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DERIVATION OF THE RADIAL DEPENDENCE OF THE QUADRUPOLE

FORCE FROM A WOODS-SAXON POTENTIAL[†]

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Abstract

For use in calculation of collective quadrupole phenomena we construct

a modified quadrupole force, whose radial dependence is related to that of a

Woods-Saxon well. Matrix elements and self-consistent interaction

strengths are compared to those of the usual quadrupole force, which is based

on a harmonic oscillator potential.

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1. Introduction

In order to study the appropriateness of the pairing-plus-quadrupole

interaction for the calculation of collective quadrupole levels, e.g. in the

adiabatic approximation we here try to remedy the obvious drawbacks of

the usual quadrupole interaction, which were pointed out by Baranger and

Kumar[2,7].

In sect. 2 we use a self-consistency argument to relate the radial

form factor of an arbitrary multipole interaction to a given average potential.

In this way the form of the usual quadrupole force follows from the harmonic

oscillator potential, whereas the self-consistency argument leading to the

magnitude of the interaction is meaningless and has to be replaced by an

estimate using a realistic density. Assuming instead a Woods-Saxon type

average potential, one obtains a really self-consistent quadrupole interaction.

We extend the argument to include also a spin-orbit term in the average

potential.

In sect. 3 this modified quadrupole interaction is compared with the

usual one, and the renormalization problems arising in numerical calculations

are discussed.

2. Self-consistent Multipole Force and Average Potential

Let us first consider a separable multipole interaction in general,

$$V(\underline{r}_1\tau_1,\underline{r}_2\tau_2) = -\chi\, P_{\tau_1}(r_1)\, P_{\tau_2}(r_2) \sum_\mu Y^*_{\lambda\mu}(\theta_1\phi_1)\, Y_{\lambda\mu}(\theta_2\phi_2) \quad , \quad (2.1)$$

where $(r\theta\phi)$ are position coordinates and τ is an isospin label giving the 3-component ($\tau = -1$ for protons). Assuming the nuclear average potential in which a proton or a neutron will be moving to arise entirely from the two-body interaction, it has to satisfy

$$V_\tau(\underline{r}) = \sum_{\tau'} \int V(\underline{r}\tau,\underline{r}'\tau')\, \rho_{\tau'}(\underline{r}')\, d\underline{r}' \qquad (2.2)$$

where $\rho_{\pm 1}$ is the neutron (proton) part of the density function

$$\rho(\underline{r}) = \sum_\tau \rho_\tau(\underline{r}) \quad . \qquad (2.3)$$

Note that although we do not assume identity between the proton and neutron average fields, we have by (2.1) in fact assumed that the two-body inter-action is isospin independent (which is the conventional choice in the type of application we are thinking about)[1-3].

We now impose a deformation of multipole order λ on the system, assuming the surface to be of the form

$$r = r'\, (1 + \sum_\mu \alpha_{\lambda\mu}\, Y^*_{\lambda\mu}(\theta\phi)\,) \qquad (2.4)$$

with r' equal to the spherical radius R_o. Provided now that the interaction

(2.1) is of sufficiently short range, the surface will adjust itself completely

to the shape of the average field, enabling us to relate the equipotential

surfaces of the deformed average potential V_τ to the spherical average

potential V_τ^{sph} which was present before the interaction was switched on

(assuming $\lambda \neq 0$),

$$V_\tau(r\theta\phi) = V_\tau^{sph}(r') \tag{2.5}$$

where the relation between $(r\theta\phi)$ and r' must be given by (2.4). If the

deformation implied by applying the residual interaction is small, we may

retain only the leading order term in a Taylor expansion of $V_\tau^{sph}(r')$

around the actual radius r at a given direction $(\theta\phi)$:

$$V_\tau^{sph}(r') = V_\tau^{sph}(r) - r \frac{\partial V_\tau^{sph}(r)}{\partial r} \sum_\mu \alpha_{\lambda\mu} Y_{\lambda\mu}^*(\theta\phi) \quad . \tag{2.6}$$

Comparing (2.6) to (2.1) and (2.2) we get for $\lambda \neq 0$

$$P_\tau(r) = r \frac{\partial V_\tau^{sph}(r)}{\partial r} \quad , \tag{2.7}$$

$$\chi \int \sum_{\tau'} P_{\tau'}(r') Y_{\lambda\mu}(\theta'\phi') \rho_{\tau'}(\underline{r}') d\underline{r}' = \alpha_{\lambda\mu} \quad , \tag{2.8}$$

which is unique up to the choice of a scale factor. Eq. (2.7) tells that

$P_\tau(r)$ does not depend on λ.

