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Authors
Sun, Peng
Isaacson, Joshua
Yuan, C-P
et al.

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Nonperturbative functions for SIDIS and Drell–Yan processes

Peng Sun,[*,†,‡,§] Joshua Isaacson,[†] C.-P. Yuan[†] and Feng Yuan[‡]

[*] *Department of Physics and Institute of Theoretical Physics, Nanjing Normal University, Nanjing, Jiangsu, 210023, China*

[†] *Department of Physics and Astronomy, Michigan State University, East Lansing, MI 48824, USA*

[‡] *Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA*

[§] *sunpeng19820124@gmail.com*

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We update the well-known BLNY fit to the low transverse momentum Drell–Yan lepton pair productions in hadronic collisions, by considering the constraints from the semi-inclusive hadron production in deep inelastic scattering (SIDIS) from HERMES and COMPASS experiments. We follow the Collins–Soper–Sterman (CSS) formalism with the b_*-prescription. A nonperturbative form factor associated with the transverse momentum dependent quark distributions is found in the analysis with a new functional form different from that of BLNY. This releases the tension between the BLNY fit to the Drell–Yan data with the SIDIS data from HERMES/COMPASS in the CSS resummation formalism.

Keywords: CSS resummation; Drell–Yan; SIDIS; nonperturbative functions.

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1. Introduction

To reliably predict the transverse momentum distribution of the final state particles in some scattering processes in hadron collisions, it may require all order resummations of large logarithms. Among these hard processes, two of the classic examples include the Drell–Yan lepton pair production and the semi-inclusive hadron production in deep inelastic scattering (SIDIS).[1,2] In both processes, there are two separate scales: the virtuality of the virtual photon Q and the transverse momentum of either final state virtual photon $q_\perp$ in Drell–Yan process or final state

[§] Corresponding author.

hadron $P_{h\perp}$ in DIS process. Large logarithms exist in high order perturbative calculations when Q is much larger than $q_\perp$: $\alpha_s^i(\ln Q^2/q_\perp^2)^{2i-1}$.[3–9] The resummation of these large logarithms are carried out by applying the transverse momentum dependent (TMD) factorization and evolutions,[3–5,10–13] where the nonperturbative form factors associated with the TMD parton distributions play an important role.[14–27] This resummation is usually referred to as the TMD resummation or Collins–Soper–Sterman (CSS) resummation. Following the QCD factorization arguments and the universality of the TMD parton distributions, we shall expect that the nonperturbative functions determined from Drell–Yan processes can be applied to the SIDIS processes as well, where the TMD distributions couple to fragmentation functions to generate final state transverse momentum distributions. Recent experimental measurements of SIDIS processes from the HERMES[28] and COMPASS[29] collaborations provide an opportunity to understand the TMD distributions in both processes, which have already attracted several theory studies.[30–35] The goal of the current paper is to investigate the universality of the TMDs in the CSS resummation formalism to simultaneously describe the transverse momentum distributions in the Drell–Yan and SIDIS processes.[a]

The well-known Brock–Landry–Nadolsky–Yuan (BLNY) fit, to the transverse momentum dependent Drell–Yan lepton pair productions in hadronic collisions,[14,15] parametrizes the nonperturbative form factors as $(g_1 + g_2 \ln(Q/2Q_0) + g_1 g_3 \ln(100 x_1 x_2))b^2$ in the impact parameter space with x_1 and x_2 donating the longitudinal momentum fractions of the incoming nucleons carried by the initial state quark and antiquark. These parameters are constrained from the combined fit to the low transverse momentum distributions of Drell–Yan lepton pair production with 4 GeV $< Q <$ 12 GeV in fixed target experiments and W/Z production ($Q \sim 90$ GeV) at the Tevatron. However, this parametrization does not apply to the SIDIS processes measured by HERMES and COMPASS collaborations: if we extrapolate the above parametrization down to the typical HERMES kinematics, where Q^2 is around 3 GeV^2, we cannot describe the transverse momentum distribution of hadron production in the experiments.[30,31]

In this paper, we provide a novel parametrization form to consistently describe the Drell–Yan data and SIDIS data in the CSS resummation formalism with a universal nonperturbative TMD function. In order to describe the SIDIS data, it is necessary to modify the original BLNY parametrization. In the original BLNY parametrization, there is a strong correlation between the x-dependence and the Q^2-dependence for the nonperturbative form factor.[14,15] This is because $x_1 x_2 = Q^2/S$ where S is the square of the center-of-mass energy of the incoming hadrons. Therefore, at the first step, we will separate out the x-dependence, and assume a

[a]The SIDIS processes in the very small-x from HERA measurements have been analyzed in Refs. 39 and 40 in the CSS resummation, where a totally different functional form has been used to describe the experimental data. Since the HERA data covers mostly the small-x region, we will come back to them in a future publication.

power-law behavior: $(x_0/x)^\lambda$. These two parametrizations (logarithmic and power law) differ strongly in the intermediate x range. Second, we modify the $\ln Q$ term in the nonperturbative form factor by following the observation of Refs. 30 and 31, which has shown that a direct integral of the evolution kernel can describe the SIDIS and Drell–Yan data of Q^2 range from a few to hundred GeV^2. Direct integral of the evolution kernel leads to a functional form of $\ln(Q)$ term as $\ln(b/b_*)\ln(Q)$, instead of $b^2\ln(Q^2)$. Therefore, we will fit the experimental data with the nonperturbative function:

$$g_1 b^2 + g_2 \ln(b/b_*)\ln(Q/Q_0) + g_3 b^2((x_0/x_1)^\lambda + (x_0/x_2)^\lambda)\,, \tag{1}$$

with b_* defined as

$$b_* = b/\sqrt{1+b^2/b_{\max}^2}\,, \quad b_{\max} < 1/\Lambda_{\rm QCD}\,. \tag{2}$$

After obtaining the TMD nonperturbative function from the fit to the Drell–Yan data, we apply the fit to the transverse momentum distributions in SIDIS processes from HERMES and COMPASS. We find that the new parametrization form can describe well the SIDIS data with some obvious modification, and therefore establish the universality property of the TMD distributions between DIS and Drell–Yan processes.

The rest of the paper is organized as follows. In Sec. 2, we present the theoretical framework of the CSS formalism and the basic setup in the calculations of the transverse momentum dependence in Drell–Yan lepton pair production and SIDIS processes. In Sec. 3, we fit the Drell–Yan data with the new parametrization form, which is named as the SIYY form, and compare its result with that from the BLNY form. In Sec. 4, we apply the newly determined nonperturbative function to the SIDIS processes and demonstrate that with some obvious modification it can consistently describe the transverse momentum distribution measurements from HERMES and COMPASS Collaborations. We will also comment on the role of the Y-terms in SIDIS at the energy range of HERMES and COMPASS. In Sec. 5, we conclude our paper and comment on the impact of the new fit.

