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A VOLTAGE INTEGRATOR FOR MAGNETIC FIELD MEASUREMENTS



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L. Jones and J. W. Rose

February 13, 1952

Berkeley, California

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# A VOLTAGE INTEGRATOR FOR MAGNETIC FIELD MEASUREMENTS

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## I. INTRODUCTION

The integrator described here was designed and built at the Radiation Laboratory to replace point by point measurements of magnetic fields. Since the e.m.f. induced in a coil is proportional to $d\Phi/dt$ (where $\Phi$ is the magnetic flux through the coil), a device which will give an output proportional to the time integral of the e.m.f. across the coil can be used to plot magnetic fields directly. This integrator has been used to make such plots to an accuracy of one percent over the range of 500 to 15,000 gauss.

## II. BASIC THEORY OF OPERATION

To understand the basic operation of the voltage integrator to be described one can first consider a simple RC integrating circuit as shown in Figure 1. The condenser C acts as a current integrator, the voltage across it being proportional to the accumulated charge.

$$V = q/c = 1/c \int i \, dt$$

To use this circuit to integrate voltages, one desires a charging current proportional to input voltage. Since

$$e_{in} = V_R + V_C$$
$$= iR + V_C$$

as $V_C$ becomes comparable to $V_R$, i will no longer be an accurate measure of $e_{in}$. If at t = o $e_{in}$ is raised from 0 to $e'$ and held constant as a function of time (i.e. a step function), the output voltage across the condenser as a function of time will be

$$e_o = e'(1 - \varepsilon^{-t/RC}) \qquad \text{(cf. Fig. 2)},$$

or expanding

$$e_o = e' \frac{1}{RC} (t - 1/2 \frac{t^2}{RC} + \ldots),$$

which shows the integration to be accurate only for times t, where $t^2 << RC$.

One would like a circuit in which the charging current is maintained equal to $e_{in}/R$ but in which the voltage across the condenser is still equal to the integral of the current. This is provided in the Feedback Amplifier RC integrator shown in Fig. 3. In this circuit the condenser appears as a feedback loop across a high gain amplified. If the amplifier is assumed to have infinite gain, then $e_g$ the input voltage to the amplifier will be held constant. Then since the voltage at one end of the condenser is held fixed we have for the current into the condenser $i = - C\frac{de_o}{dt}$, and if we assume the amplifier draws no grid current this is the current through R.

$$e_{in} = Ri = -RC\frac{de_o}{dt}$$

Hence

$$e_o = -1/RC\int e_{in}\,dt + \text{const.}$$

and the integration is completely accurate.

It is feasible to obtain what amounts to infinite gain and hence perfect integration by the use of an amplifier with positive regeneration. The inverse feedback through the condenser gives the amplifier great stability and freedom from oscillation.

## III. DESCRIPTION OF THE UNIT

The integrator unit is housed in a standard five foot relay rack. The rack contains the integrator chassis itself, an e.m.f. panel, a regulated plate voltage supply and a filament supply for the integrator tubes. The circuit of the integrator is given in Fig. 6, the e.m.f. panel circuit in Fig. 7, and the power supply circuits in Fig. 8. A block diagram showing the interconnections of these units is given in Fig. 9.

The integrator chassis has two panel meters, a 0-500 microammeter in the plate lead to the cascoded stage, and a 0-5 voltmeter across the filament of the 6SU7. The off-on switch, a high-low sensitivity switch which changes the size of the integrating condenser, and the regeneration control are brought out to the front of the panel.

The plate power supply consists of a standard 550 volts 50 ma d.c. power supply with a condenser input filter connected to two sets of voltage regulator tubes through appropriate resistors. The first set of regulator tubes (2 VR150 s and a VR105 in series) hold the output of the power supply through a resistor to a constant 405 volts. The second set of regulators (three VR105 s in series) connected through a resistor to the first set removes any remaining voltage fluctuations and delivers a constant 315 volts to the integrator. The two sliding tap resistors should be adjusted so that each string of VR tubes draws no less than 5 ma and no more than 35 ma when the input line voltage is varied between 105 and 125 volts.

