Title
Interaction of turbulence spreading and avalanching in relaxation and transport

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Interaction of turbulence spreading and avalanching in Relaxation and Transport

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What is turbulence spreading and avalanching?

- Turbulence spreading is self-scattering and expansion/ redistribition of turbulence energy from excitation zone to stable zone. Phenomenon of inhomogeneous turbulence, outside of usual realm of $K41$ paradigm.
- Avalanches are space-time localized large transport events.
- Avalanches are often observed in nature, gyrokinetic simulations, sand pile models and experiments.

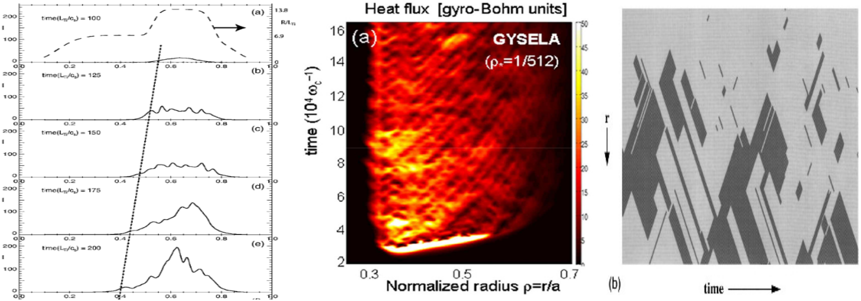


Figure: Left: Hahm etal 2005. Middle: Sarazin etal 2011. Right: Newman etal 1996

Why worry about spreading and avalanching?

- The "short fall problem" i.e., failure of G-K simulations to predict turbulence and transport in 'no man's land' and the dynamics of core edge coupling[T S hahm etal., 2005, 2018].
- Nonlocality[Ida et al., 2015], breakdown of gyroBohm transport scaling[Lin and Hahm, 2004], breakdown of Ficks law [Hahm and Diamond, 2018].
  - mixing length estimate of effective diffusivity $D \sim v_\star l$, where l is charecteristic scale, $v_\star$ is diamagnetic velocity. For $\rho_i < l < a$ yields D in between gyro-Bohm and Bohm $\rho_\star D_B < D < D_B$
  - However experiments indicate $D = \rho_\star^\alpha D_B$ with $\alpha < 1(\alpha \sim 0.6 - 0.7)) \implies$ gyro-Bohm scaling is broken !
- Turbulence spreading can invade regions with magnetic islands, impact on physics of $3d$ systems and neoclassical tearing modes. [K. Ida etal 2018]
- Any effect of spreading on avalanching ?
- Physics mechanism of avalanching and effect of spreading on avalanching?

Introduction and Motivation I

- This is an initial attempt to understand interaction of turbulence spreading and avalanching in a continuum model.
- G-K simulations have "all effects" but hard to delineate the mechanism of interaction of spreading and avalanching.
- Avalanching is well studied in sandpile paradigm with unistable supercritical toppling rule without accounting for turbulence spreading.
- Spreading in G-K but profile not self-consistent, except flux driven.
- No work on relaxation and transport with turbulence spreading effects in bisatble subcritical turbulence.

Introduction and Motivation II

A recent experiment clearly demonstrate hysteresis between flux-gradient and fluctuation-gradient in L-mode. Hysteresis suggests bistability.

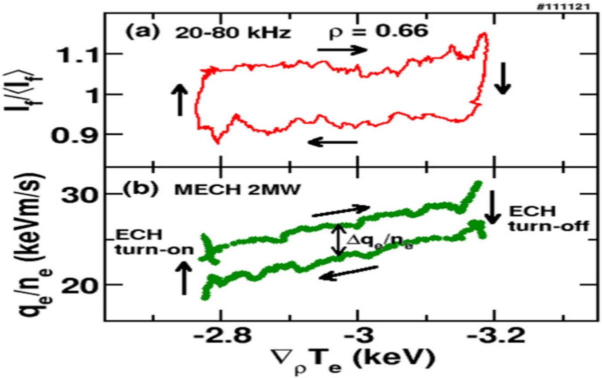


Figure: Inagaki etal 2013

What is new in THIS reduced model?

- Bistable model for turbulence consistent with the intensity-grad T hysteresis observed by Inagaki.
- Simplest setup to study interaction of spreading and avalanching with various degree of subcriticality.
- Generalization of the turbulence spreading with self-consistent profile evolution.
- This model shows that avalanching and spreading are essentailly dis-entangled.