2. Collins–Soper–Sterman Formalism for Low Transverse Momentum Drell–Yan and SIDIS Processes

In this section, we review the basic formulas of the CSS resummation formalism and the theory framework to calculate the transverse momentum distributions for the Drell–Yan lepton pair production at hadron colliders and semi-inclusive hadron production in DIS processes. In the Drell–Yan lepton pair production in hadronic collisions, we have

$$A(P_A) + B(P_B) \to \gamma^*(q) + X \to \ell^+ + \ell^- + X\,, \tag{3}$$

where P_A and P_B represent the momenta of hadrons A and B, respectively. According to the CSS resummation formalism, the differential cross-section can be expressed as

$$\frac{d^4\sigma}{dy\,dQ^2\,d^2q_\perp} = \sigma_0^{(\mathrm{DY})}\left[\int \frac{d^2b}{(2\pi)^2}\,e^{i\mathbf{q}_\perp\cdot\mathbf{b}}\tilde{W}_{UU}(Q;b) + Y_{UU}(Q;q_\perp)\right], \tag{4}$$

where $q_\perp$ and y are transverse momentum and rapidity of the lepton pair, respectively, $\sigma_0^{(\mathrm{DY})} = 4\pi\alpha_{\mathrm{em}}^2/3N_c sQ^2$ with $s = (P_A + P_B)^2$. In the above equation, the first term is dominant in the $q_\perp \ll Q$ region, while the second term is dominant in the region of $q_\perp \sim Q$ and $q_\perp > Q$. In this paper, we focus on the low transverse momentum region to constrain the nonperturbative form factors, which is embedded in the first term of the above equation.

Similarly, in the SIDIS, we have,

$$e(\ell) + p(P) \to e(\ell') + h(P_h) + X\,, \tag{5}$$

which proceeds through the exchange of a virtual photon with momentum $q_\mu = \ell_\mu - \ell'_\mu$, and invariant mass $Q^2 = -q^2$. The differential SIDIS cross-section is written as

$$\frac{d^5\sigma}{dx_B\,dy\,dz_h\,d^2\mathbf{P}_{h\perp}} = \sigma_0^{(\mathrm{DIS})}\left[\frac{1}{z_h^2}\int \frac{d^2b}{(2\pi)^2}\,e^{i\mathbf{P}_{h\perp}\cdot\mathbf{b}/z_h}\tilde{F}_{UU}(Q;b) + Y_{UU}(Q;P_{h\perp})\right], \tag{6}$$

where $\sigma_0^{(\mathrm{DIS})} = 4\pi\alpha_{\mathrm{em}}^2 S_{ep}/Q^4 \times (1-y+y^2/2)x_B$ with usual DIS kinematic variables y, x_B, Q^2, and $S_{ep} = (\ell + P)^2$. Here, $z_h = (P_h \cdot P/q \cdot P)$ and $P_{h\perp}$ represent the momentum fraction of the virtual photon carried by the final state hadron and its transverse momentum with respect to the lepton plane, respectively.

In the CSS resummation formalism, we can write down the following expressions for the cross-sections in the impact parameter space,

$$\tilde{W}_{UU}(Q;b) = e^{-\mathcal{S}_{\mathrm{pert}}(Q^2,b_*)-S_{\mathrm{NP}}(Q,b)}\Sigma_{i,j}C_{qi}^{(\mathrm{DY})} \otimes f_{i/A}(x_1,\mu = c_0/b_*)C_{\bar{q}j}^{(\mathrm{DY})} \otimes f_{j/B}(x_2,\mu = c_0/b_*)\,, \tag{7}$$

$$\tilde{F}_{UU}(Q;b) = e^{-\mathcal{S}_{\mathrm{pert}}(Q^2,b_*)-S_{\mathrm{NP}}(Q,b)}\Sigma_{i,j}C_{qi}^{(\mathrm{DIS})} \otimes f_{i/A}(x_B,\mu = c_0/b_*)\hat{C}_{qj}^{(\mathrm{DIS})} \otimes D_{h/j}(z_h,\mu = c_0/b_*)\,, \tag{8}$$

where $c_0 = 2e^{-\gamma_E}$ with γ_E denoting the Euler constant, $x_{1,2} = Qe^{\pm y}/\sqrt{s}$ represent the momentum fractions carried by the incoming quark and antiquark in the Drell–Yan processes, $f_{i/A}$ and $D_{h/j}$ for the integrated parton distribution and fragmentation functions, respectively. In the above equation, b_*-prescription is introduced[5] and b_* follows the definition in Eq. (2). The perturbative Sudakov form factor resums the large double logarithms of all order gluon radiations,

$$S_{\mathrm{pert}}(Q,b) = \int_{c_0/b}^{Q}\frac{d\bar{\mu}}{\bar{\mu}}\left[A\ln\frac{Q^2}{\bar{\mu}^2} + B\right], \tag{9}$$

with A and B calculable order by order in perturbation theory. In the following numerical calculations, we keep A and B up to 2-loop order in QCD interaction.

Meanwhile, we will keep C coefficients and Y terms at the next-to-leading order (NLO) in the numerical calculation.

In addition, the b_*-prescription in the CSS resummation formalism introduces a nonperturbative form factor, and a generic form was suggested,[5]

$$S_{\rm NP} = g_2(b) \ln Q/Q_0 + g_1(b) \,. \tag{10}$$

Here, g_1 and g_2 are functions of the impact parameter b and they also depend on the choice of $b_{\rm max}$. In the literature, these functions have been assumed Gaussian forms for simplicity, i.e. $g_{1,2} \propto b^2$. The most successful approach is the so-called BLNY parametrization mentioned in Sec. 1, which has been encoded in ResBos program[14,15] with successful applications for vector boson production at the Tevatron and LHC. We notice that the above adaption is not the only choice to apply to the CSS resummation.[17–27]

3. Updated Fits to Vector Boson Production in Hadronic Collisions

In the BLNY fit, the following functional form has been chosen,

$$S_{\rm NP} = g_1 b^2 + g_2 b^2 \ln(Q/Q_0) + g_1 g_3 b^2 \ln(100 x_1 x_2) \,, \tag{11}$$

for Drell–Yan type of processes in hadronic collisions, where $g_{1,2,3}$ are fitted parameters,[14,15] with

$$\begin{gathered} g_1 = 0.21 \,, \quad g_2 = 0.68 \,, \quad g_1 g_3 = -0.12 \,, \quad \text{with} \\ b_{\rm max} = 0.5 \ {\rm GeV}^{-1} \,, \quad Q_0 = 3.2 \ {\rm GeV} \,. \end{gathered} \tag{12}$$

Although the above parametrizations describe very well the Drell–Yan type of processes in hadronic collisions from fixed target experiments to high energy collider data, we cannot use them to describe the transverse momentum distributions of semi-inclusive hadron production in DIS processes, as explained in great detail in Refs. 30, 31, 39 and 40.

