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Contracting for Collaborative Services

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In this paper, we analyze the contracting issues that arise in collaborative services, such as consulting, financial planning, and IT outsourcing. Analyzing first a bilateral relationship, we assume that neither the buyer's nor the vendor's efforts are directly observable, resulting in double moral hazard. We investigate the efficiency of fixed-fee, time-and-materials, and performance-based contracts. We find that fixed-fee contracts are the least responsive to unplanned contingencies, time-and-material contracts are associated with high monitoring costs, and performance-based contracts do not incentivize agents to exert high levels of effort. We then show that our results are robust with respect to the number of vendors involved in the joint production process. On the other hand, the involvement of multiple buyers in the joint-production process creates an additional negative externality, akin to free riding, unless the vendor prescribes all buyers' actions. Our model highlights the trade-offs underlying the choice of contracts in a collaborative service environment and identifies service process design changes that improve contract efficiency.

Key words: services, consulting, contracting, principal/agent models

1. Introduction

Business services, despite their spectacular growth in 1992-97 (Apte and Nath 2006), are currently facing many challenges (Toppin and Czerniawska 2005, The Economist 2007). Alongside the globalization of clients, the fierce competition from emerging economies, and the blurring of the boundaries between integrators and specialists, the nature of the client relationship has also changed. In particular, "consulting is no longer simply about a relationship between clients and consultant, but about more complex issues. Clients and consultants are far more interdependent than they once were" (Toppin and Czerniawska 2005).

Consider a consulting firm that is working on a proposal. Which contract type should it offer? Fixed-fee, time-and-materials, or performance-based pricing? Clearly, the choice of contract depends on project complexity, monitoring costs, and incentive alignment, as well as on the consultant's and client's respective skills and costs. There exist however very few guidelines for choosing a particular type of contract over the others.

The goal of this paper is to offer a formalized model of contracting, to identify the key trade-offs underlying the contract choice, and to suggest strategies to improve contract efficiency. We analyze a production process involving collaboration between a buyer and a vendor that occurs in many service systems (Fuchs and Leveson 1968). Neither the buyer's nor the vendor's efforts are directly observable, resulting in double moral hazard (Bhattacharyya and Lafontaine 1995). We investigate the efficiency of fixed-fee, time-and-materials, and performance-based contracts. Fixed fee contracts (FF) consist only of a fixed transfer payment. Time-and-materials contracts (TM) include a remuneration per unit of effort in addition to a fixed transfer payment. Finally, performance-based contracts (PB) consist of a fixed fee, a remuneration per unit of effort, as well as a share of the total outcome (or a proxy).

We also investigate the implications of having multiple buyers or multiple vendors in the joint-production process. The involvement of multiple buyers creates a negative externality on the total output, similar to free riding or social loafing (Latané et al. 1979), lowering the buyers' incentives to exert large amounts of effort. On the other hand, the presence of multiple vendors does not alter the structure of the contracts.

The paper is organized as follows. We review the related literature in §2. Section 3 introduces the model and discusses its assumptions. In §4, we analyze the efficiency of the contracts in a bilateral environment, suggest improvement opportunities, and discuss how our results would help a firm select the best contract to meet a client's needs. Section §5 demonstrates that the involvement of multiple buyers creates an additional negative externality, called social loafing; by contrast, §6 shows that our results are robust with respect to the number of vendors. We present our conclusions and directions for future research in §7. All proofs appear in the electronic companion paper.

2.   Literature Review

The information service sector, in particular business services and medical and educational services, was the fastest growing sector of the US economy in 1992-97 (Apte and Nath 2006). Despite the importance of this sector, research on service operations has primarily focused on the physical aspects of services including queuing models (Allon and Federgruen 2007), logistics (Bramel and Simchi-Levi 1997), retailing (Fisher and Raman 1996), and revenue management (Talluri and van Ryzin 2004). With the exception of financial services (Melnick et al. 1999), service operations research on "white-collar" jobs is indeed still in its infancy (Hopp et al. 2007).

This paper analyzes the contractual arrangements arising in a collaborative, business-to-business setting. An early discussion of joint production in services is that by Fuchs and Levenson (1968). More recently, Karmarkar and Pitbladdo (1995) present a framework to analyze service processes, service competition, and its implications for service strategy. In this paper, we pursue this topic in a bilateral setting, investigating the efficiency of different types of contracts, accounting for project uncertainty and double moral hazard.

The study of contracting has a long history in economics, in areas such as government procurement contracts (Laffont and Tirole 1993), employer-employee relationships (Laffont and Martimort 2002) and make-or-buy decisions (Williamson 1985). Contracting often has to deal with either adverse selection (when one of the parties has private information about her costs) or moral hazard (when one party's efforts are unverifiable by the other party). Adverse selection can be mitigated by screening the buyers and/or vendors through competitive bidding and menus of contracts. Moral hazard, on the other hand, can be addressed by offering the agent a contract that links his compensation to the outcome. Inefficiencies however arise when the agent is risk-averse or has limited liability (Laffont and Martimort 2002) or when the agent's performance measure is distorted (Baker 2002). See Gibbons (2005) for an overview of recent developments in principal-agent models

In operations management, the study of contracts has been more detail-oriented, not only capturing the operational characteristics of the environment (e.g., stochastic demand, capacity constraints) but also highlighting its implementation challenges (Krishnan et al. 2004). While the study

of incentives in operations initially focused on cross-functional coordination (Porteus and Whang 1991, Kouvelis and Lariviere 2000), its scope has recently been enlarged outside the boundaries of the firm, in decentralized supply chains (Lariviere 1999, Cachon 2003, Taylor and Plambeck 2007).

In contrast to the literature on supply contracts, we study contracts on projects, not quantities. Similar contracts arise in construction, spare-part supply chains, call centers, and consulting, among others. For the construction industry, Bajari and Tadelis (2001) find that the choice of contract mostly depends on its capacity of ex-post adaptation and that cost-plus contracts are preferred to fixed-price contracts when the project is more complex. For spare-part supply chains, Kim et al. (2007) show that PB contracts can induce the first-best effort levels, even in a multi-task setting, as long as the contracting parties are risk-neutral. For call centers, Hasija et al. (2008) show that the optimal contracts should combine performance measures (i.e., service level agreements) with input measures (e.g., pay-per-time, pay-per-call) in the presence of information asymmetry about worker productivity. Xue and Field (2008) analyze pricing and effort allocation in collaborative services with information stickiness. In contrast to our model, they assume that the efforts are substitutes, rather than complementary, and that the sum of efforts is exogenous. Although our work is close in spirit to these papers, we study an environment where both the vendor and the buyer provide inputs and where the output depends on the effort exerted by both. Collaboration will not only be associated with a larger set of contracts, depending on the choice of allocation of decision rights, but will also give rise to additional negative externalities due to the complementarity of efforts.

The notion of collaboration in services was first introduced by Fuchs and Leveson (1968) and addressed in an operations management context by Chase (1981) through the concept of customer contact. Managing collaboration in organizations presents many challenges, such as the deployment of globally distributed work teams (Kumar et al. 2005) or the adoption of interorganizational (information sharing) systems (Chi and Holsapple 2005). Moreover, the prevalence of collaboration in complex, dynamic environments makes obsolete the traditional top-down approach to process design. Kogan and Muller (2006) and Hill et al. (2006) indeed report the widespread use of ad hoc collaboration and personal information management tools, while prescribed work processes serve

only as reference models. To understand where the work takes place in collaborative environments, Kieliszewski et al. (2007) present a new process model, based on practice diagrams, that captures information flows, work context, team members, information resources, and special events.

From a contracting perspective, collaboration leads to double moral hazard, i.e., both the principal's and the agent's actions are unverifiable. Bhattacharyya and Lafontaine (1995) demonstrate the linearity of optimal PB contracts for constant-elasticity of substitution (CES) functions. Corbett et al. (2005) generalize their result with linear cost functions, in a supply chain context. Although our model is similar to these papers, its scope is more general, because we not only consider PB contracts, but also FF and TM contracts in a bilateral, multi-buyer and multi-vendor settings.

Contract adoption has also been studied from an empirical perspective, in offshore drilling (Corts and Singh 2004) and in information technology (Banerjee and Duflo 2000, Gopal et al. 2003, and Kalnins and Mayer 2004). Gurbaxani (2007) reviews the empirical literature and proposes a transaction economics framework to analyze information systems outsourcing contracts. It is generally observed that contract choice depends on project complexity, project size, resource shortage, as well as the firms' reputation. The current state of the consulting industry, its challenges, and future are reviewed in Toppin and Czerniawska (2005). Our model, supported by these empirical studies, provides an analytical framework for contract selection and service process design.

3. The Model
3.1. Output

Let x and y denote the buyer's and the vendor's efforts, respectively. We model the output of the collaborative work with a Cobb-Douglas function $x^\alpha y^\beta$ exhibiting decreasing returns to scale, i.e., $\alpha + \beta < 1$. Cobb-Douglas functions display a constant elasticity of substitution (CES), similar to linear output functions, which are are common in the principal-agent literature (Baker 2002). In contrast to linear outputs, the Cobb-Douglas effort levels are complementary, rather than substitutes, consistent with the observation that "effective consultations often depend on cooperation between the client and a consultant in a spirit of teamwork" (Shenson 1990). As a client put

it: "What is the main lesson we learned about working with consultants? The power of working together." (Toppin and Czerniawska 2005, p. 241)

When the project involves multiple tasks $i = 1, ..., N$, each with output $x_i^\alpha y_i^\beta$, we model the overall service output as a linear combination of the different task outputs $\sum_{i=1}^N f_i x_i^\alpha y_i^\beta$ with $f_i \geq 0$, in the same spirit as Baker (2002).

We say that a project is *complex* if its output cannot easily be described. Very often, consulting projects start with neither the consultant nor the client having precisely identified the client's needs (Shenson 1990). Consequently, linking the vendor's remuneration to distorted measures of output can lead to undesired actions. When the project involves multiple tasks, we assume that the only available measure of output is $\sum_{i=1}^N g_i x_i^\alpha y_i^\beta$, with $g_i \geq 0$. Project complexity thus depends on the degree of distortion between the vectors $(f_i)_{i=1,...,N}$ and $(g_i)_{i=1,...,N}$, and is geometrically measured by the angle defined by these two vectors. Project complexity can typically be reduced by breaking down the project into small pieces with clearly defined objectives.

In addition to being complex, projects are also *uncertain*. Although the *output* (e.g., service level agreements) of the collaborative process is defined by $x^\alpha y^\beta$, its *outcome* (e.g., a client's market share) may be different. Outcomes can differ from outputs under the influence of external factors, as well as because of the vendor's and buyer's variable characteristics (Frei 2006). Formally, we define the project outcome as $\epsilon x^\alpha y^\beta$, in which ϵ is a nonnegative random variable that caputres not only the uncertainty in project duration but also the variability in customer satisfaction. Project uncertainty can be reduced through preliminary disambiguation work so as to clearly define the scope of the project and agree on the goals to achieve as well as on the methodology to adopt.

We furthermore assume that the noise is multiplicative. In particular, the choice of efforts not only increases the mean output, it also increases its variance. By contrast, with an additive noise, the choice of efforts would have been independent of ϵ, at least if the parties are risk-neutral. Reality probably lies between those two extremes, as an increase in effort levels is typically associated with at least an increase in the semivariance (i.e., the zero outcome, where the client is dissatisfied and the project aborts, never completely disappears).

3.2. Inputs

The inputs are the buyer's and the vendor's efforts. In contrast to the economics literature, which often assumes strictly convex costs, we assume linear costs of efforts, denoted by c_B and c_V for the buyer and the vendor. In fact, business-to-business collaboration typically involves large organizations, for which the cost of adding workforce on a particular project is relatively constant over a reasonable range of values (see Corbett et al. 2005 for a similar assumption). We furthermore assume that these costs are common knowledge, i.e., there is no adverse selection. These costs can however be uncertain (e.g., training costs), and their variability can be easily modeled through the outcome uncertainty factor ϵ (see §4.1).

We assume that neither the vendor's nor the buyer's actions are directly observable, resulting in *double moral hazard* (Bhattacharyya and Lafontaine 1995). We assume, nevertheless, that one party's actions can be observed at some cost. In consulting, this monitoring cost includes meetings, on-site work, phone calls, intermediate reports, or presentations, and are related to the degree of information stickiness (Xue and Field 2008). The monitoring costs can be reduced by improved reputation (e.g., previous collaboration, age of the firm; see Banerjee and Duflo 2000) or by designing efficient communication processes.

Furthermore, we assume that the reporting costs are proportional to the party's effort, and we denote by ϕ the unit monitoring cost. For conciseness, we will assume that these costs are billed to the client, either as direct expenses or as overhead, but most of our results hold when these costs are incurred by the vendor.

3.3. Timing of the Game

We focus on a one-period game and therefore ignore the reputation effects associated with the project outcome. We moreover assume risk neutrality of both parties and no liability constraint. Without loss of generality in the presence of fixed transfer payments, we assume that the vendor makes a take-it-or-leave-it contract offer. A contract is therefore accepted by the buyer if it gives him an expected profit larger than his reservation utility, which we assume to be zero for simplicity.

Depending on the choice of contracts, effort levels can be chosen prior to the realization of the random components ϵ and ϕ (ex-ante) or after (ex-post). The timing of the game is the following.

1. Vendor offers contract.

2. Buyer accepts it or rejects it.

3. Ex-ante effort levels are chosen.

4. Outcome uncertainty ϵ and monitoring cost uncertainty ϕ are resolved.

5. Ex-post effort levels are chosen.

6. Output is realized and payments are collected.

3.4. Contract Types

We examine three classes of contracts: fixed fee (FF), time and materials (TM), and performance-based (PB). The FF contract consists of only a fixed payment s. The TM contract remunerates the vendor for the amount of effort provided at a per-unit price p, in addition to a fixed fee s. Finally, the PB contract rewards the vendor with a share of the outcome, i.e., a bonus b, in addition to a fixed payment s and the effort compensation p. Thus, structurally, the PB contract generalizes the other two contracts.

However, their risk- and reward-sharing and their allocation of decision rights are radically different. With a PB contract, both parties choose their respective effort levels. Because the vendor receives a share of the outcome, the risks and rewards are shared between the two parties. We consider two types of PB contracts, depending on whether the vendor's efforts are monitored (at a cost ϕy) or not. With a TM contract, the buyer decides the effort levels of both parties. In fact, after the contract is signed, the vendor is committed to delivering any effort level at a price p. The vendor has therefore transferred her decision rights to the buyer, and the buyer is accordingly the residual risk bearer. Finally, FF contracts can be of two types, depending on who is the residual risk bearer: Fixed-Fee Contracts with Precommitment on Effort (FF-E) or Fixed-Fee Contracts with Precommitment on Outcome (FF-O). In an FF-E, the vendor commits to a particular effort level y, making the buyer the residual risk bearer. In contrast, in an FF-O, the vendor commits

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Table 1   Contract Characteristics

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Contracts	Payment transfers	Add'l costs	When?		Who?	
			x	y	x	y
FF-E	s	ϕy	ex-post	ex-ante	buyer	vendor
FF-O	s	ϕx	ex-ante	ex-post	vendor	vendor
TM	$s + py$	ϕy	ex-post	ex-post	buyer	buyer
PB w/ monitoring	$s + py + b\epsilon x^\alpha y^\beta$	ϕy	ex-post	ex-post	buyer	vendor
PB w/o monitoring	$s + b\epsilon x^\alpha y^\beta$	0	ex-post	ex-post	buyer	vendor

to achieving a particular level of outcome, ex-ante prescribing the buyer's level of effort x and therefore becoming the sole risk bearer. Table 1 summarizes the characteristics of the different contracts.