Model description

We consider a 2-field continuum model consisting of turbulence intensity S (order parameter OP) and pressure h (control parameter CP) à la Gil-Sornette (GS) PRL 1996.

$$\frac{\partial S}{\partial t} = \chi \left[\underbrace{\mu S}_{lin.\ growth} + \underbrace{2\beta S^2}_{nonlin.\ growth} - \underbrace{S^3}_{nonlin.\ damp} \right] + \underbrace{\sigma \frac{\partial}{\partial x} S \frac{\partial S}{\partial x}}_{turb.\ spreading}$$

- $\mu = \left(\left(\frac{\partial h}{\partial x} \right)^2 - \mu_c^2 \right)$, μ_c is the critical gradient, χ is total growth rate, β measures strength of nonlinear growth, σ is spreading coefficient.

$$\frac{\partial h}{\partial t} = \underbrace{\frac{\partial}{\partial x} \alpha S \frac{\partial h}{\partial x}}_{turb.\ diff} + \underbrace{\phi(x)}_{heating\ source} + noise$$

Boundary conditions: $\frac{\partial S}{\partial x}(x=0) = \frac{\partial S}{\partial x}(x=1) = 0$, $\frac{\partial h}{\partial x}(x=0) = 0$, $h(1) = 0$. Noise helps in trigerring the avalanches.

Model description

- This model is an extension of the original GS model with turbulence spreading effect $\sigma\frac{\partial}{\partial x}S\frac{\partial S}{\partial x}$ and a steady source $\phi(x)$ included.
- Following previously studied turbulence spreading models did not consider self-consistent evolution of pressure profile.

$$[T\ S\ Hahm\ et al., 2004]\ \frac{\partial I}{\partial t} \quad = \quad \gamma(x)I - \alpha I^{1+\beta} + \frac{\partial}{\partial x}\chi(I)\frac{\partial I}{\partial x}$$

  - Non-linear Fisher fronts, depth of penetration is limited by damping in subcriricial region

$$[Guo\ and\ Diamond\ 2017]\ \frac{\partial I}{\partial t} \quad = \quad \gamma I + \beta I^{3/2} - \gamma_{nl} I^2 + D\frac{\partial}{\partial x}I\frac{\partial I}{\partial x}$$

  - $I^{3/2}$ term appears due to profile corrugations which is usually observed in G-K simulations
  - Stronger penetration into stable zone.
- The extended Gil-Sornette model presented here considers self-consistent evolution of pressure profile and provides the simplest setting for study of interaction of trubulence spreading and avalanching.

Numerical setup

- Conservative FD/FV discretization in space

$$\left(\frac{\partial}{\partial x} S \frac{\partial h}{\partial x}\right)_i \approx \frac{\left(S\frac{\partial h}{\partial x}\right)_{i+1/2} - \left(S\frac{\partial h}{\partial x}\right)_{i-1/2}}{\Delta x}$$

$$\left(\frac{\partial h}{\partial x}\right)_{i+1/2} = \frac{h_{i+1} - h_i}{\Delta x};\ S_{i+1/2} = \frac{S_i + S_{i+1}}{2}$$

etc..

- leads to coupled stochastic ODEs which are time integrated by Euler-Maruyam method

$$Y_{n+1} = Y_n + a(Y_n)\Delta t + b\Delta W_n$$

- $\Delta W_n = W_{t_{n+1}} - W_{t_n} = \mathcal{N}(0, \Delta t/\Delta x) = \sqrt{\Delta t/\Delta x}\mathcal{N}(0,1)$ are independent random variables with zero mean.

Mean profiles variation with subcritical parameter β

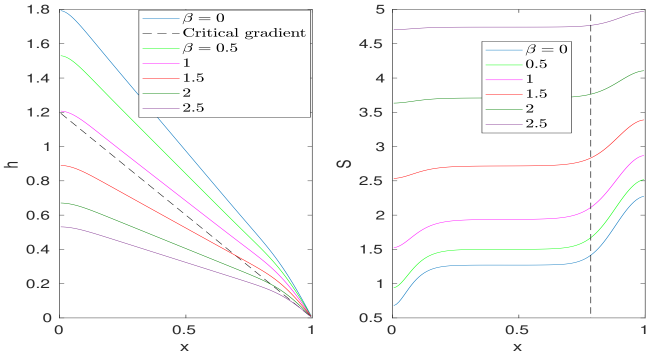
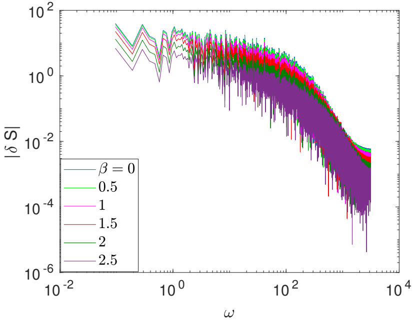


Figure: Mean profiles at $Pr = \frac{\sigma}{\alpha} = 1$

- Turbulence energy increases and internal energy decreases with β.
- Edge intensity is higher than core intensity in all cases.

