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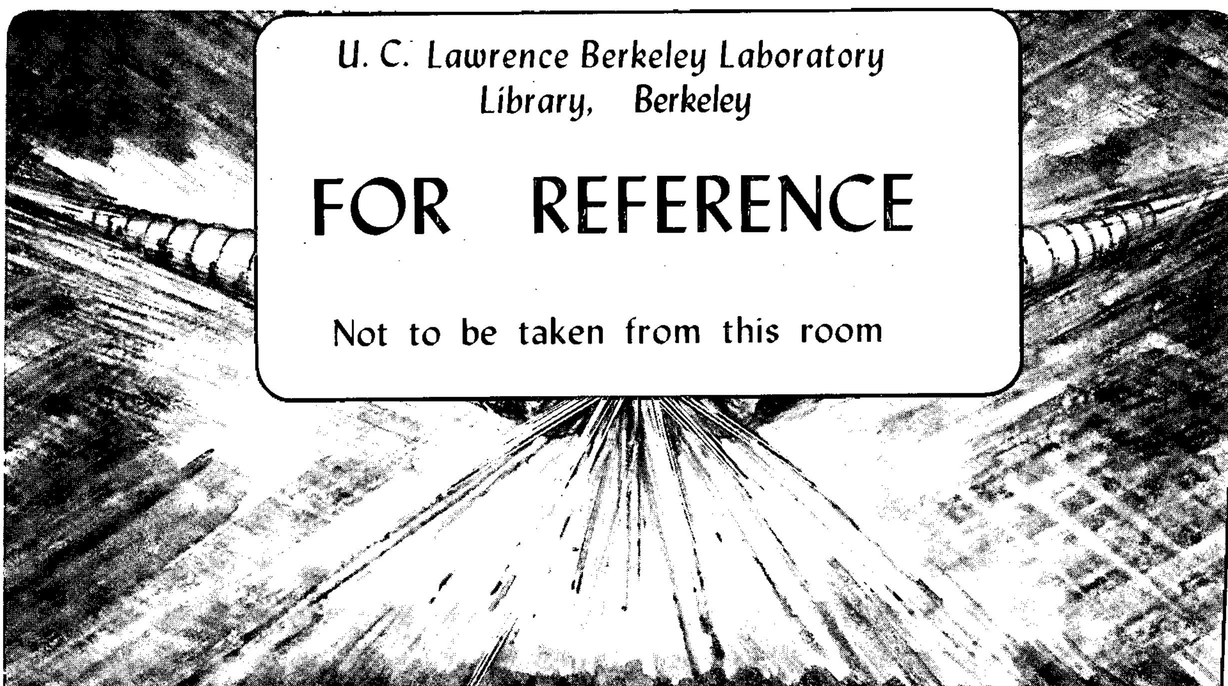

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COHERENT GENERATION OF VISIBLE RADIATION BY RELATIVISTIC ELECTRON BEAMS IN A SOLID SPACE-PERIODIC TARGET*

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COHERENT GENERATION OF VISIBLE RADIATION BY RELATIVISTIC ELECTRON BEAMS IN A SOLID SPACE-PERIODIC TARGET

I. Dubovskaya

Abstract:

The possibility of observing the coherent visible radiation generation by using the particle beam at Lawrence Berkeley Laboratory, which passes through a solid space-periodic target, is considered. The estimations for parameters of such a system and the geometry of a possible experiment is analyzed.

1. Introduction.

Cerenkov radiation, which is characterized by a high intensity and a wide spectrum of radiation, is well-known in optical region of frequencies. The condition of this radiation existence, in a target with a refractive index n_0, is written in the following form:

$$1-\beta\sqrt{\varepsilon_0-\sin^2\theta}=1-\beta n_0\cos\theta=0\,, \tag{1}$$

where ε_0 is the dielectric constant of the medium, θ is the radiation angle, $\beta = u/c$, u is the electron speed. We assume the electron velocity to be directed perpendicular to the target surface and along the Z-axis.

