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EXCHANGE EFFECTS FOR COMPOSITE PARTICLES[†]

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Abstract: Inelastic and charge-exchange scattering of light projectiles by a nucleus is considered, including the exchange effects arising from the antisymmetrization of the projectile and target wave functions and from the exchange nature of nuclear forces. It is shown that the projectile nucleon-target nucleon interaction has to be described by antisymmetrized two-body matrix elements of the nuclear force rather than by the direct ones only, as is commonly assumed. Exchange effects change only slightly the angular distribution, but may increase the cross-section strength up to a factor 4.

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1. Introduction

The microscopic description of inelastic scattering (or charge exchange) of light projectiles is generally made in terms of an effective interaction[1] between the excited nucleons of the target and the center of mass of the projectile, but neglecting exchange effects arising from antisymmetrization of the projectile and target wave functions and from the exchange nature of nuclear forces. Analysis[2] of the (He^3,t) data[3] at 30 MeV on N = 28 targets, where states with spins up to 7 were excited, shows however serious discrepancies (fig. 1). In order to fit experiment, the strength of the effective force, which should be the same for any final state, has to be strongly increased for high angular momentum transfer. The same discrepancy is found in the interpretation[4] (fig. 1) of the 37 MeV (He^3,t) reaction on Fe^{54} and Z^{90}. Such discrepancies appeared[5] also for (pp') scattering but were removed[6-12] by introducing a knock-on term which takes account of the antisymmetrization of the incident particle and target wave functions and for the exchange nature of nuclear forces. It was not possible to draw definite conclusions about the importance of exchange for composite particles from an earlier attempt[2] to introduce these exchange effects because of the approximations made, i.e., we assumed[2] that the nucleons in the He^3 and H^3 were concentrated at the center of mass of the projectile. As previously explained[13] we take account of the space extension of the projectiles (this was suggested in Ref. [2]). The formalism we propose is simple, but accurate enough to give a definitive answer about the size of the exchange effects. These are still treated approximately, but the approximation we use was introduced by Petrovich *et al.*[9] for (pp') scattering and was seen to be really a good one, especially for high spin transfers.

2. The Formalism

We assume that the particles in He^3 and triton are in relative s states. Therefore, the spatial part $\psi(\vec{r}_1,\vec{r}_2,\vec{r}_3)$ of the projectile wave function is symmetric. Furthermore, as is done for any calculation of composite projectile scattering, we write ψ as a product of a function $X(\vec{R})$ of the center of mass coordinate $\vec{R} = \frac{1}{3}(\vec{r}_1 + \vec{r}_2 + \vec{r}_3)$ and an internal function $\omega(\vec{r}_1 - \vec{R}, \vec{r}_2 - \vec{R}, \vec{r}_3 - \vec{R})$. That is, we assume that the nucleons in the projectile move much more rapidly than the projectile itself. This approximation, which is very good in the projectile interior, is less good at its surface where the nucleons have rather low energy, i.e., the projectile may be polarized as it approaches the target nucleus. We have neglected these polarization effects, although it is not obvious that they are unimportant. $X(\vec{R})$ will be calculated using the usual optical model approximation. The initial and final states (denoted $|I\rangle$ and $|F\rangle$) of the system are described as the antisymmetrized product of the projectile and the target wave functions. We consider all particles as identical and use isospin formalism. In the D.W.B.A. approximation, the transition amplitude T is (see Appendix) the matrix element $\langle F|\Sigma v|I\rangle$ where Σv is the sum of the interactions between target and projectile nucleons. The hypothesis we have to make, in order to obtain this expression for T is that the projectile wave function can be expressed in terms of Slater determinants of <u>unoccupied</u> shell model states of the target. This same hypothesis had to be made in order to get[11]) the similar formula including exchange for pp' scattering. In this case, it is possible to calculate directly the overlap of the proton optical wave function and the occupied shell model states. This overlap is found to be small in general, with however a few exceptions. Nevertheless exchange effects are quite important for pp' scattering (up to a factor of 10 with $f_{7/2}$ region[11,12])). Even for composite particles, this hypothesis is therefore probably not really restrictive.

The transition amplitude T can then be written (see Appendix) in terms of the antisymmetrized matrix elements of the two-body nucleon-nucleon force v taken between the target shell-model states A, a, B and b:

$$\langle Aa|v|Bb\rangle_{\mathcal{A}} = \langle Aa|v\{|Bb\rangle - |bB\rangle\}$$

A and B refer to the projectile nucleons (whose wave function has been expanded in terms of the target shell-model states) and a and b are states of the target nucleons.

Using a similar approximation as in Ref. 9), we replace the exchange term $\langle Aa|v|bB\rangle$ of the nucleon-nucleon matrix element by a direct matrix element of an effective force with zero range and an energy dependent strength.

$$\langle Aa|v|Bb\rangle_{\mathcal{A}} \sim \langle Aa|w|Bb\rangle$$

$$w(r) = V\, f(\vec{r}) + V'\, \delta(\vec{r}) \int e^{i\vec{k}\cdot\vec{x}}\, f(\vec{x})d\vec{x} \quad , \quad v(r) = V\, f(r)$$

V' has the same form as V (Appendix 2), but with a different mixture parameters, $\vec{k}$ is the relative momentum of the exchanged nucleons.

In the expression of the T matrix for composite particles, this matrix element is averaged over the projectile matter distribution

$$\rho(\vec{r}-\vec{R}) = \int \omega_t(\vec{r}-\vec{R},\vec{r}_2-\vec{R},\vec{r}_3-\vec{R})\omega_{He^3}(\vec{r}-\vec{R},\vec{r}_2-\vec{R},\vec{r}_3-\vec{R})\ \delta\left(\frac{\vec{r}+\vec{r}_2+\vec{r}_3}{3} - R\right)\ d\vec{r}_2 d\vec{r}_3 \quad .$$

The average over the direct term $\langle Aa|v|Bb\rangle$ leads to the usual[1]) effective force acting between the projectile center of mass and the target nucleon:

$$V^{[D]}_{He^3,t}(\vec{r}) = V \int \rho(\vec{x})\ f(\vec{r}-\vec{x})d\vec{x}$$

$f(\vec{r})$ is the radial form and V the exchange mixture of the nucleon-nucleon interaction

$$v(\vec{r}) = V\ f(\vec{r}) \quad .$$

The effective interaction for the (He^3,t) reaction when antisymmetrized wave functions are used and the exchange nature of nuclear forces is taken into account can therefore be written:

$$V^{[D+E]}_{He^3,t}(r) = \int \rho(\vec{x})\ w(\vec{x}-\vec{r})d\vec{x}$$

$$= V^{[D]}_{He^3,t}(\vec{r}) + V^{[E]}_{He^3,t}(\vec{r})$$

$$V^{[E]}_{He^3,t}(\vec{r}) = V'\ \rho(\vec{r}) \int e^{i\vec{k}\cdot\vec{x}}\ f(\vec{x})d\vec{x}$$

