A Haunting Conviction: Frege on Truth and Logic

by

James Edwin Hutchinson

A dissertation submitted in partial satisfaction of the

requirements for the degree of

Doctor of Philosophy

in

Philosophy

in the

Graduate Division

of the

University of California, Berkeley

Committee in charge:

Professor John Campbell, Co-Chair
Professor John MacFarlane, Co-Chair
Professor Line Mikkelsen

Spring 2018

A Haunting Conviction: Frege on Truth and Logic

Abstract

A Haunting Conviction: Frege on Truth and Logic

by

James Edwin Hutchinson

Doctor of Philosophy in Philosophy

University of California, Berkeley

Professor John Campbell, Co-Chair

Professor John MacFarlane, Co-Chair

Many of us think that *truth* is prior to *science:* though it belongs to the nature of science to pursue truth, the nature of truth does not involve science. Frege denies this. He thinks that to be true is to play a role in the characteristic goals of science, such as explanation.

For Frege and the Neo-Kantian philosophers in his philosophical milieu, the point of this view is that no truth is *trivial*: every one makes a contribution to a deep understanding of the world. This allows them to maintain that truth is a *value,* whose significance is rivalled only by that of the Good and the Beautiful. This view leads Frege to a normative conception of logic, on which the logical laws are the ones that tell us how we ought to judge if we are to achieve our highest scientific goals. Since the nature of truth itself is to be found in such goals, these laws tell us about truth itself.

This makes possible an epistemology for logical axioms that has frustrated readers by its apparent inconsistency: Frege seems both to rule out arguing for logical axioms, and himself to offer arguments for them. But he only means to rule out arguments that derive the axioms from other truths that we are *already justified* in accepting. His own arguments, by contrast, derive them from a goal: they show us that the axioms must be true if that goal is to be reached. The goal in question is the construction of a logical system; but because such a system is needed in every science, the goal is none other than truth itself.

Frege's discussions of truth and logic are conducted independently of any claims about language, which sits badly with his reputation as the herald of the "linguistic turn," who proclaims the philosophy of language to be the foundation of all philosophy. This calls for a re-evaluation of the role of language in Frege's philosophy. I argue that Frege's philosophy of language is a grand technological project in *linguistic engineering*: he does not aim to describe language, but to change it so that we can reach the truth more effectively. Reading Frege this way helps us understand the most puzzling passages in his early work, and, I hope, points us to a resolution of the outstanding puzzles in his later philosophy of language as well.

Preface

An undergraduate student was telling me recently about his final paper for a class on the theory of meaning. He told me that he planned to describe a counterexample to a certain theory of reference—"because," he added, "that's what you do." I puzzled over this unprompted addition as he described his paper, which centered on the description of an odd sort of case. While the theory in question implied that in such a case, the referent of a certain name is a certain person, this student thought that the referent was someone else. "Or anyway," he continued, unprompted, with a shrug and a weary smile, "That's the intuition I'm going to say that I have."

These remarks reminded me of my early undergraduate days. It seemed then that at the core of every philosophical argument stood a picturesque example, populated by Super-Spartans, colorblind neuroscientists, Gödel and Schmidt, and other characters. I was fairly certain that in branches of philosophy other than those that interested me, things worked the same way, though the examples were perhaps more gruesome—I heard grim rumours of drowning children, disturbing surgical procedures, and vehicles mowing pedestrians down *en masse.* The core of philosophical argument was to describe some characters in an odd situation, and to profess to be moved in some way, experiencing an "intuition"—an *inner compulsion*—to say something about that situation. A philosophical *theory* was the bold attempt to say something general such that no one would be able to tell a story that would produce an intuition that conflicts with that general claim. My own philosophical writing was replete with reports of strong intuitions. I expect, however, that I did not always feel those intuitions quite as strongly as I professed; and if asked why philosophers do this, I would have had little more to say than "that's what you do." I think I saw philosophy as largely like a game of strategy which, like all games, had somewhat arbitrary rules. (Not coincidentally, I was first drawn to philosophy by a formal logic class, with its usual focus on finding proof-strategies to get from one place to another.)

But by the time I graduated, I had learned that this was not all there is to philosophy. I was intrigued—even a little shocked—to read Gareth Evans' claim that

"the truth or falsity of Russell's Principle is not to be decided by evidence as to the pattern of ordinary English usage...the deliverances of untutored linguistic intuition may have to be corrected in the light of considerations of theory."[1] To use "considerations of theory" to *trump* intuitions was a major change to the rules of the game as I knew them. I had thought that "theories" were nothing more than attempts to make general claims without running afoul of intuitions. I increasingly began to read work that was characterized by *principles* rather than intuitions: the Principle of Acquaintance, the Generality Constraint, Russell's Principle, the Context Principle, and more. At first, I was excited by the power of these principles: they were new, apparently winning moves in the philosophy game.

But where do these "principles" and "considerations of theory" come from, and what explains their power? At crucial turning-points in their articles, I found philosophers asking a particular question: *why do we care about this*? In one paper that became a favourite of mine, for example, Richard Heck asks: "How...can we resolve disputes about whether someone has understood?...Let us...ask: why do we care whether we understand one another?"[2] By answering this question, we can learn something central about what understanding is: answering this question yields a *principle.* Discovering this source of principles was a big discovery, because it allowed me to come up with my own principles—potentially unlimited winning moves in the philosophy game! But if I wanted that argumentative power, I had to think about why we care about the things we talk about in philosophy. These reflections were the undoing of my thinking of philosophy as a game.

I started to use these kinds of considerations to arrive at principles, and on this basis to determine how things are. But I quickly became uneasy about the approach I was pursuing. After all, why should what we care about have any consequences for how things are? This approach to philosophy seems to presuppose a connection between what is good (or what we want) and what is so. It is perhaps not surprising that an approach characterized by such a presupposition could spark serious philosophical interest,[3] but if that really was the presupposition of this method, how could it be justified?

---

[1]Evans 1982, 70.

[2]Heck 1995, 84.

[3]Plato's Socrates describes his philosophical development in *Phaedo*, 97b-99c: "One day I heard someone reading, as he said, from a book of Anaxagoras, and saying that it is Mind that directs and is the cause of everything. I was delighted with this cause and it seemed to me good, in a way, that Mind should be the cause of all. I thought that if this were so, the directing Mind would direct everything and arrange each thing in the way that was best. If then one wishes to know the cause of each thing, why it comes to be or perishes or exists, one had to find out what was the best way for it to be, or to be acted upon, or to act. On these premises then it befitted a man to investigate only...what is best...I would gladly become the disciple of any man who taught the workings of that kind of cause."

I didn't know the answer, but I thought I knew where to look. All of my favourite philosophers—the ones who argued about language and logic from principles motivated by why we care about things—were very closely associated with Frege's writings, and especially with Michael Dummett's interpretation of them. Sure enough, the first few pages of the *Foundations of Arithmetic* finds Frege rattling off a numbered list of "fundamental principles," and at crucial points in Dummett's discussions we are sternly reminded how to pursue our questions: "Before we proceed further, let us ask what we want the notion of reference...*for*. If we do not keep asking that question, we are liable to fall victim to scholasticism."[4] I planned to study these works carefully someday, in order to understand this method better—and my resolve to do so increased whenever one of my fellow graduate students would reply to an argument of mine by saying something like: "Who cares what we want a notion of reference for? The facts about reference are what they are, independent of our wants! What *I* want is to get it right about reference."

But not only was I concerned to explore and justify this philosophical method, I also often found myself at a loss to pursue it. I recall trying to evaluate a dispute among philosophers about the identity conditions of propositions. In accordance with the method, I straightaway tried to determine why we cared about the identity conditions of propositions, but I did not get far. I recalled that this question (put in terms of "thoughts" rather than propositions) mattered very much to Frege. I hoped that Frege could show me how to connect such esoteric questions with something that I could care about.

Frege did not disappoint. I found not only a model for how to wield considerations about what we care about to settle seemingly esoteric questions,[5] but also a conception of truth and of the philosophical project that bear on the legitimacy and value of this method.[6] I learned that the philosophical method that so impressed me was in some ways the characteristic method of the Neo-Kantian period within which Frege was working, which has not yet received the attention it deserves. As one recent article puts it, "the Neo-Kantians have been prematurely silenced. A philosophical discussion which conceivably could (and should) have continued uninterrupted through the twentieth century and into our own, was effectively broken off."[7] Philosophers are now starting to talk to the Neo-Kantians again, and I hope that these chapters will play a role in the effort to open up new lines of communication between us and figures like Wilhelm Windelband and Hermann

---

[4]Dummett 1981c, 157.

[5]See the conclusions reached about grounding in Chapter One, about logicality in Chapter Three, and about language in Chapter Five.

[6]See the views of truth in Chapter Two and of justification in Chapter Four.

[7]Makkreel and Luft 2010, 9.

Lotze, with Frege himself as a mediator. The title of this dissertation itself comes not from Frege, but from Lotze, who claims that "a haunting conviction of the existence of truth pursues men everywhere."[8] The center of Frege's philosophy, as I understand it, is the idea that truth is something that it makes sense to say this kind of thing about.

The undergraduate student with whom I recently spoke is not the only philosopher pursuing certain questions in a certain way merely "because that's what you do." Philosophers always face doubts, both about whether our questions are worth asking and about whether our methods are suitable to answer them. The great thing about the philosophical method that so impressed me is it does not allow the first question to be postponed or ignored for long: it is only *by* deciding whether and why a philosophical question matters that we can take the first step toward answering it. Whether or not this is ever a good way to learn how things are is a difficult and complicated question, but the method that requires us to continually remind ourselves why we care at least helps us to find and sustain the motivation to ask it.

<div align="right">

Berkeley, California
May 2018

</div>

---

[8]See Lotze 1856-1864, Book V, Chapter IV. The phrase is Constance Jones' translation of Lotze's "die Ahnung, dass es eine Wahrheit gibt, verfolgt den Menschen überall." When I refer to works published in languages other than English in this dissertation, where the bibliographic entry gives the title in the original language, the translation is mine; when that entry gives the English title, that entry also names the translator.

Acknowledgements

Contents

Chapter 0   Introduction

Upon opening Frege's books on logic, one is greeted with unfamiliar symbols that are introduced to talk about a range of unfamiliar things, such as the "functions" of negation and identity. Frege makes various claims involving these things, such as that everything is identical with itself. Some of these claims are identified as "axioms," and much space and effort is devoted to proving the others from the axioms, according to specified "modes of inference."

The logic which these books contain is widely regarded as having great historical significance.[1] But what did Frege himself think he was doing when he invented it? Why did he think that these symbols, proofs, axioms, and so on belonged together? This is the first of two main questions of this dissertation: what did Frege think logic is?

Frege's answers to this question often point to a special relationship between logic and *truth*: "I assign to logic the task of discovering the laws of truth...the meaning of the word 'true' is spelled out in the laws of truth."[2] Beyond just *being true,* though, what relationship to the truth does the law that everything is self-identical have? Indeed, what more is there to truth, such that the meaning of the word could be "spelled out" in a logical law like that one? The second main question of this dissertation, which is connected closely with the first, is: what did Frege think truth is?

The dissertation divides into two parts. Part One provides answers to the two main questions. Part Two uses the results of Part One to provide answers to other major questions about Frege's philosophy. In this introduction, I will explain how the main questions are answered, how the other questions arise from the main ones, and how those other questions are answered. After this summary, I will discuss some overall themes that tie the various topics together, which will also serve to identify some distinctive features of the approach to Frege that I have taken.

---

[1]See, for example, Beaney 2016: "Frege's creation of modern logic...lay at the heart of what can justifiably be regarded as the 'analytic revolution' that took place in philosophy in the decades around the turn of the twentieth century."

[2]Frege 1818-1919a, 352.

Interpretive Questions and Answers

What is Logic?

Frege gives his answer to this question in terms of what is distinctive of the *laws* of logic: those axioms and theorems which concern the functions mentioned above, and which Frege's new notation is designed to effectively express. He says that these laws are special because they have a certain kind of *generality*: "they are the most general laws, prescribing how to think wherever there is thinking at all." Hence, "the task we assign logic is only that of saying what holds with the utmost generality for all thinking, whatever its subject matter." But does this really identify a special feature of logical laws? Currently, commentators go in two directions.

The largest group thinks that in making these statements, Frege does not really mean to identify anything special about logical laws at all—they amount to a sketchy gesture at some feature that logical laws tend to have, but which does not really distinguish them from the laws of the other sciences. I think that we should avoid this interpretation if we can. Frege certainly *looks* like he is trying to tell us that the laws of logic are special because of their generality.

The other group notices the reference to *prescribing* in Frege's claims about generality. They think that Frege is pointing to a special *normative* feature of these laws—something about the way they tell us what we ought to think. These readers describe various features that a law might have which has something to do with normativity, and suggest that that one such feature is what Frege has in mind—but in doing so, they depart from Frege's text in serious ways. Frege is straightforwardly saying that these laws are special because they prescribe for all thinking. But readers tend to tack additional content onto this simple claim: for example, according to one reader, he is saying that these laws are special because *to be a thinker* is to *recognize and accept* the prescription that we ought not think what is inconsistent with these laws. But Frege makes no claims about recognizing prescriptions or what it is to be a thinker in these passages. This second group of commentators complicate what Frege is saying in a textually unmotivated way, which we should avoid if we can.

We should, then, try to understand Frege as saying simply that these laws are special because they tell us how all thinking ought to be. To find out how these prescriptions distinguish the laws of logic from the other laws, we need to know what the prescriptions are. Frege makes it clear that by "thinking" here, he means "judging": the mental activity whose goal is the *truth.* The prescriptions for thinking come from its goal: they are the prescriptions we must follow if we are to reach the truth. But this makes it hard to see how a law like "every object is

identical with itself" provides us with any prescriptions for all our thinking that would not be provided by non-logical laws too. Perhaps it tells us not to think anything that is inconsistent with the law—we ought never, for example, think that some particular object is *not* identical with itself. But this kind of prescription will not distinguish logical laws from those of the other sciences. Since it is a law of biology that every object is either a mammal or not a whale, any thinking that is to be true must be consistent with that law. We ought never, then, think that some particular object is a whale and not a mammal, just as we ought never think that some particular object is not self-identical.

The key question, then, is: what prescription that specially applies to all our thinking is connected with the laws of logic? Only if there is such a prescription will we be able to take Frege at his word, when he says that the laws of logic are special because they tell us how all our thinking ought to be. It is very difficult to think of the kind of prescription we need, and I think we would never make progress without turning to the other main question of this dissertation. We need to know how Frege understands the goal of thinking, which is the source of prescriptions for it: *truth.*

What is Truth?

We must not expect anything like a *definition* of truth, because Frege says that it is too basic to admit of definition. But he does accept that there are certain things that belong to the *nature* of truth, and in that sense, he has a view about what truth is.

Commentators have spent a lot of time on various aspects of Frege's treatment of truth, but the key aspect for us has been relatively neglected. It is what he is getting at when he claims that: "in logic, we are concerned with truth in the strictest sense of the word," and that this is "truth in the scientific sense" which is "that sort of truth which it is the goal of science to discern." I argue that these clarifications show that Frege thinks of truth itself as something whose nature is to be found partly in a relation to science.

In particular, Frege thinks that it belongs to the nature of truth to figure in the ambitious *goal* of science. This is a surprising claim. Most of us think that there are truths that cannot figure in the goals of science. (For example: a goal of science is to give scientific explanations. But not all truths can figure in scientific explanations. The fictional truth that Sherlock Holmes lived on Baker Street is an example of a truth that could have no place in scientific explanations.) But Frege thinks that it is in the nature of truth to figure in these goals, and we see this by the way that he denies the status of truth to various plausible candidates on the

grounds that they cannot figure in the goal of science. He also gives arguments that presuppose that all truths can figure in the goal of science.

In holding this view, Frege is right in line with his philosophical milieu: I argue that Hermann Lotze, Hermann Cohen, and Wilhelm Windelband also all accept that it belongs to the nature of truth to figure in the goals of science.

So: Frege thinks the laws of logic are special because they issue prescriptions for all our thinking. We now know that he thinks the goal of thinking, truth, is something whose nature involves figuring in the goal of *science.* We are looking, then, for a prescription that we must always follow if we are to think something to whose nature it belongs to figure in the goal of science. But this does not help much unless we know which constraints this places on truth. What exactly does Frege think the goal of science *is,* and what can figure in it?

What is the Goal of Science?