In the nonperturbative TMDs in Eq. (11), the g_2 term is responsible for the Q^2 dependence, which we have to modify in order to describe the Drell–Yan and SIDIS processes simultaneously. Concerning this term, we follow the observation from Refs. 30 and 31 that the g_2 function should have logarithmic dependence on b rather than b^2. Therefore, we assume the following parametrization,

$$g_2 \ln(b/b_*) \ln(Q/Q_0) \,. \tag{13}$$

At small-b, the above function reduces to power behavior as b^2, which is consistent to the power counting analysis in Ref. 41. However, at large b, the logarithmic behavior will lead to different predictions, depending on Q^2. It is interesting to note that the above form has been suggested in an earlier paper by Collins and Soper,[42] which, however, was not implemented in phenomenological studies.

Table 1. The nonperturbative functions parameters fitting results. Here, $N_{\rm fit}$ is the fitted normalization factor for each experiment.

Parameter	SIYY-1 fit	SIYY-g fit
g_1	0.200	0.18084
g_2	0.810	0.16741
g_3	0.0204	0.00323
E288 (28 points) (Norm Err = 0.25)	$N_{\rm fit} = 0.82$ $\chi^2 = 52.6$	$N_{\rm fit} = 0.757$ $\chi^2 = 38$
E605 (35 points) (Norm Err = 0.15)	$N_{\rm fit} = 0.86$ $\chi^2 = 63.5$	$N_{\rm fit} = 0.824$ $\chi^2 = 61$
R209 (10 points) (Norm Err = 0.1)	$N_{\rm fit} = 1.02$ $\chi^2 = 3$	$N_{\rm fit} = 0.956$ $\chi^2 = 5$
CDF Run I (20 points) (Norm Err = 0.04)	$N_{\rm fit} = 1.06$ $\chi^2 = 10$	$N_{\rm fit} = 1.048$ $\chi^2 = 9.3$
D0 Run I (10 points) (Norm Err = 0.04)	$N_{\rm fit} = 0.93$ $\chi^2 = 7$	$N_{\rm fit} = 0.94$ $\chi^2 = 6.3$
CDF Run II (29 points) (Norm Err = 0.04)	$N_{\rm fit} = 0.990$ $\chi^2 = 30$	$N_{\rm fit} = 0.992$ $\chi^2 = 26.2$
D0 Run II (8 points) (Norm Err = 0.04)	$N_{\rm fit} = 0.94$ $\chi^2 = 3.7$	$N_{\rm fit} = 0.939$ $\chi^2 = 3.6$
χ^2	169	150
χ^2/DOF	1.21	1.07

In addition, we will modify the x-dependence in the nonperturbative function as mentioned in Sec. 1 so that

$$S_{\rm NP} = g_1 b^2 + g_2 \ln(b/b_*) \ln(Q/Q_0) + g_3 b^2 ((x_0/x_1)^\lambda + (x_0/x_2)^\lambda)\,, \tag{14}$$

where we have fixed $Q_0^2 = 2.4\ \text{GeV}^2$, $x_0 = 0.01$ and $\lambda = 0.2$. The x-dependence is motivated by a saturation picture for parton distributions at small-x. This functional form also has mild dependence on x in the intermediate x-range as compared to the original BLNY parametrization.

In the above parametrization, we have chosen $Q_0^2 = 2.4\ \text{GeV}^2$ in order to make it convenient to compare to the final state hadron distribution in SIDIS experiments from HERMES and COMPASS Collaborations. From this choice of Q_0^2, the importance of g_1 and g_3 in SIDIS is clearly illustrated.

Some comments shall follow before we present the result of the fits. First of all, in general, g_1 and g_2 are nonperturbative functions of b and x. We may have educated guesses for the functional forms. However, only experiments can tell which of these forms is correct.[b] That is why the global fit is important to constrain these functional forms. The logarithmic assumption comes from the direct TMD evolution

[b]Recent proposal of a lattice formulation of the TMDs in Euclidean space may help to solve this issue in the future,[43,44] see, also some lattice calculation attempts.[45,46]

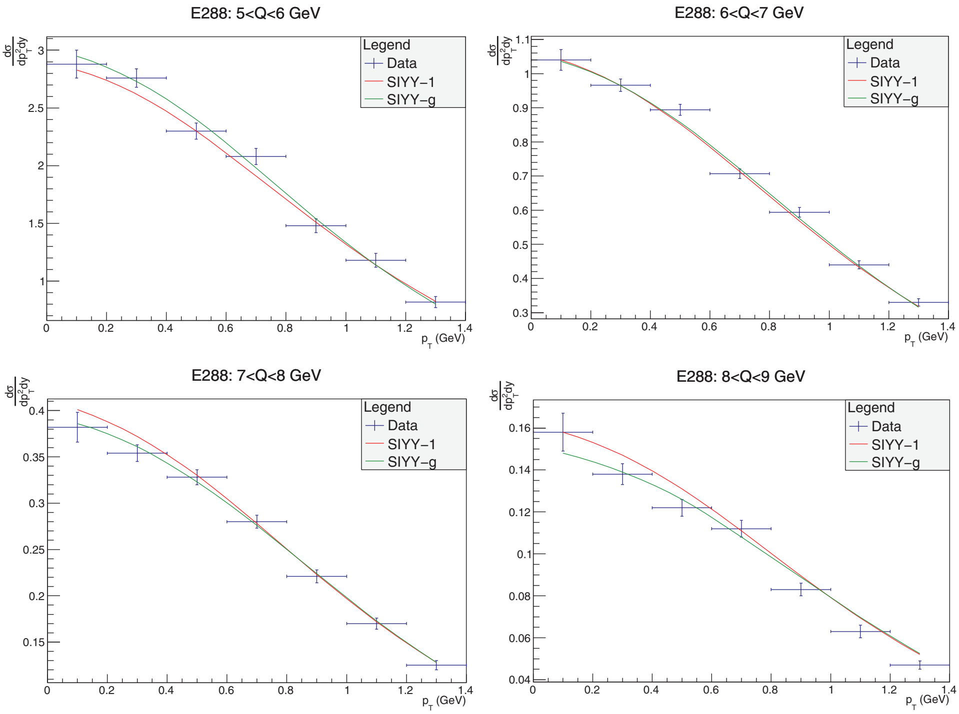

Fig. 1. Fit to the differential cross-section for Drell–Yan lepton pair production in hadronic collisions from E288 Collaboration.[49–51]

kernel which, in principle, only applies in the perturbative region. A similar idea has been applied in the Qiu–Zhang prescription in the CSS resummation.[17,18] This may introduce some theoretical uncertainties. It will be interesting to further investigate this in detail, which, however, requires more precision experimental data. Second, from the theoretical point of view, we know that at small-b Eq. (11) shall follow b^2 power law from power counting analysis.[41] That is a strong constraint for models, and our model satisfies this constraint. Most importantly, after fitting to the experimental data, the TMD evolution shall predict relevant scale dependence for various interesting observables. For example, the single transverse spin azimuthal asymmetries will be able to provide additional constraints on the evolution.[30,31] This will become possible in the near future with high precision data from JLab 12 GeV upgrade and the planned electron–ion collider.[1] In summary, introducing the logarithmic Q dependence and the mild x dependence in the intermediate x-range, as described in Eq. (14), we are able to consistently describe the transverse momentum distributions in both the Drell–Yan and SIDIS data.

