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A NOVEL APPROACH TO THE EVAPORATION OF COMPLEX FRAGMENTS[†]

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The formalisms describing the evaporation of complex fragments on the basis of the final state phase space are shown to be seriously inaccurate. The relevance of shape polarization is discussed and new expressions for the decay widths and the kinetic energy distributions are derived.

The neutron evaporation is well described by the expression introduced by Weisskopf^{1,2} on the basis of the detailed balance equation:

$$\Gamma(\epsilon)d\epsilon = \frac{2m}{\pi \hbar^2 \rho(E)} \sigma_{\text{inv}}(\epsilon) \epsilon \rho^*(E - B_N - \epsilon) d\epsilon \quad (1)$$

where E is the compound nucleus excitation energy, B_N is the neutron binding energy, ϵ and m are the neutron kinetic energy and mass respectively, σ_{inv} is the cross section for the inverse process, $\rho(E)$ and $\rho^*(E - B_N - \epsilon)$ are the level densities for the compound and residual nucleus respectively.

In a more accurate treatment, the inverse cross section is explicitly written down in terms of the penetrability coefficients T_l associated with specific l waves and the level density is considered angular momentum dependent.

Completely similar expressions are frequently used to describe the evaporation of complex particles like alpha particles and larger fragments.³ In such cases, the inverse cross section is calculated from an optical model which fits the experimental particle-induced reaction cross section. The significance of σ_{inv} in eq. (1), or of the penetrability coefficients in more sophisticated formulae, arises from a very simplified expression for the probability of the inverse transition:

$$P_{\text{inv}} = \frac{\sigma_{\text{inv}} \times \text{velocity}}{\text{normalization volume}} \quad (2)$$

The use of eq. (1) or of similar ones is proper only if σ_{inv} contains all the physical information concerning the reversible reaction path. In the case of neutron emission, the physical meaning of the optical potential may justify the use of σ_{inv} . Furthermore, the collective degrees of freedom seem to play a very minor role. In the case of the emission of a complex charged particle,

the shape polarization effects become very relevant. The shapes assumed by the system during the formation and after the separation of the two fragments are quite different from those involved in the collision of the same two fragments. Therefore the use of an experimental cross section or of equivalent quantities given by an optical model, may grossly misrepresent the number and the relevance of the degrees of freedom (especially the collective ones) involved in the process.

In order to describe the evaporation process more in detail, we assume that the decay rate is determined not by the final state but by a transition state, similar to that of fission. If a highly asymmetric mass-splitting is considered, the critical shape corresponding to a fission saddle point is highly necked-in and the mass asymmetry degree of freedom can be considered frozen because of the very large inertial mass. Furthermore, the critical point is very close to the splitting point, so that the distributions of the dynamical quantities at the two points are either the same or are very strongly correlated.

The critical point can be called the "ridge point" in analogy to the "saddle point" because it is located on a ridge of the potential energy surface plotted versus the mass asymmetry coordinate (Fig. 1A).

As an illustration, the following simple parameterization for the ridge point shapes can be chosen: a sphere (the small fragment) in contact with the pole of a spheroid (the residual nucleus). The ridge point can be found by minimizing the potential energy of the system with respect to a parameter η describing the deformation of the spheroid. If the small fragment is charged, the large fragment becomes deformed, thus making the ridge point height lower than the nominal coulomb barrier. The potential energy versus

the deformation parameter η has a minimum at the ridge point: in the quadratic approximation one can write:

$$V_T(\eta) = B + \kappa \eta^2 \quad (3)$$

where B is the height of the ridge point. Similarly the Coulomb interaction energy, which decreases monotonically with deformation, can be expanded to first order in η at the ridge point:

$$V_c = V_0 + c\eta \quad (4)$$

where V_0 is the Coulomb interaction of the two particles at the ridge point.

Two observations can be made:

(i) Since the ridge point configuration is characterized by a deformed large fragment, the light fragments can be emitted below the nominal Coulomb barrier (Coulomb interaction of two touching spheres).

(ii) The statistical fluctuation in the coordinate η involves an energy of the order of the temperature T. The fluctuation in the Coulomb interaction energy depends upon the value of c and can be much larger than T (Fig. 1B). In fact the amplification in the kinetic energy distribution is mainly dependent upon the quantity $p = \frac{c^2}{\kappa}$ which can be called amplification parameter because of the relation:

$$\left(\Delta V_{\text{coul}} \right)^2 \propto pT \quad (5)$$

This seems to provide a partial explanation to some experimental data which do indicate substantial emission below the Coulomb barrier and large kinetic energy widths as compared with the temperature.^{4,5,6} A similar effect occurs in the fission process and has been described by Nix.⁷ The kinetic energy distribution can be evaluated in more detail. The total kinetic energy can be taken as the sum of two contributions: the first contribution is due to the

statistical kinetic energy ϵ at the ridge point associated with the fission-like degree of freedom, which is of the order of the temperature; the second contribution arises from the Coulomb interaction energy of the two fragments in the vicinity of the ridge point.