When the deformation parameters $\alpha_{\lambda\mu}$ correspond to equilibrium, there is proportionality between the average field V_τ and the density distribution ρ_τ,

$$V_\tau(\underline{r}) = c_\tau \, \rho_\tau(\underline{r}) \quad , \tag{2.9}$$

implying in analogy to (2.6)

$$\rho_\tau(\underline{r}) = \rho_\tau^{sph}(r) - r \, \frac{\partial \rho_\tau^{sph}(r)}{\partial r} \sum_\mu \alpha_{\lambda\mu} \, Y_{\lambda\mu}^*(\theta\phi) \quad . \tag{2.10}$$

Inserting (2.10) into (2.8) we get

$$\chi = - \left[\int \sum_\tau r^4 \, \frac{\partial V_\tau^{sph}(r)}{\partial r} \, \frac{\partial \rho_\tau^{sph}(r)}{\partial r} \, dr \right]^{-1} \tag{2.11}$$

which is called the self-consistent value of the coupling strength χ [4]). The proportionality factors c_τ can be found by integrating (2.9),

$$c_n = - \frac{4\pi}{N} \int V_n^{sph}(r) \, r^2 \, dr \quad ,$$

$$c_p = - \frac{4\pi}{Z} \int V_p^{sph}(r) \, r^2 \, dr \quad . \tag{2.12}$$

2.1. THE USUAL QUADRUPOLE FORCE (UQF)

The standard choice for the spherical average potential[1] has been

the harmonic oscillator potential

$$V^{sph}(r) = \frac{1}{2} \hbar\omega_o \left(\frac{r}{b}\right)^2 , \qquad (2.13)$$

which leads to a quadrupole force (2.1) with $\lambda = 2$ and the radial form factor

(2.7) given by

$$P(r) = \hbar\omega_o \left(\frac{r}{b}\right)^2 . \qquad (2.14)$$

As it is, however, nonsense to assume the proportionality (2.9) between (2.14)

and the density, one usually rewrites (2.11) as a sum over all the particles

in the system

$$\chi^{-1} = \frac{1}{4\pi} \left\langle \sum_k \frac{1}{r_k^2} \frac{\partial}{\partial r_k} \left(r_k^4 \frac{\partial V^{sph}(r)}{\partial r_k} \right) \right\rangle \qquad (2.15)$$

and assume

$$\left\langle \sum_k r_k^2 \right\rangle = \frac{3}{5} A^{5/3} r_o^2 , \qquad (2.16)$$

which implies

$$\chi^{-1} = \frac{3}{4\pi} \frac{\hbar\omega_o r_o^2}{b^2} A^{5/3} \qquad (2.17)$$

where $R_0 = r_o A^{1/3}$ is the nuclear radius.

2.2. THE MODIFIED QUADRUPOLE FORCE (MQF)

        The modification of the quadrupole force which we are suggesting

assumes the average potential to be of Woods-Saxon type[5]), including a

Coulomb part and a spin-orbit part. The isospin dependence is chosen in

accordance with the symmetry energy contribution to the nuclear binding

energy

$$V_\tau^{sph}(r) = \frac{1}{2}(1 - \tau)H_C(r) + W_\tau\{f(r) - \frac{1}{2}v_{so}\lambda^2(\underline{p} \times \underline{s}) \cdot \underline{\nabla} f(r)\} \quad (2.18)$$

where

$$f(r) = \left(1 + \exp\left\{\frac{r-R_o}{a}\right\}\right)^{-1} \quad , \quad\quad (2.19)$$

and

$$H_C(r) = zZe^2\left[\frac{1}{r}\delta(r \geqslant R_c) + \frac{1}{R_c}\left(\frac{3}{2} - \frac{1}{2}\left(\frac{r}{R_c}\right)^2\right)\delta(r < R_c)\right] \quad (2.20)$$

and

$$W_\tau = V_0 + V_1\frac{\underline{t} \cdot \underline{T}}{A} \quad , \quad\quad (2.21)$$

$$\lambda = \frac{\hbar}{Mc}(1 + A^{-1}) \quad . \quad\quad (2.22)$$

Here $\underline{t}$ is the isospin vector of the nucleon and $\underline{T}$ that of the remaining

A nucleons which give rise to the average potential. The momentum and spin

vectors of the nucleon are denoted $\underline{p}$ and $\underline{s}$, and the characteristic radii

of the nuclear mass and charge distributions are denoted R_o and R_c.

The factor $(\underline{p} \times \underline{x}) \cdot \underline{V}$ in (2.18) reduces to $\frac{1}{r} \underline{\ell} \cdot \underline{s} \frac{\partial}{\partial r}$ as long as the system remains spherical. The presence of the spin-orbit term makes the form factor (2.7) state dependent, and the derivation given above in the main sect. 2 is no longer valid.

We shall for a while restrict ourselves to the case of axially symmetric deformations, following the argumentation of Chepurnov and Nemirovksy[6]), who point out that the extra components of the spin-orbit term in (2.18)[†]), which in a non-axially symmetric case would contribute to single-particle matrix elements, presumably are small. In terms of the unit vector $\underline{n}_\theta =$ $(\cos\theta \cos\phi, \cos\theta \sin\phi, -\sin\theta)$ perpendicular to $\underline{r}$, we can write the deformed field as

$$V_\tau(\underline{r}) = V_\tau^{sph}(r) - \beta Y_{20}(\theta\phi)\, r\, \frac{\partial \tilde{V}_\tau^{sph}(r)}{\partial r} + W_\tau\, v_{so}\, \lambda^2\, \beta$$

$$(2.23)$$

$$\times \left[\frac{1}{2}\, Y_{20}(\theta\phi)\, \frac{\partial^2 f(r)}{\partial r^2}\, \underline{\ell} \cdot \underline{s} - \frac{3}{2}\sqrt{\frac{5}{4\pi}}\, \cos\theta\, \sin\theta (\underline{n}_\theta \times \underline{p}) \cdot \underline{s}\, \frac{\partial f(r)}{\partial r} \right]$$