The filament voltage to the integrator is supplied by a Mallaroy 6 volt battery charger which in turn is supplied by a 30 VA Sola constant voltage transformer. A resistor in series with the charging line regulates the output voltage so that the charging current through the two storage batteries, placed as a "ballast" across the line, is zero. The batteries act in the same way that the voltage regulator tubes act: if the voltage drops lower than their charge, they discharge through the line to maintain it, if the voltage is raised, they draw charging current through the resistor, hence dropping the voltage to its stable value. A specific gravity of 1.2 is found to be a stable operating point for the battery.

The purpose of the e.m.f. panel is to supply a constant grid bias to the first stage of the 6SU7. For a given set of operating conditions (plate voltage, etc.) there is a value of grid voltage which "balances" the integrator. If this voltage is held unchanged the output voltage will remain constant. Any value of voltage on the grid higher or lower than this value will be integrated. The circuit of the e.m.f. panel includes potentiometers labeled "Balance Control", "Course Balance", and "Fine Balance". As seen in the circuit (Fig. 7) these are in voltage divider and Wheatstone bridge circuits so that any desired voltage in the range -1.5 to + 3.0 volts may be applied to the grid. Under usual operating conditions the grid is at about + 1 volt. Provision is also made on the e.m.f. panel for applying test voltages to the integrator (slow and fast buttons) and for shorting the output.

## IV. ADJUSTMENT AND OPERATION

With the batteries shown on the circuit in place, the filament supply, the 315 volt power supply, and the e.m.f. panel may be turned on. The integrator should be allowed to "warm up" before attempting to get reproducible results from it. Although we have never critically examined the exact time required, 15 minutes appears to be about a minimum.

The output can be connected to any relatively high impedance meter or recorder, either directly or through a voltage divider (if a reduced sensitivity is desired). A 160 volt battery is in series with the output lead so that the output under normal operation varies about zero volts rather than about a point 160 volts positive. We have used the unit successfully into a 30,000 ohm load, although working into such a low impedance requires readjusting the regeneration from its setting for higher impedance loads. We have used a Leeds Northrup Speedomax recorder as the output measuring device.

Balancing the integrator to the zero output drift value of grid voltage is at first tricky because of the apparent long time constant of the circuit. At balance the microammeter on the integrator panel reads between 250 and 300 microamperes. By adjusting the "Balance Control" and intermittently pressing the output short switch a point can be found where the output (as seen on an oscilloscope or zero-center meter) remains zero volts after the shorting switch is released. On a more sensitive output scale the output will then be seen to drift. By further adjusting the e.m.f. controls the bias for zero output drift may be accurately set. The output may be adjusted to zero volts then by depressing the "fast" or "slow" buttons + or - until the output is moved to zero volts or any other convenient reference.

This integrator, like any other sensitive d.c. amplifier, has, as its greatest deterent to unlimited sensitivity and reproducibility, drift. Drift of the output voltage when no signal is applied to the input and the integrator is accurately balanced is most often caused by the gradual change of some voltage or resistance in the unit. In order to minimize such voltage fluctuations the power supplies to filaments and plates of the integrator have been stabilized, and dry batteries used in all other circuit elements where a very low current is drawn (e.m.f. panel and output bucking supply). In spite of these precautions some drift occurs, and an operator must periodically readjust the balance to zero drift. The circuits containing resistors R4 and R7 (both brought to the front panel for screw driver adjustment) are intended to minimize the drift of the output due to fluctuations in filament emission, etc.

The regeneration control of the integrator should be set so that there is no drift in output along any part of the output range. With the integrator balanced for zero volts output, there will be a negative drift back toward zero if the output is driven positive (and a + drift if the output is driven -) when the regeneration is too low. Conversely, the output will tend to drift away from zero for too much regeneration. The regeneration is most easily set by balancing to zero drift at one extreme of the range, and then integrating (using the "fast" buttons on the e.m.f. panel) to the other extreme of the range and observing the drift. The regeneration may be reset and the process repeated until no drift is seen at either end of the range.

