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Authors

Bresler, Boris
Scordelis, Alex

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SHEAR STRENGTH OF REINFORCED CONCRETE BEAMS—SERIES II

BY

B. BRESLER
A. C. SCORDELIS

DECEMBER 1964

STRUCTURAL ENGINEERING LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY CALIFORNIA

Structures and Materials Research
Department of Civil Engineering
Division of Structural Engineering
and Structural Mechanics

SHEAR STRENGTH OF REINFORCED CONCRETE BEAMS

SERIES II

A Report of an Investigation

by

B. Bresler, Professor of Civil Engineering
A. C. Scordelis, Professor of Civil Engineering

to the

REINFORCED CONCRETE RESEARCH COUNCIL
BUREAU OF YARDS AND DOCKS, DEPARTMENT OF THE NAVY
OFFICE OF CHIEF OF ENGINEERS, DEPARTMENT OF THE ARMY
ENGINEERING DIVISION, DEPARTMENT OF THE AIR FORCE

Institute of Engineering Research
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I. INTRODUCTION

1. Objectives and Scope

An initial investigation on the shear strength of reinforced concrete beams was conducted at the University of California in 1960-61, during which time a series of 12 beam specimens were tested. The objectives of these tests were to observe the general behavior and to determine the cracking load and ultimate strength of beams with shear span ratios between 4 and 7 and with vertical stirrup reinforcement having rf_y values ranging from 0 to 100. The results of this research have been reported in "Shear Strength of Reinforced Concrete Beams"[1]* by B. Bresler and A. C. Scordelis, Structures and Materials Research Report, Department of Civil Engineering, University of California, Series 100, Issue 13, June 1961. A condensation of this report was subsequently published in the January 1963, ACI Journal[2]. For the beams of this first series, which failed in shear, a comparison of the test results for ultimate load with calculated values based on the ultimate shear capacity as computed from formulas which have been adopted in the 1963 ACI Code indicated test values 27 to 49% higher than calculated values.

The results of the initial investigation indicated a need for additional experimental data. The investigation reported herein was conducted in 1963 and included testing a second series of 10 additional beam specimens with the following objectives:

1. To determine the reduction in shear strength due to bar cut off in a zone of high shear.

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* Superscripts indicate references listed in Section V of this report.

2. To determine the reduction in the contribution of dowel action to the shear strength due to a change in longitudinal bar size from No. 9 to No. 7 throughout the entire beam length, while maintaining the same total area of tension reinforcement.
3. To determine the contribution to shear strength, if any, of the Howlett grip anchor nuts used at the ends of specimens of the first series to prevent bond failures.
4. To obtain additional information on shear strength of beams without web reinforcement for comparison with companion specimens with web reinforcement.

All beams tested had a 12 ft. span and were subjected to a single centerpoint load at midspan until ultimate failure.

2. Acknowledgements

The investigation reported herein was carried out in the Engineering Materials Laboratory of the University of California at Berkeley under the sponsorship of Reinforced Concrete Research Council, Bureau of Yards and Docks - Department of the Navy, Office of Chief of Engineers - Department of the Army, and Engineering Division - Department of the Air Force.

The task committee for this project appointed by the Reinforced Concrete Research Council was constituted as follows: W. E. Schaem (Chairman), C. A. Willson, D. E. Parsons, E. Hognestad, and E. Cohen. The sponsors' generous support of the investigation and the helpful suggestions of the Task Committee are gratefully acknowledged.

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in charge of the tests, and of the laboratory staff, particularly Messrs. E. H. Brown, G. Hayler, and E. L. Whittier.

3. Notation

The letter symbols used in this report are usually defined when they are introduced. They are listed below alphabetically for convenient reference:

a = Shear span = $L/2$ for beam under center point load

A_s = Area of longitudinal tension reinforcement

A'_s = Area of longitudinal compression reinforcement

A_v = Area of web reinforcement

b = Width of beam

d = Effective depth of beam

E_c = Secant modulus of elasticity of concrete at 1000 psi

E_s = Modulus of elasticity of steel

f'_c = Compressive strength of 6 x 12 in. concrete cylinder

f'_t = Modulus of rupture of concrete

f_s = Stress in longitudinal tension reinforcement

f_v = Stress in web reinforcement

f'_y = Yield point of compression steel reinforcement

f_y = Yield point of tension steel reinforcement

f_u = Ultimate strength of steel reinforcement

h = Over-all depth of beam

K = Constant depending on angle of inclination of web reinforcement; $K = 1$ for vertical stirrups

L = Span length

M = Bending moment at a section

p = Tension steel reinforcement ratio = A_s/bd

p' = Compression steel reinforcement ratio = A'_s/bd

P_{cr} = Load producing initial diagonal tension crack

P_f = Calculated ultimate load as governed by flexure

P_v = Calculated ultimate load as governed by shear

P_u = Ultimate test load

q = Longitudinal reinforcement index = $(pf_y - p'f'_y)/f'_c$

r = Web reinforcement ratio = A_v/bs

s = Longitudinal spacing of web reinforcement

v_c = Ultimate shearing stress for beams without web reinforcement

v_u = Ultimate shearing stress for beams with web reinforcement

V = Total shear at a section

V_s = Shear assumed taken by web reinforcement

Δ = Midspan deflection

II. EXPERIMENTAL PROGRAM

1. Description of Test Beams

In designing the test beams the same criteria as in Series I were used, namely:

a. Nominal rf_y values for web reinforcements were to be 0, 50, 75, and 100.

b. Nominal a/d ratio to be 4, obtained by using a centerpoint loading with a 12 ft. span for all beams.

c. Calculated ultimate loads were to be governed by shear rather than flexure.

d. Bond or anchorage failures were to be prevented by using end grip anchors with the exception of the X-series where no grip anchors were used.

e. The effective depth of all beams was to be the same.

f. The required rf_y value was to be obtained mainly by varying the width of the specimen.

g. The spacing of the stirrups was to be no greater than half the effective depth.

h. Main longitudinal reinforcement was to be made up of No. 7 or No. 9 high strength steel bars. The number of bars was to be varied to achieve the desired steel percentage.

A number of different reinforcement arrangements were considered in an attempt to satisfy the above criteria.

Cross-sectional properties for each of the 10 beams finally selected and tested to failure are given in Fig. 1 and beam elevations are shown in Fig. 2. All beams were of rectangular cross-section and had the same nominal over-all depth of 21 3/4 in. Main longitudinal reinforcement consisted of from two to four No. 9 or three to six No. 7 high strength steel deformed bars placed in the bottom of the beams at two levels. The nominal effective depth to the centroid of this reinforcement was 18 in. in all cases. Actual beam dimensions obtained by measurements prior to each test are given in Table 8. All stirrups were made from No. 2 intermediate grade steel deformed bars bent, lapped, and welded to form box-type stirrups. For beams with stirrups two No. 4 longitudinal reinforcing bars of intermediate grade steel were placed at the top of the beam to facilitate the spacing of stirrups and acted as compressive steel. Percentages of steel reinforcement and stirrups are given in Fig. 2 and in Table 8.

Three beam widths - 6, 9, and 12 in. with a constant 12 ft. length were used to obtain the desired variations in the rf_y values.

Nominal concrete strength for all specimens was 3500 psi. All beams were subjected to a single center-point load at midspan. The test beams were grouped into four series C, R, X, and O.

Series C

Specimens CA-1, CB-1, and CC-1 were identical to specimens A-1, B-1, and C-1 of the first series with the exception that half of the No. 9 longitudinal bars were cut off at a distance of 24 in. from each support. See Fig. 2C.

A difference between the test beams and the 1963 ACI Code[3] provisions should be mentioned. In beams CA-1, CB-1, and CC-1, while an appropriate length of anchorage was provided beyond the theoretical cut-off point, no additional web reinforcement, now required by Sec. 918-(c)-2 of the Code, was provided. This section, with only 2 under (c) being pertinent, reads as follows:

"(c) No flexural bar shall be terminated in a tension zone unless one of the following conditions is satisfied:

1. The shear is not over half that normally permitted, including allowance for shear reinforcement, if any.
2. Stirrups in excess of those normally required are provided each way from the cut off a distance equal to three-fourths of the depth of the beam. The excess stirrups shall be at least the minimum specified in Section 1206(b) or 1706(b). The stirrup spacing shall not exceed $d/8r_b$ where r_b is the ratio of the area of bars cut off to the total area of bars at the section.
3. The continuing bars provide double the area required for flexure at that point or double the perimeter required for flexural bond."

The test specimens in Series II were designed prior to the publication of the 1963 ACI Code.[3] It is interesting to note that in the originally proposed revision of the ACI Code (ACI Journal, February 1962, pp 187 and 188) under Sec. 918-(c)-2 the excess stirrups were to be required each way from the cut off a distance "equal to" rather than "three-fourths of" the depth of the beam and the stirrup spacing was not to exceed "$d/5$" rather than "$d/8r_b$". The changes in the original proposed revision were apparently made on the basis of a discussion presented by W. E. Kunze (ACI Journal, November 1962) which is quoted as follows:

> "A stirrup at a distance "d" from a cut-off point will have little effect on resisting diagonal tension which occurs because of stress concentrations at the cutoff. Therefore, in Line 2, after "equal to" the writer would insert "three-fourths of." In Line 5, the writer suggests changing "$d/5$" to "$d/8r_b$", where r_b is the ratio of the area of bars cut off at a section to the total area of bars at the section. The provisions of the section require an extremely close spacing of extra stirrups in addition to those normally required. Moreover, because the extra stirrups are required for a distance of 2d wherever a cutoff is made, irrespective of the number or area of the bars discontinued, such extra stirrups would have to be provided over most of the length of the beam if bars were cut off at more than two sections. There is no justification in experience for such an excessive requirement and no known justification in research."

It appears that because of a lack of experimental data, the final requirement specified in Sec. 918-(c)-2 of the 1963 ACI Code was based on engineering judgement and the results of limited tests by Ferguson and Thompson[4] which recognized a need for some additional web reinforcement in the zone of bar cut off. Additional experimental evidence is required to determine the effectiveness of varying amounts and extents of this type of additonal web reinforcement.

Series R

Specimens RA-1, RB-1, and RC-1 were similar to specimens A-1, B-1, and C-1 of the first series with the exception that No. 7 bars instead of No. 9 bars were used for longitudinal tensile reinforcement throughout the entire length. Approximately the same total area of tension reinforcement was maintained in companion specimens. See Fig. 2D.

Series X

Specimens XOB-1 and XB-1 were identical to specimens OB-1, described in paragraph 4 below, and B-1 from the first series with the exception that the Howlett grip anchors described below were omitted. See Fig. 2A.

Series O

Specimens OB-1 and OC-1 were identical to specimens B-1 and C-1 of the first series with the exception that they had no stirrups. See Fig. 2B.

To prevent possible bond failures due to insufficient anchorage after the formation of diagonal tension cracks, "Howlett" grip anchor nuts were attached to the No. 9 and the No. 7 longitudinal bars which protruded from the ends of the specimens about 6 inches. 3/4 in. thick steel plates were used at the ends of the beams to provide bearing for these nuts. Details of the bar anchorage and the "Howlett" grip anchor nuts are shown in Fig. 3.

2. Fabrication

All reinforcing steel was thoroughly cleaned before assembly into a reinforcing cage. The reinforcing cages were assembled prior to placement into the forms. The steel assembly was securely held in the proper location in the forms by means of specially fabricated individual high chairs which were spaced 3 ft. apart throughout the length of the specimen. Lifting lugs were also provided for transporting the finished specimen.

The beams were cast in wooden forms made of plywood with a plastic coating to give a smooth and impervious surface. The forms were designed so that they could be adjusted to the desired width of each test specimen.

The concrete was mixed in a 6 cu. ft. capacity horizontal, non-tilting drum-type mixer. Each batch averaged about 5 1/2 cu. ft., while the total number of batches required for a single beam together with control specimens varied between 3 and 5. Aggregates were blended and moisture contents were determined the day prior to casting. The dry materials were first blended in the mixer for one-half minute, then the water was added and the entire contents mixed for two minutes. An additional one minute mix period followed a three minute set period. The concrete was transported to the forms in buggies and placed into the forms in three layers. Each layer was vibrated internally with a high frequency vibrator (8000 to 10,000 cycles per second).

3. Materials and Control Specimens

Concrete mixes were designed by the trial batch method to achieve the desired 3500 psi mix. Type I Portland cement and locally available

Elliot sand and Fair Oaks gravel were used in all of the mixes. To compensate for the coarseness of the Elliot sand, a small amount of fine Antioch sand was added to the mix.

The cement was purchased in one lot from a single mill run. A chemical analysis of the cement is given in Table 1. As needed the cement was blended in 20 sack batches and stored in steel drums.

The results of sieve analyses on the aggregates are given in Table 2. The maximum size of the coarse aggregate was 3/4 in.

The 3500 psi concrete mix had a cement factor of 5.4 sacks per cu. yd. The water-cement ratio varied from 0.56 to 0.60 by weight or 6.32 to 6.77 gallons per sack. Mix proportions of cement:Antioch Sand: Elliot Sand:Fair Oaks gravel were 1.00:0.196:2.93:3.39 by weight. These aggregate weights are based on a saturated surface dry condition. Consistency measured by means of a Kelly-ball averaged about 3 in. slump-equivalent.

Concrete control specimens consisted of from nine to fifteen 6 x 12 in. cylinders and three to four 6 x 6 x 20 in. beams for each test specimen. The control specimens were cured in the same manner as the test beams. Values of compressive strength f'_c and secant modulus of elasticity E_c at 1000 psi obtained from the 6 x 12 in. cylinders are given in Tables 3A and 3B. Values of modulus of rupture f'_t obtained by loading the 6 x 6 x 20 in. beams at the third points of an 18 in. span are shown in Table 3C. Fig. 4 depicts the stress-strain relationship for the concrete.

Four reinforcing bar sizes were used in the beams. The bottom tension steel was made up of either No. 9 or No. 7 high strength deformed bars having a minimum specified yield point of 80 ksi. Two

No. 4 intermediate grade bars were used as compression steel for each of the beams with stirrups. No. 2 intermediate grade deformed bars were used for the stirrups. Control specimens for each bar size were tested in tension to determine the yield strength f_y, ultimate strength f_u, modulus of elasticity E_s, and per cent elongation in an 8 in. gage length. These results together with values obtained for deformation spacing and heights, weight per ft., and nominal areas are tabulated in Tables 4, 5, 6, and 7. Typical stress-strain diagrams for each bar size are shown in Fig. 5A and 5B. The reinforcing bar areas used to determine steel stresses and moduli of elasticity were computed from the weight of the bars, including that of the deformations, and thus they are nominal areas as far as effective cross-section is concerned.

4. Method of Loading and Instrumentation

The loading arrangement and instrumentation are shown in Fig. 6. The centerpoint load was applied by means of a 200,000 lb. capacity universal testing machine. An 8 in. spherical loading block was utilized at the load point. One end of the beam was supported on a 6 in. spherical bearing block while the other end was supported on a 3 in. diameter roller.

Midspan deflections were obtained by two methods. In the first method three simple dial gages with a least count of 0.001 in., supported by floor stands and bearing on the top of the beam at midspan and at each end, were used. In the second method a scale graduated in 0.01 in. and a mirror were glued to the beam on each face at midspan. A piano wire was then stretched between the support points on each face to obtain deflection readings.

Changes in the over-all depth of the beam due to diagonal cracking were measured by means of specially designed yoke extensometers. These measurements were taken at six separate stations on each beam. The yoke extensometers consisted of two 1/4 x 1-1/2 x 16 in. steel bars clamped to the beam, one across the top and one across the bottom. These two bars were connected vertically on each side of the beam by means of a 1/2 in. diameter steel rod and a dial gage. Relative movements between the top and bottom surfaces of the beam were registered on the dial gages which read to the nearest 0.0001 in. Details of the extensometers are shown in Fig. 7.

To facilitate the recording of cracks and the visual observation of the beam behavior during testing, the entire beam was first whitewashed and a ruled grid was then marked on the two sides of the beam. For beams with stirrups vertical grid lines were placed at stirrup locations so that during testing the number of stirrups being crossed by a particular crack could immediately be discerned.

5. Test Procedure

Twelve days after casting, the beam to be tested was placed in position under the testing machine after which it was whitewashed and the yoke extensometers and deflection gages were installed. All beams were tested under centerpoint load at an age of 13 days.

The beams were first loaded to about 30% of ultimate in two or three increments and then the load was removed. The load was reapplied in 10 kip increments to a point near failure and then in 5 kip increments until failure occurred.

Deflection and yoke-extensometer readings were taken at the beginning and end of each load increment. Cracks were plotted at the end of each load increment directly on the beam and also on specially prepared data sheets. After failure a careful visual inspection of the beam was made and several photographs were taken. Total testing time for a single beam varied between 1-1/2 and 3 hours.

III. EXPERIMENTAL RESULTS AND ANALYSIS OF DATA

1. General Behavior

The behavior of the beams under load was similar to that previously reported[1] for the 12-ft. span beams of the first series. Typical initial flexural cracks appeared at midspan, followed by the appearance of diagonal tension cracks in the middle third of the overall depth of the beam and in the middle third of the half span between the center-point load and the reaction. In beams with cutoff bars the initial diagonal crack inevitably occurred near the point of cutoff. As the load was increased to ultimate the beams without web reinforcement exhibited diagonal-tension (D-T) failures, while those with web reinforcement exhibited shear-compression (V-C) failures.

No basic difference in the general behavior was observed for companion specimens with or without end anchors. Likewise companion specimens with either No. 9 or No. 7 longitudinal bars throughout the entire beam length behaved similarly.

The general behavior of the various test specimens may be interpreted through a study of the crack patterns Fig. 8A to 8J, the load-deflection curves Fig. 9 and the yoke data Fig. 10A to 10J.

Diagonal Tension Failures

Diagonal tension failures occurred in beams XOB-1, OB-1, and OC-1, all of which had no stirrups. A study of the crack patterns in Figs. 8A, 8C, and 8D reveals the behavior under load of these beams. Failure occurred shortly after the formation of a main diagonal tension crack and was accompanied by a sudden longitudinal splitting crack both upwards towards the centerpoint load and downward towards the support and along the longitudinal steel. The crack patterns for XOB-1 and OB-1, Fig. 8A and 8C were remarkably similar indicating the Howlett end anchors had essentially no effect on the behavior of the beams.

The yoke data for beams XOB-1, OB-1 and OC-1 given Figs. 10A, 10C, and 10D show that there was very little relative movement between the top and bottom surfaces before failure. This is to be expected since without stirrups these beams could not accommodate any large diagonal crack openings.

Shear Compression Failures

Shear-compression failures occurred in all of the other beams. A comparison of the crack patterns in Fig. 8 with those of companion beams of the first series reveals very similar behavior. The initial diagonal tension crack occurred at about 60% of the ultimate load. No serious distress was visible under increasing load, but the diagonal cracks inclined more and more towards the horizontal and gradually approached the centerpoint load at the top of the beam. Vertical flexural cracks near midspan discontinued their progress upward from the bottom of the beam as the diagonal cracks continued to open up at higher load levels. When the depth of the flexural compression zone near midspan had been gradually reduced to a minimum of about two inches by the penetration of a major diagonal tension crack, crushing occurred near the load point almost simultaneously with a splitting along the major diagonal tension crack in the compression zone. Unlike the beams without web reinforcement there was no horizontal splitting at the bottom along the tension steel level and also in all cases failures were gradual rather than sudden.

The crack patterns of companion beams from the initial first test series and those of the present second series with reduced bar size or with cut off bars were quite similar with the exception that in beams with cutoff bars the initial diagonal tension crack inevitably occurred at the cutoff point. Beams without cutoff bars, as compared to those with cutoff bars, also appeared to have a larger number of cracks more closely spaced together.

Comparing the crack pattern of XB-1, Fig. 8B with that of companion specimen B-1 of the first series a remarkable similarity exists indicating once again that the Howlett end anchors had little effect on the behavior.

The yoke extensometer data, Fig. 10E-J, for the beams with stirrup reinforcement (CA-1, CB-1, CC-1, RA-1, RB-1, RC-1) show that extensive diagonal crack openings were sustained after the initial diagonal tension crack formed. Comparing this data with that for the companion specimens from the first series (A-1, B-1, C-1) it can be seen that beams with reduced bar size RA-1, RB-1 and RC-1 exhibited the greatest relative displacement before failure. The load at which diagonal tension cracking first occurs (Table 9) approximately corresponds to the point when the curve of vertical displacement versus load Figs. 10E-J first deviates from the vertical. It is seen that for companion specimens this occurs first for beams with No. 9 bars cutoff, CA-1, CB-1, and CC-1; then for beams with No. 7 bars throughout the entire span length RA-1, RB-1 and RC-1; and finally for beams with No. 9 bars throughout the entire length A-1, B-1, and C-1. The ultimate loads also occurred in this same order.

2. Load-Deflection Relationships

Load-deflection relationships for all of the 12-ft. span beams of Series I and II are shown in Fig. 9. Each group of curves shows the load-deflection relationship for a group of beams of the same width, therefore 12 in. for A; 9 in. for B; and 6 in. for C.

Deflection values plotted in this figure represent the average of the readings on the two faces of the beams. Only the deflections recorded during the final cycle of loading from zero to ultimate are shown.

A comparison of the curves in each series A, B, or C indicates that slope of the load-deflection curves in the lower ranges of load are essentially unaffected by lack of stirrups, removal of end anchors, cutoff bars, or reduced longitudinal bar size. Beams without stirrups, which have the lowest ultimate loads, exhibit the least ability to sustain larger deflections.

A comparison between the curves of A, B, and C demonstrates the obvious fact that beams with decreasing width have decreasing stiffnesses.

3. Evaluation of Test Results

Table 8 presents a summary of the test program and Table 9 presents a summary of test results, including values of the diagonal tension cracking load P_{cr}, ultimate load P_u, maximum deflection $\Delta_{max.}$, and failure mode for each of the beams tested in Series II. Calculated values of flexural capacity P_f, cracking load P_{cr} and shear capacity P_v are also included in Table 9.

The value of P_f for each beam was determined by trial and error using the Hognestad-McHenry-Hanson stress block with an assumed ultimate compressive unit strain of 0.003, and using experimentally determined stress-strain characteristics for the top and bottom longitudinal reinforcement. A comparison of calculated values of P_f and P_v in Table 9 indicates that for beams in Series II the calculated ultimate flexural capacity ranged from 23 to 142% higher than the calculated shear capacity. This excess flexural capacity ensured a shearing failure in all of the beams.

