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# PION CONDENSATION IN A FIELD THEORY CONSISTENT WITH BULK PROPERTIES OF NUCLEAR MATTER

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ABSTRACT

Pion condensation is investigated in a self-consistent, relativistic mean field theory that is constrained to reproduce the bulk properties of nuclear matter. This constraint and self-consistency provide stringent constraints on the existence and energy of the condensate.

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The theory of nuclear matter at densities higher than normal has attracted considerable interest in the last few years in the context of astrophysics and relativistic nuclear collisions. As the density is increased, long-range correlations can develop, which manifest themselves in collective excitations and phase transitions involving new field configurations. Particular interest has focused on the topics of pion condensation and density isomer states.[1] At sufficiently high energy density, excitations of the internal structure of the nucleons become possible,[2] leading perhaps ultimately to a quark matter phase.[3]

In this note we focus on phase transitions in the intermediate density range between normal density and three or four times normal.

Earlier estimates of the pion condensate energy and density isomer states were based on interaction Lagrangians (e.g. the chiral $\sigma$-model)[4,5] whose form and parameters were fixed from elementary particle properties such as the $\pi N$ scattering lengths and PCAC. One drawback of those calculations is that the theory was solved in the non-relativistic approximation which at higher densities and for finite pion momenta becomes suspect. However, the most disturbing aspect of those calculations is that the chiral Lagrangians used led to an unphysical equation of state for normal nuclear matter, as was shown by Kerman and Miller.[6] Accordingly in that approach, the condensate energy is defined as the difference in energy between two states of the theory with and without the condensate. It is hoped that the inherent large discrepancy between the calculated normal state and the measured nuclear matter properties cancels in the subtraction. We find that this hope is not justified and that the bulk properties of nuclear matter provide stringent constraints on the condensate state.

The approach to pion condensation that we follow here is to adopt a Lagrangian that describes correctly the saturation properties of nuclear matter at normal density. We believe that any theory that is used to extrapolate into the unknown intermediate density regime should at least reproduce the known properties of the nuclear equation of state.

This note reports on a study that makes four contributions to the theory of abnormal states:

1. We have formulated and solved self-consistently for all fields a mean field theory of the nucleus that possess pion condensate solutions.
2. The theory is constrained to reproduce the known bulk properties of nuclear matter, namely its saturation energy, density, and compressibility.
3. It is solved in its relativistically covariant form.
4. A continuous class of space-time dependent pion condensate solutions is exhibited.

Our starting point is the theory of Walecka.[7] Two representative fields, a chargeless scalar and vector meson, $\sigma$ and $V_\mu$, are introduced with Yukawa coupling to the nucleons. We incorporate also non-linear scalar interactions as did Boguta and Bodmer.[8] With such a theory the bulk properties of nuclei can be accounted for. We introduce also the pions with pseudovector coupling to the nucleons. This coupling is used rather than the pseudoscalar one, since both S- and P-wave $\pi N$ scattering lengths are then correctly given for symmetric nuclear matter. Using standard notation, the effective Lagrangian density is:

$$\mathcal{L}_{eff} = \bar{\psi}\left(i\not{\partial} - g_V \not{V} - (m_N - g_s\sigma) - g_\pi \gamma_5 \gamma^\mu \underset{\sim}{\tau} \cdot \partial_\mu \underset{\sim}{\pi}\right)\psi$$

$$+ \frac{1}{2}\left(\partial_\mu \sigma \partial^\mu \sigma - m_s^2 \sigma^2\right) + \frac{1}{2}\left(\partial_\mu \underset{\sim}{\pi} \partial^\mu \underset{\sim}{\pi} - m_\pi^2 \pi^2\right)$$

$$- \frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \frac{1}{2} m_V^2 \; V_\mu V^\mu - U(\sigma) \qquad , \qquad (1)$$

where the scalar potential density is

$$U(\sigma) = \left(\frac{1}{3}\, b\, m_N + \frac{1}{4}\, c\, g_s \sigma\right)(g_s\sigma)^3 \qquad . \qquad (2)$$