$\vec{k}$ is the relative momentum of the two exchanged nucleons (one nucleon is in the projectile, the other one in the target). The relative velocity of these two nucleons is

$$\vec{v} = \vec{v}_{proj} + \vec{V} + \vec{v}_{targ}$$

$\vec{V}$ is the projectile velocity, $\vec{v}_{proj}$ is the velocity of the projectile nucleon in the projectile system of reference and $\vec{v}_{targ}$ is the velocity of the target particle in the target system. Thus, the relation between the wave numbers

$$\vec{k} = \vec{k}_{proj} + \frac{m}{M}\vec{K} + \vec{k}_{targ}$$

m is the nucleon mass and M the projectile mass.

We can now use the fact that the reaction takes place at the nuclear surface and also at the projectile surface, i.e., the projectile does not penetrate very much into the nucleus. This property can be seen by considering that, whatever the radial effective interaction between the projectile center-of-mass and the target nucleon is, its magnitude at relative distances smaller than 2 fm is completely unimportant[2]). That is, the target nucleons remain very far from the projectile center-of-mass since the radius of the projectiles we consider never exceeds 2 fm. At the surface, k_{proj} and k_{targ} are small. A bound particle in a potential well $V(r)$ with a binding energy ε has an energy of the order of $\varepsilon(R) = \varepsilon - V(R)$ at the surface (R is the surface radius). This quantity is generally small (fig. 2) and the energy of the particle is on the average much smaller at the surface than inside. We take therefore[†]

$$\vec{k} \sim \frac{m}{M}\vec{K}$$

[†]I would like to thank N. K. Glendenning for pointing out an error on this point in an earlier stage of this work.

that is

$$k = \frac{m}{M}\sqrt{2ME} = \sqrt{\frac{m}{M}}\; k_n$$

where $k_n = \sqrt{2mE}$ is the wave number of an incident nucleon with the same energy as the composite projectile.

The direct effective potential $V^{[D]}$ has[14] a Yukawa shape at distances larger than 2 fm when a Yukawa nucleon-nucleon interaction is used. The average over the projectile leads only to a slightly larger strength. A Gaussian nucleon-nucleon force of range 1.78 fm leads to a Gaussian effective force of range 2.32 fm. For the exchange part, $\rho(r)$ can be taken as a Gaussian $\frac{4}{\sqrt{\pi}\lambda^3}\, e^{-(r/\lambda)^2}$ where λ is determined by the mean square radius. More realistic shapes[15] (fig. 3) have exactly the same tail between 2 and 4 fm, and therefore from Ref. [2] we know that a Gaussian form is perfectly adequate. The (He^3,t) reaction leads to a range $\lambda = 1.48$ fm for $\rho(r)$. Similar effective forces taking exchange into account can be derived for inelastic scattering of projectiles with masses up to 4 (in order that the nucleons can be considered in relative s states).

3. Exchange Effects for (He^3,t) Reactions

The lowest states of ^{48}Sc, excited by $^{48}Ca(He^3,t)Sc^{48}$ at 30 MeV can reasonably be assumed to be due to a recoupling of a $f_{7/2}$ neutron hole and a $f_{7/2}$ proton particle, and that makes this reaction especially interesting among others like inelastic scattering of He^3 or α particles. We shall study only the natural parity states (0^+ to 6^+) since the excitation of unnatural parity states occurs through a strong tenser force, as preliminary calculations indicated in Ref. [2]) and as it was snown in Ref. [15]). The exchange term for unnatural parity states requires therefore the introduction of a tenser term built with $\vec{k}$ instead with $\vec{r}$. The calculation of exchange effects can still be done simply by assuming that the direction of $\hat{k}$ and $\hat{r}$ are the same, which is in principle only true for larger r. We have not included such an interaction. With that restriction, the only strength parameter for an in-shell transition ($f_{7/2}$ to $f_{7/2}$) is the coefficient of $\vec{\tau}_1 \cdot \vec{\tau}_2$ in V (denoted V_J, $J = 0$ to 6, since we want to emphasize its dependence on the transferred angular momentum J), and the similar coefficient V_J' in V'. The exchange mixture of the force enters only through the parameter $p = V_J'/V_J$ in the calculation.

When exchange contributions are neglected, the usual 1.37 fm force leads[2]) to a ratio V_6/V_0 of about 10. V_0 is then about 7 MeV, which corresponds to 1.4 times the Serber strength. For (pp') very good agreement with experiment could be obtained[6,9,10]) using a Serber like force, except at lower energy where the inelastic cross-sections where slightly underestimated. A value of 7 MeV for V_0 is therefore consistent with the pp' experiments and is a good support for our assumption of simple shell model structure.

Configuration mixing is presumably important only for the 2^+ state. The matrix elements of the residual interaction decrease when the spin of the considered level increases. For the 0^+ state, however, the $f_{7/2}\, f^{-1}_{7/2}$ state is the only candidate, since the double-closed shell picture for Ca^{48} is probably very good[20]). For higher spin levels, there are in general several (proton) particle-(neutron) hole configurations which may be mixed ($f_{7/2}\, f^{-1}_{7/2}$ and $p_{3/2}\, f^{-1}_{7/2}$ mainly). Rough estimates, based on the results obtained in Ref. [21]) show that the coupling is important only for 2^+ states, where the $f_{7/2}\, f^{-1}_{7/2}$ configuration represents still more than 85% of the wave function. For usual[20,21]) particle-hole excitations, the residual interaction is attractive and configuration mixing terms add coherently in the scattering amplitude and increase the inelastic cross-section. Here, the force also has to exchange a charge and is generally repulsive. In this case the coherence is destructive and lowers therefore the cross-section. The same (rough) estimate as previously shows that the 2^+ cross-section may be lowered by a factor of 2, and therefore the corresponding V_2 is increased in this case by 40%. In the calculations which follow, we have only considered pure $f_{7/2}\, f^{-1}_{7/2}$ wave functions.

