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We calculate the Cherenkov radiation from an e^+e^- pair at small separations, as occurs shortly after a pair conversion. The radiation is reduced (compared to that from two independent particles) when the pair separation is smaller than the wavelength of the emitted light. We estimate the reduction in light in large electromagnetic showers, and discuss the implications for detectors that observe Cherenkov radiation from showers in the Earth's atmosphere, as well as in oceans and Antarctic ice.

I. INTRODUCTION

Cherenkov radiation from relativistic particles has been known for over 70 years [1]. However, to date, almost all studies have concentrated on the radiation from individual particles. Frank [2], Eidman [3] and Balazs [4] considered the Cherenkov radiation from electric and magnetic dipoles, but only in the limit of vanishing separations d. Their work was nicely reviewed by Jelley [5].

Several more recent calculations have considered Cherenkov radiation from entire electromagnetic showers, in the coherent or almost coherent limit [6]. The fields from the e^+ and e^- largely cancel, and the bulk of the coherent radiation is due to the net excess of e^- over e^+ (the Askaryan effect) [7]. Hadronic showers produce radiation through the same mechanism [8]. Coherent radiation occurs when the wavelength of the radiation is large compared to the radial extent of the shower; for real materials, this only occurs for radio waves.

Here, we consider another case, the reduction of radiation from slightly-separated oppositely-charged co-moving pairs. This includes e^+e^- pairs produced by photon conversion. When high-energy photons convert to e^+e^- pairs, the pair opening angle is small and the e^+ and e^- separate slowly.

Near the pair, the electric and magnetic fields from the e^+ and e^- must be considered separately. However, for an observer far away from the pair (compared to the pair separation d), the electric and magnetic fields from the e^+ and e^- largely cancel. Cherenkov radiation is produced at a distance of the order of the photon wavelength Λ from the charged particle trajectory. So, for $d < \Lambda$, cancellation reduces the Cherenkov radiation from a pair to below that for two independent particles. For a typical pair opening angle m/k, where k is the photon energy and m the electron mass, without multiple scattering, $\Lambda > d$ for a distance $k\Lambda/m$. For blue light ($\Lambda = 400$ nm) from a 1 TeV pair, the radiation is reduced until the pair travels a distance of 40 cm (neglecting multiple scattering).

In this paper, after a more detailed calculation of the Cherenkov radiation from e^+e^- pairs, we consider coherent optical radiation from pairs that follow realistic trajectories, and from electromagnetic showers. We consider two classes of experiments: underwater/in-ice neutrino observatories and air Cherenkov telescopes.

II. CHERENKOV RADIATION FROM PAIRS

Cherenkov radiation from closely spaced e^+e^- pairs can be derived by extending the derivation for point charges, by replacing a point charge with an oppositely charged, separated pair. We sketch the derivation for radiation from point charges, review previous work on radiation from infinitesimal dipoles, and derive the expression for Cherenkov radiation from a closely-spaced co-moving pair.

We follow the notation and derivation from Ref. [9]. In Fourier space, the charge density ρ and current $\vec{J}$ from a point charge ze propagating with speed v in the x_1 direction can be written as

$$\begin{aligned} \rho(\vec{k},\omega) &= \frac{ze}{2\pi}\delta(\omega - k_1 v) \\ \vec{J}(\vec{k},\omega) &= \vec{v}\rho(\vec{k},\omega) \end{aligned} \tag{1}$$

where $\vec{k}$ is the wave vector and ω the photon energy. This current deposits energy into the medium through electromagnetic interactions. We use Maxwell's equations beyond a radius a around the particle track, where a is comparable to the average atomic separation. Then, by conservation of energy, the Cherenkov radiation power is equal to the the energy flow through a cylinder of this radius, giving

$$\left(\frac{dE}{dx}\right) = -caRe\int_0^\infty B_3^*(\omega)E_1(\omega)d\omega\ . \tag{2}$$

E_1 is the component of $\vec{E}$ parallel to the particle track, and B_3 is the component of $\vec{B}$ in the x_3 direction, evaluated at an impact parameter b at a point with $x_2 = b$, $x_3 = 0$.

