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CHARGE-CONSTRAINED COHERENT PION STATES
PRODUCED IN NUCLEAR MATTER*

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ABSTRACT

In a recent article, Bloss, Hone and Scalapino have used a simplified model Hamiltonian and Keldysh field theoretic methods to approximately calculate the power radiated as Cherenkov charged pions whose emission is triggered when a fast nucleon traverses nuclear matter. The model used by these authors conserves charge by flipping the isospin state of the fast nucleon *alone* for each charged pion emitted. Since it is the nuclear matter excited by the fast nucleon which should be largely responsible for emitting the Cherenkov charged pions, we investigate an alternate model that allows all the disturbed nuclear matter to change its charge state when a charged pion is emitted — as an idealization, the emitting matter is assumed to have an infinite number of charge states, in contrast to the two of the fast nucleon. This modified model is shown to be exactly solvable in terms of charge-constrained coherent pion states; the power radiated as charged pions is found to be approximately twice that calculated by Bloss *et al* from their model.

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A recent publication of Bloss, Hone and Scalapino¹⁾ has treated a simplified model Hamiltonian for Cherenkov radiation of charged pions by a fast nucleon traversing nuclear matter. Based on previous work by Sawyer²⁾ indicating the possibility of such pionic Cherenkov radiation from the pion dispersion relation in nuclear matter, the simplified model Hamiltonian of Bloss *et al* suppresses consideration of a number of features — such as neutral pion emission, the detailed influence of spin-parity on the couplings, and the final state interactions of the radiated charged pions with the nuclear medium, among others — to focus on the influence of charge conservation on the charged pion emission. In particular, these authors wished to elucidate the differences between this charge-constrained case and a parallel model for neutral scalar pion Cherenkov emission. This latter model has, of course, no charge constraint and happens to be exactly solvable by the coherent state techniques well known from photon bremsstrahlung.³⁾

The model Hamiltonian of Bloss *et al* is

$$H = H_0 + H'(t) \quad (1)$$

where

$$H_0 = \sum_{\underline{q}} (\omega_{\underline{q}}^+ a_{\underline{q}^+}^\dagger a_{\underline{q}^+} + \omega_{\underline{q}}^- a_{\underline{q}^-}^\dagger a_{\underline{q}^-}) \quad (2)$$

represents the energy of "free" charged pions in the nuclear medium (the energies $\omega_{\underline{q}}^{+(-)}$ include the effect of the medium on the positive (negative) pion dispersion relation) and

$$\begin{aligned}
 H'(t) = & \sum_{\underline{q}} (2\omega_{\underline{q}}^+)^{-\frac{1}{2}} \left[j_{\underline{q}}(t) a_{\underline{q}^+} \sigma^+ + j_{\underline{q}}^*(t) a_{\underline{q}^+}^\dagger \sigma^- \right] \\
 & + \sum_{\underline{q}} (2\omega_{\underline{q}}^-)^{-\frac{1}{2}} \left[j_{\underline{q}}(t) a_{\underline{q}^-} \sigma^- + j_{\underline{q}}^*(t) a_{\underline{q}^-}^\dagger \sigma^+ \right]
 \end{aligned} \tag{3}$$

represents the coupling of the charged pions to the complex c-number source current $j_{\underline{q}}(t)$ produced by the fast nucleon traversing the nuclear medium. In the Cherenkov spirit, Bloss et al take this current to be of the form

$$j_{\underline{q}}(t) = J_0(q) \exp(i\underline{q} \cdot \underline{v} t) \tag{4}$$

with $\underline{v}$ the fast nucleon velocity. The $a_{\underline{q}^+}^\dagger(-)$ are, of course, the standard positive(negative) pion creation operators, and σ^+ changes the fast nucleon from a neutron to a proton, while σ^- does the opposite (σ^+ and σ^- have the usual 2×2 Pauli matrix representation). The charge operator for the combined fast nucleon, charged pion system

$$Q \equiv \sigma^+ \sigma^- + \sum_{\underline{q}} (a_{\underline{q}^+}^\dagger a_{\underline{q}^+} - a_{\underline{q}^-}^\dagger a_{\underline{q}^-}) \tag{5}$$

commutes with H . In this model then, the charged pions are viewed as being emitted and absorbed solely by the fast nucleon — the nuclear medium doesn't participate in the charged pion emission — for charge conservation applies to the fast nucleon plus charged pion system by itself.