The range of 1 fm for the nucleon-nucleon force is generally used for (He^3,t) reactions since it leads to better angular distributions. With such a force, the ration V_6/V_0 is about 4 when exchange effects are neglected. These effects, when a Serber mixture (p = 1) is used, increase the 6^+ cross-section by more than a factor of 2 at the first maximum (fig. 5 and Table 1). They are slightly larger at backward angles. The 0^+ cross-section on the contrary is practically not modified (fig. 4 and Table 1).

These contributions increase regularly with increasing spin transfer (Table 1) and therefore the discrepancy between theory and experiment due to the relative strengths of 6^+ and 0^+ cross-sections is partly removed (figs. 1 and 5). A Rosenfeld mixture (p = 2) provides the larger value for p when usual exchange mixtures are used. Exchange effects in that case are much more important (about a factor of 4 for 6^+ cross-section). Such a force gives however very poor results for pp' scattering calculations[6]).

The relative contribution of direct and exchange at the first maximum of the cross-section does not depend strongly on the slope of the interaction used as shown in Table 1. When the variations with r of the radial shape $f(r)$ of the nucleon-nucleon force are smooth, the direct contribution falls down more rapidly with increasing spin transfer J. The exchange term is also reduced by the factor $\hat{f}(k)$ which is smaller for smooth shapes, but this reduction is independent of J. This is illustrated by the Yukawa force of range 1.37. Compared to the calculation with a 1 fm range Yukawa exchange effects are smaller for 0^+ transitions (this is the effect of the reduction factor $\hat{f}(k)$), but larger for 6^+ transitions (reduction of the direct part for large J). The Gaussian and the Yukawa force give very similar results. The explanation was given earlier: the Fourier transforms of these two forces have the same low-energy components.

Even if the effects due to exchange processes are quite important, they are not strong enough to explain the discrepancy seen in Ref. [2]). The ratio V_6/V_0 is still of about 2.5 in the best case, i.e., with a Rosenfeld mixture and a 1 fm range Yukawa radial shape.

The energy dependence of exchange effects is very weak. There is practically no difference for 18 and 30 MeV incident particles, and even at 75 MeV, exchange effects are only slightly reduced (Table 1). Using more realistic forces (Kallio-Kolltweit for instance), the variations with energy are probably more important, as was shown[9]) for pp' scattering.

4. Exchange Effects for Inelastic Scattering of Light Particles

The exchange effects have about the same magnitude for inelastic α,He^3 and H^3 scattering, when a Serber mixture is used. We have taken the He^3 and H^3 potentials from Ref. [2]) and computed (Table 2) the ratio $\frac{\sigma_{[D+E]}}{\sigma_{[D]}}$ of the cross-sections at their first maximum for a $f_{7/2} \to f_{7/2}$ transition, with various spin transfers. The two-body interaction is taken to have a Serber mixture and a Yukawa type radial form of range 1 fm. The optical potential for α particles is taken from Ref. [17]). The small differences have their origin in the projectile size, but they may not be really significant when considering the uncertainties in the optical parameters. The magnitude of the exchange effects, as studied for the He^3,t reaction are therefore typical for inelastic scattering of composite particles with masses up to 4. In particular, their energy and range dependence are approximately the same for α,He^3 and H^3. They have also the same qualitative features as for pp' or p,n scattering[2,8,11,12]). The quantitative features differ, however, to a large extent.

For (pp') scattering exchange effects depend considerably on energy[8-12]). The strength of the effective potential $V^{[E]}_{He^3,t}$ is proportional to $\hat{f}(k)$, the Fourier transform of the radial part $f(r)$ of the nucleon-nucleon interaction. $\hat{f}$ generally decreases for increasing k, and is 1 for $k = 0$. This factor reduces the exchange effects at higher energies. For (pp') scattering, however, this reduction factor is $\hat{f}(k_n)$ whereas for composite particles it is $\hat{f}\left(\sqrt{\frac{m}{M}}\, k_n\right)$ which depends much less on energy. We see also that, at a given energy, this factor is larger for composite particles than for nucleons. The heavier is the projectile, the closer to 1 is $\hat{f}$, but there is also the strong absorption which lowers considerably the exchange effects for composite particles, so they are finally less important than for nucleons. One must however remember that it is precisely because there is a strong absorption that one could choose such a low value for $k \sim \frac{m}{M} K = \sqrt{\frac{m}{M}}\, k_n$.

For inelastic scattering, a Rosenfeld mixture would provide a value $p = -65$ for an excitation with an isospin transfer $T = 0$ (this is probably the case of all collective excitations[18])). Such a large value is probably very unrealistic since it leads to describe the inelastic scattering as dominated by the exchange contribution. For (pp') scattering therefore, very bad results were obtained[10]) with a Rosenfeld mixture, whereas a Serber mixture was successful. Moreover, the physical idea that the nucleons interact with a very low relative energy is very much in favor of a Serber type of force[19]). The latter describes[19]) to a good extent low-energy free nucleon-nucleon scattering, and also[19]) the scattering of two nucleons at the nuclear surface. We think therefore that the results obtained with a Serber mixture are the more reliable, even for He^3,t scattering.

5. Conclusion

For composite particle scattering, exchange effects arising from antisymmetrization of all nucleons and from the exchange nature of nuclear forces are very important. They increase with increasing spin transfer. Their effect is mainly a normalization effect and a typical magnitude for a spin transfer $J = 6$ is an increase of more than a factor 2 in the differential cross-section. The approximation of Petrovich et al.[9] made for the exchange part is probably very good, as was seen[9] for (pp') scattering. The major improvement of the theory is probably to study polarization effects of the incoming projectile. At the surface of the projectile, its nucleons may have velocities comparable to the velocity of the projectile itself, and the separation of its wave function into a product of an optical and an internal part is questionable.