Two values of P_{cr} and P_v were calculated for each beam and are shown in Table 9. The equations used in these calculations were as follows.

$$v_c = \frac{V_c}{bd} = 1.9\sqrt{f'_c} + 2500\,\frac{pVd}{M} \quad \text{- - - - - - - - - - - - - - -} \quad (1a)$$

$$v_u = \frac{V_u}{bd} = 1.9\sqrt{f'_c} + 2500p\frac{Vd}{M} + Krf_y \quad \text{- - - - - - - - - - - -} \quad (1b)$$

$$v_c = \frac{V_c}{bd} = 2.0\sqrt{f'_c} \quad \text{- -} \quad (3a)$$

$$v_u = \frac{V_u}{bd} = 2.0\sqrt{f'_c} + Krf_y \quad \text{- - - - - - - - - - - - - - - - - -} \quad (3b)$$

Eqs. (1a) and (3a) were used in calculating $P_{cr} = 2V_c$ and Eqs. (1b) and (3b) were used in calculating $P_v = 2V_u$. These are the same Eqs. as used in analyzing the results already reported for Series I. Eqs. (1) and (3) are the same as those adopted in Chapter 17 of the 1963 ACI Code with the exception that in the code the permissible values are reduced by multiplying the expressions by a capacity reduction factor ϕ which equals 0.85 for diagonal tension calculations.

The test results may be evaluated by a study of Tables 8 and 9 together with Fig. 11 which gives a graphical comparison of calculated and test values for the beams of both Series I and Series II.

1. It is first important to note that for all of the 22 beams tested in the two series, in only two cases, beams CA-1 and CC-1, did the test values for ultimate load fall slightly below the calculated values. Furthermore, these two beams, which had cut off bars, did not have the additional web reinforcement now required by the ACI Code as described in Sect. II.
2. With the exception of beams CA-1 and CC-1, the beams of Series II developed ultimate strengths from approximately 20 to 50% greater than the calculated values.
3. A comparison of the test value/calculated value for the ultimate strengths of beams CA-1, CB-1, CC-1 with companion beams A-1, B-1, C-1 indicates the reduction in strength due to half of the No. 9 bars being cut off at a distance of 24 in. from the support. The ratios of these ultimate strengths were CA-1/A-1 = 0.70; CB-1/B-1 = 0.80; and CC-1/C-1 = 0.73. Thus the bar cut off caused a strength reduction from 20 to 30%.

Once again it should be emphasized that the beams with cut off bars had no additional web reinforcement provided in the zone of cutoff.

4. A similar comparison for the ultimate strengths of beams RA-1, RB-1, RC-1 with companion beams A-1, B-1, C-1 indicates the reduction in strength due to a decrease in bar size from No. 9 to No. 7 throughout the entire beam length, while maintaining approximately the same tension steel areas. The ratios of these ultimate strengths were RA-1/A-1 = 0.88; RB-1/B-1 = 0.92; and RC-1/C-1 = 0.90. Thus the reduced bar size appears to have caused a strength reduction from 8 to 12%.

5. The effect of the Howlett grip anchors is obtained by comparing the ultimate strength of companion specimens with and without end anchors. These ratios are XOB-1/OB-1 = 0.96; and XB-1/B-1 = 0.92.

6. The effectiveness of web reinforcement may be estimated by comparing the shearing strengths of beams, without web reinforcement, OB-1 and OC-1 of Series II with their companion specimens, with web reinforcement, B-1 and C-1 of Series I. The test values P_u for these four beams were 57.7, 34.9, 100.0, and 70.0 kips respectively. The calculated contributions of $(Krf_y)bd$ for beams B-1 and C-1 to P_u are 22.8 and 21.2 kips respectively. Comparing these two values with the differences between the test values P_u for B-1 and OB-1 = 100.0 - 57.7 = 42.3 kips and for C-1 and OC-1 = 70.0 - 34.9 = 35.1 kips it can be seen that a simple addition of the $(Krf_y)bd$ contribution to the strength of a beam without web reinforcement

considerably underestimates the contribution of the web reinforcement. This same general result was reported[1] in comparisons made for beams OA-1 and A-1, and beams OA-2 and A-2 in Series I.

IV. CONCLUSIONS

On the basis of the experimental data obtained in the tests of the beams in Series II the following conclusions are advanced.

1. Longitudinal tensile bar cut-offs in a zone of high shear, without adding supplementary web reinforcement in the cut off zone, substantially reduces the ultimate shearing strength of reinforced concrete beams.
2. A reduction in the bar size of the longitudinal tensile steel, while maintaining the same total tensile steel area, tends to decrease the ultimate shearing strength of reinforced concrete beams.
3. The contribution of web reinforcement to the ultimate shearing strength of reinforced concrete beams with web reinforcement is substantially greater than that obtained by simply adding $(Krf_y)bd$ to the shearing strength of a similar beam without web reinforcement.

Since both bar cut-off and a reduction in bar size tend to reduce the ultimate shearing strength, it is suggested that additional tests should be conducted in which these two effects are combined in individual specimens to ascertain if these reduction effects are additive. It is also suggested that additional tests be conducted for beams with bar cut-off in which additional web reinforcement is supplied in the zone of bar cut off in varying amounts and extents. These results should then be used to determine the validity of the present requirement specified in Sec. 918-(c)-2 of the 1963 ACI Code.

V. REFERENCES

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TABLE 1 CHEMICAL ANALYSIS OF CEMENT[1]

Chemical	Percent
SiO_2	22.80
Fe_2O_3	2.64
Al_2O_3	4.90
CaO	64.30
MgO	1.57
SO_3	2.13
Ignition Loss	0.95
Total Alkalies as Na_2O	0.72
Total	100.01

[1] Type I, Light Portland Cement, Mill Analysis by Pacific Cement and Aggregate Company, Davenport, California.

TABLE 2 SIEVE ANALYSES OF AGGREGATES

Sieve Size	Percentage Retained on Sieve		
	Elliot Sand	Fair Oaks Gravel	Antioch Sand
3/4 in.		1.5	
1/2 in.		(48.3)	
3/8 in.		78.1	
No. 4	0.3	98.9	
No. 8	16.8	100.0	
No. 16	47.1		
No. 30	72.0		0
No. 50	92.0		24.0
No. 100	97.2		96.0
No. 200			99.0
Fineness Modulus	3.25	6.79	1.20

Average of 4 samples of sand and 4 samples of gravel.

TABLE 3A COMPRESSIVE STRENGTH f'_c OF CONCRETE

3500 psi mix; 6 x 12 in. cylinders

1. SSD parts by weight, C : AS : S : G* = 1 : 0.196 : 2.13 : 3.39
2. All values given in ksi
3. All tests at 13 days

Spec. No.	XOB-1	XB-1	OB-1	OC-1	CA-1	CB-1	CC-1	RA-1	RB-1	RC-1
1-A	3.62	3.74	3.43	3.20	3.81	3.68	3.90	3.71	3.52	4.04
1-B	3.54	3.81	--	3.26	3.94	3.56	3.75	3.82	3.47	4.21
1-C	3.66	3.86	3.50	3.42	3.95	3.56	3.82	3.73	3.44	4.27
2-A	3.61	3.66	3.55	4.01	3.90	3.60	3.76	3.65	3.46	4.32
2-B	3.69	3.34	3.47	4.22	3.90	3.59	4.00	3.67	3.67	4.28
2-C	3.70	3.35	3.56	4.43	3.89	3.62	3.87	3.65	3.63	4.34
3-A	3.70	3.78	3.53	4.17	3.96	3.57	3.94	3.66	3.53	4.31
3-B	3.77	3.74	3.45	4.22	3.93	3.53	3.84	3.63	3.67	4.20
3-C	3.87	3.56	3.52	4.38	3.84	3.53	4.06	3.68	3.62	4.14
4-A	--	3.57	3.40		3.62	3.53		3.35	3.54	
4-B	3.48	3.61	3.45		3.76	3.62		3.47	3.70	
4-C	3.45	3.43	3.08		3.78	3.71		3.41	3.71	
5-A	3.88	3.40	--		3.81	3.50		3.55	3.61	
5-B	--	3.21	3.26		4.02	3.56		3.57	3.45	
5-C	3.73	3.31	3.33		4.02	3.62		3.58	3.57	
Avg. f'_c	3.66	3.56	3.42	3.92	3.87	3.59	3.95	3.61	3.57	4.23
W/C	0.60	0.60	0.57	0.56	0.60	0.60	0.57	0.60	0.60	0.56

* AS = Antioch sand; S = Elliot sand.

TABLE 3B SECANT MODULUS OF ELASTICITY OF CONCRETE, E_c

3500 psi mix; 6 x 12 in. cylinders

1. E_c at 1000 psi
2. All values in ksi x 10^3
3. All tests at 13 days

Spec. No.	XOB-1	XB-1	OB-1	OC-1	CA-1	CB-1	CC-1	RA-1	RB-1	RC-1
1	3.47	3.97	3.61	4.10	3.92	3.66	4.26	3.68	3.77	3.77
2	--	4.13	3.61	4.85	3.82	4.13	--	3.53	3.64	3.86
3	3.51	4.26	3.62	3.82	3.98	3.70	3.62	3.92	3.64	4.44
Avg.	3.49	4.12	3.61	4.26	3.91	3.83	3.95	3.71	3.68	4.03

TABLE 3C MODULUS OF RUPTURE f_c' OF CONCRETE

3500 psi mix; 6 x 6 x 20 in. beams

1. All beams tested on 18 inch span under third point loading
2. All values given in ksi
3. All tests at 13 days

Spec. No.	XOB-1	XB-1	OB-1	OC-1	CA-1	CB-1	CC-1	RA-1	RB-1	RC-1
1	.644	.531	.636	.524	.632	.605	.592	.508	.594	.570
2	.565	--	.545	.564	.636	.562	.546	.595	--	.576
3	.507	.604	.541	.581	.688	.550	.610	.571	.584	.538
4	.539	.582	.545		.654	.603		.606	.543	
Avg.	.564	.572	.567	.557	.653	.580	.583	.570	.574	.561

TABLE 4 PROPERTIES OF NO. 9 HIGH STRENGTH STEEL REINFORCING BARS

Sample	#1	#2	#3
Yield strength f_y, ksi	97.7	96.0	95.8
Ultimate strength f_u, ksi	146.5	143.0	142.7
Modulus of Elasticity E_s, ksi	28.9×10^6	27.2×10^6	27.3×10^6
% elongation in 8 inches	10.6	8.5	9.8
Weight per lineal ft., lb.	3.398	3.407	3.420
Nominal area, in.^{2}	0.999	1.001	1.005
Average deformation height, in.	0.062	0.062	0.061
Average deformation spacing, in.	0.586	0.588	0.586

a. f_y computed on basis of 0.2% offset.

b. Nominal bar areas computed from the weight including that of the deformations.

c. Heat 25788; Chemical analysis supplied by Inland Steel Co., % by weight: 0.42C; 0.83 Mn; 0.022 P; 0.022 S; 0.34 Si; 0.93 Cr; 0.021 Mo.

TABLE 5 PROPERTIES OF NO. 7 HIGH STRENGTH STEEL REINFORCING BARS

Sample	#1	#2	#3
Yield strength f_y, ksi	96.6	95.7	92.9
Ultimate strength f_u, ksi	137.7	135.9	139.1
Modulus of elasticity E_s, ksi	28.1×10^6	24.6×10^6	29.0×10^6
% elongation in 8 inches	6.4*	5.0*	9.2
Weight per lineal ft., lb.	2.038	2.035	2.053
Nominal area, in.^{2}	0.599	0.598	0.603
Average deformation height, in.	0.057	0.058	0.059
Average deformation spacing, in.	0.429	0.428	0.428

a. f_y computed on basis of 0.2% offset.

b. Nominal bar areas computed from the weight including that of the deformations.

c. Heat 3180; Chemical Analysis supplied by Inland Steel Co.; % by weight: 0.43 C; 1.00 Mn; 0.011 P; 0.019 S; 0.31 Si; 0.99 Cr; 0.22 Mo.

* Specimens broke outside gage points.

TABLE 6 PROPERTIES OF NO. 4 INTERMEDIATE GRADE STEEL REINFORCING BARS

Sample	#1	#2	#3
Yield strength, f_y, ksi	50.0	51.5	49.6
Ultimate strength f_u, ksi	82.8	84.4	79.1
Modulus of elasticity E_s, ksi	27.1×10^6	30.1×10^6	27.3×10^6
% elongation in 8 inches	15.6	18.4	16.4
Weight per lineal ft., lb.	0.634	0.634	0.636
Nominal area, in.^{2}	0.186	0.186	0.187
Average deformation height, in.	0.039	0.044	0.040
Average deformation spacing, in.	0.294	0.306	0.289

a. Nominal bar areas computed from the weight including that of the deformations.

TABLE 7 PROPERTIES OF NO. 2 INTERMEDIATE GRADE STEEL REINFORCING BARS

Sample	#1	#2	#3
Yield strength f_y, ksi	48.6	48.8	50.1
Ultimate strength f_u, ksi	63.2	66.3	68.1
Modulus of elasticity E_s, ksi	29.9×10^6	27.9×10^6	30.7×10^6
% elongation in 8 inches	13.5	18.4	14.3
Weight per lineal ft., lb.	0.1749	0.1717	0.1708
Nominal area, in.^{2}	0.0514	0.0505	0.0502
Average deformation height, in.	0.016	0.015	0.016
Average deformation spacing, in.	0.183	0.179	0.178

a. Nominal bar areas computed from the weight including that of the deformations.

TABLE 8 SUMMARY OF TEST PROGRAM

	CONCRETE		BEAM DIMENSIONS				RATIO	REINFORCEMENT					
Spec. No.	f'_c (ksi)	f'_t (ksi)	b (in)	h (in)	d (in)	L (ft)	a/d	No. 9 No. 7	p %	No. 4 Bars	p'%	Spacing No. 2 Stirrups	rf_y psi
XOB-1	3.66	0.564	9.1	21.8	18.03	12.0	3.99	4-#9	2.44	0	0	-	0
XB-1	3.56	0.572	9.1	21.8	18.02	12.0	4.00	4-#9	2.44	2	0.226	7 1/2	73.0
OB-1	3.42	0.567	9.0	21.8	18.06	12.0	3.98	4-#9	2.47	0	0	-	0
OC-1	3.92	0.557	6.1	21.8	18.05	12.0	3.98	2-#9	1.82	0	0	-	0
CA-1	3.87	0.653	12.1	21.8	18.08	12.0	3.98	*4-#9	1.84	2	0.171	8 1/4	49.9
CB-1	3.59	0.580	9.0	21.8	18.04	12.0	3.99	*4-#9	2.47	2	0.229	7 1/2	73.8
CC-1	3.95	0.583	6.0	21.8	18.06	12.0	3.98	*2-#9	1.85	2	0.343	8 1/4	100.8
RA-1	3.61	0.570	12.0	21.8	18.05	12.0	3.98	6-#7	1.66	2	0.172	8 1/4	50.4
RB-1	3.57	0.574	9.0	21.8	18.06	12.0	3.98	6-#7	2.22	2	0.229	7 1/2	73.8
RC-1	4.23	0.561	6.1	21.8	18.06	12.0	3.98	3-#7	1.63	2	0.337	8 1/4	99.0

* One-half of tensile reinforcement cut off 24 inches from each support.

(1) Yield point value at 0.2% offset f_y = 96.5 ksi for No. 9 bars, tensile reinf; As = 1.002 in^2

(2) Yield point value at 0.2% offset f_y = 95.1 ksi for No. 7 bars, tensile reinf; As = 0.600 in^2

(3) Yield point value at 0.2% offset f_y = 50.4 ksi for No. 4 bars, compression steel; As = 0.186 in^2

(4) Yield point value at 0.2% offset f_y = 49.2 ksi for No. 2 bars (stirrups); As = 0.0507 in^2

TABLE 9 ANALYSIS OF TEST RESULTS

	TEST VALUES					CALCULATED VALUES				TEST VALUE/CALCULATED VALUE			
Spec. No.	P_{cr} 1) (kips)	P_u 1) (kips)	Δ_{max} (in)	Failure Mode	P_f 2) (kips)	P_{cr}		P_v 3)		P_{cr}		P_v	
						Eq. 1	Eq. 3	Eq. 1	Eq. 3	Eq. 1	Eq. 3	Eq. 1	Eq. 3
XOB-1	45	57.5	0.23	D-T	95.5	42.6	39.6	42.6	39.6	1.05	1.13	1.35	1.45
XB-1	50	90.0	0.56	V-C	100.5	42.0	39.0	66.0	63.0	1.19	1.28	1.36	1.43
OB-1	45	57.7	0.23	D-T	91.3	41.0	38.2	41.0	38.2	1.10	1.18	1.41	1.51
OC-1	25	34.9	0.22	D-T	60.6	28.6	27.6	28.6	27.6	0.87	0.91	1.22	1.27
CA-1	55	74.2	0.32	V-C	126.3	57.0	54.4	79.0	76.4	0.97	1.01	0.94	0.97
CB-1	40	79.0	0.50	V-C	100.3	42.4	39.2	66.6	63.2	0.94	1.02	1.19	1.25
CC-1	25	49.5	0.59	V-C	63.0	28.6	27.2	50.6	49.2	0.87	0.92	0.98	1.01
RA-1	60	90.0	0.48	V-C	116.6	53.8	52.0	75.6	74.0	1.11	1.15	1.19	1.22
RB-1	50	90.0	0.72	V-C	98.2	41.4	38.8	65.6	63.0	1.21	1.29	1.37	1.43
RC-1	30	61.9	0.66	V-C	65.1	29.4	28.8	51.2	50.6	1.02	1.04	1.21	1.22

(1) Applied loads - exclusive of weight of specimen.
(2) Critical section at midspan, adjustment made for weight of specimen.
(3) Critical section at midspan, requires no adjustment for weight of specimen.

Equation 1 - $v_u = \frac{Vu}{bd} = v_c + Krf_y = 1.9\sqrt{f'_c} + 2500\frac{pVd}{M} + Krf_y$

Equation 3 - $v_u = \frac{Vu}{bd} = 2\sqrt{f'_c} + Krf_y$

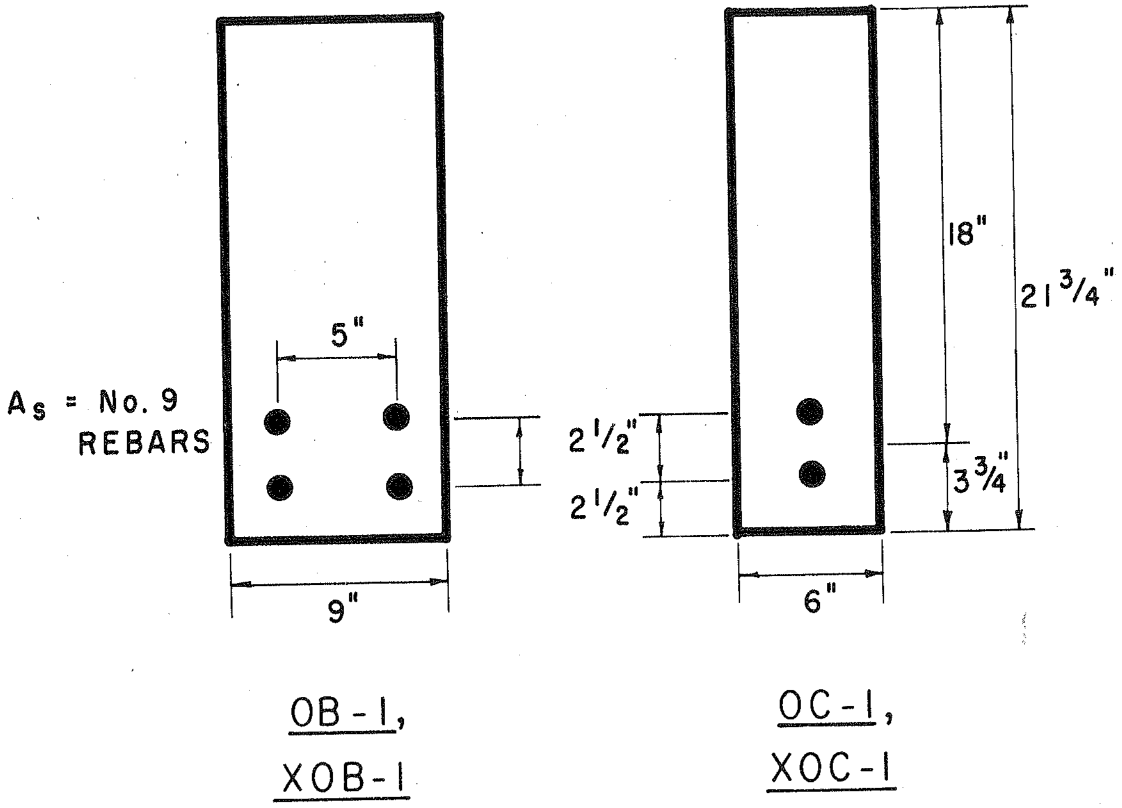
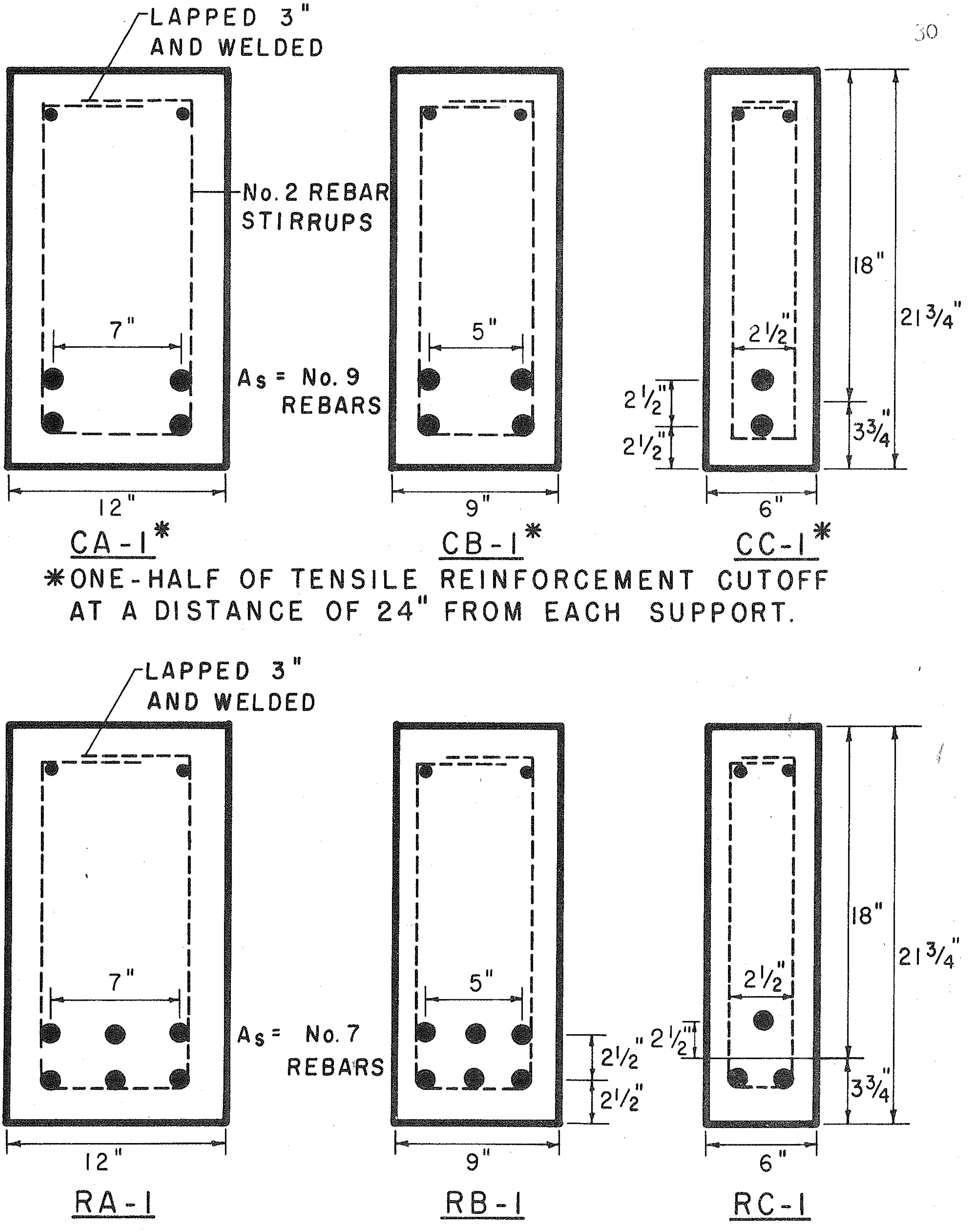


FIG 1-A SERIES O AND SERIES XO BEAM CROSS-SECTIONS.

