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Institutional determinants of the largest seat share

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Abstract

The degree of predominance of the largest party in a representative assembly affects government formation and survival. The seat share of the largest party, in turn, is constrained by the interaction of assembly size and electoral district magnitude in the following way. When all S seats in an assembly are allocated in districts of magnitude M, a logical quantitative model proposes that the largest fractional share is $s_1 = (MS)^{-1/8}$. As a curve, the model is found to fit with $R^2 = 0.509$, considering data from the averages of 46 periods in 37 countries, during which the electoral rules were essentially steady. As a worldwide average, the expression $s_1(MS)^{1/8} = 1$ holds within 1%. Deviations from this average express the impact of various country-specific political and socio-cultural factors that can be investigated once the basic institutional constraints are controlled-for. This means that the degree of largest party predominance may be engineered to hover around a desired average by adjusting assembly size, and district magnitude, while keeping country-specific factors in mind.
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Keywords: Largest seat share; Assembly size; District magnitude; Duverger's Law; Electoral systems; Political parties

1. Introduction

The seat share of the largest party is of interest, because it matters for government formation and survival. When it is less than 50%, it influences the number and weight of potential coalition partners or, if the largest party remains in opposition, its blocking ability. When it surpasses 50%, the extent of the excess still makes a difference by enhancing the largest party's

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clout, yet also by encouraging factions within it.[1] The inverse of the largest share represents a measure of the number of parties, and has been used in political analysis (e.g., by Siaroff, 2003) as a supplement to the more usual effective number of parties. The effective number itself is strongly affected by the largest share.[2]

Which institutional factors contribute to determine the seat share of the largest party, and *exactly how large* would we expect this share to be, on the average, for given values of the input variables? This is the question addressed in this study, stressing the ''exactly how large.'' We should not be satisfied by anything short of an explicit prediction that could be refuted by actual data.

One institutional factor clearly has some impact—single member districts (SMD) versus proportional representation (PR) in multi-seat districts. The SMD often leads to absolute majorities, while the largest share tends to be modest in PR, especially when the district magnitude (the number of seats allocated in the district, M) is large. This follows, of course, from the well-known Duverger's law. But this law by itself does not specify exactly how large would we expect the largest share to be, on the average, for a given value of district magnitude, all other factors being the same.

Among the SMD systems, the largest share tends to be strikingly large in tiny nations with assemblies of only 10 to 30 seats. Thus assembly size (S) may also matter, as surmised by Lijphart (1994) among others.[3] This dependence, too, may be considered self-evident, but it is so only regarding the direction of the impact. The question remains: exactly how large would we expect the largest share to be, on the average, for a given value of assembly size, all other factors being the same?

Mackie and Rose (1991, 1997) have compiled the results of 753 national assembly elections in 25 stable democracies. In about 90% of the cases, the largest share is between 21 and 69%—a range of 48 percentage points.[4] In the same country, using the same seat allocation rules, this share can fluctuate widely from one election to the next and, of course, a different party may become the largest. For single member plurality (SMP) rule, the drop from 68.8 to 33.8% in New Zealand 1925 to 1928 might be the record—35 percentage points, or by a factor of 2.0. Even with PR, the largest share went from 35.2 to 59.2% in Portugal 1985 to 1987—an increase of 24 percentage points, or by a factor of 1.7. Visibly, the moment's politics is a major factor for individual elections, overriding institutional constraints.

Over a number of elections using the same rules, however, the differential impact of these rules, and other institutional factors may make itself felt, along with socio-cultural factors. The latter may outlast institutions, or may slowly shift within the same institutional framework. Among the institutional factors, two deserve special consideration—the aforementioned assembly size and district magnitude.

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[1] Recent major monographs on electoral systems (Cox, 1997; Katz, 1997; Reynolds and Reilly, 1997; Farrell, 2001; Colomer, 2004; Norris, 2004) agree on the distinctiveness of single party absolute dominance in the assembly: ''This state of affairs is praised for providing cabinets which are unshackled from the restraints of having to bargain with a minority coalition partner'' (Reynolds and Reilly, 1997: 28).

[2] The approximation $N = s_1^{-1.5}$ has been offered by Taagepera and Shugart (1993), where N is the effective number of legislative parties and s_1 is the largest party's fractional share.

[3] Assembly size itself depends on the country's population. Whether population might affect the largest share and hence general fractionalization by channels other than assembly size will be discussed later.

[4] The lowest seat share of the largest party is 18.4 % (Belgium 1991); the highest is 97.1 % (Belgium, partial elections 1884).

We will first present a predictive model, and describe our database. We will next test the relatively numerous SMD systems for the effect of assembly size. Subsequently, we will test the joint effect of assembly size and district magnitude, and draw conclusions.

2. The model

Assembly size (the total number of its members, S) typically varies from 50 to 600 for national assemblies, but can fall to as low as 10 in tiny island countries (see Nohlen, 1993; Nohlen et al., 1999, 2001). One might expect the largest seat share to be larger in smaller assemblies, which tend to occur in smaller and possibly more homogeneous countries. This conjecture is reinforced when one considers presidential elections as a limiting case. Here $S = 1$, and the largest seat share is perforce 100%. One may conjecture that, if the number of seats were gradually expanded beyond 1, the largest share would gradually come down toward the range observed for the national assemblies.

District magnitude (M) imposes an effective threshold, in terms of votes, on the parties' ability to obtain a seat in a given district (Lijphart, 1994). This threshold is the highest for SMD ($M = 1$), and decreases as M becomes larger. As a larger magnitude enables more parties to win seats, it may whittle down the largest share. This applies, of course, only when all seats are allocated within districts, with no second-tier or nationwide adjustments or legal thresholds. When district magnitude is reduced to 1, the well-known Duverger's law predicts a 2-party system (subject to some further specifications). If so, then the largest party should reach the absolute majority of the seats, but Duverger does not specify how large a majority it would tend to have.