Appendix 1

Let us consider the reaction between the two fragments P and T: $P + T \rightarrow P' + T^*$. The total Hamiltonian of the system is generally split into two parts which describe the motion of the two fragments independently of each other and a part which describes their interaction:

$$H = H_P + H_T + \Sigma v_{PT}$$

$$H_P = \Sigma t_P + \Sigma v_{PP}$$

$$H_T = \Sigma t_T + \Sigma v_{TT}$$

t is the kinetic energy operator. H is symmetric in all coordinates, but $H_P + H_T$ is not, and the eigenfunctions of $H_P + H_T$ have no symmetry character. However H_P is symmetric in all projectile coordinates and H_T in all target coordinates. There are therefore antisymmetric eigenfunctions of H_P (denoted $|P\rangle$) and H_T (denoted $|T\rangle$), separately. The antisymmetrized product

$$|PT\rangle = \mathcal{A}|P\rangle|T\rangle$$

is a fully antisymmetric function and therefore can not be an eigenfunction of $H_P + H_T$. For that reason, the transition amplitude is generally not given by $T = \langle P'T^*|\Sigma v_{PT}|PT\rangle$, even in the D.W.B.A. approximation.

It is possible to escape this difficulty, exactly the same way as for pp' scattering[11]). Let us define a complete basis of shell model states relative to the target: $|i\rangle$ and the corresponding creation and destruction operators $\mathcal{E}_i^+$ and $\mathcal{E}_i$. The antisymmetric projectile wave function can be expanded in terms of three particle Slater determinants built with the set {i}:

$$|P\rangle = \Sigma\, K_{ijk}\ \mathcal{E}_i^+ \mathcal{E}_j^+ \mathcal{E}_k^+ |0\rangle$$

$|0\rangle$ is the vacuum relative to the operator $\mathcal{E}$. A similar expansion can be given for the N particle target states $|T\rangle$. These states are generally calculated in a restricted space of states $|i\rangle$. We shall call this space q and the corresponding states a, b, c, d .... For instance, for Ca^{48} and Sc^{48} q contains all orbitals up to the $1f_{7/2}$ orbitals. The remaining space Q is built with the states A, B, C, D .... We shall now assume that the projectile wave function is made only of states belonging to Q only

$$|P\rangle = \Sigma\, K_{ABC}\ \mathcal{E}_A^+ \mathcal{E}_B^+ \mathcal{E}_C^+ |0\rangle$$

$$|P'\rangle = \Sigma\, K'_{A'B'C'}\ \mathcal{E}_{A'}^+ \mathcal{E}_{B'}^+ \mathcal{E}_{C'}^+ |0\rangle\ .$$

The total Hamiltonian can therefore be split into

$$H = H_Q + H_q + V_{Qq}$$

$$H_Q = \sum_A \varepsilon_A\, \mathcal{E}_A^+ \mathcal{E}_A + \sum_{ABCD} \langle AB|v|CD\rangle_{\mathcal{A}}\ \mathcal{E}_A^+ \mathcal{E}_B^+ \mathcal{E}_D \mathcal{E}_C$$

$$H_q = \sum_a \varepsilon_a\, \mathcal{E}_a^+ \mathcal{E}_a + \sum_{abcd} \langle ab|v|cd\rangle_{\mathcal{A}}\ \mathcal{E}_a^+ \mathcal{E}_b^+ \mathcal{E}_d \mathcal{E}_c$$

$$V_{Qq} = \sum_{AabB} \langle Aa|v|Bb\rangle_{\mathcal{A}}\ \mathcal{E}_A^+ \mathcal{E}_a^+ \mathcal{E}_b \mathcal{E}_B + W\ .$$

W contains all terms with an odd number of $\mathcal{E}$ operators belonging either to Q or to q. The antisymmetrized two-body matrix element of the nucleon-nucleon interaction is $\langle ij|v|k\ell\rangle_{\mathcal{A}}$; $|P\rangle$ is now an eigenstate of H_Q, $|T\rangle$ an eigenstate of H_q, and $|PT\rangle = \mathcal{A}|P\rangle\ |T\rangle = \sum_{ABC} K_{ABC}\ \mathcal{E}_A^+\ \mathcal{E}_B^+\ \mathcal{E}_C^+\ |T\rangle$ is an eigenstate of $H_Q + H_q$.

In the DWBA approximation, the transition amplitude is then:

$$T = \langle P'T^*|V_{Qq}|PT\rangle$$

Since $|T\rangle$ contains no state belonging to Q, it can be easily seen that only the term

$$\Sigma\ \langle Aa|v|Bb\rangle_{\mathcal{A}}\ \ \mathcal{E}_A^+\ \mathcal{E}_a^+\ \mathcal{E}_b\ \mathcal{E}_B$$

contributes to T, that is

$$T = \sum_{ab} \langle T^*|\mathcal{E}_a^+\ \mathcal{E}_b|T\rangle \sum_{\substack{ABC\\A'}} K^{'*}_{A'BC}\ K_{ABC}\ \langle A'a|v|Ab\rangle_{\mathcal{A}}$$

$$T = \sum_{\substack{ab\\AB}} \langle T^*|\mathcal{E}_a^+\ \mathcal{E}_b|T\rangle\langle P'|\mathcal{E}_A^+\mathcal{E}_B|P\rangle\langle Aa|v|Bb\rangle_{\mathcal{A}} \quad .$$

The antisymmetrized matrix element of v: $\langle Aa|v|Bb\rangle_{\mathcal{A}}$ can be written in a direct matrix element of a non-local force $v' = v(1 - P_\sigma P_\tau P_x)$ where P_σ, P_τ and P_x are the permutation operators of spin, isospin and space coordinates. We shall make a local approximation w for v', and have therefore to consider the quantity

$$\overline{W} = \Sigma \langle P' | \mathcal{E}_A^+ \mathcal{E}_B | P \rangle \langle A | W | B \rangle$$

which, in coordinate space is

$$\overline{W} = \Sigma \langle P' | \eta^+(1')\eta(1) | P \rangle \langle 1' | W | 1 \rangle$$

$\eta^+(1)$ and $\eta(1)$ create and destroy a particle with coordinates $1 = (\vec{r}_1, \sigma_1, \tau_1)$. That is, for the spatial part g of W

$$\overline{g} = \int d\vec{r}_1 d\vec{r}_2 d\vec{r}_3 \; d\vec{r}_1' d\vec{r}_2' d\vec{r}_3' \; \psi'^* (\vec{r}_1', \vec{r}_2', \vec{r}_3') \; \psi(\vec{r}_1, \vec{r}_2, \vec{r}_3) \; \delta(\vec{r}_2 - \vec{r}_2') \; \delta(\vec{r}_3 - \vec{r}_3') \langle \vec{r}_1' | g | \vec{r}_1 \rangle$$