where $\tilde{V}_\tau^{sph}$ is the spin-orbit independent part of V_τ^{sph}. The structure of the last term in (2.23) may better be understood by rewriting it in the form

$$\cos\theta\, \sin\theta (\underline{n}_\theta \times \underline{p}) \cdot \underline{s} = \frac{\sqrt{4\pi}}{3r}\left(\frac{2}{\sqrt{5}} Y_{20}(\theta\phi) + 1 \right) - \sqrt{\frac{4\pi}{3}}\, Y_{10}(\theta\phi)(\underline{p} \times \underline{s})_3 \cdot$$

$$(2.24)$$

[†]) The 3-components in a decomposition along the axes $\underline{n}_r$, $\underline{n}_\theta$ and $\underline{n}_\phi$.

The 3-axis is chosen along the symmetry axis, and the appearance in (2.24) of a non-invariant term just reflects the fact that with the assumption of axial symmetry also the 3-component of $\underline{p} \times \underline{x}$ will be a constant of motion.

The procedure for extracting the quadrupole form factor analogous to (2.7) is now to pick up all terms proportional to $Y_{20}(\theta\phi)$, which gives

$$P_\tau(r) = \frac{1}{2}(1 - \tau)\, r\, \frac{\partial Hc}{\partial r} + W_\tau \left\{ r\, \frac{\partial f(r)}{\partial r} - \frac{1}{2}\, v_{so} \lambda^2\, \underline{\ell} \cdot \underline{s} \right.$$

$$\left. \times \left(\frac{\partial^2 f(r)}{\partial r^2} - \frac{1}{r}\, \frac{\partial f(r)}{\partial r} \right) \right. \tag{2.25}$$

As only terms which are totally invariant remain, we shall assume that (2.25) is correct also for non-axial symmetric systems, i.e., $P_\tau(r)$ can be used in the residual interaction (2.1). It should be noted that one would also arrive at the expression (2.25) if in (2.18) one replaced $(\underline{p} \times \underline{s}) \cdot \underline{\nabla}$ by $\frac{1}{r}\, \underline{\ell} \cdot \underline{s}\, \frac{\partial}{\partial r}$ and treated $(\underline{\ell} \cdot \underline{s})$ as an r-independent constant. Because of the presence of the $(\underline{\ell} \cdot \underline{s})$-term, $P_\tau(r)$ is no longer a state-independent form factor, and the significant quantities will be the reduced single-particle matrix elements,

$$\langle i \| P_\tau(r) Y_2(\theta\phi) \| j \rangle \; , \tag{2.26}$$

evaluated between the Woods-Saxon eigenstates of the spherical average potential (2.18). These matrix elements, which are the basic ingredients of a microscopic calculation using an interaction of the type (2.1), will in sect. 3 be compared to those of the UQF considered in subsect. 2.1, and the importance of using consistent wave functions in the evaluation of the matrix elements (2.26) will be stressed.

3. Comparison between UQF and MQF

Expressing the single-particle wave functions[†]) in the helicity representation[4])

$$\Big| n\ell jm \Big\rangle = \frac{1}{r} u_{n\ell j}(r) \left(\frac{2j+1}{16\pi^2} \right)^{1/2} \sum_h \alpha(\ell jh) \mathcal{D}^j_{mh}(\Omega) \chi_h (-)^{n+1} \quad (3.1)$$

one easily finds the reduced matrix elements of the quadrupole field (2.26)

$$\langle n\ell j \| P_\tau Y_2 \| n'\ell'j' \rangle = \sqrt{\frac{5}{4\pi}} \; (-)^{j-\frac{1}{2}+n+n'} \; \hat{j} \, \hat{j}'$$

$$\times \begin{pmatrix} j' & j & 2 \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{pmatrix} \langle n\ell j | P_\tau | n\ell j \rangle \qquad (3.2)$$

where $\hat{j} = (2j+1)^{1/2}$ and $\begin{pmatrix} abc \\ \alpha\beta\gamma \end{pmatrix}$ is a 3-j symbol. Fig. 1 gives for comparison the harmonic oscillator potential and the spin-orbit independent part of the Woods-Saxon potentials which are used in this section to illustrate the differences. They correspond to pure neutron ($\tau = 1$) wells for ^{120}Sn, using for the Woods-Saxon potential the parameters of table 1, assuming $R_o = R_c = r_o A^{1/3}$. The parameters of the harmonic oscillator potentials are then also fixed, since

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[†]) Phase convention: $u_{n\ell j}$ positive for $r \to 0$.

$$b = r_o \left(\frac{4}{5} \left(\frac{2A}{3} \right)^{1/3} \right)^{1/2} \tag{3.3}$$

and

$$\omega_o = \frac{\hbar}{Mb^2} \quad . \tag{3.4}$$

Fig. 2 gives a few examples of the differences in single-particle wave functions for the two potentials, in particular, the difference in range for wave functions of levels close to the top of the Woods-Saxon potential. The problem of unbound states will be considered later in this section.