Assembly size and district magnitude, therefore, appear as institutional factors that may affect the largest party's seat share. *If nothing else is known* about a given political system, except M and S, a model proposed by Taagepera and Shugart (1993) predicts that the likeliest largest fractional seat share (s_1) is the inverse of the eighth root of the product MS:

$$s_1 = (MS)^{-1/8}$$

(For seat shares in percentages, multiply this value by 100.) This is where the mean of many observations would be expected to be—*if and only if nothing else is known*. The proof is given in Appendix A. The model applies only to elections where all seats are allocated within districts of roughly equal magnitude, and a legal threshold does not override the effective threshold set by district magnitude.[5]

This model fits perfectly for presidential elections, since it yields $s_1 = 1$ at $S = 1$ (where M is perforce also 1). Such agreement may look trivial, but its absence would disqualify any logical model. For assembly elections with SMD ($M = 1$), the equation simplifies into $s_1 = S^{-1/8}$, and predicts a largest share of 0.75 (i.e., 75%) for 10-seat assemblies, but only 0.46 for 500-seat assemblies. In contrast, for these 500 seats allocated by nationwide PR unrestricted by legal thresholds, s_1 would be down to 0.21.

The range of the largest share predicted by this model, 21–75%, roughly coincides with the range observed for most national assemblies. But in principle, the model need not fit at all. The

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[5] Instead of the fractional share, one may also consider the *number* of seats for the largest party, $S_1 = s_1 S$. The model yields $S_1 = M^{-1/8} S^{7/8}$. At the one extreme, if $M = 1$, the equation reduces to $S_1 = S^{7/8}$. At the other extreme, if $M = S$, the relationship becomes $S_1 = S^{3/4}$. In sum, the *number* of seats for the largest party would mostly range from $S^{3/4}$ to $S^{7/8}$, according to the model.

key words in its presentation (see Appendix A) are "*if nothing else is known*," apart from district magnitude and assembly size. The model expresses the best probabilistic guess in such a situation. It could be stated as a null hypothesis: Once district magnitude and assembly size are set by political circumstances and processes, nothing else besides the mechanical impact of M and S affects *the average* of the largest seat share.

In reality, plenty of other political or socio-cultural factors of a universal nature can tilt the outcome away from this hypothesis.[6] Whether this is the case, and in which direction the tilt can go, is to be tested. Indeed, other factors may well be sufficiently strong to submerge the institutional impact. A thorough analysis by Lijphart (1994) yielded mixed results. "Effective" thresholds (based on district magnitude and other factors) accounted for 28% of the variance in parliamentary multipartism, as measured by the effective number of parties. Assembly size did not emerge as a significant factor in regression analysis, but did in comparable-cases analysis (Lijphart, 1994: 142).

While Lijphart's study gave credence to assembly size as a significant factor, and reinforced previous findings on the importance of magnitude, it did not test the connection of the largest seat share to M and S. Rather, it dealt with the effective number of parties, which in the Shugart and Taagepera (1993) model is connected to M and S more remotely, compared to s_1.[7] Furthermore, Lijphart's analysis included elections where all the seats were not allocated within districts. In such cases, he estimated the effective threshold by various means, some of which may be debated.

In sum, no direct and thorough test of $s_1 = (MS)^{-1/8}$ has ever been carried out. The present study uses 30 SMD systems, and 10 PR systems where all seats were allocated within districts. It tentatively adds 6 systems with nationwide PR, subject to legal thresholds. The number of PR systems may look low, but actually the number of candidates is very limited when it comes to *simple* systems where the impact of district magnitude is not masked by features like legal thresholds, and adjustments outside the basic electoral districts.

To repeat, institutional factors cannot be expected to be the main ones in determining the seat share of the largest party in any particular election. A moment's politics can obviously determine an outcome, with only mild institutional constraints. Despite Duverger's law, single member systems have witnessed fractured elections where even the largest party has only one-third of the seats (e.g., New Zealand 1928), and even nationwide PR does not prevent occasional occurrence of a large dominant electoral bloc (e.g., Israel 1969). As for country averages, durable factors other than M and S may cause some countries to have consistently higher or lower averages, compared to the theoretical expectations. These deviations are of a great deal of interest. But we cannot measure deviations from a norm, before testing the norm in the first place.

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[6] In particular, students of party politics could approach the question of how large the largest party is expected to be from a very different perspective, focusing on over-time trends, especially the phenomenon of "electoral change." In such an approach, the degree of electoral fractionalization is treated as a given that affects the size of the largest party. The logically expected, and actual average functional relationships between fractionalization and largest seat share needs more detailed study, but this requires a separate article. Here we focus on what can be predicted, on logical grounds, based on only two institutional factors, assembly size and district magnitude.

[7] Cf. Note 2. It would be mistaken to view the largest share as connected to M and S through the intermediary of overall fractionalization, as expressed by the effective number of parties, given the direct derivation presented in Appendix A. Fractionalization is not an institutional input but another feature affected by M and S, and once M and S have been decided upon, remains fairly stable.

The central questions are: (1) whether any systematic institutional impact emerges *at all* when the share of the largest party is averaged over many elections (during which the identity of the largest party may change); and *if so*, (2) whether the worldwide average of this institutional impact follows the conjecture $s_1 = (MS)^{-1/8}$. If so, then we can specify the average impact of district magnitude and assembly size. Deviations from this norm would then be the part requiring explanation in terms of other political or socio-cultural factors.

The expression above can be reformulated as a conservation rule:

$$s_1(MS)^{1/8} = 1$$

—*the product of the largest share and the* 8*th root of the product of district magnitude and assembly size is constant.* Quantities that are conserved during a transformation are of considerable interest in physical sciences, and might be so in social sciences. Such a formulation also avoids the issue of which variables are dependent, and which are independent. In the following, we will most often treat the largest seat share as formally dependent, but the model as such only posits mutual interdependence.[8]

3. The data

The theoretical conjecture above presumes an ideally simple electoral system where all seats are allocated in districts of equal magnitude, with no legal thresholds, multiple rounds, or upper tier or nationwide allocation. There would be neither primaries nor electoral alliances. The ballot would be categorical, using plurality in single-member districts, and a simple PR rule in multi-seat districts. Apart from most SMP systems, no other simple systems seem to exist. Therefore, testing with a sufficiently large number of cases calls for some relaxation of the criteria.