We can introduce the additional variables

$$\vec{R} = \frac{\sum_1^3 \vec{r}_i}{3} \quad \text{and} \quad \vec{R}' = \frac{\sum_1^3 \vec{r}_i'}{3}$$

by changing

$$\int d\vec{r}_1 d\vec{r}_2 d\vec{r}_3 d\vec{r}_1' d\vec{r}_2' d\vec{r}_3' \quad \text{into} \quad \int d\vec{r}_1 d\vec{r}_2 d\vec{r}_3 d\vec{r}_1' d\vec{r}_2' d\vec{r}_3' \; d\vec{R} d\vec{R}' \; \delta\left(\vec{R} - \frac{\Sigma \vec{r}_i}{3}\right) \delta\left(\vec{R}' - \frac{\Sigma \vec{r}_i'}{3}\right)$$

Taking now advantage of the locality of W, that is, $\langle \vec{r}_2' | g | \vec{r}_1 \rangle = \delta(\vec{r}_1 - \vec{r}_2') g(\vec{r}_1)$ and separating ψ into the product $X(\vec{R}) \; \omega(\vec{r}_1 - \vec{R}, \vec{r}_2 - \vec{R}, \vec{r}_3 - \vec{R})$, we get

$$\overline{g} = \int X'^*(\vec{R}) \; X(\vec{R}) \int \rho(\vec{r}_1 - \vec{R}) \; g(\vec{r}_1) \; d\vec{r}_1 \; d\vec{R}$$

$$\rho(\vec{r}_1 - \vec{R}) = \int \omega'^*(\vec{r}_1 - \vec{R}, \vec{r}_2 - \vec{R}, \vec{r}_3 - \vec{R}) \; \omega(\vec{r}_1 - \vec{R}, \vec{r}_2 - \vec{R}, \vec{r}_3 - \vec{R}) \; \delta\left(R - \frac{\Sigma r_i}{3}\right) dr_2 dr_3 \quad .$$

Appendix 2

The effective force for the exchange part of a two-body matrix element.

The principle of this calculation is the same as the one used by Petrovitch[9]. Some of the formulas given here can also be found in Ref. [7].

The antisymmetrized two-body matrix element

$$\langle Aa|v(|Bb\rangle - |bB\rangle)$$

can be written as a matrix element of the non-local force $v + v'$ defined such as

$$-\langle Aa|v|bB\rangle = \langle Aa|v'|Bb\rangle$$

That is

$$v' = vP_{\vec{x}}P_\sigma P_\tau$$

$P_{\vec{x}}, P_\sigma, P_\tau$ are respectively the permutation operators of the space, spin and isospin variables. We define

$$v = V\ f(r)$$

$$V = \alpha + \beta\ \vec{\sigma}_1\cdot\vec{\sigma}_2 + \gamma\ \vec{\tau}_1\cdot\vec{\tau}_2 + \delta\ \vec{\sigma}_1\cdot\vec{\sigma}_2\cdot\vec{\tau}_1\cdot\vec{\tau}_2$$

Petrovich[9] has shown that $f(\vec{r})P_{\vec{x}}$ can be replaced by $\delta(\vec{r})\int e^{i\vec{k}\cdot\vec{p}}\ f(\vec{\rho})d\vec{\rho}$.

Then, $V' = -P_\sigma P_\tau V$ has the same form as V, since

$$P_\sigma = \frac{1+\vec{\sigma}_1\vec{\sigma}_2}{2} \quad \text{and} \quad P_\tau = \frac{1+\vec{\tau}_1\cdot\vec{\tau}}{2} \quad .$$

It can therefore be written

$$V' = \alpha' + \beta' \vec{\sigma}_1 \cdot \vec{\sigma}_2 + \gamma' \vec{\tau}_1 \cdot \vec{\tau}_2 + \delta' \vec{\sigma}_1 \cdot \vec{\sigma}_2 \vec{\tau}_1 \cdot \vec{\tau}_2$$

with[7])

$$\alpha' = -\frac{1}{4}(\alpha+3\beta+3\gamma+g\delta) \qquad \beta' = -\frac{1}{4}(\alpha-\beta+3\gamma-3\delta)$$

$$\gamma' = -\frac{1}{4}(\alpha+3\beta-\gamma-3\delta) \qquad \delta' = -\frac{1}{4}(\alpha-\beta-\gamma+\delta) \quad .$$

For a Serber force, the relation between V' and V is much simpler. When $P_\sigma P_\tau$ act on a state of given spin S and isospin T, $P_\sigma P_\tau = (-)^{S+T}$. Since a Serber force acts only on states where $S+T$ is odd, $-P_\sigma P_\tau$ is in that case equal to 1, and $V' = V$. The values of the mixture parameters of V and V' are given in Table 3.

For a natural parity transition, if a and b refer to the same shell-model orbital, only γ and γ' (denoted V_J and V_J' in the text) contribute to the transition for a charge exchange reaction. The force mixture enters in the ratio of the direct and exchange cross-section to the pure direct one only through the parameter $p = \gamma'/\gamma$. Similarly, for inelastic scattering with isospin $T = 0$ transfer, the ratio of the cross-sections with and without exchange is determined by $p = \frac{\alpha'}{\alpha}$, for a Serber force, $p = 1$ in all cases.

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Table 1

Ratio $\sigma_{[D+E]}/\sigma_{[D]}$ of the cross-section including exchange effects ($\sigma_{[D+E]}$) to the one neglecting them ($\sigma_{[D]}$) at the first maximum for various spin transfers. Column 1 gives the energy of the incident He^3 particle and column 2 gives the type of radial shape used for the nucleon-nucleon interaction, that is, before averaging over the projectile. This average has been performed in the calculation. The mixtures used are the Server (S) and Rosenfeld (R) ones.

Energy (MeV)	Rad. Shape of N-N force	Mixture	0^+	2^+	4^+	6^+
30	Yuk. 1 fm	S	1.3	1.5	1.8	2.2
30	Yuk. 1 fm	R	1.6	2.1	2.8	3.9
30	Yuk. 1.37 fm	S	1.1	1.3	1.7	2.4
30	Gauss 1.78 (2.32)	S	1.4	1.6	1.9	2.5
18	Yuk. 1 fm	S	1.3	1.5	1.8	2.3
75	Yuk. 1 fm	S	1.2	1.4	1.6	1.9

Table 2

Ratio $\sigma_{[D+E]}/\sigma_{[D]}$ of the cross-section including exchange effects ($\sigma_{[D+E]}$) to the one neglecting them ($\sigma_{[D]}$) at their first maximum for various spin transfers and various projectiles. λ is the range of the Gaussian used for the projectile mass distribution. The transition is assumed to be $(f_{7/2}\ f_{7/2}^{-1})^{0^+} \rightarrow (f_{7/2}\ f_{7/2}^{-1})^{J^+}$, J = 0, 2, 4, and 6.