It is well-known that Frege is working with a *systematic conception of science.* I mentioned that the logical laws in Frege's book are organized into primitive truths from which other truths are derived by proof. Frege thinks that it is the goal of all science to construct systems of this form. The details of his thinking about this goal, however, are not well-understood, and I try to uncover enough of them to help us understand what Frege is supposing all truths will be able to do, when he supposes that it belongs to their nature to figure in the goal of science.

The construction of a system is science's goal because properly constructed systems bring us *understanding.* As Frege puts it at various points, they acquaint us with the "essence" or "nature" of sciences, afford us "mastery" and "command" over the domains of these sciences, and yield "explanations" of the phenomena belonging to these domains. They do this by effecting a certain kind of *simplification*: a completed system, in a sense, packs the whole science into the primitive truths. Frege's major requirement on acceptable systems is that they be maximally *simple,* in the sense of having as few primitive truths as possible. This requirement derives directly from the way that these systems provide us with the understanding they do: the fewer those primitive truths are, "the more perfect a mastery can we have." The goal of science is to construct maximally simple systems, in which the truths belonging to the relevant domain are proved from as few primitive truths as possible.

On the systematic conception, the boundaries between sciences are very important. Frege frequently pronounces on which bodies of truths go together to make up a science, and points out when the truths of one science do and do not appear in others. (The truths of physics, for example, will appear in the scientific system of chemistry; but they will not appear in the system of geometry.)

4

Since it belongs to the nature of truth to figure in the goal of science, this means that all truths can fit into a system of this kind. That is: every truth must be able to figure in proofs, and must belong to a body of truths with a well-defined boundary that collectively can be derived from as few primitive truths as possible.

So What is Logic?

We can now identify a way in which logical laws tell us how *all* our thinking ought to be, while the laws of other sciences do not.

Consider a truth which appears in only one scientific system. Its status as a truth depends on its having a place in that system—that is, on its relation to all the other truths in that system. By contrast, the truths that appear in multiple sciences do not relate in this way to the truths in any given system. If there were (somehow) no science of chemistry, the truths of physics would not lose their status as truths, because they also appear in other systems. On the other hand, if the laws of, say, geometry appear in every science in which the laws of physics appear, then the laws of physics bear this relation to those of geometry. The status of the laws of physics as truths would depend on the laws of geometry.

I argue that what Frege thinks is special about the logical laws is that *every* truth depends on them in this sense: they appear in every science. The prescription that brings out what is special about logical laws is that all of our thinking ought to be of what has a place in a system that includes the laws of logic. By contrast, some of our thinking may be of what does not have a place in a system alongside the laws of physics, the laws of geometry, and so on for every other set of laws. This is the prescription all our thinking must follow if it is to reach the truth. This allows for the straightforward reading of the normative generality of logical laws, and it makes sense of Frege's otherwise cryptic claims that the laws of logic are in a special sense, the laws of truth.

Why does Frege think it is distinctive of the laws of logic that they appear in every science? There are a host of sub-questions here. (Why don't the laws of geometry appear in every science? Why can't a science get along perfectly well without a few logical laws?) They are to be answered in terms of further details of Frege's conception of science. In every case, the answers are provided by drawing out what simplicity involves, what determines the borders of the different sciences, and so on. I indicate the kinds of answers that I think he would give to these questions, but finding fully satisfying ones depends on coming to a full understanding of Frege's conception of science. I think that this is the most fruitful direction in which future work on Frege will go. It is clear, however, that Frege *does* think that the laws of logic are special in that they are the ones that appear in every science,

and this provides us with the normative feature that is distinctive of them.

Let me summarize Part One of the dissertation. The main questions are: what does Frege think logic is, and what does he think truth is? These two questions turn out to depend a third: what does Frege think the goal of science is? The three chapters of Part One answer these questions in the following order:

1. **Frege on Science: Simplicity and Primitive Truths** argues that according to Frege, the goal of science is to construct a system of truths that is maximally simple.

2. **Frege on Truth** argues that according to Frege, it belongs to the nature of truth to have a place in one of those systems whose construction is the goal of science.

3. **Frege on the Generality of Logical Laws** argues that according to Frege, a logical law is a truth that figures in every one of the systems whose construction is the goal of science, and in which every truth necessarily has a place.

In Part Two, I turn to two questions that are prompted by these answers.

What Justifies Us In Accepting Logical Axioms?

It is notoriously difficult to find Frege's answer to this question, because he seems to say inconsistent things. One might hope that the new understanding of what Frege thinks logic is that we gained in Part One will help us finally find a satisfying answer, and I think that it does.

The basic difficulty is that Frege calls axioms "unproveable" and "self-evident," claims that "it is part of the concept of an axiom that it can be recognized as true independently of other truths," and claims that the justification of axioms "cannot reside in other truths which have already been recognized." One expects that when he offers an axiom, he will simply state it, note that it is self-evident, and move on. But instead, he offers arguments for his axioms that seem to appeal to other truths.

Commentators have sought to avoid the conflict between what Frege says and what he does by assuming that the arguments are not there to justify us in accepting the axioms. They have focussed on finding other purposes which the arguments might serve. But there is strong evidence that the purpose of the arguments is indeed to justify the axioms, and I suggest a new way to avoid this conflict.

I argue that Frege's claims about axioms only imply that they cannot be justified on the basis of other truths *which we are already justified in accepting.* One way to respect this point is not to appeal to any other truths in justifying the axioms, but there is another: to appeal only to truths that we are not yet justified in accepting. This opens up a new way to resolve the puzzle if we can make sense of how Frege could think it possible for an argument to justify us in accepting a conclusion even though we are not justified in accepting its premises.

As it turns out, the dominant approach to the justification of axioms in Frege's day calls for just such arguments. According to the *critical method* most associated with Wilhelm Windelband, we can justify an axiom by deriving it, not from truths we are justified in accepting, but from a characterization of a *goal,* by presupposing that this goal can be reached. If Frege's own arguments fit this model, this can explain why he thinks they can justify the axioms.

Based on what we have learned in Part One, I argue that Frege's arguments do fit this model. The claims from which Frege derives the axioms are motivated by his thinking about what the goal of judgement, truth, is—in particular, they are motivated by his thinking about the simplicity of the scientific systems in terms of which he understands truth. Frege derives those axioms, then, directly from that goal, in accordance with the critical method.

What does Language Have to Do With Anything?

Frege says a lot about language, but his linguistic claims have been given no role in my account of his views of science, truth, and logic. One might worry: how could what I have said be true if it does not integrate those linguistic claims? In any case, how do those linguistic claims fit in with what I have said?

I think that commentators have not yet separated Frege's philosophy of language sharply enough from the rest of his work. Though few today accept Michael Dummett's famous claim that Frege sees the philosophy of language as the foundation of all philosophy, many still think that Frege's linguistic claims have at least *some* role to play in his arguments for non-linguistic conclusions. I argue that they do not: Frege's linguistic claims belong to a distinct side-project that he pursues alongside his logical investigations. The aim of this side-project is to design new linguistic practices, based on the views he arrives at about truth and logic, that will help us reach the goal of thought more effectively. Frege is not trying to tell us how language is; he is trying to tell us how it would be, if it were as helpful as possible for reaching the goal of truth. He hopes to design new linguistic practices that we can adopt when our only concern is with that goal.

I argue for this view by focussing on Frege's two most famous linguistic doctrines: the *context principle* that words have meaning only in propositions, and

the *recognition requirement* that to name an object we must be able to recognize it again, distinguishing it from all other things. I argue that these claims are never premises in arguments for Frege's non-linguistic conclusions. They are, rather, prescriptions for how we ought to speak, at least when our only concern is with reaching the goal of thought.

If this is correct, it suggests a fruitful direction for further work. In order to answer the outstanding questions about Frege's philosophy of language, we must stop looking for ways in which his linguistic claims figure as premises in arguments, and start looking for ways in which we would more effectively reach our goal, as Frege understands it, by speaking the kind of language he is describing.

Part Two of the dissertation, then, looks like this:

4. **Frege's Critical Arguments for Axioms** argues that Frege tries to justify his logical axioms by deriving them from our goal, truth.

5. **Frege's Early Philosophy of Language** argues that Frege's most famous linguistic claims do not play any role in his arguments. They belong to a side-project: the construction of new linguistic practices that will help us reach our scientific goals as effectively as possible.

Overarching Themes

Let me conclude this introduction by saying something about three over-arching themes that tie together the various aspects of my reading of Frege. These themes also serve to explain why my reading has the distinctive features that it does—for reasons that I will describe, readers of Frege are sometimes reluctant to give certain aspects of Frege's work their full weight. The themes are:

1. The fundamental role of *normative claims*.

2. The close connections between Frege's own views and those of the *Neo-Kantian* philosophers of his day.

3. The exclusive concern with *science*.

These themes are connected, as comes out most clearly in Chapter Two: the fundamental role of normativity and the focus on science are both key aspects of the outlook shared between Frege and the Neo-Kantians.

The Fundamental Role of Normativity

On my reading, Frege's normative claims—his prescriptions and claims about values and goals—are fundamental aspects of his view, which are not reduced or explained away. For example, I take seriously his claim that what is special about logical laws is their normative role, and the only explanation of why these laws have this role is itself normative. I give full weight in Chapter Two to Frege's treatment of truth as a value, and in Chapter Four to the way that the premises of his arguments for the Basic Laws come from the goal of simplicity.

I think that some readers have a tendency to downplay or dilute these aspects of Frege's view. For example, others have recognized that Frege's talk of "generality" is connected with prescriptions, but they have sought semantic or metaphysical explanations of these facts, so that the distinctive feature of the logical laws would turn out to really be a semantic or metaphysical fact. I think people read Frege this way because they themselves are uneasy about the idea that the deepest level of explanation could be fully normative.

But it would be a mistake to read such uneasiness into Frege. Frege was writing in a philosophical context that was dominated by his own teacher, Hermann Lotze—the same Herman Lotze who declares in an influential passage that he is "certain of being on the right track, when I seek in that which should be the ground of that which is."[3]

The Neo-Kantian Connections

This brings me to the second theme. Chapters Two and Four have a common structure: I begin with some claims that Frege makes, and offer what seems like the only way to make sense of them. The resulting view, however, strikes us as odd. (Could Frege *really* think that it belongs to the nature of truth to figure in the goals of science?) I seek to remove this hesitation by showing that the view in question is the standard one of the time, so that we ought to be surprised neither by the fact that Frege holds it, nor by the fact that he does not make his view clearer than he does. Once we see that the odd-seeming view is shared between major figures of the day, I look to those figures to fill in some of the details that Frege does not provide. I hope this seems like a reasonable way to proceed to readers with no strong views about how to do the history of philosophy.

But many Frege scholars hold strong views about how to do the history of philosophy.[4] In the context of disputes about these views, it might seem that read-

---

[3]Lotze 1879, Volume III, Conclusion.

[4]See the opening of Heis 2013 for a good description of some of these views, which center around the wide-ranging debates between Dummett and Sluga.

ing Frege in relation to his contemporaries shows a commitment to a scholarly, historical approach, focussed on providing a scrupulously accurate understanding of what Frege is saying by detaching his writings from our current concerns and connecting his work instead to the weird obsessions of the nineteenth century context. Some readers, then, may prefer to ignore the Neo-Kantian connections that inform my readings because of their commitment to doing the history of philosophy as a way of doing *philosophy*: they want to read historical figures in a way that speaks to our current philosophical concerns.

But I myself am strongly committed to doing the history of philosophy in that way too. I think that reading Frege in the light of the Neo-Kantians *is* the way to illuminate the aspects of his views that most speak to our current concerns. I try to bring out the significance of Frege's views for us along the way.[5]

The Exclusive Concern with Science

Frege frequently claims to be exclusively concerned with science: with truth "in the scientific sense," with a concept of number "usable for the purposes of science," and so on. In combination with giving Frege's normative claims a fundamental role, taking these claims seriously leads to a reading on which he is resolutely focussed only on the most ambitious of our cognitive activities.

Readers may be reluctant to give these claims their full weight, because doing so seems to confine all of Frege's work to a branch of the philosophy of science. Then that work would seem to have no significance for a general account of language and thought, and hence no significance for the issues that are most central to our lives.

I think that it may indeed be true that Frege has little to contribute to a general theory of language and thought. (Moreover, I suspect that Wittgenstein is right that such a general theory is not really possible—which means that nobody else has much to contribute to one either.) But if my reading is correct, Frege's points are not confined to a branch of the philosophy of science, either. His talk of "science" signals that his work is concerned with the kind of normativity that is connected with our highest cognitive goals. What he says will have consequences for other aspects of philosophy to the extent that this kind of epistemology does. I think that to suppose that this deprives Frege's work of its implications for what is central to our lives would be a very serious mistake: that of underestimating the role that ideals play for us.

---

[5]See especially 2.4 about truth, 3.6 about logic, and 4.5 about justification.

Part One

Chapter 1   Frege on Science: Simplicity and Primitive Truths

Abstract

What does Frege mean when he talks about "science"? I attempt to fill in some of the details of his version of the *systematic conception of science.* In particular, I try to understand his requirement that systems be maximally *simple,* and how it relates to his idea that proofs stand in relations of *dependence* or *grounding.* I then use those considerations to resolve a disputed issue about primitive truths.

1.1   Systematic Science

At one point, Frege announces that "science only comes to fruition in a system. We shall never be able to do without systems."[1] This is no surprise. The *Posterior Analytics*-inspired, systematic conception of science is one Aristotelian doctrine that survived the transition from the medieval to the modern period, and it continued to dominate thinking about science until the 20th century.[2] According to this conception of science, the scientist's goal is not only to discover truths, but to identify some of those truths as *primitive* and others as *theorems,* and to provide a *proof* of every theorem from the primitive truths.[3]

All systematic theorists of science agree on that much, but there is a great deal of room for substantive disagreement about what requirements a scientific system must satisfy. Early modern philosophers, for example, typically relaxed the medieval requirement that only syllogisms counted as proofs, while introducing

---

[1]Frege 1914, 242.

[2]See de Jong and Betti 2010 for a discussion of this conception of science.

[3]As Frege 1914 (204-205) puts it, "Science demands that...we do not rest until we come up against something unprovable...If we assume that we have succeeded in discovering these *primitive truths*...then [the science] will appear as a system of truths that are connected with each other by logical inference," and "the totality of these inference-chains constitutes the *proof* of the theorem."

a new requirement that the truths in the system possess a certain kind of "certainty."[4] Once a theorist has laid down some *basic* requirements, those requirements can combine with each other and with independent claims (such as metaphysical claims) to imply other, *derived* requirements. For example: if all truths in the system must be certain, and the theorist independently accepts that only intellectual intuition yields certainty, there will be a derived requirement that every truth in the system be intellectually intuitable. Sometimes, the very same requirement might be laid down by one theorist as basic, and by another only because it is derived from other requirements. For example: one theorist might require that only *necessary* truths appear in the system, out of a conviction that only what is necessary can be truly understood; another theorist might derive the same requirement from the basic requirement that sciences consists of truths, combined with an independent metaphysical conviction that all truths are necessary.

Where do basic requirements come from? As just indicated, they reflect a philosopher's basic convictions about what it takes to understand something. The point of constructing a systematic science is to come to understand something in the best possible way, so that to say (for example) that proofs in systematic sciences need not take syllogistic form is to say that it is possible to understand something perfectly well even when our proofs do not take that form. The basic requirements that a philosopher places on science, then, tell us most directly what that philosopher thinks understanding something really is. This means that determining the particular version of the systematic conception of science a philosopher endorses is a very useful way to approach their thinking. If we can disentangle the various requirements from each other, determining which are derived and which are basic, as well as determining the source of the derived ones, we can learn a great deal about them.

I think that readers of Frege are missing out on this fruitful source for understanding him, because we have not gotten very far in understanding the particulars of his version of the systematic goal. So much is true for any systematic theorist of science, but there is an additional reason why it is dangerous to neglect *Frege's* conception of science in particular. This is that it puts us at risk of misunderstanding him when he makes specific reference to science in order to explain what he is saying—something he does quite often.[5] If we do not know how he is thinking

---

[4]For some discussion of variations to the systematic model in the early modern period, see, for example, Sorell 2010, Jesseph 2010, and Phemister 1993.

[5]Here are a few examples. Frege 1884 (§57) tells us that he is guided by the goal to find "a concept of number usable for science." His justifications of his *Begriffsschrift* in Frege 1880-1881 make frequent references to what science does and does not require. He repeatedly claims to be talking only about truth "in the scientific sense" (Frege 1906a, 186; Frege 1914, 232) and "that sort of truth that it is the aim of science to discern." (Frege 1918-1919a, 352.)

about "science," we may miss what he is trying to tell us.