We used Mackie and Rose (1991, 1997) as the basic data source, because it is relatively extensive. Moreover, using an existing set avoids appearances that the countries might have been chosen to suit the model. Among the election results for the 25 countries listed in Mackie and Rose (1991, 1997), we accepted all periods of at least 6 elections held under essentially the same rules, as long as all seats were allocated within districts, so that a mean district magnitude could be defined and calculated.[9] This gives us 24 separate periods: 14 SMD and 10 PR.

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[8] Once in place, assembly size and district magnitude may affect the largest seat share. But conversely, the largest seat share in the founding assembly may also affect the choice of electoral system, and of district magnitude in particular. A predominant party might push for $M = 1$, so as to assure a continuing large-party bonus in terms of seats (cf. Colomer, 2005). As for assembly size, it is strongly affected by size of population, but apparently not by the largest party's strength in the founding assembly.

[9] Temporary shifts in electoral rules (such as France using PR for the single election of 1986) were overlooked, because the party constellations hardly would have had time to react. If anything, such inclusions would make our outcomes more blurred. Shifts among different PR allocation rules (but with essentially the same M) were also overlooked. Even at the same mean magnitude, the political impact can be significantly different when individual districts have grossly divergent magnitudes (Monroe and Rose, 2002). Under such conditions, the mean M would overestimate the largest seat share. To take an extreme case, consider 100 seats allocated in SMD plus another 100 allocated in a single district of $M = 100$, so that the mean magnitude is 1.98. The prediction based on actual magnitudes leads to $s_1 = 0.44$, while the mean M predicts $s_1 = 0.47$. Thus the overestimation is likely to be limited in the actual cases, where variation is less drastic.

The degree of correlation in such a study depends very much on the range covered by the base variable, *MS* in this case. If this range is narrow, a very real effect can be swamped by random noise. Therefore, we should try to extend the database toward small SMD countries where not only *M* is minimal, but also *S* is small. In the other direction, we should try to extend it toward countries with nationwide PR, where *M* is maximal.

In order to extend the range toward smaller assemblies, we scoured Nohlen (1993), and Nohlen et al. (1999, 2001) for small and yet fairly stable SMD countries, relaxing the criteria to include some cases with only 3 or 4 elections. We found 16 such countries, ranging from St. Kitts to Cuba 1901–1954. In order to extend the range of *MS* toward higher values, we tentatively added 6 cases (all from Mackie and Rose, 1991, 1997) of nationwide PR subject to moderate legal thresholds, although such thresholds are likely to distort the impact of district magnitude.

4. Testing the model

These data will now be used to address the following questions, starting with the most general. (1) Does the worldwide average agree with the conjecture that $s_1(MS)^{1/8}$ is a conserved quantity with value 1? (2) How closely do the country averages follow this conjecture? (3) Does the institutional impact on the largest seat share emerge in a significant way in the regression of s_1 against *MS*, despite the relatively short range that $(MS)^{1/8}$ can take in the case of national assemblies?

We'll proceed in two stages. Since there is a relatively large number of systems at $M = 1$, we can use them to test for the effect of *S* alone. With $M = 1$, $s_1 = (MS)^{-1/8}$ is reduced to $s_1 = S^{-1/8}$. Once this relationship is established, we can include the multi-seat systems and test the general case.[10]

4.1. Testing for the effect of assembly size

Table 1 shows the following data for the 30 single member systems ($M = 1$): the time period and the number of elections; the type of seat allocation rule; and the geometric means, over time, of assembly size (*S*), fractional share of the largest party (s_1), and the product $s_1 S^{1/8}$. The latter is expected to be 1, on the average.[11] The table lists the systems in the order of increasing assembly size, which ranges from 10 (St. Kitts) to 643 (UK). The inevitable pattern for presidential elections is shown at the bottom of Table 1.

4.1.1. Does the worldwide average agree with the conjecture that $s_1(MS)^{1/8}$ is a conserved quantity with value 1?

For $M = 1$, this expression boils down to $s_1 S^{1/8}$. Its geometric mean for the 30 systems is 0.985, which is within 1.5% of the expected value. Table 1 shows separately the means for the 10 smallest, the 10 medium, and the 10 largest assemblies. No systematic pattern of

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[10] To test for *M* separately, we would need a large number of cases with the same assembly size, but different district magnitudes. Unfortunately, there are hardly any such cases.

[11] The expected relationship corresponds to a linear relationship between the logarithms of s_1 and *S*. Hence one should use the arithmetic means of the logarithms (*not* of the quantities themselves), which corresponds to taking the geometric means of s_1 and *S*.

Table 1
Conserved combination of assembly size (S) and largest party seat share (s_1) for single member district systems ($M = 1$)