The presence of the low-dimensional fast-nucleon charge ladder operators $\sigma^\pm$ gives H a "non-Abelian"^{1,2)} aspect — the instantaneous isospin state of the fast nucleon determines the charge of the next pion

radiated. This selection rule makes the model Hamiltonian unamenable to exact solution. However, by summing certain classes of graphs in the sophisticated field theoretic Keldysh framework⁴), Bloss et al obtained an excellent approximation for $d\langle H_0 \rangle / dt$, the power radiated as charged pions. They could then contrast their result with the same quantity calculated to lowest order (linear response theory) and to an exact result for radiated power which is obtainable from an analogous Hamiltonian describing neutral scalar pion emission (the "Abelian" case).

The present short article is prompted by the observation that Cherenkov radiation is largely emitted by the disturbed medium⁵) rather than directly by the fast particle producing that disturbance (even a fast charged lepton should trigger charged pion Cherenkov radiation in nuclear matter, though of much lower intensity than a fast nucleon because of its weaker (electromagnetic only) stimulation of the nuclear medium). Thus, the possible charge states of the emitting matter are not restricted to just the two possible charges of the fast nucleon, but must embrace quite a broad charge range. A limiting idealization of this situation is to assign the integer charge quantum number z to the emitting nuclear matter (including, of course, the fast nucleon itself), with no restriction on the integer values allowed z . This contrasts with the just two nuclear charge states, those of the fast nucleon, allowed by Bloss et al. It is interesting to note that modifying the Hamiltonian of Eqs. (1-3) to incorporate such an unbounded nuclear charge multiplicity will actually render it more tractable. On the space spanned by the orthonormal nuclear charge basis $\{|z\rangle\}$, we may readily define the net nuclear charge operator Z and the nuclear charge raising and lowering operators Z^+ and Z^- via the formulas

$$Z|z\rangle = z|z\rangle \quad (6a)$$

$$Z^+|z\rangle = |z+1\rangle \quad (6b)$$

$$Z^-|z\rangle = |z-1\rangle \quad (6c)$$

Using the orthonormality of the nuclear charge basis, $\langle z'|z\rangle = \delta_{z',z}$, plus the unrestricted integer domain of the z -values, it is straightforward to show from Eq. (6) that

$$Z^+ = Z \quad (7a)$$

$$(Z^\pm)^\dagger = Z^\mp \quad (7b)$$

$$[Z, Z^\pm] = \pm Z^\pm, \quad (7c)$$

which are the familiar properties one expects of such operators. In addition, there also follows a less familiar but very pleasant property of the nuclear charge ladders

$$Z^+Z^- = Z^-Z^+ = I \quad (8)$$

In particular, we see that Z^+ and Z^- commute. Our nuclear charge operators are seen to be formally appropriate to the description of an idealized infinite spin system, with its reducible, non-compact operator algebra.

Now, to take approximate account of the charged pion emission from nuclear matter disturbed by the fast nucleon, as well as from the fast nucleon itself, we propose to modify the Hamiltonian of Bloss *et al* by replacing the $\sigma^\pm$ operators by $Z^\pm$,

$$H = H_0 + V(t) \quad (9)$$

where H_0 is given by Eq. (2) and

$$\begin{aligned} V(t) = & \sum_{\underline{q}} (2\omega_{\underline{q}}^+)^{-\frac{1}{2}} \left[j_{\underline{q}}(t) a_{\underline{q}^+} Z^+ + j_{\underline{q}}^*(t) a_{\underline{q}^+}^\dagger Z^- \right] \\ & + \sum_{\underline{q}} (2\omega_{\underline{q}}^-)^{-\frac{1}{2}} \left[j_{\underline{q}}(t) a_{\underline{q}^-} Z^- + j_{\underline{q}}^*(t) a_{\underline{q}^-}^\dagger Z^+ \right] . \end{aligned} \quad (10)$$