	λ	0^+	2^+	4^+	6^+
He^3,t	1.48	1.3	1.5	1.8	2.2
He^3,$He^{3'}$	1.61	1.3	1.5	1.8	2.1
t,t'	1.37	1.3	1.5	1.8	2.3
α,α'	1.31	1.2	1.4	1.7	2.3

Table 3

Mixture parameters of the Serber and Rosenfeld forces. The strength of the force is assumed to be $V_0 = 40$ MeV.

	α	α'	a	β	β'	b	γ	γ'	c	δ	δ'	d
Serber	-15	-15	-30	5	5	10	5	5	10	5	5	10
Rosenfeld	-0.375	-24.375	-24	0.125	3.875	4	4.125	7.875	12	9.375	-1.375	8

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Figure Captions

Fig. 1. Ratio of the effective strengths V_J and V_0 needed in order to fit the magnitude of the experimental cross-sections. [D] is the calculation neglecting exchange effects and [D+E] the one including them. S stands for a Serber and R for a Rosenfeld mixture for the nucleon-nucleon interaction. The ^{48}Sc states and the 6^+ state of ^{54}Co are taken from the Saclay experiment[3]) at 30 MeV. The other cross sections were measured[4]) with 37 MeV incident projectiles. The strengths obtained when neglecting exchange effects are consistent† with earlier calculations[2,4]). The result for the 2^+ state of ^{48}Sc is only indicative, since this state is mixed with the 7^+ state.

Fig. 2. Comparison between Gaussian and Fermi radial forms for the He^3,t mean distribution. w and z are taken from Ref. [15]). λ and c have been chosen in order to fit the mean square radius[22]). $\rho(r)$ is normalised to $\int \rho(r) r^2 dr = 1$.

Fig. 3. Comparison of experimental and theoretical predictions for the cross-section relative to the excitation of the 0^+ state at 6.72 MeV of ^{48}Sc. [D] is the calculation neglecting and [D+E] the one including exchange. S stands for Serber mixture.

Fig. 4. Comparison of experimental and theoretical predictions for the cross-section relative to the excitation of the 6^+ ground state of ^{48}Sc. [D] is

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†The strength V_4 relative to the 4^+ state of ^{54}Co is not in agreement with the result of Ref. [4]) where a ratio $V_4/V_0 \sim 0.5$ was obtained instead of $V_4/V_0 \sim 2$.

the calculation neglecting and [D+E] the one including exchange. S stands for Serber mixture and R for a Rosenfeld mixture.

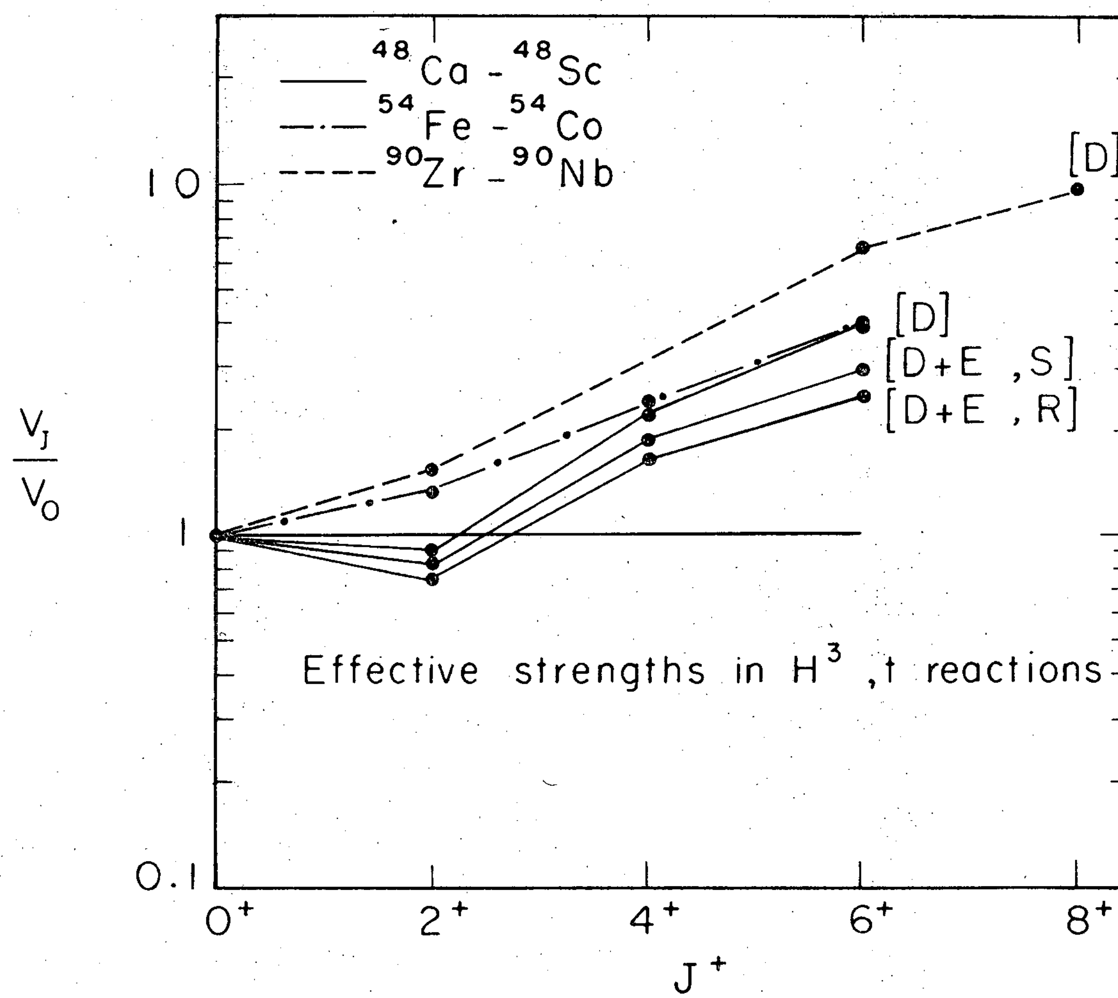

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Fig. 1.