Country, period, number of elections	Allocation rule[a]	Geometric means		
		S	s_1	$s_1 S^{1/8}$
St. Kitts 1980–89, 3	P	10.3	0.510	0.682
St. Vincent 1974–89, 4	P	13.5	0.819	1.123
Grenada 1972–90, 4	P	15.0	0.690	0.968
St. Lucia 1974–92, 6	P	17.0	0.629	0.896
Antigua 1980–89, 3	P	17.0	0.859	1.225
Dominica 1975–90, 4	P	21.0	0.693	1.014
Cook Islands 1965–99, 10	P	23.2	0.609	0.903
Belize 1979–89, 3	P	24.2	0.662	0.986
Barbados 1966–91, 6	P	25.6	0.694	1.040
Botswana 1965–94, 7	P	33.2	0.830	1.286
Geometric mean for 10 systems at small S				*0.998*
Trinidad 1961–91, 7	P	35.1	0.734	1.145
Bahamas 1972–87, 4	P	41.8	0.730	1.164
Jamaica 1944–89, 11	P	46.9	0.649	1.115
Samoa 1979–2001, 7	P	47.6	0.531	0.860
Cuba 1901–54, 23	P	64.7	0.476	0.802
Mauritius 1976–95, 6	P	68.0	0.650	1.101
Australia 1901–17, 7	P	75.0	0.483	0.829
New Zealand 1890–1993, 34	P	81.4	0.578	1.001
Netherlands 1888–1913, 8	DB	100.0	0.343	0.609
Australia 1901–96, 31	AV	106.3	0.497	0.890
Geometric mean for 10 systems at medium S				*0.934*
Norway 1882–1903, 8	P	114.4	0.593	1.073
Denmark 1901–18, 7	P	117.3	0.462	0.838
Norway 1906–18, 5	DB	123.6	0.498	0.910
Sweden 1887–1905, 8	P	226.3	0.511	1.006
Canada 1878–1993, 32	P	247.0	0.575	1.145
United States 1828–1994, 84	P	344[b]	0.588	1.220
Germany 1871–1912, 13	DB	396	0.274	0.579
France 1958–93, 10	DB	496	0.436	0.946
Italy 1895–1913, 6	DB	508	0.780	1.700
United Kingdom 1885–1992, 29	P	643	0.526	1.179
Geometric mean for 10 systems at large S				*1.024*
ALL 30 $M = 1$ SYSTEMS				0.985
Presidential elections	Any	1	1.000	1.000

[a] P, plurality; DB, double ballot; AV, alternate vote.
[b] US: S ranges from 213 to 437; in all other cases the variation is relatively minor.

decrease or increase in $s_1 S^{1/8}$ can be seen, meaning that the impact of assembly size on the largest share seems fully taken into account.

4.1.2. How closely do the country averages follow the overall pattern?

For individual countries, the product $s_1 S^{1/8}$ ranges from 0.58 (Imperial Germany) to 1.70 (pre-WWI Italy). When the distribution is approximated by a normal distribution, the standard deviation is 0.22. Italy 1895–1913 is more than 3 standard deviations off, and thus clearly

represents a special case. Germany 1871−1912, and the Netherlands 1888−1913 are almost 2 standard deviations off. Notably, these are all early double ballot systems going back to the 19th century.[12] Note that taking the 8th root reduces the range of $S^{1/8}$ severely. It only ranges from St. Kitts' $10.3^{1/8} = 1.34$ to UK's $643^{1/8} = 2.24$, so that the highest figure is less than twice the lowest. Country-specific political and socio-cultural factors could be expected to blur out this modest difference in institutional impact. The surprise is that for most twentieth century countries this is not the case: a prediction based on nothing but a probabilistic mean for given assembly size holds for individual countries with a standard error of only 22%.

4.1.3. Does the impact of assembly size on the largest seat share emerge in a significant way in the regression of largest share against assembly size?

The relatively short range that $S^{1/8}$ can take, works against it. The expression $s_1 = S^{-1/8}$ corresponds to $\log s_1 = -0.125 \log S$. Consequently, linear regression should be carried out not on s_1 and S, but on their logarithms: $\log s_1 = a_0 + b_1 \log S$, where a_0 would be expected to be close to 0, and b_1 close to -0.125. We use decimal logarithms.

The actual result is $\log s_1 = -0.039 - 0.108 \log S$ ($R^2 = 0.256$), which corresponds to $s_1 = 0.915 S^{-0.108}$. The values of the constants are close to the expected, so that the best fit line (Fig. 1) is visually close to the expected $\log s_1 = -0.125 \log S$. The R-squared is markedly reduced by the three aforementioned pre-WWI cases using double ballot (Italy 1895−1913, Germany 1871−1912, and the Netherlands 1888−1913).

We conclude that the model has passed the first test, which considers the dependence of the largest seat share on assembly size alone. The geometric mean of $s_1 S^{1/8}$ for the 30 SMD systems is within 1.5% of the expected value of 1.00. The standard error for the dispersion of individual country means around this overall mean is only 22%. And the best fit line (on log scale) is close to the predicted. The rather low value of R-squared is largely due to the impact of three pre-WWI monarchies using a double ballot.

4.2. Testing for the joint effect of assembly size and district magnitude

Table 2 shows data for the 10 single member systems, and the 6 nationwide systems. Added to the items in the previous table are the geometric means, over time, of district magnitude (M), the product MS, and the product $s_1(MS)^{1/8}$. The systems are listed in the order of increasing product MS.

For the sake of extending the range of MS, it would be useful to have cases with pure nationwide PR (meaning $M = S$), as this would lead to a testable prediction for the very low largest party seat shares. Unfortunately, all long-established nationwide systems impose some legal threshold of representation, usually in terms of a minimum percentage of votes. An approximate formula for comparing the effects of legal thresholds (T) to those inherent in district magnitude has been offered (Taagepera, 1998): effective $T = 75\%/(M + 1)$ or, reversing it, effective $M = (75\%/T) - 1$. This is the magnitude shown in Table 2, with some hesitation. While MS for the clear multi-member cases ranges from 180 to close to 3000 (a range of only 1 to 16), the cases with effective M extend the range to close to 17,000, for a total range of over 1 to 90. The widened range can be expected to reduce the impact of random fluctuations on R^2—provided that the conversion of T to effective M is valid.

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[12] Most systems tested used the plurality allocation rule. The few majority systems (double ballot and alternative vote) do not stand out, except for double ballot showing a huge dispersion in outcomes.
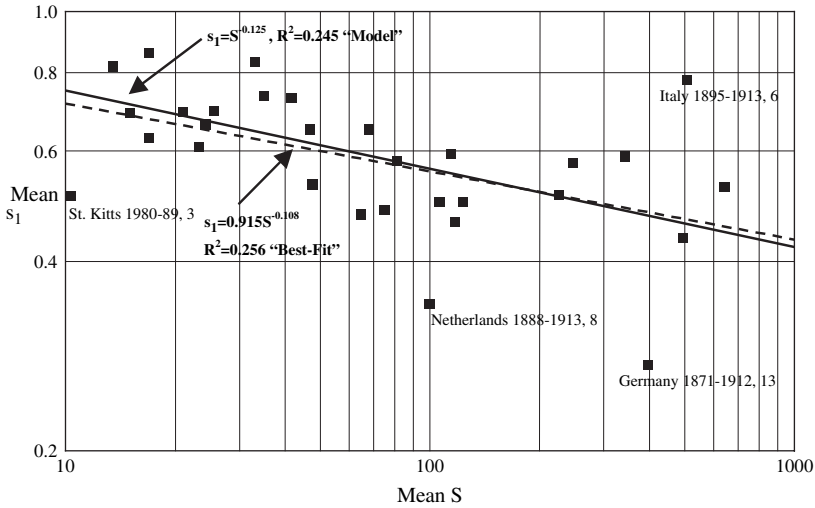
Fig. 1. Regression line of the seat share of the largest party (s_1) vs. assembly size (S) for 30 systems with single member districts.