4. Bilateral Contracting
4.1. First Best (FB)

As a benchmark, consider the first-best solution, as if there were perfect coordination between the buyer and the vendor. In particular, we assume that both the output function and the effort levels can be perfectly observed, at no cost, enabling the vendor to choose the buyer's efforts so as to maximize the total surplus. We say that a contract is *inefficient* if the total surplus generated by the contract is lower than the first-best surplus.

The first-best effort levels are chosen ex-post, after realization of ϵ has been resolved. For any realization of ϵ, the vendor chooses the buyer's effort levels so as to maximize the (concave) total surplus $\epsilon x^\alpha y^\beta - c_B x - c_V y$.

The multiplicative nature of the output function gives rise to *complementary efforts*. That is, the buyer will be more inclined to exert high efforts if the vendor exerts high efforts, consistent with what is observed in practice (Toppin and Czerniawska 2005, p. 215). In contrast to a linear output model (e.g., repetitive human resource administrative tasks), the optimal effort allocation will not be extreme (i.e., full in- or outsourcing) but will instead lead to active participation of both parties. In particular, the first-best effort levels equal

$$x^{FB} = \epsilon^{\frac{1}{1-\alpha-\beta}} \left(\frac{\alpha}{c_B}\right)^{\frac{1-\beta}{1-\alpha-\beta}} \left(\frac{\beta}{c_V}\right)^{\frac{\beta}{1-\alpha-\beta}}$$

$$y^{FB} = \epsilon^{\frac{1}{1-\alpha-\beta}} \left(\frac{\alpha}{c_B}\right)^{\frac{\alpha}{1-\alpha-\beta}} \left(\frac{\beta}{c_V}\right)^{\frac{1-\alpha}{1-\alpha-\beta}},$$

and the optimal ratio of effort levels equals $x^{FB}/y^{FB} = (\alpha c_V)/(c_B \beta)$. That is, the buyer will exert more efforts, relative to the vendor, if his cost of effort is lower and his efforts contribute more to the output. Consequently, the maximum total surplus is equal to:

$$\Pi_V^{FB} = \left(\mathbb{E}[\epsilon^{\frac{1}{1-\alpha-\beta}}]\right) \left(\frac{\alpha}{c_B}\right)^{\frac{\alpha}{1-\alpha-\beta}} \left(\frac{\beta}{c_V}\right)^{\frac{\beta}{1-\alpha-\beta}} (1-\alpha-\beta). \tag{1}$$

In addition to modeling the difference between outcome and outputs, the random term ϵ can also capture input uncertainty as follows. Suppose that the vendor's costs of effort are uncertain at the time of the contract (e.g., uncertainty about available personnel, see Gopal et al. 2003), i.e., that the vendor's costs equal $c_V \xi$, where ξ is a positive random variable. Optimizing the first-best profit would lead to a similar expression as above, with the first term equal to $\left(\mathbb{E}[(\epsilon/\xi^\beta)^{\frac{1}{1-\alpha-\beta}}]\right)$.

We next characterize the sensitivity of the total surplus to the model parameters. Consider two projects, 1 and 2, with respective output uncertainty distribution Φ_1 and Φ_2. Distribution 1 is said to be larger than distribution 2 in mean residual life if $\int_x^\infty (1-\Phi_1(\xi))d\xi \geq \int_x^\infty (1-\Phi_2(\xi))d\xi$ for all x.

PROPOSITION 1.   1. *If distribution 1 is larger than distribution 2 in mean residual life, i.e.,* $\int_x^\infty (1-\Phi_1(\xi))d\xi \geq \int_x^\infty (1-\Phi_2(\xi))d\xi$, *the total surplus is larger under distribution 1 than under distribution 2.*

2. *In addition, the total surplus Π_V^{FB} is jointly convex in α and β.*

According to Proposition 1, vendors prefer projects associated with uncertain durations and uncertain levels of customer satisfaction, according to the mean residual life order. Frei (2006) indeed argues that reducing the upward variability in service delivery could compromise some customers' experiences. Moreover, the uncertainty in outcome constitutes a tremendous learning opportunity. As a client put it, "We often asked: 'How would it be if?' Or: 'How would we get more into this?' This gave both parties an opportunity to go beyond what had been initially planned."

(Toppin and Czerniawska 2005, p. 14) When ϵ models the duration of the project, Proposition 1 suggests that the vendor prefers projects that could lead to long-term relationships.

If costs are also uncertain, e.g., if the vendor's cost of effort equals $c_V \xi$, where ξ is a nonnegative random variable, the total surplus is convex decreasing with the amount of uncertainty. Accordingly, the vendor will prefer projects associated with stochastically smaller distributions, according to the second order. That is, if the vendor considers two projects, 1 and 2, with cost distribution Φ_1 and Φ_2, where $\int_0^x (1 - \Phi_1(\xi)) d\xi \leq \int_0^x (1 - \Phi_2(\xi)) d\xi$ for all x, then the vendor will prefer the first project. Intuitively, the vendor will prefer the project associated with the largest (downward) variability.

Another feature of our model is that the total surplus is convex in the output elasticities α and β. For a fixed β (α), whether Π_V^{FB} is only decreasing, decreasing-increasing, or only increasing with α (β) depends on the magnitude of c_B (c_V). High costs of effort c_B favor low buyer's contribution (i.e., full outsourcing) while low costs of effort favor high buyer's contribution (i.e., full insourcing). There are thus *benefits of specialization.* An example is the recent development of internal consulting teams for standard consultations on the one hand, and the emergence of small specialist consulting firms on the other hand (Toppin and Czerniawska 2005).

4.2. Fixed-Fee Contracts

4.2.1. Fixed Fee with Precommitment of Effort (FF-E). We first analyze the performance of an FF contract where the vendor pre-commits to a particular level of effort (e.g., number of days of work) with no guarantee on output. These contracts abound in education, dental/medical, and domestic (e.g., household, gardening) services. For this contract to be enforceable, the vendor needs to make her effort visible (e.g., on-site work), giving rise to an additional monitoring cost ϕ per amount of effort, billed to the client as overhead.

The vendor seeks to determine the optimal contract parameters, i.e., the fixed fee s and the pre-committed level of effort y, so as to maximize her profit subject to the buyer's participation and incentive compatibility constraints. Hence, the game can be formulated as follows:

$$\max_{s,y} \quad s - c_V y$$

$$s.t. \qquad \mathbb{E}\left[\epsilon x^\alpha y^\beta - c_B x - \phi y - s\right] \geq 0$$

$$x(\epsilon) = \arg\max\left\{\epsilon x^\alpha y^\beta - c_B x - \phi y - s\right\}.$$

PROPOSITION 2. *Under an FF-E contract, the vendor's profit equals*

$$\Pi_V^{FF-E} = \left(\mathbb{E}[\epsilon^{\frac{1}{1-\alpha}}]\right)^{\frac{1-\alpha}{1-\alpha-\beta}} \left(\frac{\beta}{c_V + \mathbb{E}[\phi]}\right)^{\frac{\beta}{1-\alpha-\beta}} \left(\frac{\alpha}{c_B}\right)^{\frac{\alpha}{1-\alpha-\beta}} (1 - \alpha - \beta),$$

and the effort levels are such that $x^{FF-E} \leq x^{FB}$, $y^{FF-E} \leq y^{FB}$, *and* $\mathbb{E}[x^{FF-E}(\epsilon)]/y^{FF-E} \geq x^{FB}/y^{FB}$.

Comparing Proposition 2 with Equation (1) reveals that both effort levels are lower than optimal, especially those exerted by the vendor, not only because of (i) the lack of adaptability of FF contracts, since $\left(\mathbb{E}[\epsilon^{\frac{1}{1-\alpha}}]\right)^{\frac{1-\alpha}{1-\alpha-\beta}} \leq \mathbb{E}[\epsilon^{\frac{1}{1-\alpha-\beta}}]$, but also because of (ii) the additional monitoring cost $\mathbb{E}[\phi]$. Incidentally, the vendor's surplus would not have been different if the vendor had directly borne the monitoring costs.

Consequently, the performance of FF-E contracts can be improved by (i) working more closely ex-ante on project disambiguation so as to better define the scope of the project and to work better as a team, effectively reducing the lower tail of the distribution of ϵ and (ii) establishing efficient communication processes so as to make the vendor's effort visible at low cost $\mathbb{E}[\phi]$.

4.2.2. Fixed Fee with Precommitment of Outcome (FF-O). We next analyze the performance of an FF contract where the vendor pre-commits to a fixed level of outcome T, despite its random nature. In contrast to FF-E contracts, the vendor is here the residual risk bearer. These contracts abound in consulting, information technology provision, and tax services. Due to the collaborative nature of the work, the vendor will need to prescribe ex-ante the buyer's amount of effort necessary to attain the objective, as well as a reporting mechanism associated with a unit cost ϕ.

The vendor chooses the contract parameters, i.e., the fee s, the targeted outcome T, and the buyer's amount of effort x, so as to maximize her expected profit. There are two constraints, namely that the outcome must exceed the target T, and that the buyer obtains his reservation profit level.

$$\max_{s,x,y,T} \quad s - c_V \mathbb{E}[y]$$

$$s.t. \quad \epsilon x^\alpha y^\beta \geq T \; \forall \epsilon \tag{2}$$

$$T - (c_B + \mathbb{E}[\phi])x - s \geq 0.$$

PROPOSITION 3. *Under an FF-O contract, the vendor's profit equals*

$$\Pi_V^{FF-O} = \left(\mathbb{E}[\epsilon^{\frac{-1}{\beta}}]\right)^{\frac{-\beta}{1-\alpha-\beta}} \left(\frac{\alpha}{c_B + \mathbb{E}[\phi]}\right)^{\frac{\alpha}{1-\alpha-\beta}} \left(\frac{\beta}{c_V}\right)^{\frac{\beta}{1-\alpha-\beta}} (1 - \alpha - \beta),$$

and the effort levels are such that $x^{FF-O} \leq x^{FB}$, $\mathbb{E}[y^{FF-O}(\epsilon)] \leq y^{FB}$, and $x^{FF-O}/\mathbb{E}[y^{FF-O}(\epsilon)] \leq x^{FB}/y^{FB}$. Moreover, when $\mathbb{E}[\phi] = 0$, $x^{FF-O} \leq \mathbb{E}[x^{FF-E}(\epsilon)]$ and $\mathbb{E}[y^{FF-O}(\epsilon)] \leq y^{FF-E}$.

Comparing Proposition 3 with Equation (1) demonstrates that FF-O contracts are inefficient because of (i) the additional cost of monitoring the buyer's actions $\mathbb{E}[\phi]$, and (ii) the lack of responsiveness to future contingencies given that the vendor commits to achieve only a specific outcome, since $\left(\mathbb{E}[\epsilon^{\frac{-1}{\beta}}]\right)^{\frac{-\beta}{1-\alpha-\beta}} \leq \mathbb{E}[\epsilon^{\frac{1}{1-\alpha-\beta}}]$. In fact, FF-O contracts are less adaptable than FF-E contracts and could induce less efforts than FF-E contracts if the monitoring costs were negligible.

In reality, many FF-O contracts contain clauses to hedge the service provider against extraordinary contingencies, effectively guaranteeing the outcome T only over a restricted range of values for ϵ. For instance, hospitals who have adopted capitation contracts are insured against specific excess losses, should they incur catastrophic cases, such as organ transplants or premature babies.

Outcomes vs. Outputs? There has been a lot of business press recently advocating for measuring consulting performance with outcomes, instead of outputs (e.g., Toppin and Czerniawska 2005). With PB contracts, outcome measurement forces consultants to focus on the value they deliver (see §4.4), but it is also common practice with fixed-fee contracts, such as with incentive fixed-price contracts or performance fixed-price contracts (Shenson 1990).

We show next that, perhaps contrary to intuition, *focusing on outcomes* rather than outputs *decreases the efficiency* of FF-O contracts, because the vendor is nor properly remunerated for the additional risk burden. The vendor's problem is similar to (2) with the first constraint defined as $x^\alpha y^\beta \geq T$ and the buyer's receiving $\mathbb{E}[\epsilon]T$ from the collaborative work. Using a similar proof to the

proof of Proposition 3, one can show that the vendor's maximum profit, when she pre-commits to a certain output target, equals

$$\Pi_V^{FF-O} = (\mathbb{E}[\epsilon])^{\frac{1}{1-\alpha-\beta}} \left(\frac{\alpha}{c_B + \mathbb{E}[\phi]} \right)^{\frac{\alpha}{1-\alpha-\beta}} \left(\frac{\beta}{c_V} \right)^{\frac{\beta}{1-\alpha-\beta}} (1 - \alpha - \beta).$$

Because $\left(\mathbb{E}[\epsilon^{\frac{-1}{\beta}}] \right)^{\frac{-\beta}{1-\alpha-\beta}} \leq (\mathbb{E}[\epsilon])^{\frac{1}{1-\alpha-\beta}}$, by Jensen's inequality, the vendor's profit is larger when she commits to an output target than to an outcome target. With outcome measurement, the vendor will on average exert less effort because she is nor remunerated for the additional risk she is facing.

Consequently, FF-O contracts can be improved by (i) reducing the lower tail of the distribution of ϵ through disambiguation effort in the proposal writing process, (ii) lowering the client's effort monitoring cost $\mathbb{E}[\phi]$ through the design of effective communication processes, and either (iii) agreeing on contingency plans in case of changes in the environment or client's noncompliance to the prescribed actions, or (iii') contracting on outputs, not outcomes, in contrast to PB contracts.

4.3. Time and Materials Contracts (TM)

In this section, we investigate the performance of TM contracts, according to which the buyer pays p per unit of vendor's effort. Because the vendor's effort is not directly observable, additional monitoring costs have to be incurred. We furthermore assume that the vendor's effort levels are chosen by the buyer, after observation of ϵ and ϕ, i.e., that the vendor has transferred her decision rights to the buyer (Shenson 1990, p. 21). The vendor will not behave opportunistically ex post as long as the buyer's compensation exceeds her costs, i.e., $p \geq c_V$.

The vendor chooses the contract parameters to maximize her profit, subject to the buyer's participation and incentive compatibility constraints.

$$\max_{s, p \geq c_V} \mathbb{E}\left[s + (p - c_V)y \right]$$

$$s.t. \quad \mathbb{E}\left[\epsilon x^\alpha y^\beta - c_B x - (p + \phi)y - s \right] \geq 0$$

$$(x(\epsilon, \phi), y(\epsilon, \phi)) = \arg\max\left\{ \epsilon x^\alpha y^\beta - c_B x - (p + \phi)y - s \right\}.$$

PROPOSITION 4. *Under a TM contract, the vendor's profit satisfies*

$$\Pi_V^{TM} \geq \mathbb{E}\left[\epsilon^{\frac{1}{1-\alpha-\beta}} \right] \left(\frac{\alpha}{c_B} \right)^{\frac{\alpha}{1-\alpha-\beta}} \left(\frac{\beta}{c_V + \mathbb{E}[\phi]} \right)^{\frac{\beta}{1-\alpha-\beta}} (1 - \alpha - \beta),$$

and the bound is tight when ϕ is deterministic. In that case, $p = c_V$. The optimal effort levels satisfy

$$x^{TM}(\mathbb{E}[\epsilon], \mathbb{E}[\phi]) \leq \mathbb{E}[x^{TM}(\epsilon, \phi)] \leq \mathbb{E}[x^{FB}(\epsilon)], \text{ similarly for } y, \text{ and } x^{TM}/y^{TM} \geq x^{FB}/y^{FB}.$$

According to Proposition 4, a pure cost-reimbursement scheme is optimal. Indeed, when ϕ is deterministic, the buyer compensates the vendor only for its labor. Similarly, one can show that, when the monitoring costs are borne by the vendor, the optimal compensation is equal to $p = c_V + \phi$, i.e., the buyer compensates the vendor for the overhead in addition to labor, consistent with observed practice.

