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Essays on Quantitative Defaultable Debt Models

A dissertation submitted in partial satisfaction
of the requirements for the degree of
Doctor of Philosophy in Economics

by

Matthew Nelson Luzzetti

2012

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ABSTRACT OF THE DISSERTATION

Essays on Quantitative Defaultable Debt Models

by

Matthew Nelson Luzzetti

Doctor of Philosophy in Economics

University of California, Los Angeles, 2012

Professor Lee Ohanian, Chair

These essays contribute to the study of quantitative-theoretic equilibrium models in which agents can choose to optimally default on their debt obligations. Chapters 1 and 2, which are both joint work with Seth Neumuller, consider models with heterogeneous households. In Chapter 1, we introduce statistical learning and aggregate uncertainty into a heterogeneous households model with unsecured debt. We demonstrate that a model with learning produces credit booms during prolonged economic expansions and an endogenous severe and protracted credit crunch in response to a sequence of shocks similar to the recent financial crisis. This chapter illustrates that learning by households and creditors is an important driver of aggregate debt dynamics since the start of the Great Moderation. Chapter 2 considers an equilibrium model of mortgage and unsecured credit markets that features both long-term collateralized mortgage contracts and short-term unsecured debt. We use this framework to evaluate whether the Bankruptcy Abuse Prevention and Consumer Protection Act of 2005 (BAPCPA) contributed to the severity of the housing crisis by inducing homeowners to default on their mortgage that would have declared bankruptcy and remained in their home in the absence of the reform. We conclude that although the BAPCPA significantly increased mortgage default rates on impact in 2005, this reform had minimal impact on the severity of

the subsequent housing crash. This finding is the result of an optimal tightening of mortgage lending standards in response to heightened household default incentives. Therefore, considering the equilibrium response of mortgage prices to evolving household incentives is crucial to our findings. Finally, Chapter 3 focuses on the ability of countries to default on their debt obligations. In this chapter, I evaluate the impact of the presence of bailouts by international institutions, such as the IMF, on sovereign business cycles, default frequency, and social welfare. This analysis suggests that allowing for the presence of third-party bailouts can help explain the unique characteristics of business cycles and the historical frequency of default events in emerging economies. Moreover, this chapter concludes that bailouts tend to reduce social welfare.

The dissertation of Matthew Nelson Luzzetti is approved.

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2012

*To my wife, parents, and sister,
for your unwavering love, encouragement, and support*

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1 Endogenous Credit Booms and Busts: Learning in a Model of Consumer Default

1.1 Introduction

Between 1984 and 2004 the U.S. economy experienced an explosive rise in consumer unsecured debt and bankruptcies. The unsecured debt-to-income ratio for U.S. households almost doubled over this period, increasing from 4.9% in 1984 to 9.1% in 2004. Perhaps more surprisingly, after remaining remarkably stable for nearly twenty-five years, the consumer bankruptcy filing rate more than quadrupled, rising from 1.6 per 1,000 adults in 1984 to 7.0 in 2004.¹ The secular rise in unsecured debt reversed during the recent financial crisis as the unsecured debt-to-income ratio fell by 21.7% between 2008 and 2011 and has yet to recover. This dramatic and persistent reduction in unsecured debt was accompanied by a substantial tightening in lending standards, a reduced willingness to lend by creditors, and lower demand for consumer credit by households.

In this paper we evaluate whether a heterogeneous agent model of optimal consumer default with learning and aggregate uncertainty can help account for these facts. In our framework, aggregate states differ by the mean and variance of the idiosyncratic endowment process. Households form beliefs about the probability of transitioning between aggregate states based on the realized sequence of aggregate shocks. Creditors form expectations about household default probabilities by observing the history of default rates conditional on the loan amount, aggregate state, and household's endowment at the date of origination. Based on these beliefs, households construct their optimal decision rules and creditors determine the appropriate default premium to charge on each loan contract.

We view learning as a natural modeling choice in response to a changing economic environment like that experienced in the U.S. since 1984. As Cogley and Sargent (2008) argue, if

¹We stop our analysis in 2004 due to implementation of the Bankruptcy Abuse Prevention and Consumer Protection Act of 2005, which significantly increased the costs of declaring bankruptcy, thereby altering household incentives to both borrow and declare bankruptcy.

we assume that agents know the underlying parameters of our model with certainty, as is the standard assumption imposed by rational expectations, we are implicitly assuming that all learning is complete. Although this assumption may be innocuous and serve as a convenient simplification in many cases, there is substantial evidence that the post-1984 period was fundamentally different than that which preceded it.² Moreover, in the wake of the recent financial crisis, many observers are again wondering whether or not the underlying parameters of the economy have shifted. It is important to recognize that the first academic papers to document a reduction in aggregate volatility during the Great Moderation did not appear until the late 1990's. Given that it took quite a long time even for academic economists to begin to question whether or not the economic environment had fundamentally changed, we cannot reasonably expect the average agent in our model to know with certainty in 1984 one way or the other. We view learning as a natural response to this critique. It takes time for economic agents to change their beliefs, and we argue that we are better able to understand the behavior of households and creditors during this period by explicitly modeling their dynamic learning process.

While the theoretical model that we develop here is general enough to encompass a variety of learning algorithms, we focus on the case of constant gain learning for several reasons. First, this approach is consistent with actual creditor behavior during this period.³ Second, it has been shown that constant gain learning is preferred to recursive least squares when agents suspect that the economy may be undergoing a period of structural change.⁴ Third, constant gain learning is easier to implement and less computationally intensive than Bayesian learning.

²See, for example, Kim and Nelson (1999), McConnell and Perez-Quiros (2000), and Benati and Surico (2009).

³See Thomas (2000) for a discussion of credit industry practices during the 1980's and 1990's.

⁴As discussed in Adam, Marcet, and Nicolini (2008), constant gain learning is preferred in a changing economic environment because recent realizations are more informative about the data generating process. Therefore, households prefer to place more weight on recent realizations, as is true with constant gain learning.

As our main theoretical result, we demonstrate that when either households learn about aggregate state transition probabilities or creditors learn about household default probabilities, a rise in the unsecured debt-to-income ratio is the natural response of our economy to a sequence of favorable aggregate shocks like that experienced during the Great Moderation.⁵ Using a calibrated version of our model that allows for both types of learning, we demonstrate that learning can explain most of the boom in consumer credit experienced during the Great Moderation, but is unable to produce a sizable increase in the bankruptcy rate. Allowing for a reduction in the costs of declaring bankruptcy over this period enables our model to match the rise in the unsecured debt-to-income ratio and bankruptcy filing rate observed during the Great Moderation.⁶

The intuition for our results is the following: Realizing a string of favorable aggregate shocks leads households to become more optimistic about the future as they discount the probability of transitioning to an unfavorable state. The perception of lower endowment uncertainty reduces households' precautionary savings motive, which leads to an increase in borrowing. Moreover, for any given endowment and loan size, the fraction of households that default is increasing in the variance and decreasing in the mean of the endowment process. Hence, a sequence of favorable aggregate shocks also results in a lower than expected default rate for any given debt contract. In response, creditors revise downward their expectations about default probabilities and reduce the default premium charged on debt contracts, reinforcing the rise in the household debt-to-income ratio.

The impact of learning on the bankruptcy filing rate during a period of reduced aggregate volatility is less clear. On one hand, since the likelihood that a household will default is increasing in their debt burden, an increase in borrowing leads to a higher incidence of

⁵By 'favorable aggregate shocks' we mean those states in which households' idiosyncratic endowment process has a relatively high mean and low variance. We will refer to states with a relatively low mean and high variance for the endowment process as unfavorable.

⁶Gross and Souleles (2002) and Livshits, MacGee, and Tertilt (2010) emphasize the role of a reduction in the costs of declaring bankruptcy, broadly defined, in explaining the observed rise in bankruptcy filings during the Great Moderation.

default.⁷ In addition, because households perceive lower future endowment volatility, the punishment from default, which we model as exclusion from credit markets, is believed to be less severe. This increases incentives to declare bankruptcy. On the other hand, a decrease in the default premium makes it easier for households to refinance their debt and avoid default. Lower interest rates also increase the benefit of having access to credit markets, which reduces the attractiveness of bankruptcy. Our quantitative exercises suggest that these changes in households' incentives to declare bankruptcy offset in the aggregate, leaving the bankruptcy filing rate largely unchanged. By allowing for a reduction in the costs of bankruptcy, our model generates a simultaneous rise in the bankruptcy filing rate and unsecured debt-to-income ratio that matches the data.

The fact that our model generates realistic movements in consumer debt and bankruptcies during the Great Moderation leads us to assess whether learning can aid in explaining the dramatic and sustained contraction in consumer debt observed during the financial crisis. To address this question, we subject our model to a sequence of aggregate shocks beginning in 2005 that mimic those realized during the crisis. We find that allowing households to learn about the probability of transitioning between aggregate states amplifies both the increase in the unsecured debt-to-income ratio prior to the crisis as well as the steepness of the decline in response to the crisis.⁸ This credit crunch becomes increasingly severe and protracted when creditors also learn about household default probabilities.

Within our framework, we find that learning affects both the supply and demand for credit during the financial crisis. On the demand side, learning about the aggregate state

⁷Here we make an important distinction between a default rate and the incidence of default. A default rate is the fraction of households who find it optimal to default on a specific debt contract at a given date. The incidence of default, on the other hand, is the fraction of households who find it optimal to default across all available debt contracts at a given date. It is the incidence of default in our model that corresponds to the bankruptcy filing rate that we observe in the data.

⁸Although our model predicts an increase in the unsecured debt-to-income ratio between 2004 and the financial crisis, this measure remains relatively flat in the data during this period. We think several factors that we do not model in this paper can account for this discrepancy, among them: the implementation of bankruptcy reform in 2005 which dramatically altered household incentives to declare bankruptcy and undertake unsecured debt and a potential substitution away from unsecured debt and toward debt tied to housing during the housing boom.

during the financial crisis raises households' perceived probability of transitioning to an unfavorable aggregate state in the future. This increases households' precautionary savings motive and reduces their demand for credit. On the supply side, the financial crisis generates a substantial rise in default rates, causing creditors to raise the default premium on loan contracts, which we interpret as a tightening of lending standards. Our model is thus able to generate an endogenous credit cycle that strongly resembles that which the U.S. economy experienced during the recent financial crisis.

Our paper is closely related to several strands of literature. First, we contribute to the work on the rise in household debt and bankruptcies. The empirical literature has proposed many explanations for these findings. Boyes and Faith (1986) and Shepard (1984), for example, argue that changes in the U.S. consumer bankruptcy code made declaring bankruptcy more attractive to potential filers. Buckley and Brinig (1998), Gross and Souleles (2002), and Fay, Hurst, and White (2002), on the other hand, contend that the rise in defaults was primarily a result of a decline in the cost of filing for bankruptcy, either non-pecuniary or pecuniary in nature. Hacker (2006) and Barron, Elliehausen, and Staten (2000) argue that an increase in income volatility led more households into financial trouble; Warren and Tyagi (2003) highlight the role of greater idiosyncratic expense risk; and Barron and Staten (2003) cite credit market innovations that reduced the transaction costs associated with issuing debt.

In a recent paper, Livshits et al. (2010) evaluate the ability of several leading theories to quantitatively account for this experience in the context of an equilibrium model of consumer default. These authors point to a reduction in the transaction cost of issuing consumer credit and a simultaneous decline in the cost of filing for bankruptcy as the most likely explanation. While learning about default probabilities and changes in transaction costs can have similar implications for interest rates and household borrowing, we view learning as a simple method for endogenizing the changes in credit market conditions that exogenously varying transaction costs are intended to capture. Learning about default rates in our

model has the additional feature that creditors adjust interest rates differentially across their portfolio of available loan contracts based on differences between the realized and expected default rate for each loan type. This implies that interest rates change more for debt contracts that experienced larger forecast errors. Moreover, we demonstrate that changes in beliefs about the probability of transitioning to different aggregate states is an important driver of aggregate debt statistics.

Most current models that are able to generate a credit crunch appeal to an exogenous change in borrowing constraints or the introduction of a financial wedge. For example, Guerrieri and Lorenzoni (2011) study the effects of an exogenous and unexpected permanent tightening in consumers' borrowing capacity on consumer spending in a heterogeneous agent, incomplete-markets model. Chatterjee and Eyigungor (2011b) introduce an exogenous financial wedge into a model of the housing market to generate a persistent decline in home prices following an unexpected and permanent increase in housing supply. Our work contributes to this rapidly expanding literature by demonstrating how learning can generate similar aggregate behavior in an endogenous fashion.

We also contribute to recent literature that employs statistical learning algorithms in quantitative settings.⁹ For example, Eusepi and Preston (2011) introduce constant gain learning into a standard real business cycle model and find that it generates increased volatility in hours worked, thereby bringing the model's predictions closer to the data. Carceles-Poveda and Giannitsarou (2008) and Adam et al. (2008) use learning of this kind to shed new light on a variety of asset pricing puzzles. Our paper contributes to these findings by illustrating that learning is a quantitatively important factor in explaining unsecured debt dynamics since 1984, and it is also the first to consider statistical learning in a heterogeneous agent environment, or in a model of optimal default.

The remainder of this paper proceeds as follows. Section 1.2 introduces our full model which is later used to generate our quantitative results. Section 1.3 presents our analytical

⁹Evans and Honkapohja (2001) provide a comprehensive overview of the literature on statistical learning, its theoretical properties, and potential applications.

results along with a simplified version of the model that provides intuition for our quantitative results. In Section 1.4 we formally calibrate our model and quantify how much of the increase in consumer debt and bankruptcies over the Great Moderation can be explained by learning. In Section 1.5 we explore our model’s implications for the recent financial crisis. Finally, Section 1.6 concludes.

1.2 Model

We consider a consumer defaultable debt model in the spirit of Livshits et al. (2010) and Chatterjee, Corbae, Nakajima, and Rios-Rull (2007). Time is discrete and infinite. The economy is populated by a measure one of infinitely lived households that receive a stochastic endowment y_t each period. The process from which this endowment is drawn depends on the realization of an aggregate state variable $s_t \in S = \{s^1, \dots, s^N\}$, where S is a time-invariant set and s_t evolves according to a Markov process with transition matrix Π . The only asset is a one-period, unsecured and unconditional discount bond that trades at a price set by a pool of risk-neutral, perfectly competitive creditors.

Households choose whether or not to repay their debt each period. A defaulting household enters bankruptcy, which we model after Chapter 7 of the U.S. bankruptcy code.¹⁰ If they choose not to repay their debt obligations, the household is said to default and is relieved of their outstanding debt. Default is punished with an endowment cost, a one-time utility cost, and restricted access to credit markets. In the period of default, the household can neither save nor borrow. In the period following default, the household’s credit report is marked

¹⁰The U.S. bankruptcy code offers consumers two choices when filing for bankruptcy protection: Chapter 7 and Chapter 13. A household that chooses to file under Chapter 7 is relieved of all outstanding debt obligations in exchange for their assets net of any personal exemptions. A household that chooses to file under Chapter 13, on the other hand, agrees to pay back a portion of their outstanding debt obligations over a 3-5 year period in exchange for the ability to keep their assets. In either case, the household is not allowed to refile under the same chapter for a period of 6 years, and a record of their bankruptcy is maintained on their credit report for a period of 10 years. The conditions of default in our model are chosen to match Chapter 7 of the U.S. bankruptcy code, which accounts for approximately 70% of bankruptcy filings over the period under consideration. Moreover, given the choice between Chapters 7 and 13, a household would only choose Chapter 13 if they have assets that they would like to keep but would otherwise lose by filing under Chapter 7. Since there is only one asset in our model, a defaulting household will inevitably have a negative asset position, and therefore will always prefer to file under Chapter 7.

with a bankruptcy flag. Households with a bankruptcy flag are considered to be in a state of bad credit that persists for a random number of periods. While in bad credit standing, a household does not incur any additional costs and may save but cannot borrow.

1.2.1 Timing of Events

In any period t , the timing of events is as follows:

1. Households enter with prior beliefs about the aggregate state transition matrix, and creditors enter with prior beliefs about default probabilities.
2. The aggregate state s_t and idiosyncratic endowments y_t are realized.
3. Given their prior and the realized aggregate state, households form posterior beliefs about the aggregate state transition matrix.
4. Creditors announce a bond price schedule consistent with their beliefs.
5. With probability θ , households who are in bad credit standing have their bankruptcy flag removed and regain full access to credit markets.
6. Given their posterior beliefs and bond prices, households in good credit standing make default, consumption, and borrowing decisions, while households in bad credit standing make consumption and saving decisions.
7. Given their prior and the observed default rates, creditors form posterior beliefs about household default probabilities.

1.2.2 Household's Problem

As has become standard in the literature on statistical learning, we adopt the model of anticipated utility originally developed by Kreps (1998) and first used in applied work by Sargent (1999). In this framework, agents reoptimize at each point in time given their current

beliefs. Cogley and Sargent (2008) demonstrate that when agents are not too risk averse, anticipated utility models closely approximate the results generated by models in which agents also learn, but are considered to be fully rational in the Bayesian sense.¹¹ Moreover, anticipated utility models have the advantage of being more tractable than models that use Bayesian learning rules. To our knowledge, this paper is the first to use anticipated utility in a model of optimal default.¹²

Each period households receive a stochastic endowment y , the log of which evolves according to the following first-order autoregressive process:

$$\log(y) = (1 - \rho)\mu_s + \rho \log(y_{-1}) + \varepsilon \quad (1.1)$$

where $\varepsilon \sim N(0, \eta_s^2)$. The unconditional mean of the endowment process and the variance of the idiosyncratic endowment shock depend on the realization of the aggregate state s .

The household's state is composed of the debt with which it enters the period b (where we adopt the convention that $b > 0$ represents a household with positive assets, while $b < 0$ represents a household with negative assets, or positive debt), its idiosyncratic endowment realization in the current period y , and the realization of the aggregate state s , and is denoted by the triplet $(b, y; s)$. A household in good credit standing (G) observes the bond price schedule set by creditors and chooses whether to default (D) or repay their debt obligations (R):

$$V_t^G(b, y; s) \equiv \max_{R, D} \{V_t^R(b, y; s), V_t^D(y; s)\}, \quad (1.2)$$

where $V_t^D(y; s)$ represents the value of defaulting and $V_t^R(b, y; s)$ is the value associated with

¹¹More precisely, when agents have constant relative risk aversion preferences with a coefficient of two or less, the predictions of a model in which agents use recursive least squares are nearly identical to those generated by a model in which agents use Bayesian learning.

¹²This paper is also the first, to our knowledge, to allow for aggregate uncertainty in a model of optimal default. We demonstrate that learning in an environment with aggregate uncertainty is the key feature that allows our model to closely match the data.

repaying their debt at date t .¹³ Note that since a defaulting household is relieved of their debt obligations, the value of defaulting is independent of b .

If the household repays its debt, it then optimally chooses consumption and its asset position with which it leaves the period. The value of this option is given by:

$$V_t^R(b, y; s) = \max_{b'} u(c) + \beta \mathbb{E}_t [V_t^G(b', y'; s') | y; s] \quad (1.3)$$

subject to

$$c + q_t(b', y; s)b' = y + b$$

where $\mathbb{E}_t$ are household expectations conditional on their beliefs about the aggregate state transition matrix at date t and $q_t(b', y; s)$ is the bond price for a household leaving the period with assets b' with current endowment y in state s . A household that does not default remains in good credit standing and faces the same problem in the following period of whether or not to default and thus receives an expected continuation value of $\mathbb{E}_t [V_t^G(b', y'; s') | y; s]$.

If the household chooses to default, they are relieved of their outstanding debt obligations in exchange for an endowment cost and a one-time utility cost χ_t . The endowment cost represents a payment to satisfy the “good faith” requirement of the U.S. bankruptcy code for high-income households, and therefore, we assume that it is weakly increasing in the household’s endowment.¹⁴ The utility cost is intended to capture potential changes over time in the stigma attached to bankrupts. Households are also prohibited from saving in the period of default. Hence, a defaulting household simply consumes their endowment net of any bankruptcy costs. The value of defaulting in the current period is thus given by:

¹³The anticipated utility structure of the model implies that households reoptimize at each point in time given their current beliefs. For this reason, household decision rules and value functions, which depend on the household’s beliefs about the aggregate state transition matrix at date t , are time-dependent and thus are appropriately labeled with time subscripts.

¹⁴We think this is an important feature of our model since otherwise a household with high income and a large amount of outstanding debt would have an incentive to game the system by filing for bankruptcy. This form of endowment cost is similar to that used by Arellano (2008) in a model of sovereign default.

$$V_t^D(y; s) = u(c) - \chi_t + \beta \mathbb{E}_t [V_t^B(0, y'; s') | y; s] \quad (1.4)$$

subject to

$$c = \min \{y, \psi \mathbb{E}[y | s]\}$$

and $V_t^B(b, y; s)$ is the value of a household that has a bankruptcy flag on their credit report and so is considered to be in bad credit standing.

Households in bad credit standing are restricted from borrowing.¹⁵ But since the U.S. bankruptcy code does not prohibit asset accumulation after the discharge of debt, we allow households to save. Each period following default, the household has their bankruptcy flag removed and regains full access to credit markets with probability θ , while with probability $1 - \theta$ the bankruptcy flag remains on their credit report. The value of a household in this post-default state is given by:

$$V_t^B(b, y; s) = \max_{b' \geq 0} u(c) + \beta \mathbb{E}_t [\theta V_t^G(b', y'; s') + (1 - \theta) V_t^B(b', y'; s') | y, s] \quad (1.5)$$

subject to

$$c + q_t(b', y; s)b' = y + b.$$

1.2.3 Bond Prices

The bond price schedule is determined in equilibrium by the profit maximizing behavior of a pool of perfectly competitive, risk-neutral creditors that face an exogenously given, risk-free rate r . Creditors in our model face a proportional transaction cost $\tau > 0$ of making loans to households. One should think of τ as representing the cost to a lender of verifying a household's income prior to issuing a loan. The assumptions of risk neutrality and perfect

¹⁵Musto (2004) argues that creditors view default as an adverse signal about a household's future ability to repay their debt. Consequently, access to credit for households that have a bankruptcy flag on their credit report may be available on prohibitively tough terms or may not be available at all. Musto finds that this effect tends to last until the household's credit report is cleared of their bankruptcy flag, which occurs by law 10 years after the date at which their debt was discharged.

competition imply that creditors must earn zero expected profits on each credit contract they enter into with a household. Furthermore, the ability of creditors to price each loan based on its size, the household's income, and the aggregate state rules out cross-subsidization. As a result, bond prices fully reflect the expected default probability for a loan with these given characteristics. The bond price for a contract where $b' < 0$ is given by:

$$q_t(b', y; s) = \frac{1 - \tilde{D}_t(b', y; s)}{(1 + r)(1 + \tau)} \quad (1.6)$$

where $\tilde{D}_t(b', y; s)$ represents creditor beliefs about the default probability of a household borrowing b' with endowment y in state s at time t .¹⁶ Creditors' expectations at time t incorporate information about realized default rates up to and including time $t - 1$. Note that this object is distinct from the household's decision of whether or not to default, which is described by an indicator function taking the value of 1 if the household defaults and 0 otherwise:

$$D_t(b, y; s) = \begin{cases} 1 & \text{if } V_t^D(y; s) > V_t^R(b, y; s) \\ 0 & \text{otherwise} \end{cases} \quad (1.7)$$

Since households that save will never find it optimal to default, they carry no default risk. Moreover, we assume there are no transaction costs ($\tau = 0$) associated with accepting deposits since income verification is unnecessary in this case. Thus, the bond price for a contract where $b' > 0$ is equal to $1/(1 + r)$.

1.2.4 Information and Learning

Households and creditors have incomplete information about the underlying model parameters. Each must use the information that they possess to form beliefs about these parameters. Households learn about the aggregate state transition matrix so that they can formulate optimal decision rules. Creditors learn about household default probabilities to ap-

¹⁶Throughout we will denote objects that are creditor or household beliefs with a tilde, while the true object will be presented without a tilde.

appropriately price household debt contracts. Both agents use linear statistical learning rules when forming posterior beliefs given their prior and the realization of relevant economic variables. The details of these dynamic learning algorithms are outlined next.

Aggregate State Transition Probabilities

Households are uncertain about the transition probabilities governing the aggregate state. Let $\tilde{\Pi}_t$ represent household beliefs about the aggregate state transition matrix after the aggregate state is realized at time t . These beliefs may differ from the true aggregate state transition matrix Π . Given an initial prior $\tilde{\Pi}_0$, households learn over time about the aggregate state transition matrix by observing the realized sequence of aggregate states and using a linear updating rule to form their posterior beliefs.

Suppose that the observed transition at date t is from aggregate state s^i to s^j , and let $\tilde{\Pi}_t^k$ denote the k^{th} row of $\tilde{\Pi}_t$. If $\tilde{\Pi}_{t-1}$ is their prior belief about the aggregate state transition matrix at date t , then their posterior beliefs given the realized transition are:

$$\tilde{\Pi}_t^k = \begin{cases} \gamma_a 1^j + (1 - \gamma_a) \tilde{\Pi}_{t-1}^k & \text{if } k = i \\ \tilde{\Pi}_{t-1}^k & \text{otherwise} \end{cases} \quad (1.8)$$

where 1^j is a row vector with a 1 as the j^{th} element and 0's elsewhere and γ_a is the gain parameter that governs the weight that households place on new information about the aggregate state transition matrix relative to their prior when forming their posterior beliefs. This updating rule implies that, in response to a transition from s^i to s^j , agents increase their beliefs about the probability of transitioning from s^i to s^j relative to transitioning from s^i to any other state. Since households receive no new information about transitions from states s^k for $k \neq i$, the corresponding rows of the transition matrix are not updated.

Default Probabilities

Creditors observe the aggregate state and the endowment of any household with whom

they make a debt contract but do not observe the household's endowment in the following period nor do they know the parameters of their endowment process. While creditors can condition their loan contracts on all of the relevant state variables for the household, these assumptions imply that they are unable to compute the actual default probability for any given loan contract.

Creditors update their beliefs about default probabilities using the new information they obtain by observing actual default rates in the economy each period. When observed default rates differ from their expectations, creditors use their forecast errors to update their beliefs. Let $DR_t(b', y)$ represent the observed default rate at date t for households that borrowed an amount b' with endowment y at date $t - 1$, and recall that $\tilde{D}_t(b', y; s)$ represents creditor beliefs about the default probability of a household that borrows b' with endowment y in state s at time t . Suppose that the aggregate state at date $t - 1$ was s^i . For all b' and y , creditors' beliefs at date $t + 1$ are then:

$$\tilde{D}_{t+1}(b', y; s^k) = \begin{cases} \gamma_d DR_t(b', y) + (1 - \gamma_d) \tilde{D}_t(b', y; s^k) & \text{if } k = i \\ \tilde{D}_t(b', y; s^k) & \text{otherwise} \end{cases}. \quad (1.9)$$

The gain parameter γ_d governs the weight that creditors place on new information relative to their prior when forming their posterior beliefs. Given that creditors must announce a bond price schedule prior to household default decisions, the realized default rate at date t is the most recent default information available to creditors when they set bond prices at date $t + 1$.

The fact that creditors learn about an endogenous object gives our model a self-referential property that operates in the following way: When a favorable (unfavorable) aggregate shock occurs, default rates are below (above) creditor expectations. Given their updating rule, expectations about default probabilities are revised downward (upward). In the following period, lower (higher) interest rates make it easier (harder) for a household to roll over its debt, thus leading to even lower (higher) default rates than expected. This mechanism

imparts momentum into creditors' beliefs, amplifying the model's response to a sequence of favorable aggregate shocks.

1.2.5 Equilibrium

Definition *An equilibrium for this economy is sequences of household decision rules and beliefs $b'_t(b, y; s)$, $D_t(b, y; s)$, and $\tilde{\Pi}_t$, and creditor beliefs $\tilde{D}_t(b', y; s)$, such that, given initial beliefs for households and creditors $\tilde{\Pi}_0$ and $\tilde{D}_0(b', y; s)$, an initial distribution of households over bonds, endowments, and credit statuses Φ_0 , learning rules, and sequences of bond prices $q_t(b', y; s)$, aggregate states s_t , and endowment shocks y_t , the decision rules solve each household's problem and bond prices maximize creditors' profits at every date t .*

1.3 Analytical Results

In this section we investigate the theoretical implications of learning by households about aggregate state transition probabilities and learning by creditors about household default probabilities. In order to both maintain tractability and to isolate the effects of each type of learning, we first consider the case in which only households learn and then consider the case in which only creditors learn.¹⁷

1.3.1 Household Learning

To understand how learning by households about aggregate state transition probabilities affects their consumption, savings, and default behavior, consider the case in which bond prices are held constant and $S \equiv \{c, e\}$, where 'c' represents a contraction and 'e' represents an expansion.¹⁸ In addition, let the aggregate state transition matrix be given by:

¹⁷We analyze the effects of simultaneous learning by households and creditors later using a calibrated version of our model.

¹⁸Bond prices will remain constant in our economy if we set $\gamma_d = 0$, implying that creditors do not learn about household default probabilities.

$$\Pi = \begin{bmatrix} \pi_{ee} & 1 - \pi_{ee} \\ 1 - \pi_{cc} & \pi_{cc} \end{bmatrix},$$

where $\pi_{ij} = \Pr\{s_{t+1} = j | s_t = i\}$, and assume each household's idiosyncratic endowment takes a value $y \in \{y_L, y_H\}$, $y_L < y_H$, where

$$P^i = \begin{bmatrix} p_{HH}^i & p_{HL}^i \\ p_{LH}^i & p_{LL}^i \end{bmatrix},$$

and $p_{mj}^i = \Pr\{y' = j | y = m; s_{t+1} = i\}$. We consider the case in which the persistence of y_H and y_L are lower and higher, respectively, in contraction, $p_{HH}^e \geq p_{HH}^c$ and $p_{LL}^e \leq p_{LL}^c$. Finally, let $\tilde{\Pi}_0 = \Pi$ and suppose that $s_t = e$ for all $t \geq 0$.

At date $t = 1$, households update their beliefs about Π according to equation (1.8):

$$\tilde{\Pi}_1 = \begin{bmatrix} \gamma_a + (1 - \gamma_a)\pi_{ee} & (1 - \gamma_a)(1 - \pi_{ee}) \\ 1 - \pi_{cc} & \pi_{cc} \end{bmatrix}.$$

Given that $\gamma_a \in (0, 1)$, $\gamma_a + (1 - \gamma_a)\pi_{ee} = \pi_{ee} + \gamma_a(1 - \pi_{ee}) > \pi_{ee}$ and $(1 - \gamma_a)(1 - \pi_{ee}) < 1 - \pi_{ee}$. Therefore, households increase their perceived probability of remaining in state e and decrease their perceived probability of transitioning to state c as a result of realizing state e at date $t = 1$.