(1) ALL DIMENSIONS ARE NOMINAL; SEE TABLE 9 FOR MEASURED DISTANCES.



*ONE-HALF OF TENSILE REINFORCEMENT CUTOFF AT A DISTANCE OF 24" FROM EACH SUPPORT.

FIG. 1-B SERIES C AND SERIES R BEAM CROSS-SECTIONS.

(1) ALL DIMENSIONS ARE NOMINAL; SEE TABLE 9 FOR MEASURED DISTANCES.

(2) TOP BARS ARE No. 4; STIRRUPS ARE No. 2

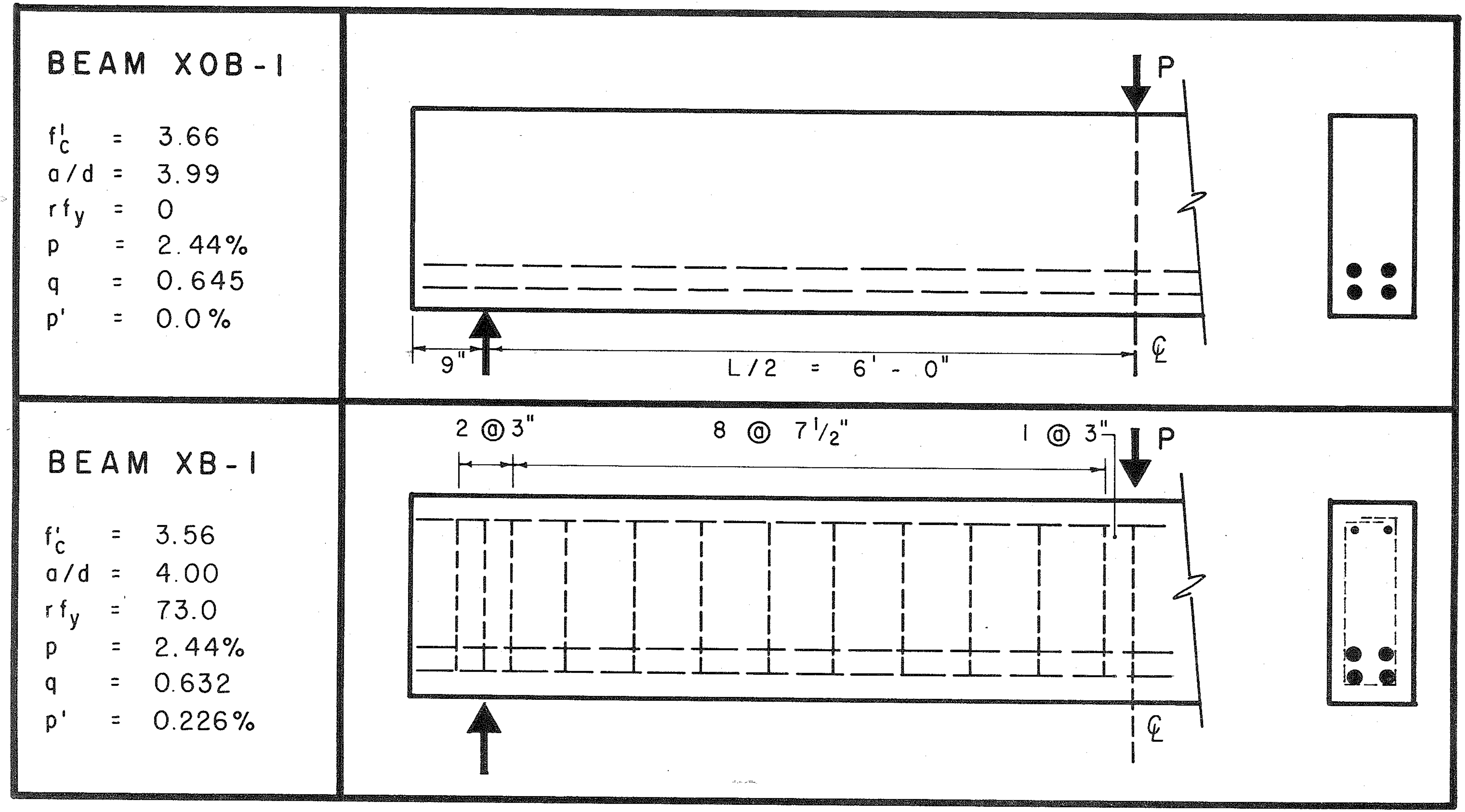


FIG. 2-A SERIES X BEAM ELEVATION

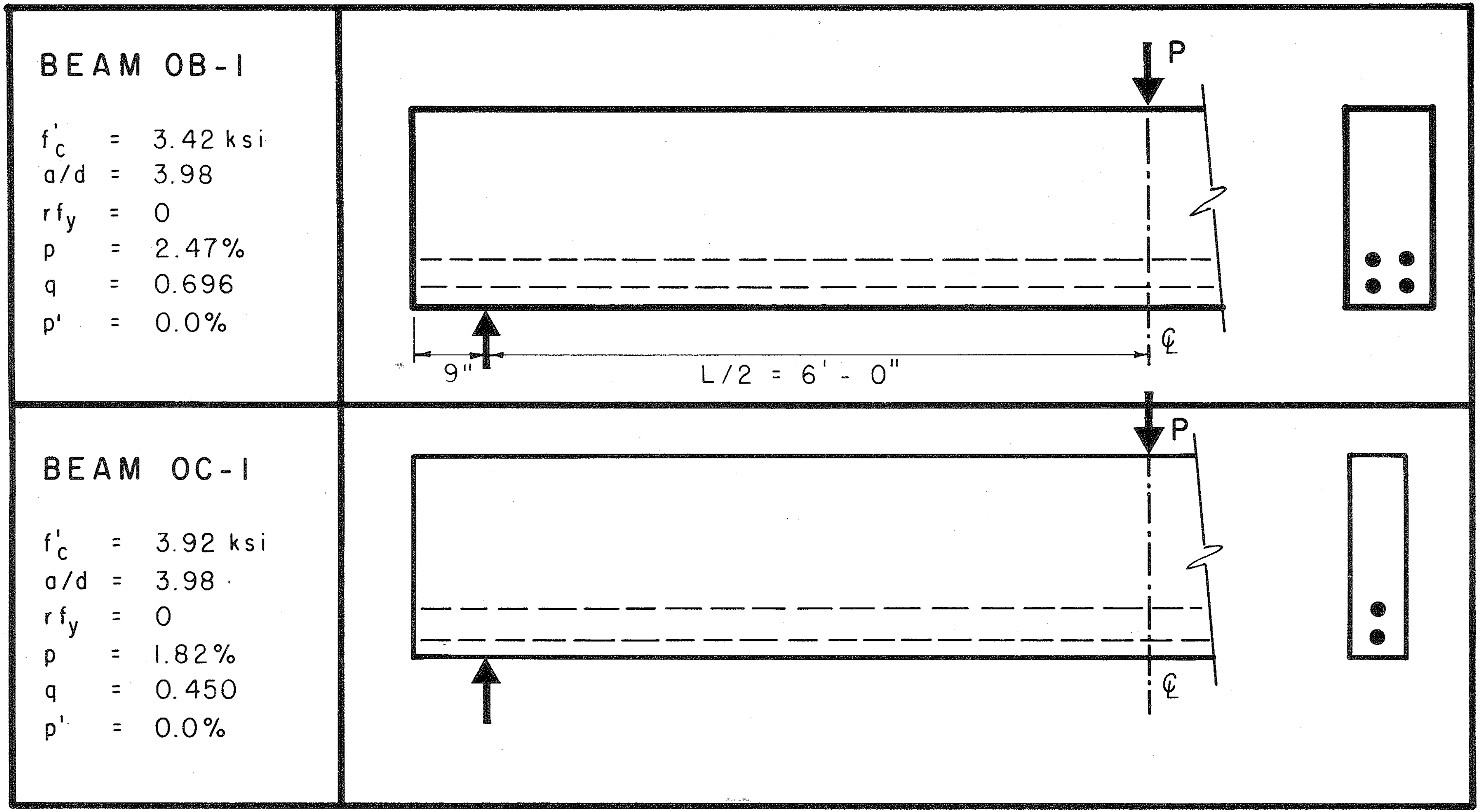
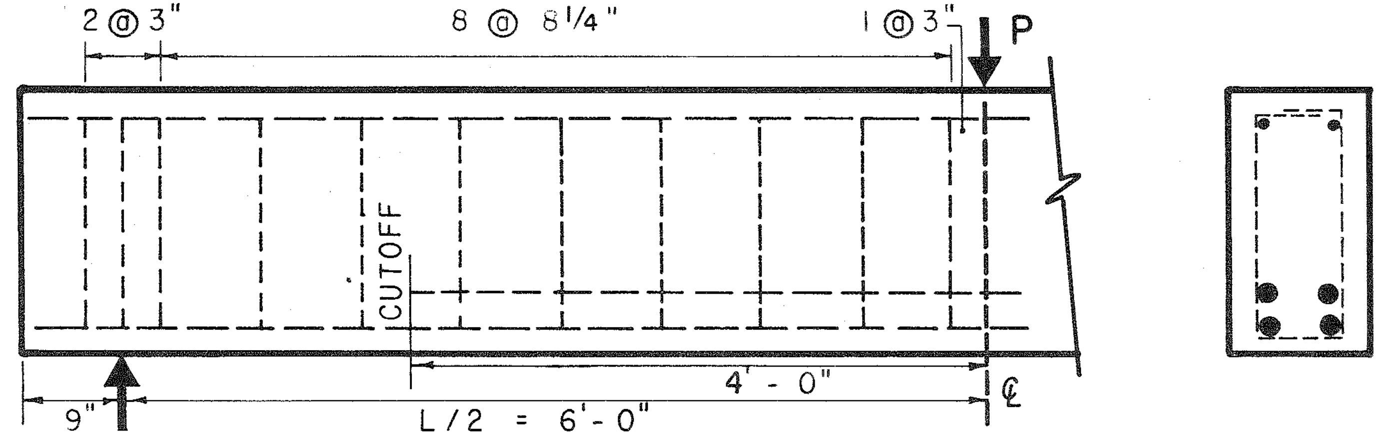


FIG. 2-B SERIES O BEAM ELEVATIONS

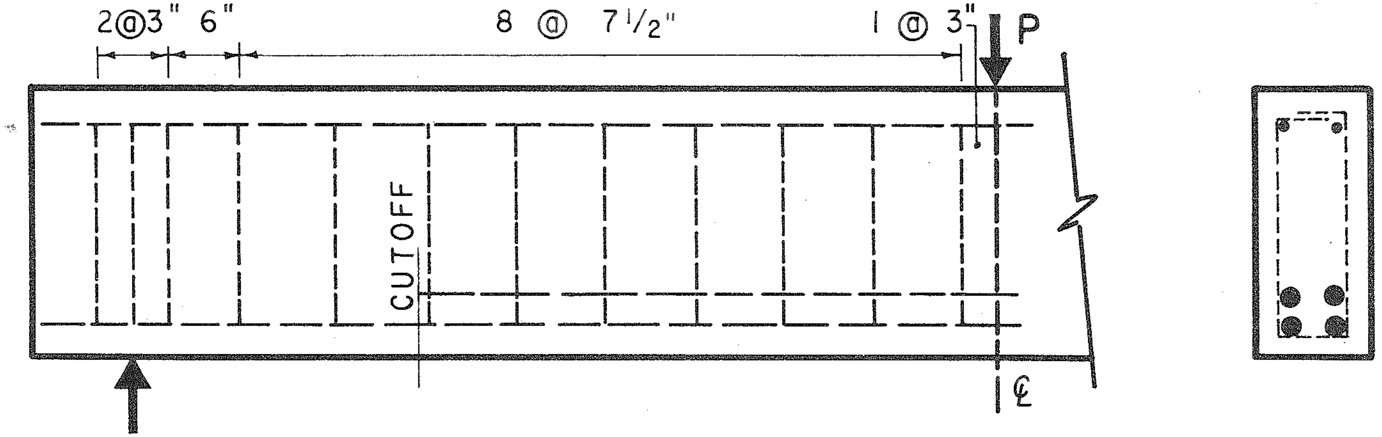
BEAM CA-1

f'_c = 3.87 ksi
a/d = 3.98
rf_y = 49.9
p = 1.84%
q = 0.430
p' = 0.171%



BEAM CB-1

f'_c = 3.59 ksi
a/d = 3.99
rf_y = 73.8
p = 2.47%
q = 0.640
p' = 0.229%



BEAM CC-1

f'_c = 3.95 ksi
a/d = 3.98
rf_y = 100.8
p = 1.85%
q = 0.410
p' = 0.343%

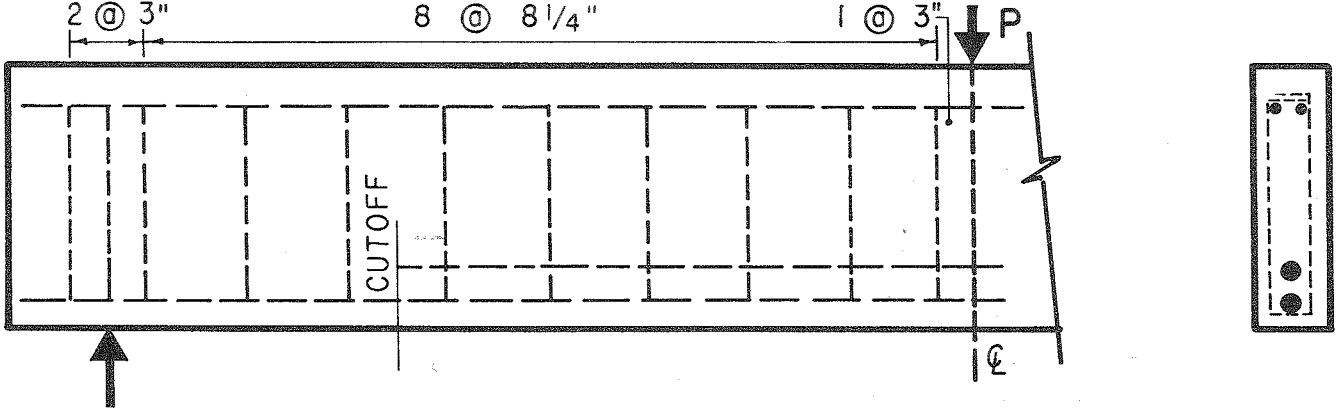


FIG. 2-C SERIES C BEAM ELEVATIONS

BEAM RA-1

f'_c = 3.61 ksi
a/d = 3.98
rf_y = 50.4
p = 1.66%
q = 0.415
p' = 0.172%

BEAM RB-1

f'_c = 3.57 ksi
a/d = 3.98
rf_y = 73.8
p = 2.22 %
q = 0.559
p' = 0.229%

BEAM RC-1

f'_c = 4.23 ksi
a/d = 3.98
rf_y = 99.0
p = 1.63 %
q = 0.326
p' = 0.337

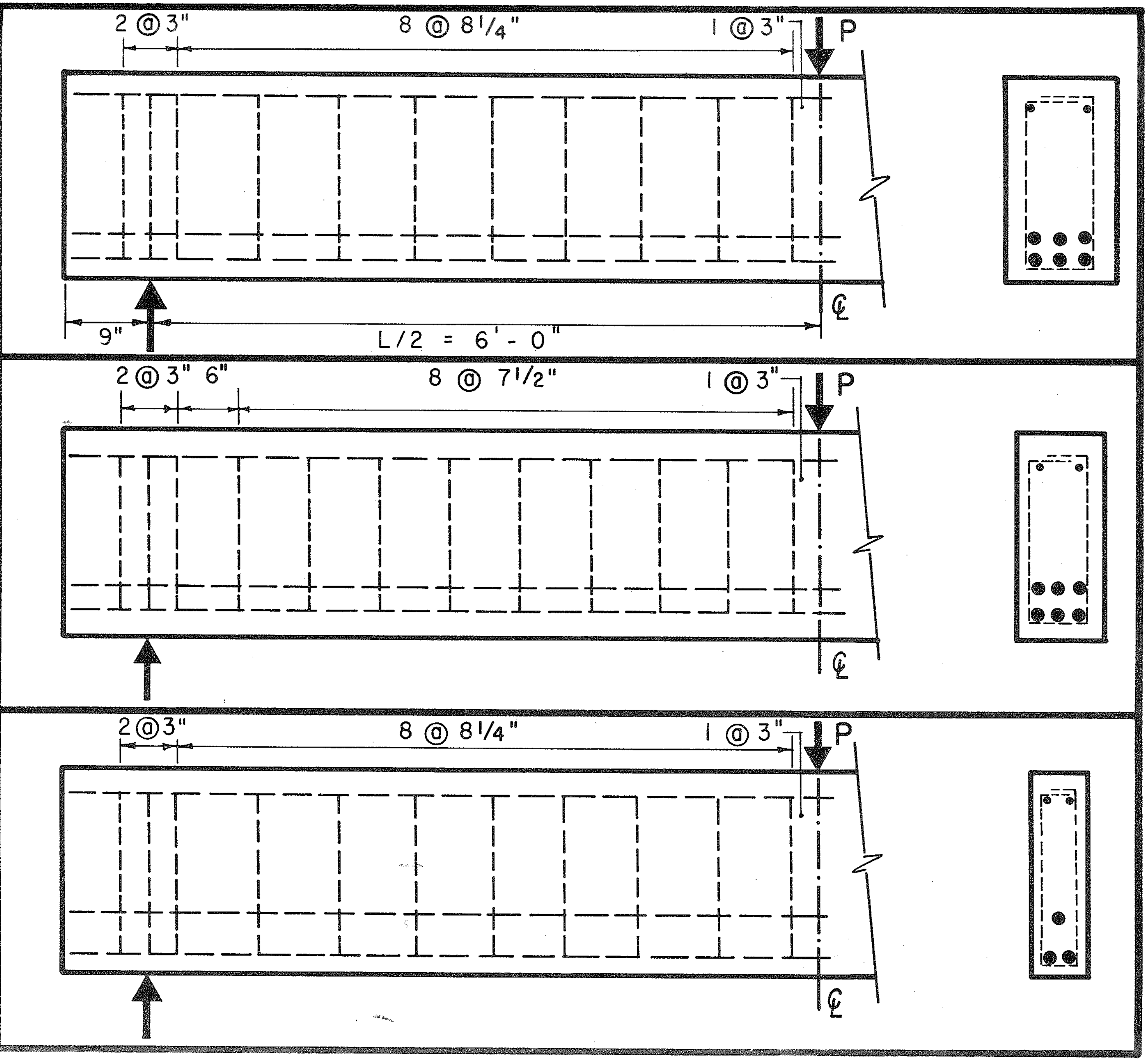
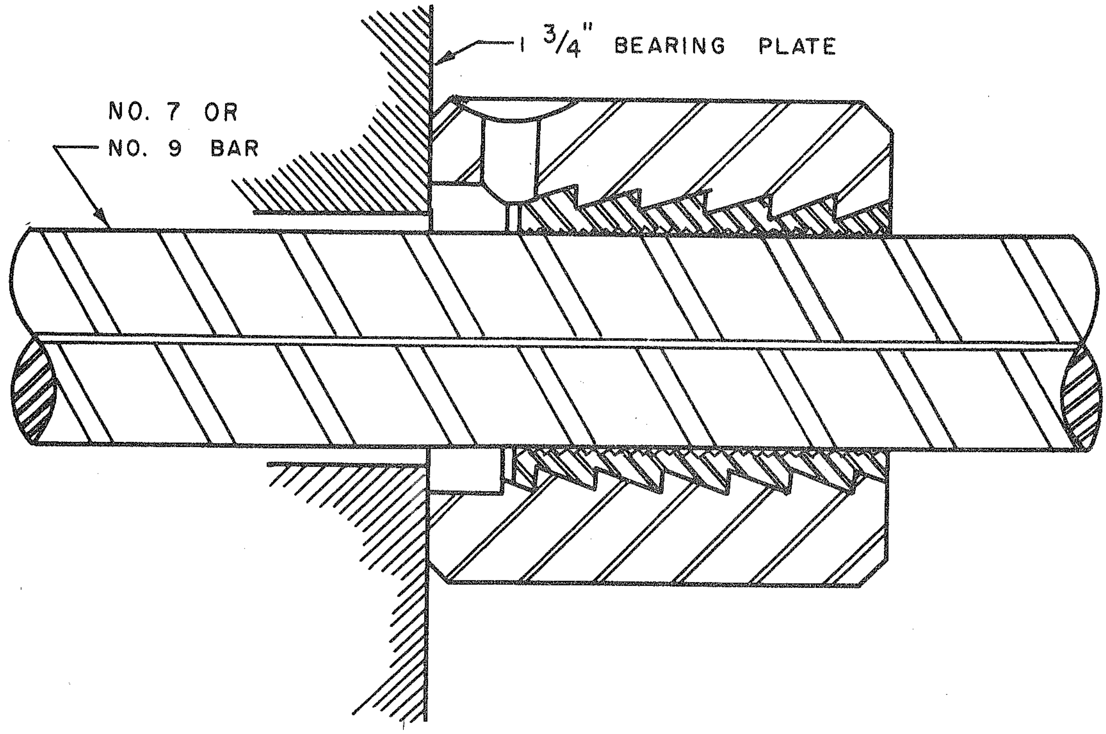
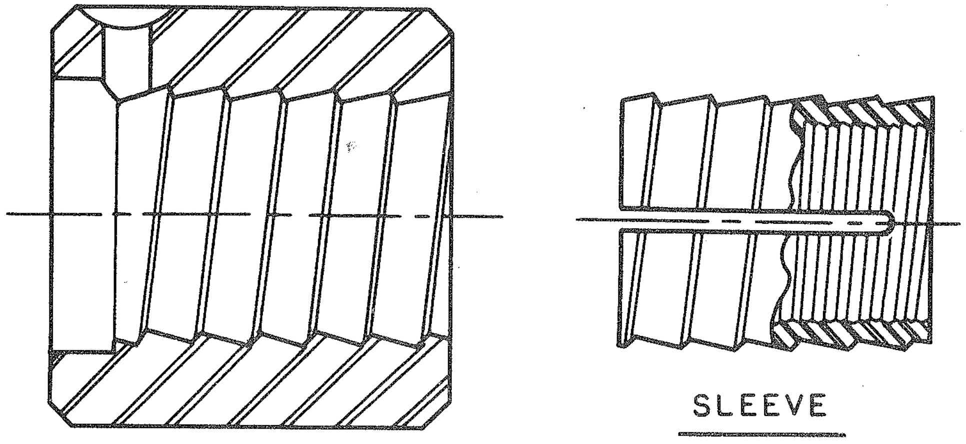