## V. CHARACTERISTICS

As the integrator appears in the circuit diagram, two integrating condensers are provided: a 0.1 microfarad and a 0.01 microfarad paper condenser with a sensitivity switch to switch from one to the other. All of the remarks concerning characteristics refer to the 0.1 microfarad condenser (low sensitivity).

If the output of the integrator is expressed in volts as a function of time, it may be related to the e.m.f. applied to the grid as follows:

$$E_o(t) = k\int_0^t E_i(\tau)d\tau + \text{constant}$$

where 0 time is taken as any time before $E_i$ was applied, and $E_i$ is an e.m.f. different from the balance e.m.f. For this integrator, k = 4 if t is expressed in seconds.

If there were a drift in the grid bias voltage such that $E_i(t) = e\tau$ (where e is some constant rate of drift), the above expression integrates to:

$$E_o(t) = k\,e\,\frac{t^2}{2}$$

We have found that drifts in any of the voltage supplies looks like a drift in grid e.m.f. For instance, if $E_o$ for constant $E_g$ is expressed as a function of plate supply voltage, $V_p$, and $V_p$ is drifting at a constant rate $\frac{dV_p}{dt}$, $E_o$ obeys a relation:

$$E_o(t) = k'\left(\frac{dV_p}{dt}\right)\frac{t^2}{2}$$

where $k' \doteq 0.02\ \text{sec}^{-1}$. A drift of one to two volts per hour in the 315 volt plate supply therefore leads to a bad drifting of the output. For this reason it was necessary to use precision resistors with low temperature coefficients in the integrator circuit and to take pains to regulate the supply voltages.

The output of the integrator should in principle be linear (the factor k relating $E_o$ to $\int E_i dt$ should remain constant throughout the region of "infinite" gain (as described in section VI on theory). For this unit, that region extends from + 90 volts output to + 230 volts output as measured at the 6SN7 cathode. In practice, linearity should be checked over the range actually used with some type of reproducible input e.m.f. A permanent bar magnet moved through a test coil a given distance and then returned with the coil shorted makes a convenient, simple, reproducible source of constant $\int E_i dt$. As usually used, the output varies over a range of only 10 or 20 volts. Within this range the unit seems linear to within a percent.

R4, which controls the cathode bias, and R7, which controls the filament voltage, both affect the gain of the 6SU7. As explained in the section on theory of operation, greatest stability against filament emission fluctuations occurs when R4 is adjusted for maximum $i_p$. This adjustment may be made by opening the integrating condenser (the high-low sensitivity switch has an intermediate neutral position) and turning the regeneration to zero. Then, for any filament voltage, R4 may be varied throughout its range and $i_p$ (the panel microammeter) and $E_o$ observed. For a range of values of filament voltage (3.3 to 4.0 volts for the unit studied), values of R4 can be found for which $i_p$ is a maximum and $E_o$ a minimum. Any of these points is in principle a stable operating point. With the regeneration at a finite value, however, current flows in R6 to the second 6SN7 cathode, hence raising the cathodes of the 6SU7 to a slightly higher voltage than was used in the test above described. Therefore $i_p$ should be noted for its maximum value during the test, and then with regeneration, $E_g$ should be readjusted (with the e.m.f. panel) for the same $i_p$ to assure return to the same grid-cathode bias. In any event, the final criterion for optimum adjustment of R4 and R7 should be stable operation with minimum drift.

## VI. THEORY OF DESIGN AND OPERATION

### A. Filament Voltage Fluctuations

The input tube of the integrator is a 6SU7 twin triode connected in a circuit due to Miller* so as to reduce any drift caused by fluctuations in cathode emission. Such variations in emission would most probably be due to heater voltage fluctuations.

Changes in cathode emission can be represented by an e.m.f. V at the cathode as shown in Fig. 4. This e.m.f. V will in general produce a change $e$ in the cathode to ground voltage. We wish to find conditions such that $e = 0$.