At date $t = 2$, households again update their beliefs about Π according to equation (1.8):

$$\begin{aligned} \tilde{\Pi}_2 &= \begin{bmatrix} \gamma_a + (1 - \gamma_a)[\gamma_a + (1 - \gamma_a)\pi_{ee}] & (1 - \gamma_a)^2(1 - \pi_{ee}) \\ 1 - \pi_{cc} & \pi_{cc} \end{bmatrix} \\ &= \begin{bmatrix} (1 - \gamma_a)^2\pi_{ee} + \gamma_a[1 + (1 - \gamma_a)] & (1 - \gamma_a)^2(1 - \pi_{ee}) \\ 1 - \pi_{cc} & \pi_{cc} \end{bmatrix}. \end{aligned}$$

Repeating this procedure, it follows that at any date t households' beliefs about Π are

given by:

$$\begin{aligned}\tilde{\Pi}_t &= \begin{bmatrix} (1 - \gamma_a)^t \pi_{ee} + \gamma_a \sum_{j=0}^{t-1} (1 - \gamma_a)^j & (1 - \gamma_a)^t (1 - \pi_{ee}) \\ 1 - \pi_{cc} & \pi_{cc} \end{bmatrix} \\ &= \begin{bmatrix} (1 - \gamma_a)^t \pi_{ee} + 1 - (1 - \gamma_a)^t & (1 - \gamma_a)^t (1 - \pi_{ee}) \\ 1 - \pi_{cc} & \pi_{cc} \end{bmatrix}.\end{aligned}$$

Note that the rate at which households adjust their beliefs is increasing in the learning gain parameter γ_a .

Thus, given $s_t = e$ for all $t \geq 0$, we have the following result in the limit:

$$\lim_{t \rightarrow \infty} \tilde{\Pi}_t = \begin{bmatrix} 1 & 0 \\ 1 - \pi_{cc} & \pi_{cc} \end{bmatrix}.$$

In response to a persistent sequence of expansionary aggregate shocks, households completely discount the possibility of transitioning to a contraction.

This result has important implications for consumption, savings, and default behavior. In particular, since expansions are associated with higher average income and lower income volatility, a string of favorable aggregate shocks leads households to become more optimistic about the future. The perception of lower endowment uncertainty reduces households' precautionary savings motive, and leads to an increase in borrowing.

1.3.2 Creditor Learning

In this section we explore the implications of creditors learning about default probabilities during an extended period of reduced aggregate volatility. To preserve analytical tractability, we consider a simplified version of our full model in which only creditors learn and assume that households do not learn about aggregate state transition probabilities. In addition, we assume that a defaulting household is forever restricted from borrowing in credit markets (i.e. $\theta = 0$).

Preliminary Results

Prior to introducing the example, we establish several preliminary results that will be useful for this section and the next. To start, let $\tilde{b}_t^{y,s}$ be the value of debt that makes a household with endowment y in state s indifferent between repaying its debt obligations and defaulting, so that $V_t^D(y; s) = V_t^R(\tilde{b}_t^{y,s}, y; s)$. Our first result establishes that households with debt greater than $\tilde{b}_t^{y,s}$ find it optimal to default.

Theorem 1.1 *A household with endowment y in state s finds it optimal to default if they have debt obligations $b < \tilde{b}_t^{y,s}$.*

Proof Notice that $V_t^D(y; s)$ is independent of b while $V_t^R(b, y; s)$ is increasing in b . Consider some $b < \tilde{b}_t^{y,s}$. Then $V_t^R(b, y; s) < V_t^R(\tilde{b}_t^{y,s}, y; s) = V_t^D(y; s)$. Hence, it is optimal for the household to default on their debt.

Our next result demonstrates that the default thresholds $\tilde{b}_t^{y,s}$ are decreasing in the bond price schedule. To start, we first establish that the household's problem is a contraction mapping and hence has a unique fixed point.

Lemma 1.2 *Define the operator T_q as follows:*

$$(T_q V_t^G)(b, y; s) = \max \left\{ V_t^D(y; s), \max_{b'} \left\{ u(y + b - q_t(b', y; s)b') + \beta \mathbb{E} [V_t^G(b', y'; s')] \right\} \right\}.$$

T_q is a contraction mapping and hence there exists a unique fixed point $V_{t,q}^*(b, y; s)$.

Proof See Appendix A for proof.

Theorem 1.3 *The default thresholds $\tilde{b}^{y,s}$ decrease in response to an increase in the bond price schedule between periods t and $t + 1$ (i.e. $q_{t+1}(b, y; s) \geq q_t(b, y; s)$ for all $\{b, y, s\}$).*

Proof See Appendix A for proof.

The intuition for this result is that as bond prices rise, interest rates fall by definition, and hence any household must be at least as well off.¹⁹ Since defaulting households are restricted from borrowing forever in this example, the value of defaulting is independent of changes in the bond price schedule. Because the value of repaying debt is increasing in the bond price schedule and the value of default is independent of this object, for any given y and s a household must take on more debt to be indifferent between repaying their debt and defaulting. Therefore, the default thresholds must fall as a result of an increase in bond prices.

Example

In this section we provide a simple example to demonstrate how creditor learning in the presence of a sequence of favorable aggregate shocks causes interest rates to fall. As in the example with only household learning, consider the case where $S \equiv \{c, e\}$ and let Π and P^i be defined as in Section 1.3.1.

We will consider a sequence of shocks in which $s_t = e$ for $t \geq 0$. Suppose that creditors begin with beliefs that are consistent with the transition matrices of the true data generating process, Π and P^i . Let $\bar{b}_t \equiv \max_{y,s} \{\tilde{b}_t^{y,s}\}$ and $\underline{b}_t \equiv \min_{y,s} \{\tilde{b}_t^{y,s}\}$. Then from the perspective of a creditor, lending a household $b' \in [\bar{b}_t, 0)$ is risk free since the household will repay their debt in the following period with probability one. Hence, the corresponding bond prices are given by

$$q_0(b', y; e | b' \in [\bar{b}_0, 0)) = \frac{1}{(1+r)(1+\tau)}.$$

On the other hand, no creditor would ever lend a household $b' < \underline{b}_t$ since the household will default with probability one in the following period. Hence, the corresponding bond prices are given by

$$q_0(b', y; e | b' < \underline{b}_0) = 0.$$

¹⁹Recall that the interest rate paid on household savings is always equal to the risk-free rate since a household with $b \geq 0$ will never find it optimal to default.

Now consider a debt contract with a non-trivial default probability. In particular, suppose that $\tilde{b}_0^{L,c} = \max_{y,s} \{\tilde{b}_0^{y,s}\}$ ²⁰ and let $\hat{b}_t \equiv \max\{\tilde{b}_t^{H,e}, \tilde{b}_t^{L,e}, \tilde{b}_t^{H,c}\}$. Then lending a household $b' \in [\hat{b}_t, \tilde{b}_t^{L,c})$ is risky since if the economy transitions to state c and the household receives endowment y_L in the following period, they will default. At date 0 creditors' expected probability of default is

$$\tilde{D}_0(b', y; e | b' \in [\hat{b}_0, \tilde{b}_0^{L,c})) = (1 - \pi_{ee})p_{yL}^c,$$

and the corresponding bond price is given by

$$q_0(b', y; e | b' \in [\hat{b}_0, \tilde{b}_0^{L,c})) = \frac{1 - (1 - \pi_{ee})p_{yL}^c}{(1 + r)(1 + \tau)}.$$

We now consider how these beliefs evolve when creditors learn. Conditional on $s_0 = e$, the household honored its debt obligation regardless of whether they receive y_L or y_H at $t = 0$ since $b' > \max\{\tilde{b}_0^{H,e}, \tilde{b}_0^{L,e}\}$. Hence, the realized default rate $DR_0(b', y | b' \in [\hat{b}_0, \tilde{b}_0^{L,c})) = 0$. Creditors observe this and update their beliefs as follows:

$$\tilde{D}_1(b', y; e | b' \in [\hat{b}_0, \tilde{b}_0^{L,c})) = (1 - \gamma_d)(1 - \pi_{ee})p_{yL}^c.$$

The bond price at date 1 is then given by

$$q_1(b', y; e | b' \in [\hat{b}_0, \tilde{b}_0^{L,c})) = \frac{1 - (1 - \gamma_d)(1 - \pi_{ee})p_{yL}^c}{(1 + r)(1 + \tau)}.$$

Given that $\gamma_d \in (0, 1)$,

$$\tilde{D}_1(b', y; e | b' \in [\hat{b}_0, \tilde{b}_0^{L,c})) < \tilde{D}_0(b', y; e | b' \in [\hat{b}_0, \tilde{b}_0^{L,c}))$$

²⁰While this condition is not necessarily true in general, it is satisfied in our calibrated model.

and

$$q_1 \left(b', y; e | b' \in [\hat{b}_0, \tilde{b}_0^{L,c}) \right) > q_0 \left(b', y; e | b' \in [\hat{b}_0, \tilde{b}_0^{L,c}) \right).$$

Recall Theorem 1.3, which states that $\tilde{b}_t^{y,s}$ is decreasing in q . Hence, at date $t \geq 1$, either $b' \in [\hat{b}_t, \tilde{b}_t^{L,c})$, in which case the loan is risky, or $b' \geq \tilde{b}_t^{L,c}$, in which case the loan is risk free. Either way, as long as the economy remains in the expansion state, we have $DR_t(b', y | b' \in [\hat{b}_0, \tilde{b}_0^{L,c})) = 0$. Iterating on equation (1.9), it follows that

$$\tilde{D}_t \left(b', y; e | b' \in [\hat{b}_0, \tilde{b}_0^{L,c}) \right) = (1 - \gamma_d)^t (1 - \pi_{ee}) p_{yL}^c$$

and

$$q_t \left(b', y; e | b' \in [\hat{b}_0, \tilde{b}_0^{L,c}) \right) = \frac{1 - (1 - \gamma_d)^t (1 - \pi_{ee}) p_{yL}^c}{(1 + r)(1 + \tau)}.$$

Thus, given $s_t = e$ for all t , we have the following results in the limit:

$$\lim_{t \rightarrow \infty} \tilde{D}_t \left(b', y; e | b' \in [\hat{b}_0, \tilde{b}_0^{L,c}) \right) = 0,$$

and

$$\lim_{t \rightarrow \infty} q_t \left(b', y; e | b' \in [\hat{b}_0, \tilde{b}_0^{L,c}) \right) = \frac{1}{(1 + r)(1 + \tau)}.$$

Creditors completely discount the probability of transitioning from expansion to contraction in the limit. This leads creditors to reduce the default premium charged on the bond to zero even though the household may default if the economy transitions to contraction and the household receives the low endowment shock.

1.3.3 Theoretical Results

In this section we prove that if creditors learn about default probabilities, then in response to a sequence of favorable aggregate shocks (1) bond prices and household debt will increase and (2) creditors' expectations and bond prices each converge in the limit.

Define the realized default rate at date t for households that borrowed an amount b' with

endowment y in the previous period as

$$DR_t(b', y) \equiv \sum_{y' \in Y} \mathbb{I}(b' < \tilde{b}_t^{y', s'}) \Pr[y' | y; s'].$$

where $\mathbb{I}(\cdot)$ is the indicator function taking a value of 1 if the interior argument is true and 0 otherwise. The following theorem states that, given the learning algorithm used by creditors in our model, the realization of aggregate states for which the actual default rate is less (greater) than expected results in higher (lower) bond prices.

Theorem 1.4 *Let $X_t(b', y) \equiv \{s' \in S : DR_t(b', y) \leq \tilde{D}_t(b', y; s)\}$. If $s_t \in X_t(b', y)$, then $\tilde{D}_{t+1}(b', y; s) \leq \tilde{D}_t(b', y; s)$ and $q_{t+1}(b', y; s) \geq q_t(b', y; s)$. Otherwise, $\tilde{D}_{t+1}(b', y; s) \geq \tilde{D}_t(b', y; s)$ and $q_{t+1}(b', y; s) \leq q_t(b', y; s)$.*

Proof See Appendix A for the proof.

The following corollary presents the first of our two main theoretical results. It states that if the realized default rate is less than expected, household borrowing will increase on both the extensive and intensive margins.

Corollary 1.5 *Suppose $s_{t-1} = s_t = s_{t+1} \equiv \hat{s}$ where $\hat{s} \in X_t(b', y)$ for all (b', y) . Let i index households. Then*

$$\int q_{t+1} b_{t+1}(i) di \leq \int q_t b_t(i) di,$$

and

$$\int \mathbb{I}(b_{t+1}(i) < 0) di \geq \int \mathbb{I}(b_t(i) < 0) di.$$

Proof Following Theorem 1.4, the bond price schedule increases in response to this sequence of shocks. Thus, the relative cost of borrowing (consumption today) declines. As a result, the income and substitution effects cause an increase in borrowing such that $q_{t+1}(b_{t+1}, y; \hat{s}) b_{t+1}(b, y; \hat{s}) \leq q_t(b_t, y; \hat{s}) b_t(b, y; \hat{s})$ for all b and y and each household. This establishes the former result, from which the latter directly follows.

We now establish what happens in the limit as the economy realizes an infinite sequence of aggregate shocks for which the actual default rate is less than the expected default rate at every date t .

Theorem 1.6 *Suppose $s_t \in X_t(b', y)$ for all $t \geq 0$. Then:*

1. $\lim_{t \rightarrow \infty} \tilde{D}_t(b', y; s) = \tilde{D}_\infty(b', y; s) \in [0, \tilde{D}_0(b', y; s)]$
2. $\lim_{t \rightarrow \infty} q_t(b', y; s) = q_\infty(b', y; s) \in [q_0(b', y; s), 1/((1+r)(1+\tau))]$.

Proof See Appendix A for the proof.

This result tells us that if the economy repeatedly experiences favorable aggregate shocks that produce default rates below expectations, then creditors' expectations and bond prices each converge in the limit. Moreover, we know that creditors' expectations are bounded above by their initial prior, while bond prices are bounded below by their initial value.

1.4 Quantitative Results

In this section we discuss our calibration exercise and the experiment that we conduct to determine how much of the simultaneous rise in consumer debt and bankruptcies between 1984 and 2004 can be accounted for by our model. To do so, we first construct a sequence of aggregate shocks through 2004 based on the unemployment rate and NBER recession dates. We then calibrate our model to match the observed unsecured debt-to-income ratio and bankruptcy filing rate in 1983 and simulate our model's response to the observed sequence of aggregate shocks between 1984 and 2004.

1.4.1 Calibration

We assume that each period in our model corresponds to one year. The aggregate state is discretized into four values by classifying the years 1890 through 2004 based on the unem-

ployment rate (low or high) and NBER recession dates (expansion or contraction).²¹ Using data from the Panel Study of Income Dynamics (PSID), Storesletten, Telmer, and Yaron (2004) establish that income dispersion increases substantially during recessions relative to expansions. Conducting a similar analysis to those authors, we find that mean income tends to be high (low) when the unemployment rate is low (high). The set of aggregate states S thus contains four elements: expansion (e) or contraction (c), combined with either high (h) or low (l) mean income. Hence, $S = \{(e, h), (c, h), (c, l), (e, l)\}$. We construct the following transition matrix for the aggregate state by counting the transitions observed between 1890 and 1983 implied by our classification of years:

$$\Pi = \begin{bmatrix} 0.55 & 0.35 & 0.10 & 0.00 \\ 0.31 & 0.35 & 0.27 & 0.07 \\ 0.20 & 0.00 & 0.27 & 0.53 \\ 0.67 & 0.16 & 0.00 & 0.17 \end{bmatrix},$$

where $[\Pi]_{ij}$ represents the probability of transitioning from state i to state j .

We take the persistence and state-dependent standard deviation of the household's income process directly from Storesletten et al. (2004), implying $\rho = 0.963$, $\eta_e = 0.088$ and $\eta_c = 0.162$. Hence, idiosyncratic income shocks are roughly twice as volatile during contractions than expansions. Our estimates for the state-dependent mean of the income process are $\mu_h = 7.55$ and $\mu_l = 7.51$.²² Given that these values are in logs, we can conclude that the mean of the income process when the unemployment rate is low is about 4% higher than when the unemployment rate is high. We discretize the endowment process for each of the four aggregate states using the method employed by Tauchen and Hussey (1991).

The remaining parameters are chosen as follows. We set $\beta = 0.95$ and assume households have CRRA preferences with a coefficient of relative risk aversion $\sigma = 2$, as is standard in the

²¹Our unemployment rate series is constructed using data from Romer (1986) for the years 1890 to 1930, Lebergott (1964) for 1931 to 1940, and the Bureau of Labor Statistics from 1941 onward.

²²See Appendix B for a detailed description of our estimation procedure.

literature. The risk-free interest rate is set to 1.7%, which is equal to the average real return on 1-year U.S. Treasury bills between 1984 and 2004. We set $\theta = 0.2$ to match an average exclusion from credit markets of six years. This implies that households in our model are, on average, able to refile for bankruptcy after six years, which is consistent with the U.S. bankruptcy code.

Finally, we choose values for ψ and τ that allow our model to most closely match the unsecured debt-to-income ratio and consumer bankruptcy filing rate in 1983. To do so we set $\tilde{D}_0(b', y; s)$ and $\tilde{\Pi}_0$ to be consistent with the true data generating process up to 1983 and then simulate the economy without household or creditor learning ($\gamma_a = \gamma_d = 0$) and assuming the utility cost of default is constant (normalized to 0) given the observed sequence of aggregate shocks from 1890 to 1983.²³ We repeatedly perform this exercise for a finite grid of ψ and τ and choose the pair of values that minimize a weighted sum of the square deviations from our targets. Our baseline parameterization is summarized in Table 1.1.

Table 1.1: Calibrated Parameters

Parameter	Value	Source / Target
σ	2.0	standard
β	0.95	standard
r	0.017	real return on 1 yr US T-bills (1983-2004)
θ	0.2	avg exclusion from credit markets of 6 yrs
ρ	0.963	Storesletten et al. (2004)
η_e	0.088	Storesletten et al. (2004)
η_c	0.162	Storesletten et al. (2004)
μ_h	7.55	own estimate using PSID data
μ_l	7.51	own estimate using PSID data
χ_t	0.0	normalization
ψ	0.37	bankruptcy filing rate in 1983 of 0.16%
τ	0.007	debt-to-income ratio in 1983 of 4.9%

²³This implies that household and creditor beliefs are consistent with the true data generating process in 1983.

1.4.2 Learning and the Rise in Unsecured Debt and Default

To analyze our model's ability to account for the rise in consumer debt and bankruptcy filing rates over this period, we must parameterize the dynamic learning process. Households and creditors are assumed to use constant gain learning and therefore assign a lower weight to past observations to protect themselves against the possibility of structural change. We think this form of learning is appropriate since it closely resembles how creditors actually behaved during this period. It has also been shown that constant gain learning results in more accurate forecasts than recursive least squares when agents are concerned about the potential for structural change and is more tractable than Bayesian learning while producing strikingly similar results.²⁴

Throughout the 1980's and 1990's it was common for lenders to use linear models to evaluate the credit-worthiness of potential borrowers and to update the parameters of these models frequently – not less than once every two years – by re-running their regressions using the most up-to-date data on consumer default decisions. Since creditors updated their models of consumer default at least once every two years and discarded past observations to protect themselves against population drift, we set $\gamma_d = 0.5$, which represents a significant departure from rational expectations.²⁵ Since at this point we have little guidance regarding the choice of γ_a , we consider a range of values that encompass both small and large deviations from rational expectations.

Our experiment consists of the following procedure using an economy of 10 million households:

1. Set $\tilde{D}_0(b', y; s)$ and $\tilde{\Pi}_0$ to be consistent with the empirically observed data generating process up until 1983.

²⁴Adam et al. (2008) make a convincing case for the use of constant gain learning rather than recursive least squares in the context of an asset pricing model, and Cogley and Sargent (2008) demonstrate that the results generated under statistical and Bayesian learning are nearly indistinguishable when agents are not too risk averse.

²⁵To be precise, $\gamma_d = 0.5$ implies that creditors place almost zero weight on their 1984 prior beliefs at the end of the simulation period.

2. Simulate the economy without learning ($\gamma_a = \gamma_d = 0$) given the observed sequence of aggregate shocks from 1890 to 1983.
3. Then simulate the economy with learning ($\gamma_a \geq 0$, $\gamma_d = 0.5$) given the observed sequence of aggregate shocks from 1984 to 2004.²⁶
4. Document the unsecured debt-to-income ratio and the bankruptcy filing rate in the economy in 2004.

The results of this experiment are summarized in Table 1.2. We find that the debt-to-income ratio is monotonically increasing in γ_a and that values between 0.15 and 0.20 match the actual data quite well. Conversely, the elasticity of the bankruptcy filing rate to changes in γ_a is negligible, as even large changes in this parameter have little, if any, impact on the bankruptcy filing rate in 2004. The case without learning refers to our simulation with $\gamma_a = 0$ and $\gamma_d = 0$. This scenario corresponds to the rational expectations equilibrium of our model under the belief that the Great Moderation was a sequence of lucky draws from an unchanged data generating process and provides a useful benchmark for understanding the effects of learning.²⁷ Note that in this case both the bankruptcy filing rate and debt-to-income ratio in 2004 are essentially unchanged from their respective values in 1983. These results suggest that while learning by households and creditors is able to account for the rise in the unsecured debt-to-income ratio, it is unable to explain the increase in bankruptcies.

In Table 1.3, we explore the sensitivity of our model's predictions to changes in γ_d holding γ_a fixed at a value of 0.20. These results suggest that the bankruptcy filing rate is relatively insensitive to changes in the rate of learning about default rates. Moreover, while γ_d has a measurable impact on the unsecured debt-to-income ratio, the elasticity of the debt-to-income ratio with respect to changes in γ_d is far smaller than that with respect to changes

²⁶In this part of the experiment, creditors update their beliefs based on realized default rates, while households update their beliefs about the aggregate state transition matrix based on the realized transitions of the aggregate state.

²⁷See Stock and Watson (2003), Sims and Zha (2006) and Gambetti, Pappa, and Canova (2008) for evidence of this “lucky draws” interpretation of the Great Moderation.

Table 1.2: The Impact of Learning

	2004 Bankruptcy Filing Rate	2004 Debt-to-Income Ratio
$\gamma_a = 0.00$	0.0017	0.051
$\gamma_a = 0.05$	0.0016	0.064
$\gamma_a = 0.10$	0.0017	0.078
$\gamma_a = 0.15$	0.0020	0.089
$\gamma_a = 0.20$	0.0018	0.092
$\gamma_a = 0.25$	0.0018	0.093
$\gamma_a = 0, \gamma_d = 0$	0.0016	0.047
Actual Data	0.0070	0.091

in γ_a , as can be seen by comparing the results between tables 1.2 and 1.3.

Table 1.3: Sensitivity Analysis for $\gamma_a = 0.20$

	2004 Bankruptcy Filing Rate	2004 Debt-to-Income Ratio
$\gamma_d = 0.00$	0.0016	0.086
$\gamma_d = 0.25$	0.0017	0.089
$\gamma_d = 0.50$	0.0018	0.092

In conclusion, our analysis suggests that learning by households about the transition matrix for the aggregate state is an important factor in generating a rise in the unsecured debt-to-income ratio like that observed in the data. Moreover, while learning about default rates by creditors has an effect on the unsecured debt-to-income ratio, this impact is relatively small. Finally, neither type of learning is able to generate a meaningful increase in the bankruptcy filing rate over this period. This finding suggests that the factors that influence households' incentives to declare bankruptcy in the presence of learning offset in the aggregate, generating virtually no change in the bankruptcy filing rate in response to this sequence of favorable shocks. In the next section we reconcile this deficiency by allowing for a reduction in the utility cost of bankruptcy.

1.4.3 Utility Cost of Default

Empirical work by Gross and Souleles (2002) and quantitative analysis by Livshits et al. (2010) has suggested that a reduction in the stigma associated with declaring bankruptcy can account for the rise in the bankruptcy filing rate that occurred after 1984. In our model, a reduction in the stigma of filing for bankruptcy corresponds to a reduction in the utility cost of default, χ_t . In this section, we consider whether allowing for a reduction in the utility cost of default can help our model account for both the rise in the unsecured debt-to-income ratio and bankruptcy filing rate from 1984 to 2004.

We calibrate a reduction in the utility cost of default by matching the observed rise in the bankruptcy filing rate between 1984 and 2004. In particular, we parameterize the time trend of the utility cost in years $t \geq 1984$ as follows:

$$\chi_{t+1} = \chi_t + m(t - 1984).$$

Given our normalization $\chi_t = 0$ for all $t \leq 1983$, we simulate the model to find values for the slope parameter m and gain parameter γ_a that most closely match the unsecured debt-to-income ratio and bankruptcy filing rate observed in 2004.²⁸ This procedure implies that $m = -0.017$ and $\gamma_a = 0.12$ allow the model to exactly match our targets for the unsecured debt-to-income ratio and the bankruptcy filing rate in 2004.

1.5 Implications for the Financial Crisis

The recent financial crisis was characterized by a significant tightening of credit standards and a drastic and protracted reduction in unsecured household debt, which has yet to recover.²⁹ As of 2011, the unsecured debt-to-income ratio is more than 21% below its

²⁸To reduce our computational burden, we assume that households believe the utility cost in year t will remain unchanged in all future periods.

²⁹Although this paper focuses on unsecured debt, we believe learning can also help explain the dramatic expansion and collapse in mortgage debt over the past decade, which played a more central role in the recent

pre-crisis level.

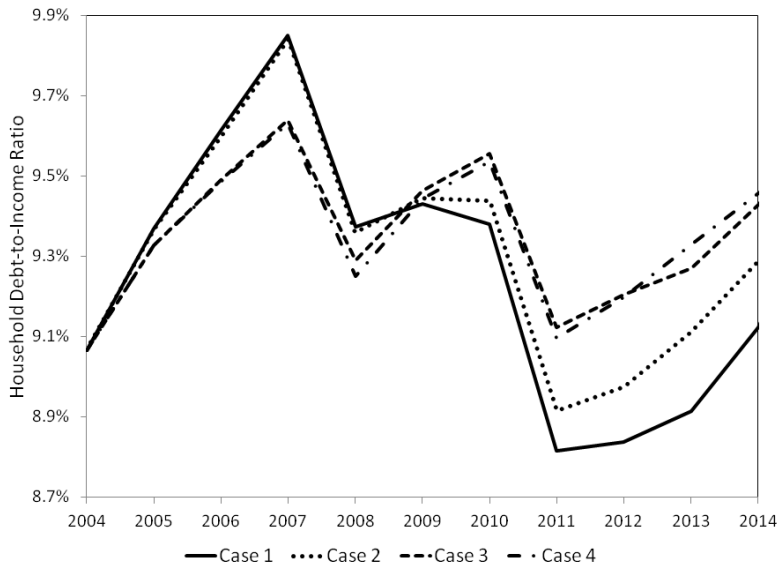
The dramatic reduction in household unsecured debt since 2008 was influenced by both supply and demand factors. According to the Federal Reserve Board’s Senior Loan Officer Opinion Survey, the recent crisis was characterized by a historically large and persistent tightening in lending standards in 2008 and 2009, a reduction in creditors’ willingness to lend, and a weakened demand for consumer loans. For instance, more than 80% of surveyed creditors tightened standards on consumer loans and more than 60% expressed a decreased willingness to lend at the peak of the crisis.³⁰ At the same time, a majority of creditors reported weaker demand for consumer loans from 2007 through the end of 2010. Given that our model with learning is able to capture the secular trends in the consumer unsecured debt-to-income ratio and bankruptcy filing rate during the Great Moderation, in this section we analyze whether learning can help us understand the debt dynamics observed during the recent financial crisis.

We study our model’s response to a sequence of aggregate shocks like those realized during the recent financial crisis. Starting with the distribution of households over states implied by our model in 2004, we fix the utility cost of default $\chi_t = \chi_{2004}$ for all $t \geq 2005$ and simulate through 2014 using the same approach as is described in the previous section.

The model implied time series for the unsecured debt-to-income ratio are depicted in Figure 1.1 for four different scenarios. Case 1 corresponds to our model with $\gamma_a = 0.12$ and $\gamma_d = 0.5$. In this scenario, the unsecured debt-to-income ratio rises by 8.6% between 2004 and 2007 and then falls by 10.5% between 2007 and 2011. This parameterization includes both the supply and demand factors that reduce aggregate borrowing that we mentioned in the introduction. Households demand less debt because they believe it is more likely the economy stays in a state with relatively low mean income and high idiosyncratic volatility. In addition, bankruptcy rates during the financial crisis exceed creditor expectations, causing

³⁰Similar patterns are seen for lending standards for only credit card loans, which are closely related to our focus on unsecured debt.

Figure 1.1: Model Implied Unsecured Debt-to-Income Ratio



creditors to tighten lending standards by demanding a higher default premium. As a result of these forces, the model with both forms of learning is able to produce a steep and protracted decline in the household debt-to-income ratio like that seen in the data.

In Case 2 we leave $\gamma_a = 0.12$ and set $\gamma_d = 0.0$. This experiment isolates the impact of learning about the aggregate state transition matrix. Whereas the unsecured debt-to-income ratio rises by about the same amount as Case 1, the subsequent drop is less severe and the recovery is faster. This suggests that creditor learning about default probabilities has an asymmetric impact on household borrowing: its influence is minimal during credit expansions but substantial during collapses.³¹ Therefore, it appears that learning by households about the aggregate state transition probabilities is most important for generating the initial boom phase and sharp reduction in lending following the crisis, while tighter lending standards amplify the depth and protracted nature of the financial crisis.

Case 3 corresponds to the scenario in which we leave $\gamma_d = 0.5$ and set $\gamma_a = 0.0$, while

³¹A recent paper by Bassett, Chosak, Driscoll, and Zakrajsek (2011) provides empirical evidence for this asymmetry, finding that a tightening shock to lending standards has a substantial impact on the macroeconomy while easing shocks have virtually no impact.

Case 4 considers the model’s predictions when $\gamma_a = \gamma_d = 0$. The time series implied by these simulations are nearly indistinguishable, and the reduction in the unsecured debt-to-income ratio is far less severe than the scenarios that consider learning about the aggregate state. Without learning about the aggregate state, the unsecured debt-to-income ratio never falls below its level in 2004 during the financial crisis, which is at odds with the data. This implies that learning about default probabilities by creditors, on its own, is insufficient to generate a credit crunch.