FIG. 2-D SERIES R BEAM ELEVATIONS

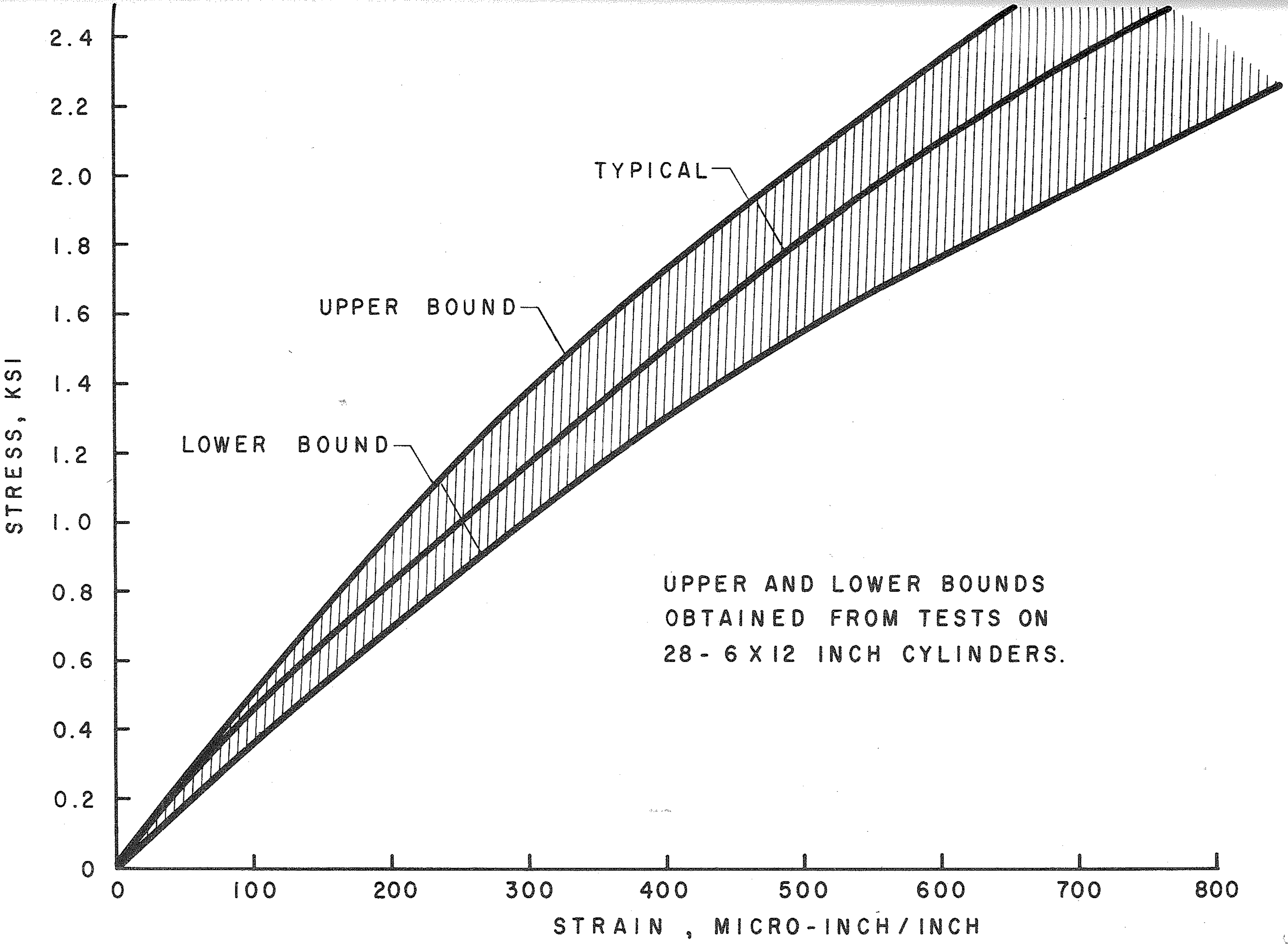


COMPLETE ASSEMBLY


NUT

SLEEVE

FIG. 3 DETAILS OF HOWLETT ANCHOR NUT



UPPER AND LOWER BOUNDS OBTAINED FROM TESTS ON 28 - 6 X 12 INCH CYLINDERS.

FIG. 4 STRESS - STRAIN RELATIONSHIPS FOR CONCRETE - 3500 PSI MIX.

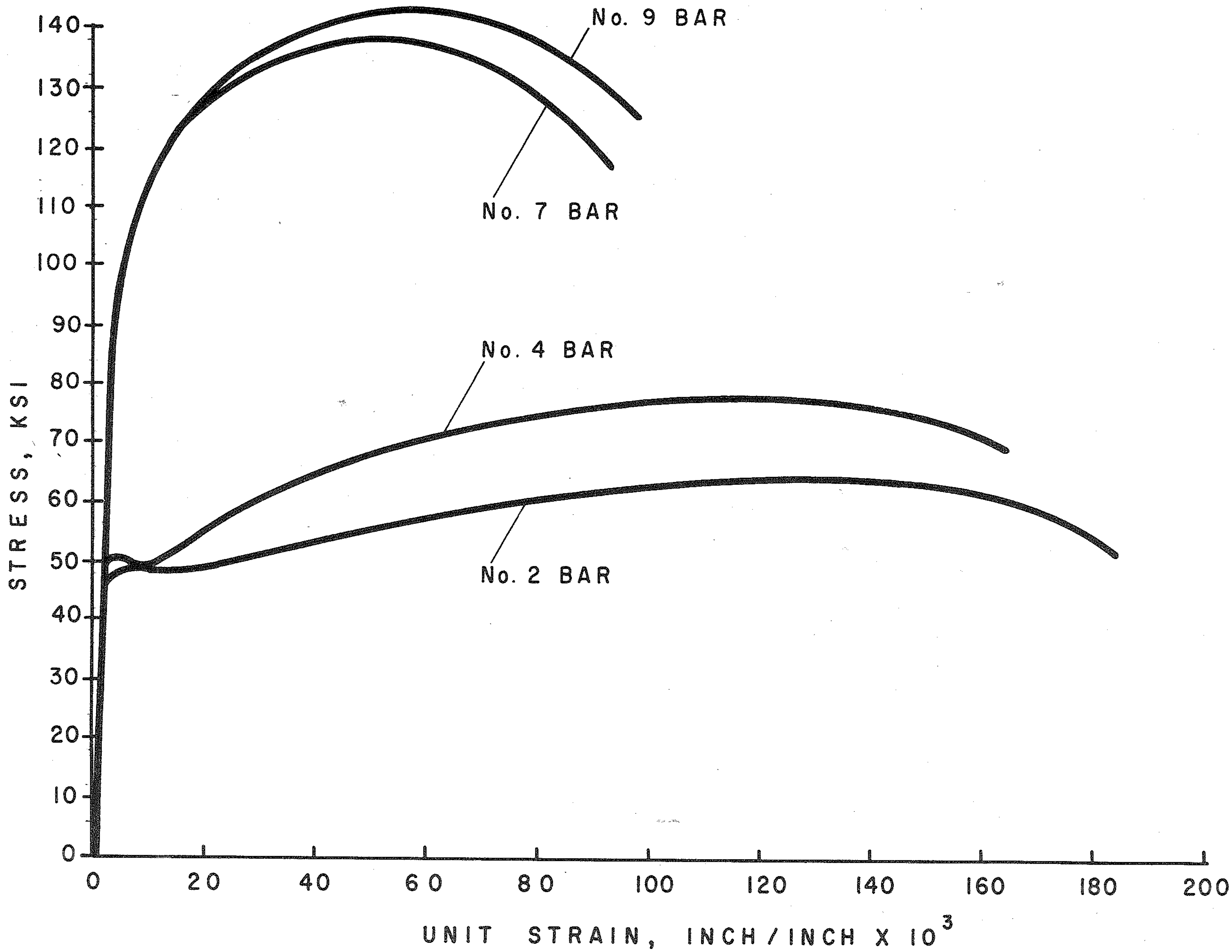


FIG. 5-A TYPICAL STRESS-STRAIN DIAGRAMS FOR STEEL REINFORCEMENT (FULL RANGE TO FAILURE)

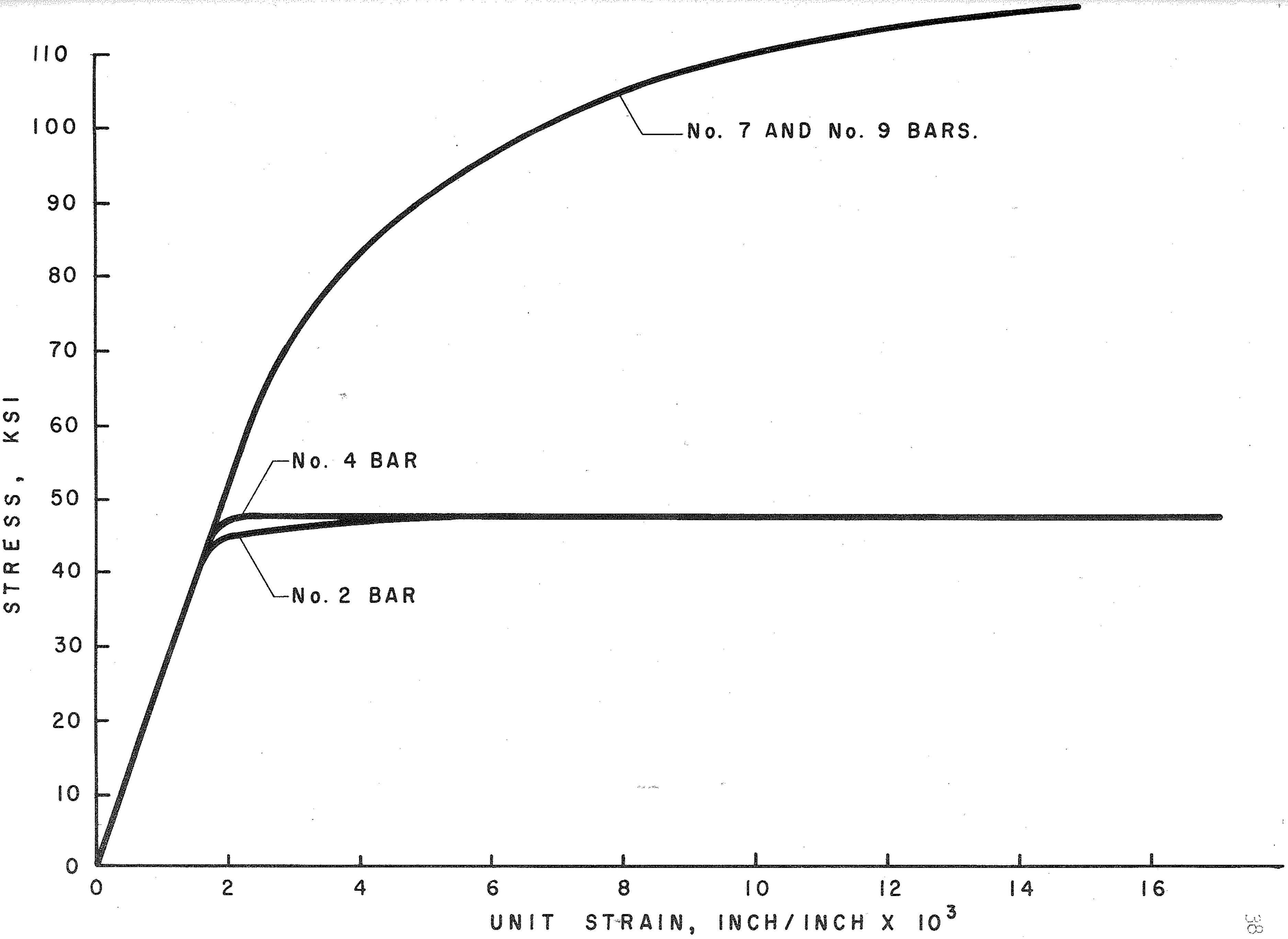


FIG. 5-B TYPICAL STRESS-STRAIN DIAGRAM FOR STEEL REINFORCEMENT (THROUGH YIELD RANGE)

