	759,689
Payable for short sales	0	80,372	98,606	39,830	1,513,664
Total Financing Transactions	72,245	432,133	111,915	39,830	3,012,854
Financial derivative instruments	8,028	13,981	595	125	8,605
Payable for investments purchased	22,016	87	4,493	1,006	145,810
Payable for TBA investments purchased	977,607	357,256	840,251	270,978	3,697,474
Deposits from counterparty	5,242	3,658	3,214	29	21,582
Payable for Fund shares redeemed	6,489	946	709	138	11,879
Other liabilities	671	689	582	110	3,359
Total Liabilities	1,092,298	808,750	961,759	312,216	6,901,563
Net Assets	1,505,063	1,421,725	652,505	172,735	4,137,931
Leverage	1.73	1.57	2.47	2.81	2.67
As a Percentage of Total Liabilities:					
Reverse repurchase agreements	0.0	41.4	1.3	0.0	10.7
TBA purchases	89.5	44.2	87.4	86.8	53.6
Sale-buyback transactions	6.6	2.1	0.1	0.0	11.0

Table 2.3

Cash flow timeline for a hypothetical 30-year Fannie Mae dollar roll. This table shows the key events and cash flows for settled mortgage-backed securities, TBAs, and dollar rolls. The outstanding principal balance of the settled MBS is B_0 in July, B_1 in August and B_2 in September. The August and September TBA prices are F_1 and F_2, respectively. The August TBA is for B_1^A dollars of mortgage-backed securities. Thus, a buyer of the August TBA receives the portion B_1^A/B_1 of the settled MBS cash flows. The roll seller sells the August TBA and buys the September TBA. If balances are chosen so that $B_1^A = B_1$ and $B_2^S = B_2$, then the roll seller continues to receive the cash flows from C_3 onward, but gives up C_2 in exchange for a loan from t_1^s to t_2^s. In reality, B_1 and B_2 are not known until after the trade date and the cash flows are slightly more complicated. The roll buyer does the opposite trade, which is approximately a loan from t_1^s to t_2^s plus the cash flow C_2.

			Settled MBS		Time t Cashflow			
Date	Event	Time	Balance	Cash Flow	Aug TBA	Sep TBA	Roll Seller*	Roll Buyer*
	Roll Date	t_0		-	-	-	-	-
August 4	Factor Date		B_1					
August 10	48-Hour Day							
August 14	Settlement Date	t_1^s		-	$-F_1B_1^A$	-	$+F_1B_1$	$-F_1B_1$
August 25	Payment Date	t_1		$+C_1$	-	-	$+C_1$	-
August 31	Record Date							
September 7	Factor Date		B_2					
September 11	48-Hour Day							
September 13	Settlement Date	t_2^s		-	-	$-F_2B_2^S$	$-F_2B_2$	$+F_2B_2$
September 25	Payment Date	t_2		$+C_2$	$+C_2B_1^A/B_1$	-	-	$+C_2$
September 30	Record Date							
October 5	Factor Date		B_3					
October 10	48-Hour Day							
October 12	Settlement Date	t_3^s		-	-	-	-	-
October 25	Payment Date	t_3		$+C_3$	$+C_3B_1^A/B_1$	$+C_3B_2^S/B_2$	$+C_3$	-
October 31	Record Date							
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

*Note: The cash flows for the roll seller and roll buyer assume that the balances are chosen to match the future balance of the settled MBS. In reality, this is not possible as B_1 and B_2 are not known at the time of trade t_0.