Frequency spectra with β



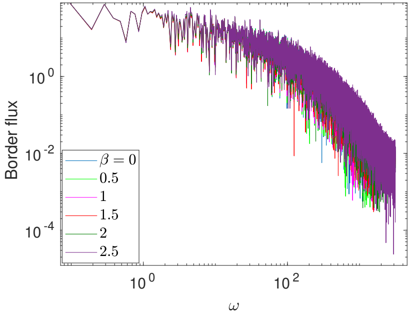


Figure: Frequency spectra with β at $Pr = 1$

- Captures typical features of spectrum as commonly seen in G-K and Sandpile simulations.
- No significant change in frequency scaling exponent with β.

Space time propagation

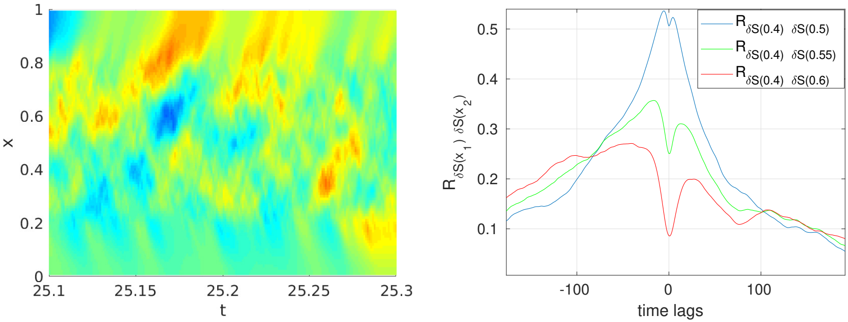


Figure: Left: Space-time plot of δS. Right: Cross-correlations at different separations, $Pr = 1$

- Clear demonstration of avalanches/ intensity pulses propagating both inward and outward.

Prandtl number $Pr(=\frac{\sigma}{\alpha})$ scan of mean profiles

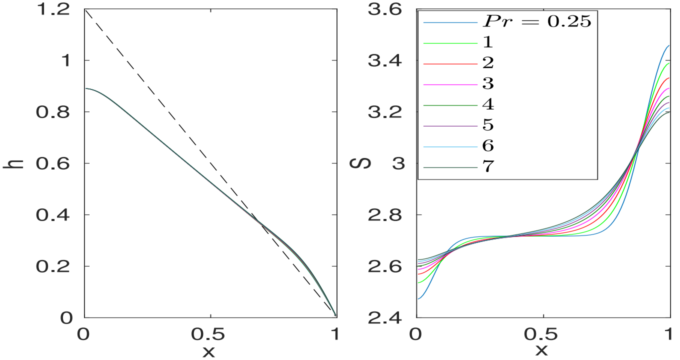


Figure: Mean profiles with Pr

- Core turbulence increases and edge turbulence decreases with increasing Pr.
- Pressure profile is weakly affected by Pr.

Frequency spectra scan with Pr

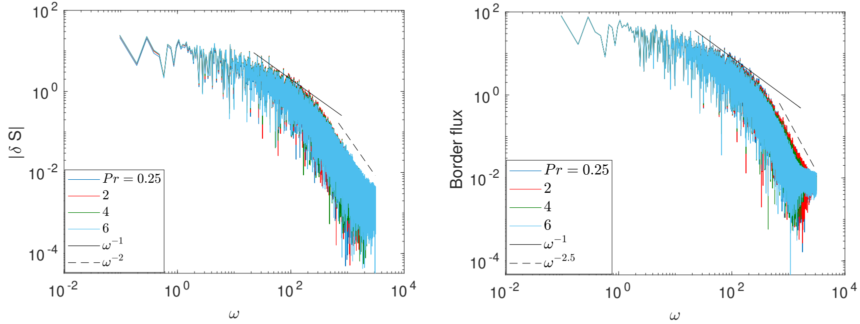


Figure: Frequency spectra with Pr

- Frequency spectrum structure is barely afffected by Pr.

Intensity autocorrelations vs Pr

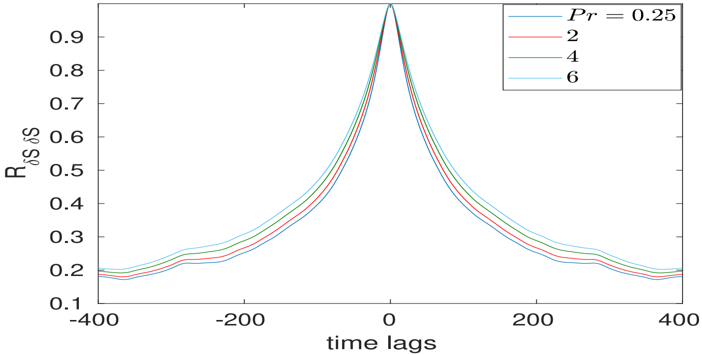


Figure: Autocorr of intensity at the edge

- Autocorrelation time (measured as e-folding time) increases with Pr. This is because spreading tries to homogenize intensity.