The vendor's profit is convex decreasing with the monitoring costs ϕ. Therefore, if the vendor considers two projects, 1 and 2, with Φ_i being the probability distribution of the monitoring costs of project $i = 1, 2$, and that Φ_1 is stochastically smaller than Φ_2 in the second order, i.e., $\int_0^x (1 - \Phi_1(\xi))d\xi \leq \int_0^x (1 - \Phi_2(\xi))d\xi$, then the vendor will prefer project 1. Intuitively, the vendor will prefer the project with the largest probability of low monitoring costs. In practice, vendors can reduce their monitoring costs by improving their reputation for fulfilling clients' requests or by designing efficient (and economical) communication processes.

Moreover, observe that $\mathbb{E}[x(\epsilon, \phi)] \geq x(\mathbb{E}[\epsilon], \mathbb{E}[\phi])$ by Jensen's inequality, and similarly for y. Hence, estimating the effort levels without accounting for the variability in the service delivery will typically underestimate the expected actual effort levels, as evidenced by the chronic budget and time overruns common in collaborative services (e.g., IT projects). The effort levels are also smaller than the first-best effort levels, and too much weight is placed on the buyer's effort, because the vendor's cost of effort from the buyer's perspective is inflated by ϕ.

Consequently, to improve the performance of TM contracts, vendors should design efficient reporting processes (e.g., on site work, frequent meetings) to increase the visibility of their work at low cost, or to enhance their reputation so as to build trust in the relationship with their clients. In particular, Gopal et al. (2003) find that, in offshore software development, clients with prior experience with a particular vendor are likely to opt for a TM contract. Similarly, Corts and Singh (2004) observe that oil and gas companies are more likely to choose a day rate contract as the frequency of interaction with a driller increases.

4.4. Performance-Based Contracts (PB)

With a performance-based contract, the vendor receives a share b of the outcome, where $0 \leq b \leq 1$. PB contracts arise frequently in legal services (e.g., medical practice cases), sales agencies (e.g., insurance), and advertising agencies (Shapiro 2002) among others.

4.4.1. Performance-Based Contracts with Monitoring of Efforts.

We first consider a PB contract where the vendor's effort level is measured at a unit cost ϕ. Suppose that the buyer incurs the monitoring costs. (The analysis is similar when the vendor incurs those costs.) The PB contract consists of three parameters, (b, p, s), representing the bonus b, the input price p, and a fixed fee s. Accordingly, the vendor's problem can be formulated as follows:

$$\max_{b,p,s} \quad \mathbb{E}\left[b\epsilon x^{\alpha}y^{\beta} + (p - c_V)y + s\right]$$

$$\mathbb{E}\left[(1-b)\epsilon x^{\alpha}y^{\beta} - c_B x - (p+\phi)y - s\right] \geq 0$$

$$x(\epsilon, \phi) = \arg\max\left\{(1-b)\epsilon x^{\alpha}y^{\beta} - c_B x - (p+\phi)y - s\right\} \quad (3)$$

$$y(\epsilon, \phi) = \arg\max\left\{b\epsilon x^{\alpha}y^{\beta} + (p - c_V)y + s\right\}.$$

PROPOSITION 5. *Under a PB contract where the vendor's efforts are monitored, the optimal contract degenerates to a time-and-materials contract with $b = 0$ and $p = c_V$. The optimal vendor's profit equals*

$$\Pi_V^{PB} = \mathbb{E}\left[\epsilon^{\frac{1}{1-\alpha-\beta}}\right]\left(\frac{\alpha}{c_B}\right)^{\frac{\alpha}{1-\alpha-\beta}}\left(\frac{\beta}{c_V + \mathbb{E}[\phi]}\right)^{\frac{\beta}{1-\alpha-\beta}}(1-\alpha-\beta),$$

and the optimal effort levels satisfy $x^{PB} \leq x^{FB}$, $y^{PB} \leq y^{FB}$, and $x^{PB}/y^{PB} \geq x^{FB}/y^{FB}$.

Proposition 5 demonstrates that the optimal PB contract tends in the limit to a time-and-materials contract in which the buyer and the vendor choose their effort levels autonomously. In fact, the effort compensation function py has the same (linear) form as the cost function $c_V y$. Hence, the vendor's cost of effort can be replicated by the buyer by properly choosing the remuneration fees. Information asymmetry or risk aversion could however alter this property, as observed by Hasija et al. (2008) and Kim et al. (2007) in a single moral hazard environments.

Comparing Propositions 4 and 5 reveals that TM contracts always perform better than PB contracts with monitoring, and that they perform equally well when the monitoring costs are deterministic. Hence, the vendor will in general prefer transferring her decision rights to the buyer.

Distorted Performance Measurement. Additional inefficiencies arise with PB contracts when the performance measure is distorted. Suppose that the service is complex and that its output cannot be accurately described. Formally, even though the outcome is $\epsilon \sum_{i=1}^{N} f_i x_i^\alpha y_i^\beta$, one can only measure $\epsilon \sum_{i=1}^{N} g_i x_i^\alpha y_i^\beta$. If $g_i = f_i$ for all i, the performance measure is perfectly aligned with the output function, in the sense that it induces the same level of efforts, and the PB contract is as efficient as a TM contract (provided ϕ is deterministic). If on the other hand $g_i \neq f_i$, for some i, the performance measure will induce distorted actions.

With a multiple-task production function $\epsilon \sum_{i=1}^{N} f_i x_i^\alpha y_i^\beta$, the first-best surplus is equal to

$$\Pi_V^{FB} = \left(\mathbb{E}[\epsilon^{\frac{1}{1-\alpha-\beta}}] \right) \left(\frac{\beta}{c_V} \right)^{\frac{\beta}{1-\alpha-\beta}} \left(\frac{\alpha}{c_B} \right)^{\frac{\alpha}{1-\alpha-\beta}} (1-\alpha-\beta) \left(\sum_{i=1}^{n} f_i^{\frac{1}{1-\alpha-\beta}} \right).$$

Under a PB contract, the vendor is evaluated based on the performance measure $\epsilon \sum_{i=1}^{N} g_i x_i^\alpha y_i^\beta$. Without loss of generality, we assume that $g_i \leq f_i$ for all i, so that the bonus $b \in [0,1]$ does not induce negative effort levels. Hence, with multiple tasks, the transfer payment to the vendor equals $s + p \sum_{i=1}^{N} x_i + b\epsilon \sum_{i=1}^{N} g_i x_i^\alpha y_i^\beta$. Maximizing the vendor's total profit in a similar fashion to the single-task case, we obtain that the optimal bonus b tends to zero, the optimal effort compensation p tends to the vendor's unit cost c_V, resulting in the following vendor's total profit:

$$\Pi_V^{PB} = \mathbb{E}\left[\epsilon^{\frac{1}{1-\alpha-\beta}} \right] \left(\frac{\alpha}{c_B} \right)^{\frac{\alpha}{1-\alpha-\beta}} \left(\frac{\beta}{c_V + \phi} \right)^{\frac{\beta}{1-\alpha-\beta}} (1-\alpha-\beta) \frac{\left(\sum_{i=1}^{N} f_i^{\frac{1-\beta}{1-\alpha-\beta}} g_i^{\frac{\beta}{1-\alpha-\beta}} \right)^{\frac{1-\alpha}{1-\alpha-\beta}}}{\left(\sum_{i=1}^{N} f_i^{\frac{\alpha}{1-\alpha-\beta}} g_i^{\frac{1-\alpha}{1-\alpha-\beta}} \right)^{\frac{\beta}{1-\alpha-\beta}}}. \quad (4)$$

By Hölder's inequality, $\sum_{i=1}^{N} f_i^{\frac{1-\beta}{1-\alpha-\beta}} g_i^{\frac{\beta}{1-\alpha-\beta}} \leq \left(\sum_{i=1}^{N} f_i^{\frac{\alpha}{1-\alpha-\beta}} g_i^{\frac{1-\alpha}{1-\alpha-\beta}} \right)^{\frac{\beta}{1-\alpha}} \left(\sum_{i=1}^{N} f_i^{\frac{1}{1-\alpha-\beta}} \right)^{\frac{1-\alpha-\beta}{1-\alpha}}$.

Therefore, comparing (4) with (1) shows that PB contracts with monitoring of the vendor's effort are inefficient because of (i) the additional monitoring cost, similar to TM contracts, and (ii) distortion in performance measures. Similar to Baker (2002), the distortion is represented by the inner

product of the vectors of weights on the different tasks, $(f_i)_{i=1,\ldots,N}$ and $(g_i)_{i=1,\ldots,N}$; geometrically, the extent of distortion appears as the magnitude of the angle between the vectors $(f_i)_{i=1,\ldots,N}$ and $(g_i)_{i=1,\ldots,N}$, weighted by the buyer's and the vendor's elasticities α and β.

4.4.2. Performance-Based Contracts without Monitoring of Efforts. We next characterize the performance of pure PB contracts, with no monitoring of efforts. This contract consists of only two parameters, namely a fixed fee s and a piece rate b. The vendor's problem can be formulated as follows:

$$\max_{b,s} \quad \mathbb{E}\left[b\epsilon x^\alpha y^\beta - c_V y + s\right]$$

$$\mathbb{E}\left[(1-b)\epsilon x^\alpha y^\beta - c_B x - s\right] \geq 0$$

$$x(\epsilon) = \arg\max\left\{(1-b)\epsilon x^\alpha y^\beta - c_B x - s\right\}$$

$$y(\epsilon) = \arg\max\left\{b\epsilon x^\alpha y^\beta - c_V y + s\right\}.$$

Similar to Bhattacharyya and Lafontaine (1995), we find that the optimal outcome share is independent of its uncertainty and the unit costs of effort.

PROPOSITION 6. *Under a PB contract with no monitoring of the vendor's efforts, the optimal vendor's profit equals*

$$\Pi_V^{PB}(b) = \mathbb{E}\left[\epsilon^{\frac{1}{1-\alpha-\beta}}\right]\left(\frac{\alpha(1-b)}{c_B}\right)^{\frac{\alpha}{1-\alpha-\beta}}\left(\frac{\beta b}{c_V}\right)^{\frac{\beta}{1-\alpha-\beta}}(1-\alpha(1-b)-\beta b).$$

with

$$b = \frac{-\beta(1-\alpha)+\sqrt{\alpha\beta(1-\alpha)(1-\beta)}}{\alpha-\beta},$$

and the optimal effort levels satisfy $x^{PB} \leq x^{FB}$, $y^{PB} \leq y^{FB}$. Moreover $x^{PB}/y^{PB} \geq x^{FB}/y^{FB}$ if and only if $\alpha \geq \beta$.

The optimal piece rate b is decreasing with α and increasing with β; that is, more incentives are needed when the service output becomes less elastic to the buyer's efforts and more elastic to the vendor's efforts. When $\alpha = 0$ (e.g., outsourcing), the vendor has high-powered incentives, i.e.,

$b = 1$. When $\beta = 0$ (e.g., insourcing), then $b = 0$ and the contract reduces to an FF contract. Finally, when $\alpha = \beta$, $b = 1/2$ and the risk is equally shared among both parties. In fact, $b \leq 1/2$ if and only if $\alpha \geq \beta$. In contrast to the other types of contracts, the relative allocation of efforts x^{PB}/y^{PB} can be smaller or larger than the first-best allocation of efforts, depending on the relative elasticities.

4.4.3. PB Contracts: A Contracting Panacea?

PB contracts have recently grown in popularity, not only in consulting (Shenson 1990) but also in other industries, such as law firms or private schools (The Economist 2008a, 2008b). The consulting industry is indeed struggling to regain credibility after the 1990's economic bubble (Toppin and Czerniawska 2005) and PB contracts offer them an opportunity to link their work to the actual value they generate.

PB contracts thus seem more effective when the vendor's performance is measured on the final *outcomes* rather than the *outputs*. When the vendor's contribution is based on outputs, her bonus equals $b\mathbb{E}[\epsilon]x^\alpha y^\beta$, instead of $b\epsilon x^\alpha y^\beta$. Solving Problem (3) with a bonus based on output rather than outcomes, we observed from numerical simulations that it was always beneficial to measure outcomes than outputs.[1] Moreover, the optimal bonus share on output depends on both the mean and the variance of ϵ, in contrast to Proposition 6, and must therefore be adapted to the scale and complexity of the project. Our model thus supports the belief that outcome measurement is more effective with PB contracts than output measurement because it makes the buyer and the vendor share the risks and the rewards of their collaboration.

However, despite their wide adoption and publicity in the business press, PB contracts must be used with caution. In fact, past research on multi-task contracts demonstrated that it is optimal to be selective in the choice of performance metrics due to diseconomies of scope (Laffont and Martimort 2002). In addition, one can show that additional inefficiencies arise when performance is measured with a distorted measure, similarly to §4.4.1. As a result, PB contracts are likely to

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[1] We ran 50^3 simulations, with randomly generated values for α in $(0.05, 0.95)$, β in $(0.05, 0.95 - \alpha)$ and the range of ϵ (between 0 and 3, with $\mathbb{E}[\epsilon] = 3$), assuming a uniform distribution for ϵ and $c_B = c_V = 0.4$. The median difference in profits was equal to 45.7% and the minimum difference to 4.15%. The performance gap increased with α and β and was non=-monotonic with the range of ϵ.

be more efficient when the performance measures are unambiguous and the project scope is well defined.

Moreover, one can show that the efficiency of PB contracts, measured by Π^{PB}/Π^{FB}, is decreasing with α for any given β, and vice versa. That is, PB contracts are the most efficient on projects involving little or no collaboration, where the degree of specialization is important.

Our analysis thus sheds light on one of the recent transformation of the consulting industry. On the one hand, clients want access to specialized skills and world-class experts, but on the other hand, they want to tie consulting firms to results, often involving them in the execution of any recommendation they make (Toppin and Czerniawska 2005, p. 167). These contradictory forces have led consulting firms to fragment and to focus on their specific skills, and clients to encourage consortia of firms to work together through multi-sourcing arrangements, which we investigate in §6.

As a result, when the vendor's efforts are not monitored, PB contracts are inefficient because of (i) their incapacity to fully transfer the output to the vendor without reducing the buyer's incentives, (ii) the focus on output rather than outcomes and (iii) the distortion between performance measures and customer's value. The performance of PB contracts can be improved by breaking down a complex project into small pieces, each with a well defined and easy to measure outcome, and by encouraging specialization from either the vendor or the buyer.