While many existing studies explore the implications of an exogenous tightening in either transaction costs or credit constraints, our model generates a credit crunch through the endogenous response of both households and creditors to a sequence of unfavorable aggregate shocks.³² In particular, a period of elevated idiosyncratic income volatility causes households to become more pessimistic about the future, increase their precautionary savings, and reduce borrowing. Increased income volatility also results in default rates on loan contracts in excess of lenders’ expectations. In response, creditors increase the default premium charged on debt contracts. This credit tightening not only reduces households’ incentives to borrow, but it also makes it more difficult for indebted households to roll over their debt, leading to a further increase in default rates. Hence, in our model, unfavorable aggregate shocks lead to an endogenous and persistent contraction in both the supply and demand for credit.

1.6 Conclusion

Is learning by households and creditors important for explaining the dynamics of household unsecured debt since the start of the Great Moderation? Our analysis suggests that it is. We develop a model of optimal default in which households and creditors learn about economic fundamentals in the presence of aggregate uncertainty. We show that in response to a sequence of favorable aggregate shocks, similar to those realized during the Great Moderation, households begin to discount the probability of transitioning to a recession and creditors

³²See, for example, Guerrieri and Lorenzoni (2011) and Buera and Moll (2012).

reduce their default expectations. As a result, households' precautionary savings motive is reduced and interest rates fall, both of which lead to increased household borrowing.

We also demonstrate that learning can help explain the severe and protracted reduction in unsecured debt that occurred during the recent financial crisis. In response to lower mean income and elevated idiosyncratic income volatility during the crisis, households increase their beliefs that the economy will remain in an unfavorable state and reduce their demand for debt accordingly. The fact that default rates exceed creditor expectations during the crisis leads creditors to tighten lending standards. The result of these forces is that the overall decline in borrowing during the crisis is more severe and the economy takes significantly longer to return to pre-crisis debt levels in a model with learning. These facts are consistent with the debt dynamics observed during the recent financial crisis. Learning therefore appears to be a fruitful avenue for understanding credit booms that often accompany prolonged economic expansions, as well as the severity and persistence of financial crises.

1.7 Appendix A: Proofs

Lemma 1.2: We show that T_q is a contraction mapping by proving that it satisfies Blackwell's sufficient conditions for a contraction. In this lemma we drop the t subscripts on the value functions and household expectations for ease of notation.

- **Monotonicity:** Suppose $W^G(b, y; s) \leq V^G(b, y; s)$ for all $\{b, y, s\}$. Then:

$$\begin{aligned} (T_q V^G)(b, y; s) &= \max \left\{ V^D(y; s), \max_{b'} \left\{ u(y + b - q(b', y; s)b') + \beta \mathbb{E} [V^G(b', y'; s')] \right\} \right\} \\ &\geq \max \left\{ V^D(y; s), \max_{b'} \left\{ u(y + b - q(b', y; s)b') + \beta \mathbb{E} [W^G(b', y'; s')] \right\} \right\} \\ &= (T_q W^G)(b, y; s) \end{aligned}$$

- **Discounting:** Let $a \in \mathbb{R}_+$. Then

$$\begin{aligned} (T_q(V^G + a))(b, y; s) - (T_q V^G)(b, y; s) &= \max \{ V^D(y; s), \max_{b'} \{ u(y + b - q(b', y; s)b') \\ &\quad + \beta \mathbb{E} [V^G(b', y'; s') + a] \} \} - (T_q V^G)(b, y; s) \\ &\leq \max \{ V^D(y; s), \max_{b'} \{ u(y + b - q(b', y; s)b') \\ &\quad + \beta \mathbb{E} [V^G(b', y'; s')] \} \} + \beta a - (T_q V^G)(b, y; s) \\ &= \beta a \end{aligned}$$

Thus, the operator is a contraction mapping, and there exists a unique fixed point by the contraction mapping theorem. Denote the fixed point associated with the operator T_q as $V_q^*(b, y; s)$.

Theorem 1.3: We will show that if $q_{t+1}(b, y; s) \geq q_t(b, y; s)$ for all $(b, y; s)$ then $V_{q_{t+1}}^*(b, y; s) \geq V_{q_t}^*(b, y; s)$, i.e. that the fixed point under the $T_{q_{t+1}}$ operator is at least as large as the fixed point under the T_{q_t} operator for the entire state space. Since the value of default is invariant to the bond price schedule, this is equivalent to showing that the value of not defaulting is at least as large under q_{t+1} as under q_t for the entire state space.

Let $V_{q_t}^*(b, y; s)$ be the unique fixed point under q_t with associated policy functions $b_{q_t}^*(b, y; s)$ and $D_{q_t}^*(b, y; s)$. Applying the operator under q_{t+1} to this fixed point gives us:

$$\begin{aligned}
(T_{q_{t+1}} V_{q_t}^*)(b, y; s) &= \max \left\{ V^D(y; s), \max_{b'} \left\{ u(y + b - q_{t+1}(b', y; s)b') + \beta \mathbb{E} [V_{q_t}^*(b', y'; s')] \right\} \right\} \\
&\geq \max \{ V^D(y; s), u(y + b - q_{t+1}(b_{q_t}^*(b, y; s), y; s)b_{q_t}^*(b, y; s)) \\
&\quad + \beta \mathbb{E} [V_{q_t}^*(b_{q_t}^*(b, y; s), y'; s')] \} \\
&\geq \max \{ V^D(y; s), u(y + b - q_t(b_{q_t}^*(b, y; s), y; s)b_{q_t}^*(b, y; s)) \\
&\quad + \beta \mathbb{E} [V_{q_t}^*(b_{q_t}^*(b, y; s), y'; s')] \} \\
&= V_{q_t}^*(b, y; s)
\end{aligned}$$

Successively applying the operator $T_{q_{t+1}}$ gives a non-decreasing sequence of functions, all at least as large as $V_{q_t}^*(b, y; s)$, that converges to some limit – the fixed point under $T_{q_{t+1}}$: $V_{q_{t+1}}^*(b, y; s)$. Thus, $V_{q_{t+1}}^*(b, y; s) \geq V_{q_t}^*(b, y; s)$. Moreover, since $V_{q_{t+1}}^D(y; s) = V_{q_t}^D(y; s)$ for all t , we conclude $V_{q_{t+1}}^R(b, y; s) \geq V_{q_t}^R(b, y; s)$. As a result, $V_{q_{t+1}}^R(\tilde{b}_t^{y,s}, y; s) \geq V_{q_t}^R(\tilde{b}_t^{y,s}, y; s) = V^D(y; s)$, and the debt thresholds under q_{t+1} are no greater than the thresholds under q_t .

Theorem 1.4: Suppose at date t , $s_t = s^j \in X_t(b', y)$ is realized. Then learning implies

$$\tilde{D}_{t+1}(b', y; s) = \gamma_d DR_t(b', y) + (1 - \gamma_d) \tilde{D}_t(b', y; s).$$

Since $s^j \in X_t(b', y)$, $DR_t(b', y) \leq \tilde{D}_t(b', y; s)$. Thus,

$$\tilde{D}_{t+1}(b', y; s) = \gamma_d DR_t(b', y) + (1 - \gamma_d) \tilde{D}_t(b', y; s) \leq \gamma_d \tilde{D}_t(b', y; s) + (1 - \gamma_d) \tilde{D}_t(b', y; s).$$

Therefore, $\tilde{D}_{t+1}(b', y; s) \leq \tilde{D}_t(b', y; s)$ and $q_{t+1}(b', y; s) \geq q_t(b', y; s)$.

Now suppose at date t , $s_t = s^j \notin X_t(b', y)$ is realized. Then learning implies

$$\tilde{D}_{t+1}(b', y; s) = \gamma_d DR_t(b', y) + (1 - \gamma_d) \tilde{D}_t(b', y; s).$$

Since $s^j \notin X_t(b', y)$, $DR_t(b', y) \geq \tilde{D}_t(b', y; s)$. Thus,

$$\tilde{D}_{t+1}(b', y; s) = \gamma_d DR_t(b', y) + (1 - \gamma_d) \tilde{D}_t(b', y; s) \geq \gamma_d \tilde{D}_t(b', y; s) + (1 - \gamma_d) \tilde{D}_t(b', y; s).$$

Therefore, $\tilde{D}_{t+1}(b', y; s) \geq \tilde{D}_t(b', y; s)$ and $q_{t+1}(b', y; s) \leq q_t(b', y; s)$.

Theorem 1.6: Suppose initial beliefs $\tilde{D}_0(b', y; s_{-1})$ are such that there exists state k such that $DR_0(b', y) \leq \tilde{D}_0(b', y; s_{-1})$. Therefore, $X_0(b', y) \neq \emptyset$. By Theorem 1.4, if $s_0 \in X_0(b', y)$, $\tilde{D}_1(b', y; s_{-1}) \leq \tilde{D}_0(b', y; s_{-1})$ and $q_1(b', y; s_{-1}) \geq q_0(b', y; s_{-1})$.

Next, we show that if the initial set is non-empty, then it is non-empty for all t . Note that $\tilde{D}_1(b', y; s_{-1}) = \gamma_d DR_0(b', y) + (1 - \gamma_d) \tilde{D}_0(b', y; s_{-1}) \geq DR_0(b', y)$ since $s^k \in X_0(b', y)$. By Theorem 1.3, the debt thresholds are decreasing over time in response to higher bond prices, so that $DR_1(b', y) \leq DR_0(b', y)$. Therefore, $DR_1(b', y) \leq DR_0(b', y) \leq \tilde{D}_1(b', y; s_{-1})$ so that $X_1(b', y) \neq \emptyset$. An analogous argument shows that if $X_t(b', y) \neq \emptyset$ for any arbitrary t , then $X_{t+1}(b', y) \neq \emptyset$, and if we continue to draw from $X_t(b', y)$, $\tilde{D}_{t+1}(b', y; s_{-1}) \leq \tilde{D}_t(b', y; s_{-1})$ and $q_{t+1}(b', y; s_{-1}) \geq q_t(b', y; s_{-1})$ for all t .

Thus, $\tilde{D}_t(b', y; s_{-1})$ is a non-increasing sequence bounded below by 0. By the monotone convergence theorem, $\tilde{D}_t(b', y; s_{-1})$ must converge to some value. Call this limiting value $\tilde{D}_\infty(b', y; s_{-1}) \geq 0$. Similarly, since $q_t(b', y; s_{-1})$ is a non-decreasing sequence bounded above by $1/((1+r)(1+\tau))$, it too must converge to some limit $q_\infty(b', y; s_{-1})$.

1.8 Appendix B: Calibrating the Mean of the Endowment Process

To estimate μ_s , we first classify the years 1969 through 1991 as either high or low average income using annual changes in the national unemployment rate. If the unemployment rate increased by more than 1.3 percentage points, then the current year is classified as low average income. If the previous year is classified as low average income, then the current year is also classified as low average income if the decrease in the unemployment rate is less than 2/3 of the increase in the previous year. All other years are classified as high average

income.

We follow Storesletten et al. (2004) and construct a repeated panel from the PSID survey years 1968-1993. We extract income data, the age of the head, and the education level of the head from the PSID main family data files, along with the 1968 interview number and relationship to the head from the PSID individual data files, for all individuals across the PSID survey years 1968-1993. We then restrict our panel to include only those individuals who are members of, or are related to, a family that was included in the 1968 SRC cross-section sample. We define income to be the log of real income in 1968 dollars (deflated by the CPI) at the family level which is the sum of the head and wife's labor income, unemployment compensation, workers compensation, and help from relatives. Income attributed to the head of the household is then defined as total income divided by the number of persons in the family unit.

We select observations on individuals in each survey year into our panel if: (1) they are in the original sample in the previous year and the following year, (2) income is positive in the previous, current, and following year, (3) income growth rate is not less than 1/20 and not larger than 20 between the previous year and the current year or between the current year and the following year, and (4) the individual's age is between 22 and 60 years in the current year.

We then perform the following regression to isolate fixed effects associated with aggregate income, education, and age:

$$y_{it}^h = \theta_0 + \theta_1 \mathbb{D}_h + \theta_2 t + \theta_3^T \mathbf{x}_{it}^h + u_{it}^h,$$

where y_{it}^h is log (per capita) income (at the household level), $\mathbb{D}_h$ is a dummy variable that we set equal to 1 if the year is classified as low unemployment (high mean income), t is a time trend, and $\mathbf{x}_{it}^h$ is a vector composed of age, age squared divided by 100, age cubed divided by 10,000, and years of education completed for individual i of age h at date t .³³

³³This regression also identifies the idiosyncratic, uninsurable component of the income process, u_{it}^h , which

Our estimates for the state-dependent mean of the income process are thus given by $\mu_h = \theta_0 + \theta_1$ for the high mean income state and $\mu_l = \theta_0$ for the low mean income state.

Storesletten et al. (2004) use to estimate ρ , η_e and η_c .

2 Bankruptcy Reform and the Housing Crisis

2.1 Introduction

Prior to 2005, the availability of debt relief through bankruptcy was widely known, the cost of filing was low, and little stigma was attached to those who filed. Bankruptcy was thus an attractive option for homeowners that wished to remain in their homes and could afford their mortgage payments if relieved of other debt obligations, such as credit card bills. This changed in 2005 as the Bankruptcy Abuse Prevention and Consumer Protection Act (BAPCPA) significantly raised the cost of filing and reduced the amount of debt that could be discharged. These changes to the bankruptcy code made it more difficult for struggling homeowners to loosen their budget constraints via bankruptcy, increasing the relative attractiveness of mortgage default. As a result, bankruptcy reform may have contributed to the severity of the housing crisis by inducing some homeowners to default that would have otherwise chosen to declare bankruptcy and keep their homes.

To understand exactly how the BAPCPA affected homeowners' incentives, consider a homeowner with negative home equity who, prior to the BAPCPA, could have had their unsecured debt discharged under Chapter 7 and remained in their home. With the introduction of the BAPCPA, this homeowner's ability to discharge their unsecured debt through Chapter 7 now depends on their income. In particular, if the homeowner has income above their state's median, they cannot file under Chapter 7 and are instead forced to file under Chapter 13 and enter into a repayment plan to which they must commit all of their non-exempt income for five years. Thus, bankruptcy became more costly for such a homeowner. If bankruptcy and mortgage default are substitutes, this higher cost will induce some households to default on their mortgage that would not have done so in the absence of the reform.³⁴ And because negative home equity is a necessary condition for mortgage default,

³⁴We think of bankruptcy and mortgage default as being complements or substitutes just as we would any other goods. That is, they are substitutes (complements) if raising the cost of one increases (decreases) the incidence of the other.

the large decline in house prices that forced many homeowners underwater on their mortgage during the recent housing crisis may have amplified this rise in the mortgage default rate.³⁵

Empirical work on the BAPCPA has reinforced this intuition. Li, White, and Zhu (2011), for example, argue that homeowners treated bankruptcy and mortgage default as substitutes in response to the BAPCPA, shifting from bankruptcy to default when the cost of the former rose. Using data on individual mortgages from LPS Analytics, these authors estimate that the BAPCPA increased the probability of default by 24% for prime borrowers and 14% for sub-prime borrowers with mortgages originated in 2004 and 2005. In a complementary study, Morgan, Iverson, and Botsch (2011) document a significant rise in the default rate of subprime mortgages in response to the BAPCPA. Although neither of these studies explicitly consider data from the housing market crash, their conclusions support the view that the BAPCPA may have increased the number of mortgage defaults during the housing crisis, thereby contributing to the severe and protracted decline in home prices.

Although this empirical work suggests that making bankruptcy more costly may have worsened the housing crash, theoretically this conclusion is ambiguous. While increasing the cost of filing for bankruptcy raises the relative attractiveness of mortgage default, rational mortgage lenders will respond by tightening lending standards on those households who are more likely to default to offset the potential for greater losses. Tighter lending standards, in turn, will tend to discourage these households from taking out a mortgage to purchase a home. Importantly, this effect is concentrated on households who bought homes in 2005 and 2006 – exactly those homeowners who are most likely to find themselves underwater as a result of a collapse in house prices – and works to reduce the mortgage default rate during the crisis. Given the presence of these opposing forces, the net impact on mortgage defaults could be either positive or negative depending on the relative magnitude of each effect.

In this paper, we quantify the effects of the BAPCPA on the housing market crash of 2007 using a quantitative-theoretic, equilibrium model of the U.S. housing market. In our

³⁵If a household has positive home equity net of transaction costs, then selling their home and repaying their mortgage will always dominate the option to default.

framework, households optimally choose between renting and owning their housing space and can finance the purchase of a home by taking out a mortgage. Households interact in credit markets with rational lenders who provide unsecured credit and mortgage loans at terms that fully reflect the general equilibrium incentives each household has to renege on their obligations. Each period, homeowners optimally choose between remaining in or selling their home, filing for bankruptcy, defaulting on their mortgage, or simultaneously declaring bankruptcy and defaulting on their mortgage. Thus, our model is rich enough to determine whether tighter mortgage lending standards in the years prior to the crisis dominated the increased attractiveness of mortgage default during the crisis.

We calibrate our model to match salient characteristics of the U.S. economy prior to 2005 and then conduct several tests to ensure that our model adequately captures key empirical facts regarding the BAPCPA and the housing market crash. First, we discipline the model to match the empirical findings of Li et al. (2011) by calibrating the bankruptcy cost under reform to produce a rise in the mortgage default rate of 21.6% for new homeowners in response to the BAPCPA.³⁶ Next, we test whether the model produces a decline in house prices and a rise in mortgage default rates, on the order of that found in the data, in response to a housing crash. Following Chatterjee and Eyigungor (2011b), we model this crash as an unexpected increase in the economy’s owner-occupied housing supply and find that our model is able to capture a decline in house prices and rise in mortgage default rates similar to the data. The model also replicates key dynamics in the bankruptcy filing rate, unsecured debt-to-income ratio, and price-rent ratio during the crash. The fact that our model is able to replicate these empirical facts gives us confidence about its implications for the counterfactual exercise that is central to our analysis. As our main quantitative experiment, we construct a counterfactual transition in the U.S. economy in which there is no bankruptcy reform in 2005 but the economy still undergoes a housing crisis in 2007. We then compare the data from this housing crisis to an economy that implemented bankruptcy

³⁶Throughout we will refer to homeowners that purchased their home in the previous period as new homeowners.

reform in 2005.

Contrary to existing arguments in the empirical literature, our results suggest that the BAPCPA did not contribute significantly to the severity of the housing crisis. In particular, the mortgage default rate is only 2.7% higher in 2007 while the path of house prices during the crisis is virtually unaltered as a result of bankruptcy reform.

In our model, bankruptcy and mortgage default appear to be treated as substitutes by households in response to the BAPCPA, as its implementation leads to lower bankruptcy and higher mortgage default rates in the aggregate. Indeed, some households find it optimal to default on their mortgage in states where they would have optimally decided to declare bankruptcy in the absence of BAPCPA. Bankruptcy and mortgage default are substitutes for these households, and by making bankruptcy more costly to file, mortgage default becomes more likely. However, there are additional forces at work in our model that generate our results.

Prior to the BAPCPA, a household's home equity in excess of their state's homestead exemption would be paid to creditors in the event of bankruptcy. Non-exempt home equity thus served as collateral for unsecured debt contracts, leading to lower interest rates for homeowners with high home equity. Under the BAPCPA, homeowners with high income relative to their non-exempt home equity are forced into a repayment plan, meaning that their non-exempt home equity no longer serves as collateral for their unsecured debt obligations. For these households, interest rates on unsecured debt are now independent of their homeownership status, which reduces the benefits of homeownership. While homeowners with negative home equity are not directly impacted by this change, the continuation value of remaining in their home falls, inducing the marginal homeowner to default on their mortgage.³⁷

Second, since the BAPCPA increased the cost of bankruptcy for high-income households,

³⁷In order for default to be optimal in our model, a homeowner must not only have negative home equity (a necessary condition for default), but must also want to move. Homeowners want to move in our model because shocks to their income, assets or house size have made their current mortgage-house combination suboptimal.

making it less likely that these households will declare bankruptcy, unsecured creditors are able to offer lower interest rates and engage in more risky lending.³⁸ If we assume that a household must repay their unsecured debt if they default on their mortgage (and do not simultaneously declare bankruptcy), bankruptcy and mortgage default are complementary as the former reduces the costs associated with the latter. A rise in risky lending that leads to an increase in bankruptcy filings will thus also tend to cause an increase in the mortgage default rate.

In our model, rational mortgage lenders internalize these changes in homeowners' incentives and respond by tightening lending standards in the years prior to the housing crisis. Higher mortgage interest rates lead new homebuyers to choose smaller homes with lower initial loan-to-income, loan-to-value, and mortgage payment-to-income ratios, on average. These mortgage contracts are inherently less risky, making these new homebuyers far less likely to default on their mortgage during the crisis. This force offsets the increased attractiveness of default and ultimately drives our conclusion that the BAPCPA caused only a slightly higher default rate during the housing crisis and had minimal effect on the severity of the drop in house prices. Accounting for the general equilibrium response of unsecured debt and mortgage interest rates to changes in households' incentives is therefore crucial to adequately assess the impact of the BAPCPA on the housing crisis.

Finally, we use our framework to consider the impact of mortgage cram down, which has been extensively discussed in policy circles and in the academic literature, on the severity of the housing crisis. Under this policy, homeowners with negative home equity are able to treat the portion of their mortgage that exceeds the value of their home as unsecured debt, which can then be discharged through the bankruptcy process. The objective of this policy would be to reduce mortgage default rates and positively impact owner-occupied house prices through a feedback effect from defaults on prices. Our analysis suggests that this policy would have slightly reduced mortgage default rates during the recent housing

³⁸Risky lending refers to unsecured debt contracts for which the household's bankruptcy decision in the following period is not trivial.

crisis, while producing a substantially higher bankruptcy rate and having minimal impact on aggregate house prices.

The remainder of the paper is structured as follows. In the next section we briefly discuss papers that are relevant to our current analysis. Section 2.3 then provides a detailed description of our full quantitative framework prior to bankruptcy reform. Next, Section 2.4 describes the BAPCPA and specifies how we model this reform in our quantitative analysis. The following section presents our parameterization and the model fit to the pre-crash period. Section 2.6 details our quantitative results, describing the effect of the BAPCPA on impact and during the housing crash and discussing the intuition for our findings. The impact of the hypothetical cram down program is then assessed in Section 2.7. Finally, Section 2.8 concludes.

2.2 Literature Review

Several recent papers aim to isolate and quantify the effects of the BAPCPA. Much of this work is empirical in nature.³⁹ As described in the introduction, the most relevant for our work are Li et al. (2011) and Morgan et al. (2011) who document that mortgage default rates increased in response to the BAPCPA. A primary benefit of our quantitative approach relative to their empirical analysis is that we are able to construct the counterfactual experiment that these authors envision.

In response to the recent housing crisis, there is a rapidly growing literature that aims to explain the rise in mortgage defaults and decline in house prices using quantitative models of the U.S. housing market. Corbae and Quintin (2011), for example, assess the importance of mortgage innovations, through the introduction of non-traditional mortgages, and conclude that this channel can explain approximately 40% of the rise in foreclosures during the crisis. Recent work by Jeske, Krueger, and Mitman (2011) evaluates the impact of in-

³⁹Two notable exceptions are Chatterjee et al. (2007) and Li and Sarte (2006), who quantitatively analyze the impact of introducing means testing, in the spirit of the BAPCPA, on the consumer bankruptcy decision but do not consider the reform's impact on mortgage default decisions.

terest rate subsidies for the government sponsored enterprises on housing market outcomes. They determine that these subsidies substantially increase mortgage origination and lower aggregate welfare, but have little impact on default rates. Closely related to our study is that of Chatterjee and Eyigungor (2011b) who demonstrate that an unexpected increase in the supply of owner-occupied housing – along with frictions in the mortgage market and foreclosure delays – can go a long way toward explaining the sharp increase in foreclosures and precipitous drop in home prices during the housing crash.

Each of these quantitative studies, though, abstracts from unsecured credit and thus from the bankruptcy versus mortgage default decision. Mitman (2011) takes up this task and exploits variations in homestead exemptions and recourse laws across states to demonstrate that while bankruptcy rates are lower in states with higher homestead exemptions, foreclosure rates are higher. Mitman also examines his model’s predictions for the long-run effects of the BAPCPA, but does not explore the implications of bankruptcy reform for the severity of the housing crisis, which is the primary focus of our analysis.

Moreover, Mitman (2011) models mortgages as one-period contracts and abstracts from the transaction costs associated with buying and selling a home. Although these assumptions improve analytical tractability and perhaps are appropriate for a steady state analysis, the inherent risks to both households and lenders in a long-term mortgage contract, such as changes in income and house prices, are of first-order importance for our dynamic analysis. It is therefore crucial that we model mortgages as long-term contracts and explicitly account for transaction costs to adequately assess how the BAPCPA impacted the subsequent housing crisis. To the best of our knowledge, ours is the first paper to allow for both short-term, unsecured debt and long-term, collateralized mortgage loans in a model of optimal consumer default.⁴⁰

⁴⁰In fact, the only other model, again to the best of our knowledge, to simultaneously consider short and long-term debt is Arellano and Ramanarayanan (2010), who consider unsecured debt instruments of different maturities in a sovereign default model.

In considering long-term debt, we build on the work introducing longer maturity bonds into models of sovereign default by Chatterjee and Eyigungor (2011a) and Hatchondo and Martinez (2009) and consumer default by Chatterjee and Eyigungor (2011b).

2.3 Model Economy

We consider an environment in which time is discrete and infinite. The economy is populated by a continuum of infinitely lived households, a pool of perfectly competitive, risk-neutral financial intermediaries, and a government. There is an exogenous and perfectly elastic supply of a homogeneous consumption good which is taken as the numeraire. The economy also has exogenous and perfectly divisible supplies of owner-occupied (K_t) and rental (H_t) housing space with prices P_t and R_t at date t , respectively. Households derive utility from consumption and the size of their housing space. Financial intermediaries accept deposits and offer competitively priced one-period unsecured debt contracts and multi-period mortgages, the latter of which households can use to help finance the purchase of housing space. The government levies income taxes on households but does not provide transfers or goods and services that affect the household's problem.

2.3.1 Households

Households are heterogeneous with respect to their homeownership status, house size k_t , mortgage payment x_t , assets a_t , endowment y_t , and credit status. We use $k_t = 0$ and $x_t = 0$ to denote a household that does not own a home and therefore does not have a mortgage.

Households in our model face three sources of uncertainty. First, each household receives an idiosyncratic and stochastic endowment y_t each period, the log of which evolves according to a first-order autoregressive process:

$$\log(y_t) = \rho \log(y_{t-1}) + \varepsilon_t$$

where $\varepsilon_t \sim N(0, \sigma)$ is i.i.d over time and across households. Second, owner-occupied housing is subject to idiosyncratic proportional depreciation shocks, δ_t , that are i.i.d. across households and time.⁴¹ The value of this shock is given by:

⁴¹We introduce this feature to capture two important characteristics of the U.S. housing market: (1) homeowners occasionally choose to default on their mortgage obligations, and (2) homeowners move fre-

$$\delta_t = \begin{cases} \bar{\delta} & \text{with probability } \phi \\ 0 & \text{otherwise} \end{cases}$$

where $\bar{\delta} > 0$. A household that exits period t with a house of size k_t and experiences depreciation shock δ_{t+1} enters period $t + 1$ with a house of size $k_{t+1} = (1 - \delta_{t+1})k_t$. Finally, households are subject to an idiosyncratic expense shock, e_t , which directly reduces the assets with which they enter the period.⁴² This expense shock is also assumed to be i.i.d. across households and time, and its value is given by:

$$e_t = \begin{cases} \bar{e} & \text{with probability } \xi \\ 0 & \text{otherwise} \end{cases}$$

where $\bar{e} > 0$. A household that exits period t with assets a_{t+1}^* , which they have optimally chosen, and experiences expense shock e_{t+1} enters period $t + 1$ with assets $a_{t+1} = a_{t+1}^* - e_{t+1}$.

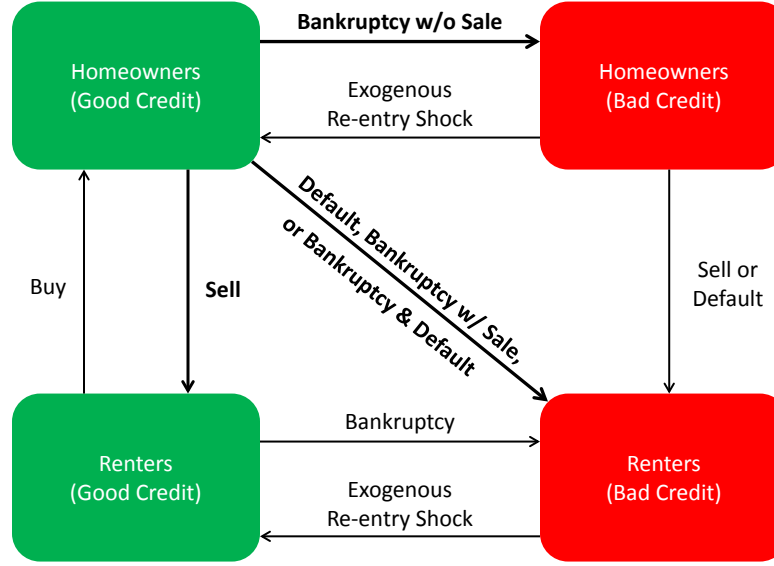
Figure 2.1 depicts how households move between different homeownership and credit statuses in our model. For example, a household that enters the period as a homeowner with good credit can become (i) a *homeowner with bad credit* by declaring bankruptcy and having home equity less than the homestead exemption, (ii) a *renter with bad credit* by defaulting on their mortgage, declaring bankruptcy and defaulting on their mortgage, or declaring bankruptcy with home equity in excess of the homestead exemption, or (iii) a *renter with good credit* by selling their home.⁴³

quently. Depreciation shocks create the potential for negative home equity, a prerequisite for mortgage default, in a steady state in which owner-occupied house prices are constant. These shocks also tend to result in a suboptimal combination of mortgage loan and house size given a household's assets and income, which is the main reason why homeowners choose to move in our model.

⁴²Expense shocks are meant to capture unanticipated household expenses relating to medical expenses, divorce costs, unexpected births of children, among others, which are commonly cited by bankrupts as contributing to their decision to file. See Livshits, MacGee, and Tertilt (2007) and Livshits et al. (2010) for further discussion on the importance of expense shocks for the consumer bankruptcy decision.

⁴³We will use the term “good credit” to mean a household that has access to credit markets and “bad credit” to mean a household that is excluded from credit markets due to a past bankruptcy filing and/or mortgage default.

Figure 2.1: Summary of Household's Problem



We begin by describing the decision problems for a homeowner with good credit because their decision to file for bankruptcy versus defaulting on their mortgage is affected by the BAPCPA and is thus the focus of our analysis.