Here $m_N$ is the nucleon mass. The field equations for $\mathcal{L}_{eff}$ in the mean field approximation are

$$m_s^2 \bar{\sigma} = g_s \langle \bar{\psi}\psi \rangle - \langle \frac{dU}{d\sigma} \rangle \qquad , \qquad (3a)$$

$$m_V^2 \bar{V}_\mu = g_V \langle \bar{\psi}\gamma_\mu \psi \rangle \qquad , \qquad (3b)$$

$$(\Box + m_\pi^2)\langle \underset{\sim}{\pi}(x) \rangle = g_\pi \partial^\mu \langle \bar{\psi}(x)\; \gamma_5 \gamma_\mu \underset{\sim}{\tau}\; \psi(x) \rangle \qquad , \qquad (3c)$$

$$\left\{ i\not{\partial} - g_V \bar{\not{V}} - (m_N - g_s \bar{\sigma}) - g_\pi \gamma_5 \gamma^\mu \underset{\sim}{\tau} \cdot \partial_\mu \langle \underset{\sim}{\pi}(x) \rangle \right\} \psi(x) = 0 \qquad , \qquad (3d)$$

where $\bar{\sigma} = \langle \sigma \rangle$ and $\bar{V}_\mu = \langle V_\mu \rangle$ are independent of x in homogeneous nuclear matter.

Equations (3a-3d) pose a self-consistency problem since the ground state expectation values of the various source currents depend implicitly on the mean field configurations. Writing the ground state wavefunctions as a single Slater determinant, this system of equations reduces to those in the Hartree approximation.

Because eq. (3) is a nonlinear system of equations, multiple field configurations may be possible. For example, $\langle \underset{\sim}{\pi} \rangle = 0$ but $\bar{\sigma} \neq 0$ and $\bar{V}_0 \neq 0$ is a self-consistent field configuration that corresponds to the non-condensed phase studied in Refs. (7,8). Our interest here is to explore whether there are any self-consistent field configurations with $\langle \underset{\sim}{\pi}(x) \rangle \neq 0$, which correspond to a pion condensed phase and which are compatible with nuclear matter properties. In particular we will study the following class of pion field configurations

$$\langle \underset{\sim}{\pi}(x) \rangle = \bar{\pi}(\underset{\sim}{u} \cos kx + \underset{\sim}{v} \times \underset{\sim}{u} \sin kx) \quad , \tag{4}$$

where $\underset{\sim}{u}$ and $\underset{\sim}{v}$ are two orthonormal isospin vectors. The case $\underset{\sim}{u} = (1,0,0)$, $\underset{\sim}{v} = (0,0,1)$ corresponds to the usual charged running wave case : $\sqrt{2}\langle \pi_\pm \rangle = \bar{\pi}\, e^{\pm ikx}$ and $\langle \pi_0 \rangle = 0$. The case $\underset{\sim}{u} = (0,0,1)$, $\underset{\sim}{v} = (0,1,0)$ corresponds to a standing wave, $\langle \pi_\pm \rangle = \bar{\pi} \sin kx$ and $\langle \pi_0 \rangle = \bar{\pi} \cos kx$.

The key to solving eq. (3d) with such a space-time dependent pion field is the identity

$$\underset{\sim}{\tau} \cdot \partial_\mu \langle \underset{\sim}{\pi}(x) \rangle = \bar{\pi} k_\mu \, R_V(kx) \, \underset{\sim}{\tau} \cdot \underset{\sim}{v} \times \underset{\sim}{u} \, R_V^{+}(kx) \quad , \tag{5}$$

where $R_V(kx)$ is a unitary (local gauge) operator of the form

$$R_V(kx) = e^{-\frac{i}{2}(kx)\underset{\sim}{\tau} \cdot \underset{\sim}{v}} \quad . \tag{6}$$

