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Analysis of Curved Folded Plate Structures

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ANALYSIS OF CURVED FOLDED PLATE STRUCTURES

by

C. MEYER

and

A. C. SCORDELIS

Report to the Sponsors: Division of Highways, Department of Public Works, State of California, and the Bureau of Public Roads, Federal Highway Administration, United States Department of Transportation.

JUNE 1970

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UNIVERSITY OF CALIFORNIA
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Structures and Materials Research
Department of Civil Engineering
Division of Structural Engineering
and
Structural Mechanics

Report No. UC SESM 70-8

ANALYSIS OF CURVED FOLDED PLATE STRUCTURES

by

C. Meyer
Graduate Student in Civil Engineering

and

A. C. Scordelis
Professor of Civil Engineering

to

The Division of Highways
Department of Public Works
State of California
Under Research Technical Agreement
No. 13945-14423

and

U. S. Department of Transportation
Federal Highway Administration
Bureau of Public Roads

College of Engineering
Office of Research Services
University of California
Berkeley, California

June 1970

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ERRATA

"Analysis of Curved Folded Plate Structures"

C. Meyer and A. C. Scordelis

Structural Engineering and Structural Mechanics Report No. UC SESM 70-8, University of California, Berkeley, June 1970

1. Revision of the Program CURSTR (Appendix A-6,A7)

(1) On page A-6, the following card was inserted before the statement no. 47:

NTPG=NTP

(2) On page A-7, the 2nd card from top has been changed to:

5 A(L40),A(L41),A(L42),A(L42),A(L43),A(L43),MH,NJT4,NOTMP,NTPG)

(3) On page A-7, the 9th card from top has been changed to:

4 NOTMP,NTPG)

2. Revision of Subroutine MAIN (Appendix A-8,A-16,)

(1) On page A-8, the 5th card from top has been changed to:

4 EDP,MH,NT4,NMP,NTPG)

(2) On page A-16, after statement no. 730, the four cards calling subroutine OPRINT have been changed to:

```
CALL OPRINT(RJDIS,MX,NX,XP,NX,O,II,IJ,IL,1,2,LIND,IO,NTPG)
CALL OPRINT(RJDIS,MX,NX,XP,NX,O,II,IJ,IL,2,2,LIND,IO,NTPG)
CALL OPRINT(RJDIS,MX,NX,XP,NX,O,II,IJ,IL,3,2,LIND,IO,NTPG)
CALL OPRINT(RJDIS,MX,NX,XP,NX,O,II,IJ,IL,4,2,LIND,IO,NTPG)
```

3. Revision of Subroutine FORCE (Appendix A-18)

(1) On page A-18, the 3rd card from top has been changed to:

2 SM,TM,STM,SN,TN,STN,U,V,W,DI,DIS,D,NX,MH,NMP,NTPG)

4. Revision of the Subroutine OPRINT (Appendix A-36)

(1) On page A-36, the top card has been changed to:

SUBROUTINE OPRINT (A,M,N,X,NX,NY,K1,K2,NCYC,L,KI,IND,IO,NTPG)

(2) On page A-36, replace the following three cards:

```
DO 25 K = 1, NCYC
J1 = N1(K)
J2 = N2(K)
```

With the following seven cards:

```
JAS = (NTPG/7) + 1
JASH = (JAS-1)*7
IF(NTPG.EQ.JASH)JAS=JAS-1
DO 25 KCOR = 1,JAS
J1 = (KCOR-1)*7+1
J2 = J1+6
IF(J2.GT.NTPG)J2=NTPG
```

ABSTRACT

A computer program has been developed which is capable of analyzing curved folded plate structures simply supported at the two ends and composed of elements that may in general be segments of conical frustra. The program is based on a harmonic analysis in the circumferential direction, with the loadings expressed by Fourier series, and on a finite element stiffness analysis in the transverse direction. The structure assembly and solution follows the direct stiffness method as it has been successfully applied previously to straight prismatic folded plates.

KEY WORDS

Curved beams; curved box girder bridges; curved folded plates; finite strip method; multi-cell bridges; shells of revolution; stresses; structural analysis; structural design.

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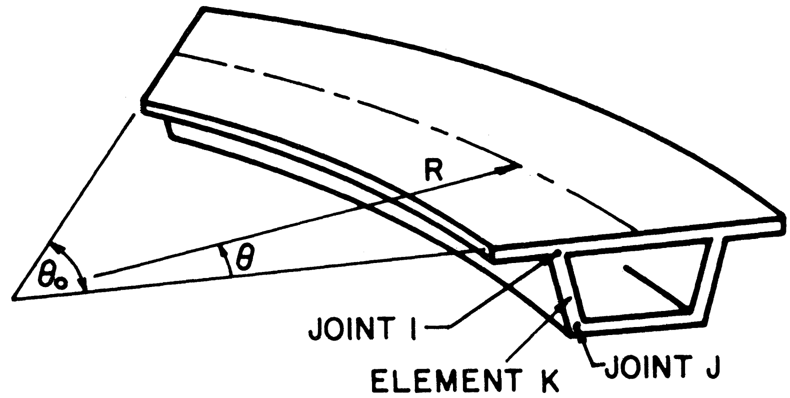
1. INTRODUCTION

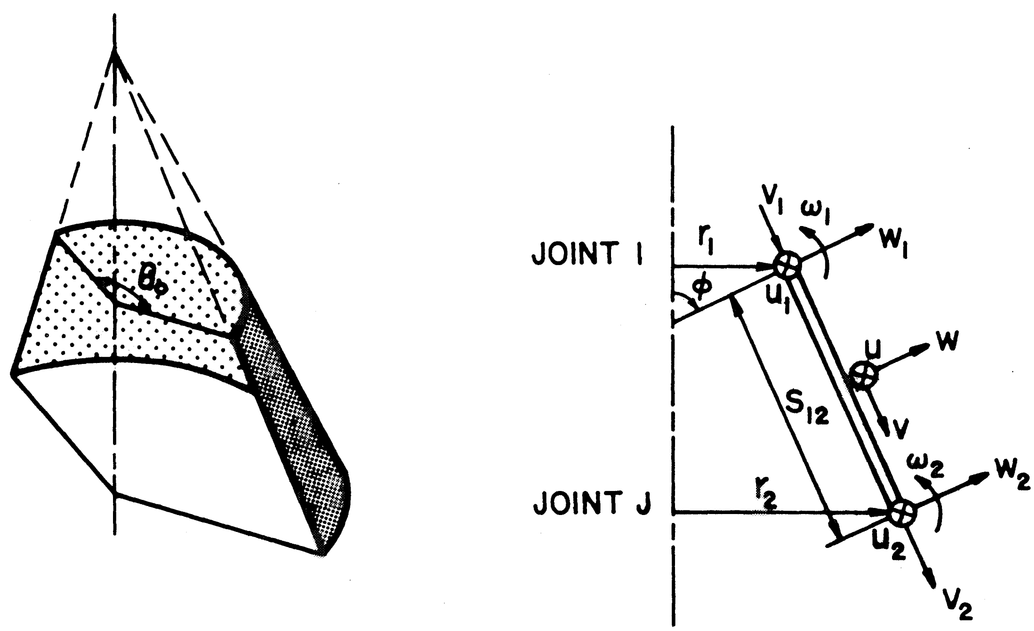
The purpose of this report is to present a computer program by means of which it is possible to analyze prismatic curved folded plate structures which are simply supported at the two ends and may be subjected to a variety of loading conditions.

Curved folded plate structures are seldom used in ordinary building construction. Curved bridges, however, can in most cases be considered as being assemblies of curved plates, each of which is in general a segment of a conical frustrum, Fig. 1.

Curved bridges are at the present usually analyzed using curved beam theory, if the curvature effects are considered at all. Hence, all common assumptions underlying beam theory are made, the most important of which is the assumption that cross sections do not distort. However, in view of the increased use of curved bridges in modern highway systems, it appears to be desirable for two reasons to have a refined method of analysis available. Firstly, the degree of accuracy expected from curved beam theory cannot be very high whenever cross sectional distortions alter the structural behavior significantly. In these cases, a refined method of analysis is really essential. Secondly, this new method of analysis may be used to establish rational criteria for simplified methods of analysis and design, which may very well be similar to the presently used curved beam theories, but which will take into account more design parameters such as cross sectional dimensions or transverse stiffnesses in order to be also applicable to bridges for which the assumptions underlying beam theory do not apply.



a) STRUCTURE



b) JOINT DEGREES OF FREEDOM OF TYPICAL ELEMENT

FIG. 1 CURVED BRIDGE

2. METHOD OF ANALYSIS

The method of analysis used in the computer program described in this report is closely related to the so-called finite strip analysis of plate type structures as well as to the finite element analysis of shells of revolution under non-axisymmetric loads. It will be called henceforth the "finite strip analysis of curved plate type structures," or simply "curved strip method."

The finite strip theory is well established and has been described by Cheung [1] [2] [8], and Willam and Scordelis [3]. Likewise, the finite element theory for shells of revolution is the topic of various publications, for example by Grafton and Strome [4] and by Percy et al. [5]. On the other hand, the theory has so far not been extended to the analysis of curved folded plate structures. This theoretical extension, however, has been developed completely in the Ph.D. dissertation by Meyer [6]. For a detailed description of the theory, reference to this dissertation should be made and thus only a brief outline will be given below.

The three displacement components of a general conical shell segment, Fig. 1b, are assumed to vary as

$$
\begin{aligned}
u &= \sum_{n=1}^{N} u_n \cos\frac{n\pi\theta}{\theta_o} = \sum_{n=1}^{N} < \Phi_u(\eta) > \{u_i\}_n \cos\frac{n\pi\theta}{\theta_o} \\
v &= \sum_{n=1}^{N} v_n \sin\frac{n\pi\theta}{\theta_o} = \sum_{n=1}^{N} < \Phi_v(\eta) > \{v_i\}_n \sin\frac{n\pi\theta}{\theta_o} \qquad (2.1) \\
w &= \sum_{n=1}^{N} w_n \sin\frac{n\pi\theta}{\theta_o} = \sum_{n=1}^{N} < \Phi_w(\eta) > \{w_i\}_n \sin\frac{n\pi\theta}{\theta_o}
\end{aligned}
$$

where

$$\{u_i\}_n = \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}_n \qquad \{v_i\}_n = \begin{Bmatrix} v_1 \\ v_2 \end{Bmatrix}_n \qquad \{w_i\}_n = \begin{Bmatrix} w_1 \\ w_2 \\ \omega_1 = \left(\frac{\partial w}{\partial \eta}\right)_1 \\ \omega_2 = \left(\frac{\partial w}{\partial \eta}\right)_2 \end{Bmatrix}_n \tag{2.2}$$

are the displacement amplitudes at the nodal joints 1 and 2 for a typical harmonic term n, and

$$< \Phi_u(\eta) > = < \Phi_v(\eta) > = \tfrac{1}{2} < (1-\eta) \quad (1+\eta) >$$

$$< \Phi_w(\eta) > = \tfrac{1}{4} < (2-3\eta+\eta^3) \quad (2+3\eta-\eta^3) \quad \frac{s_{12}}{2}(1-\eta-\eta^2+\eta^3) \quad \frac{s_{12}}{2}(-1-\eta+\eta^2+\eta^3) > \tag{2.3}$$

are the displacement interpolation polynomials, with the natural coordinate η defined such that $\eta = -1$ at joint 1 and $\eta = +1$ at joint 2.

Developing the external loads into similar Fourier series,

$$p = \sum_{n=1}^{N} p_n(\eta) \sin \frac{n\pi\theta}{\theta_o} \tag{2.4}$$

it is possible to separate variables and to perform for each harmonic n a stiffness analysis for the degrees of freedom associated with the nodal joints of the system as is done in a standard finite strip analysis for straight structures [3] as well as in the analysis of shells of revolution [5]. In analyzing curved folded plate structures, the appropriate strain-displacement and stress-strain relationships for a general conical shell element must be used.

Distributed surface loads are converted into consistent joint loads such that the work done by these consistent nodal loads going through the joint displacements equals the work done by the distributed loads while going through the displacement field.

The accuracy of the analysis can be expected to increase if the linear transverse variation for the in-plane displacements u and v assumed in Eq. (2.3) is changed to vary quadratically over the width of one element, i.e. if

$$\{u_i\}_n = \begin{Bmatrix} u_1 \\ u_2 \\ u_0 \end{Bmatrix}_n \qquad \{v_i\}_n = \begin{Bmatrix} v_1 \\ v_2 \\ v_0 \end{Bmatrix}_n \tag{2.5}$$

and

$$\langle \Phi_u(\eta) \rangle = \langle \Phi_v(\eta) \rangle = \langle \tfrac{1}{2}(1-\eta) \;\; \tfrac{1}{2}(1+\eta) \;\; (1-\eta^2) \rangle \tag{2.6}$$

where u_o and v_o are the in-plane displacement degrees of freedom associated with the joint halfway between joints 1 and 2. However, in the program version presented in this report, only the theory based on linear in-plane displacement functions is used.

3. PROGRAM DESCRIPTION

The program has been written in Fortran IV language for the CDC 6400 computer of the Computer Center at Berkeley, California. It consists of one main program called CURSTR and the 9 subroutines MAIN, FORCE, MOMPER, ADDMOM, CONE, LOADS, BANSOL, OPRINT, FL. The order of these subroutines in the deck setup is irrelevant because no overlay system has been utilized. The Fortran listing of the complete program except for subroutine FL is given in Appendix A together with short descriptions at the beginning of each subroutine on comment cards.

Subroutine FL is written in Compass language and serves two purposes. If called as CALL LWA(N), it stores into the storage cell designated by N the last word address of the program, i.e. the storage required by the complete program, excluding blank common area. If called as CALL RFL(M), it resets the required field length dynamically. Thus, having calculated the blank common area necessary to solve a specific problem, one would set

CALL RFL(M)

where M = N+L

N = program length

L = blank common length

This subroutine adjusts the total storage area for each problem to be analyzed. If for a different computer system, an equivalent subroutine is not available, a fixed amount of blank common has to be calculated as shown in the next chapter, and dimensioned for in the main program. One tape is used for temporary storage purposes.

4. PROGRAM USAGE

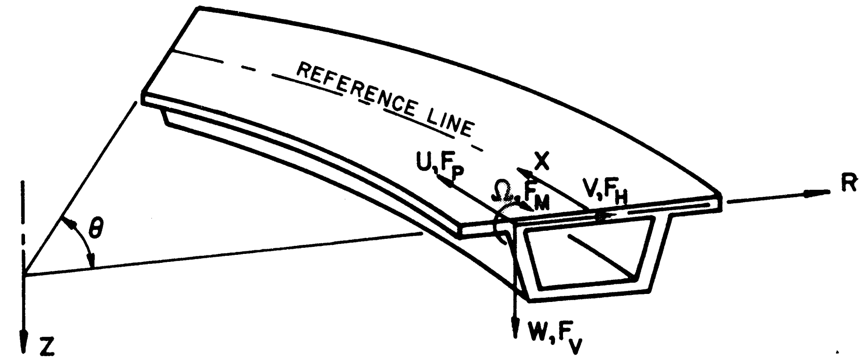
4.1 Capabilities and Restrictions

The theory underlying this computer program is limited to curved folded plate structures which are simply supported along the two straight edges at the ends of each element. These boundary conditions are equivalent to idealized end diaphragms which do not permit any displacements within their plane but offer no resistance to displacements normal to their plane.

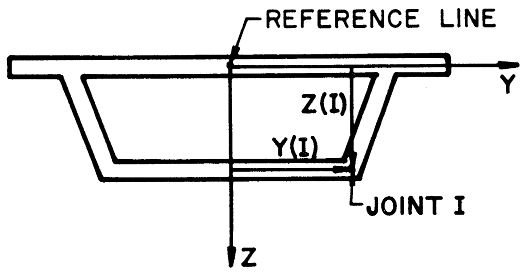
The structure to be analyzed is defined by introducing a circumferential reference line, which may in general have an arbitrary location, provided its radius is nonzero, Fig. 2a. The cross section is then defined by specifying for each nodal joint I a Y- and Z-coordinate as shown in Fig. 2b.

A typical element is then defined by specifying the joint numbers I and J. In general, an element is a segment of a conical frustrum, Fig. 1b. The material law relating stress resultants and strains is assumed to be uniform but polar-orthotropic throughout the element.

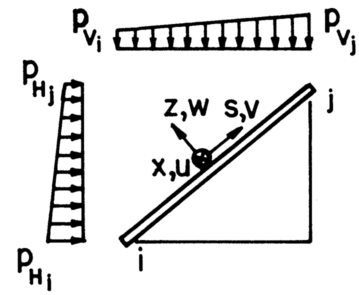
$$\begin{Bmatrix} N_s \\ N_\theta \\ N_{s\theta} \\ M_s \\ M_\theta \\ M_{s\theta} \end{Bmatrix} = \begin{bmatrix} \frac{t^m E^m_s}{1-\nu^m_{s\theta}\nu^m_{\theta s}} & \nu^m_{\theta s}\frac{t^m E^m_\theta}{1-\nu^m_{s\theta}\nu^m_{\theta s}} & 0 & 0 & 0 & 0 \\ \nu^m_{s\theta}\frac{t^m E^m_s}{1-\nu^m_{s\theta}\nu^m_{\theta s}} & \frac{t^m E^m_\theta}{1-\nu^m_{s\theta}\nu^m_{\theta s}} & 0 & 0 & 0 & 0 \\ 0 & 0 & t^m G^m & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{(t^b)^3 E^b_s}{12(1-\nu^b_{s\theta}\nu^b_{\theta s})} & \nu^b_{\theta s}\frac{(t^b)^3 E^b_\theta}{12(1-\nu^b_{s\theta}\nu^b_{\theta s})} & 0 \\ 0 & 0 & 0 & \nu^b_{s\theta}\frac{(t^b)^3 E^b_s}{12(1-\nu^b_{s\theta}\nu^b_{\theta s})} & \frac{(t^b)^3 E^b_\theta}{12(1-\nu^b_{s\theta}\nu^b_{\theta s})} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{(t^b)^3 G^b}{12} \end{bmatrix} \begin{Bmatrix} \epsilon_s \\ \epsilon_\theta \\ 2\epsilon_{s\theta} \\ \kappa_s \\ \kappa_\theta \\ 2\kappa_{s\theta} \end{Bmatrix} \tag{4.1}$$



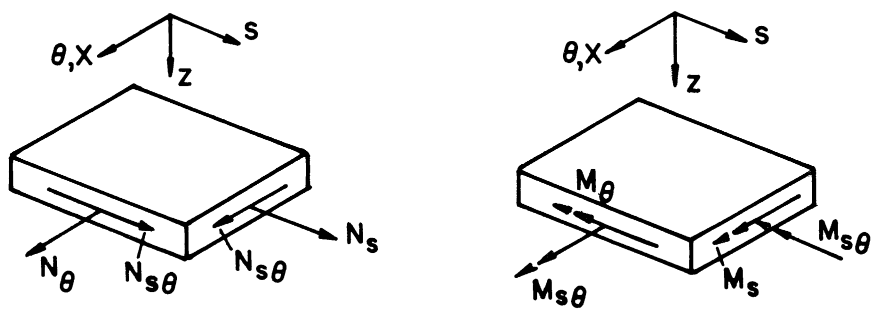
a) GLOBAL JOINT DISPLACEMENTS U,V,W,Ω AND JOINT LOADS F_P, F_H, F_V, F_M



b) JOINT COORDINATES



c) SURFACE LOADS AND ELEMENT DISPLACEMENTS



d) INTERNAL FORCES AND MOMENTS

FIG. 2 SIGN CONVENTIONS

or symbolically,

$$\begin{Bmatrix} N_s \\ N_\theta \\ N_{s\theta} \\ M_s \\ M_\theta \\ M_{s\theta} \end{Bmatrix} = \begin{bmatrix} d_{11} & d_{12} & 0 & 0 & 0 & 0 \\ d_{21} & d_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & d_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & d_{44} & d_{45} & 0 \\ 0 & 0 & 0 & d_{54} & d_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & d_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_s \\ \epsilon_\theta \\ 2\epsilon_{s\theta} \\ \varkappa_s \\ \varkappa_\theta \\ 2\varkappa_{s\theta} \end{Bmatrix} \tag{4.2}$$

where the superscript "m" denotes "membrane" or in-plane characteristics, and superscript "b" denotes "bending" properties, t is the element thickness, E and G the elastic and shear modulus, and ν Poisson's ratio. Subscripts s and θ indicate the radial and circumferential directions, respectively. Note that symmetry of Eq. (4.1) must be preserved so that

$$\nu^m_{s\theta} = \frac{E^m_\theta}{E^m_s} \nu^m_{\theta s}$$

and (4.3)

$$\nu^b_{s\theta} = \frac{E^b_\theta}{E^b_s} \nu^b_{\theta s}$$

The commonly used isotropic, homogeneous material law follows from Eq. (4.1) by setting

$$E^m_s = E^m_\theta = E^b_s = E^b_\theta = E$$

$$\nu^m_{s\theta} = \nu^m_{\theta s} = \nu^b_{s\theta} = \nu^b_{\theta s} = \nu$$

$$G^m = G^b = G = \frac{E}{2(1+\nu)} \quad , \quad t^m = t^b = t$$

The program has been written such that the material law can be input in either of two ways.

a) Input all of the following element properties,

$$t^m,\ t^b,\ E_s^m,\ E_s^b,\ E_\theta^m,\ G^m,\ G^b,\ \nu_{\theta s}^m,\ \nu_{\theta s}^b$$

for each "plate type" which is defined by a unique set of the above quantities.

b) Input the constitutive relations directly in the form of

$$d_{11},\ d_{12},\ d_{22},\ d_{33},\ d_{44},\ d_{45},\ d_{55},\ d_{66}$$

i.e. specify all independent elements of the constitutive matrix in Eq. (4.2) directly, thus also completely defining a plate type. The number of different plate types is restricted to a maximum of 30.