Problem of a Homeowner with Good Credit

A homeowner with good credit must decide between making their mortgage payment and continuing as a homeowner (O_t), selling their home (S_t), defaulting on their mortgage (D_t), filing for bankruptcy (B_t), or both filing for bankruptcy and defaulting on their mortgage (BD_t). The value of having this decision is given by:

$$V_t(k_t, x_t, a_t, y_t) = \max_{O_t, S_t, D_t, B_t, BD_t} \{O_t(k_t, x_t, a_t, y_t), S_t(k_t, x_t, a_t, y_t),$$

$$D_t(a_t, y_t), B_t(k_t, x_t, y_t), BD_t(y_t)\}$$

The value associated with making their mortgage payment and continuing as a homeowner is

$$O_t(k_t, x_t, a_t, y_t) = \max_{c_t, a_{t+1}^*} u(c_t, k_t) + \beta \mathbb{E}_t[V_{t+1}(k_{t+1}, x_{t+1}, a_{t+1}, y_{t+1}) | y_t]$$

subject to

$$c_t + q_t(k_t, x_t, a_{t+1}^*, y_t) a_{t+1}^* + x_t = y_t - g(x_t, a_t, y_t) + a_t$$

$$y_t - g(x_t, a_t, y_t) + a_t \geq x_t$$

$$k_{t+1} = (1 - \delta_{t+1})k_t, \quad x_{t+1} = \mu x_t, \quad a_{t+1} = a_{t+1}^* - e_{t+1}$$

The first constraint is the household's budget constraint, where $q_t(k_t, x_t, a_{t+1}^*, y_t)$ is the price of a one-period unsecured debt contract for a household with house size k_t , mortgage payment x_t , and endowment y_t that wishes to carry assets a_{t+1}^* into the following period. Here $g(x_t, a_t, y_t)$ represents the income tax levied by the government on a household with the given characteristics. The second constraint restricts the household from paying their mortgage with unsecured debt by ensuring that their mortgage payment does not exceed their after-tax income plus their resources from their bond holdings with which they entered the period, net of the expense shock. The final three constraints represent the laws of motion for the household's home size, mortgage payment, and assets. While we discuss in detail our assumptions about mortgage contracts in the following section, for now it suffices to convey that mortgage payments decay over time at the constant rate $\mu \in (0, 1)$. Moreover, while the household chooses assets a_{t+1}^* with which to exit the period, the assets with which they enter the following period depend on the realized expense shock e_{t+1} . Note that the expectation on the right hand side of the value function is taken with respect to all three sources of uncertainty: the household's next period endowment, depreciation shock, and expense shock.

If instead they choose to sell their home, they receive the proceeds from the sale $P_t k_t$

less a proportional transaction cost χ_S . The household must also repurchase their mortgage contract from the lender for an amount equal to the present value of the promised stream of decaying mortgage payments, discounted at the risk-free interest rate. We assume that the sale and purchase of housing space occurs at the beginning of each period, and therefore, the household must rent housing space in the current period. The value of selling is thus:

$$S_t(k_t, x_t, a_t, y_t) = \max_{c_t, a_{t+1}^*, h_t} u(c_t, h_t) + \beta \mathbb{E}_t[V_{t+1}(0, 0, a_{t+1}, y_{t+1})|y_t]$$

subject to

$$c_t + q_t(0, 0, a_{t+1}^*, y_t)a_{t+1}^* + R_th_t = y_t - g(0, a_t, y_t) + a_t + P_t k_t(1 - \chi_S) - \left(1 + \frac{\mu}{r + 1 - \mu}\right) x_t$$

$$a_{t+1} = a_{t+1}^* - e_{t+1}$$

The household may also decide to default on their mortgage. In this case they are relieved of their mortgage payment but must relinquish their home to the lender. The household must also rent housing space in the current period and is temporarily excluded from credit markets but may save. We assume that households with bad credit re-enter credit markets with probability λ each period. Hence, the value of defaulting is

$$D_t(a_t, y_t) = \max_{c_t, a_{t+1}^* \geq 0, h_t} u(c_t, h_t) + \beta \mathbb{E}_t[\lambda V_{t+1}(0, 0, a_{t+1}, y_{t+1}) + (1 - \lambda)X_{t+1}(0, 0, a_{t+1}, y_{t+1})|y_t]$$

subject to

$$c_t + q_t(0, 0, a_{t+1}^*, y_t)a_{t+1}^* + R_th_t = y_t - g(0, a_t, y_t) + a_t$$

$$a_{t+1} = a_{t+1}^* - e_{t+1}$$

where $X_{t+1}(0, 0, a_{t+1}, y_{t+1})$ is the value of being a renter with bad credit. Note that the value

of defaulting is independent of k_t and x_t since the household loses their home and is relieved of their mortgage in the current period.

Alternatively, a household may choose to file for bankruptcy and have their unsecured debt obligations discharged in exchange for a one-time utility cost $\nu > 0$ and temporary exclusion from credit markets. In addition, a household that files for bankruptcy may face either a one-time endowment cost $\omega_t(y_t)$ or be forced to sell their home. Homeowners who declare bankruptcy and are forced to sell their home are allowed to retain any home equity up to the homestead exemption ζ and must rent housing space in the current period.⁴⁴ We therefore divide the value of bankruptcy into two distinct pieces:

(1) The household is forced to sell their home:

$$B_t(k_t, x_t, y_t) = \max_{c_t, a_{t+1}^* \geq 0, h_t} u(c_t, h_t) - \nu + \beta \mathbb{E}_t[\lambda V_{t+1}(0, 0, a_{t+1}, y_{t+1}) \\ + (1 - \lambda) X_{t+1}(0, 0, a_{t+1}, y_{t+1}) | y_t]$$

subject to

$$c_t + q_t(0, 0, a_{t+1}^*, y_t) a_{t+1}^* + R_t h_t = y_t - g(0, 0, y_t) + \zeta$$

$$a_{t+1} = a_{t+1}^* - e_{t+1}$$

Note that the value of filing for bankruptcy is independent of the household's debt since it is entirely discharged.

(2) The household is allowed to keep their home:

$$B_t(k_t, x_t, y_t) = \max_{c_t, a_{t+1}^* \geq 0} u(c_t, k_t) - \nu + \beta \mathbb{E}_t[\lambda V_{t+1}(k_{t+1}, x_{t+1}, a_{t+1}, y_{t+1})$$

⁴⁴We state the conditions under which a household that declares bankruptcy is forced to sell their home in the following section.

$$+(1-\lambda)X_{t+1}(k_{t+1}, x_{t+1}, a_{t+1}, y_{t+1})|y_t]$$

subject to

$$c_t + q_t(k_t, x_t, a_{t+1}^*, y_t)a_{t+1}^* + x_t = y_t - g(x_t, 0, y_t) - \omega_t(y_t)$$

$$k_{t+1} = (1 - \delta_{t+1})k_t, \quad x_{t+1} = \mu x_t, \quad a_{t+1} = a_{t+1}^* - e_{t+1}$$

Finally, we allow a household to simultaneously file for bankruptcy and default on their mortgage. The value of doing so is:

$$BD_t(y_t) = \max_{c_t, a_{t+1}^* \geq 0, h_t} u(c_t, h_t) - \nu + \beta \mathbb{E}_t[\lambda V_{t+1}(0, 0, a_{t+1}, y_{t+1})$$

$$+(1-\lambda)X_{t+1}(0, 0, a_{t+1}, y_{t+1})|y_t]$$

subject to

$$c_t + q_t(0, 0, a_{t+1}^*, y_t)a_{t+1}^* + R_t h_t = y_t - g(0, 0, y_t) - \omega_t(y_t)$$

$$a_{t+1} = a_{t+1}^* - e_{t+1}$$

Here the value of defaulting and filing for bankruptcy together only depends on the household's endowment since defaulting results in the loss of housing space and mortgage payment, while bankruptcy relieves the household of its unsecured debt obligations.

Problem of a Homeowner with Bad Credit

Now consider the problem of a household that owns their housing space but is excluded from credit markets. Such a household necessarily has filed for bankruptcy in the past and has not yet regained access to credit markets. The decision problem of this type of household is analogous to that presented above, except that they are restricted from borrowing in unsecured credit markets and hence will not declare bankruptcy. The household chooses whether to repay their mortgage and continue as a homeowner (O_t^X), sell their home (S_t^X),

or default on their mortgage (D_t^X). Their optimal choice is the one with the highest value:

$$X_t(k_t, x_t, a_t, y_t) = \max_{O_t^X, S_t^X, D_t^X} \{O_t^X(k_t, x_t, a_t, y_t), S_t^X(k_t, x_t, a_t, y_t), D_t^X(a_t, y_t)\}.$$

The value of making their mortgage payment and continuing as a homeowner is given by

$$\begin{aligned} O_t^X(k_t, x_t, a_t, y_t) = & \max_{c_t, a_{t+1}^* \geq 0} u(c_t, k_t) + \beta \mathbb{E}_t[\lambda V_{t+1}(k_{t+1}, x_{t+1}, a_{t+1}, y_{t+1}) \\ & + (1 - \lambda)X(k_{t+1}, x_{t+1}, a_{t+1}, y_{t+1}) | y_t] \end{aligned}$$

subject to

$$c_t + q_t(k_t, x_t, a_{t+1}^*, y_t)a_{t+1}^* + x_t = y_t - g(x_t, a_t, y_t) + a_t$$

$$k_{t+1} = (1 - \delta_{t+1})k_t, \quad x_{t+1} = \mu x_t, \quad a_{t+1} = \max\{a_{t+1}^* - e_{t+1}, 0\}^{45}$$

If instead they choose to sell their home, they receive the proceeds from the sale $P_t k_t$ less a proportional transaction cost χ_S . The household must also repurchase their mortgage contract from the lender for an amount equal to the present value of the promised stream of decaying mortgage payments, discounted at the risk-free interest rate. Recall that a household that sells their home must rent housing space in the current period. The value of selling is then

$$\begin{aligned} S_t^X(k_t, x_t, a_t, y_t) = & \max_{c_t, a_{t+1}^* \geq 0, h_t} u(c_t, h_t) + \beta \mathbb{E}_t[\lambda V_{t+1}(0, 0, a_{t+1}, y_{t+1}) \\ & + (1 - \lambda)X_{t+1}(0, 0, a_{t+1}, y_{t+1}) | y_t] \end{aligned}$$

⁴⁵For simplicity, we assume that the size of the expense shock is capped by the household's positive assets when a household is excluded from credit markets. This will ensure that excluded households never want to declare bankruptcy.

subject to

$$c_t + q_t(0, 0, a_{t+1}^*, y_t)a_{t+1}^* + R_th_t = y_t - g(0, a_t, y_t) + a_t + P_t k_t(1 - \chi_S) - \left(1 + \frac{\mu}{r + 1 - \mu}\right) x_t$$

$$a_{t+1} = \max\{a_{t+1}^* - e_{t+1}, 0\}$$

The household may also decide to default on their mortgage. In this case they are relieved of their mortgage payment but must relinquish their home to the lender and remain temporarily excluded from credit markets. They must also rent housing space in the current period. The value of defaulting is

$$\begin{aligned} D_t^X(a_t, y_t) = & \max_{c_t, a_{t+1}^* \geq 0, h_t} u(c_t, h_t) + \beta \mathbb{E}_t[\lambda V_{t+1}(0, 0, a_{t+1}, y_{t+1}) \\ & + (1 - \lambda) X_{t+1}(0, 0, a_{t+1}, y_{t+1}) | y_t] \end{aligned}$$

subject to

$$c_t + q_t(0, 0, a_{t+1}^*, y_t)a_{t+1}^* + R_th_t = y_t - g(0, a_t, y_t) + a_t$$

$$a_{t+1} = \max\{a_{t+1}^* - e_{t+1}, 0\}$$

Problem of a Renter with Good Credit

Next, consider the decision problem faced by a household that does not own a home and is in good credit standing. This type of household must choose between purchasing housing space (O_t^R), continuing to rent (L_t^R), and filing for bankruptcy (B_t^R). Their optimal choice is the one with the highest value:

$$V_t(0, 0, a_t, y_t) = \max_{O_t^R, L_t^R, B_t^R} \{O_t^R(a_t, y_t), L_t^R(a_t, y_t), B_t^R(y_t)\}.$$

Households can finance the purchase of housing space using a combination of savings and a mortgage. If the household decides to purchase a house of size k_t , commits to first mortgage

payment x_t , chooses to carry assets a_{t+1}^* into the following period, and has endowment y_t , then the lender issues a mortgage with value $m_t(k_t, x_t, a_{t+1}^*, y_t)x_t$ to the household. We impose that the household must be able to afford the sum of the purchase price $P_t k_t$, a proportional moving cost χ_B , and their first mortgage payment x_t without the need to borrow in unsecured credit markets. The value of purchasing a home is thus:

$$O_t^R(a_t, y_t) = \max_{c_t, k_t, x_t, a_{t+1}^*} u(c_t, k_t) + \beta \mathbb{E}_t[V_{t+1}(k_{t+1}, x_{t+1}, a_{t+1}, y_{t+1})|y_t]$$

subject to

$$c_t + q_t(k_t, x_t, a_{t+1}^*, y_t)a_{t+1}^* + P_t k_t(1 + \chi_B) + x_t = y_t - g(x_t, a_t, y_t) + a_t + m_t(k_t, x_t, a_{t+1}^*, y_t)x_t$$

$$y_t - g(x_t, a_t, y_t) + a_t + m_t(k_t, x_t, a_{t+1}^*, y_t)x_t \geq P_t k_t(1 + \chi_B) + x_t$$

$$P_t k_t \geq m_t(k_t, x_t, a_{t+1}^*, y_t)x_t$$

$$k_{t+1} = (1 - \delta_{t+1})k_t, \quad x_{t+1} = \mu x_t, \quad a_{t+1} = a_{t+1}^* - e_{t+1}$$

where the third constraint restricts the household from taking out a mortgage that exceeds the value of the home.

If the household decides to repay their unsecured debt and continue renting housing space, the value is given by:

$$L_t^R(a_t, y_t) = \max_{c_t, a_{t+1}^*, h_t} u(c_t, h_t) + \beta \mathbb{E}_t[V_{t+1}(0, 0, a_{t+1}, y_{t+1})|y_t]$$

subject to

$$c_t + q_t(0, 0, a_{t+1}^*, y_t)a_{t+1}^* + R_t h_t = y_t - g(0, a_t, y_t) + a_t$$

$$a_{t+1} = a_{t+1}^* - e_{t+1}$$

Finally, the household can choose to file for bankruptcy subject to the same costs and

penalties described above. The value of pursuing this option is

$$B_t^R(y_t) = \max_{c_t, a_{t+1}^* \geq 0, h_t} u(c_t, h_t) - \nu + \beta \mathbb{E}_t[\lambda V_{t+1}(0, 0, a_{t+1}, y_{t+1}) \\ + (1 - \lambda) X_{t+1}(0, 0, a_{t+1}, y_{t+1}) | y_t]$$

subject to

$$c_t + q_t(0, 0, a_{t+1}^*, y_t) a_{t+1}^* + R_t h_t = y_t - g(0, 0, y_t) - \omega_t(y_t)$$

$$a_{t+1} = a_{t+1}^* - e_{t+1}$$

Problem of a Renter with Bad Credit

Lastly, consider the decision problem of a household that does not own housing space and is excluded from credit markets. To (slightly) simplify our analysis, we restrict this type of household from purchasing a home, and hence, they must rent housing space until they regain access to credit markets. The problem of this type of household is:

$$X_t(0, 0, a_t, y_t) = \max_{c_t, a_{t+1}^* \geq 0, h_t} u(c_t, h_t) + \beta \mathbb{E}_t[\lambda V_{t+1}(0, 0, a_{t+1}, y_{t+1}) \\ + (1 - \lambda) X_{t+1}(0, 0, a_{t+1}, y_{t+1}) | y_t]$$

subject to

$$c_t + q_t(0, 0, a_{t+1}^*, y_t) a_{t+1}^* + R_t h_t = y_t - g(0, a_t, y_t) + a_t$$

$$a_{t+1} = \max\{a_{t+1}^* - e_{t+1}, 0\}$$

2.3.2 Financial Intermediaries

We assume that financial intermediaries are risk neutral and competitive. For simplicity, we consider a representative financial intermediary that accepts deposits, lends to households in unsecured credit markets, and sells mortgages to help households finance the purchase of

owner-occupied housing space. The financial intermediary can also borrow or lend risk-free at the exogenously given interest rate r .

For computational tractability, we model mortgage contracts as perpetuities with payments that decay over time. In particular, when taking out a mortgage, the mortgagee agrees to the sequence of payments $\{x, \mu x, \mu^2 x, \dots\}$, where $\mu \in (0, 1)$, until they either default or sell their home. The decaying nature of mortgage payments allows households to gradually build home equity over time, even with a constant house price.

Consider a mortgage sold to a household planning to purchase a home of size k_t , with initial payment x_t , end of period assets a_{t+1}^* , and endowment y_t . The intermediary then disperses the amount $m_t(k_t, x_t, a_{t+1}^*, y_t)x_t$ to the household in the current period and receives the first payment x_t . If the household defaults in the following period, the intermediary takes control of the house and sells it through a foreclosure process, recovering a fraction $1 - \chi_S$ of its post-depreciation shock market value $P_{t+1}k_{t+1}$, where χ_S is a proportional transaction cost.

If the household decides to sell, they must repurchase their mortgage contract from the lender for an amount equal to the present value of the promised stream of decaying mortgage payments, discounted at the risk-free interest rate, or $(1 + \mu/(r + 1 - \mu))x_{t+1}$.

If the household declares bankruptcy, their unsecured debt obligations are discharged in exchange for temporary exclusion from credit markets, a one-time utility cost, and either a one-time endowment cost or the forced sale of their home.⁴⁶ If the home is liquidated as part of the bankruptcy proceedings, the intermediary receives the present value of the mortgage discounted at the risk-free interest rate. From the intermediary's perspective, bankruptcy in this case is equivalent to the sale of the home. On the other hand, if the household is allowed to keep their home, the intermediary receives the continuation value of the mortgage conditional on the household's choice of assets, realized endowment, depreciation shock, expense shock, and inability to borrow in unsecured credit markets.

⁴⁶We will discuss the details pertaining to the U.S. bankruptcy code and its treatment of homeownership in the following section.

If the household neither defaults, sells, nor declares bankruptcy, the intermediary receives the continuation value of the mortgage conditional on the household's choice of assets, realized endowment, depreciation shock, and expense shock in the following period.

Let $\mathbb{D}_t(k_t, x_t, a_t, y_t)$ be an indicator function equal to 1 if a household with these characteristics finds it optimal to default at time t and 0 otherwise. Likewise, let $\mathbb{S}_t(k_t, x_t, a_t, y_t)$ be an indicator function equal to 1 if a household with these characteristics sells their home (either because they find it optimal to sell or because their home is liquidated during bankruptcy) and 0 otherwise, and similarly, define $\mathbb{B}_t(k_t, x_t, a_t, y_t)$ for a household that declares bankruptcy but is not forced to sell their home. The zero profit condition for this mortgage contract is then:

$$\begin{aligned}
& m_t(k_t, x_t, a_{t+1}^*, y_t)x_t = x_t \\
& + \frac{1}{1+r+\alpha_t} \{ \underbrace{\mathbb{E}_t[\mathbb{D}_{t+1}(k_{t+1}, x_{t+1}, a_{t+1}, y_{t+1})P_{t+1}k_{t+1}(1-\chi_S)]}_{\text{Value if household defaults}} \\
& \quad + \underbrace{\mathbb{S}_{t+1}(k_{t+1}, x_{t+1}, a_{t+1}, y_{t+1}) \left(1 + \frac{\mu}{r+1-\mu}\right) x_{t+1}}_{\text{Value if house is sold}} \\
& \quad + \underbrace{\mathbb{B}_{t+1}(k_{t+1}, x_{t+1}, a_{t+1}, y_{t+1})m_{t+1}^X(k_{t+1}, x_{t+1}, a_{t+2}^*, y_{t+1})x_{t+1}}_{\text{Continuation value of mortgage after bankruptcy}} \\
& + (1 - \mathbb{D}_{t+1}(k_{t+1}, x_{t+1}, a_{t+1}, y_{t+1}))(1 - \mathbb{S}_{t+1}(k_{t+1}, x_{t+1}, a_{t+1}, y_{t+1})) \\
& \quad \underbrace{(1 - \mathbb{B}_{t+1}(k_{t+1}, x_{t+1}, a_{t+1}, y_{t+1}))m_{t+1}(k_{t+1}, x_{t+1}, a_{t+2}^*, y_{t+1})x_{t+1}}_{\text{Continuation value of mortgage without bankruptcy}} | y_t \}
\end{aligned}$$

where α_t is a time-varying credit wedge and the expectation is taken over the realization of the household's next period endowment, depreciation shock, and expense shock.

Since the value of a mortgage today depends on its continuation value tomorrow if the household files for bankruptcy and is allowed to keep their home, creditors must also price mortgages to households that are excluded from credit markets even though such a mortgage

is never actually sold in equilibrium.⁴⁷ A household that is excluded from credit markets, owns a home of size k_t , and has a mortgage with payment x_t will never file for bankruptcy (since they will not have any unsecured debt), but they may choose to sell their home or default on their mortgage. In addition, the household is allowed to re-enter credit markets with probability λ each period. Let $\mathbb{D}_t^X(k_t, x_t, a_t, y_t)$ and $\mathbb{S}_t^X(k_t, x_t, a_t, y_t)$ be indicator functions analogous to those described above but specific to households that are excluded from credit markets. The value of this mortgage contract is then

$$\begin{aligned}
& m_t^X(k_t, x_t, a_{t+1}^*, y_t)x_t = x_t \\
& + \frac{1}{1+r+\alpha_t} \left\{ \mathbb{E}_t[(1-\lambda) \underbrace{\{\mathbb{D}_{t+1}^X(k_{t+1}, x_{t+1}, a_{t+1}, y_{t+1})P_{t+1}k_{t+1}(1-\chi_S)\}_{}}_{\text{Value of remaining excluded and defaulting}} \right. \\
& \quad \left. + \underbrace{\mathbb{S}_{t+1}^X(k_{t+1}, x_{t+1}, a_{t+1}, y_{t+1}) \left(1 + \frac{\mu}{r+1-\mu}\right) x_{t+1}}_{\text{Value of remaining excluded and selling}} \right. \\
& \quad \left. + (1 - \mathbb{D}_{t+1}^X(k_{t+1}, x_{t+1}, a_{t+1}, y_{t+1})) \right. \\
& \quad \left. \underbrace{(1 - \mathbb{S}_{t+1}^X(k_{t+1}, x_{t+1}, a_{t+1}, y_{t+1}))m_{t+1}^X(k_{t+1}, x_{t+1}, a_{t+2}^*, y_{t+1})x_{t+1}}_{\text{Value of remaining excluded and continuing mortgage}} \right\} \\
& + \lambda \left\{ \underbrace{\mathbb{D}_{t+1}(k_{t+1}, x_{t+1}, a_{t+1}, y_{t+1})P_{t+1}k_{t+1}(1-\chi_S)}_{\text{Value of re-entering credit markets and defaulting}} \right. \\
& \quad \left. + \underbrace{\mathbb{S}_{t+1}(k_{t+1}, x_{t+1}, a_{t+1}, y_{t+1}) \left(1 + \frac{\mu}{r+1-\mu}\right) x_{t+1}}_{\text{Value of re-entering credit markets and selling}} \right. \\
& \quad \left. + (1 - \mathbb{D}_{t+1}(k_{t+1}, x_{t+1}, a_{t+1}, y_{t+1})) \right. \\
& \quad \left. \underbrace{(1 - \mathbb{S}_{t+1}(k_{t+1}, x_{t+1}, a_{t+1}, y_{t+1}))m_{t+1}(k_{t+1}, x_{t+1}, a_{t+2}^*, y_{t+1})x_{t+1}}_{\text{Value of re-entering credit markets and continuing mortgage}} \right\} | y_t]
\end{aligned}$$

⁴⁷We assume that households that are excluded from unsecured credit markets are also excluded from mortgage markets, and therefore, this type of mortgage is never sold to households in our model. Given our assumption of competitive financial intermediaries, though, one can think of an active secondary mortgage market in which this type of mortgage, along with all other active mortgages, are traded. It is this market in which the continuation value, or price, of mortgages such as this one are determined.

Together, these functional equations determine the profit maximizing, equilibrium mortgage contract pricing schedules $m_t(k_t, x_t, a_{t+1}^*, y_t)$ and $m_t^X(k_t, x_t, a_{t+1}^*, y_t)$.

The financial intermediary also offers one-period, unsecured, pure discount bonds which households cannot commit to repay. Suppose, for example, a household with house size k_t , mortgage payment x_t , and endowment y_t promises to repay an amount a_{t+1}^* in the following period. The intermediary then disperses the amount $q_t(k_t, x_t, a_{t+1}^*, y_t)a_{t+1}^*$ to the household in the current period. If the household does not declare bankruptcy in the following period, then the intermediary is repaid the amount a_{t+1}^* in full. On the other hand, if the household declares bankruptcy, the intermediary recovers an amount $\psi_{t+1}(k_{t+1}, x_{t+1}, a_{t+1}^*, y_{t+1})$ which depends on the household's characteristics and the current bankruptcy laws in place. The zero profit condition for this type of loan is:

$$q_t(k_t, x_t, a_{t+1}^*, y_t)a_{t+1}^* = \frac{1}{1+r} \left\{ \underbrace{\mathbb{E}_t[(1 - \mathbb{B}_{t+1}(k_{t+1}, x_{t+1}, a_{t+1}, y_{t+1}))a_{t+1}^*]}_{\text{Value of repaying loan}} \right. \\ \left. + \underbrace{\mathbb{B}_{t+1}(k_{t+1}, x_{t+1}, a_{t+1}, y_{t+1})\psi_{t+1}(k_{t+1}, x_{t+1}, a_{t+1}^*, y_{t+1})}_{\text{Value of declaring bankruptcy}} | y_t \right\}.$$

2.3.3 Government

There is also a government that levies income taxes on households. We include a government in our model to capture two of the primary financial benefits of homeownership in the U.S.: (1) the implicit rental income from homeownership is not taxed and (2) mortgage interest payments are tax deductible. While the former induces high-income households to purchase rather than rent their housing space, the latter gives an incentive for homebuyers to finance their purchase with debt rather than equity. For simplicity, we assume that government consumption does not provide any benefit to households and that tax revenues are not rebated to households.

The tax levied on each household (g) is modeled after the U.S. tax code. A household's

taxable income i is the sum of their current endowment and interest on deposits less the greater of (i) their mortgage interest payment $\mu r x_t / (r + 1 - \mu)$ and (ii) the standard deduction s :

$$i(x_t, a_t, y_t) = y_t + r \max\{a_t, 0\} - \max\left\{\frac{\mu r x_t}{r + 1 - \mu}, s\right\}.$$

We assume that the tax rate $\tau(i(x_t, a_t, y_t))$ is weakly increasing in the household's taxable income. The tax levied on a household is then:

$$g(x_t, a_t, y_t) = \int_0^{i(x_t, a_t, y_t)} \tau(w) dw$$

and their after-tax income is given by $y_t - g(x_t, a_t, y_t)$.

2.3.4 Market Clearing

Let $\Phi_t(k_t, x_t, a_t, y_t, cs_t)$ represent the distribution of households over owner-occupied housing space, mortgage payments, assets, endowments, and credit statuses (cs_t) entering period t . The prices P_t and R_t adjust each period so that the aggregate demands for owner-occupied and rental housing space equal their exogenous supplies:

$$K_t = \int k_t(k_t, x_t, a_t, y_t) d\Phi_t(k_t, x_t, a_t, y_t, cs_t)$$

$$H_t = \int h_t(k_t, x_t, a_t, y_t) d\Phi_t(k_t, x_t, a_t, y_t, cs_t)$$

2.3.5 Equilibrium

An equilibrium in this economy is a sequence of prices $\{P_t, R_t, q_t, m_t, m_t^x\}$, exogenous sequences of owner-occupied and rental housing stocks $\{K_t, H_t\}$, sequences of household decision rules, and a sequence of distributions of households over states $\{\Phi_t\}$, such that, taking prices, the bankruptcy code, housing supplies, and the initial distribution of households over

states Φ_0 as given, at each date t :

1. Households optimally solve their decision problems.
2. Creditors maximize profits.
3. Markets for owner-occupied and rental housing clear.

2.4 The BAPCPA

Prior to the BAPCPA, most households with significant unsecured debt obligations could benefit by filing for bankruptcy. Households who did not own a home were able to have all of their unsecured debt obligations extinguished in exchange for having a bankruptcy flag on their credit report for a period of 10 years.⁴⁸ We model this penalty as a one-time utility cost and temporary exclusion from credit markets, during which time households can neither borrow in unsecured credit markets nor purchase a home.⁴⁹ There are no other costs associated with declaring bankruptcy in this case, and unsecured creditors do not recover anything (i.e. $\omega_t(y_t) = 0$ and $\psi_t(k_t, x_t, a_t^*, y_t) = 0$).

The U.S. bankruptcy code provides exemptions that households can use to protect certain assets from seizure by creditors. The largest and most commonly used is an exemption for the home, which allows homeowners to keep their home equity up to a prespecified limit known as the homestead exemption. Homeowners with home equity less than the homestead exemption were allowed to keep their home and file under Chapter 7. Homeowners with home equity greater than the homestead exemption, on the other hand, were forced to sell their home and transfer all home equity in excess of the homestead exemption (non-exempt home equity) to their unsecured creditors.

⁴⁸The presence of a bankruptcy flag on a household's credit report has been shown to severely restrict their access to credit (see Musto (2004)).

⁴⁹The one-time utility cost is meant to capture the social stigma attached to bankrupts discussed extensively in the literature (see Fay et al. (2002) and Gross and Souleles (2002)).

In terms of our model, define the home equity of a household with house size k_t and payment x_t at time t as

$$HE_t(k_t, x_t) \equiv P_t k_t - \left(1 + \frac{\mu}{r + 1 - \mu}\right) x_t$$

and let ζ be the homestead exemption. Prior to the BAPCPA, a homeowner with $HE_t(k_t, x_t) \leq \zeta$ would be allowed to keep their home, while a homeowner with $HE_t(k_t, x_t) > \zeta$ would be forced to sell, raising an amount $P_t k_t$. Out of these funds, the mortgage lender would receive the present value of the promised stream of decaying mortgage payments, discounted at the risk-free interest rate, or $[1 + \mu/(r + 1 - \mu)]x_t$, the household would keep an amount equal to the homestead exemption ζ , and unsecured creditors would be paid all non-exempt home equity up to the original loan amount:

$$\psi_t(k_t, x_t, a_t^*, y_t) = \min \{|a_t^*|, \max \{HE_t(k_t, x_t) - \zeta, 0\}\} .^{50}$$

The BAPCPA made it more costly for households to declare bankruptcy. It raised the average total bankruptcy filing costs under both Chapter 7 and Chapter 13, capped the homestead exemption for households who have owned their home for less than 3-1/2 years, increased the number of years before a household could refile from six to eight, and introduced means testing that severely restricted high-income households' ability to benefit from bankruptcy. While all of these reforms clearly affect a homeowner's decision to file, we focus our attention on the effects of means testing as it is likely to have the largest impact on household behavior.