Using the wave equations in a dielectric medium and the definition of fields, then integrating over momenta, one finds

$$E_1(\omega) = -\frac{ize\omega}{v^2}\left(\frac{2}{\pi}\right)^{1/2}\left[\frac{1}{\epsilon(\omega)} - \beta^2\right]K_0(\lambda b) \tag{3}$$

where

$$\lambda^2 = \frac{\omega^2}{v^2}[1-\beta^2\epsilon(\omega)]\ .$$

Similarly,

$$\begin{aligned} E_2(\omega) &= \frac{ze}{v}\left(\frac{2}{\pi}\right)\frac{\lambda}{\epsilon(\omega)}K_1(\lambda b) \\ B_3(\omega) &= \epsilon(\omega)\beta E_2(\omega)\ . \end{aligned} \tag{4}$$

Since we are ultimately interested in far-field radiation we take the asymptotic form of the energy deposition at $|\lambda a| \gg 1$. Taking the case $\beta > 1/\sqrt{\epsilon(\omega)}$ for real $\epsilon(\omega)$, λ becomes completely imaginary. The asymptotic contribution of the Bessel functions in the integrand of dE/dx is finite, giving the well-known expression for the Cherenkov radiation

$$\begin{aligned} \left(\frac{dE}{dx}\right) &= \frac{(ze)^2}{c^2} \\ &\times \int_{\epsilon(\omega)>1/\beta^2} \omega\left(1-\frac{1}{\beta^2\epsilon(\omega)}\right)d\omega\ . \end{aligned} \tag{5}$$

Note how a has dropped out [9, Ch. 13]. The derivation of this Cherenkov radiation may be expanded to give the field from an pair.

The radiation from an e^+e^- pairs depends on two parameters: the separation d and the angle between the direction of motion and the orientation of the pair. For relativistic pairs created by photon conversion, the transverse (to the direction of motion) separation is important; the longitudinal separation of a highly relativistic pair can be neglected, due to Lorentz length contraction.

Balazs [4] provided an expression for Cherenkov radiation from an infinitesimal dipole D oriented transverse to its momentum. These fields are well approximated by by a linear Taylor expansion of the corresponding point-charge fields:

$$\begin{aligned} E_1^{(D)}(\omega) &= -d\frac{\partial E_1(\omega)}{\partial x_2} \\ B_3^{(D)}(\omega) &= -d\frac{\partial B_3(\omega)}{\partial x_2}\ , \end{aligned}$$

where d is the effective pair separation, so $D = zed$. Then, following the same steps as in the point-charge case, Balazs finds

$$\begin{aligned} \left(\frac{dE}{dx}\right) &= \frac{1}{2}\frac{D^2}{c^4} \\ &\times \int_{\epsilon(\omega)>1/\beta^2} \epsilon(\omega)\omega^3\left(1-\frac{1}{\beta^2\epsilon(\omega)}\right)^2 d\omega\ . \end{aligned} \tag{6}$$

Jelley [5] also provides an expression for a point dipole oriented parallel to its direction of motion,

$$\begin{aligned} \left(\frac{dE}{dx}\right) &= \frac{D^2}{c^4}(1-\beta^2) \\ &\times \int_{\epsilon(\omega)>1/\beta^2} \omega^3\left(1-\frac{1}{\beta^2\epsilon(\omega)}\right)d\omega\ . \end{aligned}$$

The dE/dx falls to zero for $\beta \to 1$, whereas in the perpendicular case the radiation is finite (for $\epsilon(\omega) > 1$). Again, this is due to Lorentz contraction.