4.5. Discussion

As a result of our analysis, a vendor can rank the different types of contracts in order of profitability (i.e., total surplus). The ranking will be client-dependent, because of the impact of the client's cost and elasticity on the total profit. The order according to which the contracts are ranked will mostly depend on (i) the project complexity, including environmental risk, client unpreparedness, and cost uncertainty, which determines the relative attractiveness of FF contracts; (ii) the monitoring costs, including on-site work, frequency of meetings, and number of intermediate reports, which affect the attractiveness of TM and FF contracts; (iii) the ability to measure outputs or outcomes affecting

Figure 1 **Dominating contracts as a function of the range of the output uncertainty around its mean, and the**

monitoring costs $\left(c_B = c_V = 0.4,\ \alpha = 0.3,\ \beta = 0.2,\ \epsilon \sim \mathcal{U}(3 - \textbf{Range}/2, 3 + \textbf{Range}/2)\right)$**.**
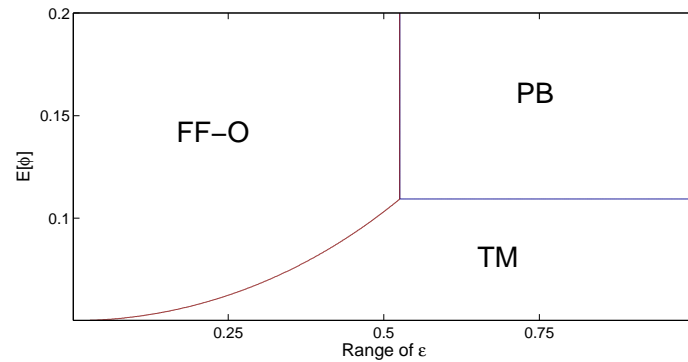
the efficiency of FF-O and PB contracts (in opposite ways); (iv) the degree of distortion in the performance measure, relative to the final value delivered to the client, affecting PB contracts; and (v) the amount of which performance-sharing reduce the buyer's incentives, also affecting PB contracts.

Figure 1 illustrates such a ranking as a function of the range of the outcome uncertainty ϵ about its mean (assuming a uniform distribution) and the magnitude of monitoring costs ϕ (assumed to be deterministic). We compared the performance of three contracts, namely FF-O (focusing on outcomes, assuming zero monitoring costs on the buyer's actions), TM, and PB (assuming no distortion between the performance measure and the outcome). Consistent with our analysis, FF contracts are preferred when the outcome uncertainty is low (because they are the least adaptable), TM contracts are preferred when the effort monitoring costs are low, and PB contracts dominate when neither of these conditions is satisfied.

Our analysis has so far assumed the same parameters for all contracts. In practice, however, the cost and elasticity parameters are likely to depend on the contractual arrangement. For instance, HCL America, a software development company, offers two types of contracts, namely TM and FF-O (O'Connell and Loveman 1995). Consistent with our analysis, TM contracts involve on-site work with daily supervision of the engineers and are typically used for projects that could not be specified in advance. With FF-O contracts, HCL commits to specific deadlines and prices; the contracts are therefore used when clients define clearly the capabilities and functions of the

program they want. FF-O contracts are associated with two types of work organization. On the one hand, FF-O contracts can be executed at HCL America facilities in the United States, enabling the client to consult frequently with the project manager and software engineers. On the other hand, FF-O contracts can also be executed in India at a 50% lower price. Different FF-O contracts may therefore co-exist, trading off the buyer's monitoring cost ϕ against the vendor's cost of effort c_V.

The ordering of contracts can also be used in a competitive setting, when multiple vendors bid on the same project. Similar to a Bertrand competition, the vendor who offers the most profitable contract will win the bid and her profit will be equal to the profit differential between her profit and the profit of the second most profitable contract. The buyer's profit will on the other hand be equal to the second largest bid and will therefore increase with the number of bidders.

5. Multiple Buyers

In this section, we show that having multiple buyers involved in the joint production process creates a negative externality, similar to free riding or social loafing (Latané et al. 1979). Examples of multi-buyer collaborative processes arise in education (e.g., the students' learning experience depends not only on their effort and the instructor's effort, but also on overall class participation) or in a supply chain management consulting project (involving, e.g., a manufacturer, a retailer, and a consultant).

We focus the analysis on FF and PM contracts, because the nature of TM contracts is not well defined in a multi-buyer environment. It is indeed unclear who should decide how much effort the vendor would have to provide in the absence of an "orchestrator" coordinating all buyers' actions. Moreover, we show in §5.3 that PB contracts reduce to TM contracts even though the vendor maintains his decision rights. We furthermore assume that the vendor offers the same type of contract (i.e., FF or PB) to all buyers but that she is nevertheless able to price-discriminate among them.

Let us respectively denote by x_i, α_i, and c_{Bi} buyer i's effort, contribution to output, and cost of effort. To simplify the analysis, we assume that there are only two buyers and that each buyer obtains the same value from the service delivery, denoted by $\epsilon x_1^{\alpha_1} x_2^{\alpha_2} y^{\beta}$, where $\alpha_1 + \alpha_2 + \beta < 1$.

5.1. First Best (FB)

Suppose that the buyers' efforts are contractible at no cost. Then, the vendor will choose all effort levels to maximize the total ex-post surplus $2\epsilon x_1^{\alpha_1} x_2^{\alpha_2} y^\beta - c_{B1}x_1 - c_{B2}x_2 - c_V y$, yielding the following optimal effort levels

$$x_1^{FB} = 2^{\frac{1}{1-\alpha_1-\alpha_2-\beta}} \epsilon^{\frac{1}{1-\alpha_1-\alpha_2-\beta}} \left(\frac{\alpha_1}{c_{B1}}\right)^{\frac{1-\alpha_2-\beta}{1-\alpha_1-\alpha_2-\beta}} \left(\frac{\alpha_2}{c_{B2}}\right)^{\frac{\alpha_2}{1-\alpha_1-\alpha_2-\beta}} \left(\frac{\beta}{c_V}\right)^{\frac{\beta}{1-\alpha_1-\alpha_2-\beta}},$$

$$x_2^{FB} = 2^{\frac{1}{1-\alpha_1-\alpha_2-\beta}} \epsilon^{\frac{1}{1-\alpha_1-\alpha_2-\beta}} \left(\frac{\alpha_1}{c_{B1}}\right)^{\frac{\alpha_1}{1-\alpha_1-\alpha_2-\beta}} \left(\frac{\alpha_2}{c_{B2}}\right)^{\frac{1-\alpha_1-\beta}{1-\alpha_1-\alpha_2-\beta}} \left(\frac{\beta}{c_V}\right)^{\frac{\beta}{1-\alpha_1-\alpha_2-\beta}},$$

$$y^{FB} = 2^{\frac{1}{1-\alpha_1-\alpha_2-\beta}} \epsilon^{\frac{1}{1-\alpha_1-\alpha_2-\beta}} \left(\frac{\alpha_1}{c_{B1}}\right)^{\frac{\alpha_1}{1-\alpha_1-\alpha_2-\beta}} \left(\frac{\alpha_2}{c_{B2}}\right)^{\frac{\alpha_2}{1-\alpha_1-\alpha_2-\beta}} \left(\frac{\beta}{c_V}\right)^{\frac{1-\alpha_1-\alpha_2}{1-\alpha_1-\alpha_2-\beta}},$$

and the optimal ratio of effort levels equals $x_i/y = (\alpha_i c_V)/(c_{Bi}\beta)$ for $i = 1,2$ and $x_1/x_2 = (\alpha_1 c_{B2})/(\alpha_2 c_{B1})$. Similarly to the bilateral contracting situation, the effort levels are complementary, i.e., move in the same direction. There is indeed strong evidence of the benefits of peer pressure on academic performance (Sacerdote 2001). But peer pressure is a double-edged sword as it can also lead to lower efforts, if it is negative.

The maximum total surplus is therefore equal to

$$\Pi_V^{FB} = 2^{\frac{1}{1-\alpha_1-\alpha_2-\beta}} \left(\mathbb{E}[\epsilon^{\frac{1}{1-\alpha_1-\alpha_2-\beta}}]\right) \left(\frac{\alpha_1}{c_{B1}}\right)^{\frac{\alpha_1}{1-\alpha_1-\alpha_2-\beta}} \left(\frac{\alpha_2}{c_{B2}}\right)^{\frac{\alpha_2}{1-\alpha_1-\alpha_2-\beta}} \left(\frac{\beta}{c_V}\right)^{\frac{\beta}{1-\alpha_1-\alpha_2-\beta}}$$
$$\times (1-\alpha_1-\alpha_2-\beta). \tag{5}$$

As in the case of bilateral contracting, the total surplus is a convex increasing function in the output uncertainty and is therefore larger for distribution of ϵ with larger mean residual life. It is also convex in α_1, α_2, and β. With low costs of effort, the total surplus would therefore benefit from having one buyer or the vendor with a high output elasticity. In education for instance, if the costs of effort are sufficiently low, the total surplus would be larger in a classroom with a large diversity in student backgrounds than in a classroom with students of equivalent backgrounds.

5.2. Fixed-Fee Contracts

5.2.1. Fixed Fee with Precommitment of Effort (FF-E).
We next analyze the performance of an FF contract where the vendor pre-commits to a particular level of effort. For this

contract to be enforceable, the vendor needs to make her effort visible, and therefore incurs an additional cost ϕ by unit of effort. (Similar results hold when the monitoring costs are billed to the buyers.)

The vendor seeks to determine the optimal contract parameters, i.e., the fixed fees s_1 and s_2, and the pre-committed level of effort y, so as to maximize her profit, subject to the buyers' participation and incentive constraints. Hence, the game can be formulated as follows

$$\max_{s_1,s_2,y} \quad s_1 + s_2 - (c_V + \mathbb{E}[\phi])y$$

$$s.t. \quad \mathbb{E}\left[\epsilon x_1^{\alpha_1} x_2^{\alpha_2} y^{\beta} - c_{Bi} x_i - s_i\right] \geq 0 \quad i = 1,2$$

$$x_i(\epsilon) = \arg\max\left\{\epsilon x_1^{\alpha_1} x_2^{\alpha_2} y^{\beta} - c_{Bi} x_i - s_i\right\}, \quad i = 1,2.$$

PROPOSITION 7. *Under an FF-E contract with multiple buyers, the vendor's profit equals*

$$\Pi_V^{FF-E} = \left(\mathbb{E}[\epsilon^{\frac{1}{1-\alpha_1-\alpha_2}}]\right)^{\frac{1-\alpha_1-\alpha_2}{1-\alpha_1-\alpha_2-\beta}} \left(\frac{\alpha_1}{c_{B1}}\right)^{\frac{\alpha_1}{1-\alpha_1-\alpha_2-\beta}} \left(\frac{\alpha_2}{c_{B2}}\right)^{\frac{\alpha_2}{1-\alpha_1-\alpha_2-\beta}} \left(\frac{\beta}{c_V + \mathbb{E}[\phi]}\right)^{\frac{\beta}{1-\alpha_1-\alpha_2-\beta}}$$

$$\times (1 - \alpha_1 - \alpha_2 - \beta)\left(\frac{2 - \alpha_1 - \alpha_2}{1 - \alpha_1 - \alpha_2}\right)^{\frac{1-\alpha_1-\alpha_2}{1-\alpha_1-\alpha_2-\beta}},$$

and, if $\phi = 0$, the effort levels are such that $x^{FF-E} \leq x^{FB}$, $y^{FF-E} \leq y^{FB}$, $x_1^{FF-E}/x_2^{FF-E} = x_1^{FB}/x_2^{FB}$, and $\mathbb{E}[x^{FF-E}(\epsilon)]/y^{FF-E} \leq x^{FB}/y^{FB}$.

Comparing Propositions 7 to Equation (5) reveals that multi-buyer FF-E contracts are associated with three sources of inefficiency: (i) the vendor's monitoring cost $\mathbb{E}[\phi]$, (ii) the lack of adaptability to changes in output because $\left(\mathbb{E}[\epsilon^{\frac{1}{1-\alpha_1-\alpha_2}}]\right)^{\frac{1-\alpha_1-\alpha_2}{1-\alpha_1-\alpha_2-\beta}} \leq \mathbb{E}[\epsilon^{\frac{1}{1-\alpha_1-\alpha_2-\beta}}]$, and (iii) social loafing because $2 \geq ((2-\zeta)/(1-\zeta))^{1-\zeta}$ for any $0 \leq \zeta \leq 1$.

Social loafing is a well-studied phenomenon in psychology (Latané et al. 1979) according to which individual efforts decrease due to the social presence of other persons. To compensate for the negative impact of social loafing, the vendor also needs to exert more effort relative to the buyers than what would be socially optimal, if the monitoring costs are negligible. Social loafing can be reduced by intensifying individual responsibility on the final output, which we investigate next.

5.2.2.   Fixed Fee with Precommitment of Outcome (FF-O). We next analyze the performance of FF contracts when the vendor pre-commits to a particular level of outcome T after prescribing ex-ante the buyer's effort levels. The vendor seeks to choose the contract parameters, i.e., the fees s_1 and s_2, the targeted outcome T, and the buyers' pre-commitment of effort x_1 and x_2, so as to maximize her expected profit. There are two sets of constraints, namely that the outcome must exceed the target T, and that the buyers obtain their reservation profit level.

$$\max_{s_1,s_2,x_1,x_2,T} \quad s_1 + s_2 - c_V \mathbb{E}[y]$$

$$s.t. \quad \epsilon x_1^{\alpha_1} x_2^{\alpha_2} y^\beta \geq T \ \forall \epsilon$$

$$T - (c_{Bi} + \mathbb{E}[\phi_i])x_i - s_i \geq 0 \ \ i = 1, 2.$$

PROPOSITION 8. *Under an FF-O contract with multiple buyers, the vendor's profit equals*

$$\Pi_V^{FF-O} = 2^{\frac{1}{1-\alpha_1-\alpha_2-\beta}} \left(\mathbb{E}[\epsilon^{\frac{-1}{\beta}}] \right)^{\frac{-\beta}{1-\alpha_1-\alpha_2-\beta}} \left(\frac{\alpha_1}{c_{B1} + \mathbb{E}[\phi_1]} \right)^{\frac{\alpha_1}{1-\alpha_1-\alpha_2-\beta}} \left(\frac{\alpha_2}{c_{B2} + \mathbb{E}[\phi_2]} \right)^{\frac{\alpha_2}{1-\alpha_1-\alpha_2-\beta}}$$

$$\times \left(\frac{\beta}{c_V} \right)^{\frac{\beta}{1-\alpha_1-\alpha_2-\beta}} (1 - \alpha_1 - \alpha_2 - \beta).$$

Comparing Proposition 8 with Equation (5) reveals that FF-O contracts are inefficient because of (i) the additional cost monitoring the buyers' actions and (ii) the lack of responsiveness to outcome variability. Similar to §4.2.2, the efficiency of FF-O contracts improves when the vendor specifies her performance target in terms of output, not outcome; in particular, one can show that, when the vendor specifies an output target, her total profit has the same expression as above, with $\left(\mathbb{E}[\epsilon^{\frac{-1}{\beta}}] \right)^{\frac{-\beta}{1-\alpha_1-\alpha_2-\beta}}$ replaced by $(\mathbb{E}[\epsilon])^{\frac{1}{1-\alpha_1-\alpha_2-\beta}}$.

In contrast to FF-E contracts, FF-O contracts do not give rise to social loafing. In fact, the vendor specifies the respective effort levels of the different buyers, getting them involved in the collaborative process without having them bear the risk associated with outcome variability. In education for instance, instructors could grade participation to mitigate social loafing and foster learning. Consequently, vendors using a fixed-fee contract must trade off the lack of adaptability of FF-O contracts with social loafing arising with FF-E contracts.

5.3.   Performance-Based Contracts (PB)

We next evaluate the performance of PB contracts in the presence of multiple buyers. For the sake of brevity, the detailed analysis is relegated to §EC.9 in the online appendix and we only present here the main insights.

