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Basic Theory and Equations Used In The Two-Phase Multidimensional Geothermal Reservoir Simulator, SHAFT79

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ABSTRACT

The algorithm of SHAFT79 is based on mass and energy balance equations for two-phase flow in a porous medium. These basic equations are formulated as Integrated Finite Difference equations. The latter formulation allows both regular and irregular discrete grid approximations of reservoir geometry. The present version of SHAFT79 solves the non-linear mass and energy equations simultaneously using an efficient linear algebra package.

The computer program is being applied to a variety of problems to study both real reservoir behavior and to better understand the physics of two-phase systems. The types of applications include idealized one-and two-phase reservoir depletion, two-phase reservoir behavior with distributed liquid, simulation of real reservoirs, matching production data, and estimating material parameters from well test data.

BASIC THEORY AND EQUATIONS

The computer program SHAFT79 was developed to compute two-phase flow phenomena in geothermal reservoirs. The program solves transient initial-value problems with prescribed boundary-conditions. The solution method is an explicit-implicit (IFD) (Narasimhan and Witherspoon, 1976) approach which does not distinguish between 1, 2, or 3-D coordinate systems and allows a flexible choice of the shape of the discrete grid elements. The mass-and-energy equations are formulated in conservative form. The stability and convergence of the algorithm can be controlled by an automatic choice of time steps or can be chosen by the user. Since the equation of state is a tabular array, fluids other than pure water can be used in the calculations. However, the pressure difference between the wetting and non-wetting phases is neglected. The relative permeabilities for the wetting and non-wetting phases are available as analytical approximations, or in tabular form and can be specified for any fluid.

The solution algorithm is based upon statements of mass and energy conservation in both the rock and two-phase fluid. (Bird et al., 1960) The porous medium is assumed to have sufficiently small pores so that the thermal equilibration between rock and fluid is instantaneous. For most geothermal reservoir problems where the time scale is days or more and reservoir dimensions are often several kilometers, this approximation is acceptable.

SHAFT79 offers a choice of several methods for solving the coupled non-linear equations for mass and energy flow. Also, different algorithms are available for solving the set of linear equations which arises at each iteration step. This flexibility allows an optimal balance between accuracy and efficiency of simulations, depending upon problem size (number of elements), occurrence of phase transitions, relative magnitude of energy-and-mass flows, and other characteristics of the problem. The preferred solution method is fully implicit, employs a Newton/Raphson (Blair and Weinaug, 1969) iteration for simultaneous solution of the non-linear mass-and-energy transport equations, and uses an efficient sparse solver (Duff, 1977). SHAFT79 has been applied to problems with up to 250 elements in three dimensions. Throughputs of up to 65 per time step have been achieved with good accuracy.*

The microscopic structure of porous rock is highly heterogeneous. The channels through which the fluids move are tortuous and have (in general) non-regular shapes. In addition, the porous rock in a geological setting generally has many structural variations, and fractures of widely varying aperture and extent. The fluids, in general, move through the fractures more rapidly than through the microscopic pores, but heterogeneous rock can usually be approximated by using macroscopic rock and fluid parameters. When the flow rates are large, or if the fracture velocities are very large relative to the microscopic pore velocities, the relationship between fluid flow-rate and macroscopic pressure gradient becomes non-linear. In the case of two phases - one wetting and the other non-wetting - the relationship between flow rate and pressure gradient can be specified in terms of a function of wetting or non-wetting volumetric saturation.

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*Throughput is defined as the ratio of fluid mass flowing across an element, divided by the fluid mass initially in place in that element.

Macroscopic equations can be derived using statistical averaging which have exactly the same form as the point equations (differential or integral equations obtained assuming a macroscopic representative elementary volume) (Whittaker, 1969; Bear, 1975) The same equations are obtained when the differential laws are integrated as when the differential-integral forms are derived directly (when the same assumptions are made). The choice of presenting the equations in one form or another is primarily a matter of style and preference. We prefer to write the macroscopic (point) differential equations, integrate them over a volume which will have special significance for discretization, and then define the particular numerical solution procedure incorporated in the algorithm called SHAFT79.

There is more than one possible choice of intensive thermodynamic variable pairs from which all other thermodynamic information can be derived. Internal energy and specific density are two such variables. When the equation of state (EOS) is known in terms of energy and density the EOS gives pressure, temperature, and fluid saturation. This completely specifies the thermodynamic state in terms of macroscopic quantities we can measure. It is also possible to use triplets of variables such as temperature, pressure, and steam saturation. Of these three variables only two are independent, namely, temperature and pressure in the one-phase region, and temperature and saturation in the two-phase region. Therefore, using such combinations becomes somewhat awkward for problems involving phase transitions. The SHAFT79 equations will be presented in terms of internal energy and density.