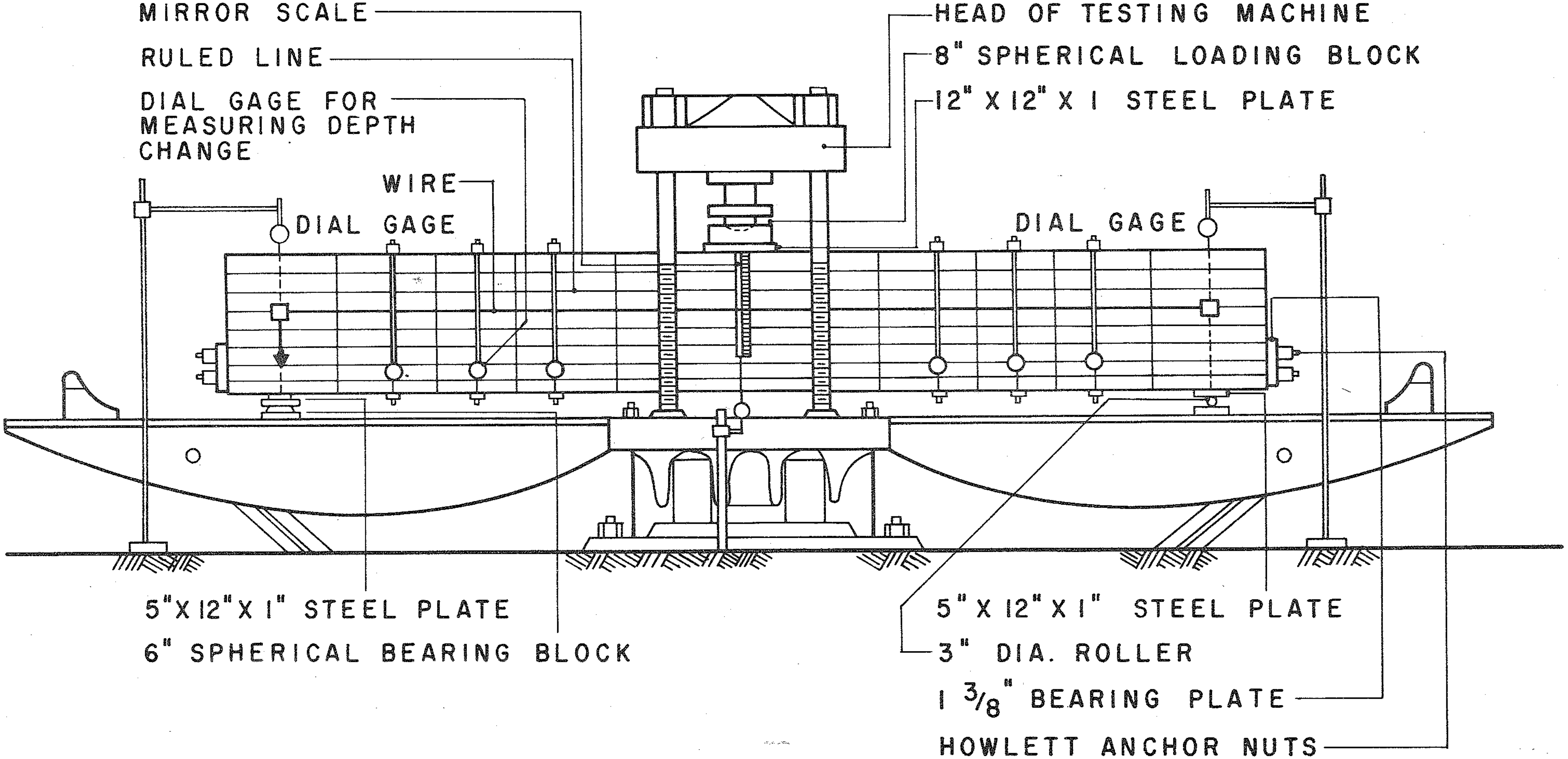


FIG. 6 LOADING ARRANGEMENT AND INSTRUMENTATION

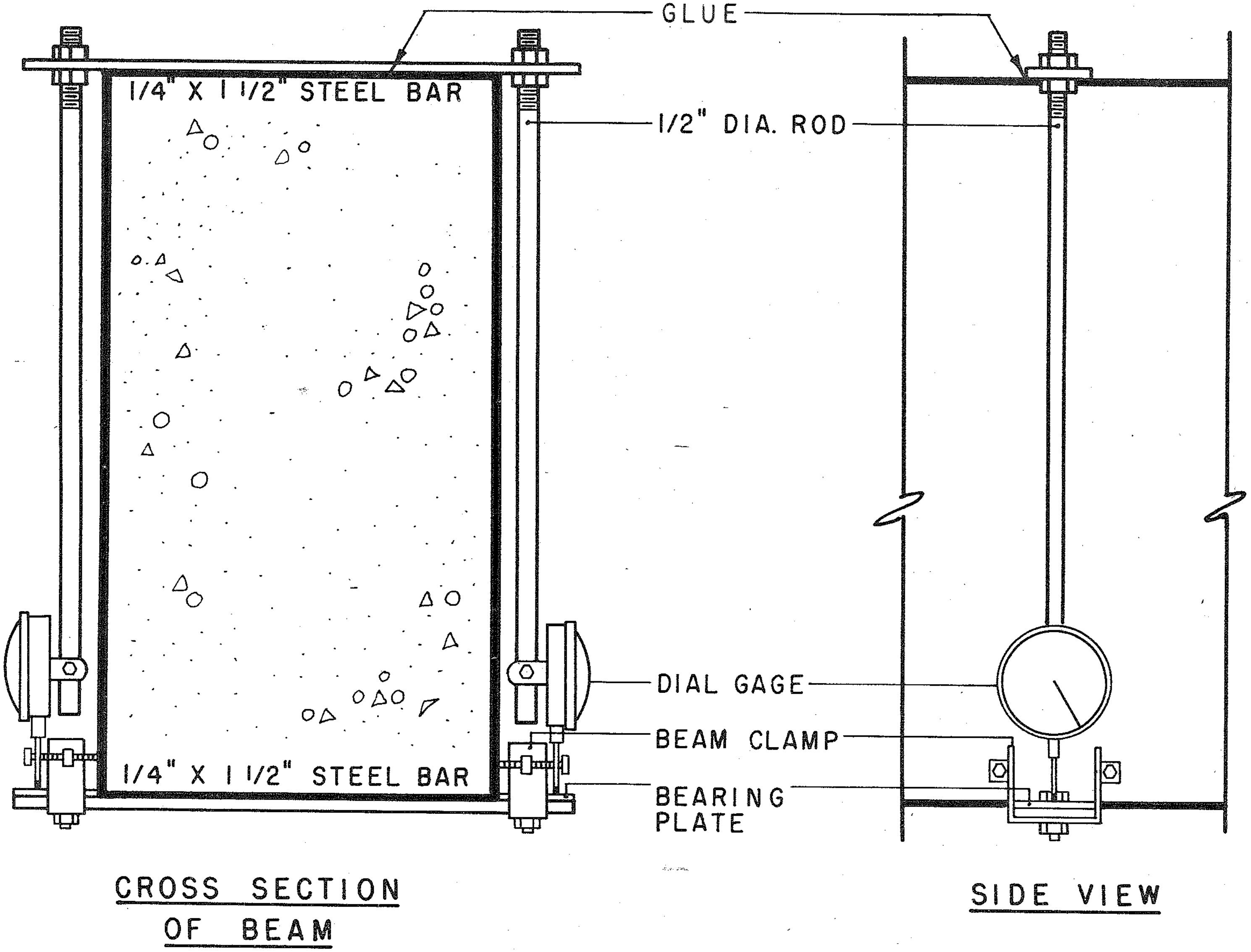
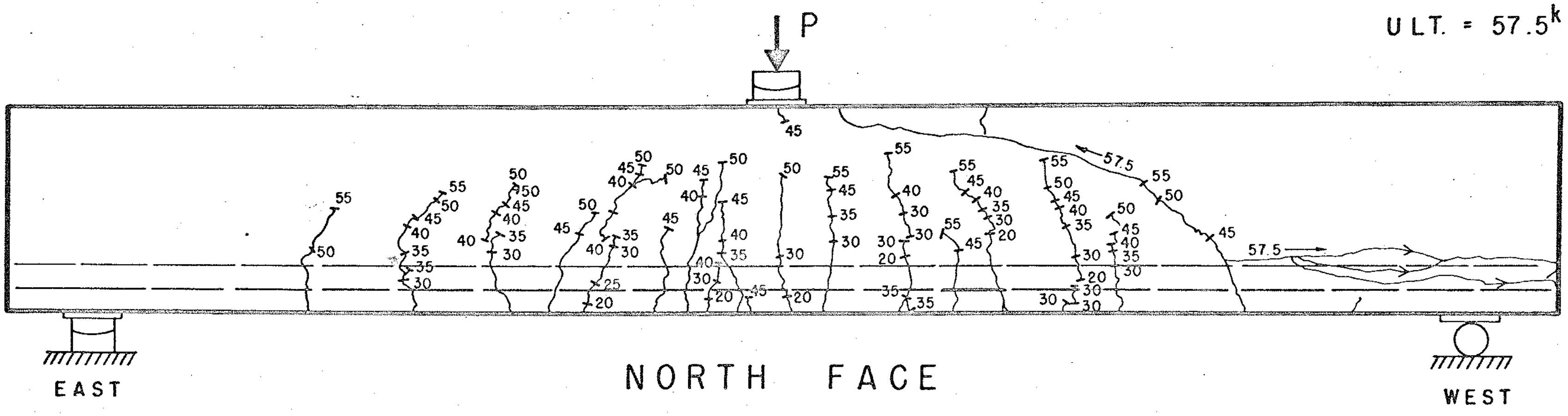
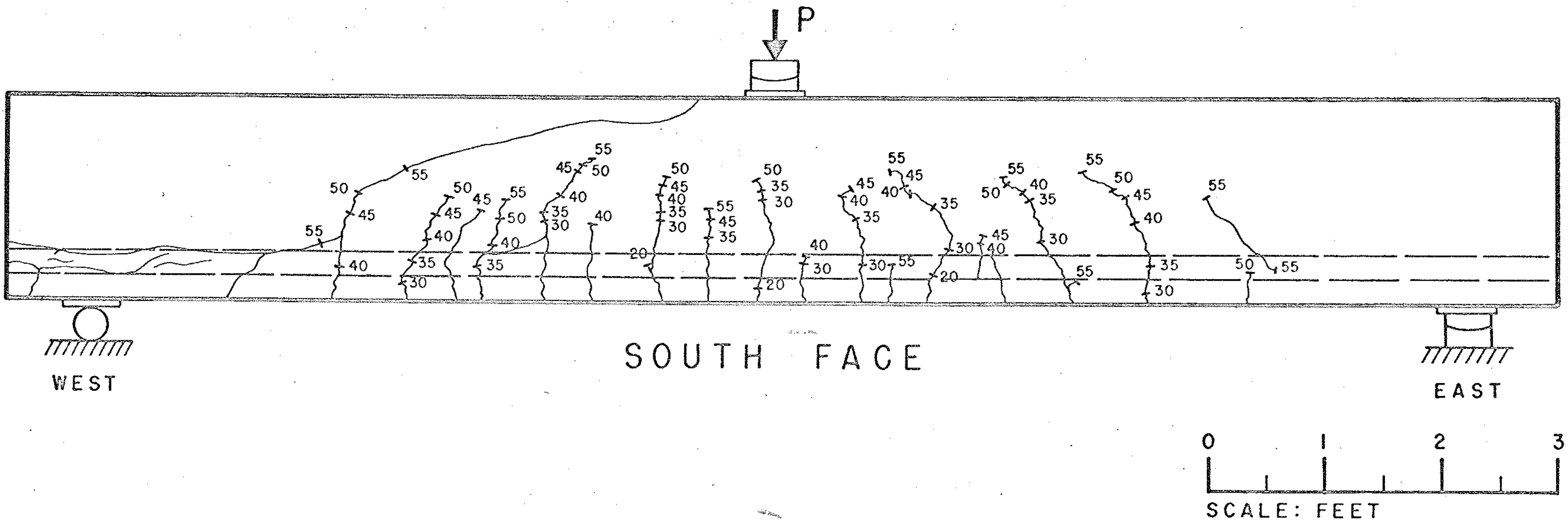


FIG. 7 DETAILS OF DIAL GAGE FOR MEASURING DEPTH CHANGE.