Similar to Proposition 5, the optimal PB contract with monitoring of the vendor's efforts degenerates into a TM contract, with zero bonuses and a full-cost reimbursement scheme. That is, the vendor is rewarded only for her efforts and not for her contribution to the output. Therefore, even if TM contracts are not well defined in a multi-buyer setting in the absence of an "orchestrator," PB contracts with monitoring effectively act as TM contracts.

Moreover, PB contracts also give rise to social loafing in the presence of multiple buyers, similar to FF-E contracts. PB contracts are therefore inefficient in the presence of multiple buyers not only because of (i) the monitoring costs $\phi_1 + \phi_2$ and (ii) the distortion in performance measure, as in §4.4.1, but also because of (iii) social loafing.

When the vendor's efforts are not monitored, we find that the optimal bonuses are positive and independent of the outcome uncertainty and the costs of effort, as in Proposition 6. The allocation of efforts x_i^{PB}/y^{PB} and x_1^{PB}/x_2^{PB} can be smaller or larger than their first-best counterparts, depending on the relative magnitude of the effort elasticities α_1, α_2 and β.

Similar to bilateral contracts, PB contracts are inefficient with multiple buyers because of (i) their incapacity to transfer a share of the outcome to the vendor without lowering the buyers' incentives, (ii) the focus on outputs rather than outcomes and (iii) distorted performance measures.

5.4.   Contract Comparison

Figure 2 depicts the zones of dominance among FF-O, TM (i.e., degenerate PB contracts), and PB contracts in the presence of multiple buyers, for various values of the monitoring costs ϕ and the range of outcome uncertainty. The data is the same as in Figure 1, except that we split the single buyer into two buyers, whose costs and elasticities sum up to the single buyer's characteristics. Comparing the two figures (and in particular the axes' ranges of values) reveals that social loafing

Figure 2 **Dominating contracts with multiple buyers, as a function of the range of the output uncertainty**

around its mean, and the monitoring costs ($c_{B1} = c_{B2} = 0.2$, $c_V = 0.4$, $\alpha_1 = 0.25$, $\alpha_2 = 0.05$, $\beta = 0.2$,

$\epsilon \sim \mathcal{U}(3 - \mathbf{Range}/2, 3 + \mathbf{Range}/2)$**).**
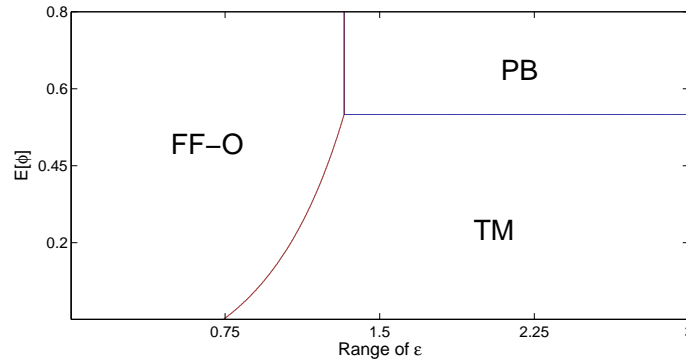
significantly expands the dominance region of FF-O contracts (despite the fact that they are based on outcomes, not outputs), relative to the other contract types. We also observed from numerical results that, by making the buyers' elasticities more balanced, the PB dominance region would shrink further, due to the convexity of the vendor's profit function, illustrating that two buyers with different elasticities generate more value than two buyers with similar elasticities.

6. Multiple Vendors

Finally, we finally demonstrate that having multiple vendors in the joint production process does not introduce additional sources of inefficiency to the ones identified in §4. Multi-vendor (or multi-sourcing) arrangements have become more prevalent in the consulting industry as a way to give clients access to world-class expertise and to make advisory work independent from system implementation (Toppin and Czerniawska 2005).

As is common in multi-sourcing arrangements, we assume that the vendors contract directly with the buyer. Moreover, we assume that the vendors negotiate the contract parameters among themselves through a Nash bargaining process. While there exist different types of bargaining process, the Nash bargaining solution is the only solution that is simultaneously Pareto-efficient, symmetric with respect to the preference relationships, and independent of irrelevant alternatives (Osborne and Rubinstein 1994).

Under these assumptions, we show in §EC.10 in the online appendix that the contract parameters

Table 2   Limitations of Contracts.

Contracts		Issues
Fixed Fee	Vendor Commits to Effort	Lack of adaptability
		Cost of monitoring vendor's actions
	Vendor Commits to Output Target	Lack of adaptability
		Cost of monitoring buyer's actions
		Focus on outcomes rather than outputs
Time and Materials		Cost of monitoring vendor's actions
Performance-Based	Monitoring of Vendor's Actions	Cost of monitoring vendor's actions
		Distorted performance measure
	No Monitoring of Vendor's Actions	Poor incentive-sharing in joint production
		Distorted performance measure
		Focus on outputs rather than outcomes

are chosen so as to maximize the total ex-post surplus, as if there were only one vendor. The inefficiencies thus arise because some of the efforts have to be chosen ex-ante (FF), because of the presence of additional monitoring costs (TM), or because of the incapacity to incentivize the vendors without lowering the buyer's incentives to exert effort (PB), as in §4, but not because of the presence of multiple vendors. The results obtained in §4 are thus robust to the presence of multiple vendors in the joint-production process.

7.   Conclusions

In this paper, we have developed an analytical framework for contract selection in a collaborative service environment. Using the double moral hazard paradigm, we evaluated the vendor's profit in a bilateral arrangement under the three following contracts: fixed-fee, time-and-materials, and performance-based contracts. We observed that fixed-fee contracts are the least responsive to outcome uncertainty, time-and-material contracts are associated with high monitoring costs, and performance-based contracts do not provide enough incentives for the parties to fully collaborate and may even induce undesired actions with distorted performance measures. Table 2 summarizes the limitations of each of these contracts.

In addition, we observed that our insights were robust to the number of vendors involved in the buying process. By contrast, the involvement of multiple buyers in the production process gives

rise to social loafing or free-riding.

Our analysis justifies the growing importance in consulting, outsourcing, and IT services, of disambiguation, specialization, focus on outcomes, breakdowns of big projects, and long-term relationships (Toppin and Czerniawska 2005). Specifically, disambiguation aims at reducing the lower tail of outcome uncertainty and agreeing on undistorted performance measures. Specialization of either the buyer or the vendor improves the collaboration output. Focus on outcomes, rather than outputs, is beneficial for PB contracts, because it links the vendor's efforts to the value that she delivers; in contrast, FF-O contracts are more efficient when the vendor targets a certain output, rather than outcome, because the lack of compensation for the additional risk may induce lower efforts. Moreover, breaking down a big project into small pieces helps define undistorted performance measures. Finally, long-term relationships reduce the communication and coordination costs between the buyer and the vendor.

In the future, we plan to extend the model in a competitive setting to evaluate the implications of competition on contracting and service design. The purpose of such analysis will be to understand how companies like HCL America (O'Connell and Loveman 1995) choose among the different service processes they offer to best satisfy their clients' needs.

References

Allon, G., A. Federgruen. 2007. Competition in services industries. *Oper. Res.* **55**(1) 37–55.

Apte, U. M., H. K. Nath. 2006. Size, structure and growth of the U.S. information economy. U. M. Apte, U. S. Karmarkar, eds., *Managing in the Information Economy*, chap. 1. Springer, New York.

Bajari, P., S. Tadelis. 2001. Incentives versus transaction costs: A theory of procurement contracts. *RAND J. Econom.* **32**(3) 387–407.

Baker, G. 2002. Distortion and risk in incentive contracts. *J. Human Resources* **37**(4) 728–751.

Banerjee, A. V., E. Duflo. 2000. Reputation effects and the limits of contracting: A study of the Indian software industry. *Quart. J. Econom.* **115**(3) 989–1017.

Bhattacharyya, S., F. Lafontaine. 1995. Double-sided moral hazard and the nature of share contracts. *RAND J. Econom.* **26**(4) 761–781.

Boyd, S., L. Vandenberghe. 2004. *Convex Optimization*. Cambridge University Press, United Kingdom.

Bramel, J., D. Simchi-Levi. 1997. *The Logic of Logistics*. Springer-Verlag, New York, NY.

Cachon, G. P. 2003. Supply chain coordination with contracts. S. Graves, T. de Kok, eds., *Handbook in Oper. Res. and Management Sci.: Supply Chain Management*, chap. 6. Elsevier, North-Holland.

Chase, R. B. 1981. The customer contact approach to services: Theoretical bases and practical extensions. *Oper. Res.* **29**(4) 698–706.

Chi, L., C. W. Holsapple. 2005. Understanding computer-mediated interorganizational collaboration: A model and framework. *J. Knowledge Management* **9**(1) 53–75.

Corbett, C. J., G. A. DeCroix, A. Y. Ha. 2005. Optimal shared-savings contracts in supply chains: Linear contracts and double moral hazard. *Eur. J. Oper. Res.* **163**(3) 653–667.

Corts, K. S., J. Singh. 2004. The effect of repeated interaction on contract choice: Evidence from offshore drilling. *J. Law Econom. Organ.* **20**(1) 230–260.

Fisher, M., A. Raman. 1996. Reducing the cost of demand uncertainty through accurate response to early sales. *Oper. Res.* **44**(1) 87–99.

Frei, F. X. 2006. Breaking the trade-off between efficiency and service. *Harvard Bus. Rev.* **84**(11) 1–12.

Fuchs, V. R., I. H. Leveson. 1968. *The Service Economy*. National Bureau of Economic Research, New York, NY.

Gibbons, R. 2005. Incentives between firms (and within). *Management Sci.* **51**(1) 2–17.

Gopal, A., K. Sivaramakrishnan, M. Krishnan, T. Mukhopadhyay. 2003. Contracts in offshore software development: An empirical analysis. *Management Sci.* **49**(12) 1671–1683.

Gurbaxani, V. 2007. Information systems outsourcing contracts: Theory & evidence. U. S. Karmarkar, U. Apte, eds., *Managing in the Information Economy: Current Research*, chap. 4. Springer-Verlag, New York, NY, 83–116.

Hasija, S., E. J. Pinker, R. A. Shumsky. 2008. Call center outsourcing contracts under information asymmetry. *Management Sci.* **54**(4) 793–807.

Hill, C., R. Yates, C. Jones, S. L. Kogan. 2006. Beyond predictable workflows: Enhancing productivity in artful business processes. *IBM Systems J.* **45**(4) 663–682.

Hopp, W. J., S. M. T. Iravani, F. Liu. 2007. White-collar workforce management: An operations-oriented survey. Working Paper.

Kalnins, A., K. Mayer. 2004. Relationships and hybrid contracts: An analysis of contract choice in information technology. *J. Law Econom. Organ.* **20**(1) 207–229.

Karmarkar, U. S., R. Pitbladdo. 1995. Service markets and competition. *J. Oper. Management* **12**(3-4) 397–411.

Kieliszewski, C., J. Bailey, J. Bloomberg. 2007. Early reflections on practice diagrams to facilitate service design. *Emergence*. Pittsburgh, USA.

Kim, S.-H., M. Cohen, S. Netessine. 2007. Performance contracting in after-sales service supply chains. *Management Sci.* **53**(12) 1843–1858.

Kogan, S. L., M. J. Muller. 2006. Ethnographic study of collaborative knowledge work. *IBM Systems J.* **45**(4) 759–771.

Kouvelis, P., M. A. Lariviere. 2000. Decentralized cross-functional decisions: Coordination through internal markets. *Management Sci.* **46**(8) 1049–1058.

Krishnan, H., R. Kapuscinski, D. A. Butz. 2004. Coordinating contracts for decentralized supply chains with retailer promotional effort. *Management Sci.* **50**(1) 48–63.

Kumar, K., P. C. van Fenema, M. A. Von Glinow. 2005. Intense collaboration in globally distributed work teams: Evolving patterns of dependencies and coordination. D. L. Shapiro, M. A. Von Glinow, J. L. C. Cheng, eds., *Managing Multinational Teams: Global Perspectives*. Elsevier/ JAI, Oxford.

Laffont, J.-J., D. Martimort. 2002. *The Theory of Incentives*. Princeton University Press, Princeton, NJ.

Laffont, J.-J., J. Tirole. 1993. *A Theory of Incentives in Procurement and Regulation*. The MIT Press, Cambridge, MA.

Lariviere, M. A. 1999. Supply chain contracting and coordination with stochastic demand. S. Tayur, M. Magazine, R. Ganeshan, eds., *Quantitative Models for Supply Chain Management*, chap. 8. Kluwer, Doordrecht, North-Holland, 233–268.

Latané, B., K. Williams, S. Harkins. 1979. Many hands make light the work: The causes and consequences of social loafing. *J. Personality and Soc. Psych.* **37**(6) 822–832.

Melnick, E. L., P. R. Nayyar, M. L. Pinedo, S. Seshadri. 1999. *Creating Value in Financial Services*. Springer-Verlag, New York, NY.

Müller, A., D. Stoyan. 2002. *Comparison Methods for Stochastic Models and Risks*. J. Wiley and Sons, New York, NY.

O'Connell, J., G. W. Loveman. 1995. HCL America. HBS 9-396-030.

Osborne, M. J., A. Rubinstein. 1994. *A Course in Game Theory*. MIT Press, Cambridge, MA.

Porteus, E. L., S. Whang. 1991. On manufacturing/marketing incentives. *Management Sci.* **37**(9) 1166–1181.

Sacerdote, B. 2001. Peer effects with random assignment: Results for Dartmouth roommates. *Quart. J. Econom.* **116**(2) 681–704.

Shapiro, B. 2002. Is performance-based pricing the right price for you? HBS 9-999-007.

Shenson, H. L. 1990. *The Contract and Fee-Setting Guide for Consultants and Professionals*. J. Wiley and Sons, New York, NY.

Talluri, K. T., G. J. van Ryzin. 2004. *The Theory and Practice of Revenue Management*. Kluwer Academic Publishers, Boston, MA.

Taylor, T. A., E. L. Plambeck. 2007. Simple relational contracts to motivate capacity investment: Price only vs. price and quantity. *Manufacturing Service Oper. Management* **9**(1) 94–113.

The Economist. 2007. External affairs. July 28, 2007.

The Economist. 2008a. Is time almost up for clockwatching lawyers? August 28, 2008.

The Economist. 2008b. The Swedish model. June 14, 2008.

Toppin, G., F. Czerniawska. 2005. *Business Consulting: A Guide to How It Works and How to Make It Work*. The Economist, in association with Profile Books Ltd., London, UK.

Williamson, O. E. 1985. *The Economic Institutions of Capitalism: Firms, Markets, Relational Contracting*. The Free Press, New York, NY.

Xue, M., J. M. Fields. 2008. Service coproduction with information stickiness and incomplete contracts: Implications for consulting services design. *Production Oper. Management* **17**(3) 357–372.

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Proofs of Statements

EC.1. Proof of Proposition 1

For part (1), observe that the function $U(\epsilon) = \epsilon^{\frac{1}{1-\alpha-\beta}}$ is convex increasing. Hence, if Φ_1 dominates Φ_2 in increasing convex order (Müller and Stoyan 2002, Definition 1.5.1), $\int_0^\infty U(\epsilon)d\Phi_1(\epsilon) \geq \int_0^\infty U(\epsilon)d\Phi_2(\epsilon)$. By Theorem 1.5.7 in Müller and Stoyan (2002), Φ_1 dominates Φ_2 in increasing convex order if and only if $\int_0^x(1-\Phi_1(\epsilon))d\epsilon \geq \int_0^x(1-\Phi_1(\epsilon))d\epsilon$ for all x.