This input option gives the user much freedom in defining his constitutive relations. Stiffeners in either direction and different amounts of reinforcement may be taken into account. In complex situations, the d-coefficients may be determined experimentally.

The structure may be subjected to surface or joint loads. Surface loads vary linearly over the width of an element and are constant over a specified portion of the circumferential length of the element. In this case they are referred to as "partial surface loads." Similarly, joint loads may also extend uniformly over the whole length of a joint or over only a fraction of it, in which case they are referred to as "partial joint loads."

The program has an option to suppress all even terms in the Fourier series whenever the applied loads are symmetric about the midspan section. Similarly, if the loads are anti-symmetric about the midspan section as for example prestress forces applied at the end of the structure, all odd harmonics may be suppressed. If the structure is a full 360° axisymmetric shell subjected to axisymmetric loading, the Fourier analysis degenerates such that only the zero-th harmonic is retained, and the program has been written to incorporate this special case.

For various reasons it might be of interest to study the results not only for a specified final sum of harmonic series contributions, but also intermediate results. For this purpose, it is possible, by specifying for example the total number of harmonics to be used as 50, to print internal element forces and displacements also after only say 10 or 20 or 30 terms of the respective Fourier series have been accumulated, or any other combination of harmonics smaller than 50.

The question of how many Fourier terms should be used to represent the loading depends on the type of loading and on the desired output quantities. Deflections usually converge very rapidly, and 5 to 10 nonzero harmonic terms are sufficient to describe most loading types. Stresses and moments do not converge as fast, especially for concentrated loads in which case at least 25 nonzero terms will be necessary for adequate accuracy.

For input/output labelling, the user has the option to specify the circumferential coordinates either in angular degrees or in arc lengths. For strongly curved structures, the user might prefer the angle option, while for large curvature radii, the arc option will usually be more convenient. If use of the arc is made, care has to be taken that the longitudinal position of a concentrated or line load (see page 19) is specified along the joint where the load is acting and not along the reference line unless they coincide.

In the case that curved bridge structures are to be analyzed, it will often be useful to know the moment that each individual girder contributes to the total statical moment at any section. In fact, the sum of these girder moments should add up to the statical moment due to the applied loads. This useful check is obtained by means of the "moment integration" option. Individual girder moments due to normal stress resultants as well as longitudinal plate bending moments may be printed out at any section for which stress and displacement output is available, together with the net compressive and tensile stress resultants within each girder.

For each problem, only one load case can be treated.

4.2 Sign Conventions

The structure is defined in global cylindrical coordinates R,Z,θ as shown in Fig. 2a. The Z-axis is defined by the axis of revolution. It has its origin in the plane of the reference line and points downward. The R-axis points from the axis of revolution outward, and the angular θ-coordinate points from one end of the structure towards the other end such that it describes a rotation vector in the negative Z-direction.

External vertical and horizontal loads are positive if acting in the positive Z- and R-directions, respectively. Longitudinal loads and applied moment vectors are positive if acting along a tangential X-axis which is normal to the Z-R plane such that X,R,Z form a right-handed system in that order, Fig. 2a.

Joint coordinates within a cross section are measured in modified global coordinates Y and Z which are positive as shown in Fig. 2b.

Joint loads and displacements are positive as shown in Fig. 2a. Figure 2c defines positive directions of element surface loads and element displacements, and Fig. 2d those of internal forces and moments.

4.3 Input Specifications

Input data are key punched on cards as specified below. It is very important that the sequential order is strictly adhered to and consistent units are used throughout a problem.

1) Title Card (8A10)

Col. 1 to 80 - TITLE(I) = Title of problem to be printed with output for identification

2) Control Card (2F10.0, 15I4)

Col. 1 to 10 - TETAO = angle of curvature (in degrees) between end supports (360 degrees for axisymmetric shell with axisymmetric loading)

Col. 11 to 20 - R = radius of curvature of reference line

Col. 21 to 24 - NPL = number of plate types, max. = 30

Col. 25 to 28 - NEL = number of elements

Col. 29 to 32 - NJT = number of joints

Col. 33 to 36 - NTP = number of transverse sections at which output results are desired

Col. 37 to 40 - MHARM = maximum Fourier series limit (zero for axisymmetric shell with axisymmetric loading)

Col. 41 to 44 - NCHECK = harmonic series type indicator,

+ 1 use odd harmonics only (symmetry)

0 use all harmonics (non-symmetry)

- 1 use even harmonics only (anti-symmetry)

(Leave blank for axisymmetric case)

Col. 45 to 48 - INTRES = number of harmonic series limits for which intermediate results are desired

Col. 49 to 52 - IO = input/output option indicator,

1 sections for input/output given by angular degrees

0 sections for input/output given by arc lengths measured from origin along the reference line or joint under consideration

Col. 53 to 56 - MCHECK = girder moment integration option,

1 statical moments of girders desired

0 statical moments of girders not desired

Col. 57 to 60 - MB = half bandwidth [= (max. joint difference in an element + 1)*4]

Col. 61 to 64 - NSURL = number of partial surface loads

Col. 65 to 68 - NCONL = number of partial joint loads

Col. 69 to 72 - NOTMP = number of sections for which statical moments of girders are desired

Col. 73 to 76 - NGIR = number of girders. The last two entries are needed only if MCHECK = 1

Col. 77 to 80 - MI = material option indicator

0 for inputing material properties

1 for inputing constitutive relations directly

3) Circumferential Coordinate Card (10F7.2)

Col. 1 to 70 - XP(I) = X-coordinates along reference line (if IO = 0) or angles measured in degrees (if IO = 1) of transverse sections at which results are desired. Use second card, if more than 10 such sections.

4) Intermediate Result Card (20I4)

Col. 1 to 80 - IRES(I) = harmonic series limits for which intermediate results are desired. Omit this card if INTRES = 0.

5) Plate Type Cards

If MI = 0, two cards (I10, 5F10.0/10X, 5F10.0) are required for each type

First Card - Membrane characteristics

Col. 1 to 10 - I = type number

Col. 11 to 20 - THM(I) = effective thickness

Col. 21 to 30 - ETM(I) = modulus of elasticity in hoop direction

Col. 31 to 40 - ESM(I) = modulus of elasticity in meridional direction

Col. 41 to 50 - GM(I) = shear modulus

Col. 51 to 60 - PRM(I) = Poisson's ratio (negative strain in hoop direction for unit strain in meridional direction)

Second Card - Bending characteristics

Col. 11 to 20 - THB(I) = effective thickness

Col. 21 to 30 - ETB(I) = modulus of elasticity in hoop direction

Col. 31 to 40 - ESB(I) = modulus of elasticity in meridional direction

Col. 41 to 50 - GB(I) = shear modulus

Col. 51 to 60 - PRB(I) = Poisson's ratio (negative strain in hoop direction for unit strain in meridional direction)

If MI = 1, two cards (I10, 4F10.0/10X, 4F10.0) are required for each type

First Card - Membrane constitutive constants

Col. 1 to 10 - I = type number

Col. 11 to 20 - D11(I)

Col. 21 to 30 - D12(I)

Col. 31 to 40 - D22(I)

Col. 41 to 50 - D33(I)

Second Card - Plate bending constitutive constants

Col. 11 to 20 - D44(I)

Col. 21 to 30 - D45(I)

Col. 31 to 40 - D55(I)

Col. 41 to 50 - D66(I)

6) Element Cards (5I4, 5F10.0)

Each element requires one card.

Col. 1 to 4 - I = element number

Col. 5 to 8 - NPI(I) = joint i of element I

Col. 9 to 12 - NPJ(I) = joint j of element I

Col. 13 to 16 - KPL(I) = plate type number

Col. 17 to 20 - NSEC(I) = number of element subdivisions for output of internal forces and displacements, max. = 4. If NSEC(I) = 0, no internal forces and displacements will be output for element I.

Col. 21 to 30 - DL(I) = dead load (force per unit plate area)

Col. 31 to 40 - HLI(I) = horizontal load intensity at joint i (force per unit vertically projected area)

Col. 41 to 50 - HLJ(I) = horizontal load intensity at joint j

Col. 51 to 60 - VLI(I) = vertical load intensity at joint i

(force per unit horizontally projected area)

Col. 61 to 70 - VLJ(I) = vertical load intensity at joint j

Note that horizontal and vertical load intensities are uniformly distributed along a joint.

7) <u>Joint Cards</u> (I10, 6F10.0, 4I2)

Each joint requires one card.

Col. 1 to 10 - I = joint number

Col. 11 to 20 - Y(I) = Y-coordinate of joint I

Col. 21 to 30 - Z(I) = Z-coordinate of joint I

Col. 31 to 40 - AJF(1,I) = applied horizontal joint force/displacement intensity

Col. 41 to 50 - AJF(2,I) = applied vertical joint force/displacement intensity

Col. 51 to 60 - AJF(3,I) = applied joint moment/rotation intensity

Col. 61 to 70 - AFJ(4,I) = applied longitudinal joint force/displacement intensity

Col. 71 to 72 - LCASE(1,I) = horizontal force/displacement index

Col. 73 to 74 - LCASE(2,I) = vertical force/displacement index

Col. 75 to 76 - LCASE(3,I) = moment/rotation index

Col. 77 to 78 - LCASE(4,I) = longitudinal force/displacement index

Force/displacement index is equal to the following (note only zero displacements may be input)

0 for given zero force or moment

1 for uniformly distributed force or moment (input uniform force per unit length for AJF)

2 for concentrated force or moment at midspan (input total force for AJF)

3 for given zero displacement or rotation

4 for prestress force P at each end (input total force P for AJF, positive towards midspan)

8) Partial Surface Load Cards (I10, 6F10.0)

Each partial surface load requires one card.

No cards are required if NSURL = 0 (see control card).

Col. 1 to 10 - LEL(I) = element number

Col. 11 to 20 - PHLI(I) = horizontal load intensity at joint i (force per unit vertically projected area or length)

Col. 21 to 30 - PHLJ(I) = horizontal load intensity at joint j

Col. 31 to 40 - PVLI(I) = vertical load intensity at joint i (force per unit horizontally projected area or length)

Col. 41 to 50 - PVLJ(I) = vertical load intensity at joint j

Col. 51 to 60 - SURT(I) = X-coordinate measured along mid-element line (if IO=0) or angle measured in degrees (if IO=1) from origin to center of loaded area

Col. 61 to 70 - SURL(I) = length measured along mid-element line (if IO=0) or angle measured in degrees (if IO=1) subtended by distributed load. Note that for SURL(I) = 0.0 (transverse line load), loads are input as force per unit length. For SURL(I) ≠ 0.0, loads are input as force per unit area.

9) Partial Joint Load Cards (I10, 6F10.0)

Each partial joint load requires one card.

No cards are required if NCONL = 0 (see control card).

Col. 1 to 10 - LJT(I) = joint number

Col. 11 to 20 - FH(I) = total horizontal force

Col. 21 to 30 - FV(I) = total vertical force

Col. 31 to 40 - FM(I) = total moment

Col. 41 to 50 - FP(I) = total longitudinal force (Note that this force must be balanced by another force FP(I) somewhere on the same joint)

Col. 51 to 60 - FTL(I) = X-coordinate measured along joint LJT (if IO = 0) or angle measured in degrees (if IO = 1) from origin to center of joint load

Col. 61 to 70 - FTT(I) = length measured along joint LJT (if IO = 0) or angle measured in degrees (if IO = 1) subtended by joint load. For concentrated joint load, FTT(I) = 0.0. Note that each joint may be loaded with more than one joint load, but each joint load requires one separate card.

10) Girder Moment Integration Data

No cards are required if MCHECK = 0 (see control card)

Section Coordinate Card (10F7.2)

Col. 1 to 70 - T(I) = X-coordinates/angles of sections at which girder moments are desired. T(I) must be a subset of the circumferential coordinates TP(I) listed on the third card. Use a second card if necessary.

Element Cards (3I5, 3F10.0) - One card for each element

Col. 1 to 5 - I = element number

Col. 6 to 10 - NGIEL(1,I) = girder which joint i of element I belongs to

Col. 11 to 15 - NGIEL(2,I) = girder which joint j of element I belongs to. Leave blank if joints i and j belong to same girder.

Col. 16 to 25 - DNAI(I) = vertical distance from neutral axis to joint i (downward is positive)

Col. 26 to 35 - DNAJ(I) = vertical distance from neutral axis to joint j (downward is positive)

Col. 36 to 45 - XDIV(I) = horizontal distance from joint i to the dividing line, if the element belongs to two girders. Leave blank if not applicable.

Repeat the same set of data cards (1) through (10) for the next problem. Two blank cards are added at the end of the data deck to terminate execution.

4.4 Output Description and Interpretation

The output of each correctly executed problem contains the following information:

1) The complete set of input data is printed out with proper headings for identification and as a check for input errors.
2) The final displacements of all joints of the structure are printed out in the global coordinate system defined in Fig. 2a.
3) For each element, the internal stress resultants and moments N_s, N_θ, $N_{s\theta}$, M_s, M_θ, $M_{s\theta}$ and element displacements u, v, w as defined in Figs. 2d and 2c, respectively, are printed for as many transverse sections and intermediate harmonics as specified by the user.

4) If use of the girder moment option is made, then for each individual girder, at any transverse section specified as such, the statical moment and the percentage of this moment compared to the total statical moment contributed by all girders at this section, as well as net axial tension and compression forces, are printed in tabular form. Thus it is possible to have insight into the load distributing characteristics of the structure. The sum of the axial stress resultants of all girders should be zero (or very small), unless prestress forces are applied at the ends. The sum of all girder moments should equal the statical moment due to the external loads.

5) Finally, for each executed problem, the elapsed computing time is printed out separately for (1) input setup and formation and solution of the structure stiffness for all the harmonics, up until the output of final joint displacements, and (2) for calculation of internal forces and displacements for plate elements as well as determination of girder moments.

In interpreting the output results, one has to bear in mind that the curved strip method is only an approximate numerical technique. However, the results become more accurate as the strip representation of the structure is refined. In particular, it has to be noted, that differential equilibrium and force boundary conditions are not satisfied. For example, Fig. 3c shows the unbalanced moment at the free edge of a cylinder. This same figure also exhibits the rapid decrease of this moment as the mesh layout is refined.

In general, stress quantities output halfway between two element edges are sufficiently accurate. In order to obtain equally reliable results at element edges, results may have to be calculated as the average of those for all elements meeting at a joint.

In addition, it is recommended that use of the moment integration option should be made whenever feasible because the user gets for practically no extra cost an invaluable check which may even in some cases help in locating input errors.

4.5 Storage Requirements and Execution Time Estimate

Program CURSTR, if compiled by the FUN compiler of the CDC 6400 of the University of California Computer Center, requires together with all subroutines a central memory area of about $26{,}500_8 \approx 11{,}600_{10}$, not counting the blank common area. This additional storage requirement can be calculated as follows.

For execution of subroutine MAIN,

$$\text{COMMON}_1 = 12*\text{NEL}+\text{NJT}*(18+4*\text{NTP}+4*\text{MB}) + \text{NTP}*(2+2*\text{MH}) + \text{NOTMP}*(2+3*\text{NGIR})+7*(\text{NSURL} + \text{NCONL})+\text{INTRES}+1$$

and for execution of subroutine FORCE,

$$\text{COMMON}_2 = \text{NEL}*(15+5*\text{MCHECK})+2*\text{NJT}+\text{NTP}*(2*\text{MH}+47) +120*\text{MH}+\text{NOTMP}*(2+3*\text{NGIR})+\text{INTRES}+1$$

where

NEL = number of elements

NJT = number of joints

NTP = number of output sections

MB = half bandwidth = (max. joint difference in an element + 1)*4

MH = number of non-zero terms of harmonic series

NOTMP = number of sections for girder moment output

NGIR = number of girders

NSURL = number of partial surface loads

NCONL = number of partial joint loads

INTRES = number of intermediate harmonic series results

MCHECK = girder moment option, 1 if girder moments are desired, 0 if girder moments are not desired.

The required blank common area is then

$$COMMON = MAX(COMMON_1, COMMON_2)$$

which may have to be converted to the octal base. The program version listed in Appendix A utilizes an automatic field length adjustment requesting for each problem only as much core storage as required for the specific problem.

Execution of small problems requires only a few seconds on the CDC 6400, therefore an execution time estimate is only feasible for structures with many elements. This time is almost directly proportional to the number of harmonic terms used to represent the applied loads. In addition, the bandwidth of the system of equations, the number of joints, and the amount of requested output are important influence factors. But many other factors affect the total execution time, so that the following formula allows only an estimate,

$$T = N_H(\alpha\ N_{Eq}\ M_B^2 + \beta\ N_{E\ell})$$

where

T = total execution time in seconds

N_H = number of non-zero harmonics

N_{Eq} = number of joints times 4

M_B = bandwidth = (maximum joint difference in a strip + 1)*4

$N_{E\ell}$ = number of elements times average number of transverse sections for output

α = $0.4.10^{-4}$ } for FUN compiler of CDC 6400

β = .004

For example, for a structure with 15 elements and 4 output sections for each, 14 joints, 50 harmonics, and a bandwidth of $M_B = 12$, execution time would be estimated as

$$T = 50 \cdot 10^{-4} [(0.4)(56)(12)^2 + (40)(60)] = 28.1 \text{ sec.}$$

5. EXAMPLES

5.1 Bending of a Cylinder

In order to demonstrate the degeneration of the theory to a shell of revolution under axisymmetric loading, a cylinder fixed at one end and subjected to a uniform radial load at the free end, Fig. 3, has been analyzed with program CURSTR using four different element representations. As a theoretical check, the free constants in Flügge's closed form solution [7] have been adjusted to the boundary conditions of this problem, and for the evaluation of displacements and moments, a small computer program has been written, the results of which are shown in Fig. 3 as "exact" solution.

As can be seen in Fig. 3, displacement results agree very well with the closed form solution, even for the coarse mesh A. Also the bending moment check is excellent throughout the shell except for the "unbalanced" moment at the free edge which stems from the fact that the finite strip theory does not satisfy force boundary conditions. The numerical values for these unbalanced moments are listed in Table 1 and show how rapidly they decrease

Table 1. Unbalanced Moments At Free Edge of Cylinder

Mesh	A	B	C	D	"Exact"
Unbalanced Moment	.053814	.022330	.007458	.002126	0.0

with mesh refinement.