I would like to start making progress on Frege's notion of science by coming to understand a central requirement that he places on systems: *simplicity.* Understanding Frege's notion of simplicity in the way I do will allow me to resolve a disputed issue about his view of primitive truths in empirical sciences: whether they include particular facts about particular objects.

1.2 Simplicity

Frege frequently emphasizes that our system must exhibit a certain kind of *simplicity:* the reduction of the number of primitive truths to as few as possible. He thinks that this reduction is "in itself a goal worth striving for," identifying it as "a basic principle of science to reduce the number of [primitive truths] to the fewest possible."[6]

Simplicity of one kind or another is widely agreed to be a goal of scientific thinking. It is, however, a requirement that is basic for some and derived for others; and many common sources for the requirement prove to be unavailable to Frege. For example:

1. Often the requirement is *derived* from the requirement that the claims in our system be true, along with an independent metaphysical principle such as Newton's famous claim that "Nature is pleased with simplicity, and affects not the pomp of superfluous causes."[7] That claim implies that the theory that is simplest (in Newton's sense) will be the one whose claims are true, or perhaps most likely to be true. But whether or not Frege agrees that nature is simple, and whether or not simplicity is a derived requirement for him, we know he is not deriving it in this way. This is because he insists upon further simplification even when the truth of the claims is already "beyond all doubt."[8]

2. Sometimes simplicity is put forward as a *basic* requirement, motivated directly by a pessimistic conception of the kind of understanding to which we can aspire. Theorists like Thomas Huxley, for example, recommend minimizing primitive truths because "In ultimate analysis everything is incomprehensible, and [therefore] the whole object of science is simply to

---

[6]Frege 1884, §2 and Frege 1880-1881, 36. For more endorsements of this idea, see Frege 1879, §13, Frege 1893, Introduction, and Frege 1914, 204-205 and 242.

[7]Newton 1687, Book III, Rule I.

[8]Frege 1884, §2. See also Frege 1879, §13: we continue to prove things "not in order to make [our truths] more certain."

reduce the fundamental incomprehensibilities to the smallest possible number."[9] Whether or not Frege thinks of simplicity as a basic principle, he is not likely to agree with this claim about why that is. Frege gives every indication that he thinks of the axioms of geometry and logic, which are the primitive truths of that science, as paradigms of comprehensibility.

Despite being so widely shared, then, it is not obvious for any particular theorist just where the simplicity requirement comes from. Frege himself does not give any very clear statement of why he accepts the simplicity requirement, and some familiar motivations for it are unavailable to him. Why, then, does Frege accept this requirement?

Deriving Simplicity: Proof and Grounding

Let us begin with a promising way to make sense of simplicity as a *derived* requirement. Simplicity, in Frege's sense, is the minimization of primitive truths. But it is an obvious fact about systems that the number of primitive truths always varies inversely with the number of theorems, since whichever of our truths is not the one is the other. To minimize primitive truths is to maximize theorems. But theorems are truths that are proved. To minimize the number of primitive truths, then, is to maximize the number of proved truths. If there is a basic requirement to have as many proved truths as possible, this would allow us to straightforwardly derive the simplicity requirement.

Is there a basic requirement to have as many proved truths as possible? Since basic requirements flow directly from what it takes to understand something in the best way, the question is: is there some direct way in which having proofs of as many truths as possible improves our understanding? Fortunately, Frege tells us how proofs contribute to our understanding: they do so in two distinct ways.[10] First, proving something can be a way of gaining *certainty* that what we prove is true; second, proofs serve to "reveal logical relations between truths." Might either of these explain why we should have as many proved truths as possible?

Though added certainty is indeed a contribution to our understanding, Frege is clear that not *all* proofs increase our certainty. As we saw above, he thinks we must continue to prove truths that are already completely certain. If anything can account for Frege's basic requirement to have as many proved truths as possible, it must be something that he thinks *every* proof does. The second function of proof, revealing logical relations, seems more promising here. Presumably Frege has in mind relations of logical *implication*: to prove B from A is to reveal that

---

[9]Huxley 1871, 165.
[10]He lists both ways at Frege 1914, 204, and Frege 1884, §2.

A logically implies B. This would explain a basic requirement to have as many proved truths as possible, if it is always a contribution to our understanding to reveal an implication relation in this way.

But Frege cannot be supposing this. That is, he cannot hold that revealing implication relations is always a contribution to our understanding. If it were, then we ought to prove truths in our systems *more than once,* and we ought also to prove the primitive truths—or if not an official "proof," we at least ought to indicate which truths imply them. But since Frege thinks that each truth is to be proved at most once,[11] he cannot be supposing that it is always a contribution to our understanding to become aware of any implication relation between truths.

Fortunately, Frege sometimes gives a different gloss on the second contribution to understanding that proof makes. Instead of talking about revealing mere logical relations, Frege sometimes says that what proof gives us is "insight into the dependence of truths upon one another."[12] Frege says that it is a truth's *ground* that proof shows us, and that "in Leibniz's words, 'the question is…[about] the connexion and natural order of truths."[13] Frege is saying that truths stand to one another in relations of *dependence* or *grounding,* and that this arranges them into an *order.* Presumably, not all logical implication relations are dependence relations.

Might Frege think that it is always a contribution to our understanding to become aware of a grounding relation? Many philosophers who believe in grounding relations think that it is. Bernard Bolzano, for example, writes that "As the establishment of the objective ground is something so useful that we should communicate it in our books as often as possible, there is no doubt that it must be regarded as a virtue of a proof if the proven truth is derived from its objective ground."[14] If Frege thinks the same way, it could account for a basic requirement to prove as many truths as possible. If all proofs must follow and thereby reveal grounding relations, and if revealing grounding relations is always a major contribution to our understanding, we ought to have as many proved truths as possible—which is to say, as few primitive truths as possible. Moreover, this line of thought seems to avoid the objection above. We can explain why Frege does not prove primitive truths or offer multiple proofs of the same truth, by an independent metaphysical fact: that every truth has at most, one ground.

Promising as it is, it seems to me that there is a decisive objection to this idea, which concerns *asymmetry.* Philosophers who believe in grounding relations typically suppose that the relation is asymmetric: if A grounds B, then B does not

---

[11]Frege 1914 (204-205)

[12]Frege 1884, §2

[13]Frege 1884, §17.

[14]Bolzano 1837, §525.

ground A.[15] But the grounding relation, as Frege sees it, is not asymmetrical. That is because Frege thinks that "it is conceivable that there should be a truth A and a truth B, each of which can be proved from the other in conjunction with truths C, D, E, F, whilst the truths C, D, E, F, are not sufficient on their own to prove either A or B...we have the choice of regarding A, C, D, E, F as axioms and B as a theorem, or B, C, D, E, F as axioms, and A as a theorem...the possibility of one system does not necessarily rule out the possibility of an alternative system."[16] Frege thinks that sometimes, we can choose between proving A from B, and B from A; either system is acceptable. If the grounding relation between truths on which the admissibility of a proof depends were asymmetrical, then what Frege is talking about here would be impossible—the legitimacy of one system would rule out the other.

For Frege, then, the grounding relation is not asymmetrical after all: sometimes, A grounds B *and* B grounds A.[17] But this leads straight to the same problem as before. In cases where A grounds B and B grounds A, why should we just pick *one* system? If we pick, say, the system in which B is proved from A, then there is one grounding relation that our system is leaving out: the relation of B's grounding A. If Frege agreed with Bolzano that "the establishment of the objective ground is something so useful that we should communicate it in our scientific texts in general as often as it is possible," then in such cases, we really ought to indicate both grounding relations somehow. Since Frege's systems do not do this, he must not be supposing that revealing grounding relations is *itself* always a contribution to understanding. Though he may still be supposing that every proved truth makes some contribution to understanding, it cannot be simply because each one reveals a grounding relation.

Even though Frege's proofs always follow grounding relations, then, it does not seem that we can derive the simplicity requirement from this fact.

_____

[15]Influenced by this assumption, some commentators automatically assume Frege's notion is this way too. For example, according to Detlefsen 1988 (98 and endnote 7) a proof for Frege "provides insight into the source or ground of a theorem's truth," and "the grounding relation is asymmetrical."

[16]Frege 1914, 205. This commitment is confirmed as late as Frege 1918-1919c (404) and as early as Frege 1879, §13. One might suppose that this case is better described as a case in which neither truth grounds, or depends on, the other. But Frege always only lists *two* reasons why it is important to prove things: increasing certainty and revealing grounding relations. Since both A and B are acceptable as axioms, we must be certain of both of them without proof; since Frege thinks we need to prove one or the other, then, there must be a symmetrical grounding relation.

[17]The kind of "dependence" between truths is less like the way each link of a hanging chain depends on the next if it is not to fall, and more like the way that when person A depends on person B, it might well be that person B also depends on person A.

Interlude: Leibniz on the Natural order(s) of Truths

Perhaps we are still missing something about the grounding relation that could allow us to derive the simplicity requirement. While Frege says very little about it, he does point us to Leibniz's discussion of "the natural order of truths." We might wonder whether he means for us to discover in Leibniz's texts what he has in mind. It is, then, worth a brief look at Leibniz's "natural order" discussion to see whether something he says can help us understand Frege's notion of grounding in a way that would allow us to derive the simplicity requirement. Ultimately, I think nothing in Leibniz's discussion helps us, and readers willing to take my word for it are welcome to skip this interlude.

As it turns out, Leibniz's discussion involves two different "natural orders." The first consists of proofs that proceed from what Leibniz calls "identities," or "first truths." These are truths that "repeat the same thing without giving us any information."[18] Each one is of the form "A is A" or "A is not not-A": they "predicate something of itself or deny the opposite of its opposite."[19] Leibniz, notoriously, thinks that "all other truths are reduced to first truths with the aid of definitions or by the analysis of concepts."[20] Hence, "there is contained in the perfect individual concepts of Peter or Judas…everything that will happen to them, whether necessarily or freely."[21] This allows for a certain kind of proof of every truth, which reaches its conclusion by invoking only identities and definitions.

Second, there is what Leibniz calls the "natural order of our knowledge." This is under discussion when Leibniz claims that "this proposition, *I am*, is an axiom, and…we may be assured that it is a *primitive truth*…in the natural order of our knowledge…"[22] What the primitive truths in *this* order have in common is that each of them "cannot be proved by anything more certain."[23] The primitive truths have "intuitive certainty," which is the maximum possible: "Whoever asks for greater certitude, knows not what he asks."[24] The order of *our* knowledge is different from the order of identities, because we humans are unable to prove everything from identities—for example, we cannot know the definitions of Judas

---

[18]Leibniz 1765, 404.

[19]Leibniz 1680-1684, 267.

[20]Leibniz 1680-1684, 267.

[21]Leibniz 1680-1684, 3.

[22]Leibniz 1765, 469. Leibniz calls "I am" the "Cartesian Principle," and notes that he says "I am" instead of "I think", because "to say, I am thinking, is already to say, *I am.*"

[23]Leibniz 1765, 410.

[24]Leibniz 1765, 404. Intuitive certainty is possessed only by "experiences and the axiom of identity," (Leibniz 1765, 13) where by "experiences," Leibniz here means "immediate internal experiences of an *immediateness of feeling*," (Leibniz 1765, 410) which for Leibniz includes "I have different thoughts" and "I exist."

and Peter. (On the other hand, "All this is known by God.")[25]

Leibniz thinks every truth proved from identities is very valuable, because such proofs give us "what I value most highly…as related to reflection…a glimpse of the true *source* of [these] truths…"[26] By seeing the source of these truths in an identity, we know *why* they are true. Hence, because "only God can see how the two items I and existence are connected," then, "it is only God who sees…why I exist."[27] The proofs in "our" order are valuable too, but for a different reason. What is good about them is that we gain the utmost *certainty* that is possible for us: the intuitive certainty of the primitive truths provides "demonstrative certainty"[28] for the theorems, which is the second-highest degree of certainty[29].

Unfortunately, this does not help to illuminate how Frege is thinking. The contribution to understanding made by proofs in the order of *our* knowledge is that they increase the *certainty* of what is proved; but Frege distinguishes this from the function of proofs to reveal grounding relations.[30] The order of identities, by contrast, is characterized by employing only proofs from identities; but Frege clearly rejects Leibniz's views that proofs from identities are always possible in logic and mathematics, and that they are the best kind of proof.[31]

Since Frege's notion of grounding does not endorse either of Leibniz's ideas here, why the approving reference to the "natural order of truths"? The specific

---

[25]Leibniz 1680-1684, 268.

[26]Leibniz 1765, 523. The speaker is Theophilus, who represents Leibniz's view.

[27]Leibniz 1765, 469.

[28]Leibniz 1765, 414.

[29]"It is not so clear as the intuitive knowledge, as the image reflected in several mirrors from one to another grows more and more faint with each reflection…" (Leibniz 1765, 411)

[30]In the natural order of our knowledge, as we would expect, there is no point in proving a truth that is already intuitively certain—we may as well take it as primitive. There are some passages in which Leibniz seems to suggest otherwise, but they are misleading. For example, he claims that "to reduce the number of axioms was always something gained," (Leibniz 1765, 464) and praises Euclid for reducing the number of axioms to as few as possible. But such claims only appear in discussion of what Leibniz calls "secondary axioms," which are truths that are *taken as* primitive in systems of geometry such as Euclid's, even though they *lack* intuitive certainty. Reducing the number of *secondary axioms* is good because since secondary axioms lack intuitive certainty, those working with them can easily "fall into errors." Secondary axioms may be in some sense "evident" but their evidence can be "seen only confusedly," hence, "it was better for [Euclid] to limit himself to a small number of truths of this nature." (Leibniz 1765, 523.) Anyway, we should not use secondary axioms at all, since proofs from identities are posible in mathematics: "it is important to demonstrate *all* our secondary axioms…by reducing them to the primitive or immediate and indemonstrable axioms, which I…called the *identicals*." (Leibniz 1765, 464.)

[31]He distances himself from Leibniz's claim in Frege 1884, §15. The logical and geometrical axioms he puts forward are not identities (See, for example, Frege 1893), and he supposes that the primitive truths of empirical sciences will include particular truths acquired by the senses. (See Frege 1884, §3 and Frege 1924/1925.)

passage that Frege quotes is concerned only to distinguish the "natural order" from the various accidental orders in which individual people happen to come to believe things. This suggests that all Frege is getting out of the reference to Leibniz is the idea that a scientific arrangement of truths need not follow the order in which people happen to come to know things.[32] It seems, then, that there is no particularly close connection between Frege and Leibniz on the specifics of the grounding relation.

Simplicity as a Basic Requirement

To recap: we wondered whether the requirement of simplicity could be derived from a basic requirement to have as many proved truths as possible. This seemed promising, given Frege's requirement that proofs follow and reveal relations of grounding. If Frege shared the widespread idea that the more relations of grounding we know about, the better our understanding is, this would explain why there would be a basic requirement to maximize proved truths: every proof would make an important contribution to our understanding. But it turned out that this is not how he is thinking. If he were, he would recommend including more proofs in a system than he does. Not only, then, does this line of thought not allow us to derive the simplicity requirement; it has led us to notice a second question. We have seen that Frege cannot hold that revealing grounding relations is always a contribution to our understanding. But that makes it puzzling why it is *ever* a contribution to our understanding to reveal them. How can every proof contribute to our understanding by revealing a grounding relation, even though we should not try to reveal every grounding relation?

It is time to try a different approach. Let us see whether we can understand simplicity as a basic requirement rather than a derived one. There is some strong evidence pointing in this direction. For example, it is suggested by Frege's claims that reducing the number of primitive truths to as few as possible is "a basic principle of science" and "in itself a goal worth striving for." Moreover, when Frege does derive some requirements on systems from others, he typically derives them from the simplicity requirement.[33]

---

[32]This might seem so basic as just to be built into the systematic conception of science, but it is not: Locke endorsed a systematic conception of science, but thought that the order followed should be the order in which truths are discovered. Leibniz and Frege make this point to register disagreement with such empiricist opponents. (See Wilson 1967.)

[33]For example, Frege derives a certain requirement on the primitive signs of our system from the fact that it would enable us to have as few primitive signs as possible, and he in turn derives the requirement that we have as few primitive signs as possible from the requirement that we have as few primitive *truths* as possible: "the more primitive signs you introduce, the more axioms you need" (Frege 1880-1881, 36.)

If the simplicity requirement is basic, we should expect to see it directly connected with what understanding is. Indeed, that is just what we find. Frege often says that to have a set of primitive truths is to grasp the "nature" or "essence" of a particular domain,[34] and that the fewer primitive truths are in the set, the greater our "command" or "mastery" of that domain is.[35] This directly connects the notion of simplicity with basic terms for the kind of understanding that a science ought to give us: a "command" or "mastery" of the "nature" or "essence" of the domain.