The radiation picture essentially changes if the target is a space-periodic medium which can be an optical grating for radiation emitted by electron beams. Such optical grating can be formed, for example, by a periodic set of plates with different refractive indices which are cemented together, or by a homogeneous medium with a refractive index modulated by the holography method. In this case, we can represent the refractive index as $n = n_0 + \eta n_0 = n_0 + \Delta n$, where $\eta = \Delta n/n_0$ is the modulation depth, $\Delta n = a\sin\left(\vec{K}_\xi\vec{\xi}\right)$, a is the modulation amplitude, $K_\xi = 2\pi/d$, d is the modulation period in the direction of a vector $\vec{\xi}$. Obviously, it is possible to construct, in such a way, the periodic structure with the reciprocal lattice vector directed in any direction.

In [1] the optical radiation by particles uniformly moving through infinite medium, formed by a periodic set of plates with different refractive indices and placed perpendicular to the particle velocity, was considered. It was shown that Cerenkov radiation, in this situation, essentially modifies and the authors referred to this new radiation as parametric radiation. The possibility of parametric (quasi-Cerenkov) radiation in the X-ray region was predicted in [2] and experimentally confirmed in [3]. The detailed investigation of optical spontaneous parametric (quasi-Cerenkov) radiation was performed in [4,5] for liquid crystals and in [6] for arbitrary finite space-periodic target. It was obtained that the spectral-angular characteristics of radiation modify when diffraction conditions are fulfilled for photons emitted by a particle uniformly moving through a periodic medium. Near the Bragg condition the dielectric properties of the medium change and, as a consequence, the formation condition of Cerenkov radiation also changes. In [6] the characteristics of parametric (quasi-Cerenkov) radiation were considered in the framework of dynamical diffraction for the case of the excitation of two strong waves with the wave vectors $\vec{K}$ and $\vec{K}_\tau = \vec{K} + \vec{\tau}$, where $\vec{\tau}$ is the reciprocal lattice vector describing the optical grating. It was obtained that the analogy of Cerenkov condition for a periodic medium can be written in the form [2]:

$$1 - \beta \cos\theta - \beta \delta_{1,2}(\alpha(\omega)) = 0 \qquad (2)$$

where $\delta_{1,2}(\alpha(\omega))$ are the magnitudes of dielectric susceptibility of a medium under diffraction. These magnitudes depend on the deviation from the exact Bragg condition $\alpha(\omega) = \left[2\vec{\tau} \cdot \vec{K} + \tau^2\right]/K^2$. Obviously, the medium is now characterized by two refractive indices and, consequently, by two thresholds of quasi-Cerenkov radiation. It means that the radiation distribution, in this case, is characterized by two Cerenkov angles and looks like two rings. The difference between these two angles can be represented as:

$$\Delta = \cos\theta_1 - \cos\theta_2 \cong \frac{2}{\beta}\sqrt{\left[g_0\left(1-\frac{1}{\beta_1}\right)+\frac{\alpha}{\beta_1}\right]^2 + 4\frac{g_\tau^2}{\beta_1}} \qquad (3)$$

where $\varepsilon_0 = 1 + g_0$ is the mean dielectric constant of the medium, β_1 is the asymmetry factor of diffraction $\beta_1 = \gamma_1/\gamma_0$, $\gamma_0 = \cos(\vec{K}\wedge\vec{n})$ and $\gamma_1 = \cos(\vec{K}_\tau \wedge \vec{n})$, $\vec{n}$ is the normal to a target surface directed inside the target, g_0, g_τ are defined by a series expansion of dielectric susceptibility in terms of the reciprocal lattice vectors $\vec{\tau}$, $\vec{\tau}$ characterizes a given diffraction grating, $\beta = u/c$.