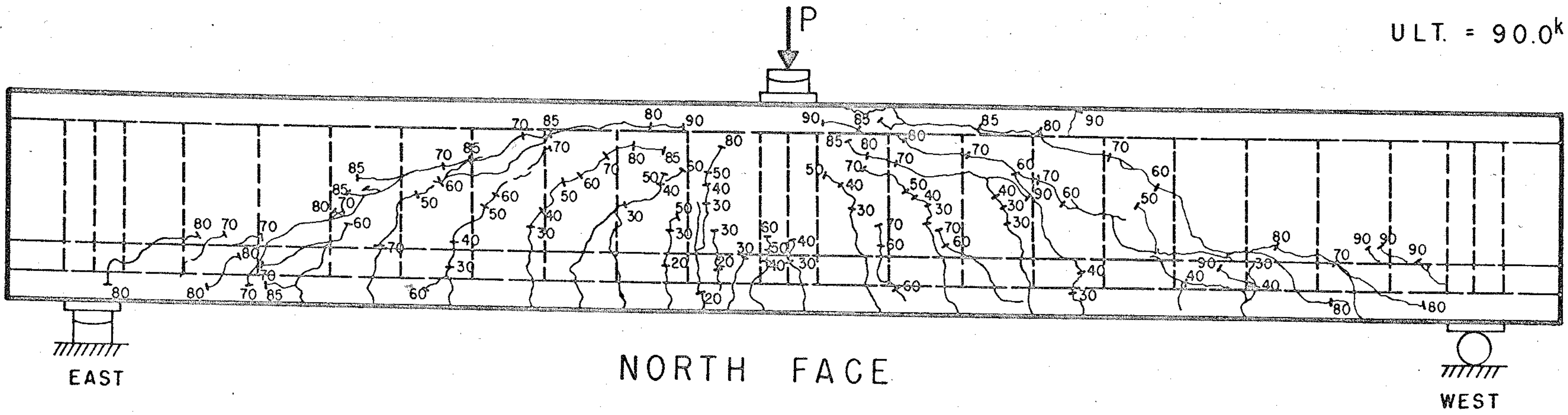


NORTH FACE

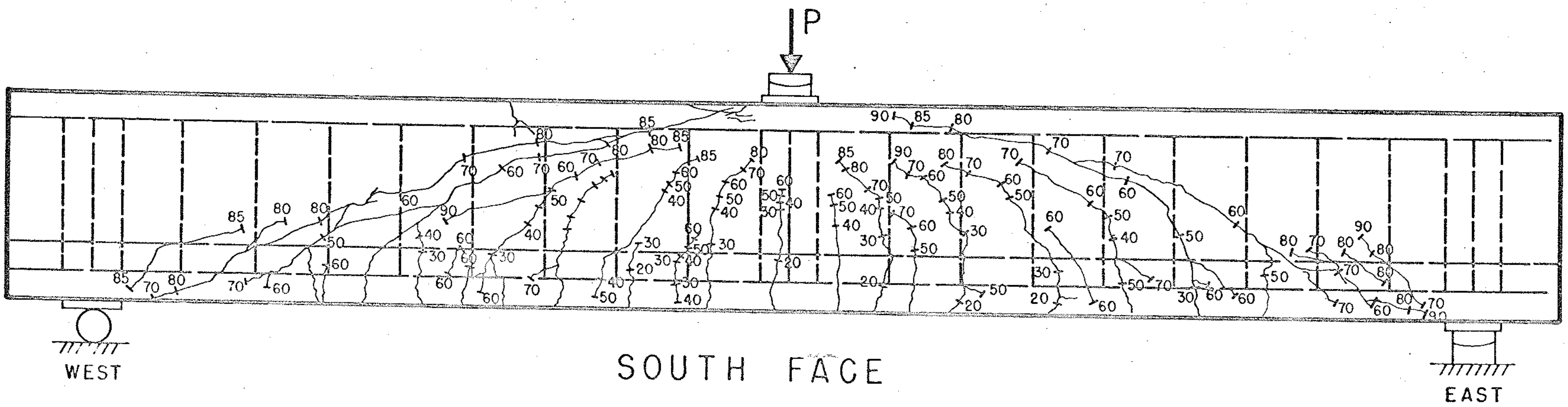


SOUTH FACE

FIG. 8A BEAM XOB-1

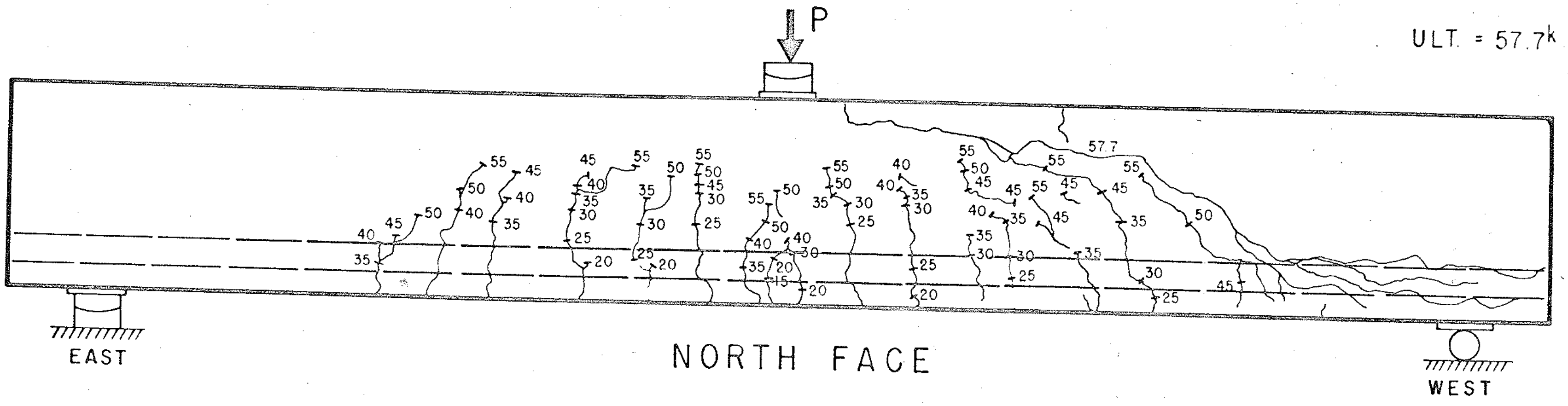


NORTH FACE

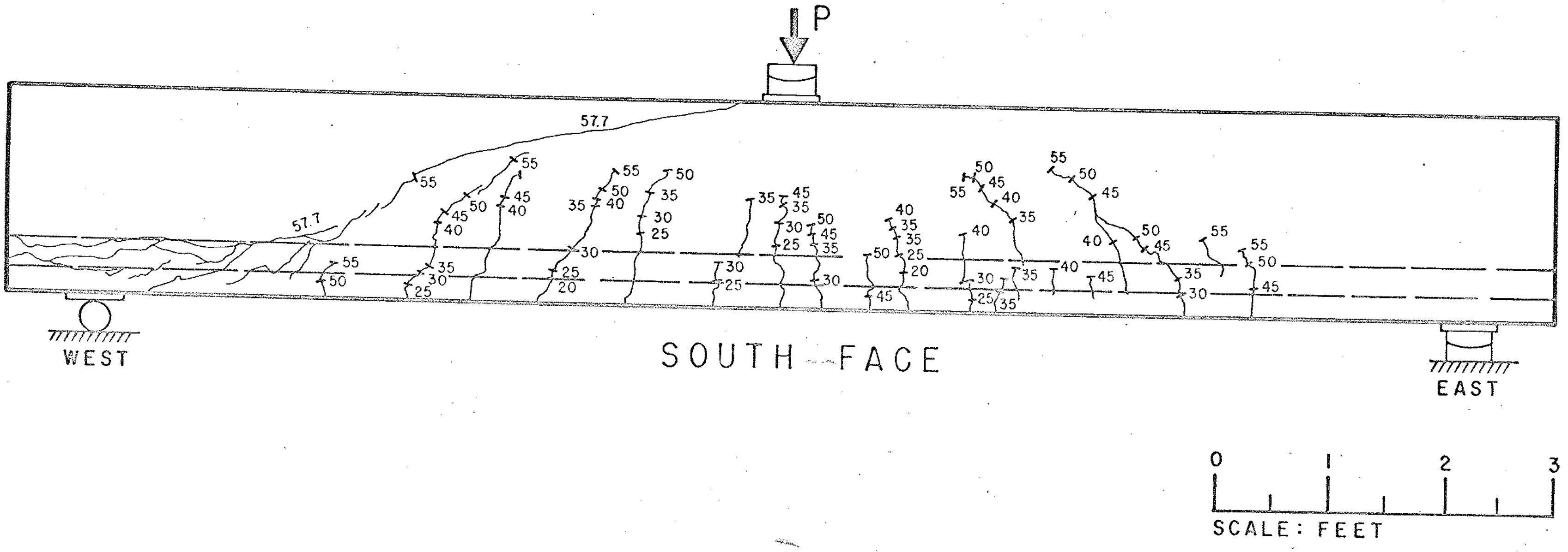


SOUTH FACE

FIG. 8B BEAM XB-1



NORTH FACE



SOUTH FACE

FIG. 8C BEAM OB-1

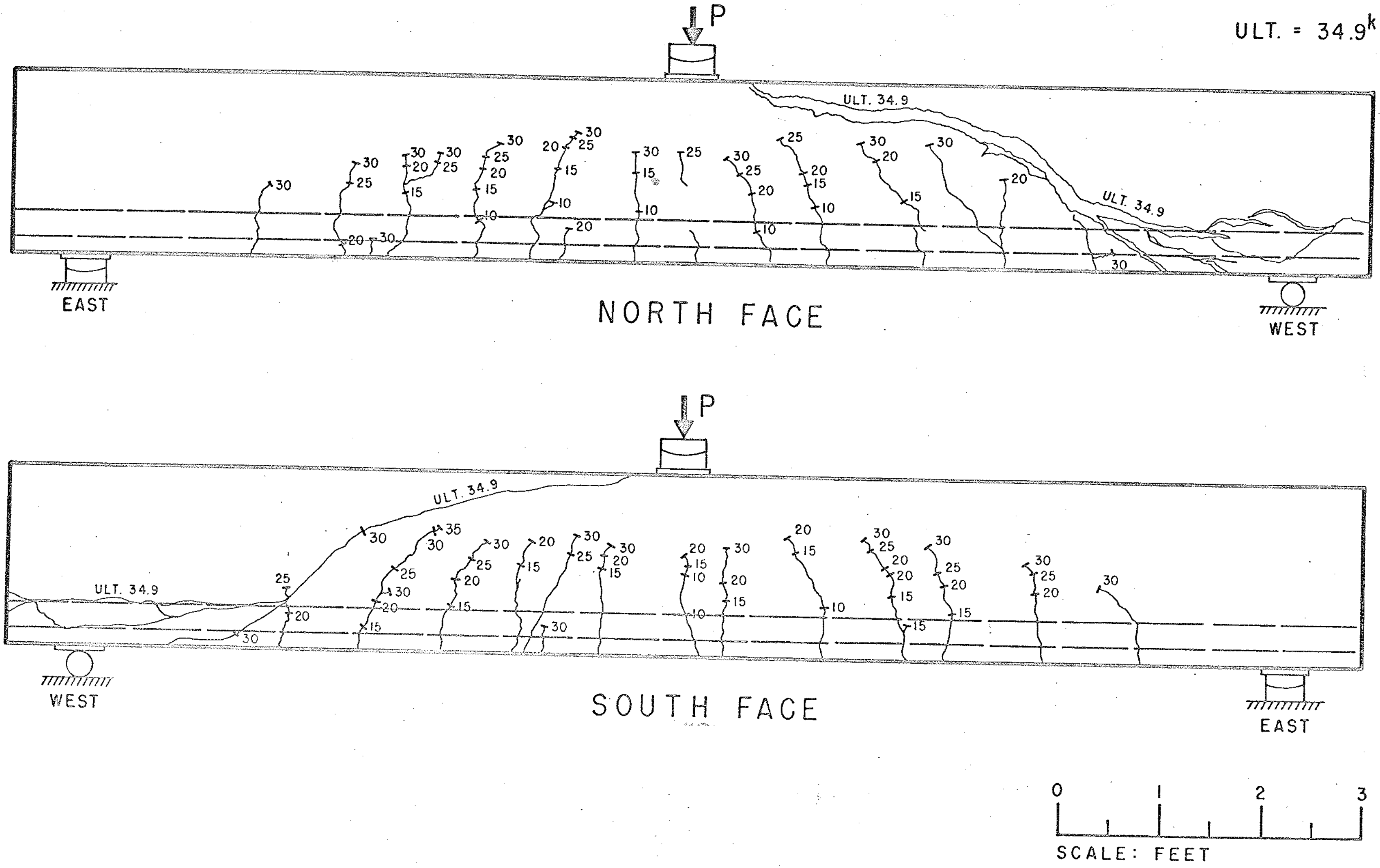


FIG. 8D BEAM OC-1

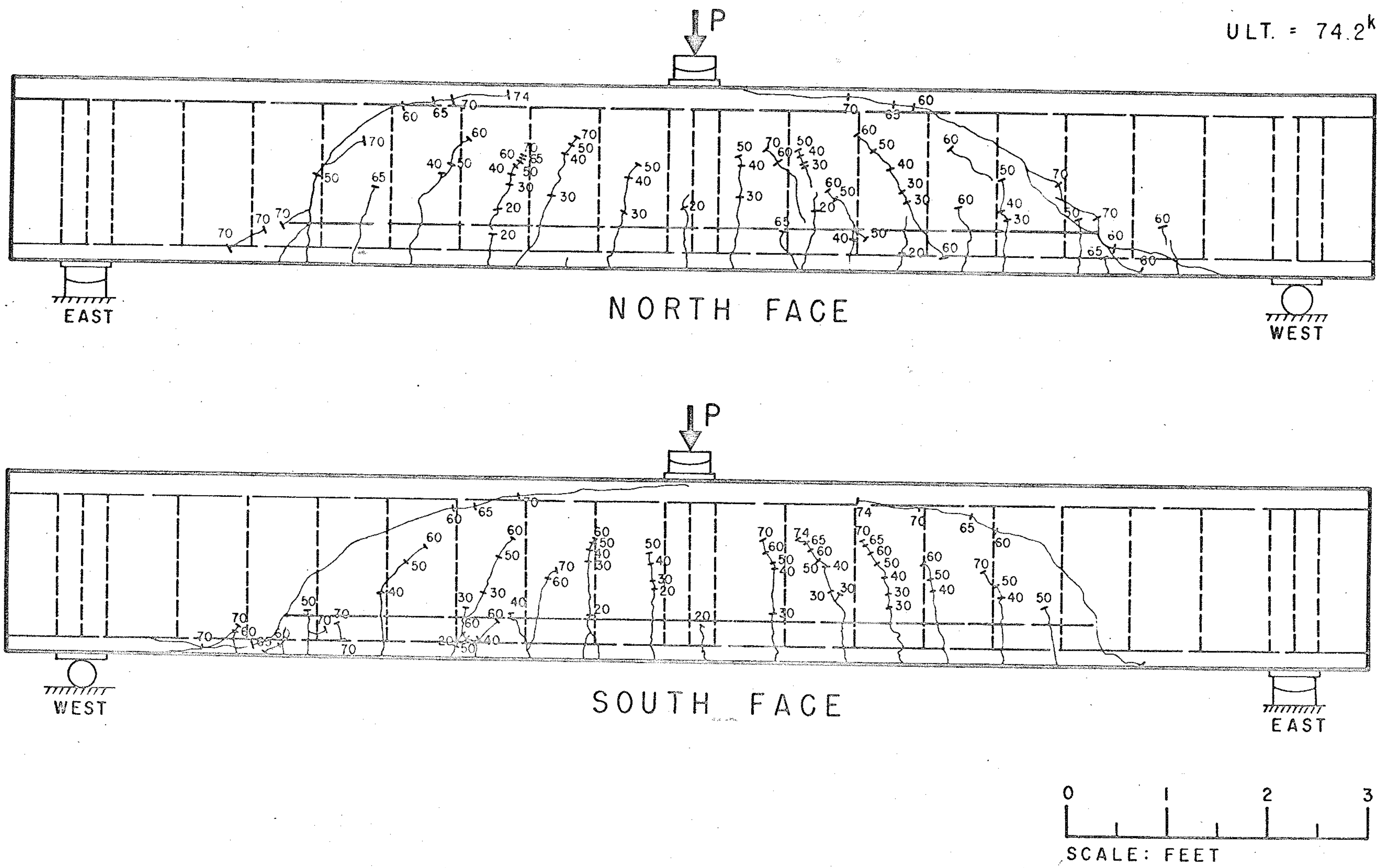


FIG. 8E BEAM CA-1

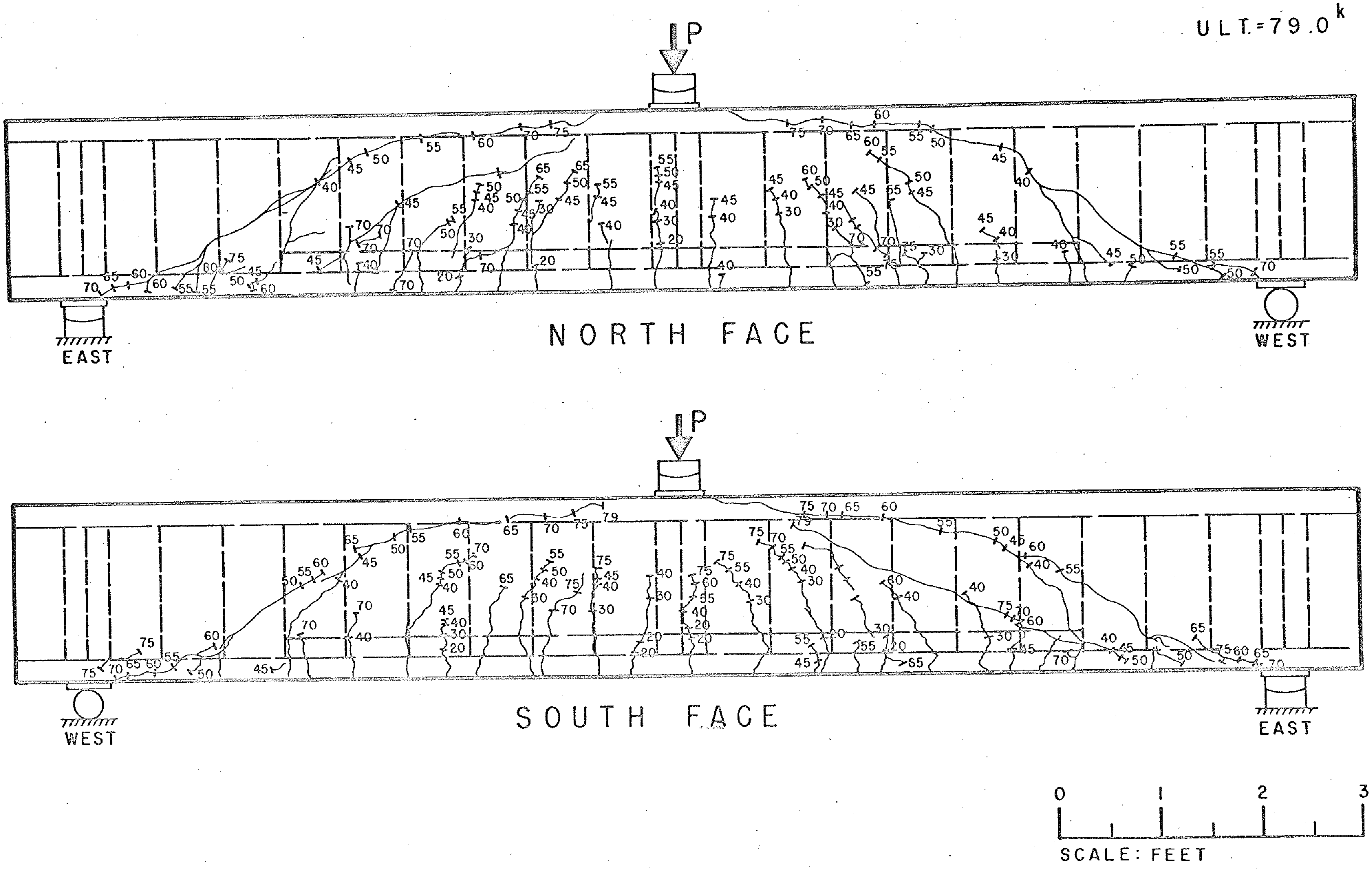
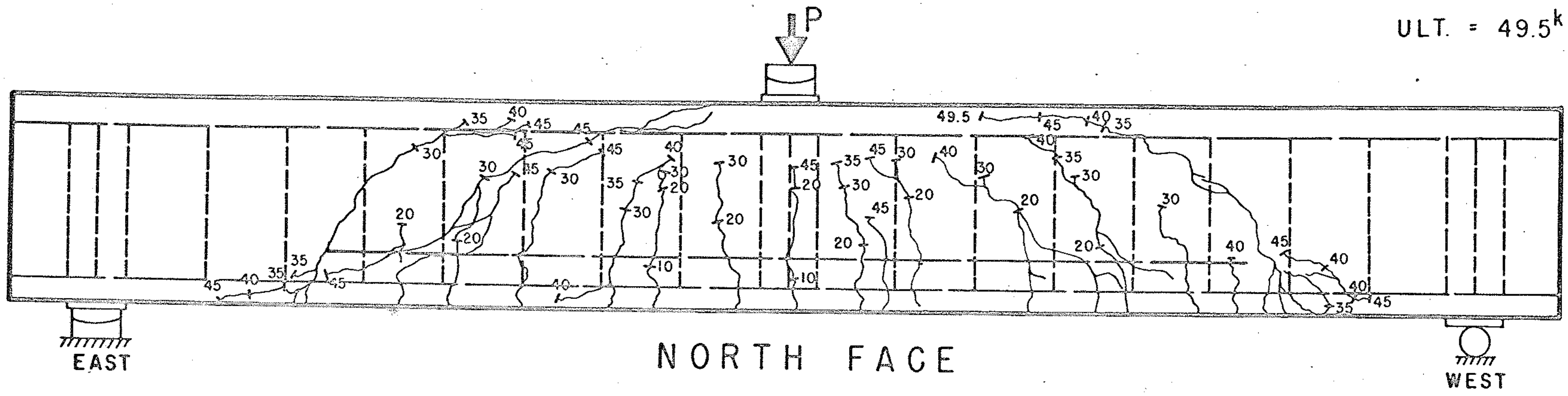
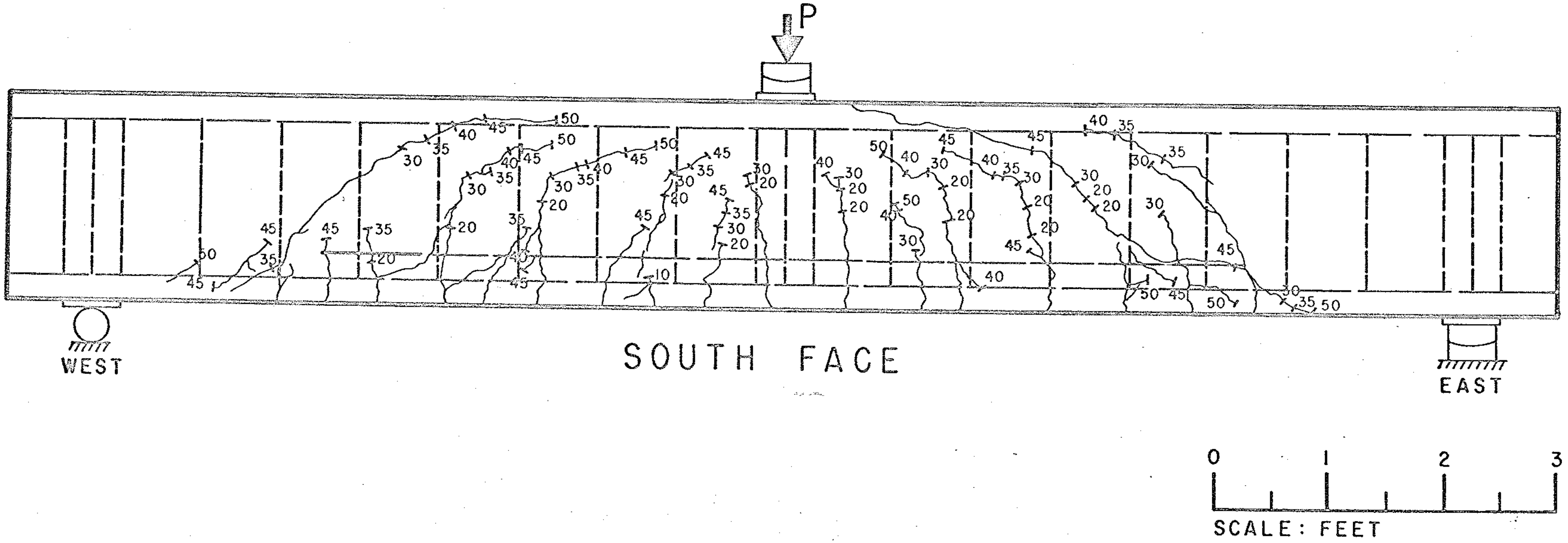


FIG. 8F BEAM CB-1



NORTH FACE



SOUTH FACE

FIG. 8G BEAM CC-I

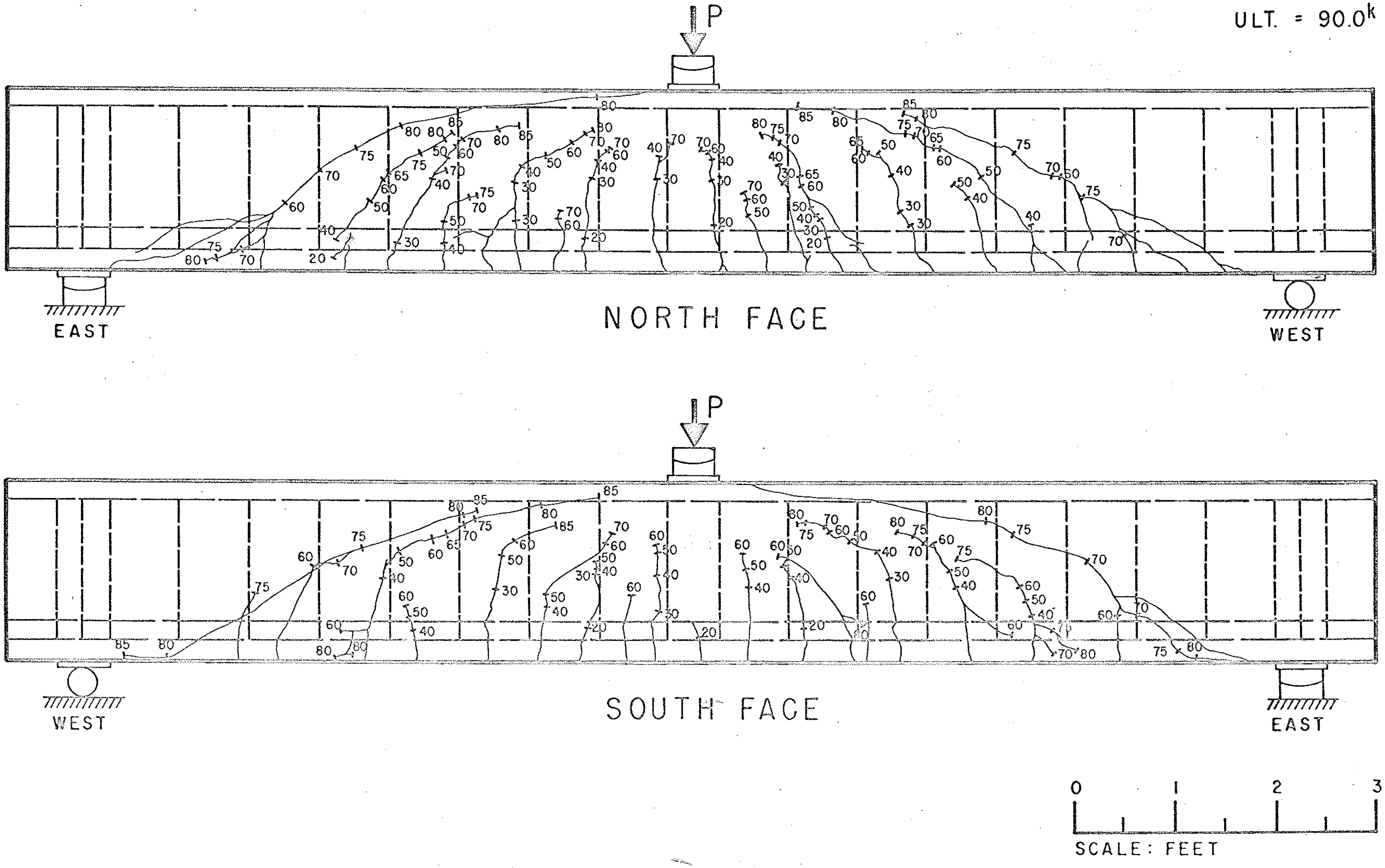
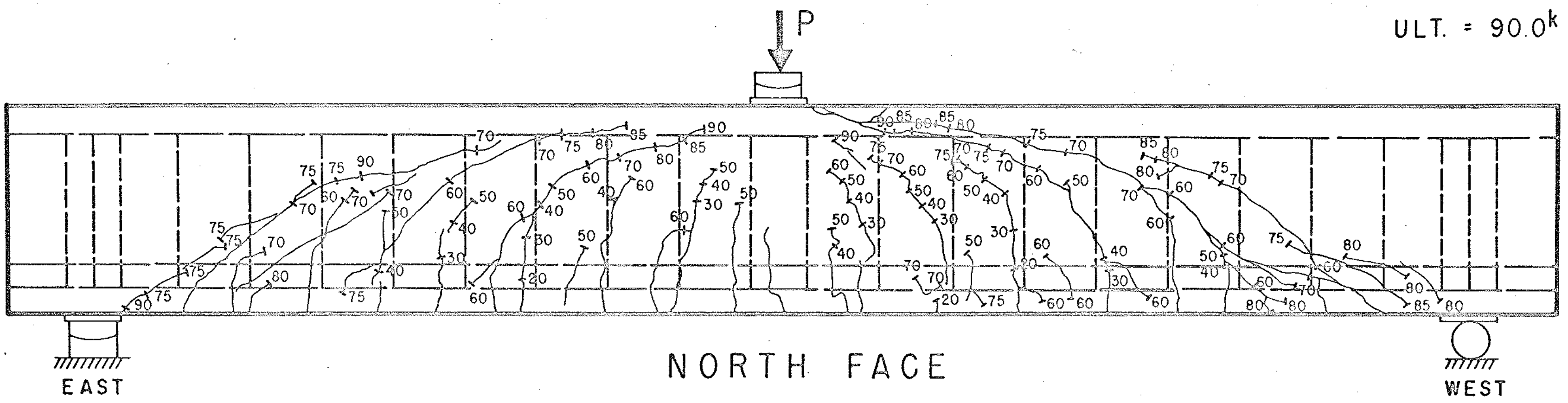
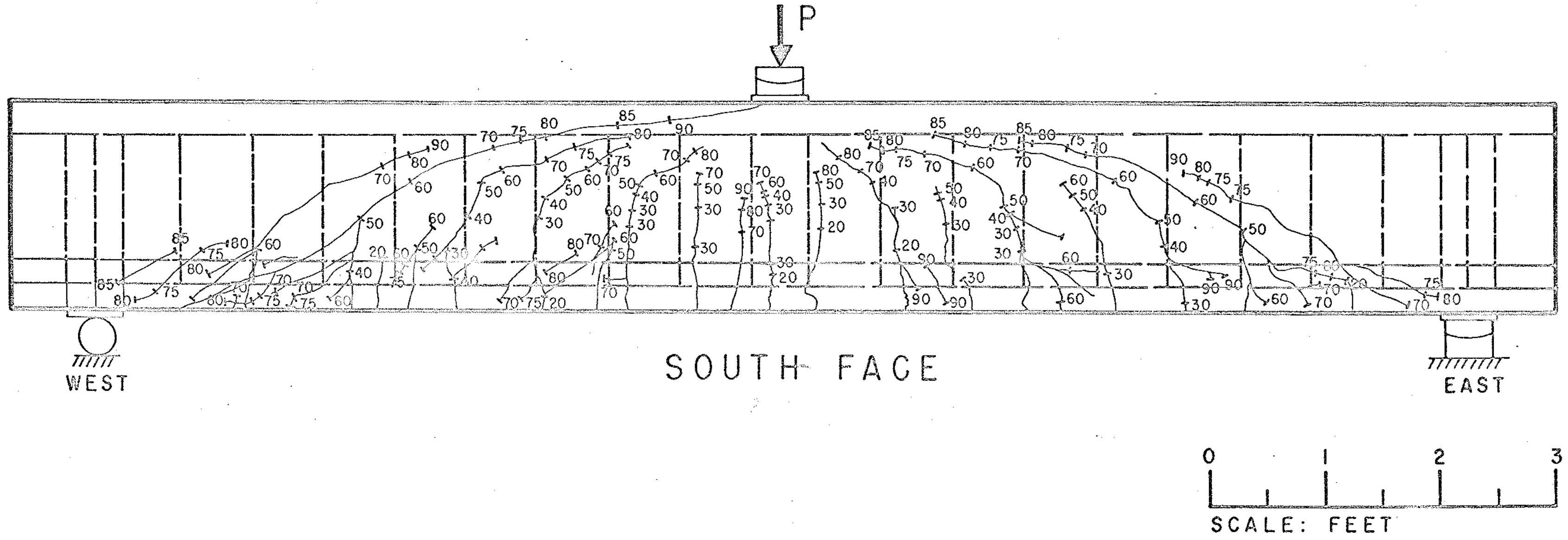


FIG. 8H BEAM RA-I



NORTH FACE



SOUTH FACE

FIG. 81 BEAM RB-1

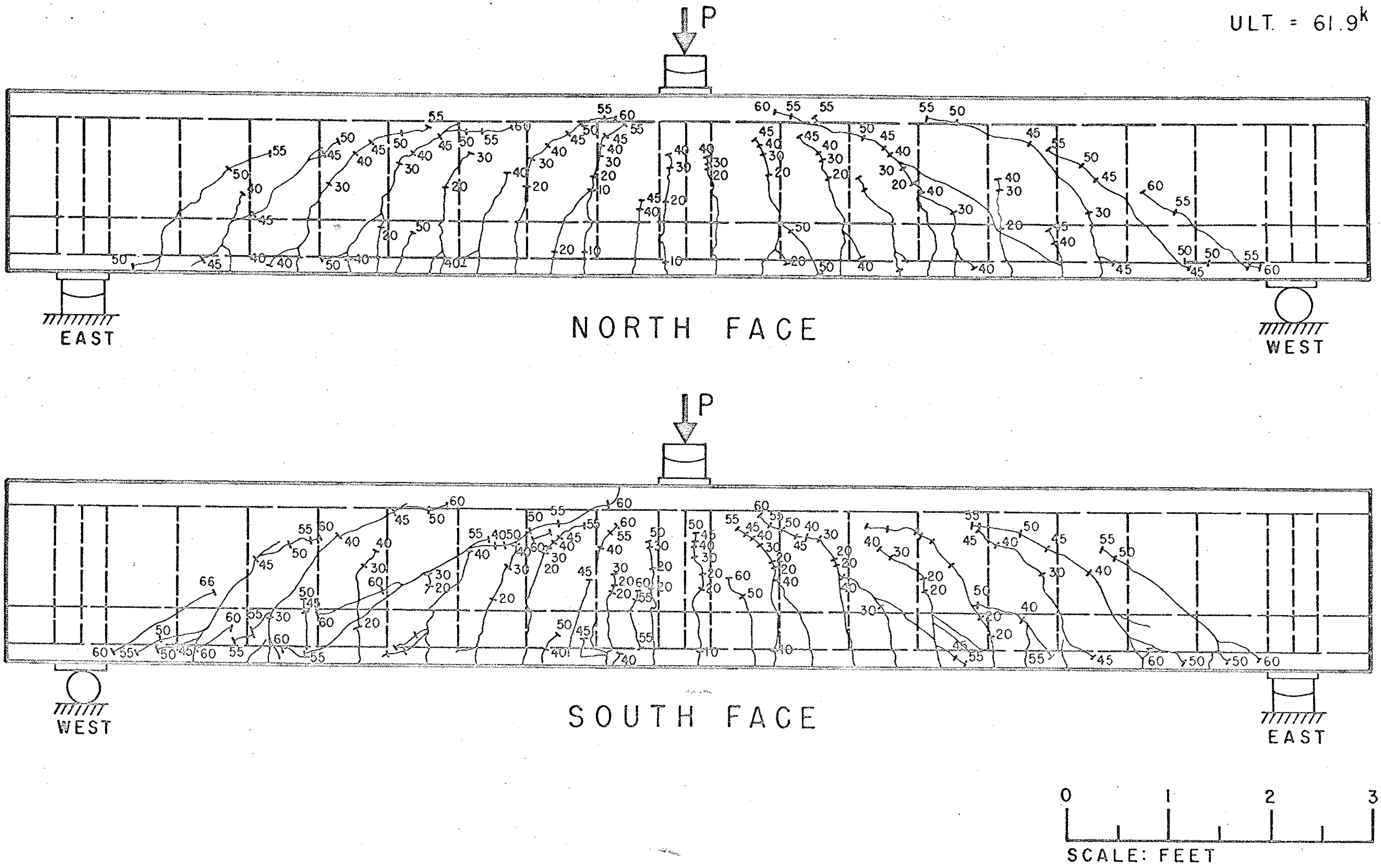


FIG. 8J BEAM RC-1

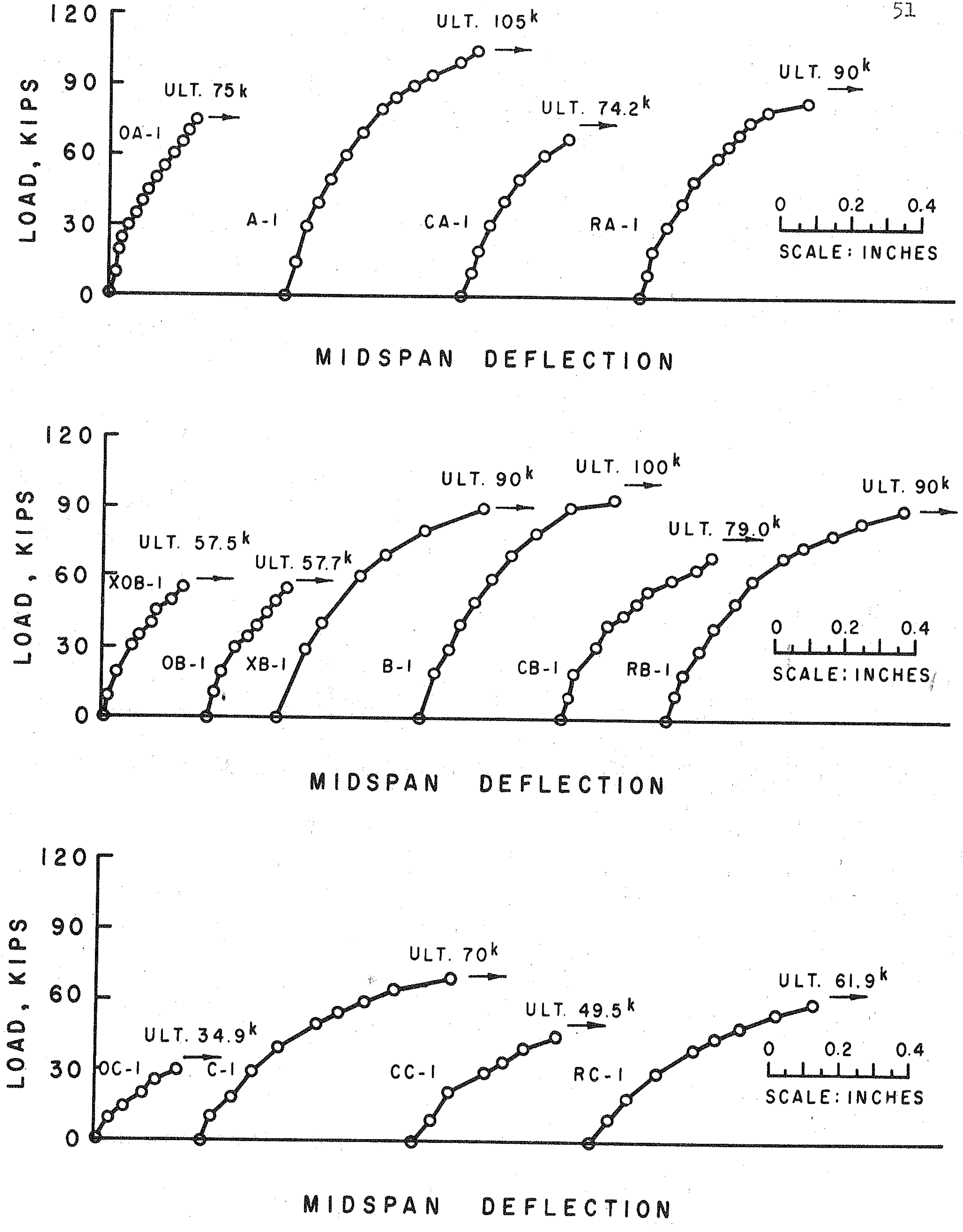
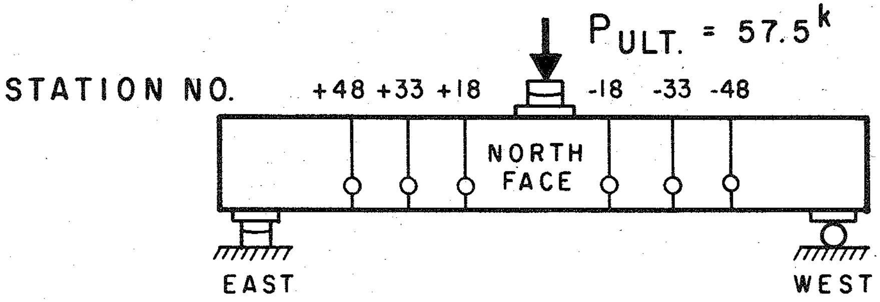