For part (2), consider the second derivative of Π_V^{FB} with respect to α:

$$\frac{d^2\Pi_V^{FB}(\alpha,\beta)}{d\alpha^2} = \epsilon^{\frac{1}{1-\alpha-\beta}}\left(\frac{\alpha}{c_B}\right)^{\frac{\alpha}{1-\alpha-\beta}}\left(\frac{\beta}{c_V}\right)^{\frac{\beta}{1-\alpha-\beta}}\frac{1}{(1-\alpha-\beta)^3}$$
$$\times\left(1-\alpha-\beta+(1-\beta)\ln\left(\frac{\alpha}{c_B}\right)+\beta\left(\frac{\beta}{c_V}\right)+\ln(\epsilon)\right)^2$$
$$+\epsilon^{\frac{1}{1-\alpha-\beta}}\left(\frac{\alpha}{c_B}\right)^{\frac{\alpha}{1-\alpha-\beta}}\left(\frac{\beta}{c_V}\right)^{\frac{\beta}{1-\alpha-\beta}}$$

and is strictly larger than zero given that $1-\alpha-\beta > 0$. By symmetry, it follows that $d^2\Pi_V^{FB}(\alpha,\beta)/d\beta^2 > 0$. Finally, comparing $d^2\Pi_V^{FB}(\alpha,\beta)/d\alpha^2 d^2\Pi_V^{FB}(\alpha,\beta)/d\beta^2$ with the cross-derivative of Π_V^{FB} with respect to α and β:

$$\frac{d^2\Pi_V^{FB}(\alpha,\beta)}{d\alpha d\beta} = \epsilon^{\frac{1}{1-\alpha-\beta}}\left(\frac{\alpha}{c_B}\right)^{\frac{\alpha}{1-\alpha-\beta}}\left(\frac{\beta}{c_V}\right)^{\frac{\beta}{1-\alpha-\beta}}\frac{1}{(1-\alpha-\beta)^3}$$
$$\times\left(1-\alpha-\beta+(1-\beta)\ln\left(\frac{\alpha}{c_B}\right)+\beta\left(\frac{\beta}{c_V}\right)+\ln(\epsilon)\right)$$
$$\times\left(1-\alpha-\beta+(1-\alpha)\left(\frac{\beta}{c_V}\right)+\alpha\ln\left(\frac{\alpha}{c_B}\right)+\ln(\epsilon)\right)$$

shows that the determinant of the Hessian is positive. $\quad\square$

EC.2. Proof of Proposition 2

The buyer's profit is concave in x. From the first-order optimality conditions, the optimal buyer's effort is given by:

$$x^{FF-E}(y) = \left(\frac{\epsilon\alpha y^\beta}{c_B}\right)^{\frac{1}{1-\alpha}}.$$

Plugging the buyer's optimal effort level into his expected profit function yields

$$\Pi_B^{FF-E}(s,y) = \mathbb{E}\left[\epsilon^{\frac{1}{1-\alpha}}y^{\frac{\beta}{1-\alpha}}\left(\frac{\alpha}{c_B}\right)^{\frac{\alpha}{1-\alpha}}(1-\alpha)-\phi y-s\right],$$

which is jointly concave in y and s. Therefore, the vendor's problem of maximizing her revenue subject to the buyer's participation constraint, is convex.

The optimal fixed fee s makes the buyer's participation constraint active. Accordingly, the vendor's expected profit can be written as a function of y only:

$$\Pi_V^{FF-E}(y) = \mathbb{E}\left[\epsilon^{\frac{1}{1-\alpha}}\right] y^{\frac{\beta}{1-\alpha}} \left(\frac{\alpha}{c_B}\right)^{\frac{\alpha}{1-\alpha}} (1-\alpha) - (c_V + \mathbb{E}[\phi])y.$$

Solving the first-order optimality conditions for y yields

$$y^{FF-E} = \left(\mathbb{E}[\epsilon^{\frac{1}{1-\alpha}}]\right)^{\frac{1-\alpha}{1-\alpha-\beta}} \left(\frac{\beta}{c_V + \mathbb{E}[\phi]}\right)^{\frac{1-\alpha}{1-\alpha-\beta}} \left(\frac{\alpha}{c_B}\right)^{\frac{\alpha}{1-\alpha-\beta}}.$$

Plugging the vendor's optimal effort into her profit function yields the desired result. By Hölder's inequality, $\left(\mathbb{E}[\epsilon^{\frac{1}{1-\alpha}}]\right)^{\frac{1-\alpha}{1-\alpha-\beta}} \leq \mathbb{E}[\epsilon^{\frac{1}{1-\alpha-\beta}}]$, so $y^{FF-E} \leq y^{FB}$. Similarly,

$$x^{FF-E} = \left(\mathbb{E}[\epsilon^{\frac{1}{1-\alpha}}]\right)^{\frac{1-\alpha}{1-\alpha-\beta}} \left(\frac{\beta}{c_V + \mathbb{E}[\phi]}\right)^{\frac{\beta}{1-\alpha-\beta}} \left(\frac{\alpha}{c_B}\right)^{\frac{1-\beta}{1-\alpha-\beta}} \leq x^{FB}.$$

Finally, $\mathbb{E}[x^{FF-E}(\epsilon)]/y^{FF-E} = (\alpha(c_V + \mathbb{E}[\phi]))/(\beta c_B) \geq x^{FB}/y^{FB}.$ $\quad\square$

EC.3. Proof of Proposition 3

The vendor's profit function is jointly concave in s and y, and all inequality constraints are concave. The optimal solution is such that both constraints are active; otherwise one could increase s and/or decrease y, thereby increasing the vendor's profit, a contradiction. Hence, $y = (T/(\epsilon x^\alpha))^{1/\beta}$ and $x = (T-s)/(c_B + \mathbb{E}[\phi])$. Thus,

$$y^{FF-O}(s,T) = \left(\frac{T}{\epsilon\left(\frac{T-s}{c_B+\mathbb{E}[\phi]}\right)^\alpha}\right)^{\frac{1}{\beta}}.$$

Plugging the optimal vendor's effort level into the vendor's objective function yields

$$\Pi_V^{FF-O}(s,T) = s - c_V \mathbb{E}[\epsilon^{\frac{-1}{\beta}}] \left(\frac{T}{(T-s)^\alpha}(c_B + \mathbb{E}[\phi])^\alpha\right)^{\frac{1}{\beta}}$$

which is concave in T. The optimal threshold is $T = s/(1-\alpha)$. Plugging the optimal value for T into the vendor's profit function yields

$$\Pi_V^{FF-O}(s) = s - c_V \mathbb{E}[\epsilon^{\frac{-1}{\beta}}] \left(\left(\frac{s}{1-\alpha}\right)^{1-\alpha} \left(\frac{c_B + \mathbb{E}[\phi]}{\alpha}\right)^\alpha\right)^{\frac{1}{\beta}},$$

which is concave in s. The optimal fixed fee thus satisfies

$$s = \left(\mathbb{E}[\epsilon^{\frac{-1}{\beta}}]\right)^{\frac{-\beta}{1-\alpha-\beta}} \left(\frac{\beta}{c_V}\right)^{\frac{\beta}{1-\alpha-\beta}} \left(\frac{\alpha}{c_B + \mathbb{E}[\phi]}\right)^{\frac{\alpha}{1-\alpha-\beta}} (1-\alpha).$$

Plugging the optimal fee into $\Pi_V^{FF-O}(s)$ yields the desired result.

By Hölder's inequality and Jensen's inequality, it turns out that $\left(\mathbb{E}[\epsilon^{\frac{-1}{\beta}}]\right)^{\frac{-\beta}{1-\alpha-\beta}} \leq (\mathbb{E}[\epsilon])^{\frac{1}{1-\alpha-\beta}} \leq$
$\left(\mathbb{E}[\epsilon^{\frac{1}{1-\alpha}}]\right)^{\frac{1-\alpha}{1-\alpha-\beta}}$. Therefore, when $\mathbb{E}[\phi] = 0$,

$$x^{FF-O} = \left(\mathbb{E}[\epsilon^{\frac{-1}{\beta}}]\right)^{\frac{-\beta}{1-\alpha-\beta}} \left(\frac{\beta}{c_V}\right)^{\frac{\beta}{1-\alpha-\beta}} \left(\frac{\alpha}{c_B}\right)^{\frac{1-\beta}{1-\alpha-\beta}}$$

$$\leq \left(\mathbb{E}[\epsilon^{\frac{1}{1-\alpha}}]\right)^{\frac{1-\alpha}{1-\alpha-\beta}} \left(\frac{\beta}{c_V}\right)^{\frac{\beta}{1-\alpha-\beta}} \left(\frac{\alpha}{c_B}\right)^{\frac{1-\beta}{1-\alpha-\beta}} = \mathbb{E}[x^{FF-E}(\epsilon)] \leq \mathbb{E}[x^{FB}(\epsilon)]$$

$$\mathbb{E}[y^{FF-O}] = \left(\mathbb{E}[\epsilon^{\frac{-1}{\beta}}]\right)^{\frac{-\beta}{1-\alpha-\beta}} \left(\frac{\beta}{c_V}\right)^{\frac{1-\alpha}{1-\alpha-\beta}} \left(\frac{\alpha}{c_B}\right)^{\frac{\alpha}{1-\alpha-\beta}}$$

$$\leq \left(\mathbb{E}[\epsilon^{\frac{1}{1-\alpha}}]\right)^{\frac{1-\alpha}{1-\alpha-\beta}} \left(\frac{\beta}{c_V}\right)^{\frac{1-\alpha}{1-\alpha-\beta}} \left(\frac{\alpha}{c_B}\right)^{\frac{\alpha}{1-\alpha-\beta}} = y^{FF-E} \leq \mathbb{E}[y^{FB}(\epsilon)].$$

Finally, observe that the ratio of expected effort levels equals $x^{FF-O}/\mathbb{E}[y^{FF-O}(\epsilon)] = (\alpha c_V)/(\beta(c_B + \mathbb{E}[\phi])) \leq x^{FB}/y^{FB}$. $\square$

EC.4.   Proof of Proposition 4

The buyer's profit function is jointly concave in x and y. Hence, the optimal effort levels are uniquely defined by the first-order optimality conditions, that is,

$$x^{TM} = \epsilon^{\frac{1}{1-\alpha-\beta}} \left(\frac{\alpha}{c_B}\right)^{\frac{1-\beta}{1-\alpha-\beta}} \left(\frac{\beta}{p+\phi}\right)^{\frac{\beta}{1-\alpha-\beta}}$$

$$y^{TM} = \epsilon^{\frac{1}{1-\alpha-\beta}} \left(\frac{\alpha}{c_B}\right)^{\frac{\alpha}{1-\alpha-\beta}} \left(\frac{\beta}{p+\phi}\right)^{\frac{1-\alpha}{1-\alpha-\beta}}.$$

For any given TM contract, $\mathbb{E}[x^{FB}(\epsilon)] \geq \mathbb{E}[x(\epsilon,\phi)] \geq x(\mathbb{E}[\epsilon], \mathbb{E}[\phi])$, by Jensen's inequality, and similarly for y. Moreover, because $p \geq c_V$ and $\phi \geq 0$, $x^{TM}/y^{TM} \geq x^{FB}/y^{FB}$.

The maximum buyer's expected profit thus equals

$$\Pi_B^{TM}(p,s) = \mathbb{E}[\epsilon^{\frac{1}{1-\alpha-\beta}}] \left(\frac{\alpha}{c_B}\right)^{\frac{\alpha}{1-\alpha-\beta}} \mathbb{E}\left[\left(\frac{\beta}{p+\phi}\right)^{\frac{\beta}{1-\alpha-\beta}}\right] (1-\alpha-\beta) - s.$$

The optimal fixed fee s makes the buyer's participation constraint active. Accordingly, the vendor's expected profit can be expressed as

$$\Pi_V^{TM}(p) = \mathbb{E}\left[\epsilon^{\frac{1}{1-\alpha-\beta}}\right]\left(\frac{\alpha}{c_B}\right)^{\frac{\alpha}{1-\alpha-\beta}}(\beta)^{\frac{\beta}{1-\alpha-\beta}}\left((1-\alpha)\mathbb{E}\left[\frac{1}{(p+\phi)^{\frac{\beta}{1-\alpha-\beta}}}\right] - \beta\mathbb{E}\left[\frac{c_V+\phi}{(p+\phi)^{\frac{1-\alpha}{1-\alpha-\beta}}}\right]\right).$$

If ϕ is deterministic, the value of p that maximizes the vendor's profit equals c_V. Plugging $p = c_V$ into the vendor's expected profit function yields the lower bound in the statement of the proposition.

The second derivative of the last term in the vendor's profit function with respect to ϕ is proportional to

$$\frac{\beta}{1-\alpha-\beta}(p+\phi)^{-\frac{1-\alpha}{1-\alpha-\beta}-2}\left\{\frac{(1-\alpha)^2}{1-\alpha-\beta}(p-c_V)+(1-\alpha)(2p+\phi-c_V)\right\}$$

and is positive when $p \geq c_V$. Hence, by Jensen's inequality, the vendor's profit when ϕ is stochastic is larger than her profit when ϕ is deterministic. $\quad\square$

EC.5. Proof of Proposition 5

The buyer's and the vendor's profit functions are respectively concave in x and y. Hence, the optimal effort levels are given by:

$$x^{PB} = \epsilon^{\frac{1}{1-\alpha-\beta}}\left(\frac{\alpha(1-b)}{c_B}\right)^{\frac{1-\beta}{1-\alpha-\beta}}\left(\frac{\beta b}{c_V-p}\right)^{\frac{\beta}{1-\alpha-\beta}}$$

$$y^{PB} = \epsilon^{\frac{1}{1-\alpha-\beta}}\left(\frac{\alpha(1-b)}{c_B}\right)^{\frac{\alpha}{1-\alpha-\beta}}\left(\frac{\beta b}{c_V-p}\right)^{\frac{1-\alpha}{1-\alpha-\beta}}.$$

Observe that p must be smaller than c_V for the effort level to be finite. The optimal fee s makes the buyer's participation constraint active. Accordingly, the vendor's expected profit is equal to the total surplus:

$$\Pi_V^{PB}(b,p) = \mathbb{E}\left[\epsilon^{\frac{1}{1-\alpha-\beta}}\right]\left(\frac{\alpha(1-b)}{c_B}\right)^{\frac{\alpha}{1-\alpha-\beta}}\left(\frac{\beta b}{c_V-p}\right)^{\frac{\beta}{1-\alpha-\beta}}\left(1-\alpha(1-b)-\beta b\frac{c_V+\mathbb{E}[\phi]}{c_V-p}\right),$$

The derivative of the vendor's profit function with respect to p is proportional to

$$\left(\frac{\alpha(1-b)}{c_B}\right)^{\frac{\alpha}{1-\alpha-\beta}}\left(\frac{\beta b}{c_V-p}\right)^{\frac{\beta}{1-\alpha-\beta}}(c_V-p)^{-2}\left((c_V-p)(1-\alpha(1-b))-b(1-\alpha)(c_V+\mathbb{E}[\phi])\right)$$

and is thus positive when

$$p \leq c_V - b\left(c_V + \mathbb{E}\left[\phi\right]\right) \frac{1 - \alpha}{1 - \alpha(1 - b)},$$

and negative otherwise. Hence, the vendor's profit function is maximized at this threshold value of p. Plugging the optimal value for p into the vendor's profit yields

$$\Pi_V^{PB}(b) = \mathbb{E}\left[\epsilon^{\frac{1}{1-\alpha-\beta}}\right]\left(\frac{\alpha(1-b)}{c_B}\right)^{\frac{\alpha}{1-\alpha-\beta}}\left(\frac{\beta}{c_V + \mathbb{E}[\phi]}\right)^{\frac{\beta}{1-\alpha-\beta}}\left(\frac{1-\alpha(1-b)}{1-\alpha}\right)^{\frac{1-\alpha}{1-\alpha-\beta}}(1-\alpha-\beta).$$

The first-order optimality conditions yield two nonnegative stationary points, $b = 0$ and $b = 1$. The second derivative of the profit function is negative at $b = 0$ and equal to zero at $b = 1$. Because the vendor's profit is equal to zero at $b = 1$, the optimal piece bonus is thus equal to $b = 0$.