Table 2
Conserved combination of district magnitude (M), assembly size (S) and largest party seat share (s_1) for multi-member district systems

Country, period, number of elections	Allocation rule[a]	Geometric means				
		M	S	MS	s_1	$s_1(MS)^{1/8}$
Malta 1921–92, 18	STV	4.8	38[b]	180	0.515	0.985
Luxembourg 1919–94, 19	L	11.9	42[c]	504	0.430	0.937
Ireland 1922–92, 25	STV	3.8	150	567	0.472	1.042
Norway 1921–93, 19	L	7.8	153	1198	0.424	1.027
Switzerland 1919–95, 21	L	7.9	196	1544	0.269	0.674
Sweden 1908–68, 20	L	6.9	231	1601	0.448	1.126
Japan 1928–93, 24	SNTV	4.0	482	1927	0.498	1.281
Spain 1977–96, 7	L	6.7	350	2356	0.492	1.298
Portugal 1975–95, 9	L	11.2	247	2769	0.425	1.145
Finland 1907–95, 32	L	14.3	200	2857	0.328	0.887
Geometric mean for 10 systems at M > 1						*1.024*
Sweden 1970–94, 9	N 4%	17.7	349	6200	0.446	1.328
Denmark 1964–94, 14	N 2%	36.5	175	6390	0.350	1.045
Germany 1961–94, 10	N 5%	14.0	527	7380	0.411	1.251
Netherlands 1918–52, 9	N 1%	74.0	100	7400	0.305	0.930
Israel 1949–96, 14	N 1%	74.0	120	8880	0.370	1.152
Netherlands 1956–94, 12	N 67%	111.5	150	16720	0.312	1.053
Geometric mean for 6 nationwide threshold systems						*1.1186*
ALL 46 SYSTEMS (INCLUDING $M = 1$)						1.0097

[a] STV, single transferable vote; L, list PR; SNTV, single non-transferable vote. N, Nationwide single district, with legal % votes threshold (T) as shown. T is converted to effective magnitude through approximation $M = (75\%/T) - 1$.
[b] Malta: S ranges from 10 (Government Council, 1939 and 1945) to 65.
[c] Luxembourg: S ranges from 25 (partial elections) to 64.

4.2.1. Does the worldwide average agree with the conjecture that $s_1(MS)^{1/8}$ is a conserved quantity with value 1?

For the 10 unambiguous PR cases, the mean value of $s_1(MS)^{1/8}$ is 1.024, which is within 2.5% of the expected 1.000. For the 6 nationwide threshold cases, the mean value of $s_1(MS)^{1/8}$ is appreciably higher: 1.119. This 12% excess, compared to the expected 1.000, might involve error in the estimate of effective magnitude. Let us now combine the SMD systems (Table 1) and the multiseat systems (Table 2). For all 40 cases without legal thresholds, the geometric mean is 0.994. For all 46 systems (including those with thresholds) it is 1.010. Either way, the mean is within 1% of the expected value of 1.000. In sum, the conjecture about the worldwide average institutional impact on the largest seat share is confirmed within 1%. This result is quite robust against the omission of individual countries, and hence it looks also robust against further additions.

4.2.2. How closely do the country averages follow the overall pattern?

For individual PR systems with no legal thresholds, the product $s_1(MS)^{1/8}$ ranges from 0.67 (Switzerland) to 1.30 (Spain). For those with legal thresholds, it ranges from 0.93 (Netherlands since 1956) to 1.33 (Sweden). These ranges remain within the previously observed range for SMD systems (0.58 to 1.70).

Fig. 2 shows the frequency distribution of country geometric means of $s_1(MS)^{1/8}$ for all the 46 systems. The zone 0.800 to 1.299 is fairly evenly populated. When the distribution is approximated by a normal distribution, the standard deviation is 0.21. Only two cases exceed two standard deviations: pre-WWI Italy (upwards), and pre-WWI Germany (downwards).

Also shown in Fig. 2 are the distributions of individual elections in two selected countries. Switzerland is the country with the narrowest distribution. The US, which involves by far the largest number of elections, is on the wider side.[13]

4.2.3. Does the impact of assembly size and district magnitude on the largest seat share emerge in a significant way in the regression of largest share against the product MS?

Using the data in Tables 1 and 2, Fig. 3 shows the mean largest party shares of all 46 systems graphed against the mean product of magnitude and assembly size. Both are on logarithmic scales, so that $s_1 = (MS)^{-1/8} = M^{-0.125} S^{-0.125}$ becomes a straight line: $s_1 = 1.000 M^{-0.125} S^{-0.125}$, which is shown as a full line in Fig. 3. By and far, the data points are scattered around this line. For regression, the presence of two independent variables forces us to make a detour, by first considering them separately, and then together. Table 3 shows the regression constants and R-squares for these three approaches, using the full set of 46 systems:

1. Regressing $\log s_1$ on $\log M$ only, meaning a relationship of form $s_1 = aM^b$.
2. Regressing $\log s_1$ on $\log S$ only, meaning a relationship of form $s_1 = aS^c$.
3. Regressing $\log s_1$ on $\log M$ and $\log S$, meaning a relationship of form $s_1 = aM^bS^c$.