FIG. 9 LOAD-DEFLECTION CURVES



GAGE PLACEMENT

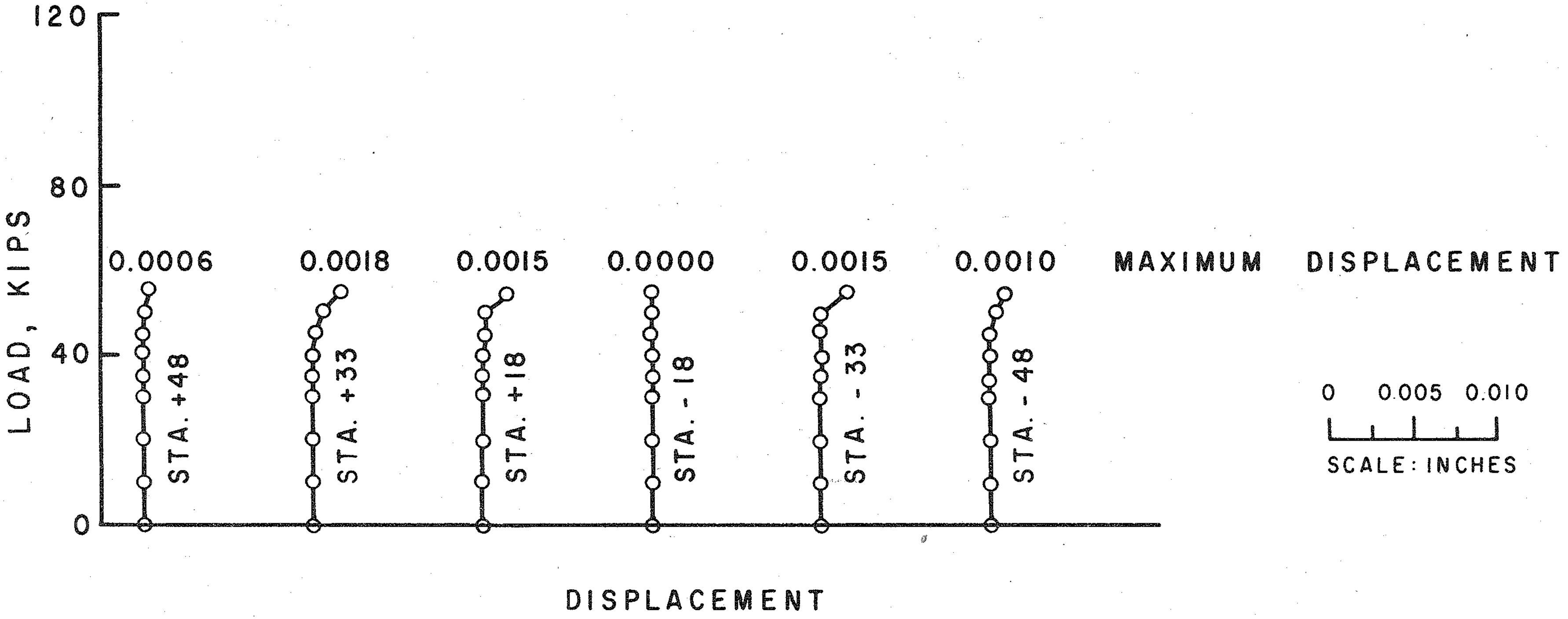
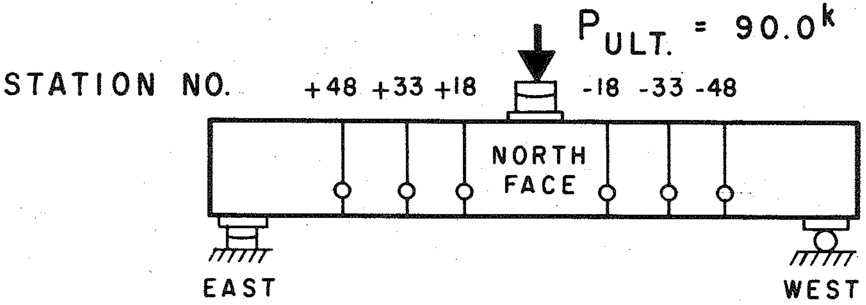
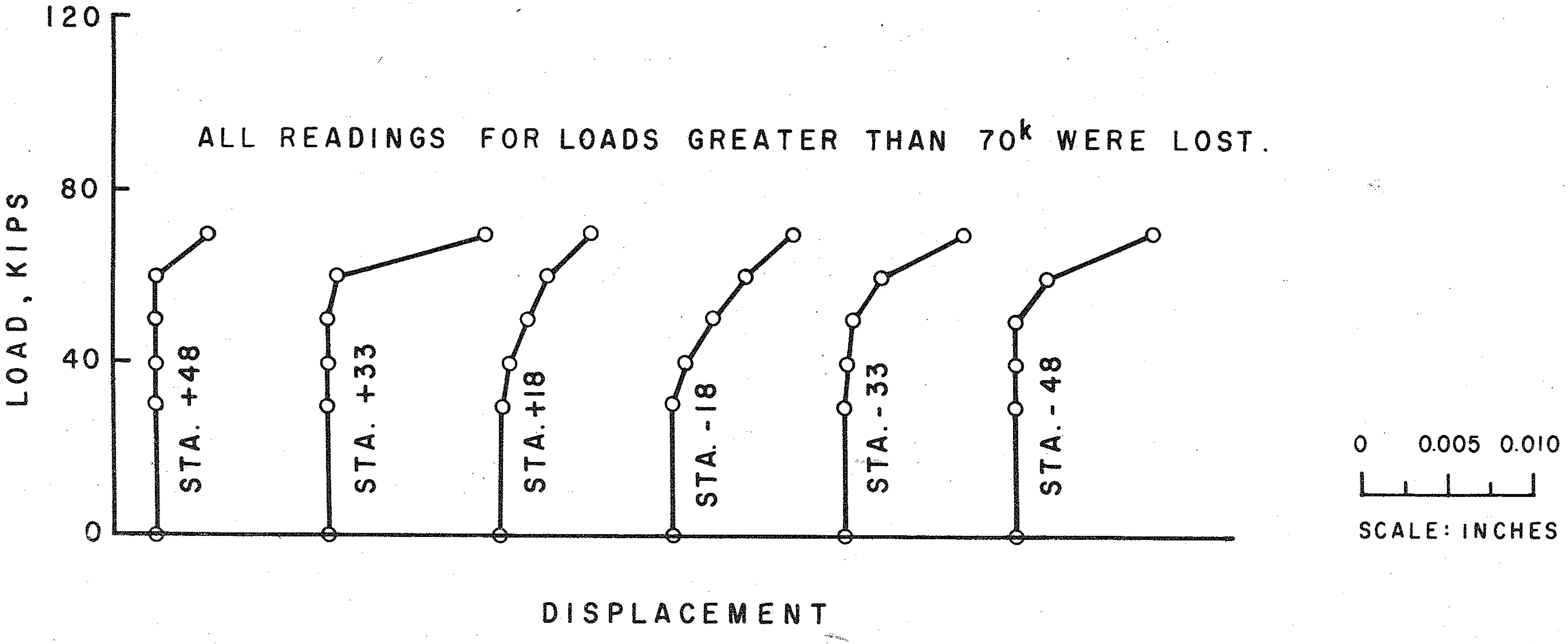


FIG. 10-A YOKE DATA, BEAM XOB-1

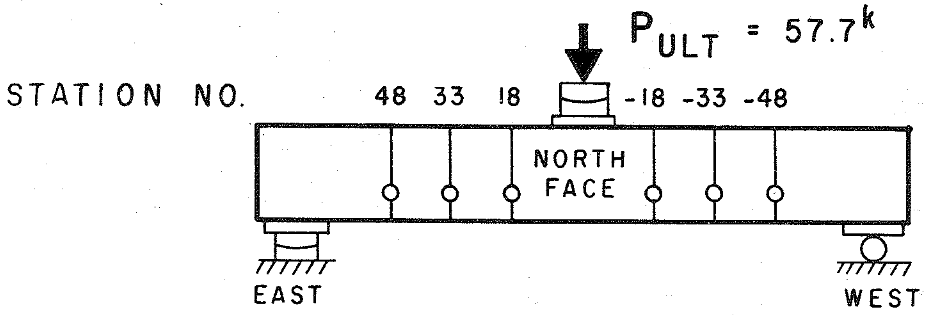


GAGE PLACEMENT



ALL READINGS FOR LOADS GREATER THAN 70^k WERE LOST.

FIG. 10-B YOKE DATA, BEAM XB-1.



GAGE PLACEMENT

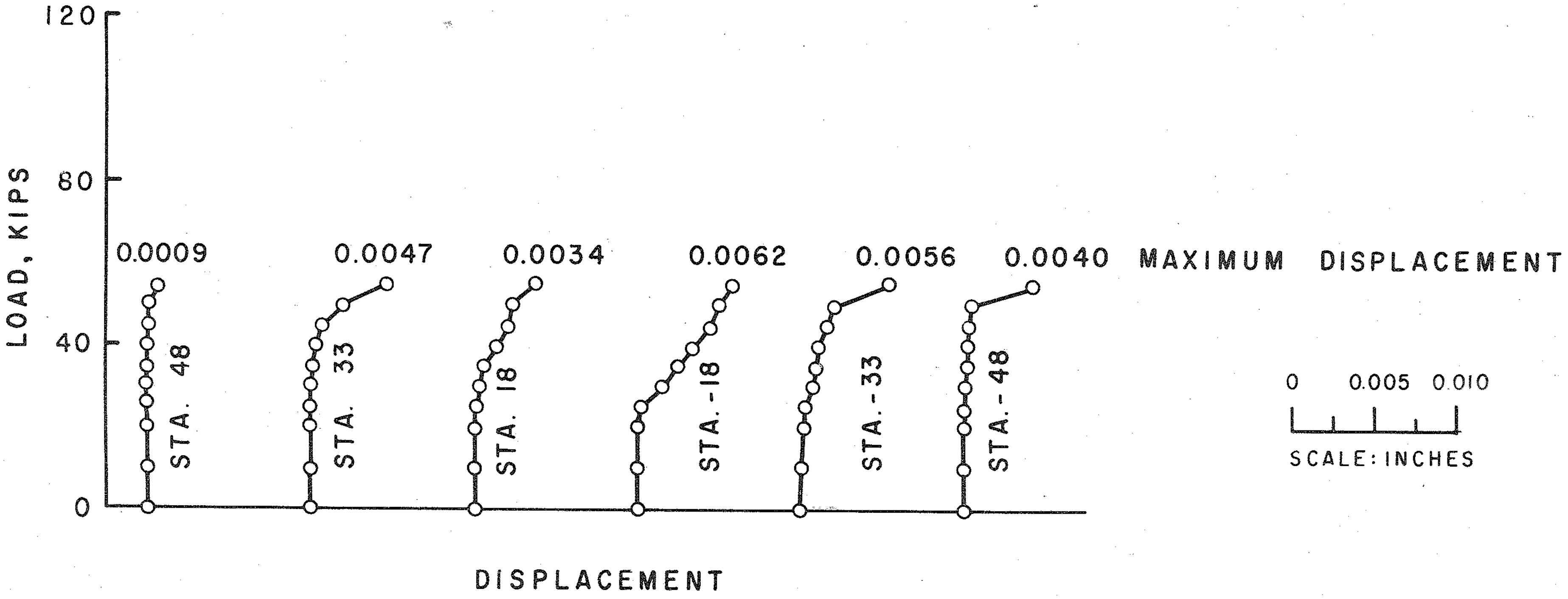
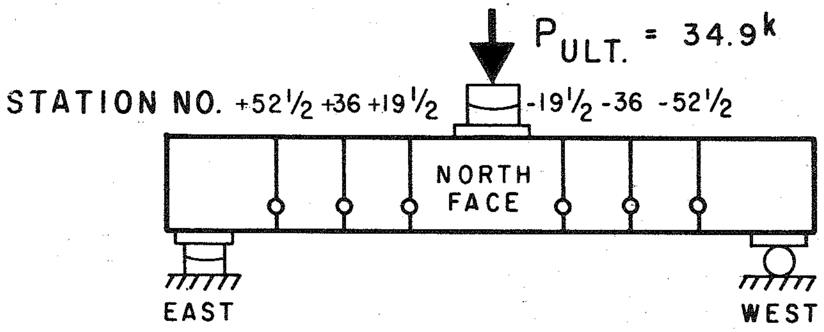
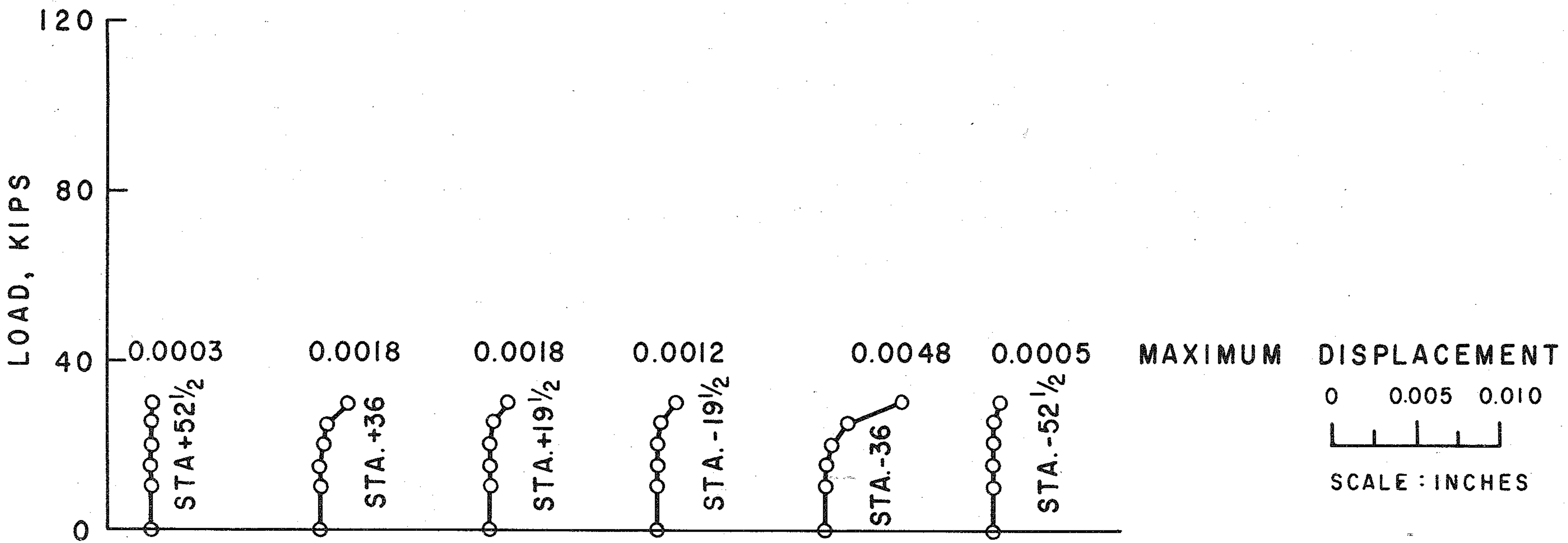


FIG. 10-C YOKE DATA, BEAM OB-1

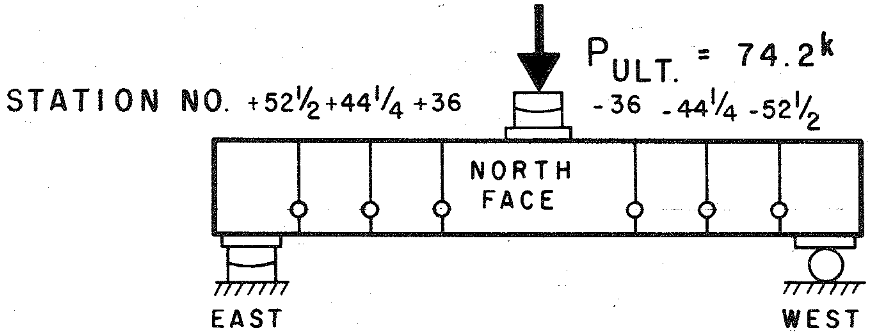


GAGE PLACEMENT



DISPLACEMENTS

FIG. 10-D YOKE DATA, BEAM OC-1



GAGE PLACEMENT

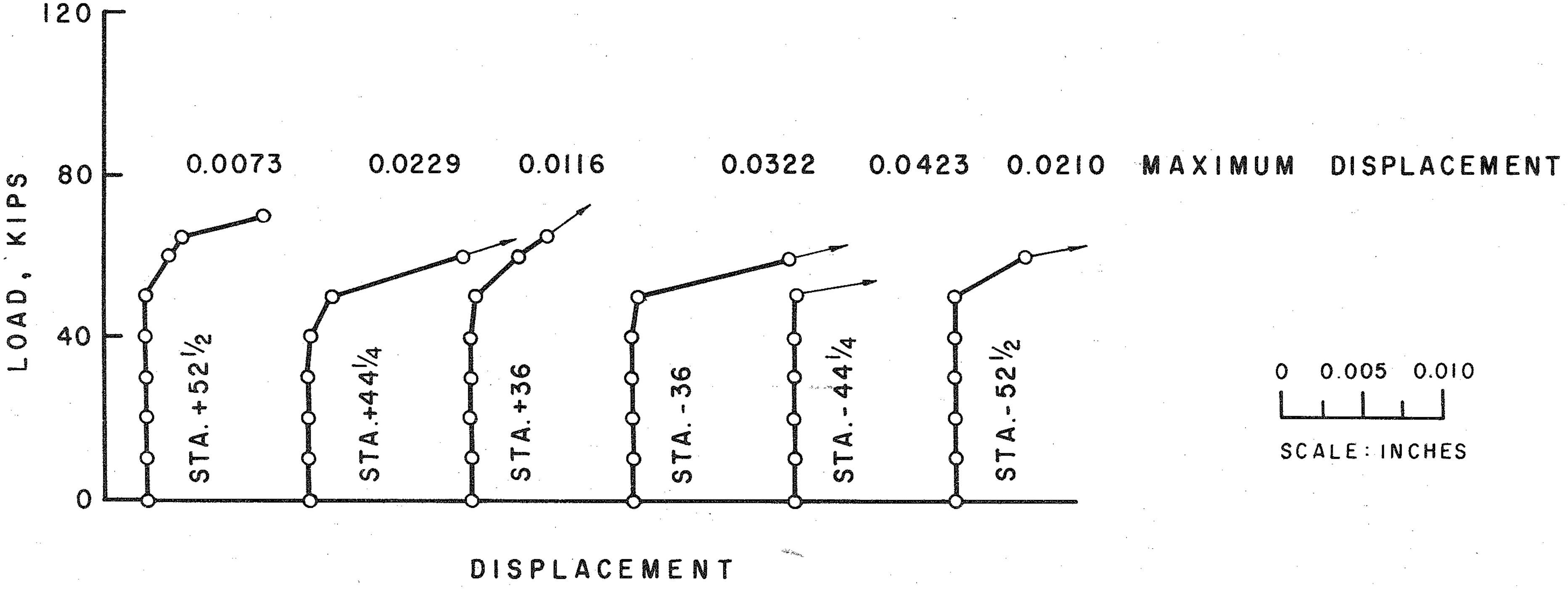
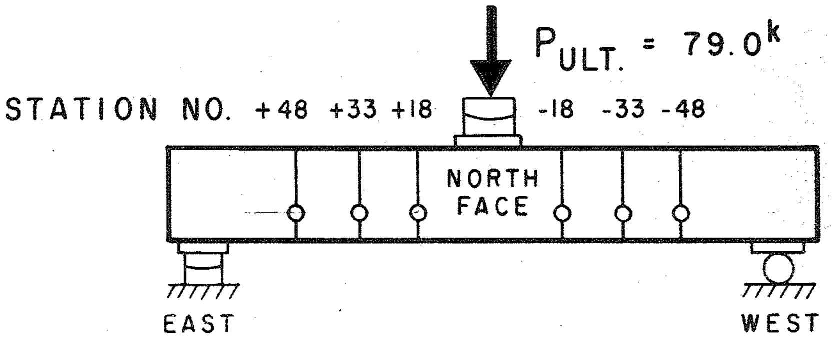