Besides two Cerenkov rings along the particle movement direction, an additional diffraction Cerenkov cone appears. The center of this cone is directed along the vector $\omega_B \vec{n} + \vec{\tau}$ and, in its turn, consists of two rings as well, ω_B is the Bragg frequency. As the frequency spectrum of radiation is now restricted by diffraction condition, $|\alpha| \le g_0$ the quasi-Cerenkov radiation spectrum is not wide but is characterized by a spectral width determined by the diffraction conditions. It is known that in the case of Bragg diffraction ($\beta_1 < 0$) the degeneration of the roots of the dispersion diffraction equation is possible near the edges of absorption region [7]. According to (3) the degeneration takes place at $\alpha_\pm = g_0 + g_0|\beta_1| \pm 2|g_\tau|\sqrt{|\beta_1|}$. At given parameters of a target and diffraction geometry, i.e. the magnitude of β_1, two radiation angles correspond to the degeneration condition:

$$\left(\theta_{ef}^{\pm}\right)^2 = \left[g_0' - \gamma^{-2} \pm |g_\tau|/\sqrt{-\beta_1}\right], \; g_0' \equiv \mathrm{Re}\, g_0 . \qquad (4)$$

According to (4), the difference between these two angles $\theta^\pm$ can be significant, especially under asymmetric diffraction ($|\beta_1| << 1$).

So, the periodic medium, under diffraction condition, not only forms the condition of Cerenkov radiation formation but also is a resonator for radiation in this region. It is therefore possibile to apply the spontaneous quasi-Cerenkov radiation for the production of coherent optical radiation generated by a relativistic electron beam passing through a space-periodic medium.

In the papers [8-10] devoted to the consideration of the coherent X-ray generation by intensive relativistic particle beams in natural three-dimensional periodic media (crystals), it was pointed out to the possibility of generation of coherent optical radiation on the basis of spontaneous quasi-Cerenkov radiation using artificial periodic targets. The estimations for threshold current density of a particle beam, which is necessary to achieve the generation regime, were given both for the X-ray and optical regions of spectrum. It is very important that the periodic medium, being a resonator, forms a distributed feedback. This allows to reduce considerably the current density of the particle beam and to decrease the size of a target which is required for the realization of generation regime. From analysis performed in [8-10], it follows that the most effective interaction between the particle beam and the radiation takes place near the point of the degeneration of dispersion equation roots. At this condition, the equations for generation threshold were obtained for various regimes of X-ray gain (weak and high gain regimes) and for "cold" and "hot" particle beams.

2. Generation Regime for Visible Radiation FEL.

The purpose of this report is to analyze the possibility of realization of the optical radiation generation by existing particle beams moving through a periodic solid target. By using the results of [10] we will give the estimations and corresponding requirements for the main parameters of such possible system.

In the X-ray region, the requirement of the coincidence of Cerenkov synchronism with degeneration of dispersion equation roots leads to the strong restriction for possible diffraction geometry, due to $g_0' < 0$. In the visible region of the spectrum there are no such restrictions and the degeneration of roots and Cerenkov synchronism can be simultaneously realized at definite angles under the arbitrary geometry of Bragg diffraction.

2.1. Choice of experimental geometry.

So let the relativistic electron beam with the following parameters ($E = 56$ MeV, $\varepsilon_n =$ 20 mm mrad, $I = 100$ A in a peak, r (beam radius) = 0.5-0.2 mm) be incident on a solid plane-parallel target with the length of L. Here ε_n is the normalized emittance, r is the beam radius. In the general case, while considering the generation in an optical range of frequencies we should take into account the waves reflected by target boundaries, because of a large magnitude of a refractive index, and the possibility of the beam synchronism with diffracted wave. However, if we restrict ourselves the consideration of the medium with dielectric susceptibility which is 5-10 times less than the unity, all parameters, which has been used for the expansion in a series or for approximations in the theory of X-ray generation, will be true also in the optical region of frequencies. New features are the possibility of generation at two radiation angles $\theta^{\pm}$ and coincidence of Cerenkov synchronism with degeneration of roots under arbitrary geometry of diffraction. Taking into account these circumstances, let us consider a possible geometry of generation process and the parameters of such system. It should be noted that the gain of quasi-Cerenkov radiation strongly depends on the beam quality, as in the case of the ordinary FEL, on the basis of undulators. It was shown in [10], that the generation condition for the "cold" particle beam, when the angular spread of particles in a beam can be neglected, essentially distinguish from the generation condition of a "hot" electron beam. A particle beam can be considered as "cold" if the following inequality is fulfilled $\theta(\psi_{\perp} + \psi_{||}) << \Gamma$, where θ is the radiation angle, $\psi_{\perp}$ and $\psi_{||}$ are the transverse and longitudinal angular divergence of electrons in a beam $(\psi_{||} = \Delta v_{||}/v = \Delta\varepsilon/\varepsilon\gamma^3)$, Γ is the quantity characterizing the width of the gain curve. Obviously, in the case of a solid target, the multiple scattering of electrons by atoms and nuclei of a target strongly affects the particle beam quality and transforms even the initially "cold" beam to the "hot" one. This, in its turn, destroys the Cerenkov synchronism condition and, consequently, the generation condition. This influence is the strongest for electron beams with moderate energies, because the mean square angle of multiple scattering is inversely proportional to the particle energy, i.e. $\overline{\theta_s^2} = E_s^2/E^2 \cdot L/L_r$, where $E_s = 21$ meV, L is the target length, L_r is the radiation length of an electron in the medium. It can be easily seen that $\overline{\theta_s} = \sqrt{\overline{\theta_s^2}} = 1.2 \cdot 10^{-1}$ rad at $\gamma = 10^2$ and $L_r = 10$ cm even at the target length of $L \sim 10^{-2}$ cm. The reduction of multiple scattering is possible, for example, by decreasing the target length.