In fig. 3 the radial functions $P(r)$ are given, for the MQF case only the spin-orbit independent part. However, the radial function multiplying the state-dependent matrix element of $\underline{\ell} \cdot \underline{s}$ is also indicated. It is seen that the MQF concentrates its effect at the surface of the nucleus[†]), and that the spin-orbit part tends to let the interaction take place a little outside or inside the surface, according to which one of the two spin-orbit partner levels is involved.

In fig. 4 we compare reduced matrix elements of the type (3.2) for MQF and UQF, using in both cases either Woods-Saxon or harmonic oscillator wave functions in the evaluation. We can thereby test whether a restricted improvement of either only the wave functions or only the force would produce

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[†])In this respect the MQF is much closer to the surface delta interaction[8]) than the UQF. Actually the radial dependence of the MQF equals that of the surface delta interaction for the diffuseness $\underline{a}$ going to zero and no spin-orbit potential. Under these approximations the surface delta interaction simply equals the sum of the self-consistent multipole forces over all multipole orders.

the main difference between the two "natural" types of matrix elements, which use consistent potential and wave function. This is seen not to be the case. In fact, the two steps of improvement go in opposite directions, and it is remarkable that the consistent use of MQF and Woods-Saxon wave functions give matrix elements which are much closer to those of UQF with harmonic oscillator wave functions than one might expect looking at the major differences in the potentials and wave functions exhibited in figs. 1-3. In most cases the differences in magnitude are only a few percent, and only for one significant matrix element there is a difference of 35%. One important feature is that the matrix elements which were zero because of the selection rules associated with the harmonic oscillator are non-zero for the MQF, for instance the matrix element between $1p_{3/2}$ and $2f_{7/2}$ is 12.8 MeV[†]). One observes through the shell structure an oscillatory behaviour of the difference between the MQF and the UQF matrix elements. For instance the matrix elements between levels just above the Fermi level $(2d_{3/2})$ are largest for the MQF, while those between levels just below the Fermi level are largest for the UQF. The gain by using MQF for levels close to the binding limit can directly be understood from figs. 1 and 3.

[†]) This is, however, still a rather small matrix element. As there was a free choice of scale for $P(r)$, the significant number is χ times the squared single particle matrix element, which in the case mentioned is only 4% of the same quantity for the strongest transition.

Fig. 5 is concerned with the effect of the spin-orbit part in the

MQF, comparing the relevant matrix elements of fig. 4 with matrix elements

calculated with the $\underline{\ell} \cdot \underline{s}$ part left out of the MQF (yet the Woods-Saxon

wave functions were not altered). The total difference conform with the sign

change of $\langle \underline{\ell} \cdot \underline{s} \rangle$ going from one spin-orbit partner to the other, but the

absolute differences are extremely small, supporting the argumentation for

the approximate treatment of the deformed spin-orbit potential presented in

sect. 2.

3.1.   THE SELF-CONSISTENT COUPLING STRENGTH

Formally, the self-consistent strength of the MQF may be expressed in analogy to (2.11)

$$\chi = - \left[\int \sum_\tau \frac{1}{c_\tau} (P_\tau(r))^2 \; r^2 \; dr \right]^{-1} \tag{3.5}$$

where the c_τ is obtained by inserting (2.18) into (2.12). However, due to the spin-dependence of P_τ (2.25), it is convenient to use the equivalent form (2.15) and second quantization

$$\chi = 4\pi \left[\sum_{\kappa,\tau} \left\langle \kappa \left| \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^3 P_\tau(r) \right) \right| \kappa \right\rangle \langle a_\kappa^+ a_\kappa \rangle \right]^{-1} \tag{3.6}$$

where the expectation value of $a^+ a$ has to be taken in a state with density distributions $\rho_\tau^{sph}(r)$ corresponding to the spherical Woods-Saxon potential. From the derivation of the MQF this is the state of the system when the quadrupole force is absent, i.e., the state in which particles occupy the lowest available Woods-Saxon orbits. Assuming these to be the states labelled $\kappa = (n\ell jm)$ in (3.6) we obtain

$$\chi = 4\pi \left[\sum_{\tau,\kappa \; occupied} \left\langle \kappa \left| \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^3 P_\tau(r) \right) \right| \kappa \right\rangle \right]^{-1}$$

$$= 4\pi \left[\sum_{i \leqslant i_F, \tau} d_i \left\langle i \left| \frac{\partial}{\partial r} \left(r^3 P_\tau^i(r) \right) \right| i \right\rangle_{radial} \right]^{-1} , \tag{3.7}$$

where d_i is the degree of filling for orbit $i = (n\ell j)$ $(d_i = 2j_i + 1$ except

for $i = i_F)$ and the radial matrix elements are to be calculated with the

radial Woods-Saxon wave functions $u_{n\ell j}(r)$ of (3.1). The label i on P_τ^i

signifies that $\underline{\ell} \cdot \underline{s}$ has to be replaced by 0.5 $[j_i(j_i + 1) - \ell_i(\ell_i + 1) -$

$0.75]$ in accordance with the discussion in sect. 2.