Note that this Hamiltonian commutes with the total charge operator

$$Q \equiv Z + \sum_{\underline{q}} (a_{\underline{q}^+}^\dagger a_{\underline{q}^+} - a_{\underline{q}^-}^\dagger a_{\underline{q}^-}) . \quad (11)$$

Nonetheless, because of the simple properties of the $Z^\pm$ operators given by Eq. (8), we shall see that the Hamiltonian of Eqs. (9) and (10) describes practically "Abelian" pion emission, in close analogy to photon bremsstrahlung.³⁾ Indeed, the interaction picture time evolution operator $U_I(t, t_0)$ for this Hamiltonian is readily written down in a useful normal ordered form very similar to the photon result.

To derive this result, we write the interaction picture Hamiltonian in the form

$$\begin{aligned} V_I(t) & \equiv \exp(iH_0 t) V(t) \exp(-iH_0 t) \\ & = A_+(t) Z^+ + A_+^\dagger(t) Z^- + A_-(t) Z^- + A_-^\dagger(t) Z^+ \end{aligned} \quad (12)$$

where

$$A_\pm(t) = \sum_{\underline{q}} (2\omega_{\underline{q}}^\pm)^{-\frac{1}{2}} j_{\underline{q}}(t) e^{-i\omega_{\underline{q}}^\pm t} a_{\underline{q}^\pm} . \quad (13)$$

Using the properties of the pion creation and annihilation operators plus

those of the $Z^\pm$ given in Eq. (8), we may derive the unequal time commutation relations among the four terms of $V_I(t)$. Of the ten unequal time pairs of these four terms, eight commute, the two exceptions being

$$[A_\pm(t_1) Z^\pm, A_\pm^\dagger(t_2) Z^\mp] = C_\pm(t_1, t_2) \quad (14)$$

where

$$C_\pm(t_1, t_2) \equiv \sum_{\underline{q}} (2\omega_{\underline{q}}^\pm)^{-1} j_{\underline{q}}(t_1) j_{\underline{q}}^*(t_2) e^{-i\omega_{\underline{q}}^\pm(t_1-t_2)} \quad (15)$$

is a c-number. In particular, $V_I(t)$ may be written as the sum of two Hermitian sub-Hamiltonians, which commute at all times, i.e.,

$$V_I(t) = V_+(t) + V_-(t) \quad (16)$$

where

$$V_\pm(t) \equiv A_\pm(t) Z^\pm + A_\pm^\dagger(t) Z^\mp \quad (17)$$

and

$$[V_+(t_1), V_-(t_2)] = 0$$

Thus, $U_I(t, t_0)$ will break up into the product of two corresponding *commuting* time evolution operators, each of which is uniquely defined by its own Schrödinger equation and initial condition,

$$U_I(t, t_0) = U_+(t, t_0) U_-(t, t_0) \quad (19)$$

where

$$i \frac{\partial U_\pm(t, t_0)}{\partial t} = V_\pm(t) U_\pm(t, t_0) \quad (20)$$

and

$$U_{\pm}(t_0, t_0) = I \quad . \quad (21)$$

Noting that the two-term structure of $V_{\pm}(t)$, Eq. (17), plus the unequal time commutation relations of these two terms, is very similar to the state of affairs one would have in the neutral scalar pion ("Abelian") theory, the useful normal ordered form of $U_{\pm}(t, t_0)$ can be immediately guessed in analogy with the photon bremsstrahlung result³⁾

$$\begin{aligned} U_{\pm}(t, t_0) = & \exp \left[- \int_{t_0}^t dt_1 \int_{t_0}^{t_1} dt_2 C_{\pm}(t_1, t_2) \right] \exp \left[-i \int_{t_0}^t dt_1 A_{\pm}^{\dagger}(t_1) Z^{\mp} \right] \\ & \times \exp \left[-i \int_{t_0}^t dt_1 A_{\pm}(t_1) Z^{\pm} \right] \quad . \end{aligned} \quad (22)$$