Let i = change in plate current in $V_2$ due to V. Then at the grid of $V_2$

$$e_g = i R_2$$

and if $\mu$ and $r_p$ are the amplification factor and plate resistance of $V_2$ $\mu e_g = \mu i R_2 = i r_p$, since the only voltage drop is produced across the tube if $e = 0$. We see that the condition for compensation is

$$R_2 = \frac{r_p}{\mu} = 1/g_m.$$

The net result is that as the emission varies the grid voltage of $V_2$ varies in such a way as to route the corresponding current variations through $V_2$ and leave the current through $V_1$ unaffected.

In practice one adjusts $R_1$ until $1/g_{m_2} \rightarrow R_2$. This condition corresponds to the value of $R_1$ for which the plate current in $V_1$ is at a maximum. This may be seen by studying Fig. 5. Proper adjustment of $R_1$

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* S. E. Miller, Electronics, 14, No. 11, 27 (1944)

gives a minimum value to the length AC which is the cathode to ground voltage, since AB = $V_{R_1}$ and BC = $V_{R_2}$. Then $V_1$ will have a maximum grid to cathode voltage and hence maximum current.

B. Cascode Amplifier

The use of cascode connection, plate tied directly to cathode of following tube, in a d.c. amplifier enables one to use two tubes to get higher gain than with just one, but without need of extra bias voltage supplies. A cascode circuit is shown in Fig. 10. The upper tube $V_2$ has a fixed grid so that the voltage of its cathode is held fairly constant. If the cathode voltage drops, the grid to cathode voltage increases, more current flows and the i R drop in $V_1$ below returns the cathode to its normal value. Now in general the gain of a tube connected with the load resistance R in its plate circuit is given by

$$G = \frac{-\mu r_p}{R + r_p} = \frac{-1}{\frac{1}{\mu} + \frac{1}{g_m R}}$$

where $r_p$, $\mu$, and $g_m$ are the plate resistance, amplification factor and transconductance. If $e_{p_1}$ were held absolutely constant the current gain of $V_1$ would be $g_m$, since $g_m = \left(\frac{di}{de_g}\right)_{e_p \text{ constant}}$, and the voltage gain would be $- g_m R$ as for a pentode where $\mu \gg g_m R$. Actually the plate load of $V_1$ is the cathode input impedance of $V_2$ which is $\frac{r_{p2} + R}{\mu_2 + 1} = z$.

Then $\Delta i_p = \frac{\Delta e_{p1}}{z + r_{p1}}$

and the current gain is $\frac{\Delta i_{p1}}{\Delta e_{g1}} = \frac{\mu_1}{r_{p1} + \frac{r_{p2} + R}{\mu_2 + 1}}$

The Voltage gain is $G = \frac{\Delta e_{p1}}{\Delta e_{g1}} = \frac{-\mu_1 R}{r_{p1} + \frac{r_{p2} + R}{\mu_2 + 1}}$

For $R >> r_{p_2}$ $\quad G \longrightarrow - \frac{1}{\frac{1}{g_{m1} R} + \frac{1}{\mu_1 \mu_2}}$

and we see $\mu_1 \mu_2$ replaces $\mu$ in the usual expression for the gain.

C. Regeneration

If one observes the output voltage of the integrator as a function of the voltage at the input grid of the amplifier one obtains curves such as those in Fig. 11 depending upon the regeneration. The slopes of the curves in the usual operating range may be explained by the following.

Let A be the gain of the amplifier without feedback and $\beta$ be the fraction of the output returned to the input to give positive regeneration. (cf. Fig. 12)

Then $\Delta E_o = - A (\Delta E_g - \beta \Delta E_o)$

or $\Delta E_g = (\beta - 1/A) \Delta E_o = - \frac{\Delta E_o}{G}$ where G = gain with feedback.

For $\beta = 1/A$ or $G = \infty$ we have correct regeneration and $E_g$ is constant in the operating range. For G finite and positive we have under regeneration; for G finite and negative we have over regeneration.

D. Drift

To balance the circuit against drift a certain constant input voltage has to be applied. Let us suppose $E_{in}$ is adjusted to a constant value $E'_{in}$ which balances the circuit against drift. Then let us suppose an increment of voltage is momentarily applied and then that $E_{in}$ is returned to its balance value $E'_{in}$. This will produce a shift in the output voltage $E_o$ and if the regeneration is not correct it is found that $E_o$ will continue to change even after $E_{in}$ is returned to $E'_{in}$. This drift may be analyzed as follows.