The idea that simplicity is a basic requirement is strongly confirmed by the way Frege connects it with another such basic term: "explanation." According to Frege, "the essence of explanation lies precisely in the fact that a wide, possibly unsurveyable, manifold is governed by one or a few sentences."[36] That is, to have an explanation *just is* to have a simple system.

It looks, then, like simplicitly does connect directly with what understanding is. This makes it a basic requirement on systems. Using this requirement, let us see whether we can derive the right requirement about grounding—one that will make sense of why Frege thinks that every proof contributes to our understanding by revealing a grounding relation, even though we should not try to reveal every grounding relation.

We could derive the right requirement, given an assumption about how the grounding relation works: we must suppose that A grounds B only when some maximally simple system derives B from A. If we make that assumption, then we can combine it with the simplicity requirement to derive the requirement that proofs follow relations of grounding. Then, it will follow that every proof reveals such a relation, and in doing so contributes to understanding *by* contributing to simplicity, but without this implying that we ought to indicate more grounding relations beyond the ones that are necessary to have a maximally simple system. I think that this is the right way to understand what Frege is doing.

If so, Frege is departing from Bolzano's conception of the role of grounding in science in quite a radical way, and the same is true of the notion of *explanation*. Someone who believes in grounding relations typically thinks that revealing each individual grounding relation increases our understanding; someone who believes

---

[34]As early as Frege 1874 (57), Frege emphasized the goal of identifying a "kernel" of primitive truths, from which the rest of mathematics would grow "as from a seed." As he puts it later on, "The essence of mathematics has to be defined by this kernel of truths, and until we have learnt what these primitive truths are, we cannot be clear about the nature of mathematics." (Frege 1914, 204-205.)

[35]We attain a "greater command of the material" when we can assemble the "large mass of detail under a more comprehensive point of view." Also, "the fewer the number of primitive sentences, the more perfect a mastery can we have." (Frege 1880-1881, 39.)

[36]Frege 1880-1881, 36.

in explanations typically think the same thing, that every explanation increases our understanding. On this view, we can see the contribution that a system makes to our understanding as just a combination of the value of each of the individual grounding relations it reveals, or each of the individual explanations it provides. But for Frege, understanding is most basically about having a maximally simple system, and the contribution of such a system to our understanding cannot be understood as the individual value of each of its proofs, added up. Each individual explanation, in which we trace out a particular fact from the primitive truths, is *worthless* on its own. It only makes a contribution to understanding when it is part of a system with all the other proofs—only then does it help a few sentences to govern a wide manifold. As Frege puts it, "the value of an explanation can be directly measured by this condensation and simplification: it is zero if the number of assumptions is as great as the number of facts to be explained."[37] This is a surprising claim! If I am puzzled by a phenomenon, and a scientist tells me the explanation, Frege thinks I am no better off from the perspective of understanding than I was before—at least, not until I can co-ordinate my new explanation with others, and begin to construct a system of the relevant science. The sense that one-off explanations can really give us understanding, for Frege, must be an illusion. All value for our understanding comes back to the simplicity of the whole system.[38]

We might wonder whether all of this makes the grounding relation *itself* in some way dependent on us or on our simple systems of proof. After all, we might say, it sure is *convenient* that there are no grounding relations out there except for those that serve our purposes—those learning about which will contribute to the kind of systems that provide us with understanding. From this, we might conclude that the grounding relation cannot be an objective relation among the truths themselves at all, but only the mind-dependent shadow cast by our drive to simplify. But there are two things wrong with this thought. First, we have not yet seen a reason to think that the grounding relation is *all that convenient.* Even if every grounding relation contributes to our goals, we have not yet said that every time a certain proof would contribute to the simplicity of the system, there is a grounding relation there too. For all we have said, we could go on to fill in more details of the grounding relation that would make it a substantive

---

[37]Frege 1880-1881, 36.

[38]Frege's idea here would be introduced into contemporary philosophy of science by Michael Friedman, under the heading of the "unification theory of explanation." Friedman 1974 (15) writes that "science increases our understanding of the world by reducing the total number of independent phenomena that we have to accept as ultimate or given. A world with fewer independent phenomena is, other things equal, more comprehensible than one with more." Friedman seems to be unaware that Frege is a predecessor, writing that "the only writer that I am aware of who has suggested that this…is the essence of explanation…is William Kneale."

constraint, ruling out our pursuing paths of proof that would be beneficial to our understanding. Second, even if that relation *were* maximally convenient for us, this would only raise a question about how this could be explained. The mind-dependence of grounding would be one explanation, but another would be along Newtonian lines: perhaps, for reasons that have nothing to do with us, "nature is pleased with simplicity, and affects not the pomp of superfluous causes."

Interlude: Kant on Simplicity

We might wonder whether there is more to say about *why* simplicity is valuable. Frege seems to identify it very closely with understanding itself: with explanation, mastery over a domain, and so on. But saying so much could still fit with a view like Huxley's, that "everything is incomprehensible, and [therefore] the whole object of science is simply to reduce the fundamental incomprehensibilities to the smallest possible number." I said that this does not fit with Frege's views about the laws of logic and geometry, but is there any more to say about why simplicity is the essence of understanding for him? What is it about reducing the number of primitive truths to as few as possible to see how "a wide, possibly unsurveyable, manifold is governed by one or a few sentences" that makes it worth our time? A brief look at Kant suggests that Frege would give a Kantian answer to this question.

Kant observes[39] that we often postulate hidden "powers" or "principles" from which the things we have observed will be derivable—for example, we postulate pure elements (fire, water) of which the things we can observe are mixtures. We are tempted to assume that there will always be such hidden powers lying behind apparent diversity, and at the limit, we believe that there must be a *single* principle that accounts for all multiplicity. Kant thinks this is a mistake: we can have no good reason to believe in such a single principle. Our susceptibility to this mistake, however, derives from a deep source: the nature of reason itself.

It is reason, for Kant, that sets us the goal of systematicity,[40] one aspect of which is a search for "parsimony of principles." This is what drives us to search

---

[39]See 1781/1787, Appendix to the Transcendental Dialectic. As Kant 1786 (3) makes clear, the systematic conception of science is definitional: "Every doctrine that is supposed to be a system…is called a science." In addition to the general conception of "science" that is the model for Frege's thinking, Kant has a more restricted notion of "proper science." For a discussion of these restrictions and their relation to the more general notion of "science", see van den Berg 2011 and Friedman's Introduction to Kant 1786.

[40]Reason "unites the manifold of concepts…by positing a certain collective unity as the goal of the understanding's actions." (Kant 1781/1787, A643/B671-A645/B673.) It "prescribes and seeks to bring about…the systematic in cognition…" (Kant 1781/1787, A644/B672.) Kant is here using "reason" for a particular faculty that is distinct from the understanding, while elsewhere, the term is more inclusive.

for hidden powers that will enable us to reduce the number of principles in our system.[41] Though we must be on guard against the mistake just discussed, reason's demand for systematicity is a genuine "regulative principle" that we must strive to satisfy as much as we can.[42] This is so because the demand stems from the nature of our faculty of reason itself. "The law of reason to seek unity is necessary, since without it we would have no reason,"[43] and "everything grounded in the nature of our powers must be purposive and consistent with their correct use, if only we can guard against…misunderstanding,"[44] like the one just described. The *correct* use Kant has in mind is to seek the system by which the multitude of the "possessions of the understanding" can be seen to follow from the fewest possible principles. To have such a system would satisfy one of the "needs of reason."

I expect that this is how Frege understands the simplicity requirement too. Frege shows his Kantian orientation when he himself invokes the "needs of reason" to say why we must have few enough primitive truths for them to be surveyable.[45] He would, then, have a bit more to say about why understanding, understood in terms of simplicity, is worth pursuing: it is what we ought to do because it is what our reason needs.

More Complicated Simplicity

Frege closely identifies simplicity—the minimization of primitive truths—with understanding and explanation. But consider the following objections to such an identification, which aim to show that such simplicity can be very easily achieved, and in ways that obviously do not result in improvements of understanding.

1. By including additional *modes of inference* we can reduce the number of primitive truths in Frege's logical system or in any other science to one—or even, if desired, to zero.[46]

---

[41]Kant 1781/1787, A650/B678.

[42]"Where it can be done, one must…bring systematic unity into cognition." Kant 1781/1787, A649/B677

[43]Kant 1781/1787, A651/B679.

[44]Kant 1781/1787, A643/B671.

[45]Frege 1884, §5.

[46]To take an extreme case, we could add a mode of inference to Frege's system that would allow an inference from Basic Law I to Basic Law II, one that permits us to infer basic Law III from Basic Law II, and so on. Contemporary logicians sometimes prefer systems which operate only with inference rules to the Hilbert-and-Frege-style axiomatic systems. (Dummett 1973 (433-434) credits Gentzen as "the first to correct [Frege's] distorted perspective.")

2. By *conjoining* the primitive truths, we can reduce the number of primitive truths in Frege's logical system (and any other science) to one conjunctive truth.[47]

Such cheap tricks seem neither to increase simplicity nor our understanding, but they seem like they would satisfy Frege's account of simplicity. Now, it may be that additional assumptions about systems—about which modes of inference are admissible, or which truths are admissible as primitive—rule out these moves. We cannot say without taking a closer look at these other aspects of systems. But it may also be that Frege needs a more complex notion of simplicity than what he usually offers. I will provide a response along these lines, because Frege shows some awareness of these points and gives some indication of how a complex notion of simplicity would go.

On modes of inference: Frege acknowledges that we can trade these off against axioms, and claims to always prefer axioms on the grounds that when we do so, we do not "make use of the same thing in different forms."[48] Frege has in mind the way that every mode of inference—whether logical or not—corresponds to a truth: an inference-mode from "P" to "not-not-P," for example, might correspond to the truth that if P, then not-not-P, and an inference-mode from "x is a whale" to "x is a mammal" might correspond to the truth that if anything is a whale, then it is a mammal. Frege seems to think that modes of inference either *are* their corresponding truths in a different "form," or at least strongly depend on those truths: "a mode of inference...must be subject to a law, and this law...can be ranked with the theorems or axioms of this science."[49] There are good questions about what he has in mind, but this suggests that the modes of inference ought to be counted along with the primitive truths in determining the overall simplicity of a system. If we complicate the notion of simplicity in this way, it seems to avoid the objection.

On conjunctions: even though it is only one truth, a conjunction does not seem any simpler than its conjuncts taken separately. Some truths, then, are simpler than others. What Frege needs is a notion of the simplicity of the system that takes into account not only how many primitive truths there are, but how simple

---

[47]As Hempel and Oppenheim 1948 (fn 28) put the point: "The core of the difficulty can be indicated briefly by reference to an example: Kepler's laws, K, may be conjoined with Boyle's law, B, to a stronger law K*B; but derivation of K from the latter would not be considered as an explanation of the regularities stated in Kepler's laws..." (Though Frege's system includes no primitive conjunction-function, one could be defined in a familiar way from Frege's conditional and negation functions.)

[48]See Frege 1880-1881, 37.

[49]Frege 1914, 203. In Frege 1893, he introduces his own inference-rules by arguing for the associated law.

those truths themselves are. As it happens, Frege seems to have such a notion. He claims that the truth "If A then B," is simpler than "A and B," because it rules out only one line of a truth-table rather than three.[50] At one point, though in a sketchy way, he even connects the simplicity of individual truths to their suitability to be primitive truths: "it seems reasonable to derive the more composite...judgements from the simpler ones."[51] Frege does not develop this kind of simplicity in detail, but this at least shows that he is aware of a sense in which truths differ in how simple they are, which seems like just the kind of complication that would need to be added to his account of simplicity.

1.3 Are there Particular Facts among the Primitive Truths?

Before I conclude, I would like to draw attention to the way that the above account of simplicity helps resolve a puzzle about Frege's conception of science that has tripped up many commentators on Frege, and sometimes in ways on which major interpretive points crucially depend.

The question is: what are the primitive truths of empirical sciences? Though Frege does not say very much about empirical sciences, understanding how he is thinking of them turns out to matter for our understanding of his notion of what is admissible as a primitive truth in general. This, in turn, has proven to have major implications for understanding his logicist project of proving the arithmetical truths from the primitive truths of logic.[52]

Many commentators have concluded that the primitive truths of the empirical sciences must be *general*—they must be the laws of the empirical science.[53] That conclusion seems to be supported by what we have seen. After all, if we had to say one-by-one how the objects of the domain are, our system could hardly be very simple; including general laws among the primitive truths seems essential to

---

[50]Frege 1880-1881, 35-36.

[51]Frege 1879, §13.

[52]For example: Jeshion 2001 argues that if we are to understand why Frege thinks he must derive the truths of arithmetic from logical truths, we must understand why he would not be satisfied with a science of arithmetic that began with more familiar arithmetical claims, such as the Peano axioms. She thinks that this requires making several important distinctions among the different things that Frege says about primitive truths, especially connected with the "self-evidence" that at least some of them must exhibit; and to correctly understand that, she thinks we must "examine what unites and differentiates primitive truths...of the mathematical and non-mathematical sciences." Jeshion 2001, 948.

[53]Jeshion 2001 (948) holds that "For Frege, laws of nature are primitive truths of empirical sciences." Textor 2011 (29-30) claims that for Frege, "A truth P can only be the ground of a truth Q if P is more general than Q," so that proofs in empirical sciences, if they are to follow grounding relations, must begin from general claims.

having few of them. This suggests that the science of physics, as Frege sees it, will look very much like the kind of axiomatization that Hilbert calls for in 1900, at the International Congress of Mathematicians in Paris: one in which we "treat in the same manner [as mathematics], by means of axioms, those physical sciences in which mathematics plays an important part."[54]

But this line of thought crashes directly into almost the only clear thing that Frege says about proofs in empirical sciences. What it *is* for a truth to be empirical, for Frege, is that it is "impossible to construct a proof of it without including an appeal to facts, i.e., to truths which cannot be proved and are not general, since they contain assertions about particular objects."[55] One reason for this seems to be that even though (as we have seen) it is not the *only* purpose of proofs to produce certainty, that is one of the main things proofs do for us, and it is something that we need them for in empirical sciences. Frege's discussions of our basic sources of knowledge afford us nothing that we can use to be certain of empirical truths except our senses, which yield general laws only through the use of *induction.*[56] I believe that Weiner is entirely right that "Frege thinks the justification of a posteriori truths—including laws of natural sciences—requires appeals to data, particular facts about particular objects. Moreover, he thinks this appeal needs to be revealed by a systematization of that science. Thus the primitive truths of a systematic natural science will...include supporting data for its laws."[57]

But is Frege *really* supposing that proofs in physics will proceed from a multitude of individual observations to the establishment of general laws by induction? How could such proofs constitute explanations, or follow grounding relations? And won't every such science end up with an unimaginably large number of primitive truths? The reason interpreters seem to ignore what Frege says about empirical sciences, I believe, is that they think those claims commit him to an absurd picture of the project of empirical science.

The first thing to note is that Frege's conception of induction does not require us to pile up of hundreds of observations in order to reach a general law. He describes induction as a process by which laws can be arrived at by combining detailed thinking about probability with a *single* observation—perhaps one made in the context of an important experiment. Because the theory of probability depends on the science of arithmetic, this account of induction is the basis of an argument that arithmetic will be needed in every empirical science: "the procedure of the sciences, according to its objective standards, will sometimes find a high probability established by a single confirmation, and sometimes regard a thousand as

---

[54]Hilbert 1902, Sixth Problem.

[55]Frege 1884, §3.

[56]See Frege 1884, §§9-10, and Frege 1924-1925.

[57]Weiner 2004, 123.

almost worthless...induction must be based on the theory of probability."[58] Empirical sciences, then, need not begin with hundreds of observations. One or a few carefully chosen facts, observed in the context of carefully devised experiments and combined with general laws of probability, are the way to arrive at empirical laws, which in turn imply "a wide, possibly unsurveyable, manifold" of other facts.

The other points that seem to preclude taking particular facts as the primitive truths depend on mistaken assumptions about Frege's views of grounding and explanation. In such a system, any particular line of proof might indeed look strange. (For example: perhaps the fact that a is F, observed in the context of a crucial experiment, combines with the laws of probability to prove that everything is F; this claim is then used to prove that b is F.) Such a proof might not look very explanatory of the fact that b is F, but we have seen that we ought not look for illumination by examining particular proofs anyway; the idea that a single derivation yields explanation is an illusion. The value for our understanding comes only from the fact that when taken all together, the system has few primitive truths and many consequences. Similarly, one might be concerned about the *grounding* relation that a proof like this presupposes. (Can the general law that everything is F really be grounded in the particular fact that a is F, while the apparently similar fact that b is F is itself grounded in the law instead? What is so metaphysically special about the fact that a is F?) But Frege's grounding relation is sometimes symmetrical, and any cases like this are likely to be ones that admit of a wide variety of alternative systematizations. This suggests that we should understand the relationship between particular facts and general laws as a reciprocal grounding relation—they each depend on each other.