We include the following data in the Drell–Yan global fit.

- Drell–Yan lepton pair production from fixed target hadronic collisions, including R209, E288 and E605 experiments.[49–53]

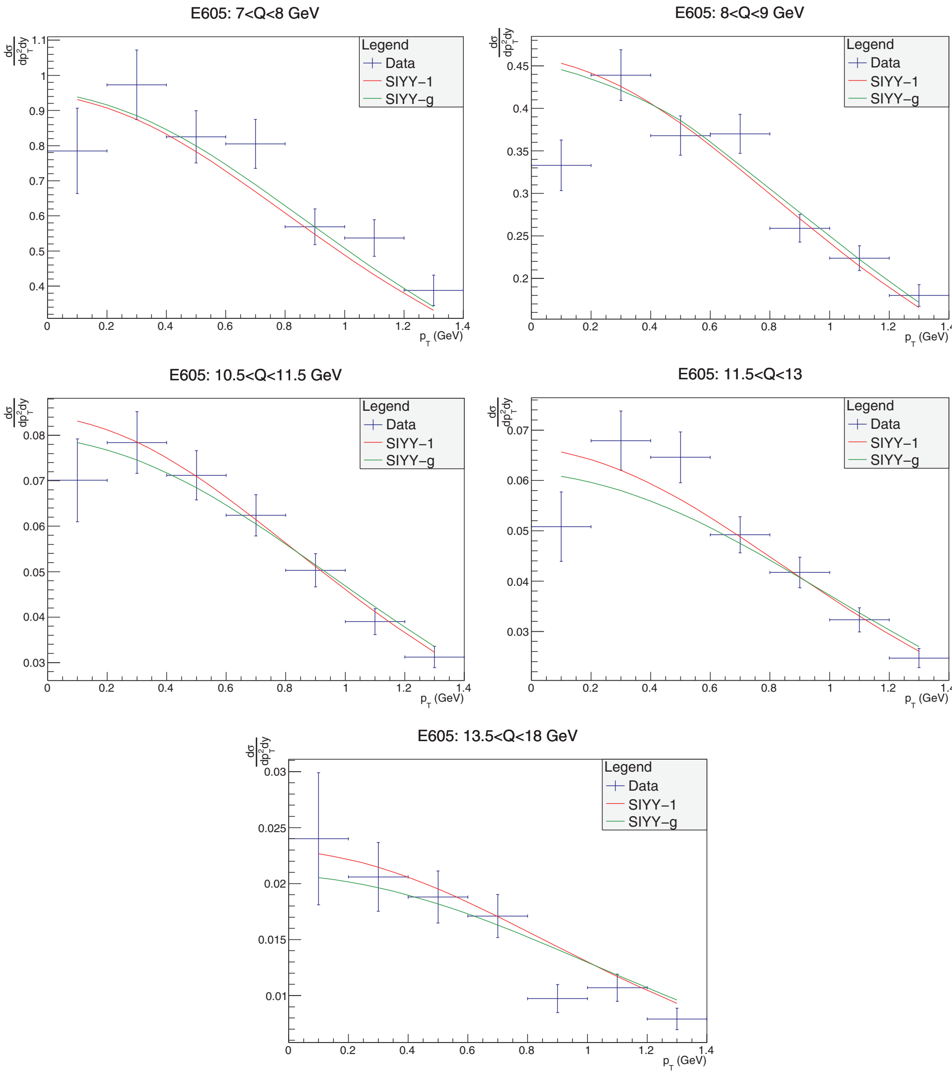

Fig. 2. Fit to the Drell–Yan data from the E605 Collaboration.[53]

- Z boson production in hadronic collisions from Tevatron Run I and Run II.[54–57] In total, we include seven Drell–Yan data sets from three fixed target experiments and four Tevatron experiments. We will then compare our predictions with the CMS and ATLAS data on Z boson production at the LHC.

We would like to emphasize that the high precision data from Z-boson production at the Tevatron Run II[57] require precision calculations of the resummation. In total, we have 140 experimental data points, and g_1, g_2, and g_3 as free parameters in the global fit, and we have chosen $b_{\max} = 1.5\ \text{GeV}^{-1}$ which is different

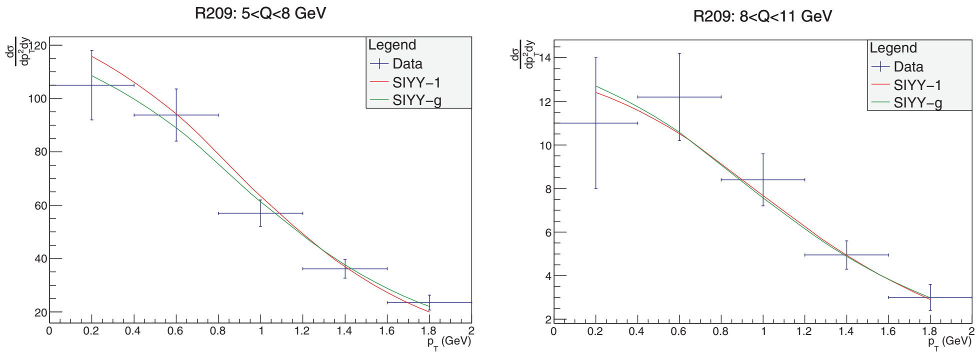

Fig. 3. Fit to the Drell–Yan data from the R209 Collaboration.[52]

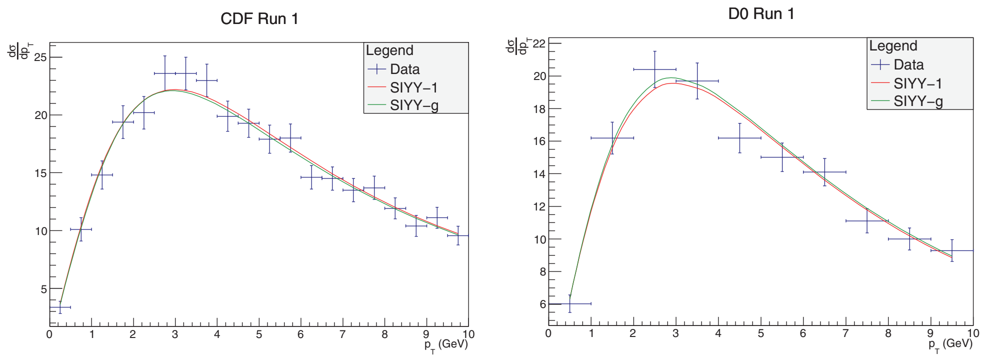

Fig. 4. Fit to the Tevatron Run I data from the CDF and D0 Collaborations.[54,55] The fits include only the $A^{(1,2)}$, $B^{(1,2)}$, and $C^{(1)}$ contributions.