Hence, under the optimal bonus scheme, p tends to c_V, and the optimal effort levels tend to

$$x^{PB} = \epsilon^{\frac{1}{1-\alpha-\beta}}\left(\frac{\alpha}{c_B}\right)^{\frac{1-\beta}{1-\alpha-\beta}}\left(\frac{\beta}{c_V + \mathbb{E}[\phi]}\right)^{\frac{\beta}{1-\alpha-\beta}} \leq x^{FB}$$

$$y^{PB} = \epsilon^{\frac{1}{1-\alpha-\beta}}\left(\frac{\alpha}{c_B}\right)^{\frac{\alpha}{1-\alpha-\beta}}\left(\frac{\beta}{c_V + \mathbb{E}[\phi]}\right)^{\frac{1-\alpha}{1-\alpha-\beta}} \leq y^{FB}.$$

$\square$

EC.6.   Proof of Proposition 6

The buyer's and the vendor's profit functions are respectively concave in x and y. Hence, the optimal effort levels are given by:

$$x^{PB} = \epsilon^{\frac{1}{1-\alpha-\beta}}\left(\frac{\alpha(1-b)}{c_B}\right)^{\frac{1-\beta}{1-\alpha-\beta}}\left(\frac{\beta b}{c_V}\right)^{\frac{\beta}{1-\alpha-\beta}}$$

$$y^{PB} = \epsilon^{\frac{1}{1-\alpha-\beta}}\left(\frac{\alpha(1-b)}{c_B}\right)^{\frac{\alpha}{1-\alpha-\beta}}\left(\frac{\beta b}{c_V}\right)^{\frac{1-\alpha}{1-\alpha-\beta}},$$

and therefore $x^{PB} \leq x^{FB}$ and $y^{PB} \leq y^{FB}$.

The fee s makes the buyer's participation constraint active. Accordingly, the vendor's profit is equal to the total surplus:

$$\Pi_V^{PB}(b) = \mathbb{E}\left[\epsilon^{\frac{1}{1-\alpha-\beta}}\right]\left(\frac{\alpha(1-b)}{c_B}\right)^{\frac{\alpha}{1-\alpha-\beta}}\left(\frac{\beta b}{c_V}\right)^{\frac{\beta}{1-\alpha-\beta}}\left(1-\alpha(1-b)-\beta b\right).$$

Maximizing $\Pi_V^{PB}(b)$ is equivalent to maximizing $\left(\Pi_V^{PB}(b)\right)^{1/3}$. Let $g_1(b) = (1-b)^{\frac{\alpha}{1-\alpha-\beta}}$, $g_2(b) = b^{\frac{\beta}{1-\alpha-\beta}}$ and $g_3(b) = (1 - \alpha(1-b) - \beta b)$. Therefore, maximizing $\Pi_V^{PB}(b)$ is equivalent to maximizing the geometric mean of $g_1(b)$, $g_2(b)$, and $g_3(b)$. Because all $g_i(b)$ are concave, for $i = 1, ..., 3$, $\left(\Pi_V^{PB}(b)\right)^{1/3}$ is also concave (Boyd and Vandenberghe 2004). The first-order optimality condition yields:

$$b = \frac{-\beta(1-\alpha) + \sqrt{\alpha\beta(1-\alpha)(1-\beta)}}{\alpha - \beta},$$

and simple algebra reveals that $0 < b < 1$, i.e., the maximizing bonus rate always lies in the interior of $[0,1]$.

One can easily check that the optimal piece rate b is decreasing with α and increasing with β. In particular, $b \leq 1/2$ if and only if $\alpha \geq \beta$. Therefore,

$$\frac{x^{PB}}{y^{PB}} = \left(\frac{\alpha}{c_B}\right)\left(\frac{c_V}{\beta}\right)\frac{1-b}{b} \geq \frac{x^{FB}}{y^{FB}}$$

if and only if $\alpha \geq \beta$. $\quad\square$

EC.7. Proof of Proposition 7

The buyers' profit functions are concave in x_1 and x_2 respectively. From the first-order optimality conditions, the optimal buyers' efforts are given by:

$$x_1^{FF-E}(y) = \epsilon^{\frac{1}{1-\alpha_1-\alpha_2}}\left(\frac{\alpha_1}{c_{B1}}\right)^{\frac{1-\alpha_2}{1-\alpha_1-\alpha_2}}\left(\frac{\alpha_2}{c_{B2}}\right)^{\frac{\alpha_2}{1-\alpha_1-\alpha_2}}y^{\frac{\beta}{1-\alpha_1-\alpha_2}},$$

$$x_2^{FF-E}(y) = \epsilon^{\frac{1}{1-\alpha_1-\alpha_2}}\left(\frac{\alpha_1}{c_{B1}}\right)^{\frac{\alpha_1}{1-\alpha_1-\alpha_2}}\left(\frac{\alpha_2}{c_{B2}}\right)^{\frac{1-\alpha_1}{1-\alpha_1-\alpha_2}}y^{\frac{\beta}{1-\alpha_1-\alpha_2}}.$$

The optimal fixed fees s_1 and s_2 make the buyers' participation constraints active. Accordingly, the vendor's expected profit can be written as a function of y only:

$$\Pi_V^{FF-E}(y) = \mathbb{E}\left[\epsilon^{\frac{1}{1-\alpha_1-\alpha_2}}\right]y^{\frac{\beta}{1-\alpha_1-\alpha_2}}\left(\frac{\alpha_1}{c_{B1}}\right)^{\frac{\alpha_1}{1-\alpha_1-\alpha_2}}\left(\frac{\alpha_2}{c_{B2}}\right)^{\frac{\alpha_2}{1-\alpha_1-\alpha_2}}(2 - \alpha_1 - \alpha_2) - (c_V + \mathbb{E}[\phi])y.$$

Solving the first-order optimality conditions for y yields

$$y^{FF-E} = \left(\mathbb{E}[\epsilon^{\frac{1}{1-\alpha_1-\alpha_2}}]\right)^{\frac{1-\alpha_1-\alpha_2}{1-\alpha_1-\alpha_2-\beta}}\left(\frac{\beta}{c_V + \mathbb{E}[\phi]}\right)^{\frac{1-\alpha_1-\alpha_2}{1-\alpha_1-\alpha_2-\beta}}\left(\frac{\alpha_1}{c_{B1}}\right)^{\frac{\alpha_1}{1-\alpha_1-\alpha_2-\beta}}\left(\frac{\alpha_2}{c_{B2}}\right)^{\frac{\alpha_2}{1-\alpha_1-\alpha_2-\beta}}$$

$$\times \left(\frac{2 - \alpha_1 - \alpha_2}{1 - \alpha_1 - \alpha_2} \right)^{\frac{1 - \alpha_1 - \alpha_2}{1 - \alpha_1 - \alpha_2 - \beta}}.$$

Plugging the vendor's optimal effort into her profit function yields the desired result. The resulting effort levels are equal to

$$\mathbb{E}[x_1^{FF-E}(\epsilon)] = \left(\mathbb{E}[\epsilon^{\frac{1}{1-\alpha_1-\alpha_2}}] \right)^{\frac{1-\alpha_1-\alpha_2}{1-\alpha_1-\alpha_2-\beta}} \left(\frac{\alpha_1}{c_{B1}} \right)^{\frac{1-\alpha_2-\beta}{1-\alpha_1-\alpha_2-\beta}} \left(\frac{\alpha_2}{c_{B2}} \right)^{\frac{\alpha_2}{1-\alpha_1-\alpha_2-\beta}} \left(\frac{\beta}{c_V + \mathbb{E}[\phi]} \right)^{\frac{\beta}{1-\alpha_1-\alpha_2-\beta}}$$

$$\times \left(\frac{2 - \alpha_1 - \alpha_2}{1 - \alpha_1 - \alpha_2} \right)^{\frac{\beta}{1-\alpha_1-\alpha_2-\beta}},$$

$$\mathbb{E}[x_2^{FF-E}(\epsilon)] = \left(\mathbb{E}[\epsilon^{\frac{1}{1-\alpha_1-\alpha_2}}] \right)^{\frac{1-\alpha_1-\alpha_2}{1-\alpha_1-\alpha_2-\beta}} \left(\frac{\alpha_1}{c_{B1}} \right)^{\frac{\alpha_1}{1-\alpha_1-\alpha_2-\beta}} \left(\frac{\alpha_2}{c_{B2}} \right)^{\frac{1-\alpha_1-\beta}{1-\alpha_1-\alpha_2-\beta}} \left(\frac{\beta}{c_V + \mathbb{E}[\phi]} \right)^{\frac{\beta}{1-\alpha_1-\alpha_2-\beta}}$$

$$\times \left(\frac{2 - \alpha_1 - \alpha_2}{1 - \alpha_1 - \alpha_2} \right)^{\frac{\beta}{1-\alpha_1-\alpha_2-\beta}},$$

.

Because $\mathbb{E}[\phi] \geq 0$, $\left(\mathbb{E}[\epsilon^{\frac{1}{1-\alpha_1-\alpha_2}}] \right)^{\frac{1-\alpha_1-\alpha_2}{1-\alpha_1-\alpha_2-\beta}} \leq \mathbb{E}[\epsilon^{\frac{1}{1-\alpha_1-\alpha_2-\beta}}]$ by Hölder's inequality, and $((2-\zeta)/(1-\zeta))^{1-\zeta} \leq 2$, for any $0 < \zeta < 1$, we have $x_i^{FF-E} \leq x_i^{FB}$ for $i = 1, 2$ and $y^{FF-E} \leq y^{FB}$. Moreover, if $\phi = 0$, $x_1^{FF-E}/x_2^{FF-E} = x_1^{FB}/x_2^{FB}$, and

$$\frac{\mathbb{E}[x_i^{FF-E}]}{y^{FF-E}} = \frac{\alpha_1 c_V}{\beta c_{B1}} \frac{1 - \alpha_1 - \alpha_2}{2 - \alpha_1 - \alpha_2} \leq \frac{x_i^{FB}}{y^{FB}}.$$

□

EC.8.   Proof of Proposition 8

The vendor's profit function is jointly concave in s_1, s_2, and y, and all inequality constraints are concave. The optimal solution is such that both constraints are active; otherwise one could increase s_1 or s_2, and/or decrease y, thereby increasing the vendor's profit, a contradiction. Hence, $y = (T/(\epsilon x_1^{\alpha_1} x_2^{\alpha_2}))^{1/\beta}$ and $x_i = (T - s_i)/(c_{Bi} + \mathbb{E}[\phi_i])$ for $i = 1, 2$. Thus,

$$y^{FF-O}(s, T) = \left(\frac{T}{\epsilon \left(\frac{T-s_1}{c_{B1}+\mathbb{E}[\phi_1]} \right)^{\alpha_1} \left(\frac{T-s_2}{c_{B2}+\mathbb{E}[\phi_2]} \right)^{\alpha_2}} \right)^{\frac{1}{\beta}}.$$

Plugging the optimal vendor's effort level into the vendor's objective function yields

$$\Pi_V^{FF-O}(s_1, s_2, T) = s_1 + s_2 - c_V \mathbb{E}[\epsilon^{\frac{-1}{\beta}}] \left(\frac{T}{\left(\frac{T-s_1}{c_{B1}+\mathbb{E}[\phi_1]} \right)^{\alpha_1} \left(\frac{T-s_2}{c_{B2}+\mathbb{E}[\phi_2]} \right)^{\alpha_2}} \right)^{\frac{1}{\beta}}$$

which is concave in s_1 and s_2. The optimal fixed fees satisfy $(T - s_1)/\alpha_1 = (T - s_2)/\alpha_2$ and

$$s_1 = T - \left(\left(\frac{\alpha_1}{\alpha_2} \right)^{\frac{\alpha_2}{\beta}} \frac{\alpha_1}{\beta} c_V \mathbb{E}[\epsilon^{\frac{-1}{\beta}}] T^{\frac{1}{\beta}} (c_{B1} + \mathbb{E}[\phi_1])^{\frac{\alpha_1}{\beta}} (c_{B2} + \mathbb{E}[\phi_2])^{\frac{\alpha_2}{\beta}} \right)^{\frac{\beta}{\alpha_1 + \alpha_2 + \beta}} .$$

Plugging the optimal fixed fees into the vendor's profit function yields

$$\Pi_V^{FF-O}(T) = 2T - T^{\frac{1}{\alpha_1 + \alpha_2 + \beta}} (\alpha_1 + \alpha_2 + \beta) \left(\mathbb{E}[\epsilon^{\frac{-1}{\beta}}] \right)^{\frac{\beta}{\alpha_1 + \alpha_2 + \beta}}$$

$$\times \left(\frac{c_{B1} + \mathbb{E}[\phi_1]}{\alpha_1} \right)^{\frac{\alpha_1}{\alpha_1 + \alpha_2 + \beta}} \left(\frac{c_{B2} + \mathbb{E}[\phi_2]}{\alpha_2} \right)^{\frac{\alpha_2}{\alpha_1 + \alpha_2 + \beta}} \left(\frac{c_V}{\beta} \right)^{\frac{\beta}{\alpha_1 + \alpha_2 + \beta}}$$

which is concave in T. Optimizing the profit function with respect to T yields the desired result.

$\square$

EC.9. Performance-Based Contracts with Multiple Buyers

In this appendix, we evaluate the performance of PB contracts in the presence of multiple buyers. We first consider a PB contract with monitoring of the vendor's efforts and then consider a PB contract with no monitoring.

EC.9.1. Performance-Based Contracts with Monitoring of Efforts.

We first consider a PB contract where the vendor's effort is monitored at a unit cost $\phi_1 + \phi_2$. With a PB contract, the vendor's problem can be formulated as follows:

$$\max_{b_1, b_2, p_1, p_2, s_1, s_2} \mathbb{E}\left[(b_1 + b_2)\epsilon x_1^{\alpha_1} x_2^{\alpha_2} y^\beta + (p_1 + p_2 - c_V)y + s1 + s2 \right]$$

$$\mathbb{E}\left[(1 - b_1)\epsilon x_1^{\alpha_1} x_2^{\alpha_2} y^\beta - c_{Bi}x_i - (p_i + \phi_i)y - s_i \right] \geq 0 \ \ i = 1, 2$$

$$x_i(\epsilon, \phi_1, \phi_2) = \arg\max\left\{ (1 - b_i)\epsilon x_1^{\alpha_1} x_2^{\alpha_2} y^\beta - c_{Bi}x_i - (p_i + \phi_i)y - s_i \right\} \ \ i = 1, 2$$

$$y(\epsilon, \phi_1, \phi_2) = \arg\max\left\{ (b_1 + b_2)\epsilon x_1^{\alpha_1} x_2^{\alpha_2} y^\beta + (p_1 + p_2 - c_V)y + s1 + s2 \right\}.$$

The next proposition shows that the optimal PB contract with monitoring rewards the vendor only for her efforts and not for her contribution to the output. Hence, even if a TM contract is not well defined in a multi-buyer setting in the absence of an "orchestrator," PB contracts with monitoring effectively act as TM contracts.