Let

$E_{in} = E'_{in} + e_{in}$ where $E'_{in}$ is the input voltage at the balance point

$E_g = E'_g + e_g$ " $E'_g$ " " grid " " " " "

$E_o = E'_o + e_o$ " $E'_o$ " " output " " " " "

$V_c = V'_c + v_c$ " $V'_c$ " " condenser " " " " "

It is to be noted that $E'_g = E'_{in}$ if there are no leakage currents. Now let us take all zeros of voltage, charge etc. at the values at balance. Define polarity of $v_c$ as in Fig. 13. If after the shift in output there is drift there will be current flow through R into C.

Summing voltage drops:

$$0 = i R + v_c + e_o \qquad (1)$$

$$\text{or } e_g = v_c + e_o \qquad (2)$$

But by the preceding section

$$e_g = -\frac{e_o}{G} \qquad (3)$$

Then

$$v_c = -\left(1 + \frac{1}{G}\right) e_o \qquad (4)$$

Substituting into Eq. (1)

$$i\,R + \frac{v_c}{1 + G} = 0 \qquad (5)$$

Differentiating: $(V_c = q/c)$

$$\frac{di}{dt} + \frac{i}{RC\,(1 + G)} = 0 \qquad (6)$$

Solving:

$$i = (\text{const.})\ \varepsilon^{-\,t/RC\,(1 + G)}$$

$$\text{or} \quad i = -\frac{e_g'}{R}\ \varepsilon^{-\,t/RC\,(1 + G)} \qquad (7)$$

where $e_g'$ is the value of $e_g$ at $t = o$ where t is measured from time when $E_{in}$ is returned to its balance value.

But $e_g = -iR$ and hence

$$e_g = e_g'\ \varepsilon^{-\,t/RC\,(1 + G)} \qquad (8)$$

But since $e_g = -\frac{e_o}{G}$ we have finally:

$$e_o = e_o'\ \varepsilon^{-\,t/RC\,(1 + G)} \qquad (9)$$

where $e_o'$ is the value of $e_o$ at $t = o$

when $E_{in}$ is returned to its balance value $E_{in}'$.

For $G = \infty$, (correct regeneration) $e_o = e_o' =$ constant and there is no drift. For G finite and positive (under regeneration) $e_o \longrightarrow o$, as t increases, and the output returns to the balance value. For G finite and negative (over regeneration) $e_o \rightarrow \infty \times e_o'$, as t increases, i.e. the output drifts in the direction away from the balance point.

BIBLIOGRAPHY

1. W. C. Elmore and M. Sands, Electronics, New York, McGraw-Hill, 1949.

2. S. E. Miller, "Sensitive D.C. Amplifier with A.C. Operation", Electronics, 14, No. 11, 27 (1941).

3. G. E. Valley and H. Wallman, Vacuum Tube Amplifiers, New York, McGraw-Hill, 1948.

4. G. A. Korn, "Design of D.C. Electronic Integrator", Electronics, 21, No. 5, 124 (1948).

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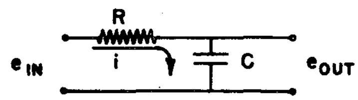