To illustrate the impact of means testing introduced under the BAPCPA, consider a household that either does not own a home or has home equity below the homestead exemption. The first step is to convert the household's income over the previous six months

⁵⁰The fact that non-exempt home equity is seized by unsecured creditors during bankruptcy should lead to lower interest rates on borrowing in unsecured credit markets for these borrowers. We find this to be a quantitatively important benefit of homeownership which was reduced by the BAPCPA.

to an annualized basis and then compare it to the median income in their home state. If their income is less than the median, they are permitted to file under Chapter 7 and are unaffected by the reform. Conversely, if their income is above the median, the household may be forced to file under Chapter 13 and commit to a repayment plan.⁵¹ In this case, the household's unsecured debt is discharged, but they are required to pay all non-exempt income to their creditors for a period of five years. For simplicity, we model this penalty as a one-time endowment cost. Specifically, if $\bar{y}$ is median income, then a household that declares bankruptcy, does not own a home or owns a home but has home equity less than the homestead exemption, and has endowment $y_t > \bar{y}$, is required to repay an amount

$$\omega_t(y_t) = \kappa(y_t - \bar{y})$$

in the current period to their unsecured creditors, in addition to facing the same one-time utility cost and temporary exclusion from credit markets discussed above. It follows in this case that,

$$\psi_t(k_t, x_t, a_t^*, y_t) = \min \{|a_t^*|, \kappa(y_t - \bar{y})\},$$

where the creditor's recovery amount is bounded above by the initial loan amount.

Now consider a household that owns their home and has home equity in excess of the homestead exemption. If the household's non-exempt home equity is greater than five times their non-exempt income, then the household is forced to sell their home and pay all non-exempt home equity to their unsecured creditors. Otherwise, the household is allowed to keep their home, but must pay all non-exempt income to their creditors for a period of five years. In both cases the household is subject to the same one-time utility cost and temporary exclusion from credit markets discussed above. In terms of our model, if

⁵¹This occurs if their income in excess of their exempt income, where exempt income includes the funds required for housing and transportation costs and personal expenses as well as additional amounts for their mortgage and car payments, exceeds \$2,000. See Li et al. (2011) for a detailed description of how a household's non-exempt income is computed.

$$HE_t(k_t, x_t) - \zeta > 5(y_t - \bar{y})$$

then the household is forced to sell their home, raising an amount $P_t k_t$. Out of these funds, the mortgage lender receives the present value of the promised stream of decaying mortgage payments, discounted at the risk-free interest rate, or $[1 + \mu/(r + 1 - \mu)]x_t$, the household receives an amount equal to the homestead exemption ζ , and unsecured creditors are paid all non-exempt home equity up to the original loan amount:

$$\psi_t(k_t, x_t, a_t^*, y_t) = \min \{|a_t^*|, \max \{HE_t(k_t, x_t) - \zeta, 0\}\}.$$

On the other hand, if

$$HE_t(k_t, x_t) - \zeta \leq 5(y_t - \bar{y})$$

the household is required to repay an amount

$$\omega_t(y_t) = \kappa(y_t - \bar{y}),$$

in the current period to their unsecured creditors, in addition to facing the same one-time utility cost and temporary exclusion from credit markets discussed above. It follows in this case that,

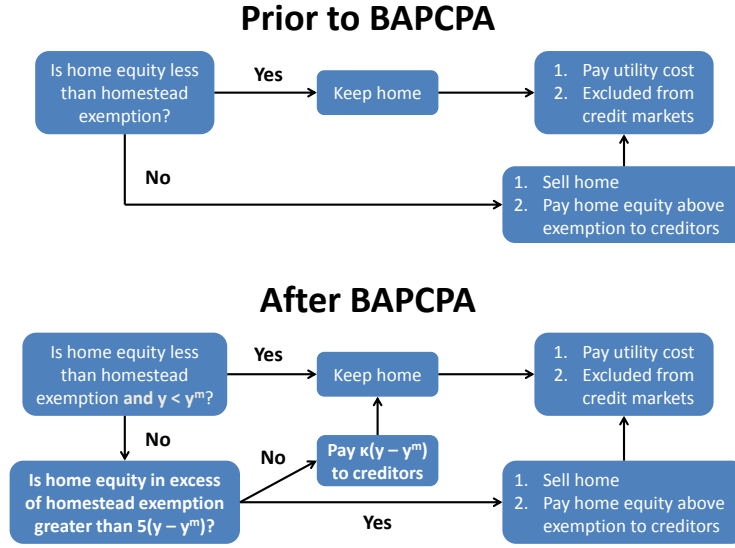
$$\psi_t(k_t, x_t, a_t^*, y_t) = \min \{|a_t^*|, \kappa(y_t - \bar{y})\},$$

where, again, the creditor's recovery amount is bounded above by the initial loan amount.

Importantly, in this case the creditor's recovery amount only depends on the household's income and is independent of the household's homeownership status. While this household would have benefited from owning a home through lower interest rates on unsecured debt prior to the BAPCPA, this benefit is no longer available under the BAPCPA. Consequently, homeowners that currently have negative home equity and expect to have high income in

the future, perceive a reduced benefit to future homeownership. This lower future benefit may induce some homeowners to default on their mortgage instead of staying in their home. Understanding this mechanism is important when discussing the effects of the BAPCPA on households' incentives to default.

Figure 2.2: Implementation of BAPCPA



The effects of means testing implemented under the BAPCPA on the costs of bankruptcy faced by households in our model is depicted in Figure 2.2. Clearly, bankruptcy reform made filing for bankruptcy much more costly for high-income households. In the following sections we calibrate our model and quantify the effects of BAPCPA on the recent housing crisis.

2.5 Parameterization

We assume that each period in our model corresponds to one year. Many of our model parameters are common in the literature and can therefore be set outside of the model. We first discuss how these parameters are chosen and then describe how we calibrate the remaining model parameters.

The parameters governing households' stochastic first-order autoregressive endowment process are set to $\rho = 0.97$ and $\sigma = 0.129$, which are consistent with the findings of Storesletten et al. (2004). We discretize this process with a 17-state Markov chain using Tauchen and Hussey (1991)'s method.

We assume that a household's flow utility at date t is given by:

$$u(c_t, h_t) = \frac{(c_t^{1-\theta} h_t^\theta)^{1-\gamma}}{1-\gamma}$$

where γ is a proxy for risk aversion and θ determines the share of income spent on housing space.⁵² We set $\gamma = 2$, which is a standard value for this parameter used in the literature. Empirical work by Davis and Ortalo-Magne (2011) indicates that household's spend approximately 24% of their income on housing services, so we set $\theta = 0.24$.

The parameters related to the housing sector that are determined outside of our model are $(\chi_B, \chi_S, \bar{\delta}, \zeta)$. The proportional transaction costs for buying and selling are set to $\chi_B = 0.025$ and $\chi_S = 0.070$, respectively, which are in line with the values reported by Gruber and Martin (2003). Pennington-Cross (2006) finds that the value received from the sale of a foreclosed home is about 78% of the market value for a similar non-foreclosed home.⁵³ Since in our model a household that chooses to default on their mortgage also often has incurred the housing depreciation shock, we set $\bar{\delta}$ such that creditors receive 78% of the value of the pre-depreciation shock home after selling transaction costs. This implies that $\bar{\delta} = 0.15$.⁵⁴ Since our model is intended to represent the U.S. economy, we compute the average homestead exemption across states, where each state is weighted by its share

⁵²Cobb-Douglas preferences imply that a renting household will choose to spend constant fractions of their wealth on non-durable consumption and housing services. Note that households do not derive any direct utility benefit from owning versus renting their housing space in this model.

⁵³This finding is in line with estimates from other work, including Shilling, Benjamin, and Sirmans (1990), who find values that range in 22%-24%.

⁵⁴More formally, the value to the creditor of a foreclosed home that received the depreciation shock is $(1 - \bar{\delta})(1 - \chi_S)P_t k_t = 0.85(0.925)P_t k_t = 0.786P_t k_t$, matching the empirical literature.

of U.S. households.⁵⁵ Using data collected by Mitman (2011), we find the weighted average homestead exemption to be 1.10 times median household income. We normalize median income in our model to 1 and therefore set $\zeta = 1.10$.

The risk-free interest rate is set to 4% as is standard in the literature. The positive value for the expense shock $\bar{e}$ is set to 3.33 times median income, which is consistent with the findings of Livshits et al. (2007). The probability of re-entering credit markets after declaring bankruptcy or defaulting on a mortgage λ is set to 12%, implying that, on average, an excluded household re-enters credit markets after 8.5 years. Although households that declared bankruptcy during the pre-reform period were only restricted from refiling for 6 years, there is empirical evidence that filing for bankruptcy impacts a household’s credit market status for as long as their credit score is adversely affected. Moreover, underwriting standards by the government-sponsored enterprises over this period suggest that access to mortgage markets is also similarly restricted after a bankruptcy or default.⁵⁶

Finally, we calibrate the tax schedule. As in Chatterjee and Eyigungor (2011b) we assume that a household in our model files their taxes as married filing separately and calibrate the model’s income tax schedule to match that of the U.S. economy in 1998. Table 2.1 presents the implied tax schedule, and we set the standard deduction $s = 0.1116$.

Table 2.1: Model Income Tax Brackets

Taxable Income (i)	Tax Rate (τ)
0 – 0.64	0.15
0.64 – 1.55	0.28
1.55 – 2.37	0.31
2.37 – 4.23	0.36
4.23 – ∞	0.396

⁵⁵We exclude states with an infinite homestead exemption from this calculation.

⁵⁶For example, Musto (2004) finds that households that declare bankruptcy face restricted access to credit markets at potentially prohibitively tough terms for 10 years after they file – at which point the bankruptcy flag is removed from their credit report. Defaulting on a mortgage also negatively impacts a household’s credit score and thus their ability to borrow in unsecured credit markets (see Christie (2010) and Brevoort and Cooper (2010)).

The remaining parameters to be calibrated are the discount factor β , the utility cost of bankruptcy ν , the rate of decay for mortgage payments μ , the probability the household receives a depreciation shock ϕ , and the probability a household receives an expense shock ξ . These parameters are jointly calibrated to match the unsecured debt-to-income ratio, bankruptcy filing rate, percentage of homeowners with less than 30% home equity, mortgage default rate, and bankruptcy rate among new homeowners in the stationary distribution of the model prior to the BAPCPA.

Since these statistics are intended to capture a steady state in the U.S. housing market prior to the BAPCPA, we choose targets that predate the substantial rise in homeownership rates and house prices that corresponded with the housing boom in the mid-2000's. The target bankruptcy filing rate is set to 1.4%, which was the total bankruptcy filing rate in 2004 as reported by Li and White (2009). The percentage of homeowners with home equity less than 30% is taken from Chatterjee and Eyigungor (2011b), who in turn compute this number from the 1998 Survey of Consumer Finances. This value is set to 23.0%. The annual foreclosure rate according to the Mortgage Banker's Association was about 1.0%. However, using data from LPS Analytics between 2001 and 2003, Herkenhoff and Ohanian (2012) find that roughly 15% of homeowners entering the foreclosure process self-cure and remain in their home. Since defaulting on a mortgage is synonymous with losing the home through a foreclosure process in our model, we exclude such households from our target statistic, implying a mortgage default rate of 0.85%. The target bankruptcy rate for new homeowners is set to 0.57%, as reported in Li et al. (2011). The target unsecured debt-to-income ratio is set to 9.6%. This statistic is computed by constructing a revolving debt-to-income ratio measure from the Flow of Funds Accounts and adjusting this series with the historical spread between the unsecured and revolving debt-to-income ratios implied by Livshits et al. (2010). Finally, we choose to target a homeownership rate of 66.4%, which matches the ten-year average in the U.S. economy prior to 2003.

The joint calibration of these five parameters $(\beta, \nu, \mu, \phi, \xi)$ is achieved by conducting a

grid search over the parameters, computing the stationary distribution of the economy for each set of parameters, and choosing the combination that minimizes a weighted sum of squared residuals between the empirical and model values for the target statistics.⁵⁷ Table 2.2 summarizes the parameter values.

Table 2.2: Parameterization

Parameter	Value	Source/Target
γ	2.0	Standard
θ	0.24	Davis and Ortalo-Magne (2011)
ρ	0.97	Storesletten et al. (2004)
σ	0.129	Storesletten et al. (2004)
r	0.04	Standard
χ_B	0.025	Gruber and Martin (2003)
χ_S	0.070	Gruber and Martin (2003)
$\bar{\delta}$	0.15	Pennington-Cross (2006)
ζ	1.10	Mitman (2011)
λ	0.12	Average Exclusion Period of 8.5 yrs
$\bar{e}$	3.33	Livshits et al. (2007)
β	0.936	Unsecured Debt-to-Income Ratio = 9.6%
R/P	0.052	Homeownership Rate = 66.4%
ν	1.6	Bankruptcy Filing Rate = 1.4%
μ	0.966	Fraction of HO with < 30% HE = 23.0%
ϕ	0.005	Mortgage Default Rate = 0.85%
ξ	0.004	New HO Bankruptcy Rate = 0.57%

Table 2.3 presents the calibration results and other relevant model statistics in the pre-reform stationary distribution of the model. The model is able to match the pre-reform empirical moments for the statistics targeted in our calibration exercise reasonably well. It also performs well in replicating several relevant statistics that are not targeted by our calibration exercise. Notably, the model replicates the home equity distribution rather well, only slightly underpredicting the fraction of homeowners with home equity less than 25 percent, in addition to matching the fraction of homeowners with less than 30 percent home equity.

⁵⁷See Appendix A for this chapter for a detailed description of our algorithm to solve for the model's stationary distribution. When computing the pre-BAPCPA steady state, we also normalize the credit wedge to be equal to zero, i.e. $\alpha_t = 0$.

Table 2.3: Steady State Results

Statistic	Data	Model
Homeownership Rate*	66.4%	71.0%
Bankruptcy Filing Rate*	1.4%	1.7%
New Homeowner Bankruptcy Filing Rate*	0.57%	0.65%
Aggregate Mortgage Default Rate*	0.85%	0.90%
Homeowners with < 25% Equity	19.0%	16.5%
Homeowners with < 30% Equity*	23.0%	23.7%
Average Loan-to-Value at Origination	?	83.5%
Average Income of Homeowners to Renters	2.02	1.77
Average Annual Home Sales	4.3%	4.8%
Loan-to-Income Ratio	3.9	3.6
Unsecured Debt-to-Income Ratio*	9.6%	11.5%

* = Calibration Target

Matching this region of the home equity distribution is particularly important because it suggests that the fraction of homeowners that are pushed underwater on their mortgage by a drop in house prices similar to the recent housing crash is the same in the model and the data – a necessary feature of a model that quantitatively evaluates the effects of an unexpected housing crash.

The initial stationary distribution of the model is also consistent with the fact that homeowners have higher income, on average, than renters. The average income of homeowners relative to renters is 1.77 in our model, compared to 2.02 in the data.

Moreover, the pre-crisis stationary distribution is consistent with several statistics regarding the relative size of mortgages. In particular, our model matches the empirical loan-to-income value rather well and generates a loan-to-value at origination of 83.5%. Although we were unable to locate an analog to this statistic in the data, this value seems reasonable. Finally, the model implies that 4.8% of all owner-occupied houses are sold each year, which is in line with the ten-year average prior to 2003 as reported by the National Association of Realtors.

Now that we have calibrated the model and determined that it is able to match key

empirical statistics in the pre-BAPCPA stationary distribution, we turn to the primary quantitative objective of this paper: assessing the impact of the BAPCPA on the U.S. economy during the recent housing crisis. The next section details how we use the model to make such an assessment and describes our quantitative results.

2.6 Quantitative Results

In this section we detail the quantitative experiment that we run to assess the impact of the BAPCPA on the housing market crash. The experiment is based on the economy experiencing two shocks: a bankruptcy reform shock in 2005 and a housing crisis in 2007. We then compute the perfect foresight transition path of the economy in response to the following sequences of events assuming each event is unanticipated by the agents in our model:

1. *Actual Timeline:* In 2005 the U.S. economy experiences an unexpected change to the bankruptcy code that mimics the BAPCPA and then suffers a housing crisis in 2007.⁵⁸
2. *Counterfactual:* The U.S. bankruptcy code is not altered in 2005 but the economy does experience a housing crisis in 2007.

The ability to run counterfactual exercises that incorporate general equilibrium effects through housing, mortgage, and unsecured debt prices, to isolate the impact of the BAPCPA on the subsequent housing crisis is a key benefit of constructing a quantitative model like that presented in this paper.

We model the BAPCPA shock as an unexpected and permanent change to the U.S. bankruptcy code as outlined in Section 2.4: an introduction of the income and asset means testing consistent with this reform. Following Chatterjee and Eyigungor (2011b) we model

⁵⁸The fact that we model bankruptcy reform as unexpected in 2005 seems reasonable given our annual calibration. Although this act was originally introduced in Congress in 1998, it gained little political support until Republican majorities increased in Congress in 2004. It was ultimately passed by the U.S. Congress on April 14, 2005 and signed into law by President Bush on April 20th of that same year. Its provisions affected bankruptcy filings on or after October 17, 2005.

the housing crisis as an unexpected increase in the owner-occupied housing supply in 2007. Unlike these authors, however, we assume that this shock is temporary and dissipates over time, which implies that the housing market eventually returns to a state consistent with the initial stationary distribution.⁵⁹ We find that a 4% shock to the supply of owner-occupied housing produces a decline in house prices similar in magnitude to the decline in the S&P Case-Shiller 20-City Home Price Index between January 2007 and January 2009. The size of our housing supply shock is also in line with the empirical estimates of excess housing supply reported by McNulty (2009).

To capture the fact that housing prices remain below their peak in 2006, we also include an exogenous credit wedge that increases the cost of issuing mortgages in the periods immediately following the housing market crash. A substantial rise in credit spreads during this period is documented in Hall (2011), who finds increases in various spreads on the order of 1.0 - 3.7%. In particular, we model this wedge as an additional spread – above the risk-free rate – that the creditor requires, represented by α_t . The effect of the credit wedge on the mortgage pricing equations is demonstrated in Section 2.3.2. To match the upper-end of Hall (2011)’s estimates, we set $\alpha_t = 0.035$ for $t = 2008, \dots, 2012$, and then allow this wedge to slowly decline back to zero by 2020. In sum, these modeling assumptions imply that the housing supply and credit markets return to their initial standing by 2020.

While solving the counterfactual perfect foresight transition path is relatively straightforward – given that the economy only experiences one unexpected shock – solving the transition under the actual timeline is more complicated. To solve for this transition, we have to compute two different transitions, and then combine the results from each to form the actual sequence of events. First, we compute a transition path of our economy starting from the pre-BAPCPA stationary distribution that experiences a bankruptcy reform-only shock in 2005, and then transitions to the post-BAPCPA steady state from there. This transition gives us the model statistics for 2005 and 2006 as well as the distribution of households over

⁵⁹In particular, we assume that the owner-occupied housing supply remains elevated between 2007 and 2012 and then declines to its original value by 2020.

states entering 2007. The second transition starts with this distribution and subjects the economy to an unexpected 4% increase in the owner-occupied housing supply. The transition from this shock to the steady state for the post-BAPCPA economy with an owner-occupied housing supply consistent with the initial stationary distribution is then computed. This transition provides us with the statistics for the economy from 2007 onward. Appendix A for this chapter presents a more detailed description of our model solution, including solving for the economy's stationary distribution given a fixed set of parameters, and also computing the perfect foresight transitions described in this section.

Prior to computing the transitions, we have to determine the value for κ , which controls the cost of filing for bankruptcy for high-income households that pass the income means test after bankruptcy reform. Recall that a household that files for bankruptcy and is forced into a repayment plan due to their high income must pay $\kappa(y_t - \bar{y})$ in the current period, where $\bar{y}$ is the economy's median income. We choose κ to match Li et al. (2011)'s findings that, on impact, bankruptcy reform increased the default probability of households that owned their home for less than three years by 21.6%.⁶⁰ To compute this statistic, we have to solve the entire perfect foresight transition of the economy in response to only the BAPCPA shock in 2005. We then find the value of κ that produces an increase in the mortgage default probability of new homeowners of 21.6%. A value of $\kappa = 1.0$ most closely matches this statistic.

2.6.1 BAPCPA and the Housing Crisis

We first consider the effect of the BAPCPA on the U.S. economy on impact when it was introduced in 2005. In Table 2.4 we compare the model implied statistics in 2005 with the BAPCPA to the statistics taken from the pre-reform stationary distribution, along with the percentage change in each of these statistics in response to the BAPCPA.

⁶⁰We compute this number from their findings that the probability of defaulting on a prime mortgage – which represented 81% of outstanding mortgages – increased by 23.4%, and the probability of defaulting on a subprime mortgage increased by 13.9%.

Table 2.4: BAPCPA on Impact in 2005

Statistic	Initial Steady State	BAPCPA	BAPCPA / Initial Steady State
P	1.0	1.0	0.0%
R	0.0520	0.0517	-0.5%
Bankruptcy Rate	1.69%	1.68%	-1.0%
Mortgage Default Rate	0.90%	1.13%	25.5%
R/P	5.20%	5.17%	-0.5%
Unsecured Debt-to-Income	11.48%	11.53%	0.5%

Upon implementation, the BAPCPA reduces the bankruptcy rate and produces a substantially higher mortgage default rate, which is consistent with the empirical literature. In particular, the bankruptcy filing rate falls by 1.0% and the mortgage default rate increases by 25.5%. However, the BAPCPA had minimal impact on house and rental prices. The price of owner-occupied housing is unchanged and the rental price only declines by 0.5% in response to the reform.⁶¹ In addition, by reducing households incentives to file for bankruptcy and increasing the expected recovery for creditors in the event that they do, the BAPCPA generates a rise in the amount of unsecured borrowing, evidenced by a 0.5% increase in the unsecured debt-to-income ratio.

We now turn to the primary quantitative question of this paper: To what extent did the BAPCPA impact the housing market crash? We begin by assessing the ability of our model to match the severity of the housing crisis. The model statistics in 2007 are presented in Table 2.5.⁶²

Evident from this table is that our model produces a housing crash that looks very much like the data. Specifically, the unexpected supply shock generates a substantial decline in house prices, by 25.5%, and a quintupling in the mortgage default rate – from 0.9% in the

⁶¹The fact that the owner-occupied house price is unchanged despite a substantial rise in the mortgage default rate suggests that there may be minimal feedback from mortgage defaults on house prices. This finding is in line with Chatterjee and Eyigungor (2011b) and will be discussed in more detail in Section 2.7.

⁶²See Appendix B for graphs depicting the transitions for relevant model statistics.

Table 2.5: The Housing Crisis

Statistic	Initial Steady State	Value in 2007
P	1.00	0.75
R	0.052	0.044
Bankruptcy Rate	1.7%	3.3%
Mortgage Default Rate	0.9%	4.5%
R/P	5.2%	5.9%
Unsecured Debt-to-Income	11.5%	10.4%

pre-crisis steady state to 4.5% in 2007 – which remains elevated for several years following the crash. By comparison, the S&P Case-Shiller 20–City Home Price Index fell by 27.6% between January 2007 and January 2009, while the adjusted annual foreclosure rate reported by the Mortgage Bankers Association reached 4.2% in 2008. Moreover, this crash is accompanied by a pronounced rise in bankruptcy filing rates and a severe and protracted decline in unsecured borrowing relative to income similar to those observed in the data during this period.

The model also captures the empirical fact that the rent-price ratio rose during the housing crash.⁶³ Our model predicts a 13.5% increase in this statistic from 2004 to 2007. Thus, to clear both the owner-occupied and rental housing markets in response to the housing supply shock, the owner-occupied house price must decline more relative to the rental price.

The fact that our model has quantitative predictions that are consistent with *both* Li et al. (2011)’s findings that bankruptcy reform caused mortgage default rates to rise by 21.6% for new homeowners *and* the response of the housing and unsecured debt markets to the recent housing crisis, gives us confidence in its implications for the impact of the BAPCPA on the severity of the housing crisis. To analyze this question, we compare the implications of our model under the actual and counterfactual timelines.

Table 2.6 compares the economy with bankruptcy reform to the counterfactual economy by contrasting the statistics in 2007 between the two simulations. These results suggest that bankruptcy reform had little impact on the severity of the housing crisis, producing only

⁶³See Davis, Lehnert, and Martin (2008) for quarterly data on this ratio.

modestly higher bankruptcy and mortgage default rates in 2007. The aggregate mortgage default rate is 2.7% higher in the economy that underwent bankruptcy reform in 2005. However, the BAPCPA had little impact on the aggregate prices for owner-occupied and rental housing. In the next section we discuss some intuition for these findings.

Table 2.6: BAPCPA's Impact on the Housing Crisis in 2007

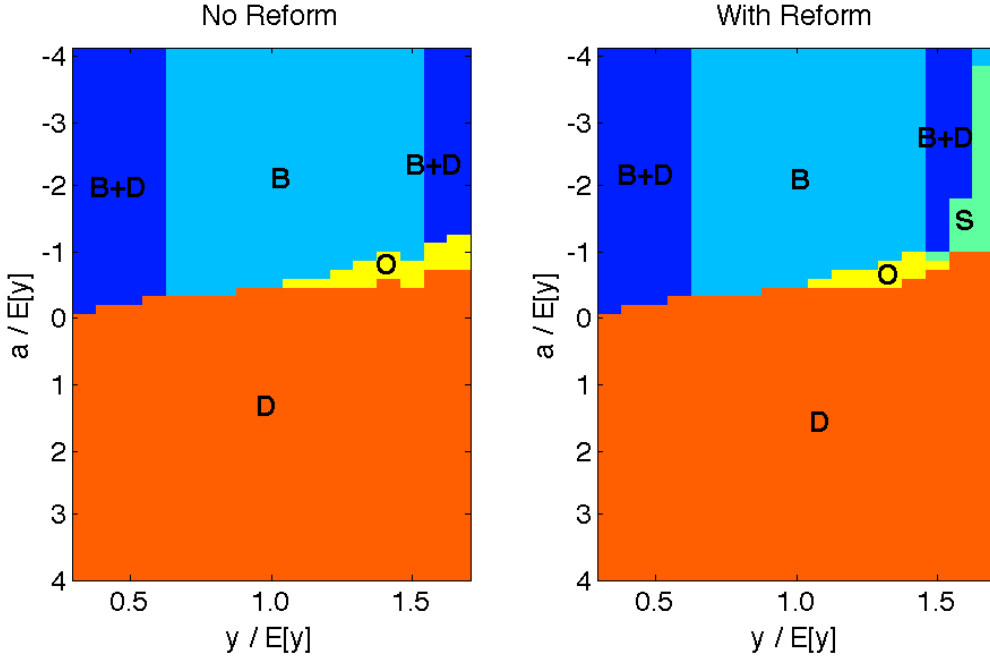
Statistic	BAPCPA	No Reform	BAPCPA / No Reform
P	0.745	0.745	0.0%
R	0.044	0.043	1.1%
Bankruptcy Rate	3.27%	3.22%	1.4%
Mortgage Default Rate	4.52%	4.40%	2.7%
R/P	5.88%	5.82%	1.1%
Unsecured Debt-to-Income	10.38%	10.51%	-1.2%

2.6.2 Discussion

Figure 2.3 depicts decision rules for a homeowner during the housing crisis in 2007 that is underwater on their mortgage both with (right figure) and without (left figure) the reform. This figure displays these decision rules fixing a household's house size and mortgage payment presented in the endowment (x-axis) - asset (y-axis) space. For ease of interpretation, assets and income are normalized by mean income, and negative values on the y-axis represent debt relative to mean income.

There are several facts about the impact of the BAPCPA on homeowner decisions that are evident from this figure. First, reform reduces the region in which continuing to own is optimal (yellow). With reform, the household now finds it optimal to default on their mortgage (orange) or to sell their home (green) in several regions where they find it optimal to continue to own their home in the absence of reform. Second, the introduction of the BAPCPA leads to a reduction in the region where it is optimal to declare bankruptcy for high-income households. By decreasing the probability the homeowner files for bankruptcy,

Figure 2.3: BAPCPA and Homeowner Decision Rules in 2007



the BAPCPA causes unsecured creditors to reduce interest rates on their lending, leading to an increase in risky lending for which bankruptcy-only and bankruptcy and default are non-trivial decisions.

Note that we can decompose the aggregate mortgage default rate into three components: households that simultaneously declare bankruptcy and default, non-excluded households that default on their mortgage, and excluded households that default on their mortgage. By considering these three components we can determine the quantitative importance of these effects in accounting for the higher mortgage default rate in the transition with the BAPCPA. We find that the reduced benefits to homeownership due to the BAPCPA, which lead to a rise in the number of non-excluded households that default on their mortgage, is the most quantitatively important factor. However, the rise in the number of households that declare bankruptcy and default on their mortgage due to the BAPCPA is also quantitatively important, while the impact of the BAPCPA on excluded homeowners is quantitatively insignificant. We now discuss these features in further detail.

Reduced Benefits to Homeownership

Prior to the BAPCPA, a homeowner with positive non-exempt home equity was able to borrow against that home equity in unsecured credit markets, as it served as collateral for unsecured debt in the event of bankruptcy. Consequently, homeowners with non-exempt home equity experienced a benefit of being able to borrow in unsecured credit markets at more favorable interest rates. This changed with the implementation of the BAPCPA and reduced the benefits of homeownership for some homeowners.

Consider a homeowner with income above the median ($y_t > \bar{y}$) and home equity above the exemption such that $5(y_t - \bar{y}) > HE_t(k_t, x_t) - \zeta > 0$. Table 2.7 depicts the amount recovered by creditors in the event of bankruptcy for a household with these characteristics if they are a homeowner or renter. Prior to the introduction of the BAPCPA, this household directly benefits by facing lower interest rates on unsecured borrowing by being a homeowner since, in the event of bankruptcy, the creditor recovers their non-exempt home equity (up to the face value of the bond). Following the introduction of the BAPCPA, creditor recovery no longer varies with the household's homeownership status, eliminating the benefit to this household of borrowing at lower interest rates because they own a home.