The differential decay width can be written as:

$$\Gamma d\gamma dp_\gamma d\epsilon = \frac{d\gamma}{2\pi\rho(E)h} \rho^* \left(E - B - \frac{p_\gamma^2}{2m_\gamma} - \kappa\gamma^2 - \epsilon \right) d\epsilon dp_\gamma \quad (6)$$

where B is the height of the ridge point, p_γ is the conjugate momentum of γ , m_γ is the inertia along the same coordinate, $\rho(E)$ is the compound nucleus level density and ρ^* is the level density at the ridge point. The canonical expansion of eq. (4) is:

$$\Gamma d\gamma dp_\gamma d\epsilon = \frac{d\gamma}{2\pi\rho(E)h} \rho^*(E-B) \exp \left(-\frac{1}{T} (\epsilon + p_\gamma^2/2m_\gamma + \kappa\gamma^2) \right) dp_\gamma d\epsilon. \quad (7)$$

By integration over p_γ one obtains:

$$\Gamma d\gamma d\epsilon = \frac{d\gamma}{2\pi\rho(E)h} \rho^*(E-B) \sqrt{2\pi m_\gamma} T^{1/2} \exp - \frac{1}{T} (\epsilon + \kappa\gamma^2) d\epsilon. \quad (8)$$

By further integration over ϵ and γ and by re-incorporating the contribution of the collective mode into the level density, one obtains the total decay width:

$$\Gamma = \frac{1}{2\pi\rho(E)} \int_0^{E-B} \rho^*(x) dx \approx \frac{1}{2\pi\rho(E)} \rho^*(E-B)T. \quad (9)$$

This decay width is completely analogous to the fission decay width and differs from the neutron decay width given in eq. (1). The reason for this difference is the following: In the case of neutron emission one considers the phase space associated with the motion of the neutron in a three dimensional cartesian space. In the charged particle emission, as in fission, one has only one unbound mode in the transition state instead of three. Therefore the phase

space is correspondingly reduced.

The final kinetic energy distribution can now be calculated. By employing the Coulomb interaction energy given by eq. (4) the total kinetic energy can be assumed to be:

$$E_K = \epsilon + V_0 + c_1 \quad (10)$$

In this expression, the contribution to the final kinetic energy of the modes normal to the fission-like coordinate is assumed to come from the potential energy alone. Substituting eq. (10) into eq. (8), and integrating over all the possible combinations yielding the same kinetic energy one obtains:

$$P(x) dx \propto \operatorname{erfc} \left(\frac{p-2x}{2\sqrt{pT}} \right) \exp \left(-\frac{x}{T} \right) dx \quad (11)$$

where $x = E_K - V_0$. This formalism falls short in reproducing the simplest case of neutron evaporation. For $p = 0$ (emission of a neutral particle) the kinetic energy distribution becomes:

$$P(E_K) \propto \exp \left(-\frac{E_K}{T} \right) \quad (12)$$

which is not a Maxwellian. This is to be expected because some relevant collective degrees of freedom have been left out. The introduction of two degenerate degree of freedom which may be taken to be oscillations of the small spherical fragment about the tip of the large spheroidal fragment, yields a more realistic picture.

Let us assume that the potential energy associated with the two new degrees of fission is:

$$V(w) = aw^2 \quad (13)$$

and that this potential energy is essentially of electrostatic nature. The

differential decay width is:

$$\Gamma = \frac{d\epsilon}{2\pi\rho(E)h^2} \rho^* \left(E-B-\epsilon-aw^2-\kappa\epsilon^2 - \frac{p_w^2}{2m_w} - \frac{p_x^2}{2m_x} \right) 2\pi p_w dp_w dp_x d\epsilon \quad (14)$$

After expansion to the canonical form and integration over the momenta one obtains:

$$\Gamma = \frac{2\pi}{h\rho(E)} \rho^* (E-B) \sqrt{2\pi m_x} m_w T^{3/2} \frac{1}{2a} e^{-\frac{1}{T}(\epsilon+\kappa\epsilon^2+\delta)} d\delta d\epsilon \quad (15)$$

where $\delta = aw^2$.