But in this case the quality of the radiation resonator deteriorates. That leads, in its turn, to the sharp increase of the current density of the particle beam which is necessary to the achievement of the generation regime. The reduction of multiple scattering is also possible by using particle beams with higher energies.

In order to make possible the application of particle beams with moderate energies for the generation process inside a solid target, let us recall that the process of emission photon with a wave vector $\vec{K}$ and the wave length λ by a particle with the energy γ, moving at a distance of $\rho \leq \lambda\gamma$ from the target boundary, is developing as the same process in the medium and is characterized by the dielectric constant of this medium. It is well-known, for example, the possibility of Cerenkov radiation by a particle moving over the target surface at a distance of $\rho \leq \lambda\gamma$ [11]. Consequently, if the radiation of photons takes place inside a channel of the diameter of $\rho = \lambda\gamma$, which is made in a solid target, the photons will perceive such target as an ordinary homogeneous medium with the corresponding dielectric constant. As a result, the diffraction process will also be the same one as inside an ordinary solid target. On the other side, the multiple scattering of a particle beam moving inside such a channel will be suppressed. For example, at $\gamma = 1.12 \cdot 10^2$ and $\lambda = 6000$ Å , the size of the channel is about $\rho \approx 67$ μm. Under the generation of the softer radiation the requirement to the channel size is not so rigid, but, for example, in the infrared region of spectrum, the absorption of photons inside the solid target becomes very strong. This leads to the decrease of the possible target length and, as a consequence, to the deterioration of conditions for threshold generation. These circumstances make the process of the generation of infrared radiation in a solid target impossible because of its strong absorption inside the medium. These facts restrict the wave length of the possible process under consideration by a soft part of visible radiation, that is the radiation with rather large wave length and the weak absorption in the medium.

For the choice of the optimal geometry of the example under consideration, it is important that the process of radiation amplification and, consequently, the generation threshold are asymmetric relative to the azimuthal angle on the plane perpendicular to the particle motion direction. The threshold condition for generation in the case of weak amplification is given by the following equation [10]:

$$-\frac{\pi^2 n^2}{4\gamma}\left(\frac{\omega_L}{\omega}\right)^2 K^2 L^2 \theta_{ef}^2\left(\theta_{ef}^2+\gamma^{-2}\right)\sin^2\varphi\, F(y) = G =$$

$$=\left(\frac{\gamma_0}{\beta}\right)^3 \frac{16\left(-\beta_1\right)\pi^2 n^2}{(K|g_d|L)^2\, KL} + \varepsilon_0''\left(\frac{1}{\sqrt{-\beta_1}}-1\right)^2, \qquad (5)$$

$$F(y) = \sin y\,\frac{(2y+\pi n)\sin y - y\,(y+\pi n)\cos y}{y^3\,(y+\pi n)^3},$$

where $\omega_L^2 = 4\pi n_0 e^2/m_e$, n_0 is the mean density of electrons in a beam, n = 1,2,... is a number of radiation harmonic, ε_0'' is the imaginary part of the mean dielectric constant of the medium, which determines the radiation absorption inside the target, θ_{ef} is the effective angle of radiation, y is expressed through the root of dispersion equation (see [10]). The function F(y) determines the width of the amplification spectral line. The gain coefficient G is defined such that the single pass field amplification is given by exp (KGL).