PROPOSITION EC.1. *Under a PB contract with monitoring of the vendor's efforts in a multi-buyer setting, the vendor's profit equals*

$$\Pi_V^{PB} = \mathbb{E}[\epsilon^{\frac{1}{1-\alpha_1-\alpha_2-\beta}}] \left(\frac{\alpha_1}{c_{B1}}\right)^{\frac{\alpha_1}{1-\alpha_1-\alpha_2-\beta}} \left(\frac{\alpha_2}{c_{B2}}\right)^{\frac{\alpha_2}{1-\alpha_1-\alpha_2-\beta}} \left(\frac{\beta}{c_V+\mathbb{E}[\phi_1+\phi_2]}\right)^{\frac{\beta}{1-\alpha_1-\alpha_2-\beta}}$$

$$\times(1-\alpha_1-\alpha_2-\beta)\left(\frac{2-\alpha_1-\alpha_2}{1-\alpha_1-\alpha_2}\right)^{\frac{1-\alpha_1-\alpha_2}{1-\alpha_1-\alpha_2-\beta}},$$

and the optimal contract degenerates to a TM contract with $b=0$ and $p_1+p_2=c_V$.

Proof. Solving the game backwards, we obtain the profit-maximizing effort levels of the buyers and the vendor. The vendor then sets the fees s_1 and s_2 so as to give zero expected profit to the buyers. Accordingly, the vendor's expected profit is equal to the total surplus:

$$\Pi_V^{PB}(p_1,p_2,b_1,b_2) = \mathbb{E}\left[\epsilon^{\frac{1}{1-\alpha_1-\alpha_2-\beta}}\right]\left(\frac{\alpha_1(1-b_1)}{c_{B1}}\right)^{\frac{\alpha_1}{1-\alpha_1-\alpha_2-\beta}}\left(\frac{\alpha_2(1-b_2)}{c_{B2}}\right)^{\frac{\alpha_2}{1-\alpha_1-\alpha_2-\beta}}\left(\frac{\beta(b_1+b_2)}{c_V-p_1-p_2}\right)^{\frac{\beta}{1-\alpha_1-\alpha_2-\beta}}$$

$$\times\left(2-\alpha_1(1-b_1)-\alpha_2(1-b_2)-\beta(b_1+b_2)\frac{c_V+\mathbb{E}[\phi_1+\phi_2]}{c_V-p_1-p_2}\right).$$

Interestingly, the vendor's profit only depends on the sum of compensations, p_1+p_2. Maximizing the vendor's profit with respect to p_1+p_2 yields the following optimality condition:

$$p_1+p_2 = c_V - (b_1+b_2)(c_V+\mathbb{E}[\phi_1+\phi_2])\frac{1-\alpha_1-\alpha_2}{2-\alpha_1(1-b_1)-\alpha_2(1-b_2)}.$$

Similarly to bilateral contracting, plugging the optimal value for p_1+p_2 into the vendor's profit and optimizing with respect to b_1 and b_2 yields that the maximum profit is attained when $b_1 = b_2 = 0$, completing the proof. $\square$

As a result, PB contracts are inefficient because of (i) the monitoring costs $\phi_1+\phi_2$ and (ii) the negative externality induced by social loafing, similar to FF-E contracts. Distortion in performance measures can further lower the efficiency of these contracts, as discussed in §4.4.

EC.9.1.1. Performance-Based Contracts without Monitoring of Efforts. We consider a PB contract without monitoring of effort, i.e., $p_i = 0$, for $i = 1, 2$, thereby saving the monitoring costs $\phi_i = 0$ for $i = 1, 2$. The next proposition shows that, similar to §4.4.2, the optimal bonuses are independent of the output uncertainty and the effort costs.

PROPOSITION EC.2. *Under a PB contract with no monitoring of the vendor's efforts, in a multi-buyer setting, the optimal vendor's profit equals*

$$\Pi_V^{PB}(b) = \mathbb{E}\left[\epsilon^{\frac{1}{1-\alpha_1-\alpha_2-\beta}}\right] \left(\frac{\alpha_1(1-b_1)}{c_{B1}}\right)^{\frac{\alpha_1}{1-\alpha_1-\alpha_2-\beta}} \left(\frac{\alpha_2(1-b_2)}{c_{B1}}\right)^{\frac{\alpha_2}{1-\alpha_1-\alpha_2-\beta}} \left(\frac{\beta(b_1+b_2)}{c_V}\right)^{\frac{\beta}{1-\alpha_1-\alpha_2-\beta}}$$

$$\times \left(2 - \alpha_1(1-b_1) - \alpha_2(1-b_2) - \beta(b_1+b_2)\right),$$

in which the optimal bonuses b_1 and b_2 solve

$$\left(-\alpha_i(b_1+b_2) + \beta(1-b_i)\right)\left(1 - \alpha_1(1-b_1) - \alpha_2(1-b_2) - \beta(b_1+b_2)\right) =$$

$$(1-\alpha_1-\alpha_2-\beta)(1-b_i)(b_1+b_2)(\beta-\alpha_i).$$

for $i = 1, 2$. The optimal bonuses are independent of the costs of effort as well as of the output uncertainty, and the optimal effort levels satisfy $x_i^{PB} \le x_i^{FB}$, for $i = 1, 2$, $y^{PB} \le y^{FB}$.

Proof. The buyers' and the vendor's profit functions are respectively concave in x_1, x_2, and y. Hence, the optimal effort levels are given by:

$$x_1^{PB} = \epsilon^{\frac{1}{1-\alpha_1-\alpha_2-\beta}} \left(\frac{\alpha_1(1-b_1)}{c_{B1}}\right)^{\frac{1-\alpha_2-\beta}{1-\alpha_1-\alpha_2-\beta}} \left(\frac{\alpha_2(1-b_2)}{c_{B2}}\right)^{\frac{\alpha_2}{1-\alpha_1-\alpha_2-\beta}} \left(\frac{\beta(b_1+b_2)}{c_V}\right)^{\frac{\beta}{1-\alpha_1-\alpha_2-\beta}}$$

$$x_2^{PB} = \epsilon^{\frac{1}{1-\alpha_1-\alpha_2-\beta}} \left(\frac{\alpha_1(1-b_1)}{c_{B1}}\right)^{\frac{\alpha_1}{1-\alpha_1-\alpha_2-\beta}} \left(\frac{\alpha_2(1-b_2)}{c_{B2}}\right)^{\frac{1-\alpha_1-\beta}{1-\alpha_1-\alpha_2-\beta}} \left(\frac{\beta(b_1+b_2)}{c_V}\right)^{\frac{\beta}{1-\alpha_1-\alpha_2-\beta}}$$

$$y^{PB} = \epsilon^{\frac{1}{1-\alpha_1-\alpha_2-\beta}} \left(\frac{\alpha_1(1-b_1)}{c_{B1}}\right)^{\frac{\alpha_1}{1-\alpha_1-\alpha_2-\beta}} \left(\frac{\alpha_2(1-b_2)}{c_{B2}}\right)^{\frac{\alpha_2}{1-\alpha_1-\alpha_2-\beta}} \left(\frac{\beta(b_1+b_2)}{c_V}\right)^{\frac{1-\alpha_1-\alpha_2}{1-\alpha_1-\alpha_2-\beta}}.$$

We now show that $x_i^{PB} \le x_i^{FB}$ for $i = 1, 2$ and $y^{PB} \le y^{FB}$. Consider first x_1^{PB}; buyer 1's effort is maximized when $(1-b_1)^{1-\alpha_2-\beta}(1-b_2)^{\alpha_2}(b_1+b_2)^{\beta}$ is maximized. Solving the first-order conditions, we find that the above function is minimized when $b_2 = \max\{0, 1 - 2\alpha_2\}$ and $b_1 = \min\{1, 2(\alpha_2 + \beta) - 1\}$. Therefore, $(1-b_1)^{1-\alpha_2-\beta}(1-b_2)^{\alpha_2}(b_1+b_2)^{\beta} \le \max_{\alpha_2 \le 0.5, \beta \le 0.5} 2\alpha_2^{\alpha_2}\beta^{\beta}(\alpha_2+\beta)^{\alpha_2+\beta} = 1$, and so $x_1^{PB} \le x_1^{FB}$. Similar reasoning applies to x_2^{PB} and y^{PB}.

The fees s_1 and s_2 make the buyers' participation constraints active. Accordingly, the vendor's profit is equal to the total surplus:

$$\Pi_V^{PB}(b_1, b_2) = \mathbb{E}\left[\epsilon^{\frac{1}{1-\alpha_1-\alpha_2-\beta}}\right] \left(\frac{\alpha_1(1-b_1)}{c_{B1}}\right)^{\frac{\alpha_1}{1-\alpha_1-\alpha_2-\beta}} \left(\frac{\alpha_2(1-b_2)}{c_{B2}}\right)^{\frac{\alpha_2}{1-\alpha_1-\alpha_2-\beta}} \left(\frac{\beta(b_1+b_2)}{c_V}\right)^{\frac{\beta}{1-\alpha_1-\alpha_2-\beta}}$$

$$\times \left(1 - \alpha_1(1-b_1) - \alpha_2(1-b_2) - \beta(b_1 + b_2)\right).$$

Maximizing $\Pi_V^{PB}(b_1, b_2)$ is equivalent to maximizing $\left(\Pi_V^{PB}(b_1, b_2)\right)^{1/4}$. Let $g_1(b_1, b_2) = (1 - b_1)^{\frac{\alpha_1}{1 - \alpha_1 - \alpha_2 - \beta}}$, $g_2(b_1, b_2) = (1 - b_2)^{\frac{\alpha_2}{1 - \alpha_1 - \alpha_2 - \beta}}$, $g_3(b_1, b_2) = (b_1 + b_2)^{\frac{\beta}{1 - \alpha_1 - \alpha_2 - \beta}}$ and $g_4(b_1, b_2) = (1 - \alpha_1(1 - b_1) - \alpha_2(1 - b_2) - \beta(b_1 + b_2))$. Therefore, maximizing $\Pi_V^{PB}(b_1, b_2)$ is equivalent to maximizing the geometric mean of all $g_i(b_1, b_2)$ for $i = 1, ..., 4$. Because all $g_i(b_1, b_2)$ are concave, for $i = 1, ..., 4$, $\left(\Pi_V^{PB}(b_1, b_2)\right)^{1/4}$ is also concave (Boyd and Vandenberghe 2004). The first-order optimality conditions yield, for $i = 1, 2$,

$$\left(-\alpha_i(b_1 + b_2) + \beta(1 - b_i)\right)\left(1 - \alpha_1(1 - b_1) - \alpha_2(1 - b_2) - \beta(b_1 + b_2)\right) =$$

$$(1 - \alpha_1 - \alpha_2 - \beta)(1 - b_i)(b_1 + b_2)(\beta - \alpha_i).$$

Therefore, b_1 and b_2 are independent of the costs of efforts and the output uncertainty. $\square$

When $\alpha_1 = \alpha_2$, we have $b_1 = b_2$. When $\alpha_1 = \beta$, we have $b_1 = (1 - b_2)/2$, and similarly for b_2 when $\alpha_2 = \beta$. In addition, if the optimal bonuses are increasing with their respective elasticities, i.e., when $b_1'(\alpha_1) \geq 0$ and $b_2'(\alpha_2) \geq 0$, then $x_i^{PB}/y^{PB} \geq x_i^{FB}/y^{FB}$, for $i = 1, 2$, if and only if $\alpha_i \geq \beta$ and $x_1^{PB} \geq x_2^{PB}$ if and only if $\alpha_1 \geq \alpha_2$.

EC.10. Contracting with Multiple-Vendors

Instead of analyzing the performance of each contract separately, we prove a general result that guarantees the robustness of the results obtained with bilateral contracting. To simplify the analysis, we assume that there are only two vendors. For any type of contract, let us define $\Pi_{Vi}^{ex-post}(\gamma_1, \gamma_2) + s_i$ vendor i's ex-post profit, for $i = 1, 2$. The profit is evaluated ex-post, i.e., evaluated after the uncertainties have been realized and the effort levels have been chosen. The profit is a function of the contract parameters γ_1 and γ_2, excluding the fixed fees, where, for $i = 1, 2$, the parameters γ_i abstractly refer to y_i (FF-E contract), p_i (TM contract), or p_i and b_i (PB contract). (FF-O contracts are not well defined in this multi-vendor environment in the absence of a rule determining the residual risk bearer among the vendors.) Similarly, let us define $\Pi_B^{ex-post}(\gamma_1, \gamma_2) - s_1 - s_2$ the buyer's ex-post profit, for these contract parameters.

Assuming zero reservation profits, the Nash solution maximizes the product of utilities, $(\Pi_{V1}^{ex-post}(\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2) + s_1) \times (\Pi_{V2}^{ex-post}(\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2) + s_2)$ (Osborne and Rubinstein 1994), subject to the buyer's participation constraint. (The incentive compatibility constraint is implicit to the ex-post formulation of the profits.) The game can be therefore formulated as follows:

$$\max_{s_1, s_2, \boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2} (\Pi_{V1}^{ex-post}(\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2) + s_1) \times (\Pi_{V2}^{ex-post}(\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2) + s_2)$$

$$s.t. \qquad \Pi_{B}^{ex-post}(\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2) - s_1 - s_2 \geq 0. \qquad (\text{EC.1})$$

PROPOSITION EC.3. *For any type of contract, let $\Pi_{Vi}^{ex-post}(\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2) + s_i$ denote vendor i's ex-post profit, for $i = 1, 2$, and $\Pi_{B}^{ex-post}(\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2) - s_1 - s_2$ denote the buyer's ex-post profit, where $(\boldsymbol{\gamma}_i, s_i)$ are the contract parameters. Then, the Nash bargaining outcome $(\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2)$ that solves (EC.1) also maximizes $\left(\Pi_{V1}^{ex-post}(\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2) + \Pi_{V2}^{ex-post}(\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2) + \Pi_{B}^{ex-post}(\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2)\right).$*

Proof. At the optimum, the buyer's participation constraint is active. Hence, $s_2 = \Pi_{B}^{ex-post}(\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2) - s_1$. Substituting s_2 into the objective function yields

$$\max_{s_1, \boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2} (\Pi_{V1}^{ex-post}(\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2) + s_1) \times (\Pi_{V2}^{ex-post}(\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2) - s_1 + \Pi_{B}^{ex-post}(\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2)).$$

The function is concave in s_1. The optimal fee s_1 is obtained from the first-order optimality conditions, i.e., $s_1 = (1/2)(\Pi_{V2}^{ex-post}(\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2) + \Pi_{B}^{ex-post}(\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2) - \Pi_{V1}^{ex-post}(\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2))$. Substituting s_1 by its optimal value yields

$$\max_{\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2} \tfrac{1}{4} \left(\Pi_{V1}^{ex-post}(\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2) + \Pi_{V2}^{ex-post}(\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2) + \Pi_{B}^{ex-post}(\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2)\right)^2.$$

$\square$

As a result, the contract parameters are chosen so as to maximize the total ex-post surplus, as if there were only one vendor.