Assuming as in eq. (10) that the final kinetic energy is given by

$$E_K = V_0 + c\epsilon + \delta + \epsilon \quad (16)$$

and integrating over all the possible modes corresponding to the same kinetic energy, the final kinetic energy distribution is obtained:

$$P(x)dx \propto (2x-p) \exp\left(-\frac{x}{T}\right) \operatorname{erfc}\left(\frac{p-2x}{2\sqrt{pT}}\right) + 2\sqrt{\frac{pT}{\pi}} \exp\left(-\frac{p^2+4x^2}{4pT}\right). \quad (17)$$

Quite remarkably, this expression does depend only on the existence of the doubly degenerate normal mode of eq. (13), and not on its inertia or stiffness. This formula does indeed reproduce the neutron evaporation spectrum. For a neutron $x = E_K$ and $p = 0$. The second term of eq. (15) vanishes and the first term becomes:

$$P(E_K) dE_K \propto E_K \exp\left(-\frac{E_K}{T}\right) dE_K \quad (18)$$

which is the expected Maxwellian distribution.

In Figs. 2, 3, 4, and 5 the calculated kinetic energy distributions are plotted for the emission of ^4He , ^9Be , ^{12}C , ^{20}Ne from the compound nucleus ^{236}U at various temperatures.

It can be observed that for small p values, (corresponding for instance to the emission of an alpha particle) the kinetic energy distribution is quite

asymmetrical and strongly resembles a Maxwellian; while for large values of p the distribution for the same temperature is much broader and more nearly symmetrical. In this calculation, the p values have been determined by using the liquid drop model and the parameterization described above. It is to be stressed that, to a large extent, eq. (17) is independent of the parameterization insofar as the quantity p can be defined unequivocally.

As the small fragment becomes more complex, the above parameterization may become insufficient. The addition of a deformation coordinate for the small fragment as well as of other collective normal modes becomes very important. Furthermore the increasing specific heat of the small fragment makes necessary to account also for its level density. However for specific fragments like perhaps ^{16}O , the shell effects may impose still a spherical shape and minimize the relevance of the intrinsic degrees of freedom. In such cases it is possible that the description presented above is still reasonably valid. At present there are no suitable experimental data to test the predictions of our theory unequivocally. For the case of ^4He emission^{3,4} the effects are rather small. The data presented in ref. 5 and 6 concern the emission of more complex fragments and seem to be consistent with our predictions. However the reactions have been induced by 5.5 GeV protons, and the resulting complex distribution of compound nuclei and excitation energies is very difficult to analyze. In order to overcome such difficulties we have started to study the emission of complex particles from moderately excited compound nuclei obtained by means of heavy ion reactions.

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FOOTNOTES AND REFERENCES

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1. V. F. Weisskopf, Phys. Rev. 52 (1937) 295; Helv. Phys. Acta 23 (1950) 187; Proc. Amer. Acad. Arts Sci. 82 (1953) 360.
2. V. F. Weisskopf and D. H. Ewing, Phys. Rev. 57 (1940) 672.
3. See for instance: G. Chenevert, I. Halpern, B. G. Harvey and D. L. Hendrie, Nucl. Phys. A122 (1968) 481, and references therein.
4. J. Muto, H. Itoh, K. Okano, N. Shiomi, K. Fukuda, Y. Omori and M. Kihara, Nucl. Phys. 47 (1963) 19.
5. A. M. Poskanzer, G. W. Butler and E. K. Hyde, Phys. Rev. C 3 (1971) 882.
6. E. K. Hyde, G. W. Butler and A. M. Poskanzer, Phys. Rev. C 5 (1971) 1795.
7. J. R. Nix, Proceedings of the Third Conference on Reactions Between Complex Nuclei, edited by A. Ghiorso, R. M. Diamond and H. E. Conzett. University of California Press, Berkeley and Los Angeles 1963. p.366.

FIGURE CAPTIONS

Fig. 1. a) Schematic illustration of the ridge points. The potential energy of a nucleus is plotted as a function of a fission-like coordinate and of a mass asymmetry coordinate. Two paths leading to the emission of two different fragments are indicated.

b) Illustration of the amplification effects occurring at the ridge point. V_T is the total potential energy as a function of the deformation coordinate η , and V_C is the Coulomb interaction energy of the two fragments. A fluctuation in the shape involving an energy of the order of the temperature T leads to a large fluctuation in the Coulomb interaction energy ΔV_C and thus in the final kinetic energy.

Fig. 2. Predicted kinetic energy spectra for ^4He emitted from ^{236}U compound nucleus at different ridge point temperatures.

Fig. 3. Predicted kinetic energy spectra for ^9Be emitted from ^{236}U compound nucleus at different ridge point temperatures.

Fig. 4. Predicted kinetic energy spectra for ^{12}C emitted from ^{236}U compound nucleus at different ridge point temperatures.

Fig. 5. Predicted kinetic energy spectra for ^{20}Ne emitted for ^{236}U compound nucleus at different ridge point temperatures.

Fig. 1

XBL 721-2027

Fig. 2

XBL72I-2028

Fig. 3

Fig. 4

XBL 721-2030

Fig. 5

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