According to Eq. 5 (this the formula (27) from [10]), which determines the generation condition for the "cold" particle beam, the gain is proportional to $\sin^2\varphi$, where φ is the angle between the projections of a photon and the reciprocal lattice vector on the plane perpendicular to the particle velocity $\varphi = (\overrightarrow{K_\perp} \wedge \overrightarrow{\tau_\perp})$. See Fig. (1). Obviously, if we choose the reciprocal lattice vector, characterizing a set of diffraction planes, on the plane parallel to the XZ plane, the photon with $\overrightarrow{K_\perp} \perp \overrightarrow{\tau_\perp}$, i.e. with wave vector $\vec{K}$ (0, K_y, K_z) will be amplified the most effectively. It means that it is convenient to have a particle beam with minimum angular spread just in the direction of the photon location, i.e. in the Y-direction. In this case such particle beam can be considered as a "cold" beam for this radiation. As a consequence, it is better to form a particle beam as a "slit" along the Y-direction. The transverse size of such beam can be determined from the requirement of conservation of the current density of the particle beam. On the other side, the size along the X-axis, it is better to make of the order of magnitude of the size of the channel $\rho = \lambda\gamma$. The analysis shows that, in this geometry of diffraction, it is possible to transfer from a channel with $\rho \sim \lambda\gamma$ to the slit also along the Y-axis. The transverse size of this slit along the X-axis is $\Delta_x \sim \rho$. In this case the effectiveness of the application of the particle beam will be high. The beam size of the Y-direction, in this case, is $\Delta y = \pi r^2/\gamma\lambda$. It means that the beam with the parameters, mentioned above, is characterized by the following angular and dimensional sizes: $\Delta\psi_{xI} = 2.9\cdot 10^{-3}$ rad, $\Delta\psi_{yI} = 1.8\cdot 10^{-5}$ rad and $\Delta y_I = 1.1$ cm at $r_I = 0.5$ mm and $\Delta\psi_{xII} = 2.9\cdot 10^{-3}$ rad, $\Delta\psi_{yII} = 1.1\cdot 10^{-4}$ rad and $\Delta y_{II} = 1.8$ mm at $r_{II} = 0.2$ mm.

In this case the direction of the most effective development of gain process corresponds to the direction of the minimum angular spread of the particle beam. This allows to consider the existing particle beam as a "cold" one. In the perpendicular direction the electron beam has a small size and a large angular spread, but the process of radiation amplification is suppressed in this direction.

As the reciprocal lattice vector $\vec{\tau}$ is now on the plane parallel to the XZ-plane, i.e. $\vec{\tau} = (\tau_x, 0, \tau_z)$, the reflection planes of the artificial grating, corresponding to this reciprocal lattice vector, are parallel to the Y-axis and inclined at a definite angle ϕ relative to the ZY-plane. The magnitude of the angle ϕ will be determined in the future. The analysis of diffraction by such grating shows that, in this case, the diffraction and, consequently, the distributed feedback, will be effective even in the presence of a slit along the Y-axis.

Thus, the geometry of the experiment in observation of the optical coherent radiation generation by a relativistic particle beam uniformly moving through a solid target with the corresponding optical grating can be represented as in Fig.1.