The total execution time for all four meshes together was 6.4 seconds.

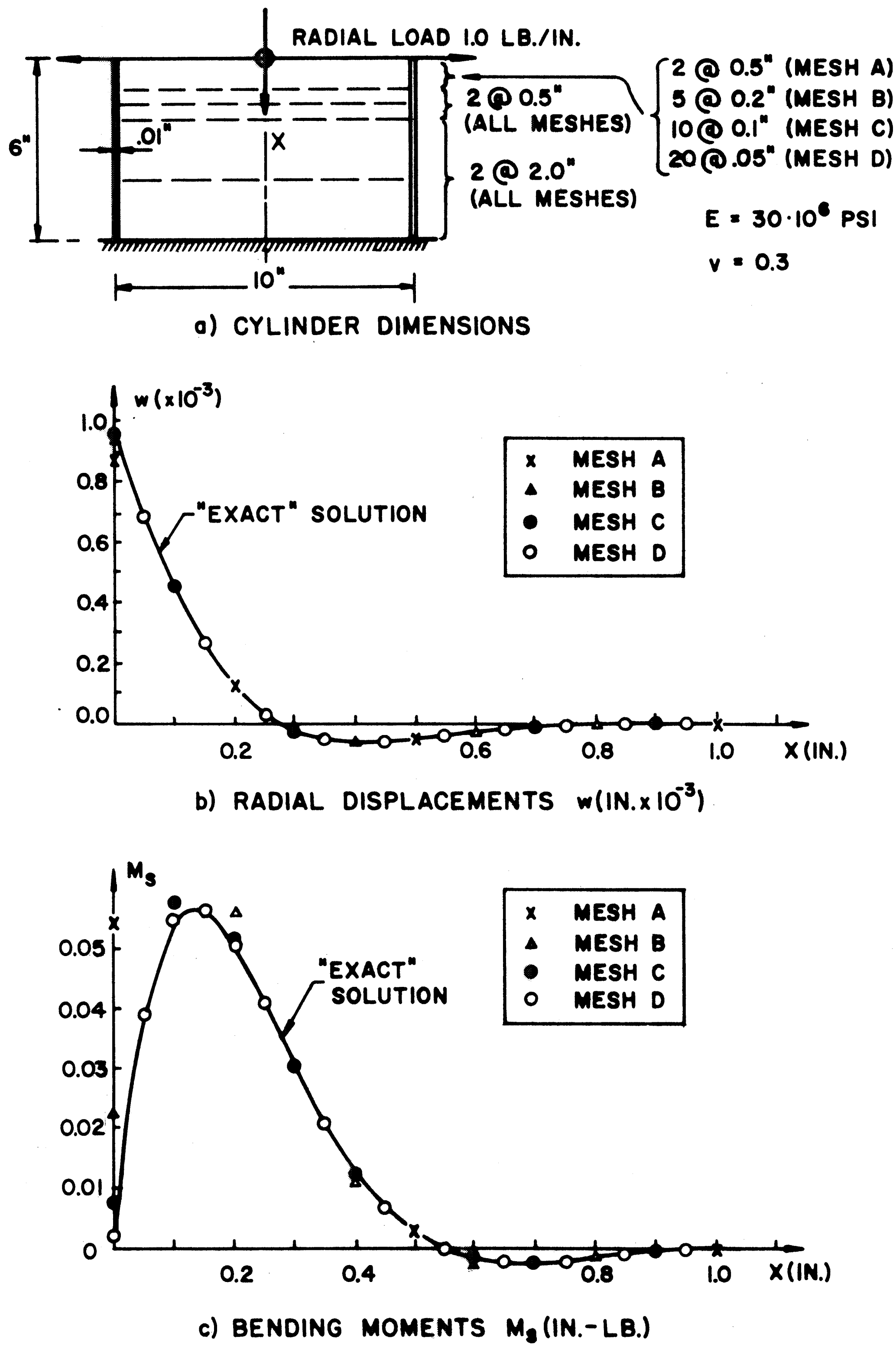


FIG. 3 EXAMPLE 1 - BENDING OF A CYLINDER

5.2 Curved Beam Problem

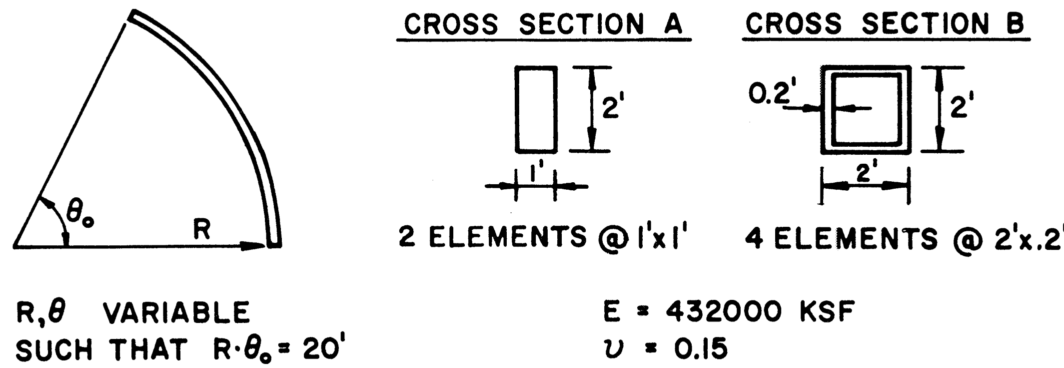
The curved beam of Fig. 4a has been analyzed using ordinary curved beam theory and also program CURSTR for a) uniform loading of q = 1.0k/ft, and b) a concentrated midspan load P = 1.0 kip, using the two cross sections shown in Fig. 4a. Denoting with

$$\rho = \frac{M_{curved}}{M_{straight}}$$

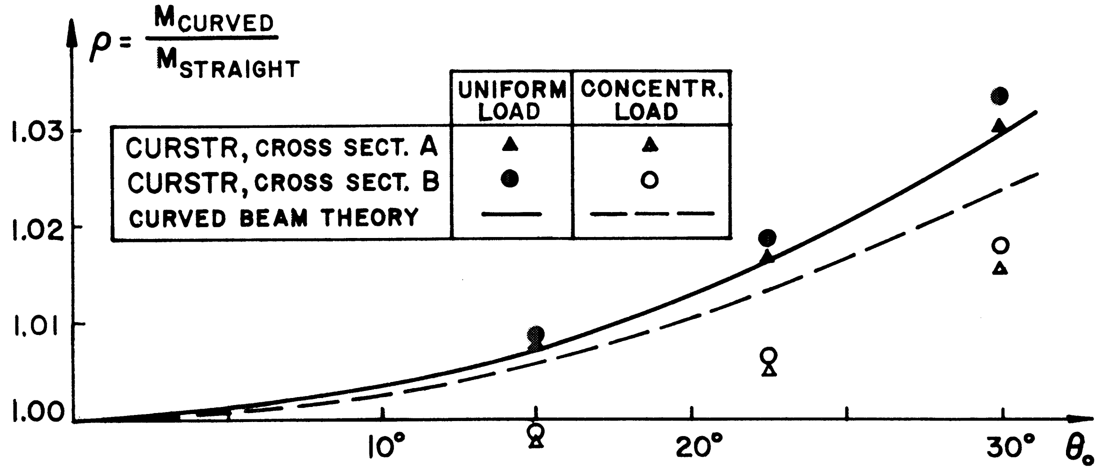
the ratio of the maximum bending moment in a curved beam to that in the corresponding straight beam of equal span length $L = R\theta_o$, this ratio ρ has been plotted in Fig. 4b as a function of the opening angle θ_o for both loading conditions. It can be seen, for uniform load, CURSTR moments are consistently higher than those predicted by curved beam theory, but for the concentrated midspan load, CURSTR moments are considerably lower. At $\theta_o = 15°$ it is even smaller than the corresponding straight beam moment which is theoretically impossible. This fact can be explained by the Fourier series representation of the concentrated load. Although 25 non-zero terms of this series have been used, the peak of the moment curve right under the load is still cut off slightly. If, therefore, the quarter-span moment ratio is plotted for the same loading, Fig. 4c, no such underestimation exists. Thus, except right at a concentrated load, CURSTR gives slightly higher moments than the curved beam theory, and the difference increases for more flexible beam cross sections.

5.3 Curved Plate Problem

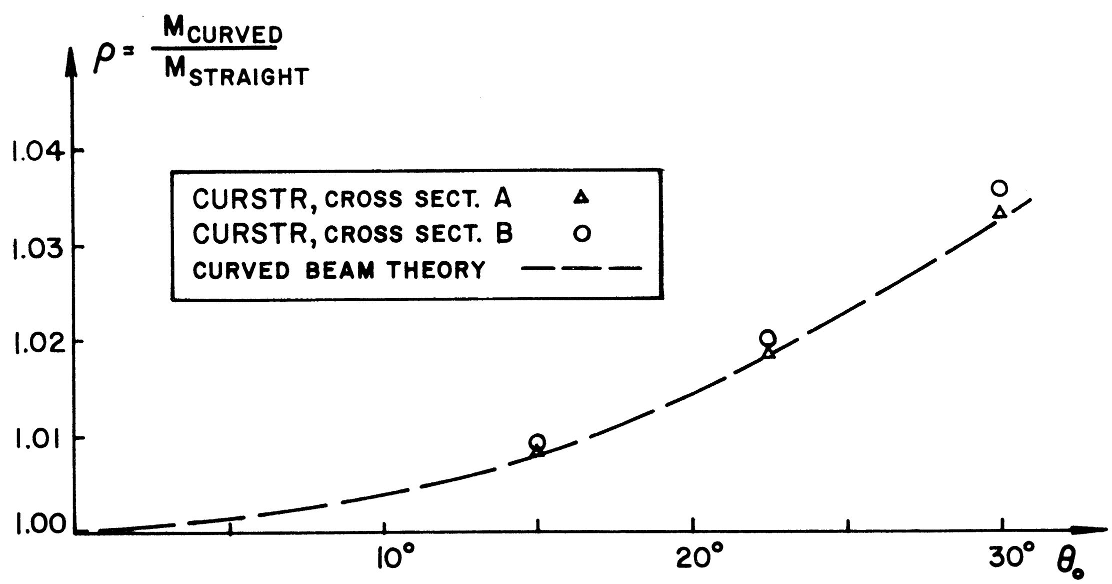
The curved plate of Fig. 5a is simply supported along the straight



a) CURVED BEAM DIMENSIONS



b) MIDSPAN MOMENTS FOR UNIFORM AND CONCENTRATED LOAD



c) QUARTERSPAN MOMENTS FOR MIDSPAN LOAD

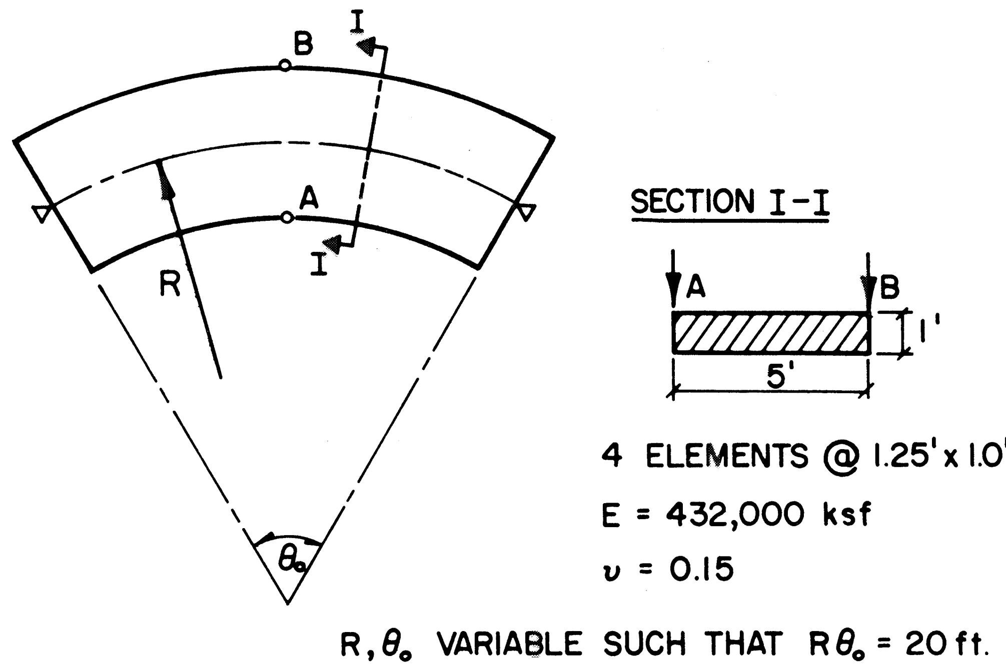
FIG. 4 EXAMPLE 2 – CURVED BEAM PROBLEM

edges and free at the curved boundaries. It has been analyzed for various opening angles θ_o (such that the span length $L = R\theta_o$ remained constant) using program CURSTR. The results are compared with those obtained from the closed form solution of the plate equation for this special boundary value problem [6].

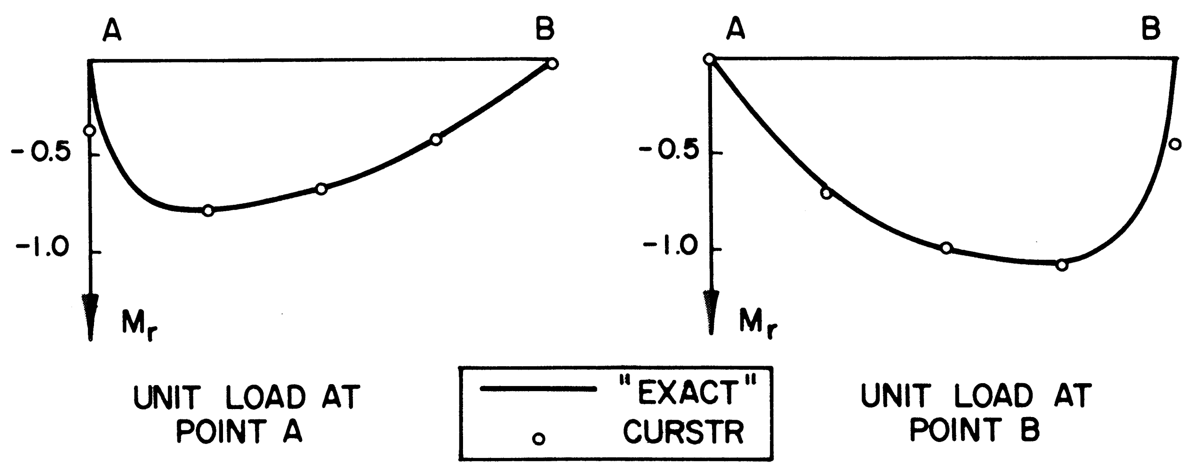
Figure 6 illustrates the effect of curvature on the statical midspan moment as well as on the longitudinal plate bending moments M_θ at midspan of the two edges. Denoting again by

$$\rho = \frac{M_{curved}}{M_{straight}}$$

the ratio between the moment quantity in the curved plate and in the corresponding rectangular plate of equal span length, then the ρ-θ_o relationships plotted in Fig. 6 reveal several interesting facts. For a unit load placed at point A, the statical moment decreases with increasing θ_o because the actual span length of the interior edge is reduced. Conversely, the statical moment for a load placed at point B increases, because the outer edge becomes longer. With increasing curvature, the inner edge of the plate becomes stiffer and the outer edge more flexible. Note that for both load positions the M_θ moments at points A (inner edge) and B (outer edge) are respectively greater than and less than the average moment across the width represented by the statical mid-span moment curves. This corroborates the fact that the inner edge becomes stiffer than the outer edge of the plate for increasing angles θ_o.



a) NOTATIONS FOR SIMPLY SUPPORTED CURVED PLATE



b) TRANSVERSE MOMENTS M_r (ft-k/ft) AT MIDSPAN FOR θ_o = 30°

FIG. 5 EXAMPLE 3 - CURVED PLATE PROBLEM

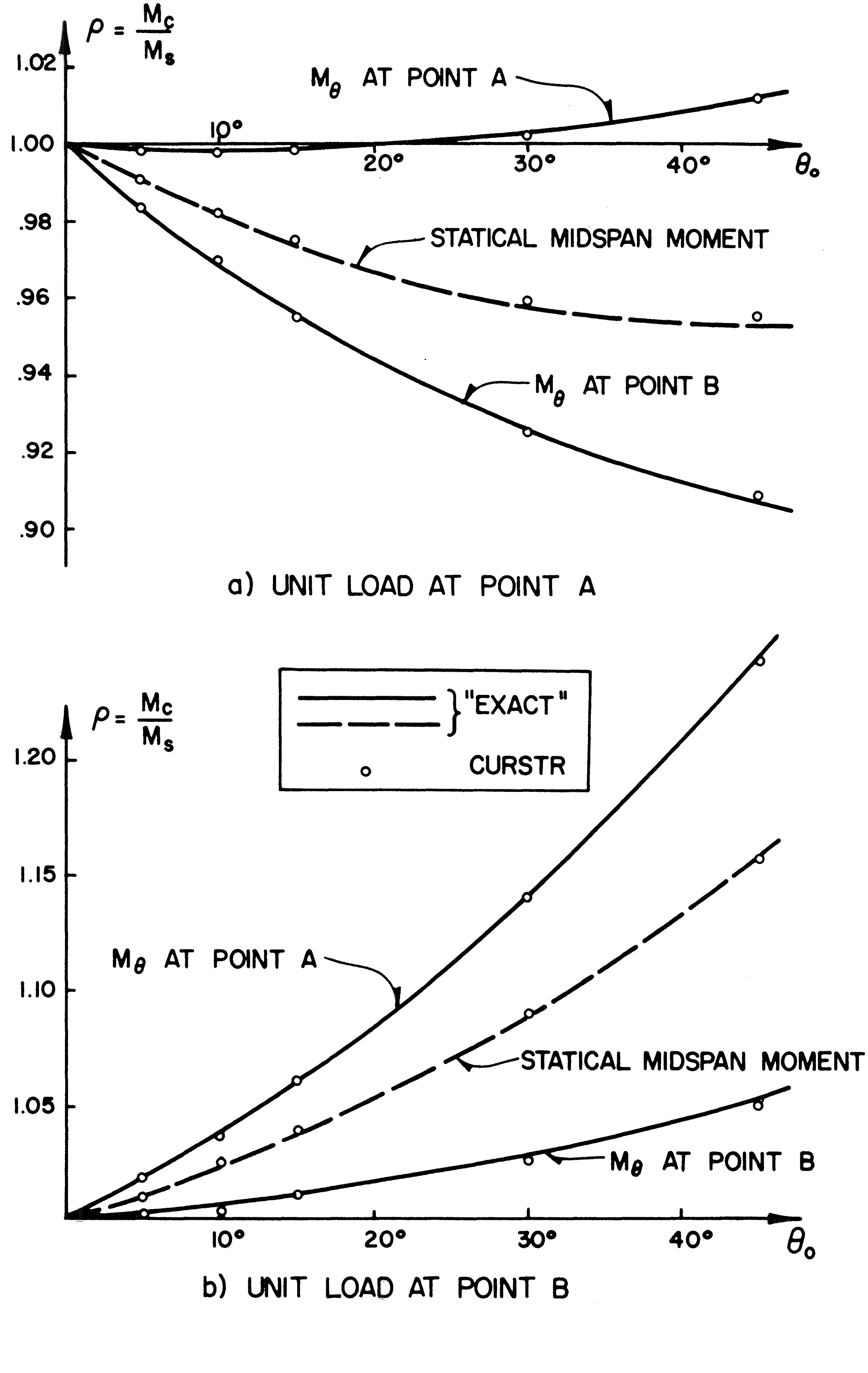


FIG. 6 EFFECT OF CURVATURE ON LONGITUDINAL MOMENTS M_θ OF PLATE

Figure 5b shows the transverse moments M_r at midspan for an opening angle of $\theta_o = 30°$, with unit loads placed at point A and B.