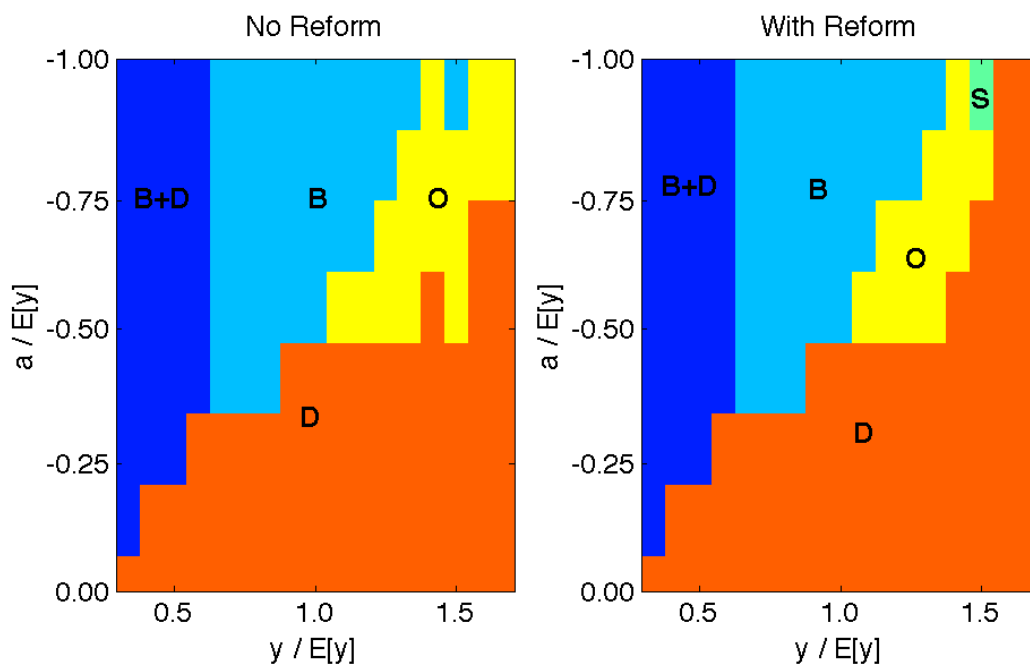
Table 2.7: Creditor Recovery and the BAPCPA

	Before Reform	After Reform
Homeowner	$HE_t(k_t, x_t) - \zeta$	$\kappa(y_t - \bar{y})$
Renter	0	$\kappa(y_t - \bar{y})$

This reduction in the benefit of homeownership is acute for homeowners with specific characteristics: relatively low current levels of home equity, expectations of high future income, and a desire to borrow in unsecured credit markets. We should expect to see households with these characteristics substituting away from homeownership in response to the BAPCPA. This intuition is reinforced by the decision rules depicted in Figure 2.4, as we see that homeowners with a higher current endowment now prefer to default on their

mortgage (orange) or sell (green) as a result of the reform rather than continuing owning (yellow).

Figure 2.4: BAPCPA and Homeowner Decision Rules in 2007



The expansion of the region in which households find it optimal to default on their mortgage is a quantitatively important determinant of the higher mortgage default rates observed with the BAPCPA compared to the economy that did not enact reform. At the peak of the crisis in 2007, an increase in the number of non-excluded households that default on their mortgage and do not declare bankruptcy accounts for 91.8% of the higher mortgage default rate with reform. Moreover, this channel accounts for more than 90% of the higher mortgage default rate with reform from 2007 through 2014, on average.

Thus, because the BAPCPA decreased the relative benefit of homeownership by reducing the dependence of unsecured interest rates on a household's homeownership status, underwater homeowners who would otherwise have decided to remain in their home now prefer to default on their mortgage. This effect tends to increase the mortgage default rate during the housing crisis.

Looser Unsecured Credit Lending Standards

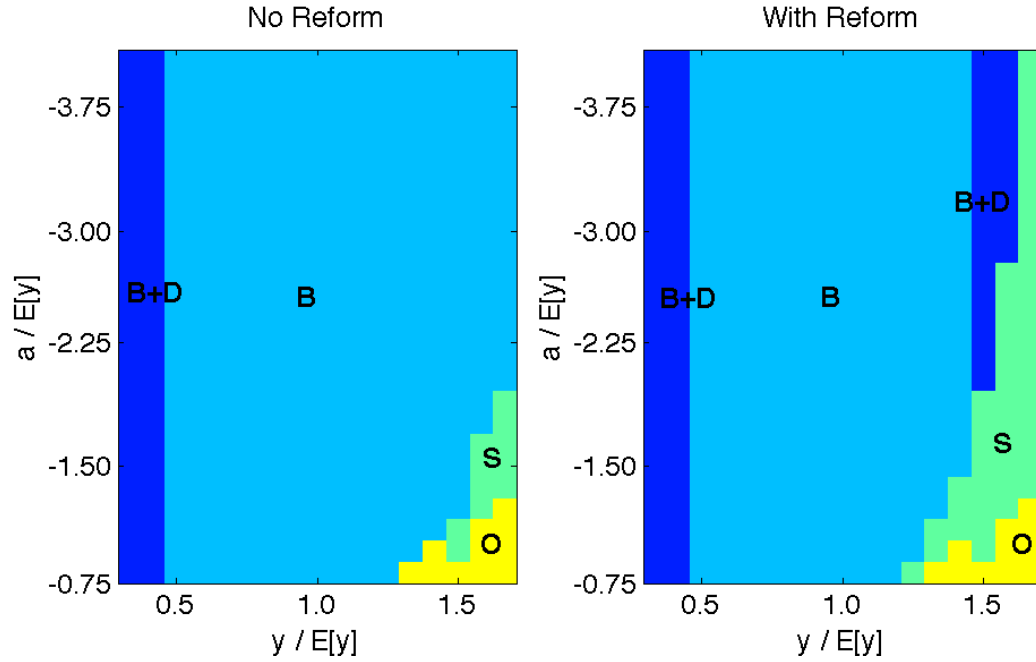
The BAPCPA significantly increased the cost of filing for bankruptcy for high-income households. This led to a dramatic reduction in the region in which households find it optimal to declare bankruptcy, as seen in Figure 2.5. This figure depicts decision rules for a homeowner in 2005 in the case with and without bankruptcy reform. Prior to the BAPCPA, this household would declare bankruptcy with near certainty (i.e. across all endowments) for large amounts of debt. This high probability of bankruptcy leads to prohibitively high interest rates for households that desire to borrow that amount of debt. As a result, it is unlikely that households would choose to borrow an amount that causes them to declare bankruptcy and default.

With the implementation of the BAPCPA, the probability that a high-income household declares bankruptcy falls dramatically. Unsecured creditors thus expect to be repaid in full with a higher probability after the reform, leading to lower interest rates and increased lending to high-income households in regions in which they may declare bankruptcy and default on their mortgage in the following period (dark blue).⁶⁴

An increase in the degree of lending for levels of debt where households are more likely to get mapped into a region where they find it optimal to simultaneously declare bankruptcy and default leads to higher mortgage default rates. Quantitatively, we find that this effect is important, although it is not as sizable as the higher default rates caused by the lower benefit of homeownership just discussed. Specifically, an increase in the number of households that simultaneously declare bankruptcy and default due to a rise in risky lending accounts for 8.2% of the difference between mortgage default rates with and without reform in 2007. This fraction is slightly higher than the average impact for this channel, which was 7.2% from 2007 through 2014. Therefore, the increased complementarity between bankruptcy and mortgage default that results from a rise in risky unsecured lending is quantitatively important in

⁶⁴Recall that the creditor is repaid in full if the household sells their home. A reduction in the area in which households find it optimal to declare bankruptcy and an increase in the optimal sell region implies higher expected returns for the creditor.

Figure 2.5: BAPCPA and Homeowner Decision Rules in 2005



accounting for the higher mortgage default rate under BAPCPA.

Tighter Mortgage Lending Standards

An important feature of our model is how bankruptcy and default incentives are fully reflected in the terms at which households can borrow in credit markets. In response to the increased incentive for households to default on their mortgage or sell, mortgage lenders expect to receive a lower return on loans to new homebuyers.⁶⁵ To continue to break even in expectation despite these changing incentives, mortgage lenders must tighten their lending standards, which in our model is accomplished by raising interest rates on those households that are now more likely to either default or sell in the future as a result of the reform.

Table 2.8 depicts how several characteristics of new homebuyers change in response to the introduction of the BAPCPA. Most of these metrics move in very intuitive directions and imply a tightening of mortgage lending standards. For example, the average house size, initial loan-to-income, loan-to-value, and mortgage payment-to-income ratios all decline.

⁶⁵More specifically, lenders face higher credit and prepayment risk after the BAPCPA.

Although the fact that the average income of new homeowners declines may at first appear to contradict a tightening of mortgage lending standards, this result is also intuitive. Since high-income households experience the higher incentive to default on their mortgage, in equilibrium, mortgage lenders tighten standards for high-income households relatively more in response to the BAPCPA, leading to lower average income for new homeowners.

Table 2.8: BAPCPA's Impact on Mortgage Lending Standards

Statistic	Change
House Size	-4.0%
Income	-1.4%
LTI	-12.8%
LTV	-0.7%
MTI	-4.0%

Tighter mortgage lending standards tend to reduce the probability households will find it optimal to default on their mortgage after the BAPCPA, offsetting some of the increased incentives homeowners have to default on their mortgage. On net, these effects nearly offset each other, implying a relatively small role for the BAPCPA in the severity of the housing crisis.

2.7 Mortgage Cram Down and the Housing Crisis

Given the depth and protracted nature of the recent housing crisis, there has been extensive discussions in policy and academic circles about policy initiatives aimed to reduce foreclosures and stabilize house prices. One such proposal, often referred to as mortgage cram down, has received attention in the economics and law literature.⁶⁶ Under this policy, homeowners with negative home equity are able to treat the portion of their mortgage that exceeds the value of their home as unsecured debt, which can then be discharged through the bankruptcy process. Thus, the value of the mortgage is reduced until the homeowner is

⁶⁶For examples, see White and Zhu (2008), Levitin (2009), and Scarberry (2010).

no longer underwater, which should reduce the number of homeowners that find it optimal to default on their mortgage. To the extent that mortgage defaults have a feedback effect on prices, lower default rates may also positively impact house prices. In this section we use our framework to evaluate the impact of this policy on the severity of the housing crisis.

To implement this policy in our model, households that are underwater on their mortgage have their mortgage payments reduced until the present value of these payments is equal to the value of their home net of selling costs. Mortgage lenders internalize this policy change, recognizing that the stream of payments they expect to receive will be reduced in the following period if the household declares bankruptcy and qualifies for a mortgage cram down.

We then compute a perfect foresight transition in our economy under the following timeline: In 2005 the BAPCPA is implemented. The economy experiences an unexpected housing crisis in 2007, in which the time series for the supply of owner-occupied housing and credit wedges are the same as in the previous transitions. In addition, the mortgage cram down policy is unexpectedly and permanently implemented, allowing households with negative home equity to reduce their payments until they are no longer underwater from 2007 onward.

Table 2.9 compares statistics in 2007 for transitions for the model with and without the cram down policy.⁶⁷ These results suggest that the cram down policy generates a reduction in the mortgage default rate relative to an economy that did not enact this policy. Specifically, the mortgage default rate is 2.1% lower in the economy that enacted cram down. This reduction in mortgage defaults comes at the expense of a 6.2% higher bankruptcy rate. Notably, however, this policy has very little impact on aggregate prices.

To gain intuition for these results, we consider how homeowner decision rules are altered by this policy. Figure 2.6 depicts homeowner decision rules in 2007 with and without the cram down policy in effect. Evident from this figure is a dramatic reduction in the

⁶⁷Not presented in this table is the fact that the cram down policy is well-utilized by homeowners. Specifically, 18.3% of homeowners that declare bankruptcy in 2007 take advantage of the benefits of cram down, and this percentage rises over the next several years after the onset of the crisis.

Table 2.9: Cram Down's Impact on the Housing Crisis in 2007

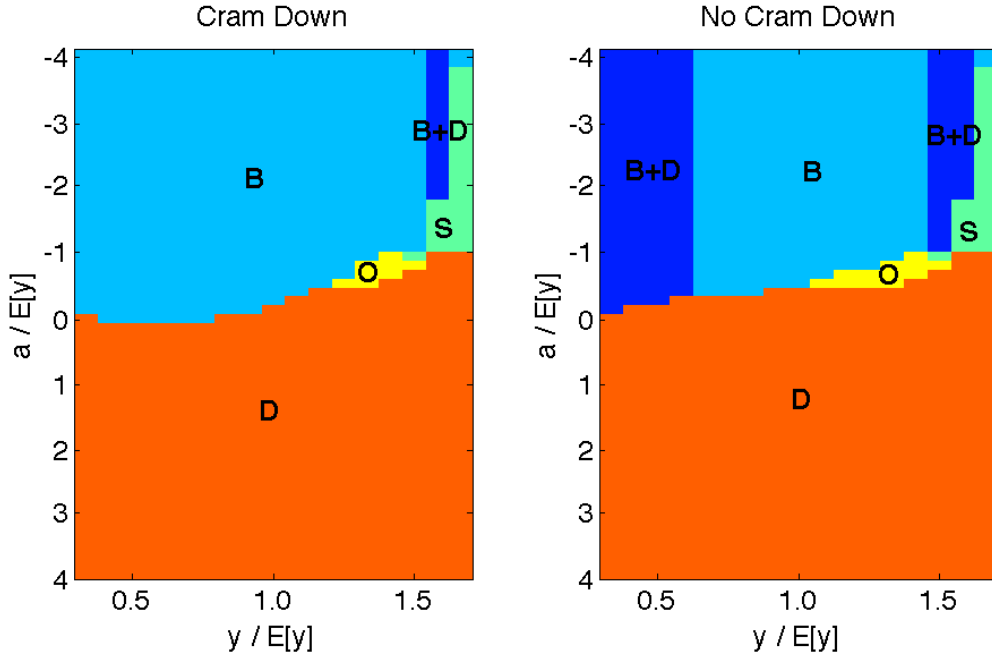
Statistic	No Cram Down	Cram Down	Cram Down / No Cram Down
P	0.745	0.745	0.0%
R	0.044	0.043	-0.9%
Bankruptcy Rate	3.27%	3.47%	6.2%
Mortgage Default Rate	4.52%	4.43%	-2.1%
R / P	0.059	0.058	-0.9%

region where the household finds it optimal to default, either by simultaneously declaring bankruptcy and defaulting on their mortgage or by only defaulting on their mortgage. By allowing homeowners to reduce their mortgage payments and no longer be underwater on their mortgage by declaring bankruptcy, the cram down policy incentivizes homeowners to declare bankruptcy in regions where they would find it optimal to default on their mortgage in the absence of the policy. Quantitatively, this produces a slightly lower mortgage default rate and a substantial increase in the bankruptcy filing rate during the peak of the crisis.

An inherent difficulty with the cram down policy is also evident from this figure. In particular, it is difficult to identify which households with negative home equity will actually default on their mortgage. Recall that while negative home equity is a necessary condition for mortgage default, it is not sufficient. Therefore, some households with negative home equity may take advantage of the lower mortgage payments under cram down even though they would not have defaulted on their mortgage in the absence of this program. In this case, the policy imposes losses on mortgage lenders and, possibly, unsecured creditors, without even reducing the number of mortgage defaults.

This can be seen in Figure 2.6, as some households that would have found it optimal to stay in their home and not declare bankruptcy in the absence of cram down, instead find it optimal to declare bankruptcy and take advantage of lower mortgage payments when cram down is available. This is true for the states that switch from own (yellow) to bankruptcy (light blue). The impact of cram down on the mortgage default rate is limited by the fact

Figure 2.6: Cram Down and Homeowner Decision Rules in 2007



that homeowners have asymmetric information about their willingness to stay in their home when they have negative home equity. As a result, we find that cram down only produces a 2.1% reduction in the mortgage default rate at the peak of the crisis.

To the extent that an objective of the cram down policy is to stabilize and raise owner-occupied home prices by decreasing the supply of housing produced by homeowners that default on their mortgage, our analysis suggests this policy would not have been successful. The owner-occupied price path for the model with cram down is virtually identical to an economy that does not enact this policy. And although rental prices fall more in the economy with cram down, due to the fact that demand for rental housing is lower because more homeowners choose to stay in their homes, this effect is also minimal. These results are in line with Chatterjee and Eyigungor (2011b), who find that the feedback effect of defaults on house prices is rather limited. In particular, these authors find that the drop in the owner-occupied house price at the peak of the crisis would still be 84% of the actual decline in house prices if mortgage defaults were completely disallowed. Therefore, our analysis suggests that although the mortgage cram down policy may have reduced mortgage default rates during

the housing crisis, its effect on aggregate prices would have been minimal.

2.8 Conclusion

This paper investigates whether the BAPCPA of 2005 exacerbated the recent housing crisis in the context of a quantitative-theoretic, equilibrium model of unsecured debt and mortgage markets. We conclude that, although the BAPCPA did produce higher mortgage default rates, it had minimal effect on the severity of the housing crisis and virtually no impact on aggregate house prices.

Understanding how unsecured debt and mortgage prices respond to new incentives to declare bankruptcy and default in response to reform is key to our findings. In particular, the BAPCPA increased homeowner incentives to default by reducing the benefit homeowners derived from borrowing against their non-exempt home equity in unsecured credit markets, leading marginal homeowners to switch from owning to defaulting on their mortgage. Moreover, a rise in risky unsecured lending brought about by a reduction in the likelihood of bankruptcy for high-income households, increased the probability that a homeowner enters a region in which they find it optimal to declare bankruptcy and default. These incentives that tend to increase the mortgage default rate were offset by the fact that mortgage lenders tightened lending standards for new homeowners by requiring the household to purchase a smaller home and/or undertake a mortgage with a lower loan-to-income, loan-to-value, and mortgage payment-to-income ratio. Tighter mortgage standards, in turn, reduce the likelihood that households will find themselves in a position in which they prefer to default on their mortgage. On net, these incentives tend to offset, implying that the BAPCPA had a limited impact on the severity of the housing crisis.

2.9 Appendix A: Solution Algorithm

In this appendix we detail the solution algorithm for our model. We begin by describing how to solve for the steady state and then discuss how we solve for the perfect foresight transition paths of our economy under the actual sequence of events and the counterfactual sequence in which there is no bankruptcy reform.

2.9.1 Solving for the Stationary Distribution

Solving for the initial stationary distribution (i.e. prior to bankruptcy reform and the housing shock) of our economy entails fixing prices for owner-occupied and rental housing and solving the following fixed point problem in our economy without bankruptcy reform. To do so we first set $P = 1.0$ and $R = 0.052$, which is in line with the historical rental price to owner-occupied ratio in the U.S. economy from 1960 to 2000 documented in Davis et al. (2008). The solution algorithm is as follows:

1. Guess initial values for $V(k, x, a, y)$, $X(k, x, a, y)$, $q(k, x, a, y)$, $m(k, x, a, y)$, $m^X(k, x, a, y)$. Denote these initial guesses with a 0 subscript.
2. Taking these guesses as given, compute household optimal decision rules. From these optimal decisions, compute implied values for $V_1(k, x, a, y)$, $X_1(k, x, a, y)$, $q_1(k, x, a, y)$, $m_1(k, x, a, y)$, and $m_1^X(k, x, a, y)$ from the functional equations outlined in Section 2.3.
3. Compute the maximum of the absolute value of the differences between the initial guesses for these functions (denoted 0) and the implied values for these functions (denoted 1) given the initial guesses. If this maximum absolute difference is less than a pre-specified tolerance level, stop value function iteration, and we have found the fixed point of the operator. Conversely, if the maximum absolute difference exceeds the tolerance level, use the implied values computed in this step as the initial guess in step 2.

4. Iterate on 2 and 3 until the maximum difference is less than the tolerance level.
5. Once value function iteration is completed, we use the resulting household optimal decision rules to simulate an economy of 30 million households over 500 periods to compute the stationary distribution. The initial supplies for owner-occupied and rental housing are determined by setting these values equal to their respective demands implied by this initial stationary distribution. Label these initial housing supplies as K_0 and H_0 respectively.

Now we have the pre-reform, pre-housing shock stationary distribution of our economy. To compute the stationary distribution under changes in the bankruptcy code, we first set the supply of owner-occupied and rental housing equal to our desired values. Then, given initial guesses for P and R , we solve for the implied demand for both types of housing using the five steps just outlined. We then adjust P and R until both housing markets clear.

2.9.2 Solving for the Perfect Foresight Transition

This section details how we solve for the perfect foresight transition in our economy under both the actual sequence of events and the counterfactual sequence of events in which the economy experiences a housing crisis in 2007 – mimicked by an unanticipated increase in the supply of owner-occupied housing – but did not implement bankruptcy reform in 2005. We begin with the actual sequence of events, as this transition is more complicated than the counterfactual.

Actual Sequence of Events

Under this transition the economy experiences an unexpected change to the bankruptcy code in 2005 and an unexpected shock to the owner-occupied housing supply in 2007. In the first few years following the housing crash, mortgage lenders experience a credit wedge that raises the cost of issuing mortgages. To compute the perfect foresight transition under this

sequence of events, we must actually compute two different transitions and then combine the results from each. Recall that we assume the following timeline:

1. 2004: Economy is in pre-bankruptcy reform steady state with housing supplies given by K_0 and H_0 .
2. 2005: An unexpected and permanent change to the bankruptcy code occurs.
3. 2007: An unexpected increase in the supply of owner-occupied housing occurs, such that the supply of owner-occupied housing becomes $\tilde{K} = 1.04K_0$ in 2007. This elevated housing supply persists until 2012 and then slowly returns to its initial value by 2020.
4. 2008: A credit wedge equal to 0.035 raises the cost of issuing new mortgages. Like the housing supply shock, this wedge persists until 2012 and then returns to zero by 2020.

Thus, to correctly compute the economy's transition given this sequence of events, we must solve for two perfect foresight transitions. The first is for an economy that begins in the pre-bankruptcy reform steady state with housing supplies given by K_0 and H_0 and experiences an unexpected and permanent change to the bankruptcy code in 2005. From this transition we derive the relevant statistics for this economy in 2005 and 2006 in addition to the distribution of households entering 2007.

Next, we compute the transition for the post-bankruptcy reform economy that experiences a housing shock in 2007, beginning from the distribution implied by the bankruptcy reform-only transition in 2007, to the steady state for the post-bankruptcy reform economy with housing supplies equal to their initial values K_0 and H_0 .

We now detail how we compute each of these transitions.

Reform-Only Transition

We assume that the economy takes $T = 40$ years to transition to its new steady state after experiencing an unexpected shock.⁶⁸ The steps for computing this transition are then:

⁶⁸This assumption is confirmed if the economy has successfully transitioned to the terminal steady state

1. Using the algorithm outlined in Section 2.9.1, solve for the steady states of the economy both pre- and post-bankruptcy reform with the housing supplies equal to their initial values K_0 and H_0 .
2. Set terminal values for $V_T(k, x, a, y)$, $q_T(k, x, a, y)$, $X_T(k, x, a, y)$, $m_T(k, x, a, y)$, and $m_T^X(k, x, a, y)$ equal to their values in the post-reform steady state.
3. Set $K_t = K_0$, $H_t = H_0$, and $\alpha_t = 0$ for all t .
4. Guess a sequence of owner-occupied house prices and rental prices $\{P_t, R_t\}_{t=1}^{39}$.
 - (a) Use the decision rules and pricing functions from the post-reform steady state to back out the $t = 39$ pricing functions from the functional equations defining these pricing functions outlined in Section 2.3.2. Given these pricing functions and the guessed house prices, compute optimal household decisions for $t = 39$ under the assumption that bankruptcy reform is in place.
 - (b) Repeat this step from $t = 38$ to $t = 1$, documenting household decision rules at each point in time along the transition, to compute the sequence of decision rules and pricing functions along the way.
5. Next, starting from the stationary distribution defined by the pre-bankruptcy reform economy, simulate the distribution of people each period given the sequences of decision rules determined in (b) from $t = 1$ to $t = 40$.
6. From the distribution of people, compute demand for owner-occupied and rental housing for each period, and compute excess demand for both types of housing at each point in time.
7. If excess housing demand and supply are below some pre-specified threshold for both owner-occupied and rental housing at each point in time along the transition, then we

in 40 years. If this is not the case, we increase T .

have successfully solved for the perfect foresight transition. If not, adjust P_t and R_t along the transition, increasing (decreasing) each slightly if the excess demand (supply) for that form of housing is too high at period t . Return to 4 with this new guess for the sequences of prices.

This algorithm gives us the perfect foresight transition of the economy under the assumption that the economy only experienced bankruptcy reform but no housing crisis. We use the statistics from the first two periods of this transition, corresponding to 2005 and 2006, in the final transition under the actual sequence of events.

Full Transition

To compute the full transition, however, the economy must experience a housing supply shock in 2007. To compute the statistics along the transition during and after the housing supply shock, we follow the algorithm just outlined, but use the value and pricing functions from the post-reform steady state as the terminal values (period T) and the distribution that is implied from the second period of the reform-only transition as the initial distribution entering 2007. We also use the time series for the housing supply and credit wedges outlined in Section 2.6.

Counterfactual

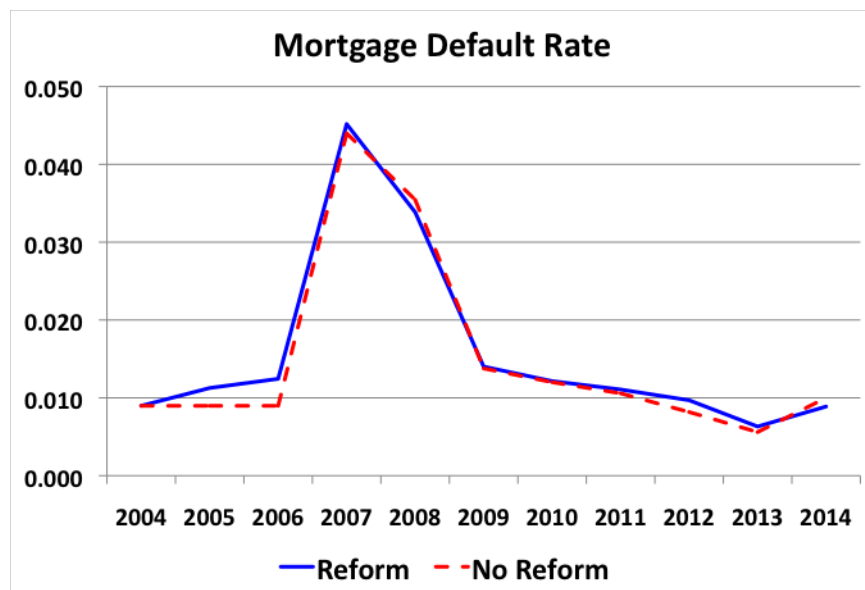
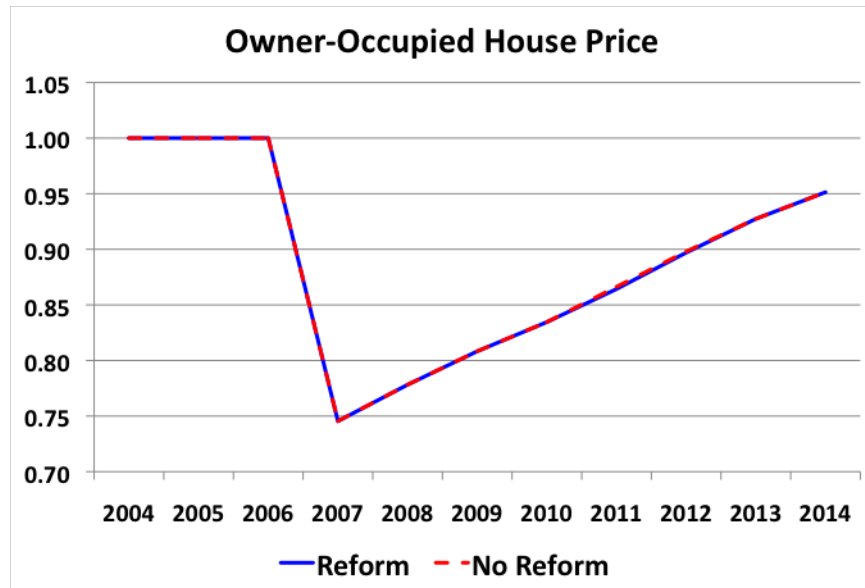
The counterfactual experiment assumes the following timeline:

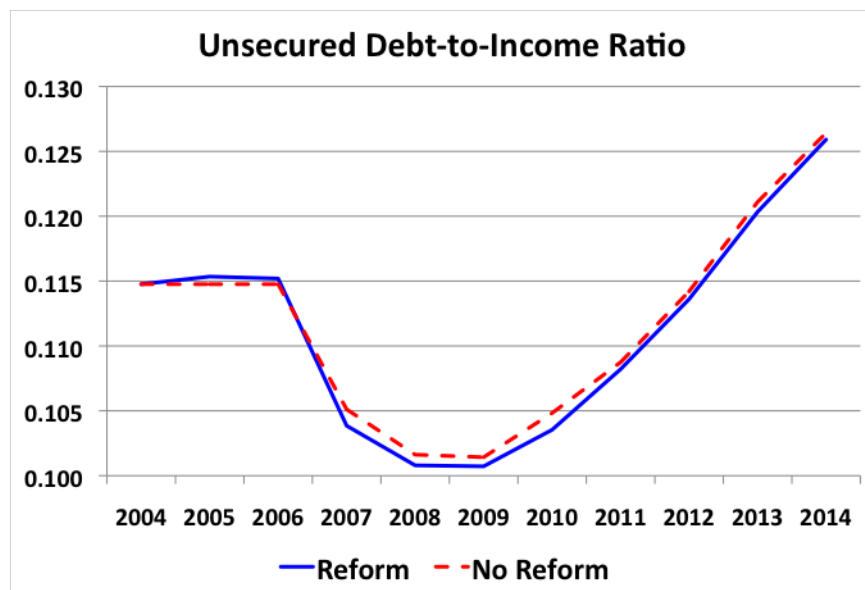
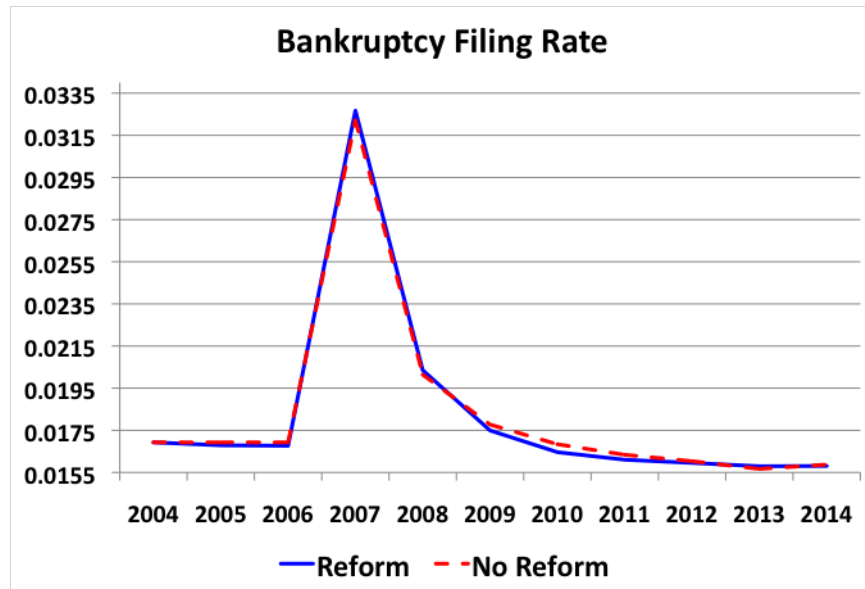
1. 2004-2006: Economy is in pre-bankruptcy reform steady state with housing supplies given by K_0 and H_0 .
2. 2007: An unexpected increase in the supply of owner-occupied housing occurs, such that the supply of owner-occupied housing becomes $\tilde{K} = 1.04K_0$ in 2007. This elevated housing supply persists until 2012 and then slowly returns to its initial value by 2020.