Given the right reading of Frege's notions of explanation and grounding, then, there ought not be any obstacles to us taking his claims about the primitive truths of empirical science at their word.

1.4 Conclusion

I hope to have made some progress in understanding the way in which Frege thinks scientific systems contribute to our understanding. I also hope to have shown how understanding this aspect of Frege's thinking can help us understand other thinks he says about science, including claims about grounding and about primitive truths in empirical sciences. We have just seen that these discoveries are relevant to understanding Frege's reasons for trying to prove arithmetical truths from logical ones. I expect that they, and other discoveries like them that are yet

---

[58]Frege 1884, §10

to be made, will be the key to making progress on other important questions about what Frege is doing.

Chapter 2   Frege on Truth

Abstract

I argue that Frege thinks that it belongs to the nature of truth to play a par-
ticular role in the goal of science. I argue for this claim first on the basis of
Frege's own text, and then on the grounds that this was the standard view in
Frege's day and he shows no signs of dissenting from it. I think that Frege's
view deserves to be taken seriously, and I attempt to bring out some inter-
esting features and motivations for it in the final section.

2.1   The Butterfly Model of Science and Truth

Consider butterfly-collecting. Those who live in butterfly-infested lands might in-
teract with butterflies a lot, treating them as objects of mild curiosity, nuisances,
playthings, or snacks. But butterflies are something very different to butterfly
collectors, who devote themselves to careful methods of catching butterflies and
housing them in enclosures, pinning them to mounting boards, or otherwise stor-
ing them. Butterfly collectors find value in these activities, and some may form
strong opinions about the proper method of catching or storing butterflies in or-
der to access that value. (e.g.: "Only by pinning them to mounting boards, *wings-
out*, can you inspect and admire them properly!") Though a mention of butterflies
would be entirely appropriate in saying what the activity of butterfly-collecting is,
the reverse is not true: it does not in any way bear on *what a butterfly is*—the "na-
ture of butterflies"—that they can play their role in (proper) butterfly-collecting.
Indeed, it would not be surprising if there were butterflies too agile to be caught in
the best butterfly nets, too fragile to be pinned to anything without falling apart,
and so on. These butterflies could bring no satisfaction to the butterfly-collector,
because they cannot figure in their activities. We might never be able to find such
butterflies and identify them as butterflies, since they might never make it to the
entomology department for study—but they might nonetheless be out there. And
even if there are no such butterflies—even if *every last butterfly* has the features

that make it suitable to figure in proper butterfly-collecting—this would be an accident. Butterflies are creatures in their own right, without any special relationship to butterfly-collecting.

Let us call this account of the relationship between an activity and the things central to it "the butterfly model." The butterfly model applies when it belongs to the nature of an activity that its goal involves a set of things, but it does not belong to the nature of those things that they be able to satisfy the practitioners of this activity by playing their role in this goal. The fact that some of the things might well not be suitable to play their role in the activity indicates that the butterfly model applies: these are things in their own right, with no special relationship to the activity.[1]

Consider, now, the relationship between science and truth. On standard ways of thinking about truth, we all live in truth-infested lands, and our daily lives are impacted by truths in one way or another. But truths are something else to scientists, who devote themselves to finding out truths by particular methods and to using those truths in various ways: to design technology, to give explanations, or simply as objects of admiration or wonder.[2] Scientists find these activities valuable in a way that goes beyond—or is at least different from—the value in our ordinary interactions with truths. Enthusiasts often have strong views about the proper way to access that value. (e.g.: "It is only a theory that is in principle *falsifiable* that can provide real explanations and genuine understanding!") Many people also see the relationship between science and truth as on the butterfly model. That is: though the nature of science involves a relation to the truth, the reverse does not hold. It does not bear on *what truths are*—the "nature" of truth—that they can

---

[1]I invite those who can to contrast the activities of a real-life butterfly-collector with those of a fictional Pokémon trainer. Pokémon trainers catch Pokémon using Poké Balls, and then strengthen them by entering them in fighting competitions. Every Pokémon can be caught, and every Pokémon can participate in fights, and this is no accident: it is part of what it is to be a Pokémon. Contrast also the relationship between the activity of playing point-based games and points. The activity of playing these games has, as a goal, the scoring of points. The fact that there cannot be any points that cannot be scored is an indication that it belongs to the nature of points to figure in these games.

[2]I leave aside for the purposes of this paper views that reject these claims. This includes the species of anti-realism on which scientific activity does not aim at truth at all, and the kind of scientific minimalism which holds that the nature of science involves nothing but a generic orientation toward the truth, with no commitments to particular methods of discovering truths or particular uses to which truths are subsequently put. (One consequence of this kind of minimalism: if God exists and divine revelation is a way of learning about him, then divine revelation ought to be part of science, because science is committed to no particular method. Another consequence: every truth, no matter how trivial, would be a suitable subject for science, because science is committed to no particular uses to which truths are to be put.)

figure in proper science.[3] Accordingly, it would not be surprising if there were truths that could not be learned by scientific methods, or that could not figure in scientific explanations, or that could not be used to design any useful technology, and so on: truths that can bring none of the distinctive value that comes from the satisfaction of scientific goals. (Of course, we may never be able to find any particular examples of such truths, but they might nonetheless be out there.) And even if there are none—even if *all* truths are suitable to figure in science—it would be a kind of accident.

There are many ways to depart from the butterfly model of the relationship between truth and science. One is to be a *scientific verificationist* about truth, who holds that it belongs to the nature of truth to be verifiable in a sense of "verify" that is tied to scientific standards. Another is to be a *scientific pragmatist* about truth, who holds that it belongs to the nature of truth to be useful in the practical activities characteristic of science, such as the design of technology and the manipulation of nature. A third is to be a *scientific explanationist* about truth, who holds that it belongs to the nature of truth to be suitable to figure in scientific explanations, whether as explanandum or explanans.[4] To hold any of these views is to depart from the butterfly model. By contrast, a natural way to adhere to the butterfly model is to think of the nature of truth as allowing for both truths so remote or elusive that they could never be discovered or verified by scientific methods— for example, truths about art, minds, infinity, or God[5]—as well as truths so boring and random that that they are useless for explanatory or technological purposes and unworthy of admiration or wonder, and therefore cannot figure in science's

---

[3]Note that the butterfly model could be accepted both by those who think of truths as mind-independent entities—propositions, or facts, perhaps—and those who think that they are things that only minds produce—sentences or ideas, perhaps. Someone who thinks that only a mind-dependent thing can be true might think that the nature of truth involves minds in some way without thinking it involves suitability for the *scientific* activity of minds in particular.

[4]Scientific verificationism about truth is what most of the early twentieth-century verificationists had in mind. I do not know of any avowed scientific pragmatists about truth, but Chapter One of Horkheimer and Adorno 1947 attributes a closely related view about knowledge to Francis Bacon. Depending on just how "reason" is understood, scientific explanationism is probably a consequence of the Principle of Sufficient Reason, which (as Leibniz states it) holds that "no fact can be true or existing and no statement truthful without a sufficient reason for its being so and not different." (Leibniz 1714, §32.)

[5]This is the position, for example, of Understanding Science 2018, a website maintained by UC Berkeley's Department of Palentology, whose mission is "to provide a fun, accessible, and free resource that accurately communicates what science is and how it really works." Their page "The many meanings of truth," claims that "Science does try to build true knowledge of how the world works, but there are other sorts of knowledge that people also call "the truth." For example, many have faith in spiritual truths, yet science cannot investigate this truth at all — or even tell us whether it exists."

distinctive activities—for example, truths about the relation of the cat vis-a-vis the mat, or fictional truths about the school uniforms worn by Harry Potter and his wizarding friends.[6]

I myself think that the butterfly model of the relationship between science and truth is probably wrong. My main point here, however, is that *Frege* thought it was wrong: on his view, the nature of truth involves figuring in science in a certain way. I hope that coming to understand Frege's view will help us figure out how we ourselves ought to think about truth. But in any case, the conclusion about Frege will have important implications for understanding the rest of Frege's philosophy, since the notion of truth is central to all of it.

I will argue that Frege thinks that necessarily, *all* truths can figure in the characteristic goal of proper science[7]—which means that no truth will disappoint a scientist by being unable to play its role in that goal, the way that unpinnable butterflies might disappoint a butterfly-collecting enthusiast. In this sense, it belongs to the nature of truth to figure in science. Now, one way to hold this view would be to hold an oddly unambitious conception of science, according to which every truth can figure in it only because it is so minimal. (Analogously, disappointment would be avoided by a maverick butterfly collector who thought that catching, pinning and storing butterflies were not part of proper butterfly-collecting, whose real goal is only to reflect on the way that all the butterflies already belong to the great collection that is existence itself. Necessarily, all butterflies can play their role in this unambitious goal.) I will argue that Frege, by contrast, maintains an ambitious conception of the goal of science.

I will give two arguments for the claim that Frege rejects the butterfly model by holding that necessarily, all truths can figure in the ambitious goal of science: a textual argument and a historical argument.[8] In the final section, on the basis of points that emerge in the course of the argument, I will take up a few questions raised by the discussion, including the question why Frege holds his view of truth.

---

[6]Such a relaxed view of truth can combine with the view that truth is a correspondence between what is said and what is so, or that the nature of truth is exhausted by an equivalence schema about sentences, so long as we do not think that the nature of the relevant kind of "correspondence," of "what is so," of "sentence," and so on involves some relation to science.

[7]Or at least, that what Frege calls "truths" have this feature. In the final section, I will address a worry that by "truth," Frege is talking about something other than *truth*.

[8]There is also a third, systematic argument, which is that by recognizing that Frege rejects the butterfly model in a particular way, we can resolve central outstanding questions about his logic: namely, what he thinks logic is and how he thinks we know logical axioms. This argument will emerge over the course of Chapters Three and Four.

2.2 Textual Argument: Truth in the Scientific Sense

The textual argument goes like this. If Frege accepts the butterfly model of the relationship between science and truth, then we must attribute to him some serious errors in argumentation. Ordinary interpretive charity motivates concluding that he rejects it. What is more, there are clarificatory passages that can easily be read as asserting that it belongs to the nature of truth to figure in the goal of science, and reading them this way reveals these apparent errors to be reasonable arguments.

The clarificatory passages I have in mind are ones in which Frege uses the notion of science to explain the notion of truth, rather than the other way around. For example, he says that "in logic, we are concerned with truth in the strictest sense of the word,"[9] which is "truth in the scientific sense"[10] and "that sort of truth which it is the goal of science to discern."[11] If such references to science are meant to tell us something about the nature of truth, then Frege is not thinking of the relationship between science and truth on the butterfly model. (No reference to butterfly-collecting should appear in a description of the nature of butterflies.)

But there is another way to understand these references to science, that is compatible with the butterfly model. Consider: there is a swimming stroke that is in a derivative way called the "butterfly," because it looks something like the insect. Suppose you are with Vladimir in a park in which butterfly-collecting and swimming were both going on, and he remarks "That man has a nice butterfly." Because it is unclear whether he means the insect or the stroke, he might clarify: "I mean a butterfly in the strict sense: the sort of butterfly that it is the goal of butterfly-collecting to collect." Ambiguity prompts him to appeal to a salient fact about the insects in order to enable you to understand that he is talking about *them.* His purpose does not require him to say anything about the *nature* of butterflies—though of course a fact about the nature of the things would have done just as well for this purpose. (e.g. He could have said, "I mean the insects.")

In some passages, the point of Frege's references to science is clearly only to disambiguate between different meanings of the word "true"—which means that these references to science do not imply a rejection of the butterfly model. For example, at one point he announces that he will explain how he uses "the word 'true', so as to exclude irrelevant uses of the word," and then says that he is using it to refer to "that sort of truth which it is the goal of science to discern." This is contrasted with uses of "true" to name "the goal of art," or as "prefixed to another word in order to show that the word is to be understood in its proper, unadul-

---

[9]Frege 1895, 226.

[10]Frege 1906a, 186; Frege 1914, 232

[11]Frege 1918-1919a, 352.

terated sense."[12] Since his aim in such passages is only to disambiguate the term, he need only name *some* fact about truth that will help us zero in on what he is talking about. That fact need not be anything about the nature of truth—though of course a fact about the nature of truth would serve this purpose too.

But now: suppose that something flutters by when you are in the park with Vladimir, and you ask him whether or not it is a butterfly. Vladimir is a butterfly-collecting enthusiast with an ordinary conception of proper butterfly-collecting, which involves pinning butterflies to mounting boards. In order to answer your question, Vladimir notes that these creatures are too fragile to be pinned, falling apart when one tries. Having determined that they cannot serve the purposes of proper butterfly-collecting, he concludes: "No, that is certainly no butterfly. I mean, not a butterfly in the strict sense: the sort of butterfly that it is the aim of butterfly-collecting to collect." Vladimir is making a mistake here: he should not conclude that this is no butterfly just because it cannot figure in proper butterfly-collecting. His claim that butterflies "in the strict sense" are those which it is the aim of butterfly-collecting to collect does not alleviate the error, because though that is true, suitability for that purpose is not a fact about the nature of butterflies. Vladimir is treating it as if it were.[13] He has forgotten that it is possible that there are some butterflies that are unsuitable to figure in the activities of proper butterfly-collecting.

Sometimes, Frege seems to reason just like Vladimir. He does so when he rules out plausible candidates for having the kind of meaning that allows a term to figure in sentences expressing truths, on the sole grounds that the term cannot be used in proper science. For example: it is common-sense, as David Lewis once wrote, that "We can truly say that Sherlock Holmes lived in Baker Street, and that he liked to show off his mental powers."[14] But when Frege considers fictional terms, he reminds us that we are "concerned with truth in the strictest sense of the word," and denies that these terms, which are "are illegitimate in science," have the kind of meaning that allows them to contribute to expressing truths.[15] He draws the same conclusion about vague terms, pointing out that when used in proofs or in the formulation of laws, they lead to "the fallacy known by the name

---

[12]Frege 1918-1919a, 352.

[13]Of course Vladimir could not have determined that all butterflies can figure in proper butterfly-collecting as a contingent matter: through an exhaustive investigation of every butterfly, for example.

[14]The opening sentence of Lewis 1983.

[15]Frege 1895, 228. It is worth pointing out that in making such a claim, Frege is making a substantial (if plausible) assumption about what could count as a science. One might have supposed that some such statements could fit into some kind of social science—perhaps a theory of archetypes. Sherlock Holmes could prove to be a useful theoretical object in such a science.

of 'Acervus'."[16] In response to the common-sense objection that "such words are used thousands of times in the language of life," he replies: "Yes; but our vernacular languages are also not made for conducting proofs." Apparently, Frege thinks that what is unsuitable to figure in science—proper, proof-based, systematic science[17]— cannot be true. Why else would the fact that our languages are not made for conducting proofs be even relevant to the question whether their terms have the kind of meaning that is needed to express truths? But if the relationship between truth and science is to be understood on the butterfly model, Frege is making mistakes here.

These are not the only passages in which Frege assumes that figuring in proof-based science belongs to the nature of truth. Consider the famous passage in which he claims that to find out where a truth belongs in the analytic/synthetic, *a priori*/empirical dichotomy, we must "find the proof and follow it back to the primitive truths."[18] Except for the primitive truths themselves, he is assuming that every truth has a proof. In another passage,[19] he supposes that certain truths about numbers might be "unprovable" and immediately concludes that in that case they would all be "primitive truths." So far, this could be trivial, because perhaps there is no more to being a "primitive truth" than being a truth with no proof, which would allow for truths that play no role in science. But Frege introduces the notion of a primitive truth in connection with his official account of proof: they are the ultimate premises of scientific proofs,[20] and he depends on this fact in arguments. For example, he denies that there can be infinitely many primitive truths, on the grounds that there is a "need of reason" that the "foundations" of a science—the

---

[16]Frege 1896, 115.

[17]See especially Frege 1884, §3, Frege 1914, and Chapter One of this dissertation for Frege's general conception of the goal of proper science, which always involves systems of proof. See de Jong and Betti 2010 for a general discussion of the prevalence of this conception of science, which Frege shares with most others at the time.

[18]Frege 1884, §3.