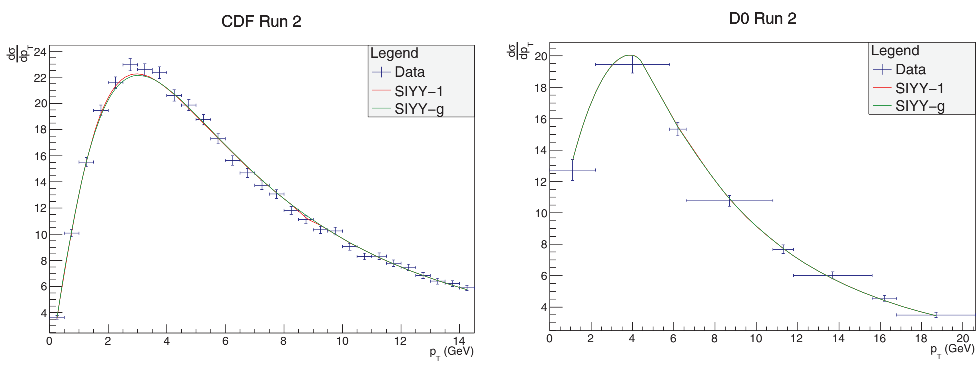

Fig. 5. Fit to the Tevatron Run II data from the CDF and D0 Collaborations.[56,57]

from the value of 0.5 GeV^{-1} adopted in the original BLNY parametrization. In the numerical calculations, we adapt the CT10-NLO parton distributions at the scale $\mu = c_0/b_*$.[48] We have also assigned an additional fitting parameter ($N_{\rm fit}$) for each experiment to account for its luminosity uncertainty.

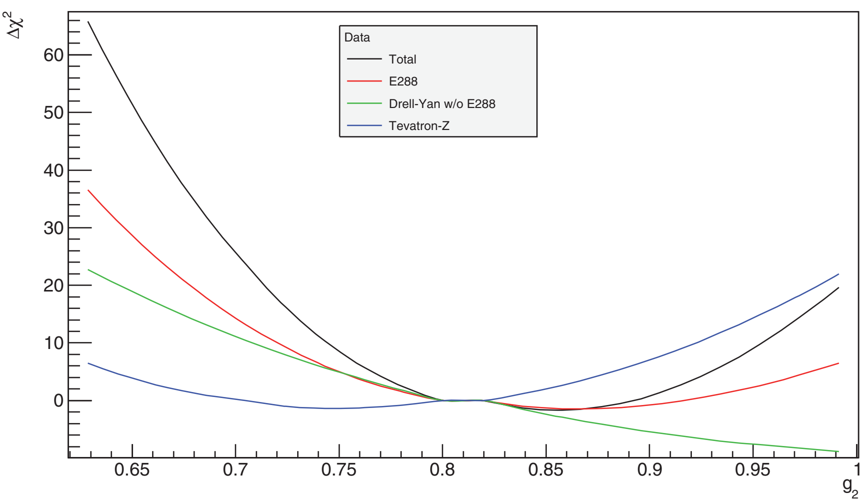

Fig. 6. $\Delta\chi^2$ distribution scanning g_2 parameter in SIYY1 fit.

In Figs. 1–5, we show the best fits to the Drell–Yan data from E288, E605 and R209 Collaborations, and Z boson production from the CDF and D0 Collaborations at the Tevatron Run I and II with two different nonperturbative parameter forms. One is the form in Eq. (14) and the fits are named as SIYY-1, another is the original BLNY form in Eq. (11) but with $b_{\max} = 1.5$ GeV^{-1}, $Q_0 = 3.1$ GeV, the fits are named as SIYY-g. The former is the one we focus on in this paper, while the latter is also showed for a comparison.

The result of our fits and their χ^2 distributions are listed in Table 1. From these plots, we see that Eq. (14) provides a reasonable fit to all seven experimental data with three nonperturbative parameters $g_{1,2,3}$ and seven independent normalization factors.

Among these parameters, the most important one, relevant to the LHC W and Z boson physics, is g_2 which controls the Q^2 dependence in the nonperturbative form factors. The increase in the χ^2 distributions, as a function of the g_2 parameter, is shown in Fig. 6, from which we obtain the uncertainty in g_2 as:

$$g_2 = 0.81 \pm 0.06 \quad (\text{at } 90\% \text{ C.L.})\,. \tag{15}$$

In order to demonstrate the sensitivities of g_2 on different experiments, in Fig. 6, we further plot the $\Delta\chi^2$ distributions as functions of g_2 from separate data sets: one from the Drell–Yan experiment E288, the combined contribution from all other Drell–Yan experiments, and one from Tevatron Z-boson experiments. From this figure, we can clearly see that the most strong constraints come from the precision Drell–Yan data at fixed target experiments, i.e. the E288 experiment. It is also interesting to note that, although Tevatron data on Z-boson production are among the most precise Drell–Yan data, they do not post a strong constraint on

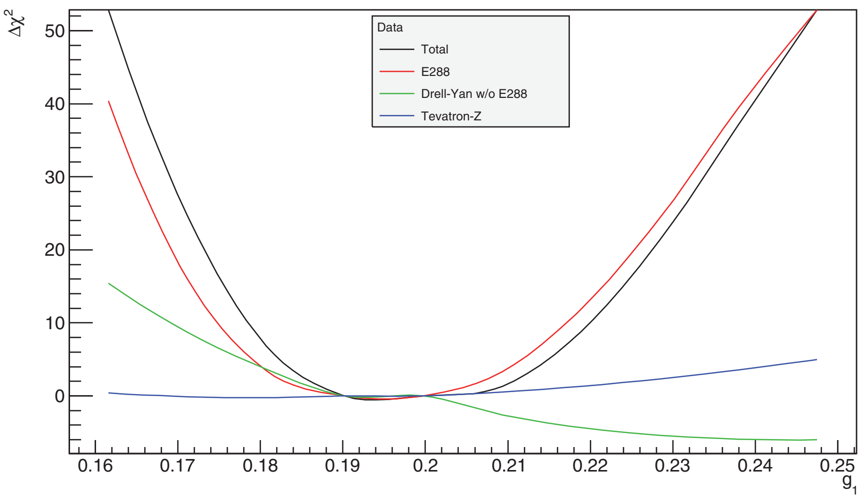

Fig. 7. Same as Fig. 6 for g_1.

the nonperturbative form factor, such as the value of the g_2 parameter. This can be understood as a result of the dominance of perturbative contribution to the transverse momentum distribution of Z-boson production at high energy hadron colliders. Similar observation has been obtained in Refs. 17 and 18 with different prescription to introduce the nonperturbative form factors.