FIG. 1

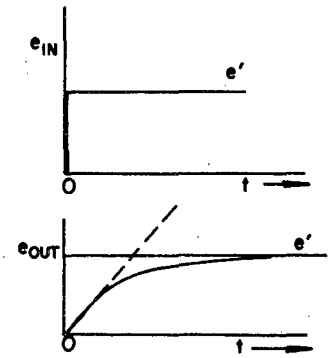


FIG. 2

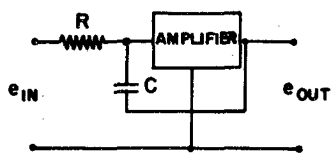


FIG. 3

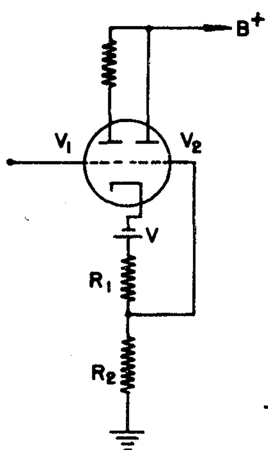


FIG. 4

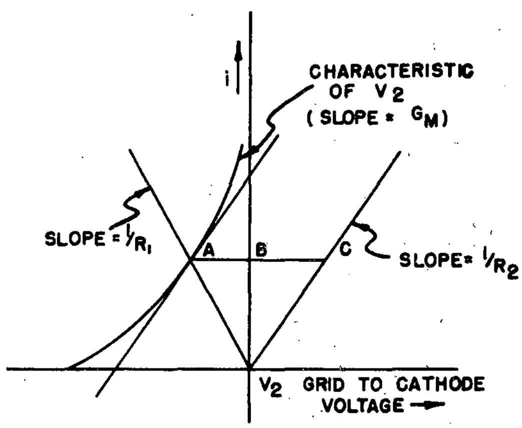


FIG. 5

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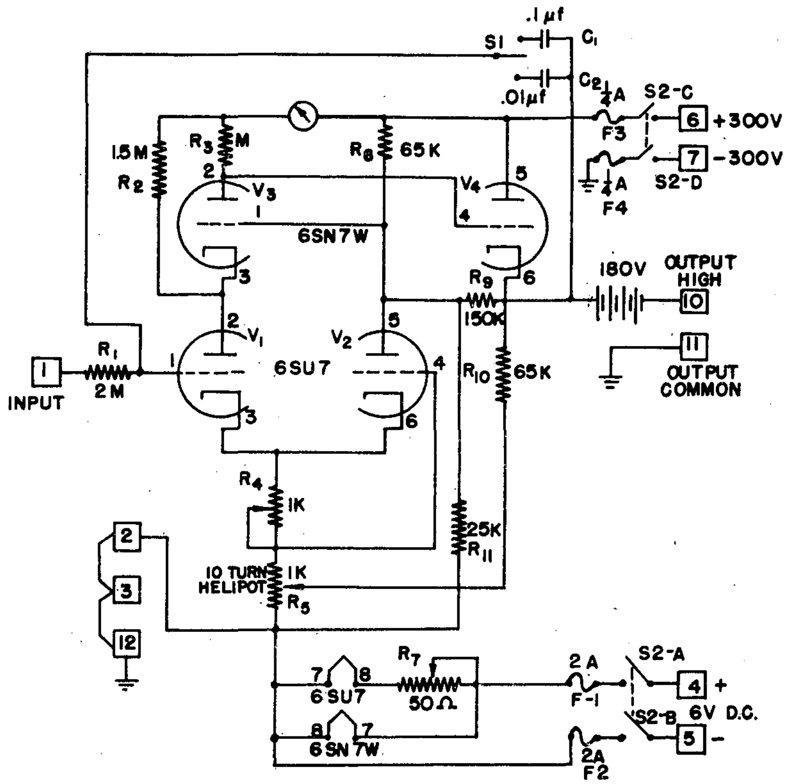