Defining the transformed Dirac field $\psi_V$ by

$$\psi(x) = R_V(kx)\,\psi_V(x) \qquad , \qquad (7)$$

eq. (3d) reduces to a space-time independent Dirac equation for $\psi_V(x)$,

$$\left\{ i\partial\!\!\!/ - g_V \not{V} - (m_N - g_S\bar{\sigma}) + k\!\!\!/\, \underset{\sim}{\tau} \cdot \left( \frac{1}{2}\underset{\sim}{v} + g_\pi\, \bar{\pi}\gamma_5 (\underset{\sim}{v}\times\underset{\sim}{u}) \right) \right\} \psi_V(x) = 0 \; . \qquad (8)$$

Therefore, the space-time dependence of the transformed nucleons or quasi-particle wavefunctions is a simple plane wave. On the other hand, the complicated spin-isospin wavefunctions, $u_V(p)$, satisfies eq. (8) with $i\partial\!\!\!/$ replaced by $p\!\!\!/$.

The quasiparticle propagator, $S_V(p)$, is obtained by inverting the Dirac operator:

$$S_V(p) = \frac{1}{D(p\ )} \left\{ p_0'^{\,2} - \varepsilon^2 - (p'k)\,\underset{\sim}{\tau}\cdot\underset{\sim}{v} - 2g_\pi\, \bar{\pi}[(p'k) + m^* k\!\!\!/]\gamma_5\, \underset{\sim}{\tau}\cdot\underset{\sim}{v}\times\underset{\sim}{u} \right\}$$

$$\times \left\{ p\!\!\!/' + m^* + k\!\!\!/\,\underset{\sim}{\tau} \cdot \left[ \frac{1}{2}\underset{\sim}{v} - g_\pi\, \bar{\pi}\gamma_5\, \underset{\sim}{v}\times\underset{\sim}{u} \right] \right\} \quad , \qquad (9a)$$

where

$$D(p_0, \underset{\sim}{p}) = D(p) = (p_0'^{\,2} - \varepsilon^2)^2 - (p'k)^2 - 4g_\pi^2\, \bar{\pi}^2[(p'k)^2 - m^{*2}(kk)] \; , \quad (9b)$$

and

$$\varepsilon^2 = |\underset{\sim}{p}'|^2 + m^{*2} - \left( \frac{1}{4} + g_\pi^2\, \bar{\pi}^2 \right)(kk) \qquad , \qquad (10)$$

$$m^* = m_N - g_S\bar{\sigma} \qquad .$$

Note that the shifted momentum $p'_\mu = p_\mu - g_V V_\mu$ appears on the right-hand side of eqs. (9) and (10) and that the effective nucleon mass $m^*$ depends on the self-consistent sigma field.

The quasiparticle spectrum, $p_0 = \omega(\underset{\sim}{p})$, follows from the solution of $D(\omega(\underset{\sim}{p}),\underset{\sim}{p}) = 0$. The bracketed terms in eq. (9a) contain all the information about the complicated spin-isospin dependence of the quasiparticle wavefunctions.

The ground state is specified by filling all quasiparticle states of momentum $\underset{\sim}{p}$ such that $\omega(\underset{\sim}{p}) \leqslant E_F$, the Fermi energy. Note that the Fermi surface is non-spherical[5] if $\overline{\pi} \neq 0$. Once a Fermi energy is specified, the ground state expectation value of any current operator $\overline{\psi}(x)\Gamma\psi(x)$ is evaluated by standard propagator techniques as

$$\langle \overline{\psi}(x)\Gamma\psi(x) \rangle = \mathrm{Tr}\left\{R_V^+(kx)\Gamma\ R_V(kx)(-i)\ S_V(\underset{\sim}{x},t;\ \underset{\sim}{x},t+0)\right\}$$