Intensity autocorrelations with Pr

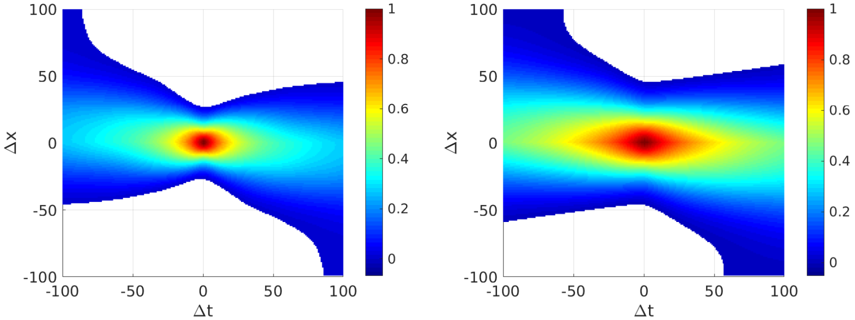


Figure: Left: 2d autocorr of intensity at $Pr = 1$. Right: 2d autocorr of intensity at $Pr = 6$. Plotted on log scale along z axis.

- Clearly inward and out ward velocities are not symmetric.
- Spatial and temporal auto correlation increase with Pr.
- Slow speeds get slower and fast speeds increase with Pr.

Effect of noise on spreading at early times

Noise promotes spreading at early times

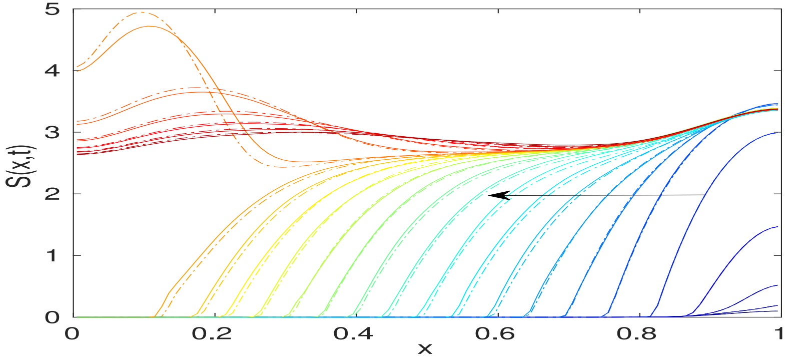


Figure: Spreading dynamics with (solid curves) and without noise (dashed curves) at $Pr = 1$. Color code: Time increasing from blue to red.

Summary

- We studied interaction of turbulence spreading and avalanching in a bistable model of turbulence using numerical simulations. Lessons learned are as follows.
  - Turbulence spreading weakly affects avalanching dynamics as see from the Prandtl number scan of frequency spectra of fluctuations and fluxes.
  - Turbulence intensity is always increasing towards the edge.
  - Mean turbulence intensity increases at core and decreases at the edge with increasing Prandtl number. Edge intensity always remains higher than the core.
  - Mean pressure profile is weakly affected by Pr.
  - Correlation time and length increases with Prandtl number.
  - β scans at fixed Pr shows that the turbulence energy increases and internal energy decreases with β.
  - Frequency spectra of fluctuations and fluxes are again weakly affected by β.

Take away message for experimentalists

- High edge turbulence intensity is a natural consequence of pressure EVOLUTION and power deposition profile. Emergent, without specialized edge modes -i.e. the RBMs.
- Turbulence spreading from edge inward can account for turbulence in 'no man's land'.
- Steady state profile is not severly affected by increasing Prandtl number. However turbulence penetrates deeper into core leading to core turbulence enhancement.
- Strength of turbulence subcriticality has severe impact on steady state profile. Higher the subcriticality parameter, higher the turbulence energy and lower the internal energy.
- Avalanching dynamics is not severly affected by turbulence spreading.

Future work

- How turbulence spreading affects spatial size and life time distribution of avalanches?
- Effect of input power and noise strength on avalanche distribution and turbulence spreading?
- Interaction of spreading and avalanching in fully subcritical and supercritical mean pressure profiles ?
- Effect of classical/neoclassical diffusion and on avalanches?
- Flux bistability, transport bifurcation and avalanches?