In order to understand the importance of various contributions of χ,

we have also considered two further approximations. One is to neglect the

spin-orbit part of P_τ, in which case (3.5) is a simple integral. If we

further neglect the Coulomb energy and the symmetry energy, we obtain a

strength χ', which only depends on A and thus may be compared with χ_{UQF}

of eq. (2.17). Carrying the integration only up to R_o, as it was done in

(2.16), we can evaluate the integral explicitly, obtaining

$$\chi'_{MQF} = \frac{16\pi a}{3V_o r_o} A^{-4/3} \approx 0.174 A^{-4/3} MeV^{-1} \quad . \tag{3.8}$$

The numerical value corresponds to the parameters listed in table 1. Using

the same parameters (which imply $b = A^{1/6}$ and $\hbar\omega_o = 59.76 r_o^{-2} A^{-1/3}$ MeV)

we get[†]) from (2.17)

$$\chi_{UQF} = 0.0702 A^{-1} MeV^{-1} \quad . \tag{3.9}$$

The two self-consistent strengths χ'_{MQF} and χ_{UQF} are compared in

fig. 6. Further the full χ_{MQF} has been calculated in a number of cases,

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[†])Note that the present definition of χ differs from the usual one by a

factor of $(\hbar\omega_o)^2$.

using eq. (3.7). The figure shows the general trends for nuclei close to the

β-stability line. The inclusion of Coulomb effects and spin-orbit terms

affects χ_{MQF} only by a few percent and the main reason for the difference

between χ_{MQF} and χ'_{MQF} is the symmetry energy, which for regions with large

neutron excess makes χ_{MQF} increase slower than $A^{-4/3}$. For the same reason

χ_{MQF} is almost constant through a series of isotopes. The gross picture is

thus a rough $A^{-4/3}$ dependence for nuclei at the β-stability line and no

A-dependence for fixed Z, in contrast to the overall A^{-1} dependence of χ_{UQF}.

It is not very easy to test the predicted A-dependence of χ against

experimental evidence. This is due to the renormalization effects which have

to be taken into account when truncating the single-particle basis. The UQF

requires the discrete but infinite harmonic oscillator basis and the MQF

requires continuum states in addition to the Woods-Saxon bound states. If

enough levels are included to make the effect of the remaining levels structure-

less, a single renormalization factor will multiply the self-consistent χ.

This factor will however be A-dependent for convenient choices of the

truncation and a further approximation in the type of many-body technique

employed may be more or less justified for different nuclei. For instance a

hypothetical RPA calculation using untruncated bases and the self-consistent

χ will presumably give reasonable fits to 2+ energies in regions with small

quadrupole matrix elements and large gaps in the single-particle spectrum,

but poor fits when leaving these regions.

Another possible calculation aimed at deformed nuclei would use

truncated bases and calculate the static deformation as function of χ by a

Hartree-Fock or Hartree-Bogoliubov method, iterating until the potential and

wave function deformation become equal. This procedure will give the correct

self-consistent χ corresponding to the truncated configuration space,

provided that dynamical effects can be neglected. This condition is similar

to the one we used to find the unrenormalized strength, but still not

equivalent since the proportionality between field and density may hold in

the complete configuration space but not in the truncated one.

We have performed such calculations for ^{150}Sm and ^{192}Os, including

all bound Woods-Saxon states above $2p_{3/2}$ for protons and above $2d_{5/2}$ for

neutrons. We thereby obtained quadrupole strengths related to the self-

consistent χ of eq. (2.7) by $\chi = 1.77\ \chi_{s.c.}$ for ^{150}Sm and $\chi = 1.60\ \chi_{s.c.}$

for ^{192}Os.

The MQF self-consistent χ values are supported by the comments

of Baranger and Kumar[7]) concerning the inappropriateness of the A^{-1}

dependence and in particular favouring a slower A-dependence within major

shells (or for a series of isotopes), which they had to introduce in their

UQF calculations[2]).

A comparison of the self-consistent χ with values which reproduce

certain experimental data is of course further affected by the possible

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insufficiency of the quadrupole force.

3.2.   THE ROLE OF THE CONTINUUM

In conventional calculations with pairing and quadrupole forces one
confines the number of single-particle levels employed to those close to
the Fermi levels (usually not more than three major shells for each kind of
particle). With the UQF one can hardly go further, and it may well be that even
the calculations which do include three shells are unreliable[7]) because of the
obvious unrealistic features of the interaction implied between distant
levels. It is also in the spirit of the pairing force with constant matrix
elements that the non-zero strength must only apply within a certain subspace,
and that all other matrix elements are zero. An enlargement of the number of
configurations is on the other hand significant in connection with the UQF, which
has infinite range. It has been suggested that already the $\Delta N = 2$ matrix
elements are questionable, and the type of correlations implied by the inclusion
of more than two major shells seems to be quite unrealistic[7]). Since the MQF
has a radial dependence peaked around the nuclear surface, one would guess that
the inclusion of more levels, either far below the Fermi level or above it,
reaching up in the continuum, would not change the correlations much, so that a
truncation of the configuration space would be justified. However, the Woods-
Saxon wave functions do have tails reaching large radii, and further the peaking
of the interaction at the surface does not apply to the Coulomb part, which in
fact has a very long range. It is seen from fig. 4, that matrix elements involv-
ing levels far below the Fermi level are in general not smaller than those of
the UQF, and although the matrix elements involving levels close to the
continuum are considerably reduced, it is not clear whether this tendency will
not be compensated by the increased level density in the continuum. In order
to estimate the magnitudes of matrix elements of the MQF involving continuum
states, let us evaluate