FIG. 10-E YOKE DATA, BEAM CA-1



GAGE PLACEMENT

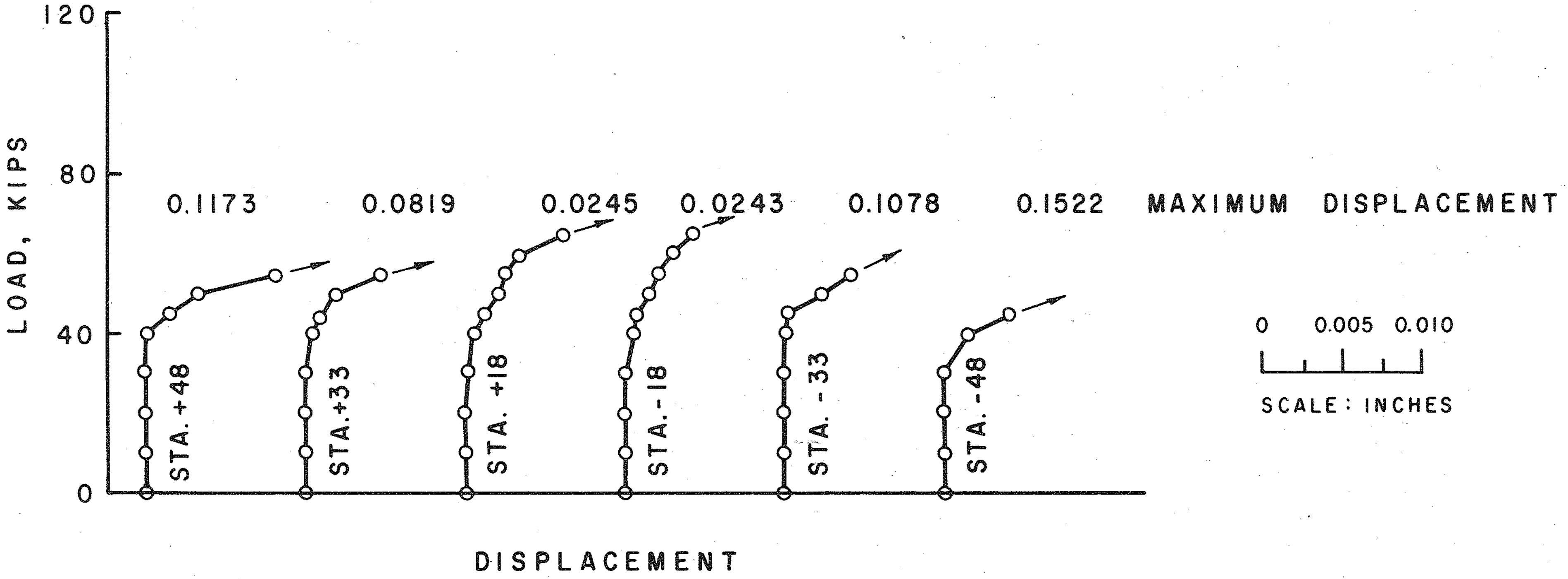
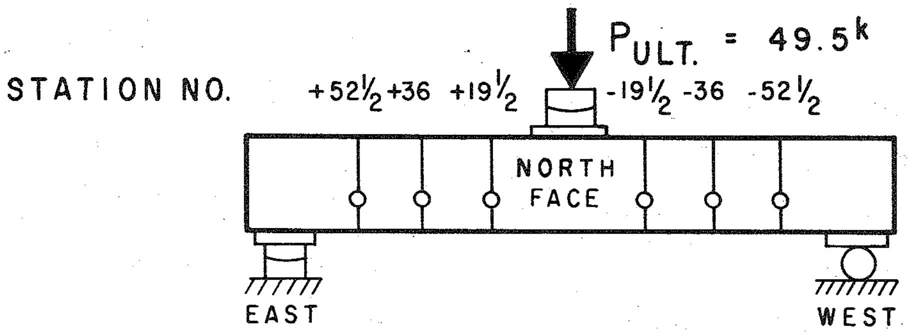


FIG. 10-F YOKE DATA, BEAM CB-1



GAGE PLACEMENT

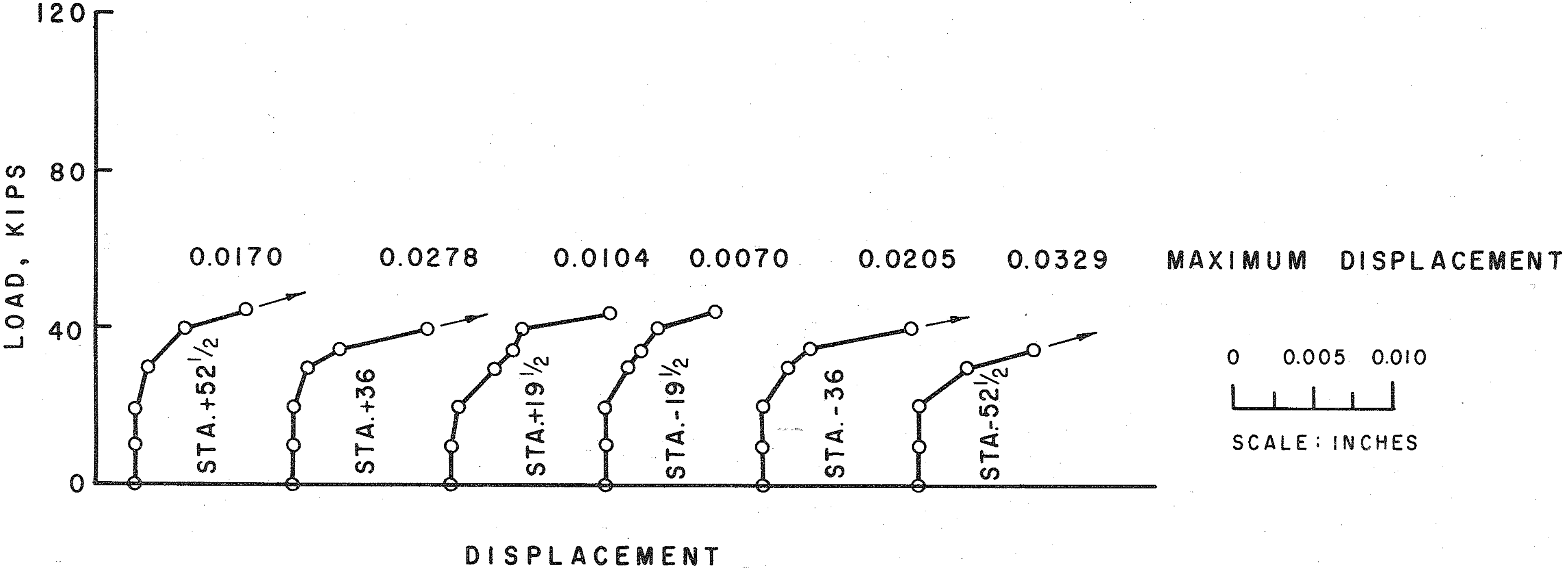
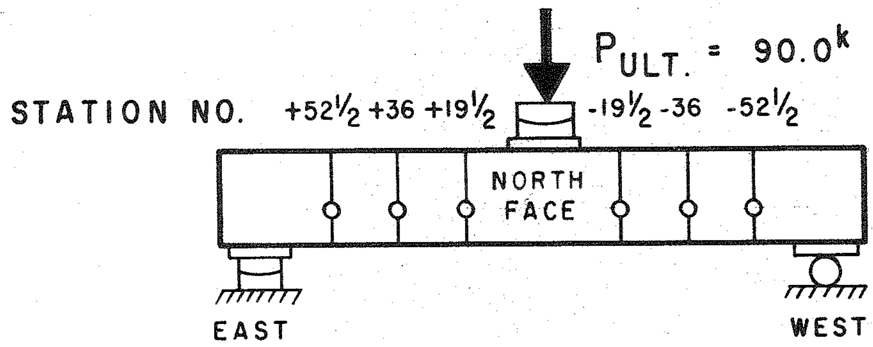


FIG. 10-G YOKE DATA, BEAM CC-1.



GAGE PLACEMENT

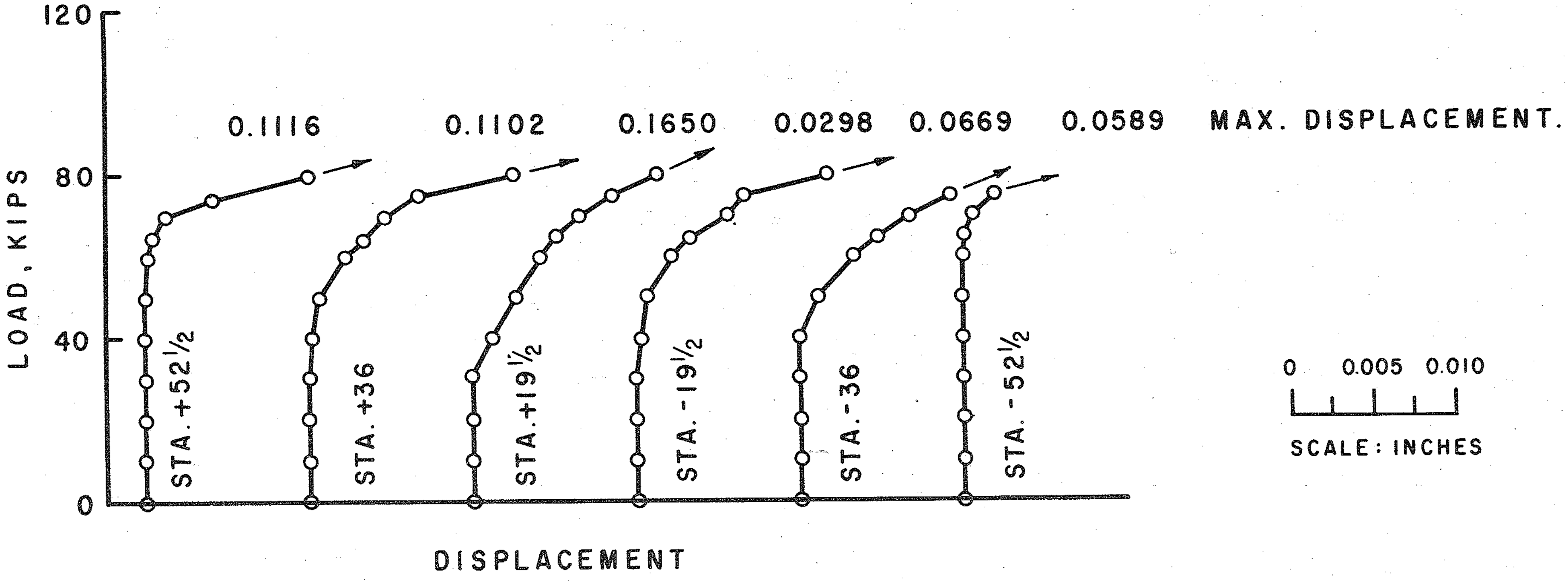
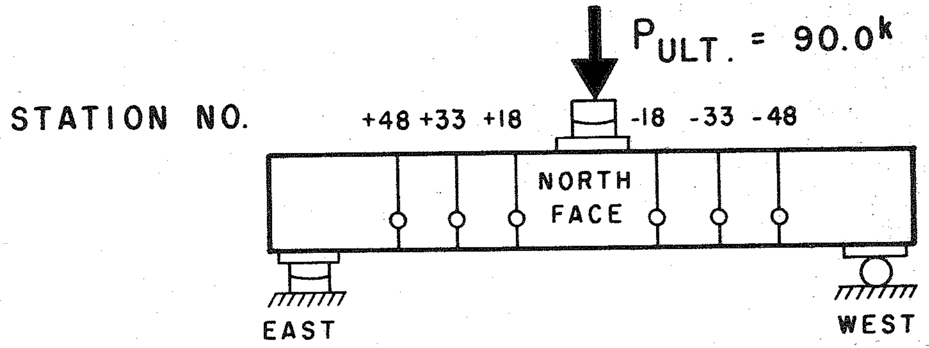


FIG. 10-H YOKE DATA, BEAM RA-I



GAGE PLACEMENT

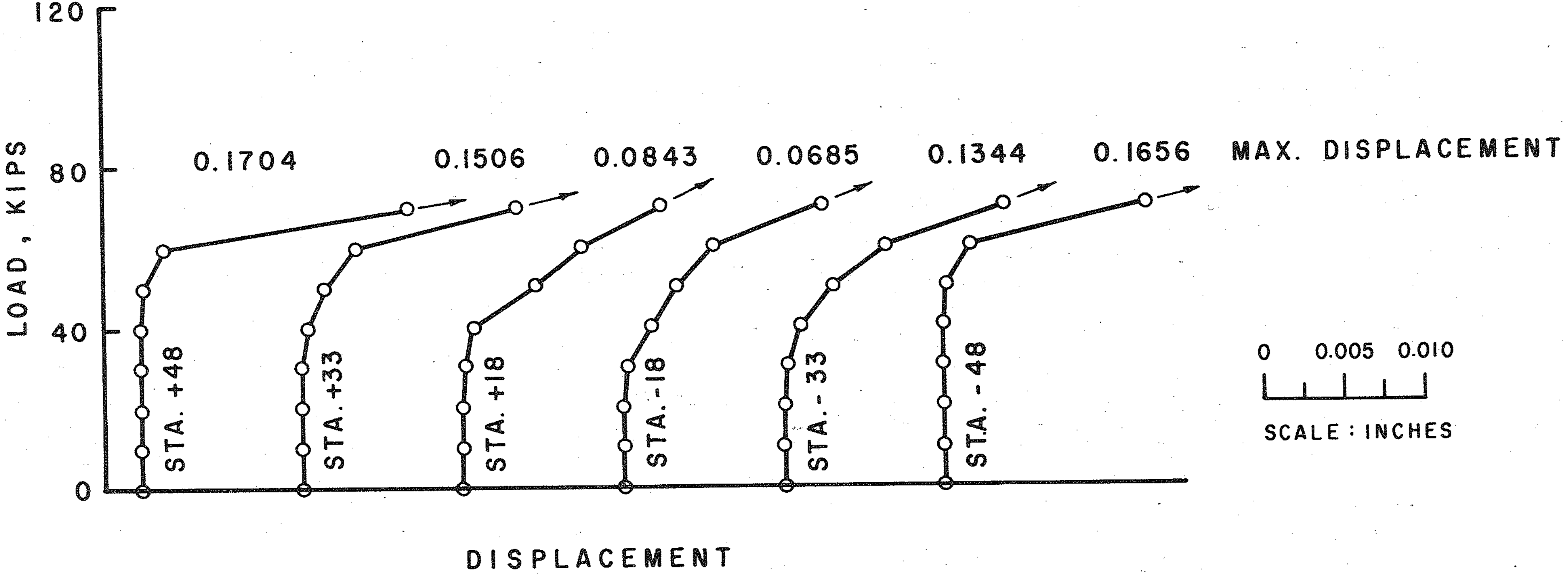
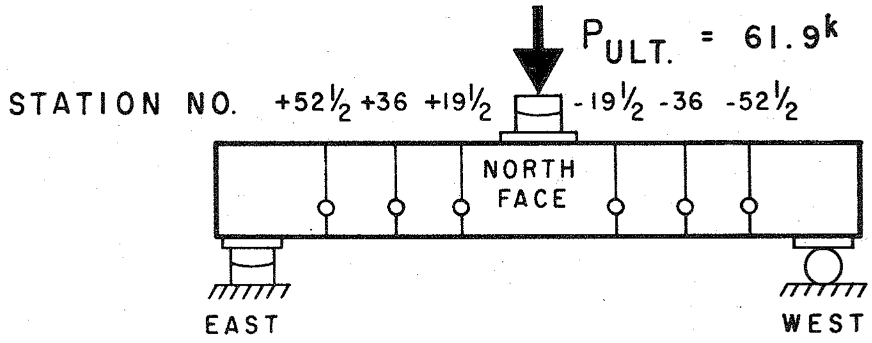


FIG. 10-I YOKE DATA, BEAM RB-1



GAGE PLACEMENT

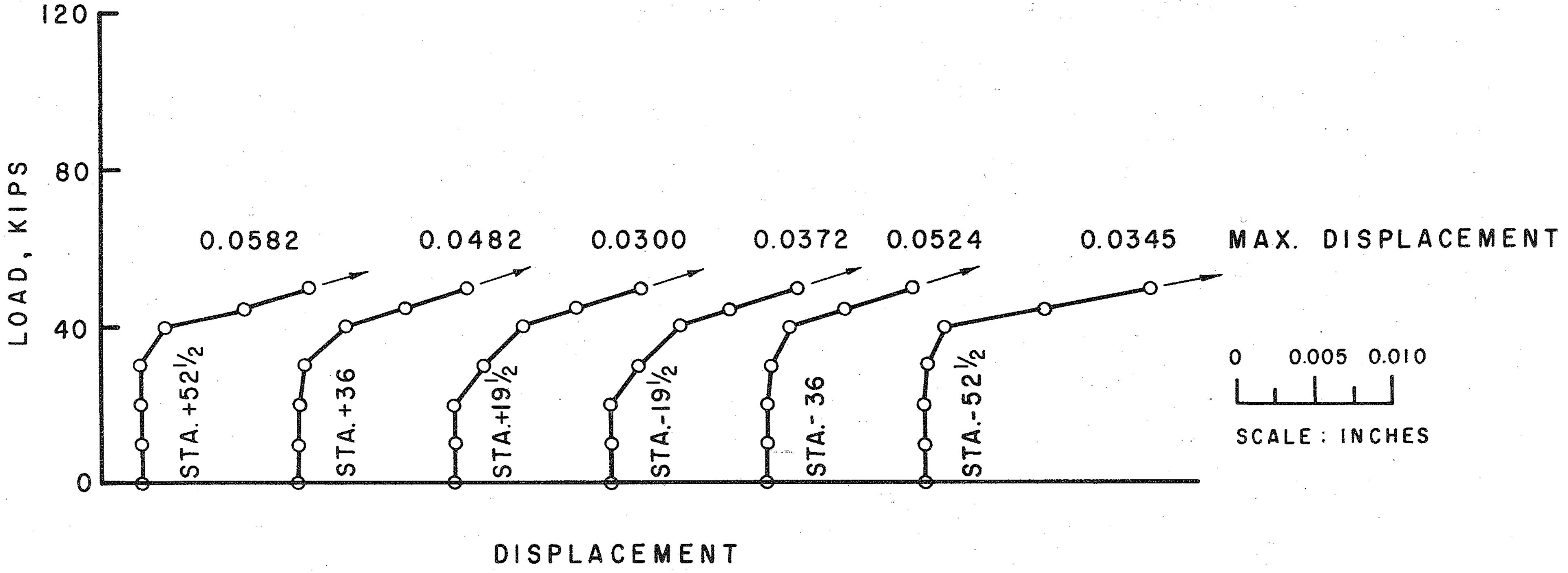
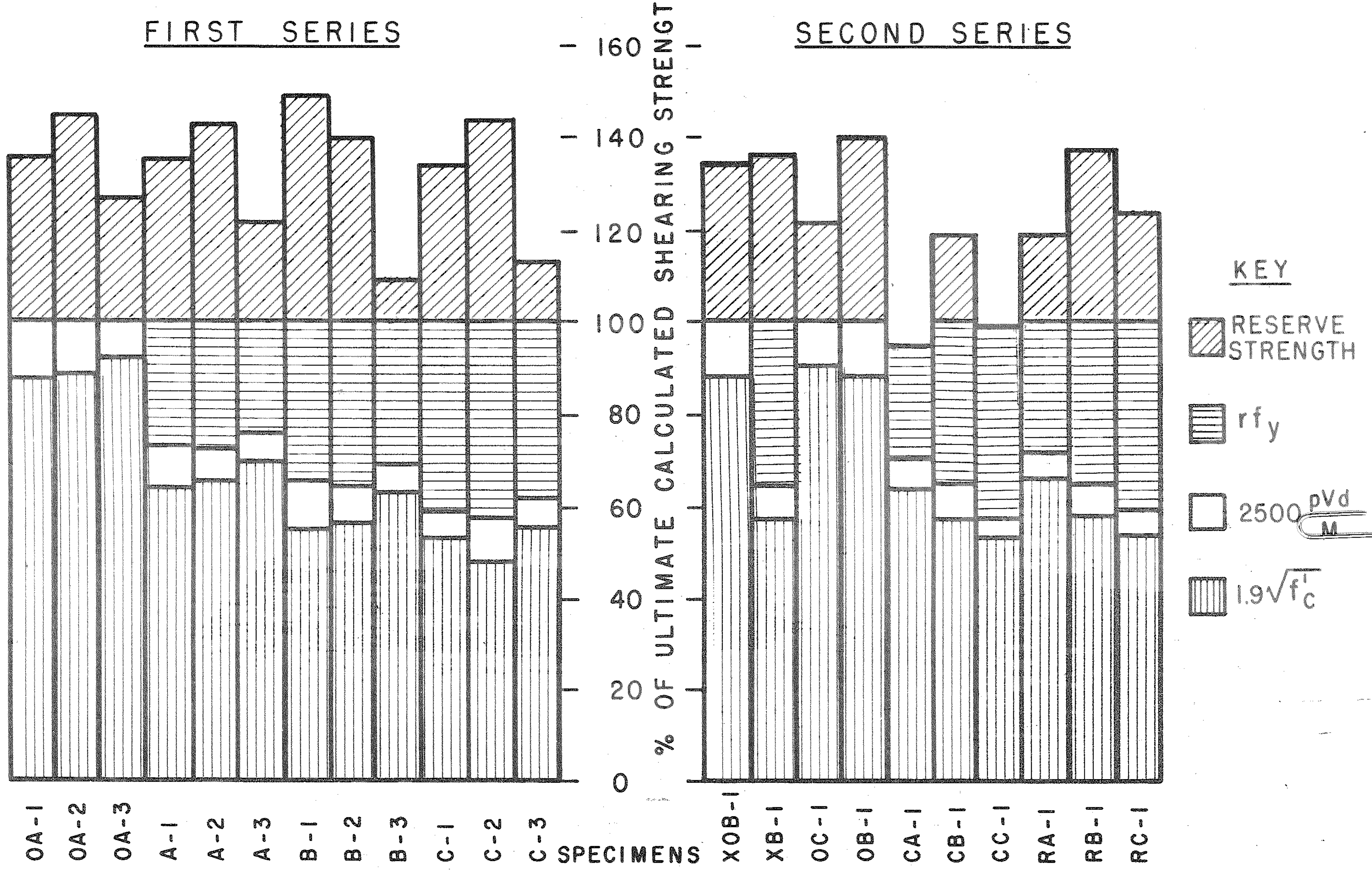


FIG 10-J YOKE DATA, BEAM RC-1



$$V_{CALC.} = bd\left(1.9\sqrt{f'_c} + 2500\,\frac{pVd}{M} + rf_y\right)$$

FIG. 11 COMPARISON OF CALCULATED AND TEST VALUES OF $P_{ULT.}$