To compute the Cherenkov radiation for finite separations d, let us consider a pair moving in the $+x$ direction. The pair lies entirely in the transverse plane y-z, with the line between them making an angle α with respect to the y-axis. Then, generalizing Eq. (1), the charge density from the pair is

$$\rho(\vec{k},\omega) = \frac{ze}{2\pi}\delta(\omega - k_1 v)\left[e^{-i(k_2 y_+ - k_3 z_+)} - e^{-i(k_2 y_- - i k_3 z_-)}\right].$$

The two charges have positions, relative to the center of mass

$$\begin{aligned} y_+ &= \frac{d}{2}\cos\alpha \qquad z_+ = -\frac{d}{2}\sin\alpha \\ y_- &= -\frac{d}{2}\cos\alpha \qquad z_- = \frac{d}{2}\sin\alpha\ . \end{aligned}$$

The angle α is then the relative azimuth between the line connecting the two charges and the azimuth of observation.

We proceed by analogy to Eq. (3). Eq. (13.60) of Ref. [9] gives the electric field from a single particle. We generalize that equation to a pair and integrate over momenta successively, so that

$$\begin{aligned} E_1(\omega) =& -\frac{2ize\omega}{(2\pi)^{3/2}v^2}\left(\frac{1}{\epsilon(\omega)}-\beta^2\right)\int dk_2 e^{ibk_2} \\ &\times \int dk_3 \frac{e^{-ik_2\frac{d}{2}\cos\alpha}e^{ik_3\frac{d}{2}\sin\alpha} - \text{c.c.}}{\lambda^2+k_2^2+k_3^2} \\ =& -\frac{ize\omega}{(2\pi)^{1/2}v^2}\left(\frac{1}{\epsilon(\omega)}-\beta^2\right) \\ &\times \int dk_2 \frac{e^{-\frac{d}{2}\sin\alpha\sqrt{\lambda^2+k_2^2}}}{\sqrt{\lambda^2+k_2^2}} \\ &\times \left(e^{i\left(b-\frac{d}{2}\cos\alpha\right)k_2} - e^{i\left(b+\frac{d}{2}\cos\alpha\right)k_2}\right). \end{aligned} \tag{7}$$

Then,

$$\begin{aligned} E_1(\omega) =& \frac{-ize\omega}{v^2}\sqrt{\frac{\pi}{2}}\left(\frac{1}{\epsilon(\omega)}-\beta^2\right) \\ &\times [K_0(\lambda b_-) - K_0(\lambda b_+)] \end{aligned} \tag{8}$$

where

$$b_\pm = \sqrt{\frac{d^2}{4}\sin^2\alpha + (b \pm \frac{d}{2}\cos\alpha)^2}\ .$$

As before, we take $|\lambda a| \gg 1$ and $a < b$, so we need only to consider $d \ll b$; there is little interference for $d < b$. Therefore, we can simplify using

$$b_\pm \simeq b \pm \frac{d}{2}\cos\alpha\ .$$

Then, as before, considering completely imaginary λ and $|\lambda a| \gg 1$,

$$E_1(\omega) = \frac{2ze\omega}{c^2}\left(1 - \frac{1}{\beta^2\epsilon(\omega)}\right) \times \sqrt{\frac{i}{|\lambda|}}\frac{e^{i|\lambda|b}}{b}\sin\left[\frac{d}{2}|\lambda|\cos\alpha\right] \tag{9}$$

where we have taken $b_- \simeq b_+ \simeq b$ in the denominator. At $\alpha = \pm\pi/2$ $E_1(\omega) = 0$. The Cherenkov radiation is no longer symmetric about the direction of motion, and vanishes at right angles to the direction of the dipole. As the charge separation increases (or the wavelength decreases), the angular distribution evolves from two wide lobes into a many-lobed structure, as shown in Fig. 1. After integration over even a narrow range of ω or d, the angular distribution becomes an almost-complete disk, with two narrow zeroes remaining at a direction perpendicular to the dipole vector.