In all cases, the excellent agreement between the elasticity solution and the curved strip theory is to be noted. The unbalanced moment M_r at the loaded edge has been discussed previously. The execution time of program CURSTR on the CDC 6400 for each individual run (using 25 non-zero terms of the harmonic series) was 4.2 seconds.

5.4 Curved Box Girder Bridge

The two-cell box girder bridge shown in Fig. 7 has a radius of curvature of R = 250 ft. and a span along the curve of 100 ft., so that the opening angle is 22.92°. It has been analyzed with program CURSTR for a standard AASHO-truck in the two different positions shown in Fig. 7, considering downward load components only. The superelevation of 0.1 ft. per ft. has hardly any influence on the resulting state of stress or deformation of the bridge, but it may be of importance if horizontal load components are to be included. The input data for this example are reproduced in Appendix B.

The longitudinal stress resultants N_θ and transverse bending moments M_r for both truck positions are shown in Fig. 8. It is interesting to note that truck A produces higher maximum stresses and transverse bending moments than truck B in spite of the smaller statical moment. This fact can be explained with some of the information obtained from the curved plate problem, Section 5.3. The outer girder of the bridge is more flexible than

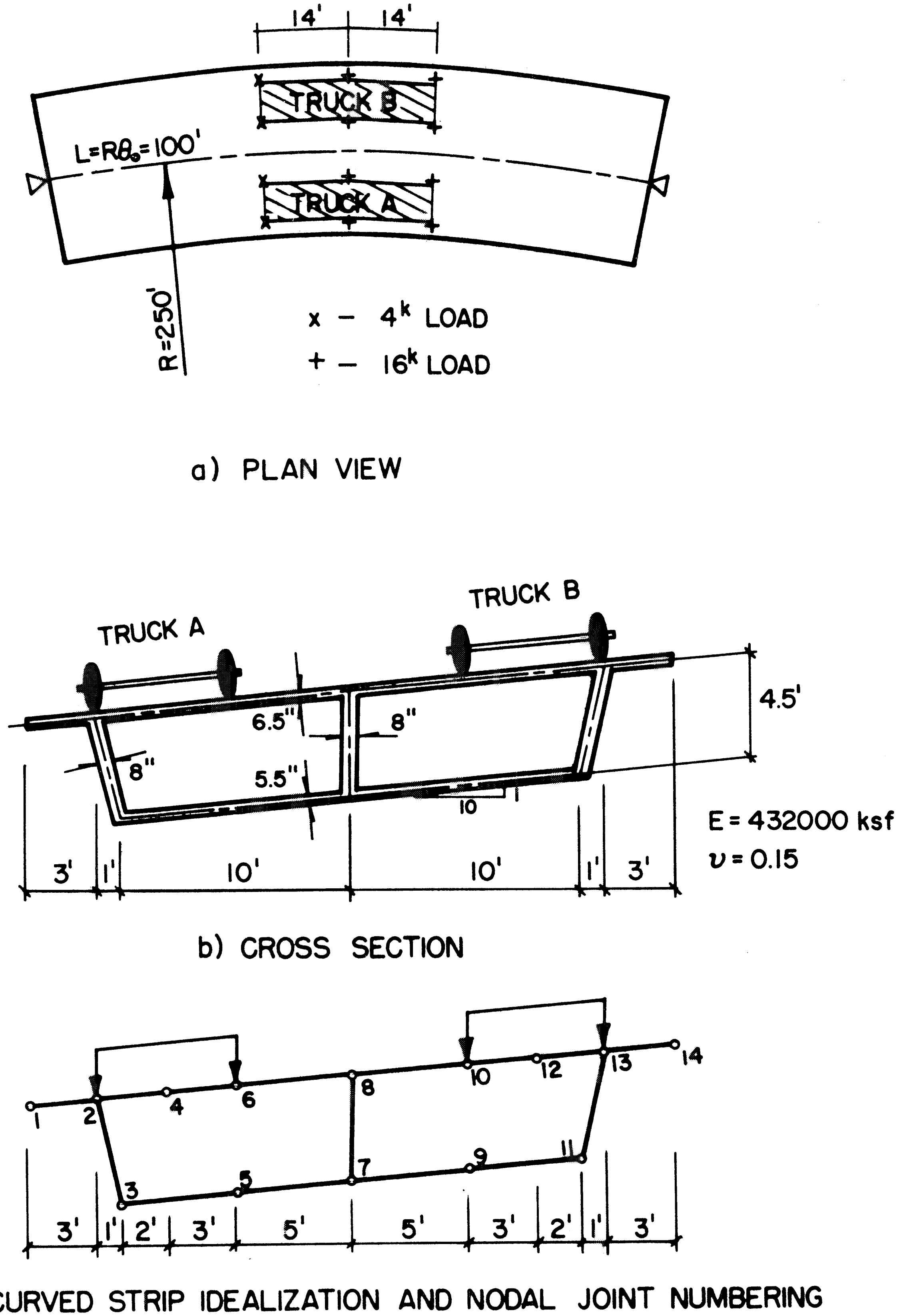
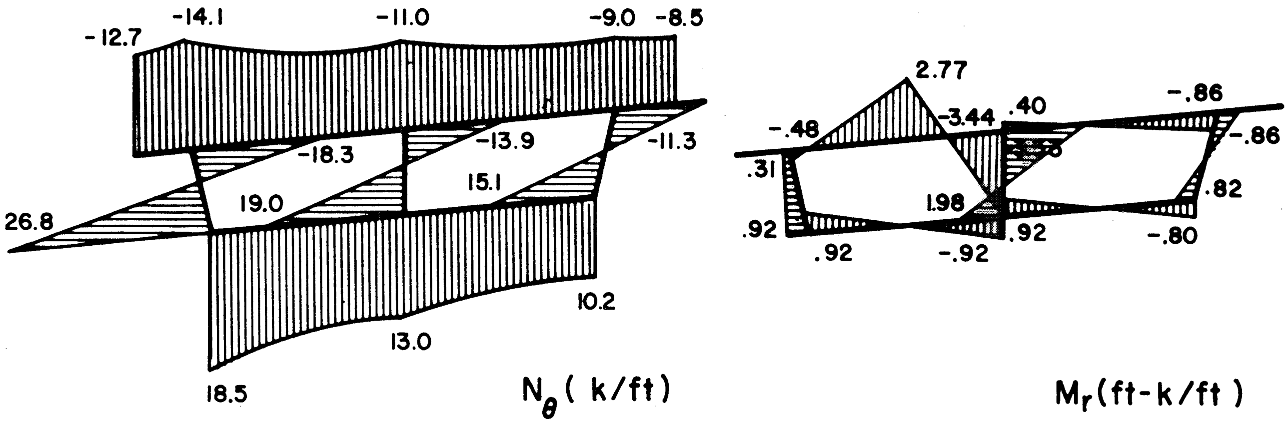
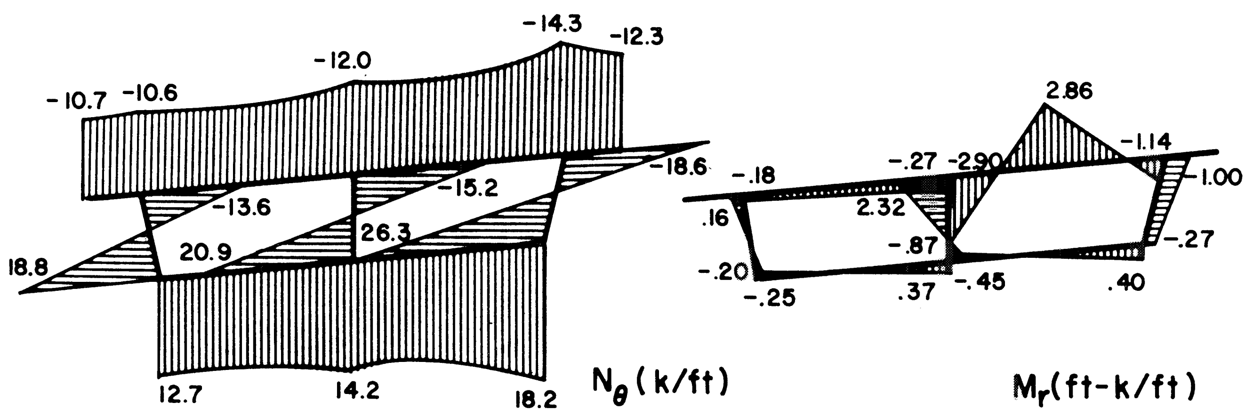


FIG. 7 EXAMPLE 4 – CURVED BOX GIRDER BRIDGE



a) TRUCK POSITION A



b) TRUCK POSITION B

FIG. 8 LONGITUDINAL STRESS RESULTANTS N_θ AND TRANSVERSE MOMENTS M_r AT MIDSPAN OF BOX GIRDER BRIDGE the inner one and therefore tries to distribute the load more evenly over the entire bridge, while the inner girder, if loaded directly, tends to carry the load alone, and load distribution is poorer.

Analyzing this curved box girder bridge using curved beam theory, one obtains a basis of comparison for the statical moments at various sections which are summarized in Table 2.

TABLE 2. STATICAL MOMENTS IN CURVED BOX GIRDER BRIDGE

	TRUCK A			TRUCK B		
X-Coordinate (Along Ref. Line)	35.36	50.00	64.64	36.64	50.00	63.36
Curved Beam Th.	1409.7	1505.1	1181.1	1463.1	1578.9	1206.9
CURSTR	1369.7	1464.9	1116.4	1486.2	1595.7	1268.1
Difference	-2.7%	-2.5%	-5.5%	+1.4%	+1.0%	+5.0%

The differences between the curved strip analysis and curved beam theory are relatively small and partially due to the slightly different truck simulation in both theories.

Approximating the wheel loads with 50 terms of the corresponding Fourier series, the total execution time on the CDC 6400 for each of the two separate runs amounted to 27.2 seconds.

6. ACKNOWLEDGEMENT

The computer program described in this report was developed as part of a continuing research project in the Department of Civil Engineering, University of California at Berkeley, under the sponsorship of the Division of Highways, Department of Public Works, State of California, and the Bureau of Public Roads, Federal Highway Administration, United States Department of Transportation. The opinions, findings, and conclusions expressed in this report are those of the authors and not necessarily those of the sponsors.

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The support of the University of California Computer Center, which provided the computer facilities, is gratefully acknowledged.

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Appendix A

FORTRAN IV Listing of Program

Considerable time, effort, and expense have gone into the development of the computer program. It is obvious that it should be used only under the conditions and assumptions for which it was developed. These are described in this research report. Although the program has been extensively tested by the authors, no warranty is made regarding the accuracy and reliability of the program and no responsibility is assumed by the authors or by the sponsors of this research project.

```
      PROGRAM  CURSTR (INPUT,OUTPUT,TAPE 1)
C
C               *********************************************
C               *                 - CURSTR -                *
C               *  SOLUTION OF CURVED FOLDED PLATE STRUCTURES *
C               *  USING THE FINITE STRIP  METHOD OF ANALYSIS *
C               *********************************************
C
C               PROGRAMMED BY CHRISTIAN MEYER
C               UNIVERSITY OF CALIFORNIA, DECEMBER 1969
C
C
C***********************************************************************
C      FORM OF INPUT DATA
C***********************************************************************
C
C     (1)  TITLE CARD (8A10)
C     COL.  1 TO 80 - TITLE OF PROBLEM TO BE PRINTED WITH OUTPUT
C
C     (2)  CONTROL CARD (2F10.0,15I4)
C     COL.  1 TO 10 - ANGLE OF CURVATURE (IN DEGR.) BETW.END SUPPORTS,TETAO
C     COL. 11 TO 20 - RADIUS OF CURVATURE OF REFERENCE LINE, R
C     COL. 21 TO 24 - NUMBER OF PLATE TYPES, NPL, MAX.=30
C     COL. 25 TO 28 - NUMBER OF ELEMENTS, NEL
C     COL. 29 TO 32 - NUMBER OF JOINTS, NJT
C     COL. 33 TO 36 - NUMBER OF TRANSVERSE SECTIONS AT WHICH OUTPUT
C                     RESULTS ARE DESIRED, NTP
C     COL. 37 TO 40 - MAXIMUM FOURIER SERIES LIMIT, MHARM
C                     (ZERO FOR AXISYMMETRIC SHELL WITH AXISYMM. LOADS)
C     COL. 41 TO 44 - CHECK ON ODD OR EVEN HARMONICS, NCHECK
C                        +1  TO WORK ON ODD HARMONICS ONLY (SYMMETRY)
C                         0  TO INCLUDE ALL HARMONICS (NON-SYMMETRY)
C                        -1  TO WORK ON EVEN HARMONICS ONLY (ANTI-SYMMETRY)
C                     (LEAVE BLANK FOR AXISYMMETRIC CASE)
C     COL. 45 TO 48 - NUMBER OF HARMONIC NUMBERS FOR WHICH INTERMEDIATE
C                     RESULTS ARE DESIRED, INTRES
C     COL. 49 TO 52 - INPUT/OUTPUT OPTION INDICATOR, IO
C                         1  SECTIONS FOR INPUT/OUTPUT GIVEN BY ANGLES
C                            THETA (DEGREES)
C                         0  SECTIONS FOR INPUT/OUTPUT GIVEN BY ARC
C                            LENGTHS OF REFERENCE LINE (FEET OR INCH)
C     COL. 53 TO 56 - GIRDER MOMENT INTEGRATION OPTION, MCHECK
C                         1  STATICAL GIRDER MOMENTS DESIRED
C                         0  STATICAL GIRDER MOMENTS NOT DESIRED
C     COL. 57 TO 60 - HALF BANDWIDTH, MB = (MAX.NODAL POINT DIFFERENCE
C                     IN AN ELEMENT + 1)*4
C     COL. 61 TO 64 - NUMBER OF PARTIAL SURFACE LOADS, NSURL
C     COL. 65 TO 68 - NUMBER OF PARTIAL JOINT LOADS, NCONL
C     COL. 69 TO 72 - NUMBER OF SECTIONS FOR WHICH STATICAL GIRDER
C                     MOMENTS ARE DESIRED, NOTMP
C     COL. 73 TO 76 - NUMBER OF GIRDERS, NGIR
C                     THE LAST TWO ENTRIES ARE NEEDED ONLY FOR MCHECK=1
C     COL. 77 TO 80 - MATERIAL OPTION INDICATOR, MI
C                         0 FOR INPUTING MATERIAL PROPERTIES
```

```
C                          1 FOR INPUTING CONSTITUTIVE RELATIONS DIRECTLY
C
C       (3)  CIRCUMFERENTIAL COORDINATE CARD (10F7.2)
C      COL.  1 TO 70 - X-COORDINATES/THETA ANGLES (DEGR.) OF TRANSVERSE
C                      SECTIONS AT WHICH RESULTS ARE DESIRED, XP(I).
C                      USE SECOND CARD IF NECESSARY.
C
C       (4)  INTERMEDIATE RESULT CARD (20I4)
C      HARMONIC NUMBERS FOR WHICH INTERMEDIATE RESULTS ARE DESIRED,
C      IRES(I).  OMIT THIS CARD IF INTRES=0.
C
C       (5)  PLATE TYPE CARDS
C      IF  MI=0, TWO CARDS (I10,5F10.0/10X,5F10.0) ARE REQUIRED FOR EACH TYPE
C      FIRST CARD   -  MEMBRANE CHARACTERISTICA
C      COL.  1 TO 10 - TYPE NUMBER, I
C      COL. 11 TO 20 - EFFECTIVE THICKNESS, THM(I)
C      COL. 21 TO 30 - MODULUS OF ELASTICITY IN TETA-DIRECTION, ETM(I)
C      COL. 31 TO 40 - MODULUS OF ELASTICITY IN MERIDIONAL DIRECTION, ESM(I)
C      COL. 41 TO 50 - SHEAR MODULUS, GM(I)
C      COL. 51 TO 60 - POISSON-S RATIO, PRM(I) (NEGATIVE STRAIN IN TETA-
C                      DIRECTION FOR UNIT STRAIN IN MERIDIONAL DIRECTION)
C
C      SECOND CARD   -  PLATE BENDING CHARACTERISTICA
C      COL. 11 TO 20 - EFFECTIVE THICKNESS, THB(I)
C      COL. 21 TO 30 - MODULUS OF ELASTICITY IN TETA-DIRECTION, ETB(I)
C      COL. 31 TO 40 - MODULUS OF ELASTICITY IN MERIDIONAL DIRECTION, ESB(I)
C      COL. 41 TO 50 - SHEAR MODULUS, GB(I)
C      COL. 51 TO 60 - POISSON-S RATIO, PRB(I)
C
C      IF  MI=1, TWO CARDS (I10,4F10.0/10X,4F10.0) ARE REQUIRED FOR EACH TYPE
C      FIRST CARD   -  MEMBRANE CONSTITUTIVE CONSTANTS
C      COL.  1 TO 10 - TYPE NUMBER, I
C      COL. 11 TO 20 - D(1,1)
C      COL. 21 TO 30 - D(1,2)
C      COL. 31 TO 40 - D(2,2)
C      COL. 41 TO 50 - D(3,3)
C
C      SECOND CARD   -  PLATE BENDING CONSTITUTIVE CONSTANTS
C      COL. 11 TO 20 - D(4,4)
C      COL. 21 TO 30 - D(4,5)
C      COL. 31 TO 40 - D(5,5)
C      COL. 41 TO 50 - D(6,6)
C
C       (6)  ELEMENT CARDS (5I4,5F10.0) - ONE CARD FOR EACH ELEMENT
C                      UNIFORM LOADS GIVEN BELOW EXIST OVER ENTIRE PLATE
C      COL.  1 TO  4 - ELEMENT NUMBER, I
C      COL.  5 TO  8 - JOINT I, NPI(I)
C      COL.  9 TO 12 - JOINT J, NPJ(I)
C      COL. 13 TO 16 - TYPE OF PLATE USED, KPL(I)
C      COL. 17 TO 20 - NUMBER OF ELEMENT SUBDIVISIONS FOR INTERNAL FORCES
C                      AND DISPLACEMENTS OUTPUT, NSEC(I), MAX.=4
C                      IF NSEC(I)=0, NO INTERNAL FORCES OR DISPLACEMENTS
C                      WILL BE OUTPUT FOR ELEMENT I
C      COL. 21 TO 30 - DEAD LOAD (FORCE PER UNIT SURFACE AREA), DL(I)
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C     COL. 31 TO 40 - HORIZONTAL LOAD INTENSITY AT JOINT I, HLI(I)
C                     (FORCE PER UNIT VERTICALLY PROJECTED AREA)
C     COL. 41 TO 50 - HORIZONTAL LOAD INTENSITY AT JOINT J, HLJ(I)
C     COL. 51 TO 60 - VERTICAL LOAD INTENSITY AT JOINT I, VLI(I)
C                     (FORCE PER UNIT HORIZONTALLY PROJECTED AREA)
C     COL. 61 TO 70 - VERTICAL LOAD INTENSITY AT JOINT J, VLJ(I)
C
C      (7)  JOINT CARDS (I10,6F10.0,4I2) - ONE CARD FOR EACH JOINT
C     COL.  1 TO 10 - JOINT NUMBER, I
C     COL. 11 TO 20 - Y-COORDINATE OF JOINT, Y(I)
C     COL. 21 TO 30 - Z-COORDINATE OF JOINT, Z(I)
C     COL. 31 TO 40 - APPLIED HORIZONTAL JOINT FORCE OR DISPL., AJF(1,I)
C     COL. 41 TO 50 - APPLIED VERTICAL JOINT FORCE OR DISPL., AJF(2,I)
C     COL. 51 TO 60 - APPLIED JOINT MOMENT OR ROTATION, AJF(3,I)
C     COL. 61 TO 70 - APPLIED LONGITUDINAL JOINT FORCE OR DISPL., AJF(4,I)
C     COL.        72 - HORIZONTAL FORCE/DISPLACEMENT INDEX, LCASE(1,I)
C     COL.        74 - VERTICAL FORCE/DISPLACEMENT INDEX, LCASE(2,I)
C     COL.        76 - MOMENT/ROTATION INDEX, LCASE(3,I)
C     COL.        78 - LONGITUDINAL FORCE/DISPLACEMENT INDEX, LCASE(4,I)
C                     FORCE/DISPLACEMENT INDEX IS EQUAL TO
C                       0  FOR GIVEN ZERO FORCE OR MOMENT
C                       1  FOR UNIFORMLY DISTRIBUTED FORCE OR MOMENT
C                          (INPUT UNIFORM FORCE OR MOMENT PER UNIT LENGTH
C                          FOR AJF)
C                       2  FOR CONCENTRATED FORCE OR MOMENT AT MIDSPAN
C                          (INPUT TOTAL FORCE FOR AJF)
C                       3  FOR GIVEN ZERO DISPLACEMENT OR ROTATION
C                       4  FOR PRESTRESS P AT EACH END  (INPUT TOTAL FORCE
C                          AT ONE END FOR AJF, POSITIVE AWAY FROM MIDSPAN)
C
C      (8)  PARTIAL SURFACE LOAD CARDS (I10,6F10.0) - ONE CARD FOR EACH
C                     PARTIAL SURFACE LOAD. NO CARDS REQUIRED IF NSURL=0.
C                     LOADS GIVEN BELOW ARE UNIFORM OVER PLATE WIDTH AND
C                     HAVE A LENGTH GIVEN UNDER SURL.
C     COL.  1 TO 10 - ELEMENT NUMBER, LEL
C     COL. 11 TO 20 - HORIZONTAL LOAD INTENSITY AT JOINT I, PHLI(I)
C                     (FORCE PER UNIT VERTICALLY PROJECTED AREA)
C     COL. 21 TO 30 - HORIZONTAL LOAD INTENSITY AT JOINT J, PHLJ(I)
C     COL. 31 TO 40 - VERTICAL LOAD INTENSITY AT JOINT I, PVLI(I)
C                     (FORCE PER UNIT VERTICALLY PROJECTED AREA)
C     COL. 41 TO 50 - VERTICAL LOAD INTENSITY AT JOINT J, PVLJ(I)
C     COL. 51 TO 60 - ARC LENGTH/ANGLE (DEGR.) FROM ORIGIN TO CENTER OF
C                     DISTRIBUTED LOAD, SURT
C     COL. 61 TO 70 - ARC LENGTH/ANGLE (DEGR.) SUBTENDED BY DISTRIBUTED
C                     LOAD, SURL (=0 FOR TRANSVERSE LINE LOAD)
C                     IF SURL NOT EQUAL 0, INPUT LOADS AS FORCE PER UNIT
C                     AREA. IF SURL=0, INPUT LOADS AS FORCE PER UNIT WIDTH
C
C      (9)  PARTIAL JOINT LOAD CARDS (I10,6F10.0) - ONE CARD FOR EACH
C                     JOINT LOAD. NO CARDS REQUIRED IF NCONL=0. MORE THAN
C                     ONE LOCATION ALONG A JOINT MAY BE LOADED, BUT EACH
C                     LOCATION REQUIRES A SEPARATE CARD.
C     COL.  1 TO 10 - JOINT NUMBER, LJT
C     COL. 11 TO 20 - TOTAL HORIZONTAL FORCE, FH
```