$$\langle \iota | \frac{5}{4\pi} P(r)^2 | \iota \rangle = \sum_{\substack{\kappa \\ M=m_\iota - m_\kappa}} \langle \iota | P(r) Y_{2M}(\hat{r}) | \kappa \rangle^2 \tag{3.10}$$

$$= \sum_{(n\ell j)\text{bound}} (2j_\iota + 1)^{-1} \langle n_\iota \ell_\iota j_\iota \| P(r) Y_2(\hat{r}) \| n\ell j \rangle^2$$

$$+ \sum_{(n\ell j)\text{unbound}} (2j_\iota + 1)^{-1} \langle n_\iota \ell_\iota j_\iota \| P(r) Y_2(\hat{r}) \| n\ell j \rangle^2$$

for a level ι close to the Fermi level, leaving as unknown the sum involving unbound states. For the case of neutrons in ^{120}Sn considered earlier we find for $\iota = (1h_{11/2})$ that $\langle \iota | \frac{5}{4\pi} P^2 | \iota \rangle = 33.7$, $\sum_{\text{bound}} = 14.1$ and thus $\sum_{\text{unbound}} = 19.6$, which is 1.4 times the bound state contribution. This may be compared to the UQF, for which we get $\sum_{\text{unbound}} = 21 - 10 = 11$, i.e., 1.1 times the "bound" state contribution. The quoted numbers are of course not directly related to the renormalization factors multiplying $\chi_{s.c.}$, but the indication is that the renormalization will be larger when using MQF than for UQF. The relative larger importance of the continuum states for the MQF as compared to the UQF is in the example given here due to the selection rules valid for harmonic oscillator states and not for Woods-Saxon states. Considering a more physical sum rule than (3.10), one which contains an energy weighting similar to that of a strength function (the energy being in the denominator), it appears reasonable that the relative importance of unbound levels in MQF will be increased further since such levels are abundant at low unbound energies, whereas the corresponding levels in an

oscillator well are separated by distances comparable to those between the
lower levels.

We thus conclude, that although the Woods-Saxon basis and the MQF
matrix elements appear much more physical than the harmonic oscillator basis
and the UQF matrix elements, the problems connected with renormalization
are just as pertinent and one will in each concrete calculation have to
consider whether the influence of the continuum can be simulated by a mere
change of force strength χ or not.

Although the magnitudes of bound state matrix elements are not very
different in MQF and UQF, there can hardly be any similarity for matrix
elements involving continuum (high-lying harmonic oscillator) states, so
should an investigation of such contributions become necessary, only the
MQF provides a physically acceptable basis.

4. Conclusions

We have proposed a modification of the quadrupole interaction, which is based on a more consistent foundation as regards the choice of radial form factor. This improvement is dictated by the need to perform various types of many-body calculations, which uses basic configurations corresponding to several major shells, for which the quadrupole matrix elements derived from the usual quadrupole force suffer from conceptual inconsistency.

It remains for reliable calculations of nuclear properties to decide, whether the modified quadrupole force can provide a satisfactory description of the currently available amount of experimental knowledge on collective quadrupole type phenomena, in the same way as the usual quadrupole force in simplified many-body calculations was able to reproduce the gross features of the information available some years ago.

We would like to express our thanks and appreciation to Professors A. Bohr and B. Mottelson, who originally pointed out the importance of consider- ing the proportionality of density and quadrupole field, and with whom we enjoyed enlightening discussions.

Table 1

Parameters of the Woods-Saxon well. We used $R_o = R_c = r_o A^{1/3}$.

r_o	a	V_o	V_1	v_{so}
1.27 fm	0.67 fm	-51.0 MeV	132.4 MeV	32.0

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Figure Captions

Fig. 1.   The spin-orbit independent neutron Woods-Saxon well for ^{120}Sn

compared with the corresponding (cfr. sect 3) harmonic oscillator well.

The numbering of the Woods-Saxon single particle states will be used in

following figures. Their spins will be given in the caption to fig. 4.

Fig. 2.   Examples of s.p. wave functions for Woods-Saxon (full) and harmonic

oscillator (dashed) wells. The circled numbers correspond to fig. 1.

Fig. 3.   The spin-orbit independent part of the radial function P(r) of

the modified quadrupole force (MQF) is shown as a solid line (a). The same

for the usual quadrupole force (UQF) is shown by a dashed line (c), and

the dotted line (b) gives the spin-orbit contribution to P(r) in MQF

divided by $\underline{\ell} \cdot \underline{s}$ (i.e. the factor multiplying this operator).