$$= \mathrm{Tr}\left\{R_V^+(kx)\Gamma\ R_V(kx)\int \frac{d^3p}{(2\pi)^3} \sum_{\omega(\underset{\sim}{p}) > \overline{E}} \theta(E_F - \omega(\underset{\sim}{p}))\ \underset{p_0=\omega(\underset{\sim}{p})}{\mathrm{Res}}\ S_V(p_0,\underset{\sim}{p})\right\}\ , \quad (11)$$

where the trace is over spin and isospin labels. The first line follows from eq. (7) and $iS_V(x,y) = \langle T(\psi_V(x)\overline{\psi}_V(y))\rangle$. In the second line, $S_V$ is expressed in momentum space with $S_V(p_0,\underset{\sim}{p})$ given by eq. (9). The sum in eq. (11) is over all quasiproton and quasineutron frequencies below the Fermi energy. The condition $\omega(\underset{\sim}{p}) > \overline{E}$ indicates that quasi-antiparticle states are <u>not</u> to be included in the sum. Explicit evaluation of eq. (11) will be given in detail elsewhere. To describe symmetric nuclear matter we must set $k_0 = 0$, in which case one finds $\underset{\sim}{V} \equiv 0$. The important point to note here is that once $\overline{\sigma}$, $\overline{V}_0$, $\overline{\pi}$ are fixed, the right-hand side of eq. (11) is readily reduced to a one-dimensional numerical integration. In this way, the source currents in eqs. (3a-3c) are calculated.

Self-consistency of the pion field configuration in eq. (4) can be checked by evaluating eq. (11) for $\Gamma = \gamma_5\gamma_\mu\underset{\sim}{\tau}$ and inserting the result into eq. (3c). We find that eq. (4) is a self-consistent solution provided that $\underset{\sim}{u}\cdot\underset{\sim}{v} = 0$. In that case, $\partial^\mu \langle \bar{\psi}(x)\gamma_5\gamma_\mu \underset{\sim}{\tau}\, \psi(x)\rangle = \langle \underset{\sim}{\pi}(x)\rangle\, f(\bar{\pi},\bar{\sigma},k,E_F)$, and the strength of the field is determined by

$$(-k^2 + m_\pi^2)\bar{\pi} \;=\; g_\pi\, \bar{\pi}\; f(\bar{\pi},\bar{\sigma},k,E_F) \qquad . \tag{12}$$

The transcendental function $f$ can be expressed as a one-dimensional integral whose form will be displayed elsewhere. Here we only note that $\bar{\pi}f \to 0$ as $\bar{\pi} \to 0$, and that for a certain range of $E_F$, $f$ has a logarithmic singularity at $\bar{\pi} = 0$.

In general, only $\bar{\pi} = 0$ will solve eq. (12). However, under suitable conditions there may be a $\bar{\pi} \neq 0$ condensate solution as well. We now indicate under what conditions a condensate solution exists.

First we note that the binding energy per nucleon, B/A, as a function of density depends on the following five parameters

$$g_\pi,\ (g_s/m_s),\ (g_v/m_v),\ b,\ c$$

The last four parameters are constrained by requiring that the saturation binding and density be B/A = 15.96 MeV and $\rho_0 = 0.145\ \mathrm{Fm}^{-3}$, which are known to high precision,[9] and that the compressibility lies in the range K = 200-300 MeV. Further, the binding is required to vanish somewhere in the range of 2 to 3 times $\rho_0$. The last constraint is a statement of the softness of the equation of state at higher densities. The free space $\pi$N coupling, $g_\pi$ = 1.41 fm, gives the correct P-wave scattering length. However, we have also varied $g_\pi$ below this vacuum value to take into account effectively nucleon-nucleon correlations and $\Delta_{33}$ intermediate states as in Ref. 10.