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C     COL. 21 TO 30 - TOTAL VERTICAL FORCE, FV
C     COL. 31 TO 40 - TOTAL MOMENT, FM
C     COL. 41 TO 50 - TOTAL LONGITUDINAL FORCE, FP
C     COL. 51 TO 60 - ARC LENGTH/ANGLE (DEGR.) FROM ORIGIN TO CENTER OF
C                     DISTRIBUTED LOAD, FTL
C     COL. 61 TO 70 - ARC LENGTH/ANGLE (DEGR.) SUBTENDED BY DISTRIBUTED
C                     LOAD, FTT (=0 FOR CONCENTRATED LOAD)
C
C      (10)  GIRDER MOMENT INTEGRATION DATA - NO CARDS REQUIRED IF MCHECK=0
C     FIRST CARD  (10F7.2)
C     COL.  1 TO 70 - X-COORDINATES/THETA ANGLES (DEGR.) OF SECTIONS AT
C                     WHICH GIRDER MOMENTS ARE DESIRED, T(I). MUST BE A
C                     SUBSET OF THOSE COORDINATES LISTED ON THE THIRD CARD.
C                     USE SECOND CARD IF NECESSARY
C
C     ELEMENT CARDS (3I5,3F10.0) - ONE CARD FOR EACH ELEMENT
C     COL.  1 TO  5 - ELEMENT NUMBER, I
C     COL.  6 TO 10 - GIRDER WHICH JOINT I OF THIS ELEMENT BELONGS TO,
C                     NGIEL(1,I)
C     COL. 11 TO 15 - GIRDER WHICH JOINT J OF THIS ELEMENT BELONGS TO,
C                     NGIEL(2,I).  LEAVE BLANK IF NO SECOND GIRDER.
C     COL. 16 TO 25 - VERTICAL DISTANCE FROM NEUTRAL AXIS TO NODE I, DNAI(I)
C     COL. 26 TO 35 - VERTICAL DISTANCE FROM NEUTRAL AXIS TO NODE J, DNAJ(I)
C     COL. 36 TO 45 - HORIZONTAL DISTANCE FROM NODE I TO THE DIVIDING
C                     POINT, IF THE ELEMENT BELONGS TO TWO GIRDERS, XDIV(I)
C                     PUNCH ZERO OR LEAVE BLANK IF NOT APPLICABLE.
C
C     REPEAT ALL ABOVE DECKS (1)  THROUGH (10)  FOR THE NEXT PROBLEM.
C     TWO BLANK CARDS ARE ADDED FOLLOWING THE LAST PROBLEM.
C
C***********************************************************************
C
C      COMMON AND DIMENSION STATEMENTS
C
      COMMON A(1)
      COMMON/SETUP/TETAO,R,NPL,NEL,NJT,NTP,MHARM,NCHECK,INTRES,IO,MB,
     1   MCHECK,MM,N1,N2,PI,NNM,NXBAND,II,IJ,IL,IAX,NSURL,NCONL,NOTMP,
     2   NGIR,MI
      DIMENSION TITLE(8)
      LOGICAL EVEN,IO
C
C      READ AND PRINT CONTROL CARDS
C
      CALL LWA(NNN)
      NNM=0
    1 READ 1000, (TITLE(I), I=1,8)
      READ 1001,TETAO,R,NPL,NEL,NJT,NTP,MHARM,NCHECK,INTRES,IIO,MCHECK,
     1   MB,NSURL,NCONL,NOTMP,NGIR,MI
      IF(R.EQ.O.) GO TO 999
      CALL SECOND(T1)
      MH=(MHARM/2)*2
      IF(MHARM.EQ.MH) GO TO 15
      EVEN=.FALSE.
      GO TO 20
```

```
   15 EVEN=.TRUE.
   20 PRINT 2000, (TITLE(I), I=1,8)
      PRINT 2002, TETAO,R,NPL,NEL,NJT,NTP,MHARM,MB,NSURL,NCONL,MI
      IF(MCHECK.EQ.1) PRINT 2010, NOTMP,NGIR
      IO=.TRUE.
      IF(IIO.EQ.0) IO=.FALSE.
      IF(.NOT.IO) PRINT 2007
      IF(IO) PRINT 2006
      IAX=1
      IF(MHARM.GT.0) GO TO 23
      PRINT 2009
      MH=1
      IAX=0
      GO TO 40
   23 CONTINUE
      MH=MHARM
      IF(NCHECK) 25,40,35
   25 PRINT 2003
      MH=MH/2
      IF(EVEN) GO TO 40
   30 MHARM=MHARM-1
      PRINT 2004, MHARM
      GO TO 40
   35 PRINT 2005
      MH=MH/2
      IF(EVEN) GO TO 30
      MH=MH+1
C
C     DETERMINE REQUIRED STORAGE FOR MAIN SUBROUTINE
C
   40 NJT4=4*NJT
      L1=1
      L2=L1+NEL
      L3=L2+NEL
      L4=L3+NEL
      L5=L4+NEL
      L6=L5+NEL
      L7=L6+NEL
      L8=L7+NEL
      L9=L8+NJT
      L10=L9+NJT
      L11=L10+MH*NTP
      L12=L11+MH*NTP
      L13=L12+NTP
      L14=L13+NTP
      L15=L14+INTRES+1
      L16=L15+NEL
      L17=L16+NEL
      L18=L17+NEL
      L19=L18+NEL
      L20=L19+NEL
      L21=L20+NOTMP
      L22=L21+NOTMP
      L23=L22+NOTMP*NGIR
```