As mentioned above, the g_2 term in the nonperturbative form factor scales as $b^2 \ln(Q)$ at small b. By using the above parameter we find that the small-b behavior of our fit can be written as $0.28b^2 \ln(Q)$ which is in the similar range of the fit found in Ref. 16 with the same $b_{\max} = 1.5\ \text{GeV}^{-1}$ choice. The difference comes from the fact that Ref. 16 fits the data with a complete Gaussian form, which only agrees with our form at small b. We also notice that the g_2 term can be estimated from fixed order calculations, from which we find that $g_2 \approx 4C_F\alpha_s/\pi$.[42] Therefore, the value of g_2 found in the SIYY-1 fit implies that $\alpha_s(1/b_{\max}) \sim 0.49$, which is consistent with the running coupling used in our fit. All these arguments support the conclusion that the SIYY-1 fit captures the QCD dynamics associated with the nonperturbative form factors in the CSS formalism.

Similarly, we find that Z boson production is not sensitive to g_1 parameter, as shown in Fig. 7. The major contribution to the $\Delta\chi^2$ comes from fixed target Drell–Yan experiments. From the plot, we obtain very strong constraints on g_1: $g_1 = 0.20 \pm 0.01$ at 90% C.L.

Recently, both CMS and ATLAS Collaborations have published their data on Z boson production at the LHC. We compare our predictions to the ATLAS data[58] in Fig. 8. From this figure, we can see that our fit can describe the LHC data well.

Before we check the consistency between the above fitted results with the SIDIS data from HERMES/COMPASS, we would like to emphasize that the above

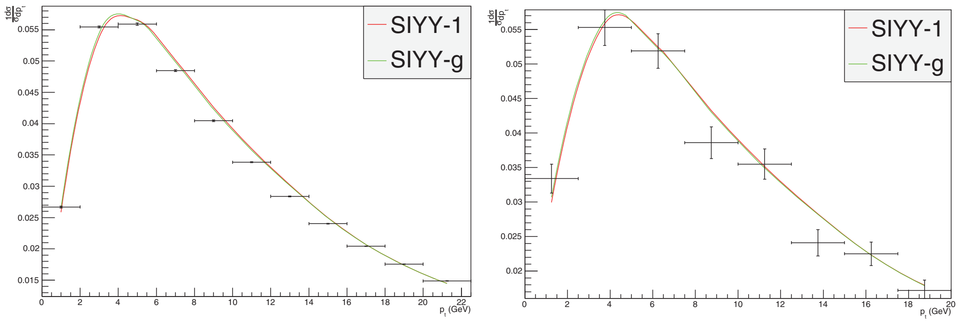

Fig. 8. Compare the resummation prediction for Z boson production at the LHC.[49–51] The data in left one is from the ATLAS collaboration, the right one is for CMS collaboration. These data are not included in our fit.

parameters are fitted only with the Drell–Yan type data. From the comparison to the experimental data, we can see that the new form is equally good as compared to the original BLNY parametrization.

4. Fitting Semi-Inclusive DIS Data with New Parametrization

The universality of the parton distribution functions (PDFs) is a powerful prediction from QCD factorization. According to the TMD factorization, the nonperturbative functions determined for the TMD quark distributions from the Drell–Yan type of processes shall apply to that in the SIDIS processes. Of course, the transverse momentum distribution of hadron production in DIS processes also depends on the final state fragmentation functions, which we will parametrize. Following the universality argument, we introduce the following parametrization form to describe the nonperturbative form factors for SIDIS processes,

$$S_{\rm NP}^{\rm (DIS)} = g_2 \ln(b/b_*) \ln(Q/Q_0) + g_1 b^2/2 + g_3 (x_0/x_B)^\lambda + g_h b^2/z_h^2 \,. \qquad (16)$$

In the above parametrization, named as SIYY-2 form, g_1, g_2 and g_3 have been determined from the experimental data of Drell–Yan lepton pair production. The only unknown parameter g_h will be determined by fitting to the HERMES and COMPASS data. Although there has been evidence from a recent study[34] that g_h could be different for the so-called favored and dis-favored fragmentation functions, we will take them to be the same in this study, for simplicity. With more data coming out in the future, we should be able to fit with separate parameters.

In principle, we can fit g_1, g_2, g_3, and g_h together to both Drell–Yan and SIDIS data. However, the DIS data do not cover large range of Q^2. In addition, the differential cross-sections in SIDIS depend on the fragmentation function, which themselves are not well constrained at the present time. Therefore, in this paper, we will take the parameters $g_{(1,2,3)}$ fitted to the Drell–Yan data to compare to the SIDIS to check if they are consistent with the SIDIS data.