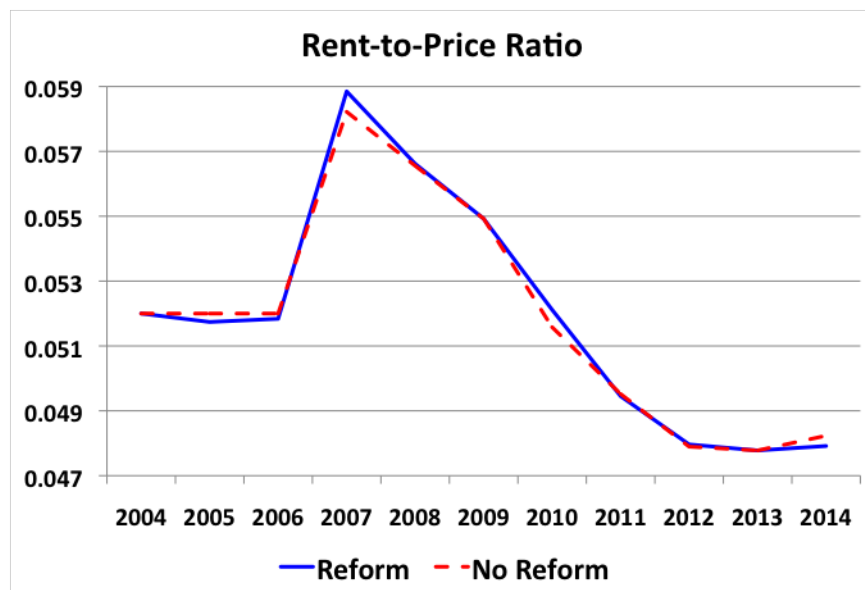
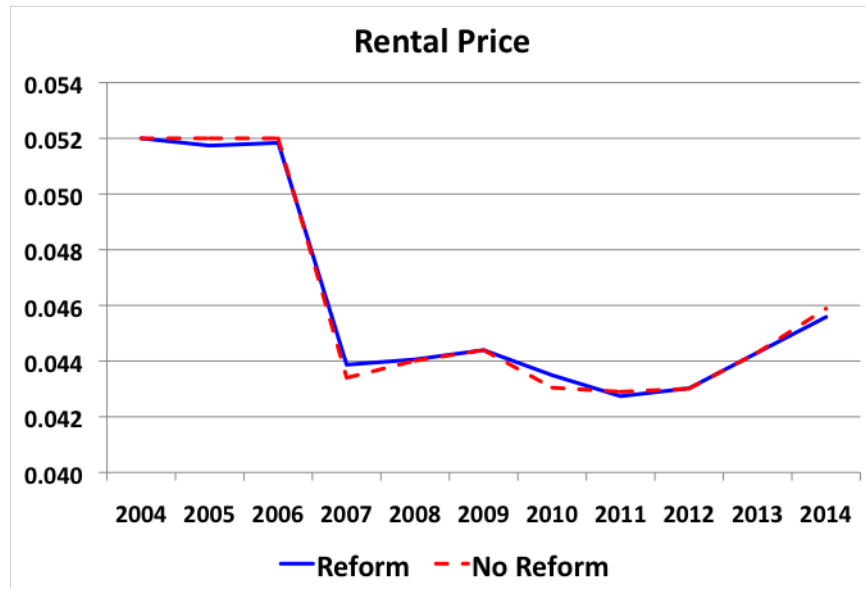
3. 2008: A credit wedge equal to 0.035 raises the cost of issuing new mortgages. Like the housing supply shock, this wedge persists until 2012 and then returns to zero by 2020.

To solve for the counterfactual transition we follow the detailed transition algorithm outlined in the preceding section, but use the steady state corresponding to an economy that does not undergo bankruptcy reform for the terminal values for value and pricing functions. The initial distribution of households is taken as the pre-reform stationary distribution with housing supplies given by K_0 and H_0 .

2.10 Appendix B: Transitions







3 The Impact of Third-Party Bailouts on Business Cycles, Sovereign Default, and Welfare

3.1 Introduction

Interventions by third parties in sovereign default events have been substantial historically. In the past decade alone, Argentina received a \$40 billion bailout from the IMF in 2001 – an amount nearly 15% of Argentine GDP that year. More recently, Ireland agreed to a bailout package exceeding \$100 billion. And while the ultimate cost of the Greek bailout is yet to be determined, it will certainly be immense. Indeed, the presence of third parties in sovereign default events and bond markets has become common. It is perhaps surprising then that little quantitative work has been conducted analyzing the effects of bailouts of this size on business cycle statistics, the incidence of default, and social welfare. Moreover, virtually no positive analysis has been conducted testing the sensitivity of the conclusions about the impact of bailouts to alternative specifications. This paper takes up this task by quantitatively analyzing the impact of introducing bailouts into an otherwise standard model of sovereign default under a broad class of assumptions about the form of bailouts, sovereign discount factors, and stochastic processes for sovereign endowments.

Previous work that has considered the impact of introducing bailouts into quantitative models of sovereign default suggests that they play a significant role. For example, Aguiar and Gopinath (2006), demonstrate that the number of defaults in their model increases nearly eight-fold from 24 to 191 occurrences per 10,000 quarters when bailouts are introduced.⁶⁹

Unlike previous work, I consider multiple specifications for bailouts. First, I consider bailouts that are implemented stochastically. In addition, the presence of these stochastic bailouts may be conditional on current observables, such as the sovereign's external debt-to-

⁶⁹In their paper the authors report the increase in defaults after raising the discount factor of the original model from 0.80 to 0.95. They report that sovereign defaults increase four-fold. The results mentioned here are found by taking their model and fixing the discount factor between the two simulations at 0.80.

output ratio. Second, I consider the impact of modeling bailouts that are not pure transfers. I find that the impact of sovereign bailouts on equilibrium objects, sovereign default, and welfare is particularly sensitive to the bailout specification. For instance, simply modeling bailouts as stochastically occurring with 25% probability – a realistic assumption according to Benjamin and Wright (2009) – reduces the incidence of sovereign default by more than 85% compared to the case when bailouts occur with probability one.

The intuition for the dramatically different results derived when introducing bailouts is the following. The interest rate on sovereign borrowing is far less sensitive to sovereign indebtedness when bailouts are present. This fact is manifested in a substantially flatter cost of debt curve in the model with certain bailouts.

This result is directly related to the lending specification common to many models of defaultable debt. It is assumed that there exists a population of risk-neutral, perfectly competitive foreign creditors that can lend to sovereigns at rates consistent with their expected recovery rate of the loan in the following period. Since the creditor is assumed to receive nothing when the borrower defaults (in the absence of bailouts), a zero-profit condition implies that he requires a greater return when the probability the sovereign defaults is higher. Therefore, the sovereign interest rate is positively related to the probability that the country does not repay its debt – the probability of default.

In the model without bailouts, each additional unit of debt increases the probability of default (holding endowments constant), thereby decreasing the expected return for the creditor. Consequently, the interest rate reacts to each unit of debt in the absence of bailouts, creating a steeper interest rate profile.

Conversely, in the model with bailouts every unit of debt up to the pre-specified bailout limit does not decrease the creditor's expected return, since that debt is guaranteed by the bailout. As a result, the interest rate is less sensitive to the sovereign's debt position when bailouts are present.

Under stochastic bailouts, the price of debt can be shown to be a linear combination of

the price of debt without bailouts and with certain bailouts. The weight placed on each of these components is the probability there is no bailout and the probability there is a bailout, respectively. This result implies that, by varying the probability of a bailout, the model in this paper can produce default levels ranging from those found without bailouts to those found with certain bailouts.

The motivation for considering the impact of different bailout specifications on equilibrium objects, incidence of default, and welfare comes from recent theoretical and empirical work. As noted by several authors, including Benjamin and Wright (2009), the existence of a bailout around sovereign default events has historically been uncertain. At the time of the default decision the sovereign is often unsure of whether a bailout will be proposed in the following period.

The second aspect abstracted from in previous models considering the impact of bailouts is the presence of conditionality in bailout lending. Increasingly, third-party interventions have been accompanied by either ex-ante or ex-post conditionality. In the case of the Vienna Initiative in March 2009, significant fiscal restrictions were required from Serbia, Hungary, and Romania to receive continued access to unfettered funding. The same is true with the recent austerity measures required by European Union members in the bailout of Greece. Many have argued that conditionality helps to mitigate problems of moral hazard and promote efficiency. Some, such as Ostry and Zettelmeyer (2005), propose a three-tiered lending mechanism in which access to IMF funds in times of crisis depends on meeting observable governmental restrictions in non-crisis periods. Here, I explicitly model ex-ante third-party conditionality by allowing the probability of a bailout to be conditioned on current observables such as the sovereign's external debt-to-output ratio.

Finally, the work of Joshi and Zettelmeyer (2005) motivates the extension into a setting where bailouts are not pure transfers. Since IMF lending usually takes the form of loans that are almost always repaid, implicit transfers arise only from the possibility that rates on IMF loans are below market rates for loans of similar default risk. These authors estimate

that the *cumulative* implicit transfers in IMF lending to emerging market economies from 1973 to 2001 range from 0% to 4% of the sovereign's 2002 GDP – significantly below the thresholds considered in other quantitative work.⁷⁰

To preview the results, I find that the effect of third-party bailouts on business cycle objects and welfare depends greatly on the form of these bailouts. I illustrate that a model with stochastic bailouts can improve on the ability of existing sovereign default models to match Argentine business cycle statistics in several dimensions, including: (i) the incidence of default, (ii) volatility of trade balance-to-output ratio, (iii) the correlation between the trade balance-to-output ratio and output, and (iv) the correlation between consumption and output. Moreover, third-party bailouts occurring with 75% probability are able to match the incidence of default and trade balance-to-output ratio found in Argentine data almost exactly. Similar to many models in this strand of the literature, however, this model produces equilibrium interest rates with counterfactually low volatility. Finally, I find that the welfare implications of introducing bailouts are particularly sensitive to assumptions about discount rates, bailout limits, and the stochastic endowment process. But we can still draw some general conclusions: the benefit to sovereigns is increasing in the probability of a bailout and the discount rate, and welfare is also higher if sovereign output follows a stochastic trend as suggested by Aguiar and Gopinath (2007). Perhaps more importantly, after considering the resource costs of providing bailouts, they generally reduce social welfare.

The remainder of this paper is structured as follows. Section 3.2 discusses related literature. Section 3.3 presents the baseline model and bailout specifications in this paper. Section 3.4 presents the quantitative results for business cycle statistics and welfare. Finally, Section 3.5 concludes.

⁷⁰The authors characterize the following countries as emerging markets: Turkey, Bulgaria, Argentina, Ukraine, Brazil, Uruguay, Algeria, Russia, Poland, Croatia, Nigeria, Colombia, Mexico, Venezuela, Hungary, Tunisia, Malaysia, China, Panama, Peru, South Africa, Chile, Thailand, Ecuador, Egypt, Morocco, and Philippines.

3.2 Related Literature

This paper is closely related to the strand of literature concerned with quantitatively replicating the salient characteristics of emerging market business cycles in a defaultable debt framework. Papers in this area include Aguiar and Gopinath (2006), Arellano (2008), Hatchondo and Martinez (2009), and Chatterjee and Eyigungor (2011a). These papers seek to quantitatively capture the distinct characteristics of emerging market business cycles documented by Uribe and Yue (2006) and Neumeyer and Perri (2005). These authors established that, unlike developed, small, open economies, business cycles in emerging market economies exhibit more volatile and countercyclical interest rates, countercyclical current account balances, and positively correlated interest rates and current account balances.

The model considered in this paper is most closely related to the Aguiar and Gopinath (2006) specification. In particular, I take this model as the baseline framework before introducing bailouts. By assumption, the sovereign only has access to one-period, unconditional bonds. Unlike this market assumption, Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2011a) allow for longer term structures in order to capture a more volatile interest rate.

This paper is also related to the strand of literature studying the role and impact of third parties in international financial markets. Most of this work has focused on the IMF. Empirically, Joshi and Zettelmeyer (2005) document the size of transfers implicit in different forms of IMF lending over the period 1973-2003. They document that cumulative implicit transfers in IMF lending to emerging market economies over this period are relatively minor: only 0-4% of 2002 GDP. Theoretically, Ostry and Zettelmeyer (2005) and Jeanne, Ostry, and Zettelmeyer (2008) analyze the optimal structure of IMF crisis prevention lending, in terms of minimizing moral hazard and promoting social welfare. Both papers provide theoretical justification for third-party conditionality. The latter work defines conditions under which moral hazard is present with bailouts, as well as describes how ex-ante conditionality helps to mitigate moral hazard. The former paper argues for ex-ante condi-

tionality in which IMF lending in times of crisis is conditional on the sovereign's ability to meet observable objectives in non-crisis periods.

Finally, Aguiar and Gopinath (2006) and Benjamin and Wright (2009) consider the impact of third-party bailouts on business cycle statistics and welfare, respectively. As previously mentioned, this work concludes that the impact of third-party bailouts on the rate of sovereign default and business cycle statistics is significant. Benjamin and Wright conclude that, not only does the presence of third-party bailouts reduce social welfare, bailouts also reduce sovereign welfare, which contrasts with the conclusions of this paper. As I show below, these differences can be mostly accounted for by the different endowment processes considered between this paper and Benjamin and Wright's work.

This paper is motivated by the work mentioned above in its analysis of the impact of different structures for third-party bailouts, including stochastic and ex-ante conditionality. However, I deviate from the above work in several important respects. First, I consider the business cycle implications of stochastic, and possibly conditional, bailouts. Second, I explicitly consider the welfare implications of third-party bailouts. Unlike the previously mentioned work, I also explicitly consider the impact of ex-ante conditionality. To my knowledge, this is the first paper that introduces conditionality in third-party bailouts into a quantitative model of sovereign default in order to assess its impact on business cycles and sovereign welfare.

3.3 Model

3.3.1 Endowment Process

The basic setup of the model is a dynamic, stochastic, general equilibrium extension of the original work by Eaton and Gersovitz (1981) and mirrors that found in Aguiar and Gopinath (2006). Consider a stochastic endowment economy with a representative agent referred to as the sovereign. The sovereign receives an endowment stream y that evolves according to the following stochastic process:

$$y = e^z \Gamma \quad (3.1)$$

where

$$\Gamma = g\Gamma_{-1} \quad (3.2)$$

and

$$\ln(g) = (1 - \rho_g) \left(\ln(\mu_g) - \frac{1}{2} \frac{\sigma_g^2}{1 - \rho_g^2} \right) + \rho_g \ln(g_{-1}) + \epsilon^g. \quad (3.3)$$

Finally, I assume $|\rho_g| < 1$ and $\epsilon_t^g \sim N(0, \sigma_g^2)$.

For most of this paper z will be deterministic and equal to zero. When analyzing the welfare implications of this model, I will consider both the stochastic trend ($z = 0$) and the stable trend in which Γ is deterministic but z is stochastic. Aguiar and Gopinath (2006) demonstrate that the model with a stochastic trend more closely replicates sovereign default data.⁷¹ These authors explain that with a stable trend the incidence of default and equilibrium debt-to-output ratios are counterfactually low. The intuition for this result is that with a stable trend the default decision, and therefore the interest rate faced by the sovereign, depends more on the level of debt held by the sovereign than the current endowment shock realization. As a result, the interest rate profile is particularly sensitive to the sovereign's debt level, providing incentives for the sovereign to reduce its equilibrium level of debt. Thus, sovereigns do not accumulate enough debt in equilibrium for default to be preferred with a stable trend.

With a stochastic trend current endowment realizations result in significant differences in the value of defaulting and not defaulting. Therefore, the default decision is more sensitive to current shock realizations and less sensitive to the sovereign's current level of debt. This produces a flatter interest rate profile than in the case with a stable trend. The sovereign

⁷¹In addition, empirical support for a stochastic trend is provided in Aguiar and Gopinath (2007).

reacts to this flatter profile by taking on more debt and defaulting more often in equilibrium. In Section 3.4 we will see that the welfare implications of introducing third-party bailouts are sensitive to the endowment specification.

3.3.2 Sovereign's Problem

I first consider the model specification in a world without bailouts. The sovereign can trade a single, one-period, uncontingent asset with a perfectly competitive, risk-neutral foreign creditor. Let b denote the net foreign assets of the sovereign at time t , so that if b is negative, the sovereign has borrowed on net at time t . The sovereign can use the resources provided by its current endowment net of its asset position for consumption or savings. The model can thus be written with two state variables: b and y . Denote the value function for a sovereign currently in good credit standing (not in default), net foreign assets of b , and endowment y , as

$$V^G(b, y) \equiv \max\{V^D(y), V^R(b, y)\}$$

where $V^D(y)$ is the value of a defaulting country with endowment y , and $V^R(b, y)$ is the value function of a sovereign that does not default, repays its debt, and remains in good credit standing. In the case of default, the sovereign loses its net foreign asset position and access to financial markets and, therefore, the value of the defaulting sovereign is not a function of b .

As is common in this literature, after defaulting, the sovereign loses a portion of output δ each period it is not in good credit standing and regains access to international credit markets with exogenous probability λ each period in default. The output loss is a reduced form way of capturing empirically documented lower output for sovereigns around default events (see, for example, Cohen (1992) and Tomz and Wright (2007)). And the exogenous re-entry probability captures the fact that sovereigns are frequently excluded from international capital markets for an extended period following default (see Sandleris, Gelos, and Sahay (2004) and Benjamin and Wright (2009)). The value function for a sovereign in default is

thus:

$$V^D(y) = u((1 - \delta)y) + \beta \mathbb{E} [\lambda V^G(0, y') + (1 - \lambda) V^D(y')] . \quad (3.4)$$

The sovereign in good credit standing that chooses to repay its debt optimally chooses its current consumption, c , taking the price of debt, $q(b', y)$, as given. The value function for this choice is given by:

$$V^R(b, y) = \max_{\{c, b'\}} \{u(c) + \beta \mathbb{E} [V^G(b', y')]\} \quad (3.5)$$

s.t.

$$c + q(b', y)b' = y + b.$$

As an alternative to lending to sovereigns, foreign creditors can earn a risk-free rate of return $1 + r^*$. Since foreign creditors are assumed to be perfectly competitive and risk neutral, the expected return on sovereign lending must equal this risk-free rate. A simple no arbitrage argument implies the following relationship:

$$1 + r^* = (1 - \mathbb{E}[D(b', y')|y]) (1 + r) \quad (3.6)$$

where $D(b', y')$ is the default function. Specifically,

$$D(b, y) = \begin{cases} 1 & \text{if } V^D(y) > V^R(b, y) \\ 0 & \text{otherwise} \end{cases} .$$

Since $1 + r \equiv 1/q(b', y)$, (3.6) implies that the price of debt must satisfy:

$$q(b', y) = \frac{1 - \mathbb{E}[D(b', y')|y]}{1 + r^*} . \quad (3.7)$$

Equation (3.7) shows the explicit relationship between the equilibrium price of debt and

the expected probability of default. Clearly, this relationship is negative. We can now define a recursive competitive equilibrium for this economy.

Definition *A recursive competitive equilibrium for this economy consists of a set of sovereign decision rules for consumption, savings, and default: $c(b, y), b'(b, y), D(b, y)$ and a bond price schedule $q(b', y)$, such that, (i) sovereigns use their decision rules to optimally solve their problem, (ii) taking the risk-free interest rate as given, sovereign interest rates are determined by the creditor's zero-profit condition, and (iii) default expectations are correct, $\mathbb{E}[D(b', y')|y] \equiv \int_{y'} D(b', y') dM(y'|y)$ where $M(\cdot|\cdot)$ is the conditional distribution of y' given y .*

The only modifications required to this definition when bailouts are present is in the definition of the equilibrium interest rate.

3.3.3 Model with Bailouts

In this subsection I describe the different bailout specifications that will be considered in the subsequent quantitative exercises.

Stochastic Bailouts as Pure Transfers

I first consider the following generalization: There exists an unmodelled third party that can commit ex-ante to provide a bailout with some probability in the subsequent period.⁷² This probability is denoted $\psi(b', y)$, and it may be conditional on the current realization of sovereign debt and endowment. The bailout, if it occurs, ensures that the creditor receives repayment on its lending in the case of default up to a predetermined, time-invariant amount b^* . Both $\psi(b', y)$ and b^* are assumed to be common knowledge to creditors and the sovereign. Moreover, I assume that bailouts take the form of pure transfers from the third party to

⁷²Admittedly, this assumption may be problematic, as the IMF may find it optimal to provide a bailout in a period after deciding that a bailout should be unlikely in the previous period. Issues concerning time inconsistency for the IMF will be abstracted from at present.

foreign creditors and are not repaid by the sovereign. This bailout may be certain, in which case $\psi(b', y) = 1$ for all b' and y ; stochastic and unconditional, in which case $\psi(b', y) = \kappa$, $\kappa \in (0, 1)$ for all b' and y ; or stochastic and conditional, in which case $\psi(b', y)$ is a function of b' and/or y .

When bailouts are present at probability $\psi(\cdot, \cdot)$, the creditors are repaid in two different circumstances. First, they are repaid in full if the sovereign does not default. This occurs with expected probability $1 - \mathbb{E}[D(b', y')|y]$. They are also repaid up to the bailout limit in the event that the sovereign defaults but a bailout occurs. The expected repayment for this event is $\psi(b', y)\mathbb{E}[D(b', y')|y] \min\{1, \frac{b^*}{b'}\}$. Therefore, the general formula for the price of debt with stochastic bailouts is given by equation (3.8).

$$q(b', y) = \frac{1 - \mathbb{E}[D(b', y')|y] + \psi(b', y)\mathbb{E}[D(b', y')|y] \min\{1, \frac{b^*}{b'}\}}{1 + r^*}. \quad (3.8)$$

When bailouts are certain, i.e. when $\psi(b', y) = 1$ for all values of b' and y , this formula reduces to the following two expressions depending on whether or not the sovereign has borrowed more than the bailout threshold:

(i) If $|b'| \leq |b^*|$:

$$q(b', y) = \frac{1 - \mathbb{E}[D(b', y')|y] + \mathbb{E}[D(b', y')|y]}{1 + r^*} = \frac{1}{1 + r^*} \quad (3.9)$$

(ii) If $|b'| > |b^*|$:

$$q(b', y) = \frac{1 - (1 - \frac{b^*}{b'}) \mathbb{E}[D(b', y')|y]}{1 + r^*}. \quad (3.10)$$

These equations have intuitive interpretations. In the first case with $|b'| < |b^*|$ and certain bailouts, the creditor is ensured of being repaid and so zero profit dictates that it must charge the sovereign the risk-free rate. In the second case where borrowing by the sovereign exceeds the bailout threshold, $|b'| > |b^*|$, the creditor loses a portion of his lending $(1 - \frac{b^*}{b'})$ if the sovereign defaults. This portion is discounted by the probability the sovereign defaults to

derive the appropriate value for the price of debt.

In this case, with $\psi(b', y) = 1$, the general formula for the price of debt described above is equivalent to:

$$q(b', y) = \frac{\min \{1, \frac{b^*}{b'}\} + (1 - \mathbb{E}[D(b', y')|y]) \max \{1 - \frac{b^*}{b'}, 0\}}{1 + r^*}. \quad (3.11)$$

I will use $\hat{q}(b', y)$ below as shorthand for this expression. In what follows, I consider stochastic and possibly conditional bailouts which imply the following formula for the price of debt:

$$\tilde{q}(b', y) = (1 - \psi(b', y)) \frac{(1 - \mathbb{E}[D(b', y')|y])}{1 + r^*} + \psi(b', y) \hat{q}(b', y) \quad (3.12)$$

where $\hat{q}(b', y)$ is given by (3.11). This formula is equivalent to (3.8). However, equation (3.12) shows that the price of debt in this general case is a linear combination of the price of debt without bailouts $\frac{1 - \mathbb{E}[D(b', y')|y]}{1 + r^*}$ and the price of debt with certain bailouts $\hat{q}(b', y)$.

Relaxing the Pure Transfer Assumption

In practice, IMF lending often takes the form of loans to the sovereign that are almost always repaid. Transfers implicit in this lending are present to the extent that such lending takes place at below market interest rates consistent with the expected default probability on IMF loans, which may be below the expected probability of default on private loans. As previously mentioned, Joshi and Zettelmeyer (2005) analyze the level of cumulative implicit transfers in IMF lending to emerging market economies like Argentina over the period 1973 to 2003. These authors estimate the cumulative implicit transfer of IMF lending to countries like Argentina – as a percentage of 2002 GDP – in the range of 2-4%. Note that this is cumulative transfers over this period, not simply the implicit transfer of a single bailout. Therefore, a transfer assumption of 18% of output as has been used in previous quantitative analysis of the impact of bailouts on business cycles, dramatically overstates transfers implicit

in IMF lending.⁷³ I aim to provide a more realistic model of IMF lending in this section.

Suppose that, in the period of default, the third party makes a loan to the sovereign in the amount $\hat{b} \equiv \max\{b^*, b\}$ which can only be used to repay the foreign creditor.⁷⁴ Because, in practice, the IMF is almost always repaid, it is assumed that the sovereign cannot default on loans from the third party. In the following period, the sovereign must repay a portion of this loan equal to $(1 - \omega)\hat{b}$, where ω represents the transfer implicit in third-party lending. In the specification laid out in the previous subsection, $\omega = 1$, so that the sovereign does not repay any portion of the payment from the third party to the creditor. The specification in this section requires that we carry an additional state variable because the timing of sovereign default is now relevant. Let D_{-1} be an indicator variable, taking the value of 1 if the sovereign defaulted in the previous period and 0 otherwise.

In this case, the value function for a country in good credit standing with $D_{-1} = 0$ can be written as:

$$V^G(b, y, 0) = \max\{V^D(b, y, 0), V^R(b, y, 0)\} \quad (3.13)$$

where

$$V^R(b, y, 0) = \max_{\{c, b'\}} \{u(c) + \beta \mathbb{E}[V(b', y', 0)]\}$$

s.t.

$$c + \tilde{q}(b', y)b' = y + b$$

and

⁷³Joshi and Zettelmeyer document that transfers implicit in IMF lending to highly-indebted poor countries are estimated to be significantly higher.

⁷⁴Since borrowing is denoted by a negative value of b , this condition states that the third party provides a loan equal to the lesser of the amount of debt outstanding by the sovereign and the prespecified threshold b^* .

$$V^D(b, y, 0) = u((1 - \delta)y) + \beta \mathbb{E} \left[\lambda V^R((1 - \omega)\hat{b}, y', 1) + (1 - \lambda)V^D((1 - \omega)\hat{b}, y', 1) \right].$$

The value functions for a sovereign at period $t + 1$ that defaulted in period t are given by:

$$V^R((1 - \omega)\hat{b}, y', 1) = \max_{\{c', b''\}} \{u(c') + \beta \mathbb{E} [V^G(b'', y'', 0)]\}$$

s.t.

$$c' + \tilde{q}(b'', y')b'' = y' + (1 - \omega)\hat{b}$$

and, finally,

$$V^D((1 - \omega)\hat{b}, y', 1) = u((1 - \delta)y' + (1 - \omega)\hat{b}) + \beta \mathbb{E} [\lambda V^G(0, y'', 0) + (1 - \lambda)V^D(y'', 0)].$$

Here I have assumed that once a sovereign defaults it cannot default on its third-party obligations in the following period, represented by $(1 - \omega)\hat{b}$. Thus, the value function of a sovereign in good credit standing but with $D_{-1} = 1$ does not allow for a default decision by the sovereign.

3.3.4 Parameterization and Solution Algorithm

In this section I present the parameterization for the model with a stochastic endowment trend since this is the relevant model for the business cycle statistics presented. Given the trend in the endowment process, this model is not stationary as presented. Aguiar and Gopinath (2006) demonstrate that it can be stationarized and represented recur-

sively by dividing through the sovereign’s problem by Γ or, equivalently, by $\mu_g\Gamma_{-1}$. I follow this detrending procedure and denote detrended objects with a tilde.

The model is calibrated at a quarterly frequency to match long-run Argentine business cycle data so as to make the results presented below directly comparable to previous work. Utility is assumed to be of the constant relative risk aversion form with risk aversion parameter γ :

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}.$$

The calibrated parameter values are provided in Table 3.1.

Table 3.1: Parameterization: Stochastic Trend

Parameter	Value
β	0.80
λ	0.10
b^*	-0.18
σ_g	0.03
ρ_g	0.17
γ	2
δ	0.02
r^*	0.01
μ_g	1.006

The value of the discount factor in this parameterization is low compared to standard values. However, as noted in Aguiar and Gopinath (2006) and Arellano (2008) for sovereign default and Chatterjee et al. (2007) for consumer default, low discount factors are required to capture the empirically observed incidence of default. The exogenous probability of re-entry after default, λ , implies that the sovereign remains in financial autarky for an average of 2.5 years. Moreover, it is assumed that default implies an output loss of 2% of the sovereign’s average detrended output. The parameters governing the endowment process are calibrated to match Argentine data. Specifically, the mean quarterly growth rate, μ_g , is calibrated to 0.6%.