In Figs. 1 and 2 we show the results for two sets of parameters which yield reasonable equations of state in the normal phase, $\langle \underset{\sim}{\pi} \rangle = 0$. Their essential difference is their high density behavior. Self-consistent pion condensate solutions, $\langle \underset{\sim}{\pi} \rangle \neq 0$, are also found that lower the ground state energy. The phase transition density is a strong function of $g_\pi$, as is the condensate energy. We consider variations in the effective $g_\pi$ to simulate the effect of short-range correlations and $\Delta$ production. It is known that these competing effects tend to drive the critical density to higher values.[10] In our model this is accomplished by lowering $g_\pi$ below 1.41 fm. Moreover since there is no conclusive evidence that the normal state is a condensed state, we insist that $g_\pi$ be chosen so that the critical density is greater than normal saturation density. For the two equations of state shown this implies, as shown, that $g_\pi < 1.2$ fm in Fig. 1 and $g_\pi < 1.14$ fm in Fig. 2.

We draw attention to the extreme dependence of the condensate energy, for fixed $g_\pi$, on the softness at high density of the equation of state, even though the two cases we show are only moderately different at higher density in the normal state. There is another respect in which the condensate solution behaves differently in the two cases. For the case in Fig. 1, the condensate solution exists only over a finite range of density, with $\langle \underset{\sim}{\pi} \rangle$ going to zero at a lower and upper density, depending on $g_\pi$. Moreover no condensate solution exists for $g_\pi \lesssim 1.177$ fm in Fig. 1. In contrast, for the somewhat softer (at high density) equation of state, the condensate energy is an increasing function of density, at least up to $\rho = 10\rho_0$. beyond which internal nucleon degrees of freedom must certainly be considered anyway. As $g_\pi$ is decreased in this case, the onset of condensation is shifted to higher density rather than disappearing. We have found that the main reason for this difference in dependence of the condensate energy in the two cases is that the effective nucleon mass for the parameters in Fig. 1 is smaller than for the parameters in Fig. 2. This sensitivity to the effective mass is in accord with the observations of Ref. 11. The new feature here is that the effective mass we use is determined self-consistently as a function of density. As usual, the condensate energy is maximized for condensate momentum $|k| \sim 2m_\pi$.

The extreme sensitivity of the condensate energy to even modest changes in acceptable nuclear equations of state implies that the commonly employed procedure of grafting a condensate energy onto a non-self-consistently calculated equation of state is unreliable.

In summary, we find that self-consistency of the theory, and compatibility with the bulk properties of nuclear matter are very strong constraints on the existence and persistence of the condensate phase and on the magnitude of the pion condensate energy. As concerns the possible existence of new phases of matter, it is encouraging that at least a weak condensate is compatible with these constraints. The condensate energy consistent with the constraints is less than 15 MeV up to density $\rho = 4\rho_0$, in contrast to other estimates of 75 MeV at $\rho = 4\rho_0$ found in a calculation[5] that is not constrained to reproduce nuclear matter properties.

Future work along these lines should include nucleon correlations and $\Delta$ production self consistently. In addition similar calculations for neutron stars including the $\rho$ meson field are underway. Here again we expect that self-consistency and compatibility with nuclear matter properties are crucial constraints.