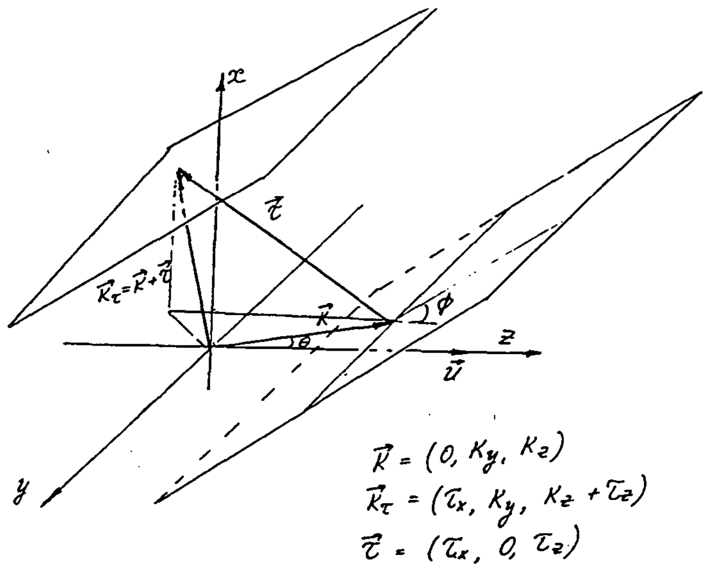

Fig.1

2.2. Parameters and requirements of possible experimental system.

Let us now estimate the parameters of the considered system in various situations. In the case of the generation of optical radiation and by taking into account two possible effective radiation angles (see (4)) we can rewrite the equation of the generation threshold (5) in the following form:

$$-\frac{\pi^2 n^2}{4\gamma}\left(\frac{\omega_L}{\omega}\right)^2 (KL)^2 \left(g_0' - \gamma^{-2} \pm \frac{|g_\tau|}{\sqrt{-\beta_1}}\right)\left(g_0' \pm \frac{|g_\tau|}{\sqrt{-\beta_1}}\right)\sin^2\varphi F(y) = G =$$

$$= \left(\frac{\gamma_0}{\beta}\right)^3 \frac{16(-\beta_1)\pi^2 n^2}{(K|g_\tau|L)^2 KL} + \varepsilon_0''\left(1 + \frac{1}{|\beta_1|}\right). \tag{6}$$

Here we also take into account that in the considered situation the inequality of $g_\tau'' \ll g_0''$ is fulfilled, because the depth of the refractive index modulation is assumed to be 1% or 2%.

Restrict ourselves to the consideration of the generation of the first radiation harmonic only, i.e. n = 1, with the wave length, for example, $\lambda = 6000$ Å, $K = 10^5$ cm^{-1}. The length of a target is chosen more than the diffraction extinction length $L > L_{exc} = (\lambda|g_\tau||\beta_1|)^{-1}$, in order to supply the effective distributed feedback. For this photon wave length and the target length the magnitude of the width of the amplification line approximately is $r = \frac{\Delta\omega}{\omega} = 10^{-4}$.

Let us consider the target with the following parameters: the refractive indices - $n_1 = 1.1$ and $n_2 = 1.05$; the depthsof modulation of the refractive index - $\eta_1 = \Delta n/n = 1\%$ and $\eta_2 = 2\%$; the length of target is 2 mm. As the effective radiation angles $\theta^{\pm}$ are distinguished rather strongly, then the generation process at the angles θ^{+} and θ^{-} can be considered separately. Moreover, the optimal geometry of diffraction can be different for these two angles.

It should be noted that, in the case of the target length of 2 mm, the term, which corresponds to radiation losses through the target boundaries, is much less than the term, connected with the radiation absorption inside the target, if $\varepsilon_0'' > 10^{-8}$. Besides, even in the weakly absorbing medium, where the radiation losses through the target boundaries can become the main ones, these losses always can be reduced by introducing additional external mirrors [12]. In the visible region of spectrum it is not a problem. That is why we will consider the losses of radiation to be only due to the radiation absorption inside the target.

Although the diffraction geometry in the optical region is not restricted, for its optimization, it is necessary to know the parameters of a periodic structure (g_0', g_0'', g_τ', g_τ''). That is why we consider two opposite cases of diffraction geometry: the symmetric Bragg diffraction - $\beta_1 = -1$ (the waves $\vec{k}$ and $\vec{k}_\tau$ make the angles θ and $\pi - \theta$ with the normal to the target surface) and the case asymmetric Bragg diffraction $\beta_1 = -4\, g_\tau'^2 / g_0'^2$ (the diffracted wave propagates at a rather small angle relative to the target surface).