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[13] The lowest observed value of $s_1(MS)^{1/8}$ for any individual elections (0.445) occurs in the Netherlands 1901–1913, when s_1 stayed at 25%. The highest value (1.878) occurs in Italy 1895 when s_1 reached 86.2%. Both occur in early double ballot systems.
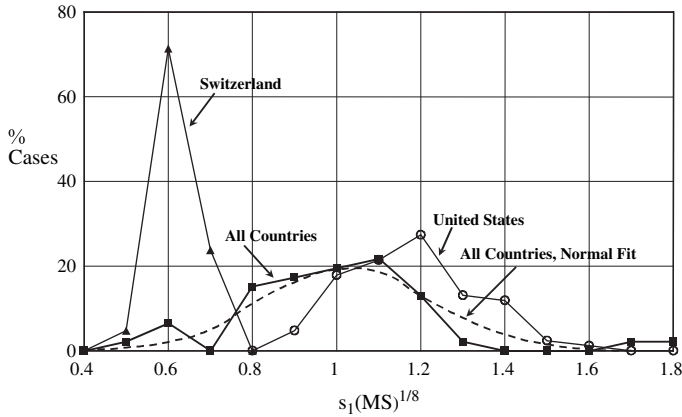
Fig. 2. Frequency distribution of the conserved quantity $s_1(MS)^{1/8}$ for the geometric means of all countries and for individual elections in selected countries.

District magnitude by itself has $R^2 = 0.432$, while assembly size taken alone has $R^2 = 0.277$.[14] However, when district magnitude and assembly size are both considered, $R^2 = 0.538$. The pattern resulting from the combined approach is shown in Fig. 3, as a dashed line.[15] How well does the theoretical equation ($s_1 = 1.000M^{-0.125} S^{-0.125}$) fit the actual data points? As is seen in Fig. 3, the R-squared drops only very slightly, compared to the best fit, from 0.538 to 0.509.

We conclude that the model passes the test with assembly size and district magnitude: (1) The geometric mean of $s_1(MS)^{1/8}$ for all 46 systems is within 1% of the expected value of 1.00. (2) The standard error for the dispersion of individual country means around this overall mean is only 21%. (3) The best fit (on log scale) is so close to the predicted line that the R^2 value for the predicted line is only marginally lower than for the best fit (0.51 vs. 0.54). This means that the model provides practically as good a prediction of the seat share of the largest party as does fitting with separate exponents for district magnitude and assembly size.

5. Factors other than district magnitude and assembly size

Do any other general factors apart from assembly size and average district magnitude affect the seat share of the largest party? If so, their impact would show up in the deviation of $s_1(MS)^{1/8}$ from the expected value of 1. Inspection of the five systems with markedly low values of $s_1(MS)^{1/8}$ (pre-WWI Germany and Netherlands, Switzerland, St. Kitts, pre-Castro Cuba) shows

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[14] Why is it that magnitude accounts for almost double of the variation, compared to assembly size, when both enter the model in a symmetrical form? It may be taken to confirm the predominance of Duvergerian processes. But two circumstantial considerations also enter. First, the range of variation of the dependent variable affects R^2. Assembly size varies only by a factor of $643/10.3 = 62$, from St. Kitts to UK (Table 1), while magnitude varies by a factor of 111.5, from SMD to the Netherlands in Table 2. Second, three widely deviant pre-WWI double ballot systems (Italy, Germany, Netherlands) markedly deflate R^2 for assembly (cf. Fig. 1). These three systems also affect the combined fit in Fig. 3.

[15] How can we graph the equation $s_1 = 0.847M^{-0.113}S^{-0.090}$ (with different exponents for M and S) against MS (where M and S have the same exponent)? A given value of $x = MS$ could represent combinations ranging from $M = 1$ and $S = x$ to $M = S = x^{0.5}$, with a mean of $M = x^{1/4}$, $S = x^{3/4}$. We used this mean, which leads to $s_1 = 0.847(MS)^{-0.096}$. The exponents -0.113 and -0.090 are sufficiently close to each other; hence, using even the extreme values would alter the value of $s_1 = 0.847M^{-0.113} S^{-0.090}$ by less than 3%.
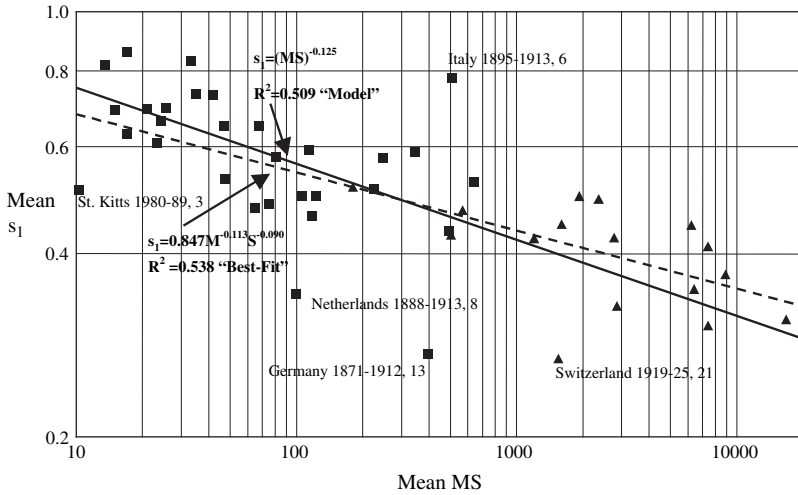
Fig. 3. Seat share of the largest party (s_1) vs. the product of district magnitude (M) and assembly size (S) for the full set of 46 systems. Squares: $M = 1$; Triangles: $M > 1$.

no obvious commonalties. The same is the case for the five systems with markedly high values of $s_1(MS)^{1/8}$ (pre-WW1 Italy, post-1969 Sweden, Spain, Botswana, Japan). In particular, no connection can be seen to country size or population, political culture, ethnic homogeneity, any particular electoral rule (except that double ballot is most erratic), or other institutional features. It seems that, once the effect of average M and S is factored out, the residue is largely due to path-dependent, and otherwise individual country characteristics.[16]

What about trends in time? If we sort the 46 systems by the center point of their duration, there is a hint of an increase over time, but it may be random fluctuation:

Time bracket	No. of cases	Mean $s_1(MS)^{1/8}$
1800s	3	0.886
1900–1950	12	1.024
1951–1975	12	0.999
1976 on	19	1.077

As a particular system matures, does the product $s_1(MS)^{1/8}$ tend to increase or decrease? Do its fluctuations tend to dampen? No such change is noted. Two of the longest time series (the US and Finland) go in opposite directions, as shown below.