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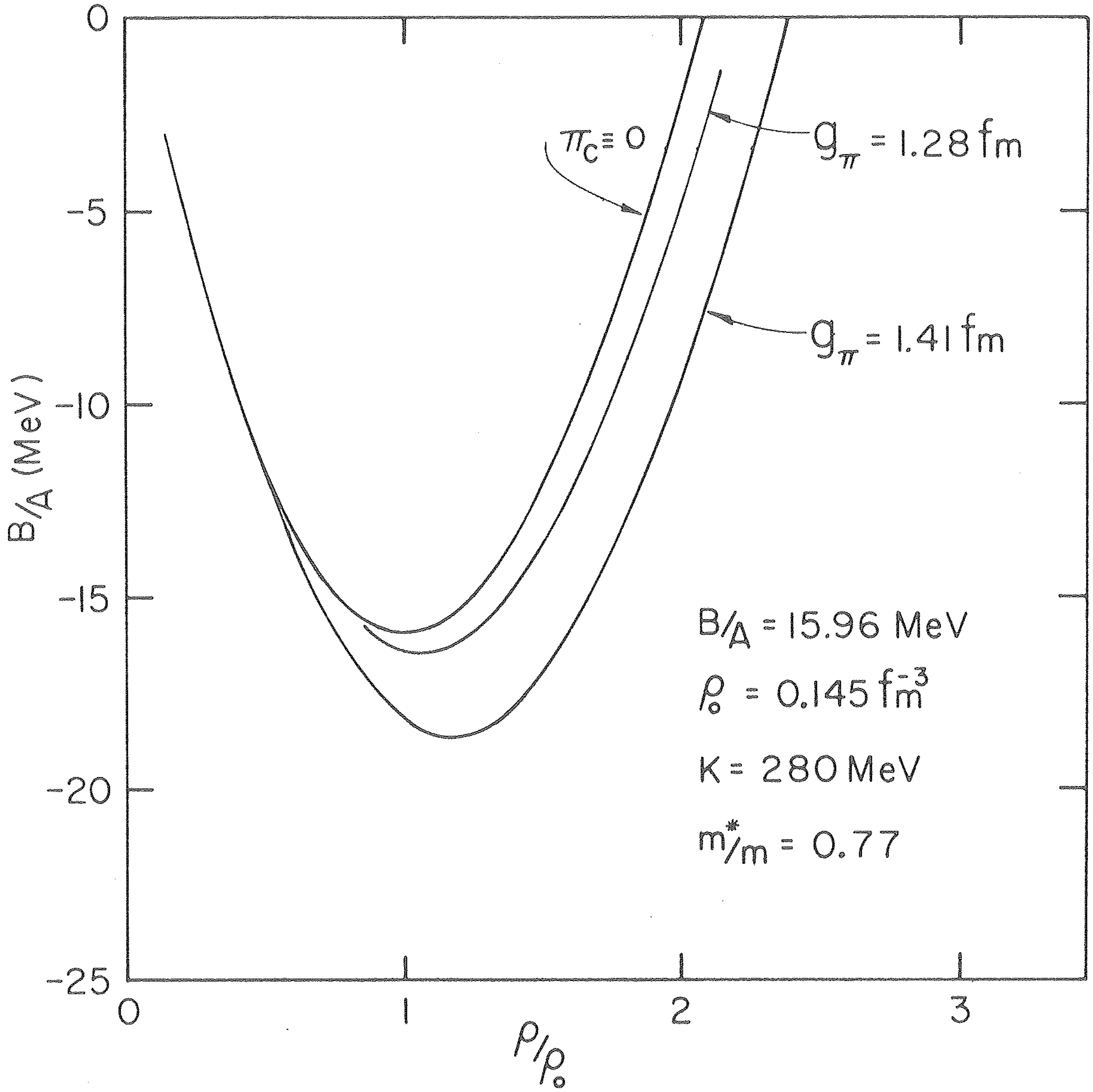