Table 2.4

Financing rates for mortgage-backed securities implied from dealer prepayment forecasts. This table shows the PSA forecasts provided by Bloomberg prepayment survey participants on October 15, 2010 for a 6% 30-year Fannie Mae TBA, obtained from Diep, Eisfeldt, and Richardson (2016). On that day, the front TBA price was \$108.1719 and the back TBA price was \$107.8906; the settlement dates were 11/10/2010 and 12/13/2010, respectively. The PSA forecasts shown are for the current term structure. Maturity denotes the weighted average maturity of the mortgages underlying the Fannie Mae mortgage-backed security. Coupon denotes the pass-though rate for the Fannie Mae mortgage-backed security. WAC denotes the weighted-average coupon or mortgage rate of the mortgages underlying the Fannie Mae mortgage-backed security. PSA is the percentage of the Public Securities Association prepayment model. This model assumes an annualized prepayment rate of 0.2% in month 1, a rate increase by 0.2% each month until reaching 6% in month 30. From the 30th month onward, the model assumes an annualized prepayment rate of 6% of the remaining balance. CPR denotes the conditional prepayment rate implied by the PSA speed and the maturity plus one month. The financing Rate is the break-even implied financing rate in basis points from the roll analysis model. The CPR forecasted by dealers ranges from 7.8% to 44.5% leading to financing rates ranging from -127 bp to 207 bp, a 334 bp range.

Dealer	Maturity	Coupon	WAC	PSA	CPR	Financing Rate (bp)
Credit Suisse	330	6.00	6.52	743	44.5	-127
Deutche Bank	335	6.00	6.56	675	35.1	-26
UBS	345	6.00	6.56	845	27.0	51
SAL	357	6.00	6.63	980	7.8	207
RBS	324	6.00	6.56	548	32.8	-4
Morgan Stanley	327	6.00	6.56	628	37.6	-52
JP Morgan Chase	334	6.00	6.63	708	38.2	-57
Bank of America	336	6.00	6.50	544	27.2	49
Goldman Sachs	345	6.00	6.57	771	24.6	72
Barclays	334	6.00	6.50	565	30.5	19
BNP	333	6.00	6.53	588	32.9	-4

Table 2.5

Summary statistics for Fannie Mae TBA mortgage-backed securities. This table reports summary statistics for Fannie Mae mortgage-backed securities with the indicated coupon rates from the to-be-announced (TBA) market. The sample consists of daily observations for the period from January 1998 to August 2017 for the front and back TBA contracts. Each front contract is matched to a back contract, so that N, the number of observations. is the same for the front and back contracts. The drop is the price of the front contract minus the price of the back contract. Average Moneyness denotes the average difference between the coupon rate and the primary mortgage rate. Average CPR denotes the average monthly conditional prepayment rate.

Coupon	Average Moneyness	Average CPR	Minimum Price	Average Price	Maximum Price	Minimum Drop	Average Drop	Maximum Drop	N
2.00	-0.866	0.580	94.008	96.560	101.322	0.137	0.286	0.625	552
2.50	-0.609	3.162	94.008	97.940	103.812	0.117	0.230	0.625	1229
3.00	-0.259	3.103	93.781	100.314	106.016	0.145	0.255	0.750	1600
3.50	-0.026	6.354	92.367	101.778	107.562	0.105	0.275	0.781	2026
4.00	0.371	11.366	93.465	103.656	107.938	0.051	0.255	0.875	2228
4.50	0.169	11.279	89.789	101.434	109.617	-0.047	0.198	0.875	3588
5.00	0.659	17.020	92.617	103.454	111.594	-0.375	0.200	0.531	3766
5.50	0.899	18.162	93.344	104.061	113.219	-0.078	0.187	0.453	4286
6.00	0.592	17.027	88.438	101.560	111.422	-0.449	0.186	0.469	3717
6.50	0.906	21.245	91.234	102.695	113.547	-0.844	0.182	0.473	3465
7.00	1.088	28.356	93.641	102.747	106.312	-0.062	0.141	0.531	2802
7.50	1.353	29.840	95.969	103.205	106.609	-0.188	0.094	0.391	2255
8.00	1.895	29.190	98.203	104.052	108.047	-1.750	0.078	0.438	2221
8.50	2.561	26.720	100.219	104.296	106.562	-1.750	0.064	0.594	1641
9.00	2.938	36.272	102.062	104.670	106.500	-0.094	0.060	0.219	1071
9.50	3.903	11.279	102.812	104.656	105.969	-0.031	0.088	0.188	730

Table 2.6

Estimates of model parameters. This table reports the estimate of the model parameters along with their asymptotic standard errors.