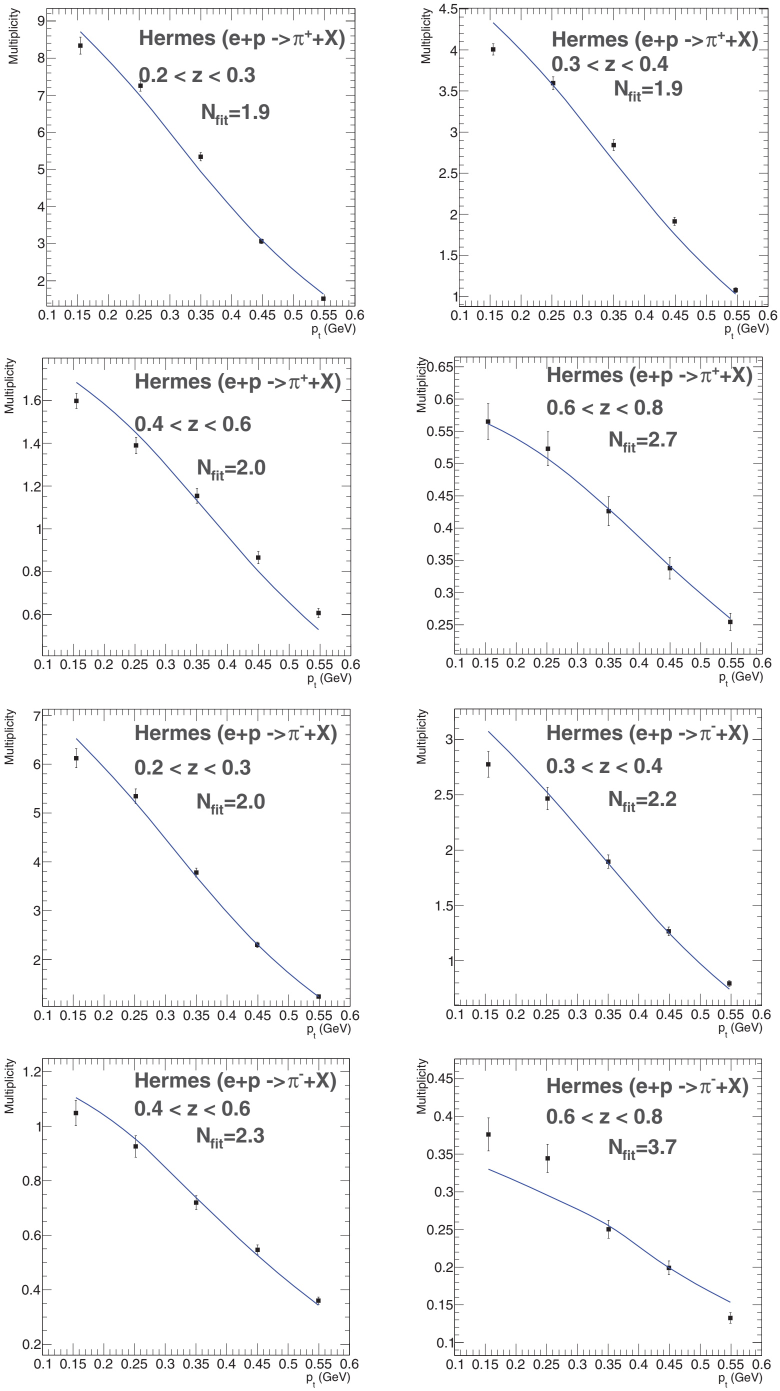

Fig. 9. SIYY-2 fit to the multiplicity distribution as function of transverse momentum in SIDIS data from HERMES Collaboration at $Q^2 = 3.14$ GeV^2.

We will make use of data for charged pion and charged kaon multiplicities taken by the HERMES experiment.[28] The multiplicities $(1/N_{\text{DIS}})dN^H/dzdQ^2$ are defined as the ratio of the semi-inclusive deep-inelastic scattering (SIDIS) cross-section in a certain bin of Q^2 and z, to the totally inclusive DIS rate.[59] The particular value of this data in the global analysis of fragmentation function emerges from the sensitivity to individual quark and antiquark flavors in the fragmentation process which is not accessible from e^-e^+ annihilation processes. The differential cross-section for SIDIS process depends on the hadron fragmentation functions, for which we adopt the parametrization from DSS fit.[59] It has been noticed that the integrated (over transverse momentum) multiplicity distribution from HERMES data shows some tension with the DSS fit. To account for this issue, we allow the normalization of data in each z-bin to float independently when fitting the HERMES data by Eq. (16) to determine the nonperturbative parameter g_h. Our result is shown in Fig. 9, where the fitted normalization factor (N_{fit}) in each z bin is also listed.

In addition, our calculations do not include the Y-term contributions. We will discuss their contributions in the following subsection. The result of the comparison between the SIYY-2 predictions with $g_h = 0.041$ and the SIDIS data from HERMES is shown in Fig. 9.

Figures 1–9 clearly illustrate that we have obtained a universal nonperturbative TMD function which can be used to describe both Drell–Yan lepton pair production and semi-inclusive hadron production in DIS processes in the CSS resummation framework. This is only possible by introducing the new functional form for the nonperturbative function, as given in both Eqs. (14) and (16).

4.1. *Issue with the Y-term in SIDIS for HERMES and COMPASS*

In Fig. 9, we have neglected the contribution from the Y-term, cf. Eq. (6). This is not likely to be a good approximation for describing the HERMES and COMPASS experiments because their data are typically in the relative low Q^2 range. Indeed, we find that the numeric contributions from Y-term are important for both HERMES and COMPASS experiments. One example is shown in Fig. 10 for π^+ production with $z_h = 0.6$–0.8. The dashed curve represents the Y-term contribution, whereas the solid curve stands for the resummation prediction without including the Y-term, as done in Fig. 10. It appears that adding the Y-term contribution will worsen the agreement between the theory prediction and the experimental data. Numerically, Y-term contributions are the same order of magnitudes as compared to the leading power TMD results (resummation without Y-term) with a totally different shape. Including the $Y^{(1)}$ contribution, the resummation formula cannot describe the SIDIS data. Quantitatively, the χ^2 will increase by orders of magnitude as compared to the fit without $Y^{(1)}$ contributions. At smaller z_h, relative dominance of Y-term becomes stronger, as expected.

This is an important observation, and imposes a concern on the interpretation of the SIDIS data from current low energy experiments. Theoretically, it indicates that

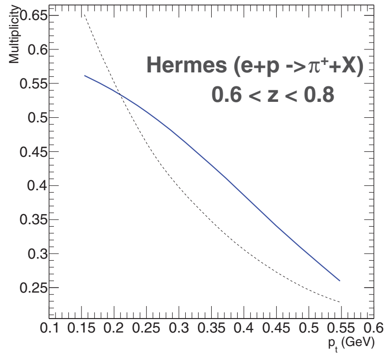

Fig. 10. Y-term contribution (dashed curve) to the multiplicity distribution as a function of transverse momentum, compared to the leading power TMD resummation prediction (as given in Fig. 9). Here, $Q^2 = 3.14\ \text{GeV}^2$.

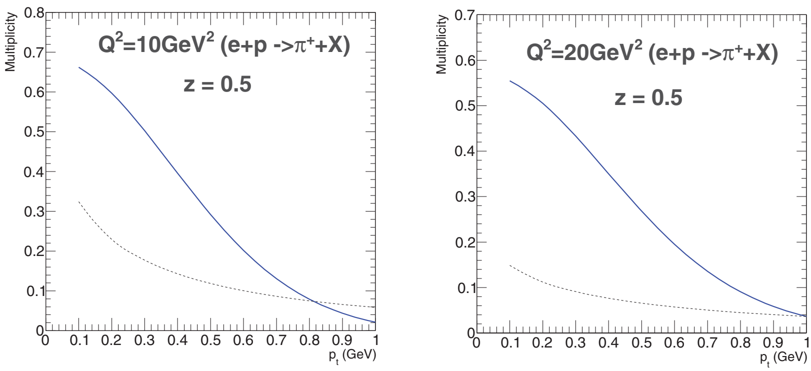

Fig. 11. Comparison between the leading power TMD calculations (solid curves) and the Y-term contributions (dashed curves) for $Q^2 = 10\ \text{GeV}^2$ (left) and $Q^2 = 20\ \text{GeV}^2$ (right) for typical $x_B = 0.1$ and $z_h = 0.5$.

higher order corrections in Y-terms are important and may have to be taken into account to interpret the experimental data. The results in Fig. 10 only include $Y^{(1)}$ contribution. $Y^{(2)}$ for SIDIS has not yet been calculated in the literature. We hope to carry out this computation and come back to this issue in the near future. This may also indicate that we need to take into account higher power corrections in the TMD resummation calculation for describing the SIDIS processes in the relative low Q^2 range. This is similar to what has been discussed in Ref. 47 for higher-twist contributions to the SIDIS, where $\cos\phi$ and $\cos 2\phi$ azimuthal asymmetries in SIDIS process come from higher-twist effects in the TMD framework. Unfortunately, the factorization for higher-twist contribution in the TMD framework is not fully understood at the present.