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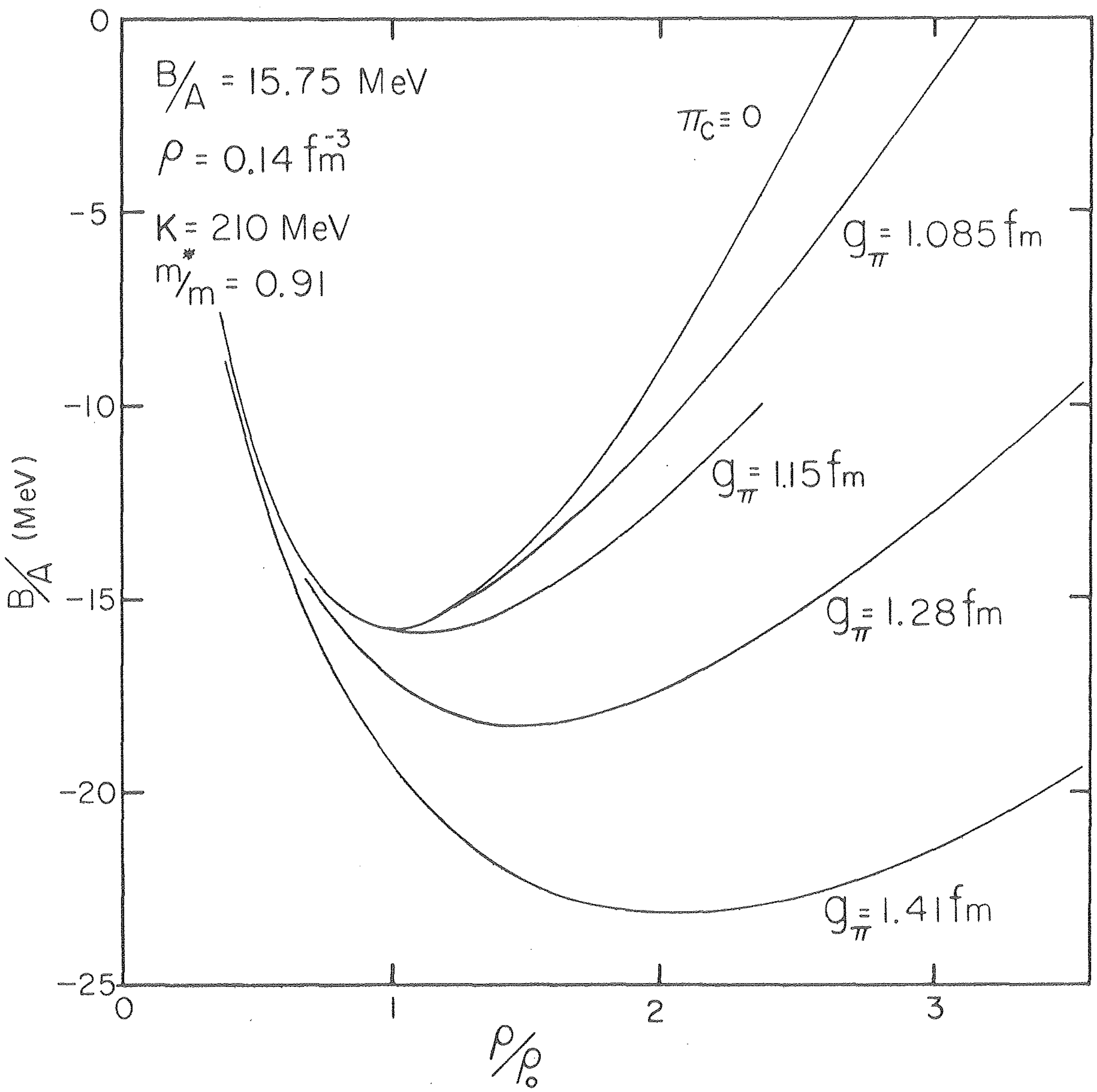
Fig. 1. Binding energy as a function of density in the absence of a pion condensate ($\bar{\pi} \equiv 0$) and for several self-consistent condensate solutions. The coupling constants and potential parameters are $g_S/m_S = 15/m_N$, $g_V/m_V = 11/m_N$, b = 0.004, c = 0.008, where $m_N = 4.77\ fm^{-1}$ is the nucleon mass. The pion momentum that minimizes the energy is $|\underset{\sim}{k}| = 1.5\ fm^{-1}$. The effective mass at the saturation density is indicated on figure.




XBL 7912-5247

Fig. 2. As in Fig. 1, but $g_s/m_s = 9/m_N$, $g_v/m_v = 5/m_N$, b = -0.192, and c = 2.47.