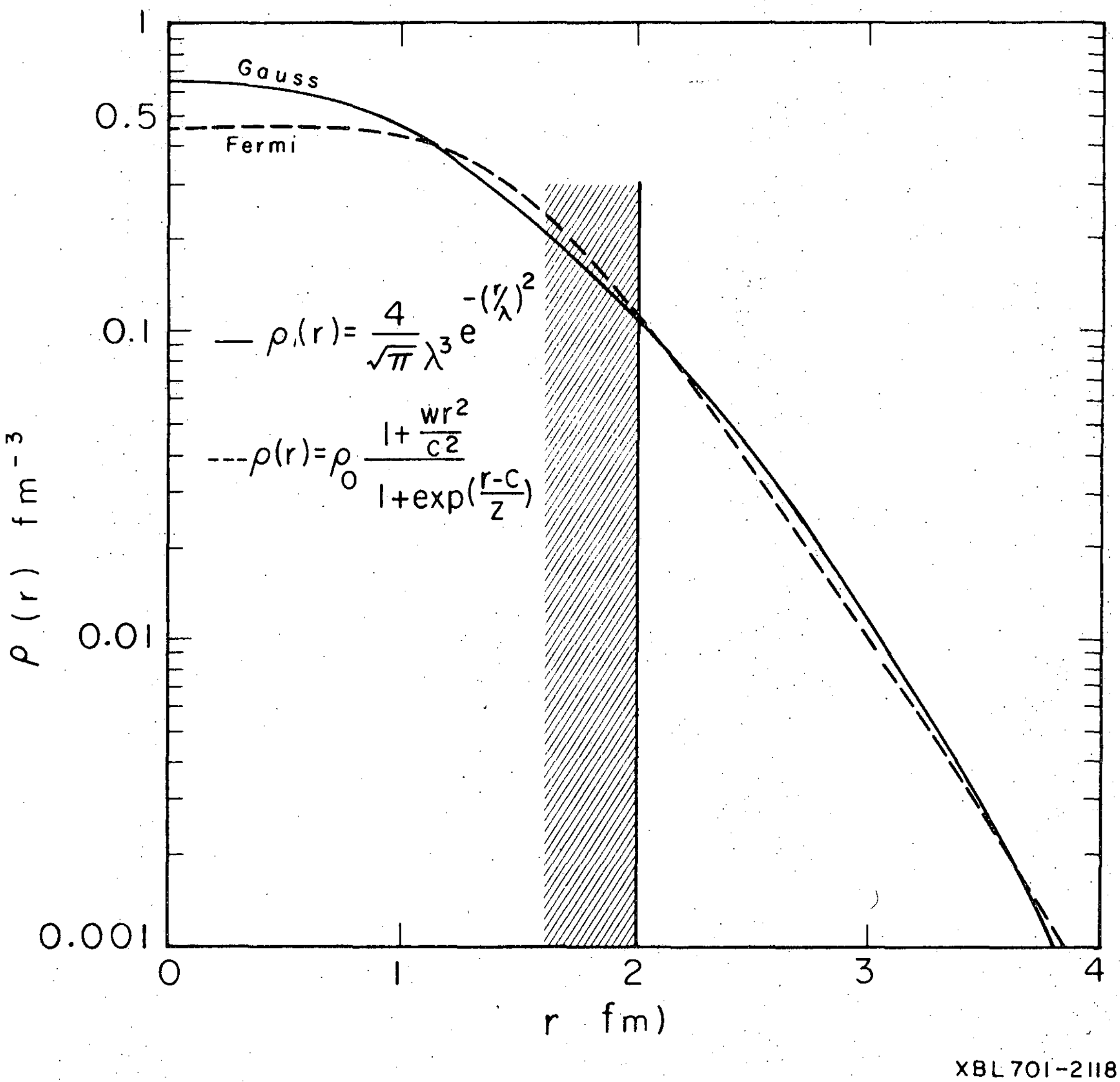
Gauss
Fermi
$\rho_1(r) = \frac{4}{\sqrt{\pi}\lambda^3} e^{-(r/\lambda)^2}$
$\rho(r) = \rho_0 \frac{1+\frac{wr^2}{c^2}}{1+\exp(\frac{r-c}{z})}$
1
0.5
0.1
0.01
0.001
0
1
2
3
4
$\rho(r)$ fm^{-3}
r fm)
XBL 701-2118

Fig. 2.