[19]Frege 1884, §5

[20]As de Jong 1996 (300) comments on the dichotomy of truths above: "Frege places and presents these distinctions from the outset within the framework of the aristotelian [i.e. proof-based] model of science. Without this model, what Frege says about these distinctions is nearly incomprehensible." Frege 1914, 204-205 introduces primitive truths as follows: "If we start from a theorem and trace the chains of inference backwards until we arrive at other theorems or at axioms, postulates, or definitions, we discover chains of inference starting with known theorems, axioms, postulates, or definitions, and terminating with the theorem in question. The totality of these inference-chains constitutes the *proof* of the theorem...Science demands that...we do not rest until we come up against something unprovable...If we assume that we have succeeded in discovering these *primitive truths*, and that [the science] has been developed from them, then it will appear as a system of truths that are connected with each other by logical inference."

ultimate premises of its proofs—be "surveyable."[21] This means that to assume that all truths are provable or primitive is to assume that all truths can figure in proofs, whether as premise or as conclusion. Once again, this is like assuming that all butterflies can be pinned to boards. It is a mistake, if the butterfly model applies to the relationship between truth and science.[22]

There is no good reason to suppose that Frege repeatedly makes obvious mistakes. Ordinary interpretive charity requires us instead to conclude that he rejects the butterfly model of the relationship between truth and science. In particular, we must suppose that he thinks that necessarily, all truths can figure in the ambitious goal of proper science, the construction of a system of proofs. That makes it reasonable for him to deny that something is true that is "illegitimate" in science or in proofs, and to treat the categories of proveable truth and primitive truth as exhausting the truths, even though both categories apply only to truths within a science. When he explains what kind of truth he is talking about by reference to the goal of science, then, he is not merely helping us to zero in on what he means—he is doing so *by* saying something about the nature of the thing he is talking about.[23]

2.3   Historical Argument: Truth Among the Neo-Kantians

The historical argument goes like this. Rejecting the butterfly model of the relationship between truth and science was standard among philosophers in Frege's philosophical milieu: everyone took themselves to be talking about something whose nature involves figuring in the ambitious goal of proper science. Since Frege wants to be understood by the others writing in this tradition, we should expect him to be writing under this assumption as well. If he did not mean to share

---

[21]Frege 1884, §5.

[22]Accordingly, commentators are puzzled by some of Frege's claims about primitive truths. Textor 2011 (29) for example, dismisses the argument of §5 as an mistake: how could Frege assume that every unprovable truth is a "primitive truth," in a sense that implies that they satisfy these "requirements of reason" on science?

[23]One might worry that rejecting the butterfly model is ruled out by the passages in which Frege denies that there is a *definition* of truth, and/or by those in which he claims that the word "true" is dispensable, since those passages commit Frege to a *deflationary* view of truth which precludes saying anything substantive about its nature. I am convinced that these passages have no such implications by Levine 1996 and Heck and May (forthcoming.) (From the latter: "Frege clearly would not agree with the common deflationist thesis that there is nothing substantial to be said about what it is...to be true.") For readers worried about this issue, I recommend taking a look at those papers. (Briefly: Frege does think that truth is not definable, but being definable requires more than having a nature about which things can be said; and as for Frege's claims about the word "true," they are only about the *word*.)

it, he would need to indicate this clearly—but as we have just seen, if anything, his text shows that he shares this view.

Kant, Logic and Psychology

First: what was Frege's "philosophical milieu"? Discussions in late nineteenth-century German philosophy were dominated by Kantian themes, Kantian terminology, and outright Kant interpretation. Putting forward a philosophical view as correct often went hand-in-hand with putting it forward as the true interpretation of Kant. Frege himself does this when he discusses the proper relationship between logic and psychology. He claims that Kant's "true view was made...difficult to discover" by some poorly chosen terminology, but what Kant *really* means to be talking about in the *Critique of Pure Reason* is something to which psychology is irrelevant. (Kant's choice of "idea" misleads us into thinking otherwise.) So understood, Frege thinks that Kant has the right approach.

The issue of the relevance of psychology to philosophy was *the* pivotal interpretive and philosophical issue in the years leading up to Frege's writings. Many nineteenth-century figures had seen themselves as engaging with Kantian questions, and even defending Kantian views, through psychology.[24] But though the psychological readings of Kant that made this possible continued to be taken seriously throughout the nineteenth century, they were in major decline by the time Frege started writing. By the mid-century, the most influential figures increasingly insisted that Kant's concerns—the proper concerns of philosophy—were with the justification and evaluation of what we think rather than its causes, and that psychology had next to nothing to contribute to these questions of justification. As one historian describes the period when Frege was writing, "psychologism, which seemed to conflate the realms of validity and matter of fact, had now become passé."[25] Frege's anti-psychological polemics reflect his allegiance to this dominant movement.

My claim is that the leading figures of this movement rejected the butterfly model of the relationship between truth and science.[26] They too held that nec-

---

[24]Some, like Jakob Fries and Friedrich Beneke, used introspective psychology, while later figures like Johannes Müller and Hermann Helmholtz used experimental psychology. Helmholtz, for example, writes about the bearing of experiments with distorting lenses on the idea that the space described by Euclidean geometry is our form of sensibility.

[25]Beiser 2014, 463.

[26]It is not of central importance what Kant himself really thought, but he does seem to have seen an essential connection between the understanding's goal of truth and reason's goal of systematic science. This seems to be the best way to understand his claims that "the law of reason to seek [systematic] unity is necessary, since without it we would have...no coherent use of the understanding," and that reason "prepares the field for the understanding." (Kant 1781/1787, A651/B679.

essarily, all truths can figure in the ambitious goals of science. I will argue, in particular, that this is the view of Hermann Lotze, Hermann Cohen, and Wilhelm Windelband. We are fairly certain that Frege himself read all of these figures,[27] but what is more important is that they are the leading figures of the dominant anti-psychological movement to which Frege belonged,[28] and who determined the shared background assumptions for those who belonged to it.

Lotze

One of Lotze's most detailed discussions of truth takes the form of an examination of the mental capacities of different creatures, in search of "the distinction...by which the intelligence of the human soul...surpasses the activity of the animal soul."[29] Lotze is very generous toward non-human animals, attributing to them souls with extensive cognitive capacities,[30] but neither nonhuman mental activities nor those of the "unstudied healthy human," (which includes both "savage peoples" and children) are "directed toward the truth of things."[31]

---

[27]and A657-8/B685-6) As Abela 2006 (421) puts it, the fact that "lacking the top-down component of rational [i.e. systematic] unity, the understanding has no employment...puts the demands of systematicity near the center of Kant's account of cognition."

[27]Frege claims Lotze as a teacher, attended his lectures in Göttingen, and (as Dummett 1981a shows) read and took notes on Lotze 1874. (See Gabriel 1989.) He probably read Windelband, since he seems to have borrowed the term "truth-value" from him. (See Sluga 1997; Gabriel 2002.) There are also strong textual echoes in Frege's writings. Frege 1879-1891 (2) claims that the causes of our judgements take place in accordance with psychological laws, which "can just as well lead to error as to truth"; Windelband 1882, 47 had written the same thing. ("kann...ebensosehr zum Irrtum wie zur Wahrheit führen" versus "können ebenso wohl zum Irrtum wie zur Wahrheit führen.") Frege 1885 is a review of Cohen 1883, which is "Perhaps Cohen's most sustained and systematic attempt to present Kantian philosophy in historical connection to mathematical natural science." (Richardson 2006, 218.)

[28]Lotze, in particular, "reigned as the single most influential philosopher in Germany, perhaps even the world." (Sullivan 2017, Section 2.) Lotze 1874 was "perhaps the most widely read logic text in Germany during Frege's early career," (Heis 2013, 122.) Windelband and Cohen were the two most important active figures when Frege was writing, whose views "became the new orthodoxy of the 1880s and 1890s." (Beiser 2014, 492.)

[29]§3 of Lotze 1858, Book 5 (Mind), Chapter 4 (Knowledge and Truth). Quotes in this section from Lotze 1858 are from this chapter.

[30]"One must admit to animals...much deliberation, much combining of thoughts, and many surprising strokes of brilliance...they dream in sleep, while awake they undoubtedly think of the past and the future." (Lotze 1858, §1.) "Their souls compare the present with the absent, the defective reality with the perfect image of what is desired," and not all of this mental activity is mere self-preservatory instinct: "Without doubt to the animal soul belongs a range of distinterested...activity": a "lopsided and inefficient ability to learn," exercised for its own sake.(Lotze 1858, §3.)

[31]Lotze 1858, §3.

Lotze identifies the important distinction between human and animal mental lives as follows. "The human mind distinguishes itself by becoming aware, through reflection on the acts of knowledge that it has mechanically carried out, that they contain laws that reach out indefinitely beyond the particular cases in which inner experience finds them to be complied with."[32] Lotze is describing the process by which we discover the laws of logic, including "the Law of Identity, that every simple content is identical with itself, and the Law of Causality, according to which every alteration demands a cause."[33] It is only when we come to recognize these laws that we come to gain "the concept of truth": these laws show us "that there is anything at all to be called truth, not in the parochial sense of an agreement between our ideas and their contents, but meaning a coherence and consequence."[34]

So we discover what truth is only with advanced reflection on the logical laws. But what exactly does such reflection teach us? What kind of "coherence and consequence" constitutes truth? The important point is that the laws of logic are *all-embracing*: unlike the kinds of judgements that we made before we discovered the laws of logic, these ones reach out indefinitely beyond any particular cases to embrace everything.[35] The reason we can only obtain the idea of truth by becoming aware of laws that connect everything in this way is that truth is what has a place in an all-embracing network of connections. That is why to learn truths is to become more "aware how unified and inevitable the network of relations in which [all things] rest is."[36] while we fail to pursue the truth whenever we "carry on a train of thought" without taking any "overview of the whole," or "thinking of finally unifying it" with the rest of our thought.[37] The way to pursue the truth is to pay attention to the whole, because to be true is to fit into the whole.

The attempt to become aware of the this all-embracing network is the task

---

[32]Lotze 1858, §5.

[33]Lotze 1858, §5.

[34]Lotze 1858, §5. "The concept of truth" is Lotze's heading for this section. I agree with Constance Jones' in Lotze 1856-1864 that Lotze's "Folgerichtigkeit" implies both of coherence and consequence. Lotze's passage shows that he is aware of another meaning of the word "truth," but that described here is the way he uses it.

[35]Before he discussed the notion of truth, Lotze 1858, (§4) introduces "A single feature [that] may be peculiar to human ideation," which is a particular use to which we put the "general images" that we share with animals: such an image "is regarded by us as a commanding law...the *general* is thought of as the form-giving law of the particular." These classifications, when unaccompanied by the idea of truth, are usually a bad thing: we categorize, for example, human beings into different kinds on the basis of superficial features, such as "differences of social position" and then we think that a law explains these accidents and resist attempts to change these law-given social relations, sometimes "with terrible brutality."

[36]Lotze 1858, §6

[37]Lotze 1858, §6

of "science"—or sometimes of "philosophy," which Lotze identifies as part of science.[38] Hence, "philosophy [is] the human endeavour...to work out for ourselves an insight into that all-embracing system."[39] Lotze warns that there is no sure route to discovering the contents of the system—"even science commits errors when it tries to group [its discoveries] together in a systematic way"[40]—but nonetheless, that is its ultimate goal.[41]

This, then, constitutes a departure from the butterfly model. Lotze's talk of elucidating "the concept of truth" makes it clear that he means to be telling us something that belongs to the nature of truth, and what he tells us is that it is what has a place in an all-embracing system. But it is the ambitious goal of science to gain insight into that system. Thus it is necessary that all truths can figure in this ambitious goal of science.

Cohen

In the relevant period, much of Cohen's philosophy is developed in the course of commentaries on Kant: in the characteristic philosophical mode of the time, he expresses his own views largely in the course of interpreting and correcting Kant's own. In these writings, truth is not a concept that receives any clear or detailed treatment. But these writings reveal a general philosophical approach that gives us good reason to suppose that Cohen rejects the butterfly model of science and truth.

---

[38]E.g.: "We should, therefore, count psychology as the last and most difficult product of philosophical investigation, or of scientific investigation more generally." (Lotze 1880, 468-469) "There is no special royal road in science. Science, philosophy above all, possess no mysterious methodical road..." (Lotze 1880,474)

[39]Lotze 1880, 452

[40]1858, §6

[41]It is worth mentioning that Lotze often makes anti-systematic statements—as, for example, when he claims that the "partiality to systematic classification is a mischievous prejudice." (Lotze 1880, 479.) But what he is objecting to under the name of "system" is making unmotivated assumptions about the structure he calls "truth": for example, the assumption that "no theory of the world should pass for truth and science which was unable to explain all the particular parts of the world's history as independent consequences of a *single principle*." (Lotze 1880, 451.) He also denies that the features of the system—including the logical laws that structure them—can be laid down in advance of actually doing science: "those in the highest degree universal truths, which we regard as an innate possession of our mind, do not stand before its consciousness from the beginning onwards as a complete well-ordered series...their systematic collection for the construction of a theory of knowledge would, therefore, not be a possible beginning for the work of philosophy." (Lotze 1880, 471) These particular errors aside, "every philosophy seeks quite naturally to unite its results in a systematic whole, and no just objection can be made against the necessity of such an attempt." (Lotze 1880 477.)

According to Cohen, Kant showed us that the only proper philosophy is critical, and the object of critical philosophy is always "*reason in science*...The critique discovers...the *conditions of certainty* on which *knowledge as science* rests."[42] In general, the "natural question of all philosophy" and "the problem of that philosophy which matured in Kant" is "what science makes into science, which conditions its certainty presupposes, from which fundamental principles its realization...as science is made possible."[43] Philosophy must begin with the achievements of science, and then explain how these achievements are possible by uncovering *a priori* "foundations" and "presuppositions" on which they depend. Proper philosophy, accordingly, was *impossible* before Newton: "philosophical maturity came with the maturity of science that began with Galileo and concluded with Newton. Since Newton, there has been a science built on principles...now the object was given, at which *the transcendental question of the possibility of a priori knowledge* could be directed."[44]

Reading these concerns into Kant's ambiguous texts requires Cohen to provide non-obvious interpretations of Kant's key terms. The following are some illustrative passages explaining the bolded terms:

- Kant "discovered a new concept of **experience**," on which "experience...must count as the total expression of all the facts and methods of scientific knowledge."[45]

- Kant's questions are "not about **knowledge** as such...but rather, mathematical-scientific knowledge." They are about "knowledge not as a manner of consciousness, rather as a *fact*, which happened in *science*...not on the process and apparatus of knowledge, but on the result, science...The investigation of knowledge is directed at the facts of science."[46]

- "The **intuition** and the **thought** are *abbreviations* for scientific methods...*general presuppositions of all scientific research.*"[47]

- "A **pure concept of the understanding**" is "a basic form of scientific thinking."[48]

---

[42]Cohen 1883, Introduction, §8.

[43]Cohen 1883, Introduction, §9.

[44]Cohen 1883, Introduction, §10. A central criticism of German idealism is that it drops this connection to science, since "without connection to mathematical natural science the concept of the transcendental becomes nonsense." Cohen 1914, 121. Presumably Cohen's verdict would be the same on much Kant-inspired philosophy in our own day.

[45]Cohen 1871, 3 and Cohen 1885, 59.

[46]Cohen 1885, 56 and Cohen 1883, Introduction, §7.

[47]Cohen 1883, Introduction, §3

[48]Cohen 1885, 409.

And so on. Cohen reminds us that much of what we ordinarily suppose falls under these terms does not qualify—for example, Cohen reminds us that many of the things we have in mind when we use "the popular expression *experience is the best teacher*" are not going to be experiences, in Kant's sense.

Given the influential conception of the proper task of philosophy that Cohen is providing, if philosophers are to concern themselves with truth at all, it can only be something which necessarily figures in science. If truth is any kind of cognitive success or goal, it must be what is possessed by "mathematical-scientific knowledge."

Windelband

Windelband, by contrast, does give truth a central place in his theoretical philosophy, or "logic." It will, however, help to begin with what he says about philosophy and science more generally. His account of the proper philosophical method largely fits with Cohen's:[49] the point of logic is to examine science and "discover the ultimate grounds on which...its knowledge rests, to understand the inner structure of the intellectual work in all the particular disciplines, and to gain the objective presuppositions that their ground of validity contains."[50] Cohen and Windelband agree on a great deal, but there are two distinctive emphases in Windelband's discussion.