On the other hand, the consistency between the leading power TMD results and the experimental data from HERMES and COMPASS collaborations (cf. Fig. 9) supports the application of the TMD factorization in the relative low Q^2 range of these two experiments. To further identify the above issue, and demonstrate validity of applying the TMD resummation formalism to describe the data in these experiments, we need more data from SIDIS experiments with large Q^2, where we expect that the Y-term contribution becomes less important. To illustrate this point, we show in Fig. 11 some numerical comparisons. In Fig. 11, we show some numeric results for $Q^2 = 10$, 20 GeV^2. In particular, for $Q^2 = 20$ GeV^2, its contribution is negligible for all $p_\perp$ range of interest. This can be well tested at the planned electron–ion collider.[1]

5. Discussion and Conclusion

In this paper, we have re-analyzed the transverse momentum distribution of the Drell–Yan type of lepton pair production processes in hadronic collisions in the framework of CSS resummation formalism. Our goal is to find a new form for the nonperturbative function which can be used to simultaneously describe the semi-inclusive hadron production in DIS processes (such as from HERMES and COMPASS Collaborations) and all the Drell–Yan type processes (such as W, Z and Drell–Yan pair productions). In Secs. 2 and 3, we argue for a new parametrization form, Eq. (14), for describing Drell–Yan processes. For clarity, we recap our findings, which was named it as the SIYY-1 form, as follows:

$$S_{\rm NP}^{\rm SIYY\text{-}1} = g_1 b^2 + g_2 \ln(b/b_*)\ln(Q/Q_0) + g_3 b^2((x_0/x_1)^\lambda + (x_0/x_2)^\lambda)\,, \qquad (17)$$

where we adopted the b_* description, cf. Eq. (2), with $b_{\rm max} = 1.5$ GeV^{-1}, and have fixed $Q_0 = 1.55$ GeV, $x_0 = 0.01$ and $\lambda = 0.2$ in a global analysis of the low energy Drell–Yan data from E288, E605, R209, and Z boson data from CDF and D0 at the Tevatron (in both Run I and II). In total, we have included 140 data points, fitted with three shape parameters (g_1, g_2, g_3) and seven normalization parameters (one for each data set). The chi-square per degree of freedom is about 1.3, cf. Table 1. The detailed comparison of the fit to the experimental data can be found in Figs. 1–7. Using the result of the fit, we showed in Fig. 8 that the LHC data can also be well-described by the SIYY-1 fit. In Table 1, we also showed the comparison of this new SIYY-1 fit to the SIYY-g fit which adopts the same form as the original BLNY form, but with a different values of $b_{\rm max}$ and Q_0. Though the SIYY-g fit has a smaller χ^2 than SIYY-1 fit for Drell–Yan processes, it failed to describe the SIDIS data.[30,31]

After obtaining the satisfactory fit to the Drell–Yan type data, we proposed to add an additional term to the SIYY-1 form with the z_h dependence for describing the transverse momentum distribution of the semi-inclusive hadron production in DIS processes, cf. Sec. 4. We have named that as the SIYY-2 form, which is

$$S_{\rm NP}^{\rm SIYY\text{-}2} = \frac{g_1}{2} b^2 + g_2 \ln(b/b_*)\ln(Q/Q_0) + g_3 b^2 (x_0/x_B)^\lambda + \frac{g_h}{z_h^2} b^2\,, \qquad (18)$$

where the factor 1/2 associated with the g_1 coefficient is due to the fact that only one hadron beam is involved in the SIDIS processes, in contrast to two hadron beams in the Drell–Yan type processes. Furthermore, the additional g_h term is to parametrize the nonperturbative effect associated with the fragmentation of the final state parton into the observed hadron. z_h represents the momentum fraction of the virtual photon carried by the final state hadron in the SIDIS process. Using the three shape parameters (g_1, g_2, g_3), determined by the global fit to the Drell–Yan data using the SIYY-1 form, as discussed above, we found that the experimental data from HERMES and COMPASS can be well described by the SIYY-2 form with

$$g_h = 0.041\,. \tag{19}$$

Here, we are not performing a fit for the lack of more precise data. Instead, we merely find a value of g_h to show that the proposed SIYY-2 form can describe the existing SIDIS data if only the leading power TMD resummation prediction (defined as the resummation result without including the Y-term) is used for the comparison, cf. Fig. 9. The reason for not including the Y-term in this comparison is that the typical energy scales (Q) of the SIDIS data from HERMES and COMPASS experiments are low, at a few GeV. Hence, the theoretical uncertainties in applying the CSS formalism is not well under control, and the Y-term contribution is expected to be sizable as compared to the leading power TMD resummation contribution. This is illustrated in Fig. 10. Followed by that, we showed in Fig. 11 that for future SIDIS data with a larger Q^2 value, the CSS formalism will provide a better description of the data, where the Y-term contribution is expected to be small in the region that the resummation effect is important, i.e. in the low transverse momentum region.

In conclusion, we have demonstrated that the proposed SIYY-1 and SIYY-2 nonperturbative forms can be used in the CSS resummation formalism to simultaneously describe the Drell–Yan and SIDIS data. Since the Q^2 dependence in the nonperturbative functions is universal among the spin-independent and spin-dependent observables in the hard scattering processes, including Drell–Yan lepton pair production in hadronic collisions, semi-inclusive hadron production in DIS, and di-hadron production in e^+e^- annihilations, we expect that the new nonperturbative functions obtained in this paper shall have broad applications in the analysis of the spin asymmetries in the above-mentioned processes. One particular example is the so-called Sivers single transverse spin asymmetries in SIDIS and Drell–Yan processes. To understand the sign change of the asymmetries in these two processes is one of the important task in hadron physics. With the proposed SIYY-1 and SIYY-2 forms, we could further test the universality property of the TMD formalism.

Acknowledgments

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