Fig. 4.   Reduced single particle matrix elements of the MQF or UQF

quadrupole fields using harmonic oscillator (HO) or Woods-Saxon (SW)

wave functions. The relevant s.p. levels are identified by numbers

corresponding to the following orbits: $1 = 1s_{1/2}$, $2 = 1p_{3/2}$, $3 = 1p_{1/2}$,

$4 = 1d_{5/2}$, $5 = 2s_{1/2}$, $6 = 1d_{3/2}$, $7 = 1f_{7/2}$, $8 = 2p_{3/2}$, $9 = 1f_{5/2}$,

$10 = 2p_{1/2}$, $11 = 1g_{9/2}$, $12 = 2d_{5/2}$, $13 = 1g_{7/2}$, $14 = 3s_{1/2}$, $15 = 2d_{3/2}$,

$16 = 1h_{11/2}$, $17 = 2f_{7/2}$, $18 = 3p_{3/2}$, $19 = 3p_{1/2}$.

Fig. 5.   The differences between the full MQF matrix elements and those with

the spin-orbit contribution left out, calculated with the same Woods-

Saxon wave functions.

Fig. 6.   Self-consistent quadrupole strength for UQF and MQF, the latter

in two versions. One, (χ'_{MQF}), neglects Coulomb, symmetry and spin-orbit terms

in order to obtain a simple A-dependence, the other, (χ_{MQF}), includes these

effects and is thus dependent on both N and Z, for which reason

we have only given the average dependence for nuclei at the β-stability

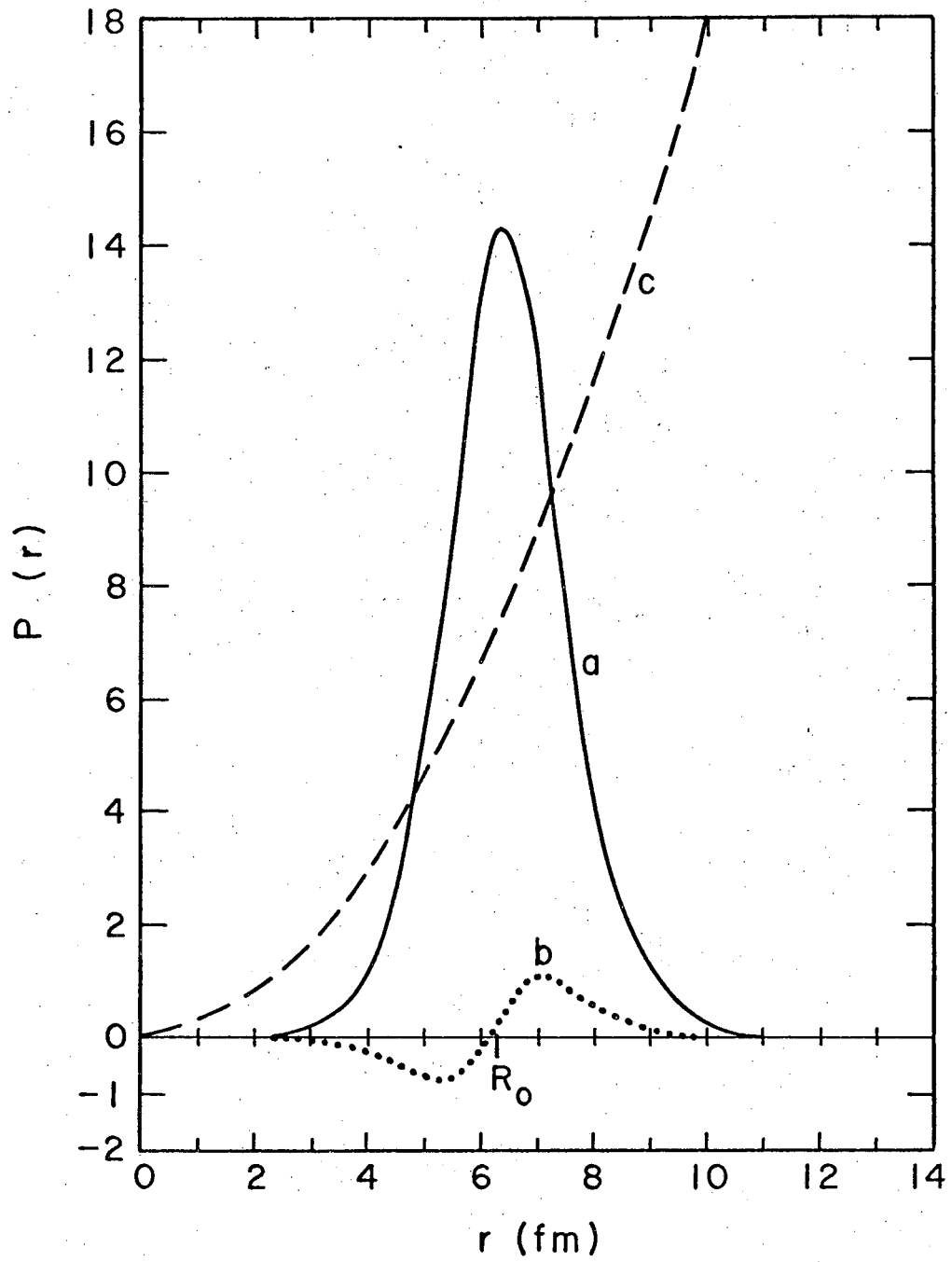line together with a few crossing lines indicating the N-dependence for
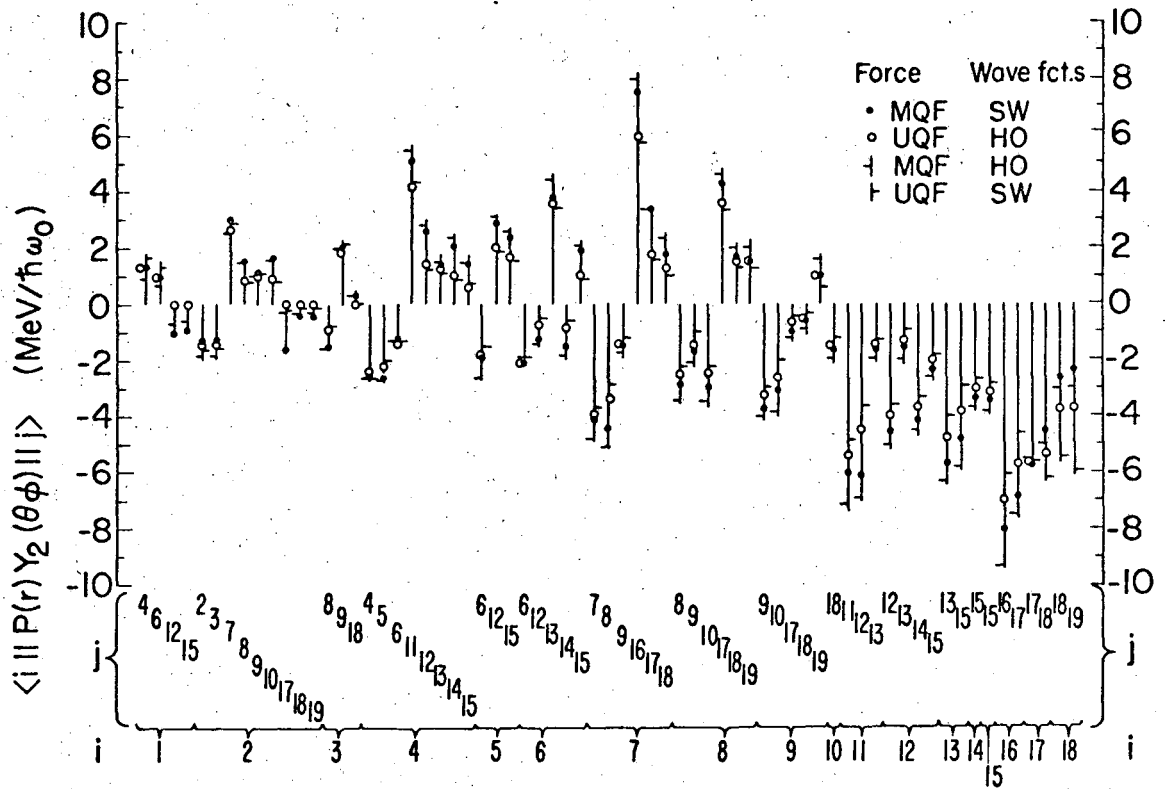
fixed Z.