The initial condition, Eq. (21), is clearly fulfilled, while verification that Schrödinger's equation, Eq. (20), is satisfied proceeds by direct time differentiation followed by use of the unequal time commutation relations and the general operator identity

$$e^D F = \left\{ F + [D, F] + \frac{1}{2!} [D, [D, F]] + \dots \right\} e^D \quad (23)$$

with

$$D = -i \int_{t_0}^t dt_1 A_{\pm}^{\dagger}(t_1) Z^{\mp} \quad (24)$$

and

$$F = A_{\pm}(t) Z^{\pm} \quad . \quad (25)$$

Thus, using Eq. (19) plus the unequal time commutation relations, the time evolution operator in normal ordered form is

$$\begin{aligned}
 U_I(t, t_0) = & \exp \left[- \int_{t_0}^t dt_1 \int_{t_0}^{t_1} dt_2 (C_+(t_1, t_2) + C_-(t_1, t_2)) \right] \\
 & \times \exp \left[-i \int_{t_0}^t dt_1 (A_+^\dagger(t_1) Z^- + A_-^\dagger(t_1) Z^+) \right] \\
 & \times \exp \left[-i \int_{t_0}^t dt_1 (A_+(t_1) Z^+ + A_-(t_1) Z^-) \right] . \quad (26)
 \end{aligned}$$

In particular, if the initial state at time t_0 contains no charged pions, with the medium having charge z , the interaction picture state at time t is

$$\begin{aligned}
 U_I(t, t_0) |0, z\rangle = & \exp \left[- \int_{t_0}^t dt_1 \int_{t_0}^{t_1} dt_2 (C_+(t_1, t_2) + C_-(t_1, t_2)) \right] \\
 & \times \exp \left[-i \int_{t_0}^t dt_1 \sum_{\underline{q}} \left((2\omega_{\underline{q}}^+)^{-\frac{1}{2}} j_{\underline{q}}^*(t_1) e^{+i\omega_{\underline{q}}^+ t_1} a_{\underline{q}^+}^\dagger Z^- \right. \right. \\
 & \left. \left. + (2\omega_{\underline{q}}^-)^{-\frac{1}{2}} j_{\underline{q}}^*(t_1) e^{+i\omega_{\underline{q}}^- t_1} a_{\underline{q}^-}^\dagger Z^+ \right) \right] |0, z\rangle , \quad (27)
 \end{aligned}$$

which is an eigenstate of the *charge conserving* pion annihilation operators

$$(a_{\underline{q}^\pm} Z^\pm) U_I(t, t_0) |0, z\rangle = \left(-i (2\omega_{\underline{q}}^\pm)^{-\frac{1}{2}} \int_{t_0}^t dt_1 j_{\underline{q}}^*(t_1) e^{+i\omega_{\underline{q}}^\pm t_1} \right) U_I(t, t_0) |0, z\rangle , \quad (28)$$

as may be verified using the pion operator commutation relations, the properties of $Z^\pm$ given in Eq. (8), and an operator identity of the type

shown in Eq. (23).

Equations (27) and (28) show that the state $U_I(t, t_0)|0, z\rangle$ (which at time t_0 has no pions and is always an eigenstate of the total charge operator Q with eigenvalue z) is a *charge-constrained coherent state* of both positive and negative pions. This concept was recently introduced in Ref. 6, where it is applied to pion interferometry in nuclear collisions. For this state, use of Eq. (28) allows straightforward calculation of the power radiated as charged pions,

$$\frac{d\langle H_0 \rangle}{dt} = i \langle 0, z | U_I^\dagger(t, t_0) [V_I(t), H_0] U_I(t, t_0) | 0, z \rangle \quad (29a)$$