In Tables 1 and 2 the parameters of such system, allowing the achievement of the generation regime, for both angles of quasi-Cerenkov radiation are given. Because the main question of this analysis is the possibility of the realization of the generation regime with the help of existing particle beams, the estimations have been obtained for the maximum magnitude of the absorption coefficients which permit to achieve the generation threshold by using these particle beams. The minimum values of the absorption length, L_{abs}, are shown. The subscripts I and II refer respectively to the case $\gamma_I = 0.5$ mm and $\gamma_{II} = 0.2$ mm.

The analysis of the obtained results shows that, for the generation process at the θ^- angle, the symmetric diffraction is the optimal one. In this case the reflection planes are perpendicular to the radiating particle motion direction. However, the interaction of the particle beam with radiation is the most effective at the angle of θ^+. In this case, the symmetric and asymmetric diffraction geometries give the comparable results, in the case of target parameters chosen by us. But the analysis of the analytical expression (6) shows that under three-dimensional distributed feedback it is possible to optimize the condition of the generation threshold achievement by changing the parameters of a target and asymmetry factor of diffraction. For the following optimization, it is necessary to analyze the particular target.

So, on the basis of the obtained results one can conclude that there is a possibility to obtain the generation regime of coherent optical radiation using the existing particle beams uniformly moving inside the channel through a periodic artificial target, if the radiation absorption lengths of these media are more than several centimeters.

It is worthwhile to stress that the increase of the target length, even in several times, essentially improves the conditions of the threshold achievement ($G \sim L^2$) and allows to realize the regime of a high gain for the beam with the higher current density (in Tables it is labelled as II). This allows, in its turn, to obtain the high intensity of coherent optical radiation after a single passage of a particle beam through a target. This is very important because the time of particle flight through the target is, in our case, comparable with the particle beam duration ($\tau = 20$ ps). Let us estimate the output of coherent radiation for the target with the length, for example, 5 mm, the absorption coefficient $\varepsilon_0' \cong 10^{-7}$, $n_1 = 1.1$, $\eta = 2\%$, $|\beta| = 0.2$ and $\theta = \theta^+$. In this case, according to Ex. (6), the generation threshold is overcome in the high-gain regime for the particle beam with the higher current density (with the radius r_{II}). It means that the output of coherent radiation can be, approximately, represented as $I = I_0 e^{kL(G-G_{th})}$, where I_0 is the intensity of spontaneous quasi-Cerenkov radiation, G_{th} is the gain corresponding to the fulfillment of the generation threshold condition for the considered target.

It should be pointed out that in the high-gain regime, the width of spectral and angular amplification line is determined by the magnitude of gain, which doesn't now depend on the target length. We can obtain the magnitude of spectral and angular divergence of a coherent photon beam from the synchronism condition, that is $\frac{\Delta\omega}{\omega} \sim 5 \cdot 10^{-4}$, $\Delta\theta_D \sim 5 \cdot 10^{-4}$ and $\Delta\varphi_D \sim 10^{-2}$ rad, for the parameters of the particle beam and target, pointed above. To obtain the total number of photons that is possible to be observed by the detector with the angular size of $\Delta\theta_D \cdot \Delta\varphi_D$, we should estimate the number of spontaneous quasi-Cerenkov photons emitted within this angular interval:

$$N_{sp} \cong \frac{e^2 k_B L}{16\pi \sin^2\theta_B} \frac{\left|g'_\tau(\omega_B)\right|^2}{\theta_{ef}} \Delta\theta_D \cdot \Delta\varphi_D \sim 10^{-7}\ \gamma/e^- ,\ \theta_{ef}^2 = 1/\gamma^2 + g'_0 ,$$

where ω_B is the frequency of a photon, radiated at the angle of $\theta = \theta^+$ and satisfying the Bragg condition and θ_B is the Bragg angle. Transition radiation is directed at the angle $\theta \sim \frac{1}{\gamma} << \theta^+$ and the number of photons emitted within the angular interval $\Delta\theta_D \cdot \Delta\varphi_D$ is 10^2 times smaller than the number of quasi-Cerenkov photons. As a result, the total number of photons, emitted by a particle beam bunch containing $Ne \sim 10^{10}$ electrons, can be represented as

$$N = N_{sp} e^{(G-G_{th})kL} = 10^3 e^{(G-G_{th})kL} = R \cdot 10^3\ \gamma/\text{bunch} .$$