The partial differential equations which model the flow of steam and water mixtures in porous rock are forms of the conservation laws for mass, energy, and force (Newton's law). For porous media it was shown empirically that the fluid flux is proportional to pressure drop (Darcy's law) (Scheidegger, 1974). These equations are summarized in (1) to (3).

$$\frac{\partial\phi\rho}{\partial t} = \frac{\partial\phi S\rho_v}{\partial t} + \frac{\partial\phi(1-S)\rho_\ell}{\partial t} = -\nabla\cdot\overline{F} + q_\rho \tag{1}$$

$$\frac{\partial(\text{energy/volume})}{\partial t} = \frac{\partial(u\rho\phi+u_s\rho_s(1-\phi))}{\partial t} = -\nabla\cdot G + \left(\frac{\overline{F}_v}{\rho_v}+\frac{\overline{F}_\ell}{\rho_\ell}\right)\nabla p + Q_u \tag{2}$$

$$\overline{F}_\alpha = -\rho_\alpha \frac{k\,k_\alpha}{\mu_\alpha}(\nabla p - \rho_\alpha \overline{g}) \tag{3}$$

where the symbols are defined by

ρ_α = density (mass per volume) of phase α

ϕ = porosity (V_p/V)

V = volume element of rock/fluid mixture

V_p = pore volume in V

S = vapor saturation (V_v/V_p)

V_v = vapor volume in V

t = time

$\overline{F}$ = mass flux vector

q_ρ = external sources (mass rate/volume, negative source corresponds to mass being withdrawn)

u = specific internal energy of the fluid

u_s = specific internal energy of the solid

ρ_s = specific density of the solid

$\overline{G}$ = energy flux

k = absolute permeability

k_α = relative permeability of phase α

μ_α = fluid viscosity of phase α

p = pressure

$\overline{g}$ = gravitational acceleration vector

These equations are integrated over a polyhedral partition of the calculation space as shown in Figure 1 and fluxes and sources are averaged over the polygonal areas and polyhedral volumes respectively. The general equations used in the SHAFT79 algorithm are summarized in Table 1.

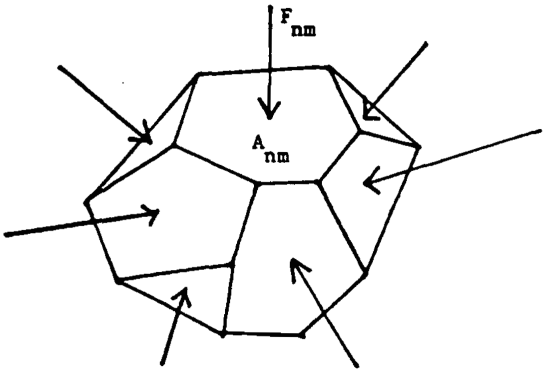

Figure 1. A three-dimensional polyhedron with a polygonal face of area A_{nm} and flux through that area F_{nm} from an adjoining polyhedral volume.

$$\frac{\partial \phi \rho}{\partial t} = \frac{\int_\tau \frac{\partial \phi' \rho'}{\partial t} \, dv}{\tau}$$

$$F_m = \frac{\int_{A_m} \overline{F} \cdot \overline{n} \, da}{A_m}$$

$$\frac{\partial \phi \rho}{\partial t} = \sum_{m=1}^{N} \frac{F_m A_m}{\tau} + q$$

$$\rho = S\rho_v + (1-S)\rho_\ell$$

$$\overline{F} = \overline{F}_v + \overline{F}_\ell$$

$$\overline{F}_v = - \frac{\rho_v k \, k_v}{\mu_v} (\nabla p - \rho_v \overline{g})$$

$$\overline{F}_\ell = - \frac{\rho_\ell k \, k_\ell}{\mu_\ell} (\nabla p - \rho_\ell \overline{g})$$

$$G_m = -K_m \, \nabla T_m + F_{vm} h_{vm} + F_{\ell m} \, h_{\ell m}$$

$$\phi \rho \, \frac{\partial u}{\partial t} + \rho_s \frac{\partial (1-\phi) u_s}{\partial t} = \sum_{m=1}^{N} \frac{(G_m - uF_m) A_m}{\tau} + (Q - \tilde{Q})$$

$$q = \frac{\int_\tau q'_\rho \, dv}{\tau} \, , \; Q = \frac{\int_\tau q_u dv}{\tau} \, , \; \tilde{Q} = \frac{\int_\tau u q'_\rho \, dv}{\tau}$$

Table 1. A summary of the basic equations used in the SHAFT79 model for two-phase flow in porous media.

CURRENT APPLICATIONS OF SHAFT79

The current version of the LBL simulator SHAFT79 is being implemented at several industrial and government laboratories. Since the newest version is the most efficient version, no applications outside LBL have been completed to date. We have verified the accuracy of the calculations and have been using the program for both idealized problems and real applications. In Table 2 the problems that have been completed and are currently underway are described briefly.

Future applications and new developments include studying the evolution of geothermal systems, estimation of reserves from depletion studies, detailed studies of the effects of injection on energy recovery, and the effects of gases on reservoir depletion behavior. The formulation of the gas equations and the method of incorporation into SHAFT79 has been completed, and only the implementation into the program is required.

Table 2. A Review of Problems Using SHAFT79

Completed problems (some were calculated using SHAFT78)

- A study of the propagation of phase fronts near a producing well (Pruess et al, 1978)
- A study of the propagation of phase fronts through a depleting reservoir (Pruess et al, 1979)
- Reservoir simulation of the Krafla, Iceland geothermal zone (V. Jonsson, 1979)

Current Calculations (SHAFT79)

- A study of phenomena occurring during injection or influx in a depleting reservoir
- A study of production and injection fronts in a 5-spot pattern
- A history match of the production data from the Serrazzano Zone at Larderello, Italy (Part of the Italian-U.S. Cooperative Research Agreement) (Bodvarsson et al, 1979)
- Determination of the material parameters using well test data from the Cerro Prieto Field (Part of the Mexican-U.S. Cooperative Research Agreement) (Benson and Schroeder, 1979)
- Additional reservoir simulation of the Krafla Reservoir, Iceland (V. Jonsson, 1979)

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