$$\begin{aligned} = & i \langle 0, z | U_I^\dagger(t, t_0) \sum_{\tilde{q}} \left\{ (\omega_{\tilde{q}}^+/2)^{1/2} \left[j_{\tilde{q}}(t) e^{-i\omega_{\tilde{q}}^+ t} a_{\tilde{q}^+}^{Z^+} - j_{\tilde{q}}^*(t) e^{+i\omega_{\tilde{q}}^+ t} a_{\tilde{q}^+}^{Z^-} \right] \right. \\ & + \left. (\omega_{\tilde{q}}^-/2)^{1/2} \left[j_{\tilde{q}}(t) e^{-i\omega_{\tilde{q}}^- t} a_{\tilde{q}^-}^{Z^-} - j_{\tilde{q}}^*(t) e^{+i\omega_{\tilde{q}}^- t} a_{\tilde{q}^-}^{Z^+} \right] \right\} U_I(t, t_0) | 0, z \rangle \end{aligned} \quad (29b)$$

$$\begin{aligned} = & \text{Re} \left\{ \sum_{\tilde{q}} \left[j_{\tilde{q}}(t) e^{-i\omega_{\tilde{q}}^+ t} \left(\int_{t_0}^t dt_1 j_{\tilde{q}}^*(t_1) e^{+i\omega_{\tilde{q}}^+ t_1} \right) \right. \right. \\ & + \left. \left. j_{\tilde{q}}(t) e^{-i\omega_{\tilde{q}}^- t} \left(\int_{t_0}^t dt_1 j_{\tilde{q}}^*(t_1) e^{+i\omega_{\tilde{q}}^- t_1} \right) \right] \right\} \quad (29c) \end{aligned}$$

If we now specialize to the Cherenkov form of $j_{\tilde{q}}(t)$ used by Bloss *et al*, as given by Eq. (4), we obtain

$$\frac{d\langle H_0 \rangle}{dt} = \sum_{\tilde{q}} \left[J_0^2(q) \frac{\sin((\omega_{\tilde{q}}^+ - \tilde{q} \cdot \tilde{v})(t - t_0))}{(\omega_{\tilde{q}}^+ - \tilde{q} \cdot \tilde{v})} + J_0^2(q) \frac{\sin((\omega_{\tilde{q}}^- - \tilde{q} \cdot \tilde{v})(t - t_0))}{(\omega_{\tilde{q}}^- - \tilde{q} \cdot \tilde{v})} \right] \quad (30)$$

Thus, for the charged pion power radiated in infinite nuclear matter at asymptotic time, we obtain

$$\lim_{(t-t_0) \rightarrow +\infty} \frac{d\langle H_0 \rangle}{dt} = \sum_{\underline{q}} \pi J_0^2(\underline{q}) \delta(\omega_{\underline{q}}^+ - \underline{q} \cdot \underline{v}) + \sum_{\underline{q}} \pi J_0^2(\underline{q}) \delta(\omega_{\underline{q}}^- - \underline{q} \cdot \underline{v}) . \quad (31)$$

This is the same as the lowest order (linear response) result for our Hamiltonian, and in the case $\omega_{\underline{q}}^+ \approx \omega_{\underline{q}}^-$, approximately twice the charged pion power obtained by Bloss *et al* from their Hamiltonian, Eqs. (1-4).

The disparity may be roughly understood by noting that, at any given instant, the fast nucleon is forbidden to radiate a pion of the "wrong" charge (in the proton(neutron) state it may only radiate a positive (negative) pion), while there is no such restriction for charged pion emission from the fast nucleon *plus* the nuclear matter it disturbs.

Given a "natural" emission rate for pions of either charge, estimated, say, from the neutral scalar pion ("Abelian") theory, our Hamiltonian may be expected to produce about twice that rate (because of the *two* types of charged pions which can be emitted), while the instantaneous charge selection rule for pion emission inherent in the Bloss *et al* Hamiltonian may be expected to produce a rate which is cut to roughly half of ours.

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