The calculation of a gain in the high-gain regime [10] gives the amplification factor $R \sim 10^3 \div 10^4$ for the parameters pointed above. It means that we can observe $N_\gamma \sim 10^6 \div 10^7$ photons per particle beam bunch.

In conclusion, it should be stressed that the advantage of three-dimensional geometry of distributed feedback manifests itself first of all in weak-absorbing media.

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Table 1

for $(\theta^+)^2 = g_0' + \frac{g\tau}{\sqrt{-\beta_1}} - \gamma^{-2}$

parameters	$\lvert\beta_1\rvert = 4\, g_\tau'^2 / g_0'^2$				$\lvert\beta_1\rvert = 1$	
	$n_1 = 1.1$ $\eta_1 = 1\%$	$n_1 = 1.1$ $\eta_2 = 2\%$	$n_2 = 1.05$ $\eta_1 = 1\%$	$n_2 = 1.05$ $\eta_2 = 2\%$	$n_1 = 1.1$ $\eta_1 = 1\%$	$n_2 = 1.05$ $\eta_1 = 1\%$
θ_{eff}	34°	34°	23°	23°	28°40′	20°30′
G_I	$2\cdot10^{-5}$	$2\cdot10^{-5}$	$5\cdot10^{-6}$	$5\cdot10^{-6}$	10^{-5}	$3\cdot10^{-6}$
$L_{abs}^{I} >$	7 cm	3 cm	12 cm	4 cm	2 cm	7 cm
G_{II}	$1.4\cdot10^{-4}$	$1.4\cdot10^{-4}$	$3.3\cdot10^{-5}$	$3.3\cdot10^{-5}$	$7\cdot10^{-5}$	$2\cdot10^{-5}$
$L_{abs}^{II} >$	1 cm	0.2 cm	1.5 cm	1 cm	0.5 cm	2 cm
d	4100 Å	3820 Å	3960 Å	3220 Å	3140 Å	3140 Å
ϕ	43°30′	47°	48°30′	65°20′	90°	90°

Table 2

$$\text{for } (\theta^{-})^2 = g_0' - \frac{g_\tau'}{\sqrt{-\beta_1}} - \gamma^{-2}$$

parameters	$\lvert\beta_1\rvert = 4\, g_\tau'^{2}/g_0'^{2}$				$\beta_1 = -1$	
	$n_1 = 1.1$ $\eta_1 = 1\%$	$n_1 = 1.1$ $\eta_2 = 2\%$	$n_2 = 1.05$ $\eta_1 = 1\%$	$n_2 = 1.05$ $\eta_2 = 2\%$	$n_1 = 1.1$ $\eta_1 = 1\%$	$n_2 = 1.05$ $\eta_1 = 1\%$
θ_{eff}	18°30′	18°30′	12°40′	12°40′	25°30′	16°10′
G_I	$2.5 \cdot 10^{-6}$	$2.5 \cdot 10^{-6}$	$5.7 \cdot 10^{-7}$	$5.7 \cdot 10^{-7}$	$7 \cdot 10^{-6}$	$1.3 \cdot 10^{-6}$
$L_{abs}^{I} >$	1 m	40 cm	180 cm	80 cm	6 cm	30 cm
G_{II}	$1.5 \cdot 10^{-5}$	$1.5 \cdot 10^{-5}$	$3.6 \cdot 10^{-6}$	$3.6 \cdot 10^{-6}$	$5 \cdot 10^{-5}$	$9 \cdot 10^{-6}$
$L_{abs}^{II} >$	20 cm	6 cm	30 cm	13 cm	cm	5 cm

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