Fig. 1.

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Fig. 2.

Fig. 3.

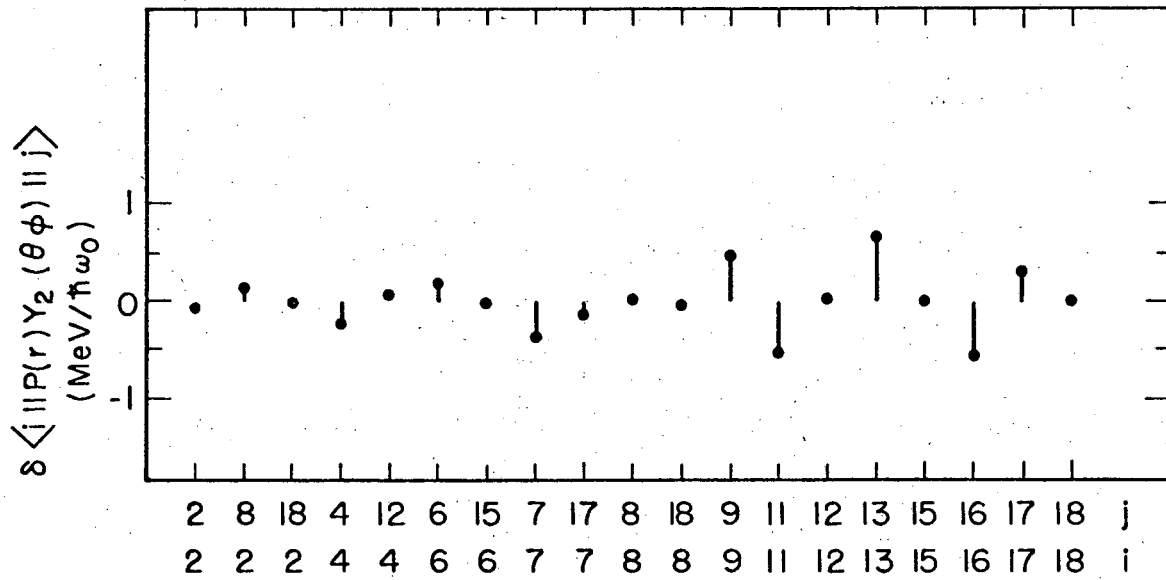XBL 68II-727I

Fig. 4.

Fig. 5.

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Fig. 6.

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