FIG. 6

CASCODE INTEGRATOR

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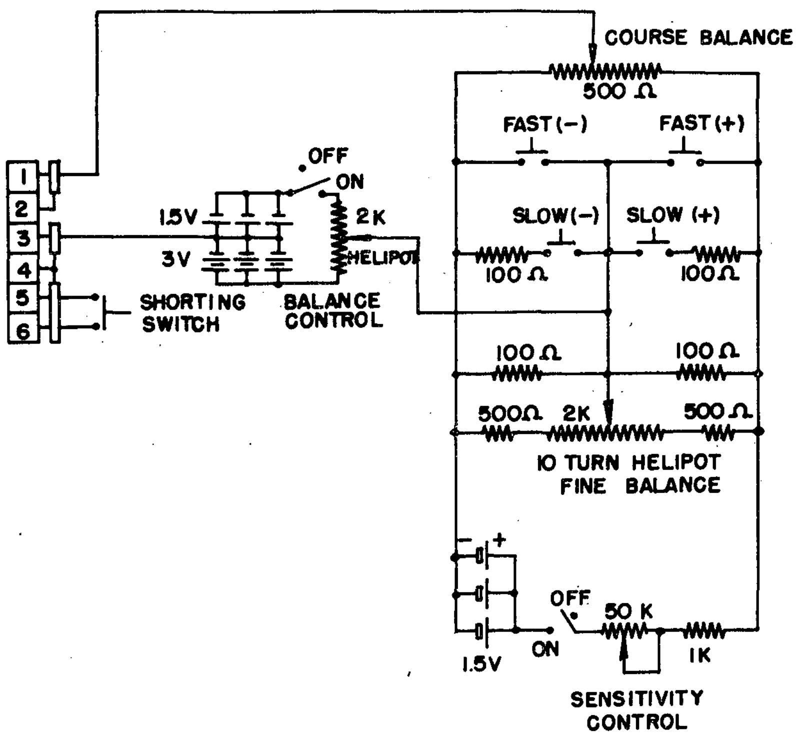


FIG. 7

EMF PANEL

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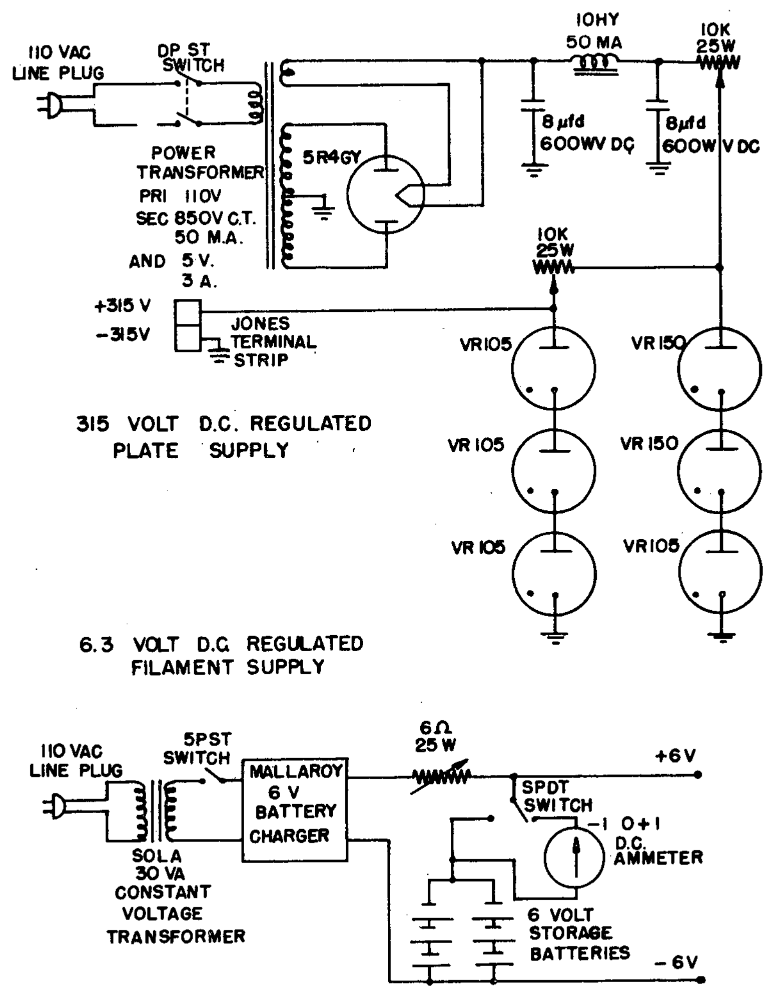