Table 4 shows the pattern for the longest time series, the US. Four time periods of equal duration are distinguished. Assembly size increased markedly during the first two periods, but contrary to theoretical expectations, the geometric mean of s_1 did not increase. Instead, the geometric mean of $s_1(MS)^{1/8}$, rather than staying constant, has crept up, though only minimally (from 1.16 to 1.27). The same is broadly the case for minimum and maximum values during the periods. Steady state seems to have prevailed early on.

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[16] As pointed out in Note 9, the mean M could slightly overestimate the largest seat share, if there is a variation in magnitude across the districts. In the present case, such variance is non-zero only for the 10 multi-member systems in Table 2. These cases show no visible connection between variance in district magnitude, and the value of $s_1(MS)^{1/8}$.

Table 3
Regression constants for the three approaches run on the full set of 46 systems as logarithms

Regression approach	Logarithmic regression constants ($log\ s_1 = a_0 + b_1\ log\ M + b_2\ log\ S$)	Converted regression equation ($s_1 = aM^{b1}\ S^{b2}$)	R^2 for logarithms
(1) $log\ s_1$ vs. $log\ M$	$a_0 = -0.239$ $b_1 = -0.137$	$s_1 = 0.577M^{-0.137}$	0.432
(2) $log\ s_1$ vs. $log\ S$	$a_0 = -0.025$ $b_2 = -0.135$	$s_1 = 0.946S^{-0.135}$	0.277
(3) $log\ s_1$ vs. $log\ M$ and $log\ S$	$a_0 = -0.072$ $b_1 = -0.113$ $b_2 = -0.090$	$s_1 = 0.847M^{-0.113}\ S^{-0.090}$	0.538

Finland is the only country where two clear stances with different ranges of $s_1(MS)^{1/8}$ emerge, even while S and M did not change. It so happens that $s_1(MS)^{1/8}$ was markedly below the general expectation when the communist party (under whatever name) was allowed to participate in the elections, thus splitting the Left (1922−1929 and from 1945 on). During these times, $s_1(MS)^{1/8}$ ranged from 0.68 to 0.85, with a geometric mean of 0.75. In contrast, when the communist party was prohibited (1907−1919 and 1930−1939), $s_1(MS)^{1/8}$ was markedly above the general expectation, ranging from 0.90 to 1.4 (geometric mean: 1.14). We make note of this peculiarity without drawing any conclusions.

Does a change in electoral rules have an effect? When a country goes from single member districts to PR, its product MS expands markedly, and the largest party's seat share is expected to decrease. However, the large party's previous strength may have some staying power, so that this decrease may be less than predicted by conservation of $s_1(MS)^{1/8}$. If so, then this product would increase. Indeed, in all three cases of such a shift in electoral rules (Netherlands, Norway, Sweden), the largest party's seat share did decrease, but also $s_1(MS)^{1/8}$ increased by an average of 0.19. To draw any conclusions, more cases would be needed.[17]

6. Conclusion

This study has tested a model that predicts the seat share of the largest party on the basis of assembly size and district magnitude. As a worldwide average, the model holds within 1%. The standard error for individual country averages around this overall average is 21%, and $R^2 = 0.509$. Within this error range, the product $s_1(MS)^{1/8} = 1$ is a conserved quantity. This result is robust against deletion of individual systems, and hence is probably robust against further additions. For the atypical countries, the present results would motivate further inquiry into the political or socio-cultural causes for deviation. No predictive ability is claimed for individual elections.

6.1. What are the political processes involved, and what do the findings tell to the political practitioners and scientists?

To the political practitioner we can tell the following. If for any reason you wish to alter the degree of largest party predominance in your national assembly so that it would hover around

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[17] Denmark and Germany also offer periods of $M = 1$ and $M > 1$, but the two are separated by a long interval during which M was difficult to evaluate. Changes of rules within the SMD framework (Australia, Norway) or the PR framework (Netherlands, Sweden) reduced $s_1(MS)^{1/8}$ for Norway while increasing it for the three other countries.

Table 4
The United States: Changes in the product $s_1(MS)^{1/8}$ over two centuries

Period, and number of elections	Assembly size	Largest share (mean)	$s_1(MS)^{1/8}$		
			Minimum	Geom. mean	Maximum
1828–1868, 21	184 to 237	0.591	0.984	1.161	1.497
1870–1910, 21	243 to 391	0.575	1.021	1.189	1.477
1912–1952, 21	435 to 437	0.589	1.061	1.259	1.643
1954–1994, 21	435	0.595	1.136	1.273	1.456

a desired average, your best bet is a specified combination of assembly size and district magnitude, corrected for your country's previous deviation from the world average—as long as no other features of electoral law blunt the impact of magnitude. Some political practitioners do not take political science seriously, partly because of the dearth of specific predictions it offers, as compared to other fields of science (or in the case of social sciences—economics). This study does offer specific numbers for such political engineering. The practitioner will not ask why a formula works, if it works, and is reliable.

For the political scientist the "why?" matters. In principle, our model need not work at all. It only expresses the best probabilistic guess, *in the absence of any other information*, except assembly size and district magnitude. Plenty of other political or cultural factors of a universal nature could tilt the outcome away from this probabilistic best guess. The wonder is that they do not, for the worldwide average. Even the individual countries deviate from this expectation with a standard error of only 21%. What does it tell us?

It tells us that institutions really matter for some purposes. Some of their effects can be predicted more precisely, going beyond a fuzzy description of Duvergerian processes. True, assembly size and district magnitude are themselves creatures of political circumstances and processes (even while population places limitations on choice of assembly size). But once they are put in place, the path toward determining the average largest seat share seems to be pretty much on autopilot. The mechanisms within the autopilot "black box" follow from the basic Duvergerian notion that size of representational units matters. To include not only district magnitude, but also assembly size is a relatively minor extra step.