Parameter	Value	Standard Error
a	0.013655	0.00142
b	0.802559	0.01485
β	0.083151	0.03286
α_v	0.000006	0.00019
α_w	0.000002	0.00013
α_x	0.001033	0.00082
α_y	0.033034	0.00135
β_v	0.001145	0.03357
β_w	0.001447	0.01574
β_x	0.009925	0.01357
β_y	0.001994	0.01479
σ_v	0.000732	0.00165
σ_w	0.001983	0.00817
σ_x	0.010146	0.00921
σ_y	0.088293	0.03510
$\rho_{r,x}$	-0.222616	0.14225
$\rho_{r,y}$	-0.023684	0.23152
$\rho_{x,y}$	0.082942	0.15451

Table 2.7

Summary statistics for the mortgage-backed security pricing factors. This table reports summary statistics for the liquidity spread, the credit spread, the turnover rate (Turnover), and the rate response factor (Rate Response). The factors are estimated from the cross section of mortgage-backed security prices. The liquidity spread and credit spread are expressed in basis points. Turnover is expressed as a percentage. Rate Response is expressed as a multiplier for the refinancing incentive. The sample consists of daily observations for the period from January 1998 to August 2017.

Statistic	Liquidity Spread	Credit Spread	Turnover	Rate Response
Mean	20.802	48.712	8.813	12.973
Minimum	-127.531	1.965	2.230	2.091
Median	21.555	37.670	7.792	12.887
Maximum	166.405	361.496	29.063	39.324
Standard Deviation	39.722	38.201	3.859	6.123
Serial Correlation	0.980	0.972	0.986	0.992
Number	4919	4919	4919	4919

Table 2.8

Results from the regression of monthly changes in the credit spread on explanatory variables. This table reports the results from the regression of the monthly change in the credit spread (measured in basis points) on it lagged value, the change in the US corporate BBB minus 10-year Treasury spread (measured in basis points), the change in the Treasury-Eurodollar (TED) spread (measured in basis points), and the lagged change in the Treasury-Eurodollar (TED) spread (measured in basis points). The t-statistics are based on the Newey and West (1986) estimator of the covariance matrix (with three lags). ** denotes significance at the 5% level. * denotes significance at the 10% level. The sample period is November 2002 to August 2017.

	Credit spread	
Variable	Coefficient	t-statistic
Intercept	-0.0940	-0.06
Lagged change in credit spread	-0.5347	-10.05**
Change in BBB - Treasury spread	0.3497	4.06**
Change in Treasury-Eurodollar (TED) spread	-0.2823	-5.33**
Lagged change in TED spread	-0.4948	-7.32**
Adj. R^2		0.422
N		178

Table 2.9

Results from the regression of monthly changes in the funding liquidity spread on explanatory variables. This table reports the results from the regression of the monthly change in the funding liquidity spread (measured in basis points) on its lagged value, the percentage change in net primary dealer positions in mortgage backed securities (measured in percentage points), the percentage change in gross issuance of mortgage backed securities (measured in percentage points), the change in the Treasury-Eurodollar (TED) spread (measured in basis points), the lagged change in the Treasury-Eurodollar (TED) spread (measured in basis points), the lagged change in the turn-of-year premium (measured in basis points), and the change in the VIX index (measured in percentage points). The t-statistics are based on the Newey and West (1986) estimator of the covariance matrix (with three lags). ** denotes significance at the 5% level. * denotes significance at the 10% level. The sample period is January 1998 to August 2017.

	Funding liquidity spread	
Variable	Coefficient	t-statistic
Intercept	0.2468	0.158
Lagged change in funding liquidity spread	-0.4658	-9.49**
Change in primary dealer net MBS holdings (%)	0.0332	22.63**
Lagged change in gross MBS issuance (%)	0.2123	1.45
Change in Treasury-Eurodollar (TED) Spread	0.2166	3.17**
Lagged change in TED Spread	0.3513	6.21**
Lagged change in the turn-of-year premium	5.3783	1.92*
Change in the VIX index	0.2464	4.26**
Adj. R^2		0.386
N		230

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