FIG. 8

POWER SUPPLIES

MU 3222

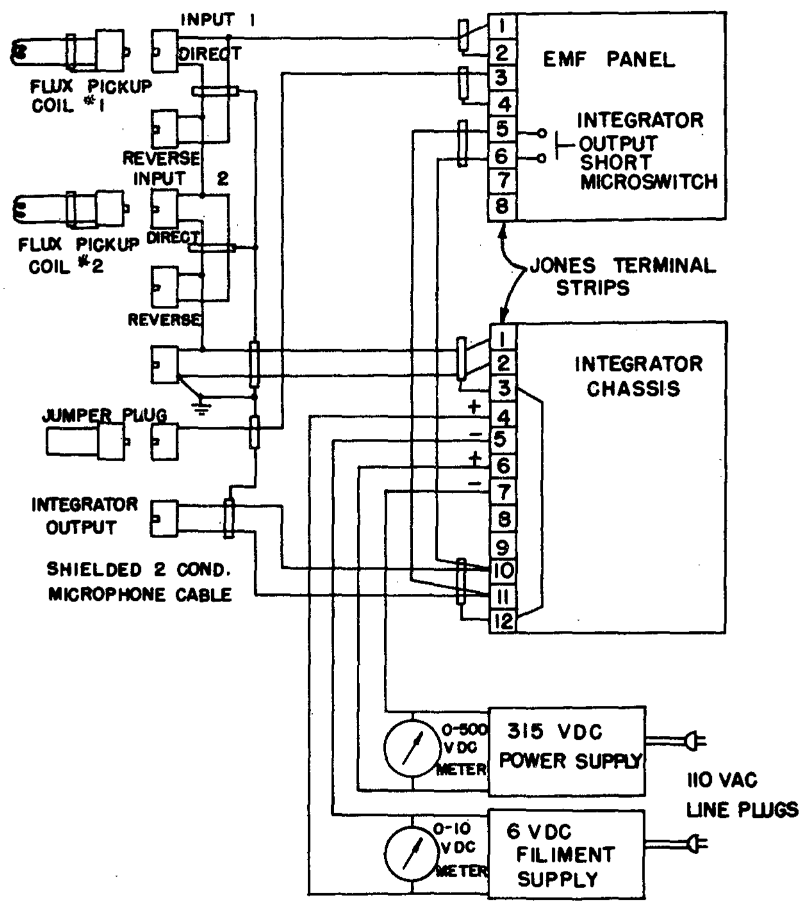


FIG. 9

BLOCK DIAGRAM OF INTERCONNECTIONS

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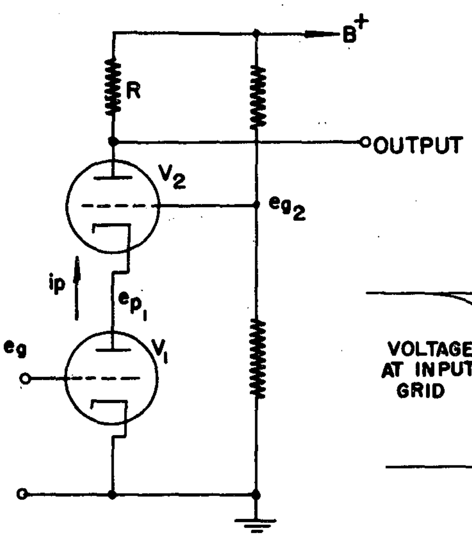


FIG. 10

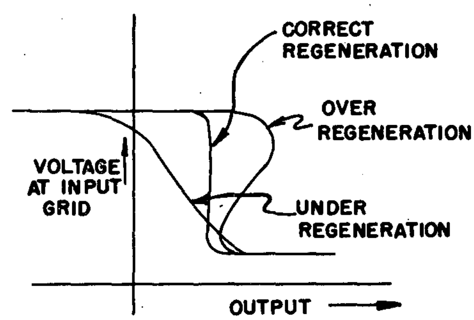


FIG. 11

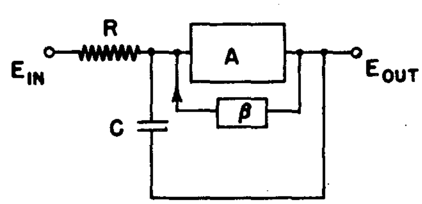


FIG. 12

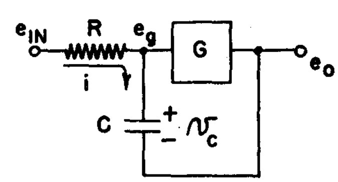


FIG. 13

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