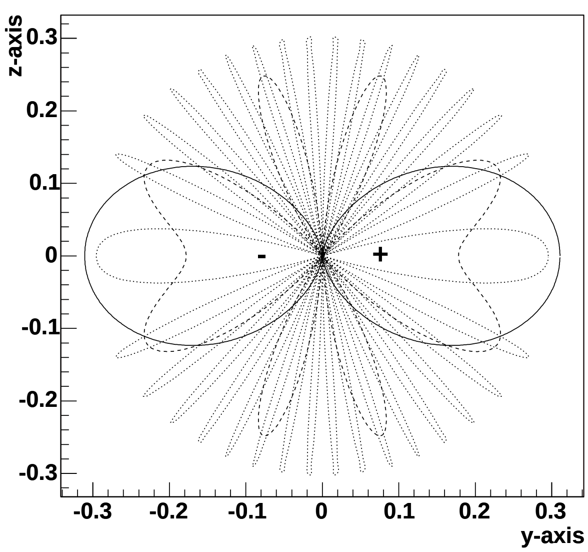

FIG. 1. The azimuthal angular distribution (transverse to the direction of motion) of Cherenkov radiation for 500 nm photons from a pair of charges oriented as shown in the Figure. Distributions are shown for pair separations 100 nm (solid line), 1 μm (dashed line) and 5 μm (dotted line), with $\sqrt{\epsilon(\omega)} = n = 1.3$ and $\beta = 1$.

After assembling the pieces, and averaging over α, we find the generalization of Eq. (5),

$$\left(\frac{dE}{dx}\right) = \frac{(ze)^2}{c^2}\int_{\epsilon(\omega)>1/\beta^2} d\omega \; \omega\left(1 - \frac{1}{\beta\epsilon(\omega)}\right) \times 2\left[1 - J_0(\lambda d)\right] \; . \tag{10}$$

For $\lambda d \ll 1$, this reproduces Eq. (6). For $\lambda d \gg 1$, the dE/dx is twice that expected for an independent particle (Eq. (5)), as expected. The transition is shown in Fig. 2. As the emission wavelength Λ approaches d, the pair spectrum converges to the point-charge spectrum in an oscillatory fashion, characteristic of the Bessel function. For certain values of λd, the radiation exceeds that of two independent charged particles.

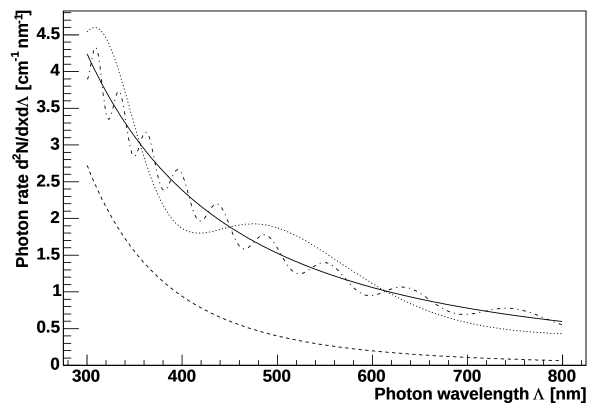

FIG. 2. The spectrum of Cherenkov radiation at $\beta = 1$, $\sqrt{\epsilon(\omega)} = n = 1.3$. Solid line is for e^+e^- with the particles considered independently, and the dashed lines are for pairs treated coherently, with separations 100 nm, 1 μm and 5 μm.

For the remainder of the paper, we assume that media satisfy $\sqrt{\epsilon(\omega)} = n$, where n is independent of frequency. In realistic detection media, any variation of n with frequency is small, and would have little effect on Cherenkov radiation from relativistic particles.

With real e^+e^- pairs, two effects should be considered. Radiation is not emitted instantaneously, but over a period known as the formation time, $\tau = 1/c\lambda$. If the pair separation varies significantly during this time, loss of longitudinal coherence may reduce the radiation. This may be significant when the change in λd is of order one. Where the coherence is important, $c\tau$ and d are comparable, and the e^+ and e^- follow roughly parallel tracks, so d will not change significantly during the formation time.

Second, the Cherenkov radiation produced at a point (x-coordinate) depends on the fields emitted by the charged particles at earlier times, when d may be different than at the point of radiation. For full rigor, these retarded separations should be used in the calculation. Again, this has a negligible effect on the results.