After calibrating and detrending the model, I conduct value function iteration by first

discretizing the state space. In this case, the state space consists of the current net asset position and detrended endowment shock, $\tilde{b}$ and $\tilde{y}$ respectively. I use the procedure developed in Tauchen and Hussey (1991) to approximate the sovereign's autoregressive endowment process using a 25-state Markov chain. The model is then solved using the following steps:

- (i) Discretize the state space and choose an initial guess for the price of debt schedule, $\tilde{q}^0(\tilde{b}', \tilde{y})$. I began with $\tilde{q}^0(\tilde{b}', \tilde{y}) = 1/(1 + r^*)$ for all elements.
- (ii) Given $\tilde{q}^0(\tilde{b}', \tilde{y})$, compute the sovereign's optimal decisions for each point of the state space.
- (iii) Iterate on the value function until convergence.
- (iv) Compute the implied expected probability of default given the optimal decisions by $\mathbb{E}[D(\tilde{b}', \tilde{y}')|\tilde{y}] \equiv \int D(\tilde{b}', \tilde{y}')dM(\tilde{y}'|\tilde{y})$.
- (v) Compute the implied value for $\tilde{q}^1(\tilde{b}', \tilde{y}) = \frac{1 - \mathbb{E}[D(\tilde{b}', \tilde{y}')|\tilde{y}]}{1 + r^*}$. If $|\tilde{q}^1(\tilde{b}', \tilde{y}) - \tilde{q}^0(\tilde{b}', \tilde{y})| < \varepsilon$ where ε is a prespecified tolerance level, then stop, and $\tilde{q}^1(\tilde{b}', \tilde{y})$ represents the equilibrium value of $\tilde{q}$. If not, return to (i) with $\tilde{q}^1(\tilde{b}', \tilde{y})$ as the new guess and iterate until convergence.

The model is then simulated over a period of 10,000 quarters. One thousand simulations were used to compute average values for the model variables. In addition, the model data are transformed into their empirical analogues for comparison with Argentine data by considering percentage deviations from a Hodrick-Prescott Filter trend for both data.

3.4 Results

3.4.1 Stochastic Bailouts as Pure Transfers

This section first considers the ability of the models outlined above to capture the distinct features of emerging market business cycles, using Argentina as the primary source of data for comparison. I first consider the case in which third-party lending is a 100% transfer, i.e.

$\omega = 1$, but $\psi(\cdot, \cdot)$ takes on several specifications. This allows me to directly compare the different specifications with the results found in previous work.

Below, Model I is the model without bailouts; Model II is the model with certain bailouts ($\psi(\tilde{b}', \tilde{y}) = 1$); Model III is the model with stochastic and unconditional bailouts ($\psi(\tilde{b}', \tilde{y}) = \kappa \in (0, 1)$); Model IV is the model with stochastic bailouts conditional on $\tilde{b}'$ ($\psi(\tilde{b}', \tilde{y}) = 1 - \tau|\tilde{b}'|^\theta$); and, finally, Model V has stochastic bailouts conditional on the current net external debt-to-output ratio $\tilde{b}'/\tilde{y}$ ($\psi(\tilde{b}', \tilde{y}) = 1 - \xi|\tilde{b}'/\tilde{y}|^\eta$).

The parameters of the above functional forms were chosen to target average bailout probabilities and to provide for a non-linear relationship between the bailout probability and the sovereign's current net foreign asset position. This specification is intended to capture Ostry and Zettelmeyer's (2005) suggested tiered IMF lending in which sovereigns meeting certain conditions, low debt levels in this case, are provided more favorable future lending conditions than sovereigns that have not met these conditions (i.e. more indebted nations). This aspect is manifested with higher bailout probabilities for sovereigns with more favorable current debt positions in Model IV and for sovereigns with more favorable debt-to-income positions in Model V.

Business Cycle Implications

Tables 3.2 through 3.5 display business cycle statistics for Models I through V compared to Argentine data. The tables differ in the average bailout probability, which are 25, 50, 75, and 90 percent in tables 3.2 through 3.5, respectively.⁷⁵ In these tables, “*TB*” represents the trade balance and “Default” represents the number of default occurrences in 10,000 quarters. Lastly, standard deviations are reported in percentages in each of the tables.

These tables show that the probability of a bailout significantly impacts many business cycle statistics. In fact, we see that specifying stochastic bailouts can improve on the model

⁷⁵Parameters for the different average bailout probabilities are as follows ($\theta = \eta = 1/3$ in all cases): For $\psi = .25$, $\tau = 1.65$, $\xi = 1.648$; for $\psi = .5$, $\tau = 1.105$, and $\xi = 1.104$; for $\psi = .75$, $\tau = .5525$, $\xi = .552$; and, finally, for $\psi = .9$, $\tau = \xi = .221$.

without bailouts and with certain bailouts in a number of dimensions including its ability to capture (i) the occurrence of default in equilibrium, (ii) the volatility of the trade balance-to-output ratio, (iii) the correlation between the trade balance-to-output ratio and output, and (iv) the correlation between consumption and output. Furthermore, when $\psi(\cdot, \cdot)$ is calibrated for a 75% probability of bailout, the model hits the empirical occurrence of defaults and the volatility of the trade balance-to-output ratio found in Argentine data.

Table 3.2: Business Cycle Statistics ($\omega = 1, \psi = 0.25$)

	Data	Model I	Model II	Model III	Model IV	Model V
$\sigma(y)$	4.08	4.48	4.43	4.45	4.43	4.41
$\sigma(c)$	4.85	4.74	4.93	4.71	4.69	4.66
$\sigma(TB/Y)$	1.36	0.99	2.11	1.03	1.00	0.97
$\sigma(r)$	3.17	0.33	0.40	0.30	0.32	0.31
$\rho(C, Y)$	0.96	0.98	0.91	0.98	0.98	0.98
$\rho(TB/Y, Y)$	-0.89	-0.17	-0.04	-0.18	-0.17	-0.17
$\rho(r, Y)$	-0.59	-0.02	0.14	-0.06	0.01	-0.02
$\rho(r, TB/Y)$	0.68	0.11	0.18	0.21	0.04	0.11
Default	75	23	191	28	24	24

Table 3.3: Business Cycle Statistics ($\omega = 1, \psi = 0.50$)

	Data	Model III	Model IV	Model V
$\sigma(y)$	4.08	4.43	4.48	4.43
$\sigma(c)$	4.85	4.72	4.76	4.71
$\sigma(TB/Y)$	1.36	1.17	1.14	1.16
$\sigma(r)$	3.17	0.33	0.33	0.33
$\rho(C, Y)$	0.96	0.97	0.97	0.97
$\rho(TB/Y, Y)$	-0.89	-0.14	-0.14	-0.14
$\rho(r, Y)$	-0.59	0.01	0.07	0.07
$\rho(r, TB/Y)$	0.68	0.10	0.00	-0.01
Default	75	45	36	37

The standard deviation of the trade balance-to-output ratio is particularly sensitive to the probability of a bailout because in periods of default the sovereign is relieved of its debt. Thus, the trade balance-to-output ratio experiences more large swings when default episodes

Table 3.4: Business Cycle Statistics ($\omega = 1, \psi = 0.75$)

	Data	Model III	Model IV	Model V
$\sigma(y)$	4.08	4.45	4.41	4.45
$\sigma(c)$	4.85	4.77	4.72	4.75
$\sigma(TB/Y)$	1.36	1.42	1.37	1.33
$\sigma(r)$	3.17	0.33	0.33	0.32
$\rho(C, Y)$	0.96	0.96	0.96	0.96
$\rho(TB/Y, Y)$	-0.89	-0.10	-0.11	-0.10
$\rho(r, Y)$	-0.59	0.11	0.05	0.07
$\rho(r, TB/Y)$	0.68	0.06	0.10	0.07
Default	75	75	61	63

Table 3.5: Business Cycle Statistics ($\omega = 1, \psi = 0.90$)

	Data	Model III	Model IV	Model V
$\sigma(y)$	4.08	4.48	4.43	4.48
$\sigma(c)$	4.85	4.85	4.81	4.87
$\sigma(TB/Y)$	1.36	1.75	1.70	1.73
$\sigma(r)$	3.17	0.37	0.36	0.36
$\rho(C, Y)$	0.96	0.94	0.94	0.94
$\rho(TB/Y, Y)$	-0.89	-0.05	-0.07	-0.06
$\rho(r, Y)$	-0.59	0.09	0.06	0.07
$\rho(r, TB/Y)$	0.68	0.18	0.18	0.18
Default	75	124	111	110

are more frequent. Since specifying stochastic bailouts significantly impacts the occurrence of default in equilibrium, it also affects the volatility of the trade balance-to-output ratio. Intuitively, we see that during simulations with more default the standard deviation of the trade balance-to-output ratio is higher.

These tables also display that conditionality reduces the occurrence of default in equilibrium and thus also the volatility of the trade balance-to-output ratio. However, specifying conditional bailouts does not materially impact the other business cycle statistics beyond specifying stochastic bailouts. Hence, most of the action comes from bailouts being uncertain, not conditional.

One characteristic of Argentine business cycles that this model is unable to capture is the volatility of the sovereign interest rate. A number of papers have successfully replicated this volatility by allowing for bonds of different maturities (Chatterjee and Eyigungor (2010), Hatchondo and Martinez (2009)) or specifying a risk-averse foreign creditor (Arelano (2008)). Another possibility is suggested by the work of Uribe and Yue (2006). These authors demonstrate empirically that variations in the world interest rate account for nearly 25% of the variation in country spreads. Abstracting from this source of volatility, therefore, may help account for the counterfactually low volatility in the country interest rate produced by this model.

Welfare Implications

I now address the following questions: Are bailouts socially welfare-improving? If so, which bailout specification results in the largest welfare improvement over an economy without bailouts? These questions are of critical importance to policy-makers concerned with whether and how to provide and structure sovereign bailouts.

Theoretically, the answers to these questions are ambiguous. On the one hand, bailouts tend to transfer resources to sovereigns that have experienced adverse shocks, thereby providing a form of insurance. In the current setting, the existence of bailouts is also fully

internalized by creditors. As a result, sovereigns face lower borrowing costs when bailouts are present. These benefits are manifested in the above quantitative results, since bailouts tend to reduce the volatility of consumption and interest rates.

On the other hand, bailouts tend to induce greater rates of default, leading sovereigns to incur output losses and episodes of financial autarky more frequently. While removed from financial markets, sovereigns are completely unable to insure against idiosyncratic shocks. The welfare implications of bailouts in this setting are therefore not immediately obvious.

I take the following steps to assess the welfare implications of introducing bailouts into an economy without them. First, an economy without bailouts is simulated with a large number of sovereigns using the policy functions derived from the solution to that problem. This economy is simulated over a sufficient period to ensure it reaches its stationary distribution over states. I then compute the fraction of lifetime consumption required by the sovereign in each state to make them indifferent between remaining in an economy without bailouts (but receiving this extra consumption), and transitioning to an economy with bailouts.

To be more precise, denote the states in this model by $s \in S$, where each s is a realization of the triplet $\{b, y, D\}$. Further, let $V^w(s)$ and $V^o(s)$ denote the value of a sovereign in state s in the economy with and without bailouts, respectively. Define the consumption level $c^*(s)$ as that consumption that provides utility equivalent to the sovereign's continuation value in the world without bailouts, $V^o(s)$. That is, $\frac{c^*(s)^{1-\gamma}}{1-\gamma} = V^o(s)$. Let $\zeta(s)$ denote the fraction of its future stream of consumption that a sovereign currently in state s requires as compensation for not introducing bailouts into the economy. By definition, this fraction satisfies: $\frac{(c^*(s)(1+\zeta(s)))^{1-\gamma}}{1-\gamma} = V^w(s)$. Solving for $\zeta(s)$ implies:

$$\zeta(s) = \left(\frac{V^w(s)}{V^o(s)} \right)^{\frac{1}{1-\gamma}} - 1. \quad (3.14)$$

Once this value is computed for each state, I then compute the expected, discounted, lifetime benefit to sovereigns from introducing bailouts and compare this value to the expected, discounted, lifetime cost to the third party of providing these bailouts. For consistency, each

of these streams are valued from the perspective of the third party by discounting according to the third parties opportunity cost: the world interest rate. First, define the expected total benefit of introducing bailouts by:

$$\mathbb{E}[B] = \int_b \int_y \sum_{D=0,1} \zeta(b, y, D) \mathbb{E} \left[\sum_{t=0}^{\infty} \left(\frac{1}{1+r^*} \right)^t c_{B,t}(b, y, D_t) \right] \Lambda(b, y, D) \quad (3.15)$$

where $c_{B,t}(b, y, D)$ is consumption at time t , in the state defined by the triplet (b, y, D) . Consumption is derived from the policy function associated with the economy with bailouts and $\Lambda(b, y, D)$ is the stationary distribution over states in the economy without bailouts. Equation (3.15) computes the expected lifetime benefit to a sovereign from introducing bailouts once the no-bailout economy is in the stationary distribution and then transitions to the stationary distribution of an economy with bailouts.

Likewise, denote the expected discounted cost of bailouts to the third party by $\mathbb{E}[C]$. Finally, define expected lifetime consumption in the economy with bailouts, beginning from the stationary distribution without bailouts, by:

$$\int_a \int_y \sum_{D=0,1} \mathbb{E} \left[\sum_{t=0}^{\infty} \left(\frac{1}{1+r^*} \right)^t c_{B,t}(b, y, D_t) \right] \Lambda(a, y, D). \quad (3.16)$$

The social welfare implications of introducing bailouts are then derived by taking the ratio of the net benefit ($\mathbb{E}[B] - \mathbb{E}[C]$) and this expected lifetime consumption.

Tables 3.7 through 3.10 present the welfare results for different specifications for discount rates, bailout limits, stochastic processes for the endowment shocks, and bailout probabilities. In this section I only present the results for unconditional stochastic bailouts. The results for conditional bailouts are not materially different. The tables are broken down by two different specifications for the endowment process: stochastic trend and stable trend, and by bailout limit. I consider both the bailout limit considered above of 18% of average output (as in Aguiar and Gopinath (2006)) as well as an estimate of 1.44% of output which is considered in Benjamin and Wright (2009). The parameterization of the endowment process for the

stochastic trend is given above in Section 3.3.4. The parameterization for the stable trend is presented below in Table 3.6.

Table 3.6: Parameterization: Stable Trend

Parameter	Value
σ_z	3.4%
ρ_z	0.90
μ_z	$-(1/2)\sigma_z^2$

Importantly, these welfare figures were computed by comparing the appropriate specification of the economy without bailouts, in terms of discount factors and stochastic processes, to a similar economy with bailouts. These figures present the social welfare calculation and expected benefit, defined as above, in percentage terms, with columns labeled $\mathbb{E}[B]$ and SW, respectively.

Table 3.7: Net Benefit of Bailouts: Stochastic Trend, Bailout Limit -0.18

	β			
	0.8	0.95	0.8	0.95
ψ	$\mathbb{E}[B]$		SW	
0.10	0.015	0.001	-0.033	-0.010
0.25	0.044	0.004	-0.007	-0.008
0.50	0.093	0.010	0.078	-0.002
0.75	0.184	0.020	0.180	0.006
0.90	0.261	0.032	0.260	0.028
1.00	0.340	0.047	0.340	0.044

Several general results can be inferred from these tables. First, bailouts provide a greater benefit to sovereigns when sovereigns are more impatient and when the bailout occurs with higher probability. This first inference is perhaps intuitive. When bailouts are first introduced, very few sovereigns are currently in default. Those sovereigns that are not in default have higher current endowments on average, and since endowment realizations are persistent, tend to have higher endowments in the near future. Once bailouts are introduced, these

Table 3.8: Net Benefit of Bailouts: Stochastic Trend, Bailout Limit -0.0144

β				
	0.8	0.95	0.8	0.95
ψ	$\mathbb{E}[B]$		SW	
0.10	0.000	0.000	-0.004	-0.001
0.25	0.001	0.000	-0.003	-0.001
0.50	0.002	0.000	-0.002	-0.001
0.75	0.003	0.001	-0.001	-0.000
0.90	0.014	0.001	0.010	-0.000
1.00	0.014	0.001	0.010	-0.000

Table 3.9: Net Benefit of Bailouts: Stable Trend, Bailout Limit -0.18

β				
	0.8	0.95	0.8	0.95
ψ	$\mathbb{E}[B]$		SW	
0.10	0.000	0.000	-0.520	-0.410
0.25	0.001	0.000	-0.550	-0.410
0.50	0.002	0.001	-0.500	-0.600
0.75	0.006	0.001	-0.570	-0.490
0.90	0.008	0.002	-0.310	-0.400
1.00	0.018	0.002	-0.470	-0.420

Table 3.10: Net Benefit of Bailouts: Stable Trend, Bailout Limit -0.0144

β				
	0.8	0.95	0.8	0.95
ψ	$\mathbb{E}[B]$		SW	
0.10	0.000	0.000	-0.044	-0.033
0.25	0.000	0.000	-0.026	-0.034
0.50	0.000	0.000	-0.040	-0.034
0.75	0.000	0.000	-0.046	-0.034
0.90	0.000	0.000	-0.042	-0.033
1.00	0.000	0.000	-0.025	-0.033

sovereigns immediately benefit from lower borrowing costs. The negative effects of bailouts for these sovereigns – output loss and financial autarky due to higher default rates – are thus not immediately felt. Therefore, sovereigns that discount the future at greater rates, receive a larger benefit from the introduction of bailouts.

The fact that the benefit received by sovereigns is increasing in the probability of a bailout is less intuitive and actually runs counter to the results found in Benjamin and Wright (2009). These results suggest that the benefit of borrowing at lower rates due to the higher probability of bailouts outweighs the higher frequency of default evident in tables 3.2 through 3.5. Indeed, this result is consistent across the entirety of specifications considered in this section.

A third striking result is the dramatic differences between economies with endowments evolving according to stochastic versus stable trends. Specifically, bailouts present a larger benefit to sovereigns when their endowments evolve according to a stochastic trend. These differences close the gap between the results in Benjamin and Wright (2009) and this paper. Importantly, Benjamin and Wright (2009) consider an endowment process with a stable trend, consistent with tables 3.9 and 3.10 below. These tables demonstrate that the benefit of introducing bailouts to sovereigns is significantly lower – if not non-existent – when the sovereign’s endowment process is given by a stable rather than stochastic trend. And social welfare is always significantly reduced when introducing bailouts in an economy whose endowment process is given by a stable trend, regardless of the bailout probability, bailout limit, and discount rate. These results beg the question of why do bailouts provide a larger benefit to sovereigns when their endowment evolves according to a stochastic rather than stable trend. This may seem particularly puzzling given that permanent shocks are hard to insure against, so that we may think sovereigns should derive greater welfare gain when shocks are temporary and thus borrowing can more effectively smooth consumption. The next section considers this question. It shows that the answer to this result lies in which sovereigns benefit most from lower borrowing costs.

Why Does a Stochastic Trend Imply Larger Welfare Gains from Introducing Bailouts?

To derive some intuition about this result, first consider the borrowing habits of a sovereign with a deterministic trend in endowment under two different growth factors, μ and $\tilde{\mu}$ with $\tilde{\mu} > \mu > 1$ and $0 < \beta\tilde{\mu} < 1$. In an environment with no uncertainty and a natural borrowing limit, the sovereign will choose a constant stream of consumption that equates the present discounted value of consumption to the present discounted value of the endowment.

More formally, consider an economy with present discounted value of endowment given by

$$\sum_{t=0}^{\infty} \beta^t \mu^t = \frac{1}{1 - \beta\mu}. \quad (3.17)$$

In this environment, the sovereign will choose a constant stream of consumption $\bar{c}$ satisfying:

$$\frac{\bar{c}}{1 - \beta} = \frac{1}{1 - \beta\mu} \implies \bar{c} = \frac{1 - \beta}{1 - \beta\mu}. \quad (3.18)$$

The present discounted value of the sovereign's debt holdings is then given by $\beta/(1 - \beta\mu)$.

We can consider the same problem for the sovereign under the alternative growth factor $\tilde{\mu}$. In this case, the present discounted value of debt holdings is given by $\beta/(1 - \beta\tilde{\mu})$. Under the assumption that $\tilde{\mu} > \mu$, we find that the present discounted value of debt holdings is greater for the sovereign when it has an endowment stream characterized by a higher growth rate. That is, the sovereign with a higher growth rate borrows more in discounted terms in response to a steeper lifetime endowment profile. This result is intuitive since without uncertainty, the sovereign with a higher growth rate desires to smooth consumption, and thus borrow more against its higher future endowment realizations.

This example tells us that, in a non-stochastic environment, when we are considering

different trend growth rates, “richer” (in terms of steeper future endowment realizations) sovereigns should borrow more. The ability to borrow at a lower rate is therefore more valuable to richer sovereigns.

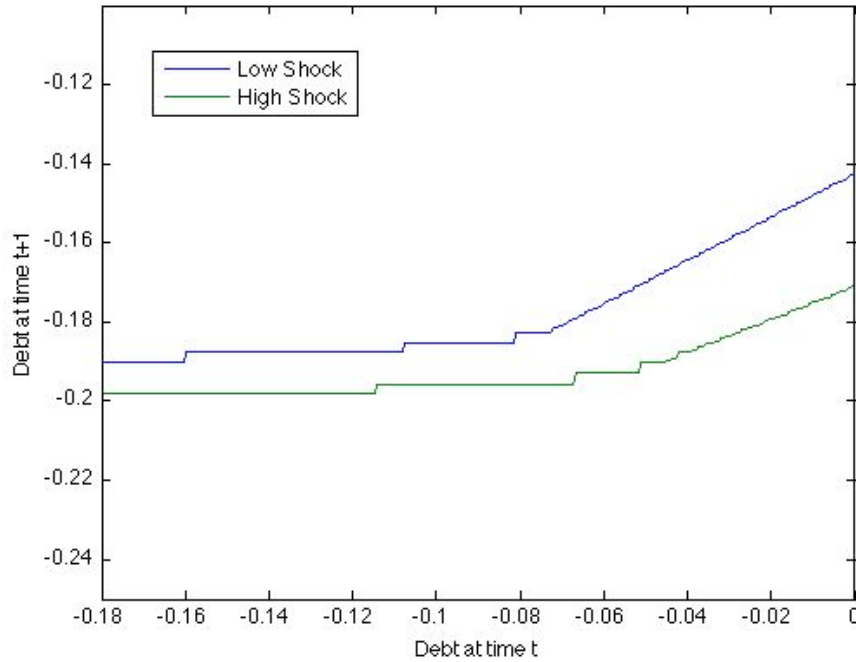
However, can this intuition be applied to the case of a stochastic trend? In particular, is it true that sovereigns experiencing higher trend growth rates borrow more than their lower growth counterparts in a stochastic environment? Figure 3.1 suggests that this intuition carries over to the stochastic environment. This figure depicts the policy functions for sovereigns with the highest endowment shock (the green curve) and lowest endowment shock (the blue curve) as a function of the sovereign’s current net foreign asset position in the standard model specification without bailouts and with a stochastic trend.⁷⁶ For all current asset positions, sovereigns with the highest endowment shock borrow more than those with the lowest endowment shock. This suggests that sovereigns with higher endowment realizations value the ability to interact in international credit markets more than sovereigns with lower endowment realizations. And, since these sovereigns are also less likely to default in the near future given a high current endowment realization, the costs incurred from higher default rates in the presence of bailouts are less severe.⁷⁷ As a result, the ability to borrow at lower interest rates in the presence of third-party bailouts implies relatively large welfare improvements for sovereigns that are currently in good credit standing.

In order for this to be the cause of the observed differences in welfare implications, however, this result must differ from the case with a stable endowment trend. Indeed, in the model with a stable trend we find the exact opposite: “poorer” sovereigns borrow more than their richer counterparts. Therefore, since in a model with a stable trend, the sovereigns that experience greater benefits from lower borrowing costs when third-party bailouts are present, are the same sovereigns that are more likely to default and thus incur the higher costs of these bailouts, the impact of introducing bailouts on sovereign welfare is muted.

⁷⁶The result persists for the specifications with bailouts.

⁷⁷This statement follows from the fact that the default probability is decreasing in the current endowment realization.

Figure 3.1: Policy Functions: Stochastic Trend and No Bailouts



The reason that third-party bailouts are welfare-improving when sovereign endowments evolve according to a stochastic trend is that the sovereigns that benefit most from lower borrowing costs: rich sovereigns, do not experience the cost of higher default rates. Consequently, these sovereigns experience significant welfare gains when bailouts are introduced. On the other hand, when endowments evolve according to a stable trend, rich sovereigns do not value the ability to borrow as much and therefore do not receive a significant welfare improvement from the introduction of bailouts. Poorer sovereigns that value lower borrowing costs, find their welfare gains offset by the fact that they will now spend more time in default, thereby experiencing more frequent output loss and an inability to smooth consumption through international credit markets. The welfare gains in this environment are minimal as a result.

This discussion also has some interesting implications for policy decisions about whether or not to offer bailouts. For, the above suggests that if sovereign endowments evolve according

to stochastic trends, the welfare benefits of introducing bailouts are realized by sovereigns that will not default with high probability in the near future – a group that most likely is not the target for introducing bailouts. It is true, however, that this environment does not capture possible additional benefits of introducing bailouts, such as a higher probability for re-entry into international credit markets upon default. In any case, these results suggest that if sovereigns' endowments evolve according to stochastic trends, introducing third-party bailouts often reduces social welfare, and even when social welfare is increased, the welfare benefits are enjoyed (through lower borrowing costs) by sovereigns that are most likely not the targets of the bailouts: richer sovereigns.

Given this, identifying the stochastic process governing emerging market output is particularly important when considering whether or not to implement a bailout program. The work by Aguiar and Gopinath (2007) argues that emerging market economies differ from developed economies in that frequent changes in macroeconomic policy lead to shifts in trend growth in the former, rather than movements around a stable trend like in the latter. If this is the case, bailout programs may lead to lower social welfare.

3.4.2 Model with Partial Sovereign Repayment

In this section I briefly discuss the business cycle implications of the model in which third-party bailouts are not pure transfers. Instead, $\omega \in [0, 1)$ in this section. Below, I consider several specifications for ω that are consistent with the empirical research of Joshi and Zettelmeyer. In particular, I consider bailouts that have implicit transfers of 0, 1, and 5% of average output. A stochastic trend is considered in this section, along with $\beta = 0.8$ and $b^* = -.18$.

Tables 3.11 through 3.13 present the business cycle implications of the model with bailouts that are not pure transfers. I have chosen to present only the results derived from the model in which bailouts are stochastic and unconditional; however, the general implications of conditional bailouts that are not pure transfers is consistent with the results presented in

this section.

These tables show that the specification with transfers on the order of 0 - 5% does not seem to materially impact much of the business cycle implications of the model. In general, when bailouts are certain ($\psi = 1$) the occurrence of default in equilibrium is less than the model where bailouts are pure transfers. This result, however, is not uniform across the spectrum of stochastic bailouts. Instead, when bailouts are implemented stochastically, the fact that bailouts are not pure transfers does not seem to induce a significant decrease in sovereign default.

The small impact of the implicit transfer on the frequency of default is somewhat counter-intuitive. We should expect that reducing the implicit transfer in third-party bailouts would decrease the value of default relative to not defaulting, leading to a decrease in sovereign default. One possible explanation for the limited action found with this specification is that the high rate of discount reduces the impact of a one-time payment by the sovereign on the difference between the value of defaulting relative to maintaining good credit standing. Thus, since the sovereign discounts the future greatly, the ability to delay repayment of debt less some implicit transfer still makes default an attractive option compared to immediate repayment of an entire debt.

Table 3.11: Business Cycle Statistics ($\omega = 0.05$)

	Data	$\psi = 0.25$	$\psi = 0.50$	$\psi = 1.00$
$\sigma(y)$	4.08	4.47	4.42	4.43
$\sigma(c)$	4.85	4.73	4.71	4.91
$\sigma(TB/Y)$	1.36	1.04	1.15	2.09
$\sigma(r)$	3.17	0.30	0.33	0.40
$\rho(C, Y)$	0.96	0.98	0.97	0.92
$\rho(TB/Y, Y)$	-0.89	-0.17	-0.15	-0.03
$\rho(r, Y)$	-0.59	-0.05	0.01	0.15
$\rho(r, TB/Y)$	0.68	0.21	0.10	0.18
Default	75	27	46	187

Table 3.12: Business Cycle Statistics ($\omega = 0.01$)

	Data	$\psi = 0.25$	$\psi = 0.50$	$\psi = 1.00$
$\sigma(y)$	4.08	4.42	4.48	4.45
$\sigma(c)$	4.85	4.68	4.77	4.93
$\sigma(TB/Y)$	1.36	1.00	1.21	2.11
$\sigma(r)$	3.17	0.30	0.33	0.40
$\rho(C, Y)$	0.96	0.98	0.97	0.92
$\rho(TB/Y, Y)$	-0.89	-0.17	-0.12	-0.03
$\rho(r, Y)$	-0.59	-0.06	0.02	0.15
$\rho(r, TB/Y)$	0.68	0.21	0.11	0.18
Default	75	28	46	189

Table 3.13: Business Cycle Statistics ($\omega = 0.00$)

	Data	$\psi = 0.25$	$\psi = 0.50$	$\psi = 1.00$
$\sigma(y)$	4.08	4.47	4.46	4.41
$\sigma(c)$	4.85	4.72	4.76	4.91
$\sigma(TB/Y)$	1.36	1.01	1.20	2.12
$\sigma(r)$	3.17	0.30	0.33	0.40
$\rho(C, Y)$	0.96	0.98	0.97	0.91
$\rho(TB/Y, Y)$	-0.89	-0.17	-0.14	-0.04
$\rho(r, Y)$	-0.59	-0.05	0.01	0.15
$\rho(r, TB/Y)$	0.68	0.21	0.11	0.18
Default	75	29	46	185

3.5 Conclusion

This paper has demonstrated that the implications of third-party bailouts for business cycles and welfare are particularly sensitive to how bailouts are implemented. In particular, implementing bailouts stochastically instead of with certainty significantly decreases the occurrence of sovereign default. Moreover, conditioning the probability of a bailout on sovereign observables, such as their debt-to-output ratio, induces sovereigns to default even less frequently. Allowing for the fact that bailouts are not pure transfers, on the other hand, does not materially impact the implications of the model.

It has also shown that modeling bailouts stochastically and/or conditionally can help capture some of the salient characteristics of emerging market economy business cycles. In fact, modeling bailouts as occurring with 75% probability enhances the ability of the model to match the following facts of Argentine business cycles: (i) the incidence of default, (ii) volatility of trade balance-to-output ratio, (iii) the correlation between the trade balance-to-output ratio and output, and (iv) the correlation between consumption and output.

In terms of welfare, I find that introducing third-party bailouts generally reduces social welfare. The benefit of this introduction for sovereigns is increasing in the probability of default, the bailout limit, and the rate of discount. Although the finding that bailouts reduce social welfare is consistent across almost all specifications, welfare implications are strikingly sensitive to the specification of the endowment process. The above results suggest that the benefits to sovereigns of introducing bailouts are significantly less, and as a result the social welfare losses much greater, when endowments evolve according to a stable rather than stochastic trend. This finding is a direct result of the fact that when sovereign endowments evolve according to stochastic trends, richer sovereigns benefit greatly from lower borrowing costs when third-party bailouts are present. Since these sovereigns do not incur the additional cost of higher default probabilities until well out into the future, they experience significant welfare gains from the introduction of these bailouts.

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