```
      L24=L23+NOTMP*NGIR
      L25=L24+NOTMP*NGIR
      L26=L25+NSURL
      L27=L26+NSURL
      L28=L27+NSURL
      L29=L28+NSURL
      L30=L29+NSURL
      L31=L30+NSURL
      L32=L31+NSURL
      L33=L32+NCONL
      L34=L33+NCONL
      L35=L34+NCONL
      L36=L35+NCONL
      L37=L36+NCONL
      L38=L37+NCONL
      L39=L38+NCONL
      L40=L39+NJT4
      L41=L40+NJT4
      L42=L41+NJT4*NTP
      L43=L42+NJT4
      NMAIN=NNN+L43+NJT4*(MB+1)
C
C     DETERMINE REQUIRED STORAGE FOR FORCE SUBROUTINE
C
      NX5=5*NTP
      N25=L25
      IF(MCHECK.EQ.0) N25=L15
      N26=N25+NX5
      N27=N26+NX5
      N28=N27+NX5
      N29=N28+NX5
      N30=N29+NX5
      N31=N30+NX5
      N32=N31+NX5
      N33=N32+NX5
      N34=N33+NX5
      N35=N34+8*NEL
      NFORC=NNN+N35+120*MH
C
C     RESET FIELD LENGTH REQUIRED FOR THIS PROBLEM
C
      IF(NFORC.GT.NMAIN) NMAIN=NFORC
      IF(NMAIN.GT.NNM) GO TO 45
      NNQ=NMAIN+1000
      IF(NNQ.GT.NNM) GO TO 47
   45 NNM=NMAIN
      CALL RFL(NNM)
C
C     CALL PROGRAM SUBROUTINES
C
   47 CALL MAIN (A(L1),A(L2),A(L3),A(L4),A(L5),A(L6),A(L7),A(L8),A(L9),
     1    A(L10),A(L11),A(L12),A(L13),A(L14),A(L15),A(L15),A(L16),A(L17),
     2    A(L17),A(L18),A(L18),A(L19),A(L19),A(L20),A(L21),A(L22),A(L23),
     3    A(L24),A(L25),A(L26),A(L27),A(L28),A(L29),A(L30),A(L31),A(L32),
```

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     4   A(L33),A(L34),A(L35),A(L36),A(L37),A(L38),A(L39),A(L39),A(L40),
     5   A(L40),A(L41),A(L42),A(L42),A(L43),A(L43),MH,NJT4,NOTMP)
      CALL SECOND (T2)
C
      CALL FORCE (A(L1),A(L2),A(L3),A(L4),A(L5),A(L6),A(L7),A(L8),A(L9),
     1   A(L10),A(L11),A(L12),A(L13),A(L14),A(L15),A(L17),A(L18),A(L19),
     2   A(L20),A(L21),A(L22),A(L23),A(L24),A(N25),A(N26),A(N27),A(N28),
     3   A(N29),A(N30),A(N31),A(N32),A(N33),A(N34),A(N34),A(N35),NTP,MH,
     4   NOTMP)
      CALL SECOND(T3)
      T1=T2-T1
      T2=T3-T2
      T3=T1+T2
      PRINT 2008, T1,T2,T3
      GO TO 1
C
C     FORMAT STATEMENTS
C
 1000 FORMAT(8A10)
 1001 FORMAT(2F10.0,15I4)
 2000 FORMAT(1H1,20X,8A10)
 2002 FORMAT(//37H ANGLE BETWEEN END SUPPORTS (DEGR.) =F10.5/
     1 25H RADIUS OF REFERENCE LINE,11X,1H=F10.2/22H NUMBER OF PLATE TYP
     2ES,14X,1H= I5/19H NUMBER OF ELEMENTS,17X,1H=I5/17H NUMBER OF JOINTS
     319X,1H=I5/26H NUMBER OF OUTPUT SECTIONS,10X,1H=I5/29H MAXIMUM FOUR
     4IER SERIES LIMIT,7X,1H=I5/15H HALF BANDWIDTH,21X,1H=I5/37H NUMBER
     5OF PARTIAL SURFACE LOADS    =I5/37H NUMBER OF PARTIAL JOINT LOADS
     6     =I5/26H MATERIAL OPTION INDICATOR,10X,1H=I5)
 2003 FORMAT(//38H   CALCULATIONS SKIP ALL ODD HARMONICS)
 2004 FORMAT(//36H   NUMBER OF HARMONICS SET EQUAL TO ,I4)
 2005 FORMAT(//39H   CALCULATIONS SKIP ALL EVEN HARMONICS)
 2006 FORMAT(//49H   TRANSVERSE SECTIONS DEFINED IN ANGULAR DEGREES)
 2007 FORMAT(//45H   TRANSVERSE SECTICNS DEFINED IN ARC LENGTHS)
 2008 FORMAT(/////34H  EXECUTION TIMES FOR THIS PROBLEM//,21H  SUBROUTIN
     1E MAIN   =F10.3,9H  SECONDS/21H  SUBROUTINE FORCE  =F10.3,9H  SECO
     2NDS//7H  TOTAL,13X,1H=F10.3,9H  SECONDS)
 2009 FORMAT(//46H   AXISYMMETRIC CASE  -  ZERO-TH HARMONIC ONLY)
 2010 FORMAT(37H NO. OF SECTIONS FOR GIRDER MOMENTS =I5/
     1        37H NUMBER OF GIRDERS IN CROSS SECTION =I5)
C
  999 STOP
      END
```

```
      SUBROUTINE MAIN (NPI,NPJ,KPL,NSEC,PWTH,SINEL,COSEL,Y,Z,SINKX,
     1   COSKX,XP,TP,IRES,DL,NGIEL,HLI,HLJ,DNAI,VLI,DNAJ,VLJ,XDIV,T,
     2   MOPT,GIRMOM,TENS,COMP,LEL,PHLI,PHLJ,PVLI,PVLJ,SURT,SURL,LJT,
     3   FH,FV,FM,FP,FTL,FTT,AJF,AJP,LCASE,LIND,RJDIS,PTOT,DISP,BIGK,
     4   EDP,MH,NT4,NMP)
C
C***********************************************************************
C     THIS SUBROUTINE READS AND PRINTS ALL INPUT DATA, FORMS THE
C     STRUCTURE STIFFNESS AND LOAD VECTOR AND SOLVES THE RESULTING
C     EQUATIONS FOR ONE HARMONIC AT A TIME, AND PRINTS OUT THE FINAL
C     JOINT DISPLACEMENTS.
C***********************************************************************
C
C     COMMON, DIMENSION, AND EQUIVALENCE STATEMENTS
C
      COMMON/SETUP/TETAO,R,NPL,NEL,NJT,NTP,MHARM,NCHECK,INTRES,IO,MB,
     1   MCHECK,MM,N1,N2,PI,NNM,NXBAND,II,IJ,IL,IAX,NSURL,NCONL,NOTMP,
     2   NGIR,MI
      COMMON/PROPT/THM(30),THB(30),ETM(30),ETB(30),ESM(30),ESB(30),
     1   GM(30),GB(30),PRM(30),PRB(30)
      COMMON/STIFF/SMALLK(8,8)
      DIMENSION NPI(1),NPJ(1),KPL(1),NSEC(1),PWTH(1),SINEL(1),COSEL(1),
     1   Y(1),Z(1),SINKX(MH,1),COSKX(MH,1),XP(1),TP(1),IRES(1),DL(1),
     2   NGIEL(2,1),HLI(1),HLJ(1),DNAI(1),VLI(1),DNAJ(1),VLJ(1),XDIV(1),
     3   T(1),MOPT(1),GIRMOM(NMP,1),TENS(NMP,1),COMP(NMP,1),LEL(1),
     4   PHLI(1),PHLJ(1),PVLI(1),PVLJ(1),SURT(1),SURL(1),LJT(1),FH(1),
     5   FV(1),FM(1),FP(1),FTL(1),FTT(1),AJF(4,1),AJP(1),LCASE(4,1),
     6   LIND(1),RJDIS(NT4,1),PTOT(1),DISP(1),BIGK(NT4,1),EDP(1),NQ(2),
     7   D11(30),D12(30),D22(30),D33(30),D44(30),D45(30),D55(30),D66(30)
C
C     EQUIVALENCED ARRAYS HAVING SAME FIRST WORD ADDRESS IN BLANK COMMON
C     (NGIEL(1,1),DL),(NGIEL(1,NEL/2),HLI),(HLJ,DNAI),(VLI,DNAJ),
C     (VLJ,XDIV),(AJF,AJP),(LCASE,LIND),(PTOT,DISP),(BIGK,EDP)
      EQUIVALENCE (THM,D11),(THB,D12),(ETM,D22),(ETB,D33),(ESM,D44),
     1            (ESB,D45),(GM ,D55),(GB ,D66)
C
      LOGICAL IO
C
C     READ AND PRINT INPUT DATA
C
      READ 1002, (XP(I), I=1,NTP)
      IF(.NOT.IO) PRINT 2005
      IF(IO) PRINT 2006
      PRINT 2004, (XP(I), I=1,NTP)
C
      IF(INTRES.EQ.0) GO TO 45
      READ 1003, (IRES(I), I=1,INTRES)
      PRINT 2007, (IRES(I), I=1,INTRES)
   45 IRES(INTRES+1)=0
C
      IF(MI.EQ.1) GO TO 47
      DO 46 J=1,NPL
      READ 1007, I,THM(I),ETM(I),ESM(I),GM(I),PRM(I)
   46 READ 1005, THB(I),ETB(I),ESB(I),GB(I),PRB(I)
```

```
      PRINT 2008
      PRINT 2009, (I,THM(I),ETM(I),ESM(I),GM(I),PRM(I),
     1            THB(I),ETB(I),ESB(I),GB(I),PRB(I), I=1,NPL)
      GO TO 49
   47 DO 48 J=1,NPL
      READ 1009, I,D11(I),D12(I),D22(I),D33(I)
   48 READ 1005,   D44(I),D45(I),D55(I),D66(I)
      PRINT 2002
      PRINT 2003, (I,D11(I),D12(I),D22(I),D33(I),D44(I),D55(I),D66(I),
     1   I=1,NPL)
C
   49 READ 1006, (I,NPI(I),NPJ(I),KPL(I),NSEC(I),DL(I),
     1            HLI(I),HLJ(I),VLI(I),VLJ(I), J=1,NEL)
      PRINT 2010
      PRINT 2011, (I,NPI(I),NPJ(I),KPL(I),NSEC(I),DL(I),
     1             HLI(I),HLJ(I),VLI(I),VLJ(I), I=1,NEL)
C
      DO 50 L=1,NJT
   50 READ 1007, I,Y(I),Z(I),(AJF(J,I),J=1,4),(LCASE(K,I),K=1,4)
      PRINT 2012
      DO 55 I=1,NJT
      PRINT 2013, I,Y(I),Z(I),(AJF(J,I),LCASE(J,I),J=1,4)
   55 CONTINUE
      PRINT 2014
C
      IF(NSURL.LE.0) GO TO 60
      READ 1007, (LEL(I),PHLI(I),PHLJ(I),PVLI(I),PVLJ(I),
     1            SURT(I),SURL(I), I=1,NSURL)
      PRINT 2015
      PRINT 2016, (LEL(I),PHLI(I),PHLJ(I),PVLI(I),PVLJ(I),
     1             SURT(I),SURL(I), I=1,NSURL)
C
   60 IF(NCONL.LE.0) GO TO 65
      DO 62 I=1,NCONL
   62 READ 1007, LJT(I),FH(I),FV(I),FM(I),FP(I),FTL(I),FTT(I)
      PRINT 2017
      PRINT 2024, (LJT(I),FH(I),FV(I),FM(I),FP(I),FTL(I),FTT(I),
     1  I=1,NCONL)
C
C     OUTPUT IS CLEARED
C
   65 PI=3.14159265358979
      MX=4*NJT
      DO 115 I=1,MX
      DO 115 J=1,NTP
  115 RJDIS(I,J)=0.0
C
C     COMPUTE PLATE WIDTHS AND SIN AND COS OF INCLINATION ANGLES
C
      DO 120 I=1,NEL
      II=NPI(I)
      IJ=NPJ(I)
      HH=Y(IJ)-Y(II)
      VV=Z(IJ)-Z(II)
```

```
      PWTH(I)=SQRT(HH*HH+VV*VV)
      SINEL(I)=VV/PWTH(I)
  120 COSEL(I)=HH/PWTH(I)
C
C     CALCULATE JOINT RADII   Y(I)=Y(I)+R
C
      FAC=PI/180.
      TETAO=TETAO*FAC
      T2=0.5*TETAO
      DO 121 I=1,NJT
  121 Y(I)=Y(I)+R
C
C     CONVERT CIRCUMFERENTEIAL COORDINATES TO RADIAN
C
      S=1./R
      IF(IO) S=FAC
      DO 122 I=1,NTP
  122 TP(I)=XP(I)*S
      IF(NSURL.LE.0) GO TO 124
      DO 123 I=1,NSURL
      IK=LEL(I)
      II=NPI(IK)
      IJ=NPJ(IK)
      RR=(Y(II)+Y(IJ))/2.
      S=1./RR
      IF(IO) S=FAC
      SURT(I)=SURT(I)*S
  123 SURL(I)=SURL(I)*S
  124 IF(NCONL.LE.0) GO TO 126
      DO 125 I=1,NCONL
      II=LJT(I)
      S=1./Y(II)
      IF(IO) S=FAC
      FTL(I)=FTL(I)*S
  125 FTT(I)=FTT(I)*S
C
C     TRANSFORM SURFACE LOADS TO ELEMENT COORDINATES
C     AND CHECK FOR MAXIMUM BAND WIDTH
C
  126 NXBAND=0
      DO 130 I=1,NEL
      S=SINEL(I)
      C=COSEL(I)
      AS=ABS(S)
      AC=ABS(C)
      VV=VLI(I)*AC+DL(I)
      HH=HLI(I)*AS
      VLI(I)=S*HH-C*VV
      HLI(I)=C*HH+S*VV
      VV=VLJ(I)*AC+DL(I)
      HH=HLJ(I)*AS
      VLJ(I)=S*HH-C*VV
      HLJ(I)=C*HH+S*VV
      NPDIF=NPJ(I)-NPI(I)
```

```
      K=IABS(NPDIF)
      IF(K.GT.NXBAND) NXBAND=K
  130 CONTINUE
      NXBAND=NXBAND*4+4
      IF(NXBAND.LE.MB) GO TO 131
      PRINT 2034, NXBAND
      NNM=NNM+(NXBAND-MB)*MX
      CALL RFL(NNM)
C
C     TRANSFORM PARTIAL SURFACE LOADS TO ELEMENT COORDINATES
C
  131 IF(NSURL.LE.0) GO TO 135
      DO 133 I=1,NSURL
      K=LEL(I)
      S=SINEL(K)
      C=COSEL(K)
      AS=ABS(S)
      AC=ABS(C)
      VV=PVLI(I)*AC
      HH=PHLI(I)*AS
      PVLI(I)=S*HH-C*VV
      PHLI(I)=C*HH+S*VV
      VV=PVLJ(I)*AC
      HH=PHLJ(I)*AS
      PVLJ(I)=S*HH-C*VV
  133 PHLJ(I)=C*HH+S*VV
C
C     MODIFY LCASE (=LIND) MATRIX
C
  135 DO 136 I=1,MX
  136 LIND(I)=LIND(I)+1
C
C     CYCLE FOR EACH HARMONIC IS INITIATED
C
      REWIND 1
      IF(NCHECK) 140,141,142
  140 N1=2
      GO TO 143
  141 N1=1
      N2=1
      IF(IAX.EQ.0) MHARM=1
      GO TO 144
  142 N1=1
  143 N2=2
  144 MM=0
C
      DO 700 NN=N1,MHARM,N2
      MM=MM+1
      DO 145 I=1,MX
      DO 145 J=1,NXBAND
  145 BIGK(I,J)=0.0
      IF(IAX.EQ.0) GO TO 154
      FN=NN
      FK=FN*PI/TETAO
```

```
C
C     HARMONIC AND FOURIER MULTIPLIERS ARE COMPUTED
C
      DO 150 I=1,NTP
      XX=FK*TP(I)
      SINKX(MM,I)=SIN(XX)
  150 COSKX(MM,I)=COS(XX)
      N3=(-1)**NN
      S1=4./(FN*PI)
      S2=(-1.)**((NN+3)/2)
      GO TO 155
  154 NN=0
      N3=0
      S1=1.0
      S2=1.0
C
C     CALCULATE ELEMENT STIFFNESSES AND STORE THEM INTO BIGK
C
  155 DO 210 IE=1,NEL
      IJ=KPL(IE)
      I=NPI(IE)
      J=NPJ(IE)
      R1=Y(I)
      R2=Y(J)
      S=SINEL(IE)
      C=COSEL(IE)
      CALL CONE (IJ,R1,R2,S,C,PWTH(IE),NN)
C
      NQ(1)=I
      NQ(2)=J
      DO 170 L=1,2
      LL=4*NQ(L)-4
      LV=4*L-4
      DO 170 K=1,2
      KK=4*NQ(K)-4
      IF(KK.LT.LL) GO TO 170
      KH=4*K-4
      DO 160 II=1,4
      LLI=LL+II
      LVI=LV+II
      DO 160 JJ=1,4
      KKJ=KK+JJ
      IF(KKJ.LT.LLI) GO TO 160
      KKJ=KKJ-LLI+1
      KHJ=KH+JJ
      BIGK(LLI,KKJ)=BIGK(LLI,KKJ)+SMALLK(LVI,KHJ)
  160 CONTINUE
  170 CONTINUE
  210 CONTINUE
C
C     SET UP LOAD VECTOR
C
      DO 215 I=1,MX
  215 PTOT(I)=0.0
```

```
      IF(N3.GT.0) GO TO 221
C
C     FIND CONSISTENT NODAL LOADS FOR UNIFORM PLATE FORCES
C     AND STORE THESE INTO LOAD VECTOR PTOT
C
      DO 220 I=1,NEL
      HI=HLI(I)*S1
      HJ=HLJ(I)*S1
      VI=VLI(I)*S1
      VJ=VLJ(I)*S1
      IF(HI.EQ.0..AND.HJ.EQ.0..AND.VI.EQ.0..AND.VJ.EQ.0.) GO TO 220
      II=NPI(I)
      IJ=NPJ(I)
      R1=Y(II)
      R2=Y(IJ)
      S12=PWTH(I)
      S=SINEL(I)
      C=COSEL(I)
      CALL LOADS (II,IJ,HI,HJ,VI,VJ,R1,R2,S12,S,C,PTOT,TETA0)
  220 CONTINUE
C
C     ADD CONTRIBUTIONS DUE TO PARTIAL SURFACE LOADS
C
  221 IF(NSURL.LE.0) GO TO 231
      DO 230 I=1,NSURL
      L=LEL(I)
      II=NPI(L)
      IJ=NPJ(L)
      R1=Y(II)
      R2=Y(IJ)
      IF(SURL(I).EQ.0.) GO TO 223
      C1=S1*SIN(FK*SURT(I))*SIN(.5*FK*SURL(I))
      C2=C1
      GO TO 225
  223 IF(SURT(I).EQ.T2) GO TO 224
      C=SIN(FK*SURT(I))/T2
      C1=C/R1
      C2=C/R2
      GO TO 225
  224 IF(N3.GT.0) GO TO 230
      C1=S2/(R1*T2)
      C2=S2/(R2*T2)
  225 HI=PHLI(I)*C1
      HJ=PHLJ(I)*C2
      VI=PVLI(I)*C1
      VJ=PVLJ(I)*C2
      S12=PWTH(L)
      S=SINEL(L)
      C=COSEL(L)
      CALL LOADS (II,IJ,HI,HJ,VI,VJ,R1,R2,S12,S,C,PTOT,TETA0)
  230 CONTINUE
C
C     ADD JOINT LOADS INTO PTOT
C
```

```
  231 IF(N3.GT.0) GO TO 241
      I=0
      DO 238 J=1,NJT
      R1=Y(J)
      XX=R1*T2
      DO 238 L=1,4
      I=I+1
      K=LIND(I)
      GO TO (238,234,235,238,236), K
  234 PTOT(I)=PTOT(I)+AJP(I)*S1*XX
      GO TO 238
  235 PTOT(I)=PTOT(I)+AJP(I)*S2
      GO TO 238
  236 PTOT(I)=PTOT(I)+AJP(I)*2.
  238 CONTINUE
C
  241 IF(NCONL.LE.0) GO TO 251
      DO 250 I=1,NCONL
      L=LJT(I)
      J=L*4-4
      R1=Y(L)
      C=FK*FTL(I)
      IF(FTT(I).LE.0.) GO TO 244
      XX=FK*FTT(I)/2.
      EQH=S1*R1*T2*SIN(XX)
      EQS=EQH*COS(C)
      EQH=EQH*SIN(C)
      GO TO 245
  244 EQH=SIN(C)
      EQS=COS(C)
  245 PTOT(J+1)=PTOT(J+1)+EQH*FH(I)
      PTOT(J+2)=PTOT(J+2)+EQH*FV(I)
      PTOT(J+3)=PTOT(J+3)+EQH*FM(I)
  250 PTOT(J+4)=PTOT(J+4)+EQS*FP(I)
C
C     MODIFY BIGK AND PTOT MATRICES DUE TO BOUNDARY CONDITIONS
C
  251 DO 260 J=1,NJT
      DO 260 I=1,4
      IF(LCASE(I,J).NE.4) GO TO 260
      IL=J*4-4+I
      IJ=IL-NXBAND+1
      IF(IJ.LT.1) IJ=1
      DO 253 L=IJ,IL
      K=IL-L+1
  253 BIGK(L,K)=0.0
      DO 255 L=2,NXBAND
  255 BIGK(IL,L)=0.0
      PTOT(IL)=0.0
  260 CONTINUE
C
C     SOLVE EQUATIONS FOR UNKNOWN JOINT DISPLACEMENTS
C
      II=NXBAND+1
```

```
      CALL BANSOL (BIGK,PTOT,BIGK(1,II),MX,NXBAND,1)
      DO 510 II=1,NTP
      IF(IAX.GT.0) GO TO 270
      S=1.0
      C=0.0
      GO TO 275
  270 C=COSKX(MM,II)
      S=SINKX(MM,II)
  275 DO 510 L=4,MX,4
      I=L-3
      J=L-1
      DO 505 K=I,J
  505 RJDIS(K,II)=RJDIS(K,II)+DISP(K)*S
  510 RJDIS(L,II)=RJDIS(L,II)+DISP(L)*C
C
C     CALCULATE EDGE DISPLACEMENTS FOR EACH ELEMENT AND STORE ON TAPE 1
C
      N=0
      DO 600 L=1,NEL
      K=KPL(L)
      I=NPI(L)*4-4
      J=NPJ(L)*4-4
      C=COSEL(L)
      S=SINEL(L)
      EDP(N+1)=DISP(I+4)
      EDP(N+2)=DISP(J+4)
      EDP(N+3)=DISP(I+1)*C+DISP(I+2)*S
      EDP(N+4)=DISP(J+1)*C+DISP(J+2)*S
      EDP(N+5)=DISP(I+1)*S-DISP(I+2)*C
      EDP(N+6)=DISP(J+1)*S-DISP(J+2)*C
      EDP(N+7)=-DISP(I+3)
      EDP(N+8)=-DISP(J+3)
  600 N=N+8
      WRITE (1) (EDP(I), I=1,N)
      IF(NN.EQ.0) NN=1
  700 CONTINUE
C
C     READ AND PRINT GIRDER MOMENT DATA
C
      IF(MCHECK.EQ.0) GO TO 709
      READ 1002, (T(I), I=1,NOTMP)
      READ 1008, (I,NGIEL(1,I),NGIEL(2,I),DNAI(I),DNAJ(I),XDIV(I),
     1             J=1,NEL)
      PRINT 2019
      PRINT 2004, (T(I), I=1,NCTMP)
C
      DO 705 I=1,NOTMP
      DO 702 J=1,NTP
      IF(T(I).NE.XP(J)) GO TO 702
      MOPT(I)=J
      GO TO 705
  702 CONTINUE
      PRINT 2022, T(I)
      MOPT(I)=0
```

```
  705 CONTINUE
C
      PRINT 2020
      PRINT 2021, (I,NGIEL(1,I),NGIEL(2,I),DNAI(I),DNAJ(I),XDIV(I),
     1             I=1,NEL)
      DO 708 I=1,NOTMP
      DO 708 J=1,NGIR
      GIRMOM(I,J)=0.0
      TENS(I,J)=0.0
  708 COMP(I,J)=0.0
C
C     PRINT RESULTS FOR JOINT DISPLACEMENTS
C
  709 DO 710 I=1,NJT
      J=4*I
      LIND(J)=I
      LIND(J-1)=I
      LIND(J-2)=I
  710 LIND(J-3)=I
      IF(NTP.GT.7) GO TO 721
      II=NTP
      IL=1
      GO TO 730
  721 II=7
      IJ=NTP
      IL=(NTP-1)/7+1
  730 PRINT 2030
      CALL OPRINT (RJDIS,MX,NX,XP,NX,0,II,IJ,IL,1,2,LIND,IO)
      PRINT 2031
      CALL OPRINT (RJDIS,MX,NX,XP,NX,0,II,IJ,IL,2,2,LIND,IO)
      PRINT 2032
      CALL OPRINT (RJDIS,MX,NX,XP,NX,0,II,IJ,IL,3,2,LIND,IO)
      PRINT 2033
      CALL OPRINT (RJDIS,MX,NX,XP,NX,0,II,IJ,IL,4,2,LIND,IO)
C
C     FORMAT STATEMENTS
C
 1002 FORMAT(10F7.2)
 1003 FORMAT(20I4)
 1005 FORMAT(10X,5F10.0)
 1006 FORMAT(5I4,5F10.0)
 1007 FORMAT(I10,6F10.0,4I2)
 1008 FORMAT(3I5,3F10.0)
 1009 FORMAT(I10,4F10.0)
C
 2002 FORMAT(1H1,50X,20H PLATE ELEMENT TYPES//120HNUMBER        D11
    1      D12           D22           D33           D44           D45
    2           D55           D66      /)
 2003 FORMAT(I6,2X,8E14.6)
 2004 FORMAT(7F12.5)
 2005 FORMAT(////42H PRINT RESULTS AT SECTIONS WITH X EQUAL TO/)
 2006 FORMAT(////46H PRINT RESULTS AT SECTIONS WITH THETA EQUAL TO/)
 2007 FORMAT(////40H PRINT INTERMEDIATE RESULTS AT HARMONICS/20I5)
 2008 FORMAT(1H1,50X,20H PLATE ELEMENT TYPES//14X,20H MEMBRANE PROPERTIE
```

```
     1S,35X,25H PLATE BENDING PROPERTIES/120H     NO.       TH        E-
     2T        E-S         G           NU          TH        E-T       E-
     3S          G         NU     /)
 2009 FORMAT(I7,3X,10E11.4)
 2010 FORMAT(1H2,52X,15H PLATE ELEMENTS//10X,100H  ELE NO    NODE I     N
     1ODE J  PLATE TYPE   NSEC         DL        HLI        HLJ       VLI
     2     VLJ      /)
 2011 FORMAT(6X,5I10,5F10.3)
 2012 FORMAT(39H2INPUT LOADS OR DISPLACEMENTS AT JOINTS//106H JOINT   Y-
     1COORD   Z-COORD      HORIZONTAL IH         VERTICAL IV       ROTA
     2TIONAL IM     LONGITUDINAL IS/)
 2013 FORMAT(I6,2F10.2,4(E17.6,I3))
 2014 FORMAT(//37H IH,IV,IM,IS = 0 FOR GIVEN ZERO FORCE/14X,30H 1 FOR UN
     1IF. DISTRIBUTED FORCE/14X,46H 2 MEANS CONC. FORCE AT MIDSPAN FOR I
     2H, IV, IM/14X,30H 3 FOR GIVEN ZERO DISPLACEMENT/14X,19H 4 PRESTRES
     3S FOR IS)
 2015 FORMAT(22H1PARTIAL SURFACE LOADS//71H ELE      HLI       HLJ
     1 VLI        VLJ    CENTER COORD  LOAD WIDTH)
 2016 FORMAT(I3,4F10.3,2F12.3)
 2017 FORMAT(20H1PARTIAL JOINT LOADS//102H JOINT          H-LOAD
     1  V-LOAD            MOMENT      LONG. FORCE      CENTER COORD    LOAD
     2 WIDTH)
 2019 FORMAT(24H1PRINT GIRDER MOMENTS AT/)
 2020 FORMAT(////73H ELEMENT     1ST GIRDER   2ND GIRDER       DNAI
     1     DNAJ          XDIV//)
 2021 FORMAT(I5,2I13,3X,3F13.4)
 2022 FORMAT(////15H SECTION COORD.F7.3,59H DOES NOT BELONG TO OUTPUT SE
     1CTIONS AND WILL BE DISREGARDED)
 2023 FORMAT(I6,6F16.3)
 2024 FORMAT(I4,6F16.3)
 2030 FORMAT(26H1FINAL JOINT DISPLACEMENTS////10X,25H HORIZONTAL DISPLAC
     1EMENTS)
 2031 FORMAT(////10X,23H VERTICAL DISPLACEMENTS)
 2032 FORMAT(////10X,10H ROTATIONS)
 2033 FORMAT(////10X,27H LONGITUDINAL DISPLACEMENTS)
 2034 FORMAT(///41H INPUT BANDWIDTH TOO SMALL - TRUE VALUE =,I5/
     1 33H FIELD LENGTH RESET AUTOMATICALLY)
C
      RETURN
      END
```

```
      SUBROUTINE FORCE (NPI,NPJ,KPL,NSEC,PWTH,SINEL,COSEL,Y,Z,SINKX,
     1   COSKX,XP,TP,IRES,NGIEL,DNAI,DNAJ,XDIV,T,MOPT,GIRMOM,TENS,COMP,
     2   SM,TM,STM,SN,TN,STN,U,V,W,DI,DIS,D,NX,MH,NMP)
C
C***********************************************************************
C     THIS SUBROUTINE READS ELEMENT EDGE DISPLACEMENTS FROM TAPE 1 AND
C     CALCULATES INTERNAL FORCES FOR ONE PLATE ELEMENT AT A TIME,SUMMING
C     UP THE SINGLE HARMONIC CONTRIBUTIONS, AND PRINTS THEM OUT. MOMENTS
C     OF INDIVIDUAL GIRDERS ARE CALCULATED AND PRINTED IF SO DESIRED.
C***********************************************************************
C
C     COMMON, DIMENSION, AND EQUIVALENCE STATEMENTS
C
      COMMON/SETUP/TETAO,R,NPL,NEL,NJT,NTP,MHARM,NCHECK,INTRES,IO,MB,
     1   MCHECK,MM,N1,N2,PI,NNM,NXBAND,II,IJ,IL,IAX,NSURL,NCONL,NOTMP,
     2   NGIR,MI
      COMMON/PROPT/THM(30),THB(30),ETM(30),ETB(30),ESM(30),ESB(30),
     1   GM(30),GB(30),PRM(30),PRB(30)
      DIMENSION NPI(1),NPJ(1),KPL(1),NSEC(1),PWTH(1),SINEL(1),COSEL(1),
     1   Y(1),Z(1),SINKX(MH,1),COSKX(MH,1),XP(1),TP(1),IRES(1),
     2   NGIEL(2,1),DNAI(1),DNAJ(1),XDIV(1),T(1),MOPT(1),GIRMOM(NMP,1),
     3   TENS(NMP,1),COMP(NMP,1),SM(NX,1),TM(NX,1),STM(NX,1),SN(NX,1),
     4   TN(NX,1),STN(NX,1),U(NX,1),V(NX,1),W(NX,1),DI(1),DIS(8,1),
     5   D(8,1),DISP(8)
      LOGICAL IO
C
C     EQUIVALENCE (DI,DIS)   (SAME FIRST WORD ADDRESS IN BLANK COMMON)
C
C     READ EDGE DISPLACEMENTS FROM TAPE 1
C
      NOFPL=0
      NEL2=0
   30 NEL1=NEL2+1
      IF(NEL1.GT.NEL) GO TO 100
      NEL2=MINO((NEL1+14),NEL)
      NDI=NEL2*8
      REWIND 1
      L=0
      DO 35 I=1,MM
      READ (1) (DI(J), J=1,NDI)
      DO 35 J=NEL1,NEL2
      L=L+1
      DO 35 K=1,8
   35 D(K,L)=DIS(K,J)
C
C     FOR EACH ELEMENT
C
      NDI=NEL2-NEL1+1
      DO 99 IE=NEL1,NEL2
      FN=NSEC(IE)
      IF(FN.LE.0.) GO TO 99
      NUMY=NSEC(IE)+1
      IEPL=KPL(IE)
      XL=2./FN
```

```
C
C     INITIALIZE INTERNAL FORCE AND DISPLACEMENT ARRAYS
C
      J=[illegible]P 45*NTP
      DO 40 I=1,J
   40 SM(I)=0.0
C
      IF(IEPL.EQ.NOFPL) GO TO 45
      IF(MI.EQ.1) GO TO 43
      FM=PRM(IEPL)*ETM(IEPL)/ESM(IEPL)
      FB=PRB(IEPL)*ETB(IEPL)/ESB(IEPL)
      DM=1.-FM*PRM(IEPL)
      DB=1.-FB*PRB(IEPL)
      TH3=THB(IEPL)**3/12.
      D11=THM(IEPL)*ESM(IEPL)/DM
      D22=THM(IEPL)*ETM(IEPL)/DM
      D12=D22*PRM(IEPL)
      D33=GM(IEPL)*THM(IEPL)
      D44=TH3*ESB(IEPL)/DB
      D55=TH3*ETB(IEPL)/DB
      D45=D55*PRB(IEPL)
      D66=GB(IEPL)*TH3*4.0
      GO TO 45
   43 D11=THM(IEPL)
      D12=THB(IEPL)
      D22=ETM(IEPL)
      D33=ETB(IEPL)
      D44=ESM(IEPL)
      D45=ESB(IEPL)
      D55=GM(IEPL)
      D66=GB(IEPL)
C
   45 S12=PWTH(IE)
      S122=S12*S12
      I=NPI(IE)
      J=NPJ(IE)
      R1=Y(I)
      R2=Y(J)
      A=0.5*(R2-R1)
      B=0.5*(R2+R1)
      SP=SINEL(IE)
      CP=COSEL(IE)
C
C     FOR EACH HARMONIC
C
      N=0
      KJK=1
      DO 90 NN=N1,MHARM,N2
      N=N+1
      I=NDI*(N-1)+IE-NEL1+1
      DO 50 J=1,8
   50 DISP(J)=D(J,I)
      FN=NN
      FK=FN*PI/TETA0
```

```
C
C     FOR EACH TRANSVERSE SECTION
C