These mechanisms have been covered by Cox (1997) and others, and will not be re-analyzed here. The essential point is that only institutional-based processes enter the determination of the average largest share in a predictable way—no processes or factors separate from assembly size and district magnitude have as yet been located. Country-specific, and possibly path-dependent factors, are of a postdictive nature. Politics enters again directly in the outcomes of individual elections, which are only loosely restrained by the institutional pressures. The overall sequence could be expressed as follows:

Politics etc. → Institutions (assembly size and district magnitude) → *Duvergerian mechanisms on autopilot* → Mean largest share → *Politics* → Largest seat share in given election → *Politics*.

This type of study seems to cause unease to some students of politics. The feeling may be that it indulges in playing with numbers in a mechanical way, with no political intuition or substance, and that it has nothing whatsoever to do with party competition as normally understood by political scientists. It may be felt that the method of analysis is unconvincing, even if the results seem to fit.

Once one feels that the model should not fit, one may look for reasons why it could happen to fit for wrong or artificial reasons. Is the number of cases sufficient? But 46 periods in 37

countries is not an unusually small number in a world where only 36 countries are presently stable democracies by Lijphart's (1999) criteria. Are the case selection criteria haphazard? But essentially all periods in stable democracies were included, excluding only those where other features of electoral rules override the impact of district magnitude. Has the study failed to include other factors that might affect the largest seat share? There are plenty that *might*—but failure to include significant factors would blur the results, not make them more clear-cut. And are the input variables really dependent, and the output ones independent? But the formulation $s_1(MS)^{1/8} = 1$ allows for interdependence without rigid directions.

The one critique that we are powerless against is that the study is not politically literate in spelling out the Duvergerian processes that lead from assembly size and district magnitude to the largest seat share. This study is limited, indeed, to testing a precise connection between the inputs and outputs of the Duvergerian "black box", without philosophizing about its contents. We observe that the relationship $s_1(MS)^{1/8} = 1$ does fit, even when by some canons of political research it should not.

Appendix: Derivation of $s_1 = (MS)^{-1/8}$

This derivation is based on Taagepera and Shugart (1993), but is presented in a somewhat different way.

Consider the number of parties (p') that could win seats in a district of magnitude M. At the least, 1 party must win seats (all of them, in this case). At the most, M parties could win one seat each. The actual number could be anything from 1 to M, if nothing else is known but M. Our ignorance seems complete. Yet we do know something very important, namely that the number of seat-winning parties cannot be smaller than 1 nor larger than M.

If nothing else is known, except for the lower and higher limits of what is possible, the best guess for p' is the one that equalizes the possible error upwards and downwards. This means that the factor by which the upper limit exceeds p' should equal the factor by which p' exceeds the lower limit: $M/p' = p'/1$. Hence $p' = M^{1/2}$, the geometric mean of the limits. As an illustrative example, the Netherlands 1918−1952 had 9 elections where the entire country was a single district of $M = 100$. The model predicts that $100^{1/2} = 10$ parties would win seats. The actual range was 8 to 17, with a geometric mean (10.4) close to the predicted.[18]

Next, consider the number of parties (p) that could win seats in an assembly of S members elected in districts of magnitude M. The nationwide number of parties is at least equal to the number in a district, which can range from 1 to M. It is at most equal to what it would be, if the entire country were made a single district of magnitude S; then the number of seat-winning parties could take the wider range from 1 to S. If nothing else is known besides M and S, the best guess for p is the one that balances the possible upward and downward errors for this set of 2-by-2 constraints (1 and M; 1 and S). This means taking their geometric mean: $p = (MS)^{1/4}$.

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[18] As an alternative approach, consider the expected *number of seats* per party (m'). It could range from 1 (when M parties win 1 seat each) to M (when one party wins all the seats). The previous reasoning leads to the expectation $m' = M^{1/2}$. Then the product is $p'm' = M$, as it should. The two approaches lead to mutually congruent results. Such congruence is not to be taken for granted. Suppose someone argued that the likeliest p' is the arithmetic mean of 1 and M, so that $p' = (M + 1)/2$. By the same reasoning, the likeliest m' would be $m' = (M + 1)/2$. But then the product is $p'm' = (M + 1)/4$, which exceeds M whenever $M > 1$. In the illustrative example of the Netherlands, with $p' = 50.5$ parties winning seats and $m' = 50.5$ seats per party, the product would be 2550 seats, much above the actual 100. Thus, the approach by arithmetic means leads to incongruent results.

As an illustrative example, Malta 1947−1955 had 5 elections with $S = 40$ and $M = 5$ in all districts. The model predicts that $(5 \times 40)^{1/4} = 3.76$ parties would win seats The actual range of the number of seat-winning parties, nationwide, was 2 to 6. The geometric mean (3.73) was close to the predicted.[19]

Now consider the *number* of seats (S_1) going to the largest among these $p = (MS)^{1/4}$ parties. If all parties have equal shares, $S_1 = S/p$, and if all other parties have only 1 seat, $S_1 = S - p + 1$ can be approximated as $S_1 = S$. If nothing else is known, the best guess for S_1 is the one that equalizes the possible upward and downward errors between the limits S/p and S. Hence, $S_1 = S/p^{1/2}$.

The *fractional* seat share (s_1) of the largest party is $s_1 = S_1/S = 1/p^{1/2}$. Since $p = (MS)^{1/4}$, the expected largest share is $s_1 = (MS)^{-1/8}$, so that $s_1(MS)^{1/8} = 1$. In view of the approximation *max* $S_1 = S$, this is an overestimate when assemblies are small. At $M = 1$ and $S = 10$, the more precise expectation would be $s_1(MS)^{1/8} = 0.98$.

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[19] Here too, an alternative approach in terms of the nationwide number of seats per party could be used. Once more, an approach by arithmetic means would lead to inconsistency.