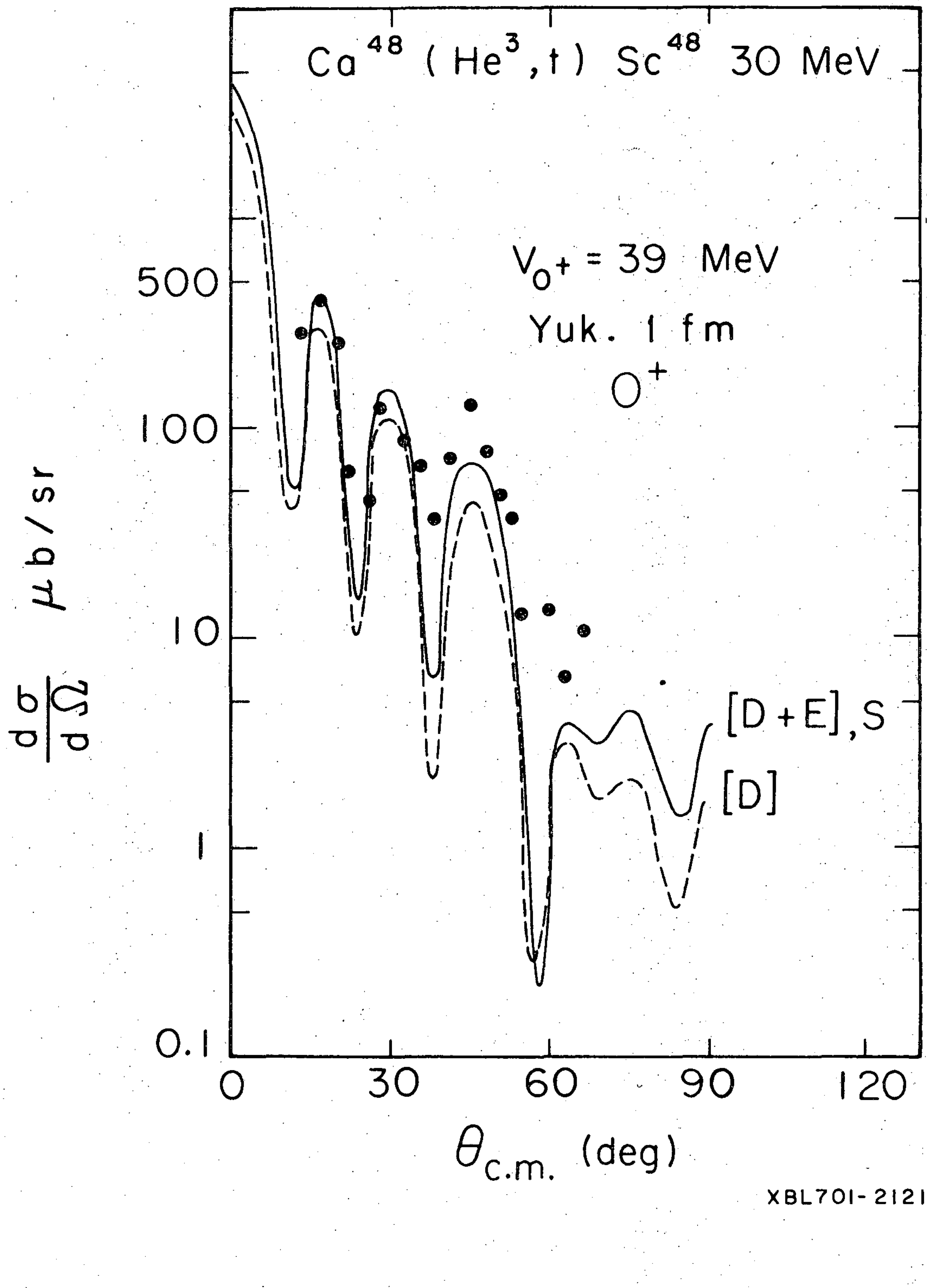
Ca48 (He3, t) Sc48 30 MeV
V0+ = 39 MeV
Yuk. 1 fm
0+
[D+E], S
[D]
dσ/dΩ μb/sr
500
100
10
1
0.1
0
30
60
90
120
θc.m. (deg)
XBL701-2121

Fig. 3.

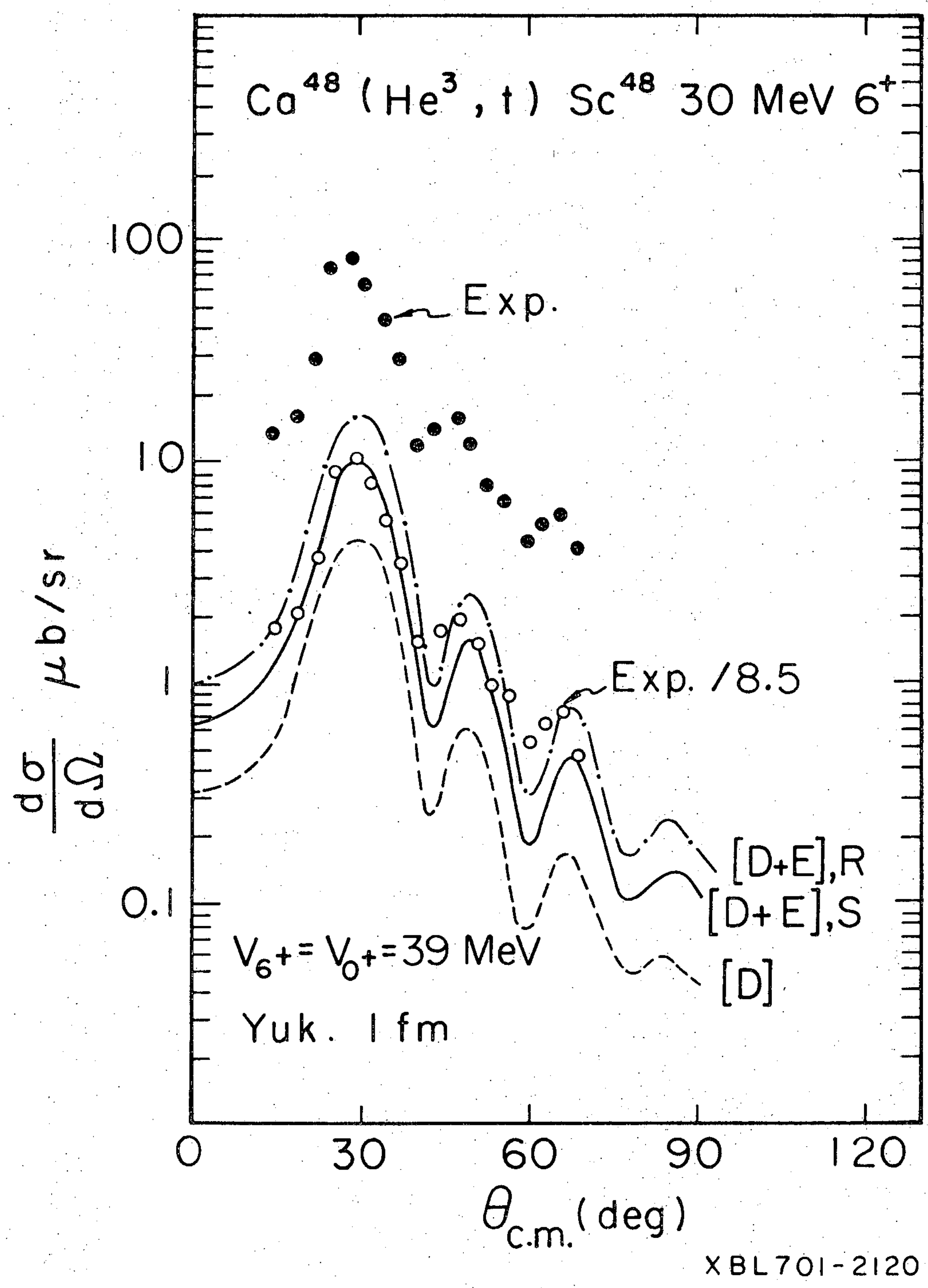
Ca48 (He3, t) Sc48 30 MeV 6+
100
10
1
0.1
Exp.
Exp. /8.5
[D+E],R
[D+E],S
[D]
V6+ = V0+ = 39 MeV
Yuk. 1 fm
dσ/dΩ μb/sr
0
30
60
90
120
θc.m.(deg)
XBL701-2120

Fig. 4.

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