      DO 80 IY=1,NUMY
      FY=IY-1
      ETA=XL*FY-1.
      E2=ETA*ETA
      E3=E2*ETA
      PU1=0.5*(1.-ETA)
      PU2=0.5*(1.+ETA)
      PU3=-1./S12
      PU4=-PU3
      P1=PU1*DISP(1)+PU2*DISP(2)
      P2=PU1*DISP(3)+PU2*DISP(4)
      P3=PU3*DISP(1)+PU4*DISP(2)
      P4=PU3*DISP(3)+PU4*DISP(4)
      PU1=.25*(2.-3.*ETA+E3)
      PU2=.25*(2.+3.*ETA-E3)
      PU3=.125*S12*(1.-ETA-E2+E3)
      PU4=.125*S12*(-1.-ETA+E2+E3)
      XW=PU1*DISP(5)+PU2*DISP(6)
      P5=XW+PU3*DISP(7)+PU4*DISP(8)
      PU1=1.5/S12*(E2-1.)
      PU2=-PU1
      PU3=.25*(-1.-2.*ETA+3.*E2)
      PU4=.25*(-1.+2.*ETA+3.*E2)
      P6=PU1*DISP(5)+PU2*DISP(6)+PU3*DISP(7)+PU4*DISP(8)
      PU1=6.*ETA/S122
      PU2=-PU1
      PU3=(3.*ETA-1.)/S12
      PU4=(3.*ETA+1.)/S12
      P7=PU1*DISP(5)+PU2*DISP(6)+PU3*DISP(7)+PU4*DISP(8)
      RR=1./(A*ETA+B)
      SPR=SP*RR
      CPR=CP*RR
      BN=FK*RR
      IF(IAX.EQ.0) BN=0.0
C
      XX=-BN*P1+CPR*P2+SPR*P5
      XNS=D11*P4+D12*XX
      XNT=D12*P4+D22*XX
      XNST=(P3-CPR*P1+BN*P2)*D33
      XX=BN*(BN*P5-SPR*P1)-CPR*P6
      XMS=D45*XX-D44*P7
      XMT=D55*XX-D45*P7
      XMST=D66*(SPR*(P3-CPR*P1)+BN*(CPR*P5-P6))
C
C     SUM UP INTERNAL FORCES AND DISPLACEMENTS
C
      DO 75 I=1,NTP
      IF(IAX.GT.0) GO TO 60
      S=1.0
      C=0.0
      GO TO 65
```

```
   60 S=SINKX(N,I)
      C=COSKX(N,I)
   65 U(I,IY)=U(I,IY)+P1*C
      V(I,IY)=V(I,IY)+P2*S
      W(I,IY)=W(I,IY)+XW*S
      SN(I,IY)=SN(I,IY)+XNS*S
      TN(I,IY)=TN(I,IY)+XNT*S
      STN(I,IY)=STN(I,IY)+XNST*C
      SM(I,IY)=SM(I,IY)-XMS*S
      TM(I,IY)=TM(I,IY)-XMT*S
   75 STM(I,IY)=STM(I,IY)-XMST*C
   80 CONTINUE
C
C     PRINT INTERNAL FORCES FOR EACH ELEMENT
C
   81 I=IRES(KJK)
      IF(I.LE.0) GO TO 83
      IF(NN-I) 83,84,82
   82 KJK=KJK+1
      GO TO 81
   83 IF(NN.NE.MHARM) GO TO 90
   84 I=NPI(IE)
      J=NPJ(IE)
      PRINT 1000, IE,I,J,NN
      PRINT 1001
      CALL OPRINT  (SN,NX,5,XP,NX,NUMY,II,IJ,IL,0,1,IO)
      PRINT 1002
      CALL OPRINT  (TN,NX,5,XP,NX,NUMY,II,IJ,IL,0,1,IO)
      PRINT 1003
      CALL OPRINT (STN,NX,5,XP,NX,NUMY,II,IJ,IL,0,1,IO)
      PRINT 1004
      CALL OPRINT  (SM,NX,5,XP,NX,NUMY,II,IJ,IL,0,1,IO)
      PRINT 1005
      CALL OPRINT  (TM,NX,5,XP,NX,NUMY,II,IJ,IL,0,1,IO)
      PRINT 1006
      CALL OPRINT (STM,NX,5,XP,NX,NUMY,II,IJ,IL,0,1,IO)
      PRINT 1007
      CALL OPRINT   (U,NX,5,XP,NX,NUMY,II,IJ,IL,0,1,IO)
      PRINT 1008
      CALL OPRINT   (V,NX,5,XP,NX,NUMY,II,IJ,IL,0,1,IO)
      PRINT 1009
      CALL OPRINT   (W,NX,5,XP,NX,NUMY,II,IJ,IL,0,1,IO)
C
C     CALCULATE GIRDER MOMENTS
C
      IF(MCHECK.LE.0) GO TO 90
      IF(NN.NE.MHARM) GO TO 90
      PLW=PWTH(IE)
      S=DNAI(IE)
      C=DNAJ(IE)
      HH=R2-R1
      VV=Z(I)-Z(J)
      I=NGIEL(1,IE)
      J=NGIEL(2,IE)
```

```
      XX=XDIV(IE)
      CALL MOMPER (TN,TM,PLW,    NUMY,I,J,S,C,HH,VV,XX,NTP,GIRMOM,TENS,
     1   COMP,MOPT,NOTMP)
   90 CONTINUE
      NOFPL=IEPL
   99 CONTINUE
      GO TO 30
C
C     CALCULATE GIRDER MOMENT PERCENTAGES
C
  100 IF(MCHECK.LE.0) GO TO 150
      PRINT 1015
      DO 140 I=1,NOTMP
      P1=0.0
      P2=0.0
      P3=0.0
      P4=0.0
      DO 110 J=1,NGIR
      P2=P2+GIRMOM(I,J)
      P3=P3+TENS(I,J)
  110 P4=P4+COMP(I,J)
      IF(P2.EQ.0.) GO TO 130
      PRINT 1010, T(I)
      DO 120 J=1,NGIR
      P5=GIRMOM(I,J)/P2*100.
      P1=P1+P5
  120 PRINT 1011, J,GIRMOM(I,J),P5,TENS(I,J),COMP(I,J)
      PRINT 1012, P2,P1,P3,P4
      GO TO 140
  130 PRINT 1014, T(I)
  140 PRINT 1016
C
C     FORMAT STATEMENTS
C
 1000 FORMAT(1H1,48H INTERNAL FORCES PER UNIT LENGTH FOR ELEMENT NO. I4,
     1 18H   BETWEEN JOINTS I3,6H  AND I3, 9H    AFTERI5,11H  HARMONICS)
 1001 FORMAT(////10X,5H N(S))
 1002 FORMAT(////10X,9H N(THETA))
 1003 FORMAT(////10X,11H N(S-THETA))
 1004 FORMAT(////10X,5H M(S))
 1005 FORMAT(////10X,9H M(THETA))
 1006 FORMAT(////10X,11H M(S-THETA))
 1007 FORMAT(////10X,2H U)
 1008 FORMAT(////10X,2H V)
 1009 FORMAT(////10X,2H W)
 1010 FORMAT(65H GIRDER MOMENTS AND AXIAL STRESS RESULTANTS AT SECTION
     1 X/THETA=F8.4//062H GIRDER NO.   MOMENT    PERCENTAGE     TENSION
     2    COMPRESSION/)
 1011 FORMAT(I6,E16.6,F9.2,2E16.6)
 1012 FORMAT(//6H TOTAL,E16.6,F9.2,2E16.6)
 1014 FORMAT(////28H STATICAL MOMENT AT X/THETA=F10.4,9H  IS ZERO)
 1015 FORMAT(1H1)
 1016 FORMAT(1H2)
C
```

```
150 RETURN
    END
```

```
      SUBROUTINE MOMPER (XN,XM,W,  NY,N1,N2,DI,DJ,H,V,XDIV,NX,GIRMOM,
     1   TENS,COMP,MOPT,NT)
C
C*********************************************************************
C     THIS SUBROUTINE STORES THE CONTRIBUTIONS OF ELEMENT I INTO THE
C     MOMENTS OF GIRDERS  N1  AND  N2  TO WHICH IT BELONGS.
C*********************************************************************
C
      DIMENSION XN(NX,1),XM(NX,1),GIRMOM(NT,1),TENS(NT,1),COMP(NT,1),
     1   MOPT(1)
C
      NSC=NY-1
      SC=NSC
      DEL=W/SC
      DEV=(DJ-DI)/SC
      AXDIV=ABS(XDIV)
      IF(DEV.EQ.0.) GO TO 5
      DEH=ABS(DEV*H/V)
      GO TO 7
    5 DEH=DEL
C
    7 DO 100 J=1,NT
      IT=MOPT(J)
      IF(IT.EQ.0) GO TO 100
      X1=DI
      IF(N2.GT.0) GO TO 20
C
C     ELEMENT CONTRIBUTES TO ONE GIRDER ONLY
C
      DO 10 NN=1,NSC
      X2=X1+DEV
      CALL ADDMOM (X1,X2,DEL,DEH,XN(IT,NN),XN(IT,NN+1),XM(IT,NN),
     1   XM(IT,NN+1),GIRMOM(J,N1),TENS(J,N1),COMP(J,N1))
   10 X1=X2
      GO TO 100
C
C     STORE ELEMENT CONTRIBUTION TO FIRST OF TWO GIRDERS
C
   20 NN=1
      HH=0.
   30 HH=HH+DEH
      IF(HH.GT.AXDIV) GO TO 40
      X2=X1+DEV
      CALL ADDMOM (X1,X2,DEL,DEH,XN(IT,NN),XN(IT,NN+1),XM(IT,NN),
     1   XM(IT,NN+1),GIRMOM(J,N1),TENS(J,N1),COMP(J,N1))
      X1=X2
      NN=NN+1
      GO TO 30
C
   40 FA=(XDIV+DEH-HH)/DEH
      XL=FA*DEL
      XH=FA*DEH
      X2=X1+FA*DEV
      XN2=XN(IT,NN)+FA*(XN(IT,NN+1)-XN(IT,NN))
```

```
      XM2=XM(IT,NN)+FA*(XM(IT,NN+1)-XM(IT,NN))
      CALL ADDMOM (X1,X2,XL,XH,XN(IT,NN),XN2,XM(IT,NN),XM2,
     1   GIRMOM(J,N1),TENS(J,N1),COMP(J,N1))
C
C     STORE ELEMENT CONTRIBUTICN TO SECOND OF TWO GIRDERS
C
      X3=X1+DEV
      XL=DEL-XL
      XH=DEH-XH
      CALL ADDMOM (X2,X3,XL,XH,XN2,XN(IT,NN+1),XM2,XM(IT,NN+1),
     1   GIRMOM(J,N2),TENS(J,N2),COMP(J,N2))
C
      X1=X3
   50 NN=NN+1
      IF(NN.GT.NSC) GO TO 100
      X2=X1+DEV
      CALL ADDMOM (X1,X2,DEL,DEH,XN(IT,NN),XN(IT,NN+1),XM(IT,NN),
     1   XM(IT,NN+1),GIRMOM(J,N2),TENS(J,N2),COMP(J,N2))
      X1=X2
      GO TO 50
  100 CONTINUE
C
      RETURN
      END
```

```
      SUBROUTINE ADDMOM      (X1,X2,XL,XH,XN1,XN2,XM1,XM2,G,T,C)
C
      F1=.5*XN1*XL
      F2=.5*XN2*XL
      XM=(F1*(X2+2.*X1)+F2*(X1+2.*X2))/3.
      F=F1+F2
      XM=XM+.5*(XM1+XM2)*XH
      G=G+XM
      IF(F.LT.0.) GO TO 10
      T=T+F
      GO TO 20
   10 C=C+F
   20 RETURN
      END
```

```
      SUBROUTINE CONE (NT,R1,R2,S,C,S12,N)
C
C***********************************************************************
C     THIS SUBROUTINE CALCULATES THE GLOBAL ELEMENT STIFFNESS OF A
C     THIN SHELL CONICAL SEGMENT, USING LINEAR IN-PLANE AND CUBIC
C     OUT-OF-PLANE DISPLACEMENT FUNCTIONS.
C     BASED ON NOVOZHILOV-S STRAIN-DISPLACEMENT RELATIONS
C
C                               -  INPUT  -
C
C     THM,ESM,ETM,GM,PRM - MATERIAL CONSTANTS FOR MEMBRANE BEHAVIOR
C     THB,ESB,ETB,GB,PRB - MATERIAL CONSTANTS FOR SHELL BENDING BEHAVIOR
C                          IF  MI=1 THEN  THM=D11, THB=D12, ESM=D22,
C                          ESB=D33, ETM=D44, ETB=D45, GM=D55, GB=D66
C                          ARE THE ELEMENTS OF THE CONSTITUTIVE MATRIX
C     R1, R2             - RADII OF CURVATURE OF JOINT 1 AND 2
C     S, C               - SINE AND COSINE OF INCLINATION ANGLE
C     S12                - ELEMENT WIDTH BETWEEN JOINT 1 AND 2
C     N                  - HARMONIC NUMBER
C     TETA0              - SEGMENT ANGLE (IN RADIAN)
C
C                               -  OUTPUT  -
C
C     T(8,8)             - GLOBAL ELEMENT STIFFNESS
C***********************************************************************
C
      COMMON/SETUP/TETA0,RR,NPL,NEL,NJT,NTP,MHARM,NCHECK,INTRES,IO,MB,
     1   MCHECK,MM,N1,N2,PI,NNM,NXBAND,II,IJ,IL,IAX,NSURL,NCONL,NOTMP,
     2   NGIR,MI
      COMMON/PROPT/THM(30),THB(30),ETM(30),ETB(30),ESM(30),ESB(30),
     1   GM(30),GB(30),PRM(30),PRB(30)
      COMMON/STIFF/T(8,8)
      DIMENSION X(8),W(8),F(3,7)
      EQUIVALENCE (F(1,1),F11),(F(1,2),F12),(F(1,3),F13),(F(1,4),F14),
     1            (F(1,5),F15),(F(1,6),F16),(F(1,7),F17),(F(2,1),F21),
     2            (F(2,2),F22),(F(2,3),F23),(F(2,4),F24),(F(2,5),F25),
     3            (F(2,6),F26),(F(2,7),F27),(F(3,1),F31),(F(3,2),F32),
     4            (F(3,3),F33),(F(3,4),F34),(F(3,5),F35),(F(3,6),F36),
     5            (F(3,7),F37)
      DATA X/
     1 .183434642495650,-.183434642495650, .525532409916329,
     2-.525532409916329, .796666477413627,-.796666477413627,
     3 .960289856497536,-.960289856497536/, W/
     4 .362683783378362, .362683783378362, .313706645877887,
     5 .313706645877887, .222381034453374, .222381034453374,
     6 .101228536290376, .101228536290376/
C
C     SET UP MATRIX OF MATERIAL CONSTANTS
C
      IF(MI.EQ.1) GO TO 1
      FM=PRM(NT)*ETM(NT)/ESM(NT)
      FB=PRB(NT)*ETB(NT)/ESB(NT)
      DM=1./(1.-FM*PRM(NT))
      DB=1./(1.-FB*PRB(NT))
```

```
      TH3=THB(NT)**3/12.
      D11=THM(NT)*ESM(NT)*DM
      D22=THM(NT)*ETM(NT)*DM
      D12=D22*PRM(NT)
      D33=GM(NT)*THM(NT)
      D44=TH3*ESB(NT)*DB
      D55=TH3*ETB(NT)*DB
      D45=D55*PRM(NT)
      D66=GB(NT)*TH3*4.0
      GO TO 2
    1 D11=THM(NT)
      D12=THB(NT)
      D22=ETM(NT)
      D33=ETB(NT)
      D44=ESM(NT)
      D45=ESB(NT)
      D55=GM(NT)
      D66=GB(NT)
C
C     INITIALIZATION
C
    2 A=0.5*(R2-R1)
      B=0.5*(R2+R1)
      SS=S*S
      CC=C*C
      XN=N
      P=XN*PI/TETA0
      PP=P*P
      S2=0.5*S12
      S122=S12*S12
      S123=S12*S122
      S124=S122*S122
C
C     INTEGRALS
C
      F1=2.
      F3=2./3.
      F5=.4
      F7=2./7.
      F8=2.*B
      F9=F3*A
      F10=F3*B
      DO 3 I=1,3
      DO 3 J=1,7
    3 F(I,J)=0.0
      DO 4 K=1,8
      XX=X(K)
      R=A*XX+B
      WR=W(K)
      DO 4 I=1,3
      WR=WR/R
      WX=WR/XX
      DO 4 J=1,7
      WX=WX*XX
```

```
    4 F(I,J)=F(I,J)+WX
C
C     STIFFNESS COEFFICIENTS
C
      H1=.25*(D22*PP+D33*CC)
      H2=.25*(D55*PP+D66*CC)*SS
      H3=C*D33/S12
      H4=D66*C*SS/S12
      H5=D33/S122
      H6=D66*SS/S122
      G1=H5*F8+H6*F11
      G2=H3*F1
      G3=F11-2.*F12+F13
      G4=F11+2.*F12+F13
      G5=F11-F13
      T(1,1)=H1*G3+H2*(F31-2.*F32+F33)+G2-H4*(F22-F21)+G1
      T(1,2)=H1*G5+H2*(F31-F33)+H4*F22-G1
      T(2,2)=H1*G4+H2*(F31+2.*F32+F33)-G2-H4*(F21+F22)+G1
C
      H1=-.25*P*C*(D22+D33)
      H2=P/(2.*S12)
      G1=H2*(D33+D12)*F1
      G2=H2*(D33-D12)*F1
      T(1,3)=H1*G3-G2
      T(1,4)=H1*G5-G1
      T(2,3)=H1*G5+G1
      T(2,4)=H1*G4+G2
C
      H1=.25*(D22*CC+PP*D33)
      G1=C*D12/S12*F1
      G2=D11/S122*F8
      T(3,3)=H1*G3-G1+G2
      T(3,4)=H1*G5-G2
      T(4,4)=H1*G4+G1+G2
C
      H1=-.125*S*P*D22
      H2=.125*S*P*(D55*PP+D66*CC)
      U1=S*C*P/(4.*S12)
      U2=S*P/(2.*S122)
      H3=U1*(D55+D66)
      H4=U2*D45
      H5=U1*D66
      H6=U2*D66
      G1=2.*F11-5.*F12+3.*F13+F14-F15
      G2=2.*F11+F12-3.*F13-F14+F15
      G3=F11-2.*F12+2.*F14-F15
      G4=-F11+2.*F13-F15
      G5=2.*F11-F12-3.*F13+F14+F15
      G6=2.*F11+5.*F12+3.*F13-F14-F15
      G7=-F11-2.*F12+2.*F14+F15
      G8=-F21+F22+F23-F24
      G9=-F21-F22+F23+F24
      T1=H1*S2
      T2=H2*S2
```

```
      T3=H3*S2
      T4=H4*S2
      T5=H5*S2
      T6=H6*S2
      U1=3.*H6*(F11-F13)
      U2=6.*H4*(F12-F13)+3.*H3*G8
      U3=6.*H4*(F12+F13)+3.*H3*G9
      U4=T5*G8-T6*(F11+2.*F12-3.*F13)
      U5=T5*G9+T6*(F11-2.*F12-3.*F13)
      T(1,5)=H1*G1-H2*(2.*F31-5.*F32+3.*F33+F34-F35)+U2+H5*(-2.*F21
     1          +3.*F22-F24)-U1
      T(1,6)=H1*G2-H2*(2.*F31+F32-3.*F33-F34+F35)-U2+H5*(-2.*F21
     1          -3.*F22+F24)+U1
      T(1,7)=T1*G3-T2*(F31-2.*F32+2.*F34-F35)+T3*(-F21-F22+5.*F23
     1          -3.*F24)+T4*(-2.*F11+8.*F12-6.*F13)+U4
      T(1,8)=T1*G4-T2*(-F31+2.*F33-F35)+T3*(-F21+3.*F22+F23-3.*F24)
     1          +T4*(2.*F11+4.*F12-6.*F13)-U5
      T(2,5)=H1*G5-H2*(2.*F31-F32-3.*F33+F34+F35)+U3+H5*(2.*F21-3.*F22
     1          +F24)+U1
      T(2,6)=H1*G6-H2*(2.*F31+5.*F32+3.*F33-F34-F35)-U3+H5*(2.*F21
     1          +3.*F22-F24)-U1
      T(2,7)=-T1*G4-T2*(F31-2.*F33+F35)+T3*(-F21-3.*F22+F23+3.*F24)
     1          +T4*(-2.*F11+4.*F12+6.*F13)-U4
      T(2,8)=T1*G7-T2*(-F31-2.*F32+2.*F34+F35)+T3*(-F21+F22+5.*F23
     1          +3.*F24)+T4*(2.*F11+8.*F12+6.*F13)+U5
C
      H1=.125*S*C*D22
      H2=S*D12/(4.*S12)
      T1=H1*S2
      T2=H2*S2
      G8=2.*H2*F1
      G9=T2*(F1-F3)
      T(3,5)=H1*G1-G8
      T(3,6)=H1*G2-G8
      T(3,7)=T1*G3-G9
      T(3,8)=T1*G4+G9
      T(4,5)=H1*G5+G8
      T(4,6)=H1*G6+G8
      T(4,7)=G9-T1*G4
      T(4,8)=T1*G7-G9
C
      H1=.0625*SS*D22
      H2=.0625*PP*(D55*PP+D66*CC)
      H3=C*PP*(D55+D66)/(8.*S12)
      H4=PP*D45/(4.*S122)
      H5=(CC*D55+PP*D66)/(4.*S122)
      H6=C*D45/(2.*S123)
      H7=D44/S124
      T1=H1*S2
      T2=H2*S2
      T3=H3*S2
      T4=H4*S2
      T5=H5*S2
      T6=H6*S2
```

```
      T7=H7*S2
      U1=T1*S2
      U2=T2*S2
      U3=T3*S2
      U4=T4*S2
      U5=T5*S2
      U6=T6*S2
      U7=T7*S2
      G1=6.*T6*(F1-3.*F3)
      G2=4.*U6*(F1-9.*F3)
      G3=H5*9.*(F11-2.*F13+F15)+36.*H7*F10
      G4=3.*F11-12.*F13+9.*F15
      G5=12.*T7*(F9-3.*F10)
      G6=12.*T7*(F9+3.*F10)
      G7=6.*(F12-F14)
      G8=F11-F13-F15+F17
      G9=F12-2.*F14+F16
      G10=2.*F11-5.*F13+4.*F15-F17
      G11=F31-F33-F35+F37
      G12=F32-2.*F34+F36
      G13=2.*F31-5.*F33+4.*F35-F37
      T(5,5)=H1*(4.*F11-12.*F12+9.*F13+4.*F14-6.*F15+F17)+H2*(4.*F31
     1        -12.*F32+9.*F33+4.*F34-6.*F35+F37)-H3*(-12.*F21+18.*F22
     2        +12.*F23-24.*F24+6.*F26)-H4*(24.*F12-36.*F13+12.*F15)+G3
      T(5,6)=H1*(4.*F11-9.*F13+6.*F15-F17)+H2*(4.*F31-9.*F33+6.*F35-F37)
     1        -H3*(-18.*F22+24.*F24-6.*F26)-H4*12.*(3.*F13-F15)-G3
      T(5,7)=T1*(2.*F11-5.*F12+F13+6.*F14-4.*F15-F16+F17)+T2*(2.*F31
     1        -5.*F32+F33+6.*F34-4.*F35-F36+F37)-T3*(-5.*F21+2.*F22
     2        +18.*F23-16.*F24-5.*F25+6.*F26)-T4*(-4.*F11+24.*F12
     3        -24.*F13-8.*F14+12.*F15)+T5*(G4+G7)+G1-G5
      T(5,8)=T1*(G9-G10)+T2*(G12-G13)-T3*(F21+10.*F22-6.*F23-16.*F24
     1    +5.*F25+6.*F26)-T4*(4.*F11-24.*F13+8.*F14+12.*F15)+T5*(G4
     2    -G7)-G1+G6
      T(6,6)=H1*(4.*F11+12.*F12+9.*F13-4.*F14-6.*F15+F17)+H2*(4.*F31
     1        +12.*F32+9.*F33-4.*F34-6.*F35+F37)-H3*(12.*F21+18.*F22
     2        -12.*F23-24.*F24+6.*F26)-12.*H4*(-2.*F12-3.*F13+F15)+G3
      T(6,7)=T1*(G9+G10)+T2*(G12+G13)-T3*(F21-10.*F22-6.*F23+16.*F24
     1    +5.*F25-6.*F26)-T4*(-4.*F11+24.*F13+8.*F14-12.*F15)-T5*(G4
     2    +G7)-G1+G5
      T(6,8)=T1*(-2.*F11-5.*F12-F13+6.*F14+4.*F15-F16-F17)+T2*(-2.*F31
     1        -5.*F32-F33+6.*F34+4.*F35-F36-F37)-T3*(-5.*F21-2.*F22
     2        +18.*F23+16.*F24-5.*F25-6.*F26)-T4*(4.*F11+24.*F12
     3        +24.*F13-8.*F14-12.*F15)+T5*(G7-G4)+G1-G6
      T(7,7)=U1*(G8-2.*G9)+U2*(G11-2.*G12)-U3*(-2.*F21-2.*F22+12.*F23
     1        -4.*F24-10.*F25+6.*F26)-U4*(-4.*F11+16.*F12-8.*F13
     2        -16.*F14+12.*F15)+U5*(F11+4.*F12-2.*F13-12.*F14+9.*F15)
     3        +G2+U7*(4.*F8-24.*F9+36.*F10)
      T(7,8)=U1*(-F11+3.*F13-3.*F15+F17)+U2*(-F31+3.*F33-3.*F35+F37)
     1        -U3*6.*(F22-2.*F24+F26)-U4*(4.*F11-16.*F13+12.*F15)
     2        +U5*(F11-10.*F13+9.*F15)+U7*(-4.*F8+36.*F10)
      T(8,8)=U1*(G8+2.*G9)+U2*(G11+2.*G12)-U3*(2.*F21-2.*F22-12.*F23
     1        -4.*F24+10.*F25+6.*F26)-U4*(-4.*F11-16.*F12-8.*F13
     2        +16.*F14+12.*F15)+U5*(F11-4.*F12-2.*F13+12.*F14+9.*F15)
     3        -G2+U7*(4.*F8+24.*F9+36.*F10)
```

```
C
      FAC=.25*TETA0*S12
      DO 9 I=1,8
      DO 9 J=I,8
      T(I,J)=FAC*T(I,J)
    9 T(J,I)=T(I,J)
C
C     TRANSFORMATION TO GLOBAL COORDINATES
C
      DO 10 I=1,8
      TX=T(I,1)
      T(I,1)=T(I,3)*C+T(I,5)*S
      TY=T(I,2)
      T(I,2)=T(I,3)*S-T(I,5)*C
      T(I,3)=-T(I,7)
      T(I,5)=T(I,4)*C+T(I,6)*S
      T(I,6)=T(I,4)*S-T(I,6)*C
      T(I,4)=TX
      T(I,7)=-T(I,8)
   10 T(I,8)=TY
      DO 20 I=1,8
      TX=T(1,I)
      T(1,I)=T(3,I)*C+T(5,I)*S
      TY=T(2,I)
      T(2,I)=T(3,I)*S-T(5,I)*C
      T(3,I)=-T(7,I)
      T(5,I)=T(4,I)*C+T(6,I)*S
      T(6,I)=T(4,I)*S-T(6,I)*C
      T(4,I)=TX
      T(7,I)=-T(8,I)
   20 T(8,I)=TY
      RETURN
      END
```

```
      SUBROUTINE BANSOL (A,B,NBL,NEQ,MBAND,KKK)
C
C*****************************************************************************
C      IN-CORE EQUATION SOLVER FOR BANDED, SYMMETRIC, POSITIVE DEFINITE
C      SYSTEMS, TAKING ACCOUNT OF VARIABLE BAND WIDTH AND AN ARBITRARY
C                          NUMBER OF LOAD VECTORS.
C
C                                 -  INPUT  -
C      A(NEQ*MBAND) - UPPER HALF OF RECTIFIED COEFFICIENT MATRIX BAND
C                     IN ONE-DIMENSIONAL FORM
C      B(NEQ)       - SINGLE LOAD VECTOR
C      NEQ          - NUMBER OF EQUATIONS
C      MBAND        - MAXIMUM WIDTH OF HALF BAND
C      KKK          - LOAD CASE INDICATOR, EQUAL TO
C                       1 FOR FIRST LOAD CASE (REDUCTION OF A AND B WITH
C                         BACKSUBSTITUTION
C                       2 FOR ANY SUBSEQUENT LOAD VECTOR (REDUCTION OF B
C                         WITH BACKSUBSTITUTION)
C
C                                 -  OUTPUT  -
C      B(NEQ)       - SOLUTION VECTOR
C      A(NEQ*MBAND) - REDUCED STIFFNESS MATRIX
C      NBL(NEQ)     - VECTOR DEFINING BAND WIDTH OF EACH EQUATION
C*****************************************************************************
C
      DIMENSION A(1),B(NEQ),NBL(NEQ)
C
      NM = NEQ*MBAND
      NE = NEQ-1
      GO TO (100,200) KKK
C
C         DECOMPOSITION OF BAND MATRIX A
C
  100 DO 160 I=1,NE
      D = A(I)
      IF (D) 110,160,120
  110 PRINT 1000, I,D
 1000 FORMAT (// 20H PIVOT IS NEGATIVE    /
     *            26H DIAGONAL TERM OF EQUATION  I8,8H EQUALS E20.6//)
C
C         ESTABLISH VARIABLE BAND WIDTH
C
  120 DO 130 J=NEQ,NM,NEQ
      IF (A(NM-J+I).NE.0.0) GO TO 135
  130 CONTINUE
  135 NBL(I) = NM-J+I
C
C         REDUCTION OF MATRIX A
C
      JL = I+1
      II = I
      MAX = NBL(I)
      JH = (MAX-1)/NEQ+I
      DO 140 J=JL,JH
```

```
      II = II+NEQ
      C = A(II)/D
      IF (C.EQ.0.0) GO TO 140
      KK = J
      DO 150 JJ=II,MAX,NEQ
      A(KK) = A(KK)-C*A(JJ)
  150 KK = KK+NEQ
  140 A(II) = C
  160 CONTINUE
C
C         REDUCTION OF LOAD VECTOR B
C
  200 DO 260 I=1,NE
      IF (A(I).EQ.0.0) GO TO 260
      JL = I+1
      II = I
      JH = (NBL(I)-1)/NEQ+I
      C = B(I)
      IF (C.EQ.0.0) GO TO 260
      DO 230 J=JL,JH
      II = II+NEQ
  230 B(J) = B(J)-C*A(II)
      B(I) = B(I)/A(I)
  260 CONTINUE
      IF (A(NEQ).EQ.0.0) GO TO 320
      B(NEQ) = B(NEQ)/A(NEQ)
C
C         SOLUTION BY BACKSUBSTITUTION
C
  320 DO 360 I=1,NE
      JI = NEQ-I
      IF (A(JI).EQ.0.0) GO TO 360
      IL = JI+NEQ
      MAX = NBL(JI)
      C = B(JI)
      JN = JI+1
      DO 340 II=IL,MAX,NEQ
      C = C-A(II)*B(JN)
  340 JN = JN+1
      B(JI) = C
  360 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE LOADS (I,J,HI,HJ,VI,VJ,R1,R2,S12,S,C,PTOT,T)
C
C     THIS SUBROUTINE TRANSFORMS DISTRIBUTED SURFACE LOADS INTO
C     CONSISTENT NODAL LOADS AND ADDS THEM INTO THE LOAD VECTOR
C
      DIMENSION PTOT(1)
      A=.5*(R2-R1)
      B=.5*(R2+R1)
      P=S12*T/120.
      RVI=P*10.*((2.*B-A)*HI+B*HJ)
      RVJ=P*10.*(B*HI+(2.*B+A)*HJ)
      RWI=P*((21.*B-11.*A)*VI+(9.*B-A)*VJ)
      RWJ=P*((9.*B+A)*VI+(21.*B+11.*A)*VJ)
      RTI=P*S12*((3.*B-A)*VI+2.*B*VJ)
      RTJ=-P*S12*(2.*B*VI+(3.*B+A)*VJ)
      K=I*4-4
      L=J*4-4
      PTOT(K+1)=PTOT(K+1)+RVI*C+RWI*S
      PTOT(K+2)=PTOT(K+2)+RVI*S-RWI*C
      PTOT(K+3)=PTOT(K+3)-RTI
      PTOT(L+1)=PTOT(L+1)+RVJ*C+RWJ*S
      PTOT(L+2)=PTOT(L+2)+RVJ*S-RWJ*C
      PTOT(L+3)=PTOT(L+3)-RTJ
      RETURN
      END
```

```
      SUBROUTINE OPRINT (A,M,N,X,NX,NY,K1,K2,NCYC,L,KI,IND,IO)
C
C     THIS SUBROUTINE PRINTS JOINT DISPLACEMENTS IF KI=2,
C              AND INTERNAL PLATE FORCES OF  KI=2
C
      DIMENSION IND(M),A(M,N),X(NX),N1(2),N2(2)
      LOGICAL IO
      DATA N1/1,8/
    1 FORMAT(I6,1P7E16.7)
    2 FORMAT(6H SECT.,7(6H     X=F10.3))
    3 FORMAT(6H SECT.,7(8H  THETA=F8.4))
    4 FORMAT(6H JOINT,7(6H     X=F10.3))
    5 FORMAT(6H JOINT,7(8H  THETA=F8.4))
      N2(1)=K1
      N2(2)=K2
      DO 25 K= 1,NCYC
      J1=N1(K)
      J2=N2(K)
      GO TO (6,15), KI
    6 IF(.NOT.IO) PRINT 2, (X(I), I=J1,J2)
      IF(IO)      PRINT 3, (X(I), I=J1,J2)
      DO 10 I=1,NY
   10 PRINT 1, I, (A(J,I), J=J1,J2)
      GO TO 25
   15 IF(.NOT.IO) PRINT 4, (X(I), I=J1,J2)
      IF(IO)      PRINT 5, (X(I), I=J1,J2)
      DO 20 I=L,M,4
   20 PRINT 1, IND(I), (A(I,J), J=J1,J2)
   25 CONTINUE
      RETURN
      END
```