The first is on value. On Windelband's telling, "what was essentially new and decisive was that Kant recognized the *inadequacy of the psychological method* for the solution of philosophical problems," and his response was to wholly separate "the questions which surround the *origin* and the actual development of man's rational activities, from those which relate to their *value*."[51] This gives him a conception of philosophy's task: "Ideas come and go; how they do so, psychology explains. Philosophy investigates which value they have."[52] In particular, philosophy studies the *universal* values: it begins with "the conviction: there are universal values...that which is worthy of universal recognition and acceptance." The particular business of *theoretical* philosophy— "logic"—is to investigate *thinking* that has this universal value. (Other branches of philosophy will study *willing* and *feeling*

---

[49]Section 3 of Anderson 2005 makes a case that Cohen's early writings influenced Windelband's understanding of what anti-psychological philosophy should be. Windelband engages in Cohen-esque glossarizing when explaining what kind of experience we (and Kant) are talking about: since neither science "can be satisfied with what the naive man usually means by experience" we as philosophers concern ourselves only with "a scientifically purified, critically trained, and conceptually tested experience." (Windelband 1894, 367.)

[50]Windelband 1907, 10

[51]Windelband 1892, 419

[52]Windelband 1882, 48

with universal value.) Because of this focus, the task of finding the foundations of a science becomes that of accounting for the universal value that scientific thinking has: theoretical philosophy's job is to explain "on what [this value] rests," and "how one has to proceed in science in order to secure this value."[53]

The second emphasis is that Windelband has little to say about the project of finding the foundations of *physics,* leaving that to Kant himself and to Cohen. He himself takes up a new, distinctively nineteenth-century task for "logical theory," which he sees as "the point on which it most needs reform."[54] Windelband thinks that relatively recently, the "historical disciplines" that used to belong to the general mass of "*belles lettres*" have been made into true sciences. This has given us an "extended concept of science," compared to Kant. The problem is that logic has not kept up. Kant found the "foundations for the Newtonian natural philosophy—that is, for mathematical-physical theory,"[55] but no one has yet found the foundations of *history.* Since such foundation-finding is the proper task of theoretical philosophy, Windelband claims that "history must become just as much the organon of philosophy, as [in Kant's day] only natural science was."[56]

Setting out to account for the value of historical thinking, Windelband immediately notices that it is not valuable in the same way as natural science: it does not seek the "purely logical value...generalization."[57] Rather than generalization, history aims at a "faithful delineation of the particulars."[58] But having universal value is part of what it is to be a science; hence, since these discoveries "lack the basic logical value of generalization," these particulars "can only be objects of [scientific] knowledge in case they possess some other value."[59]

Observation of historical practice shows Windelband that not just *any* particular is historically significant: "the object of historical science is always something that stands out from accompanying events by reason of its relation to some high standard of value in life."[60] His idea is that the value of historical thinking con-

---

[53]Windelband 1882, 46.

[54]Windelband 1894, 366.

[55]Windelband 1907, 14.

[56]Windleband 1907, 21.

[57]Windleband 1920, 239. This value is achieved to the extent that we uncover the law-governed, systematic whole that Lotze described and called "truth." Since this is the goal of natural sciences, a "basic presupposition" of these sciences is "that the world consists out of persisting substances, whose states are reciprocally determined by each other in accordance with general laws." Windelband 1907, 19.

[58]Windelband 1894, 368: "In the one [kind of science,] thought pushes from the establishing of particulars to the comprehension of general relationships; in the other, it is held fast by the fond peculiarity of the particulars." Again, when Windelband 1894 (363) divides empirical sciences: "One [kind] seeks general laws, the other particular historical facts."

[59]Windelband 1920, 240

[60]Windelband 1920, 205

sists in the way it shows us in detail something about *other* values. These values must themselves be universal if they are to explain the universal value of historical thinking: hence, the science of history "needs as its highest presupposition...a system of generally valid values": "values of reason."[61] We must "interpret the sequence of the historical life as the realization of values which, in their turn, transcend in their validity the life of man, in whose mind they attain recognition."[62] So, for example: the thoughts we have in the course of doing the history of the French Revolution have the universal value of scientific thought because of what it shows us about the values of freedom, equality, and fraternity; and this presupposes that freedom, equality, and fraternity are themselves universal values.[63] Windelband's next question is what it takes for there to be such values, and for us to be aware of them, and he goes on to uncover further presuppositions from there.

Now let us see how truth fits in. The universal values that our mental lives can exhibit—the values that it is philosophy's task to investigate—are "truth in thought, goodness in willing and acting, beauty in feeling."[64] Philosophy studies only scientific thought, because that is the thought with universal value: its value is the value of truth. Windelband claims that scientific thought just *is* "thought that, with general and necessary validity, possesses the value of truth."[65] Since there are (at least) two different ways for thought to have universal value—the historical and the natural-scientific way—there are accordingly two different kinds of truth: Kant's "entire epistemology was in fact attuned to the concept of mathematical-natural scientific theory and its sort of truth," because he was yet unaware of the "sort of truth" belonging to "the other sciences, in particular the historical."[66] The identification of scientific thought with true thought means that necessarily, all truths can figure in the goals of science. This is so whether that goal is the generalizing characteristic of natural science, is showing us something about another universally valid values the way historical science does, or is some goal characteristic of some as-yet undiscovered science.

Though Windelband's account of science is deliberately open-ended, he says enough to imply that the goals of a science are always ambitious. He is prepared to make general claims about what all scientific thought will have in common:

---

[61]Windleband 1907, 20-21.

[62]Windelband 1920, 205 Windelband further thinks that these values "are known to us only in such structures as...belong to the life of man...the structures of civilized life."

[63]Note that though the way in which historical thought is valuable might depend on other values, that does not make it valuable in the same way: the history of the French Revolution is not be valuable in the same way that freedom itself is valuable.

[64]Windelband 1883, 341.

[65]Windelband 1882, 49.

[66]Windelband 1909, 17

for example, it will always involve "methodical work," including the construction of systems of *proof* that employ the logical laws.[67] Accordingly, he denies that common-sense candidates for truth qualify as such.[68]

Windelband, then, rejects the butterfly model, because by his identification of scientific thought with true thought, he guarantees that necessarily, every truth can figure in the goal of science. Moreover, he does so while maintaining an ambitious, if open-ended, conception of the goals of science.

2.4   Scientific Truth

What I have argued is that Frege thinks that necessarily, all truths can figure in the ambitious goal of science: in this sense, it belongs to the nature of truth to figure in that goal. If Frege is right, the relationship of truths to science is not like that between butterflies and butterfly-collecting: while there might well be butterflies that can bring no satisfaction to the butterfly-collecting enthusiast, this is not possible with truths and scientists. We have also seen that this view is right in line with the most influential figures of the day, all of whom think of truth and science as connected in the this way.

I want to conclude by asking three important questions about this view.

---

[67]Windelband 1907, 11 claims that only the "methodical work of science" can give the contents of our minds the "conceptual clarity and distinctness" needed to "have a claim to necessary validity and general recognition." Windelband 1894 specifies that both history and empirical natural science "have experience for their starting points—or, from a logical perspective, the premises of their proofs." (367; see also the discussion of "historical proofs" at 375.) Windelband 1883 notes that both sciences involve "subsuming those sensations under these universal propositions by means of logical forms of connection," (321-322) even though the ultimate goals are different. This is why "Logic can say to everyone: you want truth? Then remember, you must recognize the validity of these [axioms], if your desire is ever to be fulfilled," (330) even though axioms' "sense consists in the fact that through them alone can anything about facts and from facts be proven." (326) The reason we need logical axioms for truth is that we need them for proofs, and all truths figure in proofs. See Chapter Four of this dissertation for more on Windelband's claims about justifying axioms.

[68]According to Windelband 1920 (201-202) "originally a man only thinks in order to act...[this thinking] is permeated with processes of feeling and volition," motivated by "feelings of pleasure and displeasure, hope and fear." But this thinking is really a kind of "opining" whose value "is only relative and restricted to the [opiner.]" Its value is not universal if it involves "a purely individual value-inclination, which makes no claim to generally valid interests," which means that it is not true. (An example: if I think that the cat is on the mat, I am probably paying attention to the cat largely because he is *my* cat, or my friend's cat, or a cat with a colour that I like, or something like that. But that is not enough for everyone to care about the cat, so unless there is some other reason that the cat has a claim to universally valid interest, what I think is not true.)

1. Is Frege talking about *truth*? Or is he using a special, technical term, "scientific truth," to talk about something else?

2. Does Frege think that figuring in science belongs to the nature of truth *only* in the sense discussed above, or in a stronger sense too?

3. *Why* does Frege hold his view of truth?

The answers to all these questions must be somewhat speculative, since no passage I know of clearly answers them. In attempting to provide answers, I will appeal to the views of the other figures we have discussed, where doing so may shed light on what Frege has in mind.

Is "truth" *truth*?

Cohen reads Kant's great methodological innovation into his text by identifying special uses of "experience," "knowledge," and other terms—uses that are especially connected to science. But this means that someone who opens a volume of Kant out of an interest in experience might be disappointed: she might feel that Kant has nothing to say about *experience*, but only about "experience" in some new, special sense. One might wonder along these lines whether Frege is talking about *truth* when he says "truth." We can usefully think about this question as one about *reference*: the question is whether the word "truth," as Frege uses it, refers to the same thing as the word "truth" when we use it.

We saw Frege call what he is talking about a particular "sort of truth"—which might suggest a kind of genus-and-species relationship between truth and what he means by "truth"—but we also saw him call it truth "in the strictest sense of the word," which suggests a metaphorical-and-literal relationship between other things we call "truth" and what he is talking about, truth. It is hard to know just what he has in mind.

In fact, it is not only with truth that this question proves difficult—it is *never* clear what relationship Frege thinks holds between the referents of the central terms he uses in his theorizing and those of ordinary language.[69] Frege thinks he need not concern himself about this question any more than other scientists do: "Like other sciences, logic is allowed to form technical terms, without worrying whether the words are used just that way in the language of life. In determining their meaning, it is not important to match ordinary use or etymology, as long

---

[69]Indeed, it is not even clear whether he thinks terms in ordinary language *have referents at all.* See, for example, van Heijenoort 1986 (39): "Frege seems to think that words of ordinary language have neither *Sinn* [sense] or *Bedeutung* [reference] because they are vague."

as the word is as suitable as possible for the expression of laws."[70] This passage suggests that Frege intends for our question about truth to be settled in the same way such questions are settled for technical terms in other sciences. But what is that? Let us consider a typical case.

> Suppose Melville opens a volume named "Fish Biology," because he is interested in the things to which he applies the word "fish": namely, everything whose meat he can buy from the fishmonger, which includes whales and octopuses. Melville is disappointed with what he reads. He later complains: the biologist's book does not concern fish in general, but only *some* fish—and not even the most interesting, tentacled ones! The biologist (he complains) is using the term "fish" in a special sense.[71]

Though we can all sympathize with Melville's disappointment, it is not clear that he is right in his linguistic claim. It seems to me that the biologist might well be using the word "fish" in the ordinary way, and that Melville has been wrongly applying it to some things. But my main point here is that it is not obvious *which* relationship technical terms in sciences bear to ordinary language. Since this is not the place to argue for a general theory of reference—nor to wheel in a theory of reference without arguing for it—it seems that we will have to leave this question unanswered.

However, I think that Frege gives us a bit more guidance than this when it comes to "truth" in particular. Once the answers to the next two questions are in place, I will give a reason to think that by "truth," he really is talking about truth.

In what sense of "nature"?

Some mathematicians believe that every even number is the sum of two primes. If this is so, then it is necessary: being the sum of two primes would "belong to the nature" of even numbers in the sense that I have been using. But compare this claim with another: every even number is a multiple of two. Both identify features shared of necessity by all even numbers, but the latter seems to have a stronger claim to tell us the "nature" of even numbers. We might put the difference this way: while one who knows either claim understands something of the nature of even numbers, anyone who *fails* to know that even numbers are the sum of two primes cannot understand what they are. While the claim about primes adds to our understanding of what even number are, if we did not know it, we could

---

[70]Frege 1897a, 148.

[71]The example derives from Carnap 1962, §3. Carnap's linguistic judgement agrees with Melville's.

still understand what they are. But the other claim must be part of any reasonable understanding of even numbers; if we do not know it, we cannot be said to understand what even numbers are at all. Let us call this the distinction between belonging to the nature of something in the *weak* sense, and in the *stronger* sense.[72]

So far, I have only argued that Frege thinks that figuring in the goal of science belongs to the nature of truth in the weak sense. Does he think it also belongs to the nature of truth in the stronger sense? Would he claim that one cannot understand what truth is without realizing that necessarily, all of them can figure in science in that way? I think that he would.

There are few things such that understanding what they are involves understanding something that one can do with them, but one traditional model is that of *values*.[73] As Mackie puts it, if there is such a thing as a universal value, it would be something "such that knowledge of it provides the knower with both a direction and an overriding motive…[it] would be sought by anyone who was acquainted with it…because [it] has to-be-pursuedness somehow built into it."[74] If there are universal values, knowing what they are makes us "pursue" them: to set what the values call for us to do as our goal, to hope that it is possible to reach that goal, etc. This is a very common idea about what values are: for example, Korsgaard emphasizes that if there is such a value, "the force that [it] exerts upon us is attractive,"[75] and Murdoch puts the "magnetic pull" of values at the center of her practical philosophy.[76] This means that one cannot understand what a value is without also understanding how it figures in the relevant goal.[77] It follows that if

---

[72]One classic discussion of the distinction between merely modal accounts of natures or essences and stronger ones is Fine 1994. Fine argues that the interesting notion of a "nature" for metaphysics is one that is closely connected with the notion of a *definition* of something. A definition of X is something that you can give someone who has no idea what X is that will provide them with an understanding what it is. We know that Frege does not think truth admits of a definition in this sense; but even if there are things that cannot be defined, there might still be features of them that one needs to know in order to understand what they are.

[73]There is another traditional model that I believe does *not* apply here: that of man-made *tools*. Though one cannot understand what a hammer is without understanding that one can hammer with it, none of these figures think that truth is man-made in the way tools are.

[74]Mackie 1977, 38-40. (Mackie himself doesn't think there can be such things.)

[75]Korsgaard 1996, Introduction.

[76]E.g., Murdoch 1970, 41 and 73.

[77]Suppose that deliciousness is a value, and what it calls for is tasting things that exhibit deliciousness. It would follow that one cannot understand what deliciousness is without being motivated to taste delicious things—to set this goal. This requires understanding that all delicious things, can, in some sense, be tasted. (Though of course there might be obstacles to *my* tasting some of them, the idea of a delicious thing which is in principle untasteable, or in principle could not satisfy one who tasted it, does not seem to make sense.) Note that this claim applies to values, which are different from mere things which are valu-*able.* A certain metal might in fact be valuable

truth is a value, then by understanding what it is, we understand that truths can figure in the relevant goals; which is to say that figuring in those goals belongs to the nature of truth in the stronger sense.

Lotze thinks that truth is a value, in this sense. One way that he has of characterizing "the essential feature of human thought" is as an urge to see the world in a law-governed, ordered way. Before we know what truth is, this urge leads us to impose a kind of order "in its simplest and roughest form," onto cases which only admit of "a much more subtle application."[78] But once the laws of logic acquaint us with truth, we know what it is we have been longing for: the precise kind of order exhibited by the all-embracing system in which all truths have their place. As human beings, we will always be driven to pursue this truth: as Lotze puts it, "a haunting conviction of the existence of truth pursues men everywhere."[79] Lotze's onetime students, Windelband and Frege, are explicit about this aspect of their view of truth, calling it a "value"[80] and treating the word "true" as belonging with the other value-words, "beautiful" and "good."[81] These figures, then, presumably think that one cannot understand what truth is without realizing that each one is suitable to figure in the goal as they conceive it. For them, figuring in those goals belongs to the nature of truth in the stronger sense.

Why This View of Truth?

Why reject the butterfly model of the relationship between science and truth in the way these figures do? Why suppose that it belongs to the nature of truth to figure in the ambitious goals of science?

We find one kind of answer in the methodological discussions of Cohen and Windelband. It is because the business of theoretical philosophy is to uncover the foundations and presuppositions of successful science that Cohen unfolds his range of claims about how we must understand certain philosophically relevant

---

because it is expensive; but a scientist might know fully what the metal is without realizing that people would pay a lot of money for it.

[78]Lotze 1858, §4.

[79]Lotze 1858, §6

[80]Glock 2015 argues that despite the connections with Windelband and Lotze, Frege's calling the True and the False truth-values could be a co-incidence, and imply no particular connection with value in the sense I have been discussing. After all, Frege's logic "was based on a generalization of mathematical function-theory, and the idea of a function is that of a mapping of arguments onto values." But it is highly unlikely that Frege calls them "truth-values" simply because they are the values of certain functions for certain arguments. For one thing, they are equally the *arguments* of certain functions that yield certain values, and he does not call them "truth-arguments." For another, *everything* is the value of certain functions for certain arguments, and he does not call everything a "-value."