III. RADIATION FROM e^+e^- PAIRS IN SHOWERS

Many experiments study Cherenkov radiation from large electromagnetic showers. The radiation from a shower may be less than would be expected if every particle were treated as independent. We use a simple simulation to consider 300 to 800 nm radiation from electromagnetic showers. This frequency range is typical for photomultiplier based Cherenkov detectors; at longer wavelength, there is little radiation, while shorter wavelength light is absorbed by the glass in the phototube.

We simulated 1000 γ conversions to e^+e^- pairs with total energies from 10^8 to 10^{20} eV. Pairs were produced with the energy partitioned between the e^+ and e^- following the Bethe-Heitler differential cross section $d\sigma \approx E_\pm(1-E_\pm)$, where $E_\pm$ is the electron (or positron energy) [11]. At high energies in dense media (above 10^{16} eV in water or ice), the LPM effect becomes important, and more asymmetric pairs predominate [10]. The pairs are generated with initial opening angle of m/k; the fixed angle is a simplification, but the pair separation is dominated by multiple scattering, so it has little effect on our results.

The e^- and e^+ are tracked through a water medium (with $n=\sqrt{\epsilon}=1.3$) in steps of $0.02X_0$, where X_0 is the radiation length. At each step, the particles multiple-scatter, following a Gaussian approximation [12, Ch. 27]. The particles radiate bremsstrahlung photons, using a simplified model where photon emission follows a Poisson distribution, with mean free path X_0. Although this model has almost no soft bremsstrahlung, soft emission has little effect on Cherenkov radiation, since the electron or positron velocity is only slightly affected.

At each step, we compute the Cherenkov radiation for each pair. They are treated coherently when the $d < 2\Lambda$; at larger separations the particles radiate independently.

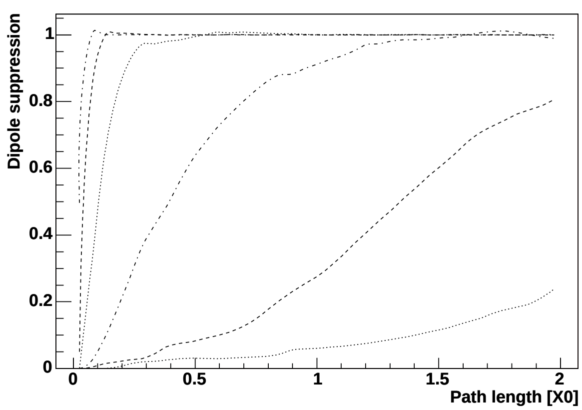

FIG. 3. Average Cherenkov photon emission rate for pairs with energies from 10^{10} (leftmost, dot-dashed curve) to 10^{15} eV (rightmost, solid curve) vs. the distance travelled by the pair in water, relative to emission from two independent particles.

As shown in Fig. 3, the particles in lower energy pairs ($< 10^{10}$ eV) radiate almost independently. In contrast, the radiation from very high energy pairs ($> 10^{15}$ eV) is largely suppressed. The broad excursions slightly above unity occur when $J_0(\lambda d) > 1$ for many of the scattered pairs.

IV. IMPLICATIONS FOR EXPERIMENTS

At least two types of astrophysical observatories depend on Cherenkov radiation. Water and ice based neutrino observatories observe Cherenkov radiation from the charged particles produced in neutrino interactions, and air Cherenkov telescopes look for γ-ray induced electromagnetic showers in the Earth's atmosphere.

Current neutrino observatories can search for electron neutrinos with energies above 50 TeV (for ν_μ, the threshold is much lower) [14]. They use large arrays of photomultiplier tubes to observe the Cherenkov radiation from ν_e induced showers. For water, $n \approx 1.3$, and Fig. 3 shows that $\lambda d < 1$ while the pair travels significant distances. Ice is similar to water, with a slightly lower density; n of ice depends on its structure, and is typically ≈ 1.29 [16].