Appendix B

Input Data for Example 4A

```
  REPORT EXAMPLE  4 A  -  2-CELL BOX GIRCER BRIDGE WITH TRUCK ON LEFT
22.9183118      250.   3  15  14   5  50   0   0   0   1  12   0   6   5  3
   25.0 35.356   50.0 64.644    75.0
          1   .541667   432000.   432000.   187826.       .15
              .541667   432000.   432000.   187826.       .15
          2   .666667   432000.   432000.   187826.       .15
              .666667   432000.   432000.   187826.       .15
          3   .458333   432000.   432000.   187826.       .15
              .458333   432000.   432000.   187826.       .15
   1   1   2   1   4
   2   2   4   1   4
   3   4   6   1   4
   4   6   8   1   4
   5   8  10   1   4
   6  10  12   1   4
   7  12  13   1   4
   8  13  14   1   4
   9   2   3   2   4
  10   8   7   2   4
  11  13  11   2   4
  12   3   5   3   4
  13   5   7   3   4
  14   7   9   3   4
  15   9  11   3   4
          1      -14.        1.4
          2      -11.        1.1
          3      -10.        5.5
          4      -8.0        0.8
          5      -5.0        5.0
          6      -5.0        0.5
          7       0.0        4.5
          8       0.0        0.0
          9       5.0        4.0
         10       5.0       -0.5
         11      10.0        3.5
         12       8.0       -0.8
         13      11.0       -1.1
         14      14.0       -1.4
          2       0.0       16.0        0.0        0.0       33.8        1.0
          6       0.0       16.0        0.0        0.0       33.8        1.0
          2       0.0       16.0        0.0        0.0       47.8        1.0
          6       0.0       16.0        0.0        0.0       47.8        1.0
          2       0.0        4.0        0.0        0.0       61.8        1.0
          6       0.0        4.0        0.0        0.0       61.8        1.0
   25.0 35.356   50.0 64.644    75.0
   1     1     0    -1.847    -1.847
   2     1     0    -1.847    -1.847
   3     1     2    -1.847    -1.847       2.5
   4     2     0    -1.847    -1.847
   5     2     0    -1.847    -1.847
   6     2     3    -1.847    -1.847       0.5
   7     3     0    -1.847    -1.847
   8     3     0    -1.847    -1.847
   9     1     0    -1.847     2.653
```

10	2	0	-1.847	2.653
11	3	0	-1.847	2.653
12	1	0	2.653	2.653
13	2	0	2.653	2.653
14	2	0	2.653	2.653
15	3	0	2.653	2.653