To quantify the effect of Cherenkov radiation from ν_e interactions, we use a toy model of an electromagnetic shower. The shower evolves through generations, with each generation having twice as many particles as the preceding generation, with half the energy. Each generation evolves over a distance of X_0; other simulations have evolved generations over a shorter distance $(\ln 2)X_0$, leading to a more compact shower [15]. In these showers, most of the particles are produced in the last radiation lengths.

Fig. 4 shows the Cherenkov radiation expected from a model 10^{20} eV shower with coherent Cherenkov radiation (solid line) and in a model where all particles radiate independently (dotted line). This model does not include the LPM effect, so it should be considered only illustrative. The LPM effect lengthens the high-energy (above 10^{17} eV) portion of the shower. By spreading out the shower longitudinally, the LPM effect will give the electrons and positrons more time to separate, and so will somewhat lessen the difference between the two results. However, it is clear from Fig. 4 that coherence has a significant effect for the first $\approx 22X_0$. Since the front of the shower contains relatively few particles, it will not affect the measured energy. However, by suppressing radiation from the front of the shower, it could affect the measurement of the shower development and the reconstruction of the shower position. This effect can only be assessed with a detailed model of a particular experiment.

Atmospheric Cherenkov telescopes like the Whipple observatory study astrophysical γ-rays with energies from 100 GeV to 10 TeV. These telescopes observe Cherenkov radiation from pairs in the upper atmosphere; for a 1 TeV shower, the maximum particle density occurs at an altitude of 8 km above sea level (asl) [13], where the density is about 1/3 that at sea level. Since $n-1$ depends linearly on the density, at 8 km asl $n-1 \approx 1\times10^{-4}$, so for 500 nm photons radiated from ultra-relativistic particles, $\lambda d < 1$ only for $d < 6$ μm. In this low-density medium, the effect of the pair opening angle is significant and multiple scattering is less important. Pairs with $k < 1$ TeV will separate by 30 μm in a distance less than 30 meters;

at 8 km asl, this is 3% of a radiation length. This distance is too short to affect the radiation pattern from the shower.

Cherenkov radiation is also studied in lead-glass block calorimetry, and in Cherenkov counters for particle identification; their response to photon conversions may be affected by this coherence.

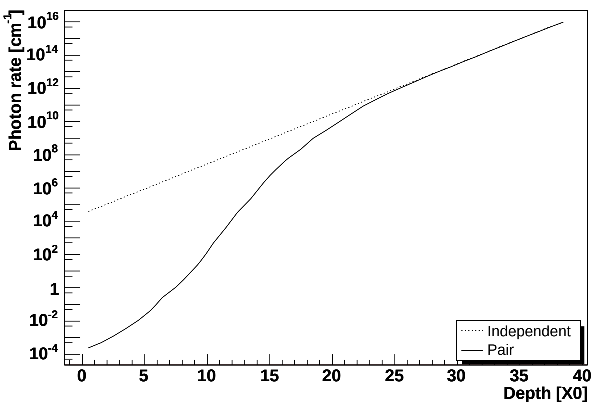

FIG. 4. Cherenkov radiation from a 10^{20} eV shower in water, using the Heitler toy model, versus shower depth (smoothed). The two curves compare the radiation for e^+e^- calculated as independent particles and as coherent pairs.

Although this calculation applies for Cherenkov radiation, a similar effect should occur for e^+e^- pair energy loss through dE/dx. When d is smaller than the typical impact parameter to cause ionization of a medium, the electric fields from the e^+ and e^- will largely cancel, and the energy loss will be reduced.

V. CONCLUSION

We have calculated the Cherenkov radiation from e^+e^- pairs as a function of the pair separation d. When $d^2 < v^2/(\omega^2[1-\beta^2\epsilon(\omega)])$, the radiation is suppressed compared to that from two independent particles.

This suppression affects the radiation from electromagnetic showers in dense media. Although the total radiation from a shower is not affected, emission from the front part of the shower is greatly reduced; this will affect studies of the shower development, and may affect measurements of the position of the shower.

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