[81]e.g. Frege 1918-1919a, 351.

things: *all* of them must ultimately be understood in terms of some relation to science. Frege seems to endorse the Cohen-Windelband conception of theoretical philosophy, or "logic." He endorses it with claims like "scientific workshops are the true field of study for logic,"[82] and his own project to uncover the *Foundations of Arithmetic* is just what this conception of philosophy calls for.[83] One reason, then, to think about truth in the way these figures do is if we share their conception of theoretical philosophy.

But there is also a more direct motivation for this view of truth, that links up more directly with the other two questions. It is a traditional part of our thinking about truth to hold that it is a value—perhaps the most important value. Plato, for example, claims that "Truth heads the list of all things good, for gods and men alike. Let anyone who intends to be happy and blessed be its partner from the start, so that he may live as much of his life as possible a man of truth."[84] But despite the tradition, this aspect of our thinking about truth proves very difficult for philosophers to hold onto. Put bluntly, the problem is that some truths are very boring, do not matter, and no one cares about them. It is difficult to say with a straight face that someone who is not motivated in the slightest to find out how many grains of sand there are on a particular beach is making any mistake; those who manage to say it invite the incredulity of their interlocutors.[85] Accordingly, it is standard these days for epistemologists to renounce or qualify the traditional claim that truth is a value.[86]

Understanding the nature of truth in terms of the goals of science represents a

---

[82]Frege 1880-1881, 33

[83]In accordance with this philosophical method, when he discovers such a presupposition, he puts it forward as how things are. (See, for example, the motivation for Basic Law V in Appendix 2 of Frege 1893: "I do not see how arithmetic can be founded scientifically...if it is not...permissible to pass from a concept to its extension.")

[84]Plato, *Laws* 730c. Lynch 2004 (13) is attempting to report a common-sense "truism" when he writes that "the very word 'true' has an evaluative dimension. Part of what you are doing when you say something is true is commending it as something good to believe...we are guided by the value of truth: other things being equal, it is good to believe a proposition when and only when it is true." (In more recent discussions, it is often said that *believing* the truth is valuable. Earlier, it was more typical to say that the truth is valuable, and that the way you access that value is by doing something with it, like believing it.)

[85]Lynch 2004 (55) maintains that it is "prima facie good to believe even the most trivial truth." David 2005 responds: "Of course, there is the 'prima facie' qualifier to soften the blow. But still, one might well think: 'Come on, that's not even prima facie good!'." Sosa 2003 (156) says of such a view: "This view is hard to take seriously. [A truly trivial truth] would not interest most of us in the slightest...absent such antecedent interest...it is hard to see any sort of value in one's having that truth."

[86]For example, Goldman 2001 (38) takes the problem of trivial truths to motivate a "slight revision" to the straightforward claim that truth is a value: instead, "the core epistemic value is a high degree of truth-possession *on topics of interest.*"

straightforward way—and perhaps the only way—to continue to affirm the value of truth. As Windelband sees it, paradigmatic "trivial truths" contribute neither to the generalizations that give natural scientific thought its value, nor do they show us something about other universal values, and nor are they universally valuable in any other way. Accordingly, they are not truths at all. Just as an aesthetic philosopher who is serious about beauty may conclude that the ordinary standards that lead most of us to call things beautiful are defective, a philosopher who is serious about truth ought to criticize our ordinary standards for it.

Upholding the status of truth as a value may matter for much more than vindicating tradition. Windelband came to think that the great philosophical task of his day was to combat certain cultural movements which drew inspiration from Nietzsche and Schopenhauer. As Beiser puts it, Windelband's great fear was "that the new *Lebensphilosophie* of Nietzsche and Schopenhauer was corrupting the youth" and that Germans would devote their lives to ultimately irrational expressions of their wills rather than to truth and the other universal values. He argues that the Neo-Kantian philosophy "was indeed an entire *Weltanschauung*, whose purpose was to answer the fundamental questions of life."[87] The universal value of the True, the Good, and the Beautiful stood at the center of the answers provided by this *Weltanschauung*. Strongly upholding the value of truth may well be an important part of a philosophical response to "post-truth" cultural movements, whether of the nineteenth or the twenty-first century.[88]

With this in mind, let me return to the first question: whether Frege is really talking about *truth* or not. We saw that in general, he thinks that the technical terms of logic should be understood on the model of those in other sciences, but that this does not seem to be enough to determine whether he is referring to the same thing as us with the word "true." But we also saw that he gives more specific guidance on "true": it is a *value*-term, to be understood on the model of "good" and "beautiful." Presumably, if ordinary-language value-terms refer to anything, they refer to values. But we have just seen that understanding truth in terms of science

---

[87]See the chapter on Windelband in Beiser 2011.

[88]Let me give an indication of what I have in mind. Suppose you are faced with someone who does not care about the truth, instead accepting what his favourite political figure says, regardless of how implausible it is. It might seem that a *philosophical* response to such a person is impossible. Certainly we cannot offer an *argument* since someone who does not care about the truth will not accept our premises. But if truth is a value, there is another strategy. In that case, what we need to do is describe this value in such a way that our opponent knows what it would be, if it existed. This will be enough to have an effect: our opponent will be motivated to pursue the truth even in the absence of a belief that it exist; they will *hope* that it exists. The fact that acquaintance with values can motivate us to pursue them, even without any assurance that they really exist or are possible for us to reach, is a major theme for writers on value—see especially Windelband 1883 and Murdoch 1970.

the way Frege does may be the only way to make sense of its being a value. If so, then we have the following reason to think that Frege is really talking about truth. Our ordinary value-term "truth," if it has a referent at all, must refer to a value; but the only value it could be is the same one that Frege identifies, so the ordinary value-term must be referring to that same value if it refers to anything at all.

2.5   Conclusion

I hope to have shown that for Frege, it belongs to the nature of truth to figure in our scientific goals, in a very strong sense. If he is right, one cannot understand what truth is without understanding that all truths can figure in the ambitious goals of proper science, conceived on Frege's systematic model. In holding this view, he accepts a central tenet of the Neo-Kantian movement of his day.

Chapter 3   Frege on the Generality of Logical Laws

Abstract

Frege claims that the laws of logic are characterized by their "generality," but it is hard to see how this is supposed to identify a special feature of those laws. Many have concluded that Frege does not really have an account of what is special about logical laws. I argue that he does, but that understanding what it is requires connecting his talk of generality with his thinking about truth and science. He means to be identifying the laws of logic as those that appear in every one of the scientific systems whose construction is the ultimate aim of science, and in which all truths have a place. Though an account of logic in terms of scientific systems might seem hopelessly antiquated, I argue that it is not: a basically Fregean account of the nature of logic is available to us today, and indeed looks to be quite promising.

3.1   A Normative Science

Does logic study how thinkers *ought* to think, or how they do, in *fact*, think? Is it the "ethics" or the "physics" of thinking? These were the terms in which the nature of logic was being disputed when Frege formulated his views.[1] He affirms that "like ethics, logic can also be called a normative science,"[2] and diagnoses a confusion that leads people to mistakenly take up the opposing position. It is "commonly granted that the logical laws are guidelines which thought should follow" but "it is only too easily forgotten,"[3] because, though we correctly "define the task of logic as the investigation of the laws of thought,"[4] we become confused by "the ambiguity of the word 'law'…In one sense [a law] says what is; in the other it

---

[1]See Chapter 1 of Carl 1994 for the state of the debate at the time.
[2]Frege 1897a, 128
[3]Frege 1893, xv.
[4]Frege 1879-1891, 4

prescribes what ought to be."[5] By applying the wrong sense in the claim that logic investigates the laws of thought, we wrongly conclude that logic studies "laws in accordance with which thinking actually takes place."[6]

Frege thinks that what makes the error particularly hard to avoid is that the laws of logic are actually laws in *both* senses: they both assert what is *and* prescribe what ought to be. As statements of what is, however, they have no special relationship to *thought*: they tell us, for example, that everything is self-identical. But by stating what is, they also tell us how we ought to think, because "any law asserting what is, can be conceived as prescribing that one ought to think in conformity with it, and is thus in that sense a law of thought." This way of being a law of thought, Frege notices, "holds for geometrical and physical laws no less than for the logical."[7]

But if *every* law is a law of thought, then Frege should not be able to define the task of logic as the investigation of the laws of thought. He holds onto that definition by claiming that the laws of logic are those with a special claim to being laws of thought, because "they are the most general laws, prescribing how to think wherever there is thinking at all."[8] Hence, "the task we assign logic is only that of saying what holds with the utmost generality for all thinking, whatever its subject matter."[9]

The laws of logic are special because they are the most general—but what exactly does this "generality" amount to? One might assume that he is saying that the logical laws tell us about *everything* there is: they are *universal generalizations.* But that kind of generality is not special. On Frege's analysis, a law like "All whales are mammals" is a universal generalization, saying that every object is such that *if* it is a whale, it is a mammal. One might point out that the laws of other sciences are often, in a sense, "restricted" by their conditional form: the above law is in this sense "restricted to whales." But many of Frege's logical laws have this conditional form too.[10] One might point out that many logical laws involve *higher-order* generality as well as first-order generality: they tell us about all concepts (or all properties) as well as all objects. But not all logical laws involve second-order generality.[11] So what does Frege have in mind?

---

[5]Frege 1893, xv.

[6]Frege 1879-1891, 4

[7]Frege 1893, xv.

[8]Frege 1893, xv.

[9]Frege 1897a, 128

[10]An example is Basic Law I, which says something like "For all x and all y, if x is the True then if y is the True then x is the True."

[11]Again, Basic Law I does not. Macbeth 2005 (103-108) argues that the logicality of Basic Law I and the other laws that lack second-level generality depends on the fact that it "can be construed"

As we will see in the following two sections, we do not yet have a good answer to this question. This strikes me as a scandal. Until we know what Frege thinks logical truths are, we cannot so much as understand the central claim of his life's work, that arithmetical truths are logical truths. Nor can we understand what he took himself to be doing when he put forward the logical system whose essentials we still teach every undergraduate philosophy major today. This is why it matters what *Frege* thinks logic is, and most of this paper is dedicated to answering that question. In the final section, I will discuss why it matters what logic *really* is, and argue that we should take Frege's answer seriously.

3.2   Generality: The Required Vocabulary Interpretation

Frege thinks that "logic...has its own concepts and relations...To logic, for example, there belong the following: negation, identity, subsumption, subordination of concepts..."[12] Thomas Ricketts has suggested that the generality of the logical laws is inherited from special features of the vocabulary that refers to these concepts and relations: "logical laws are maximally general in that the only vocabulary required for their expression is the topic-universal vocabulary required for statements on any topic whatsoever."[13] We can call this the *Required Vocabulary Interpretation* of generality.

In what sense is the vocabulary of logic "required" for statements on every topic? Ricketts does not offer a precise view, and he thinks Frege himself had none in mind.[14] And in any case, Ricketts thinks it would have been obvious to Frege that no special feature of the vocabulary used to express the logical laws, including this one, can really distinguish the logical laws from those of the other sciences. After all, many obviously non-logical truths can be stated using only the logical vocabulary: for example, that there exist five non-logical objects.[15] Ultimately, then, Ricketts thinks that "Frege has only a retail conception of logic,

---

so that it involves multiple levels of generality. But *any* law can be "construed," along the lines she suggests, and she offers no reason to think Frege attaches any importance to the possibility of reconstruals of his logical laws.

[12]Frege 1906b, 338.

[13]Ricketts 1996, 123. In earlier papers, Ricketts seem to have missed the fact that logic has its own concepts and objects, claiming that "the laws of logic do not mention this or that thing." (Ricketts 1985, 4.) Later papers seem to express a slightly weaker characterization of these concepts: they are said to "appear in thought and discourse on every subject matter whatever." (Ricketts 1998, 138.)

[14]Ricketts thinks that the notions that Frege uses to describe logic are "irremediably fuzzy." (Ricketts 1998, 151.)

[15]i.e., objects that are not value-ranges, truth-values, etc. This kind of example appears in Heck 2012, 35-36.

not a wholesale one. He tells us what logic is by identifying specific laws and inferences as logical…[he] does not state a defining criterion of the logical."[16] In particular, "generality" is not such a criterion.

The idea that Frege did not successfully distinguish the laws of logic from the others is widely accepted, and by a range of commentators who disagree about central aspects of Frege interpretation, including Warren Goldfarb,[17] Patricia Blanchette,[18] Jamie Tappenden,[19] and Richard Heck.[20] Some of these authors think Frege was actively seeking a distinguishing feature that he had not yet found, while others think he had principled reasons for thinking a genuine demarcation impossible,[21] but they agree that what he says about "generality" is not sufficient, and that he knew it.

This agreement is premature. Frege never acknowledges that some non-logical truths have the same generality as logical laws, and before invoking the generality that gives the laws of logic a special claim to the name "laws of thought," he emphasizes that "how the logical laws are conceived" will be "decisive for the treatment of this science." This gives the strong impression that he at least *thought* that he was providing a distinguishing feature of the logical laws, through which we can conceive them.[22] This means that we should look for an understanding of "gener-

---

[16]Ricketts 1996, 124.

[17]"Frege's conception of logic is retail, not wholesale. He simply presents various laws of logic and logical inference rules, and then demonstrates other logical laws on the basis of these. He frames no overarching characteristic that demarcates the logical laws from others." (Goldfarb 2001, 30.)

[18]"He gives no general characterization of the principles or truths of logic. He simply exhibits a small handful of what he takes to be self-evidently logical truths and inference-rules…" (Blanchette 2012, 147.)

[19]"It is true that nowhere does Frege give a criterion of the logical, although this could simply reflect that he had not arrived at one." (Tappenden 1997, 213.)

[20]Frege "was struggling with…questions about the nature of logic…he was developing a conception of logic in which [semantic notions] would play a fundamental role," but he never arrived at a settled view. (Heck 2012, 38.)

[21]Goldfarb, for example, thinks that Frege's principled commitment not to do "semantic" theorizing would rule out a substantive criterion of the logical. (See, e.g., Goldfarb 2001.) But even if Frege holds such commitments, they at most rule out particular *kinds* of demarcation for logic, like those involving a substantive use of a truth-predicate applied to sentences.

[22]Goldfarb tries to dissolve this tension by quoting Frege's claim that he has provided only a "rough indication of the goal of logic." (Goldfarb 2001, 31.) This sounds to Goldfarb like an admission that what he has said about generality is not really a way of demarcating the logical laws. But for one thing, the "roughness" claim is from one of the unpublished discussions of generality, while the more complete, published discussion in Frege 1893 contains no indication that anything is rough in what he is saying. Moreover, what Frege strictly says in the unpublished discussion is that he is just trying to "roughly make the goal recognizable." ["ungefähr das Ziel kenntlich…machen."] Frege emphasizes that any "roughness," derives from "the author's inadequacy and the awkward-

ality" according to which it at least plausibly distinguishes the logical laws from the others—unless, of course, the evidence for an interpretation like the *Required Vocabulary Interpretation* is very powerful.

For that interpretation, however, the evidence is in fact very thin. The mere fact that logic *has* its own vocabulary does not motivate interpreting the generality of its laws by reference to that vocabulary. The passages in which Frege claims that the logical laws are special because of their generality makes no mention at all of the vocabulary of logic, nor of the concepts and objects to which that vocabulary refers. I think the source of this interpretation is actually someone *else's* writings about logic—it is *Quine* who endorses the "widely applicable method of demarcating a branch of science...the method of listing the vocabulary," so that for Quine, "The logical truths...are the truths in which only the logical...vocabulary occurs essentially."[23] Since there is no textual reason to think Frege had this in mind with his talk of generality, we should not accept the *Required Vocabulary Interpretation.*

3.3   Generality: The Normative Interpretation

Frege says that the task of logic is "that of saying what holds with the utmost generality for all thinking, whatever its subject matter." But if logic's generality has to do with "all thinking," it must have to do with the laws in their *prescriptive* aspect, because it is only in that aspect that these laws are especially concerned with thinking at all. This is confirmed by Frege's claim that "they are the most general laws, prescribing how to think wherever there is thinking at all." The generality is elucidated in terms of a universal *prescription* for *thinking*. This leads us to the *Normative Interpretation* of generality: as John MacFarlane puts it, "the kind of generality that distinguishes logic... is a generality in the applicability of the norms it provides."[24] *All* thinking ought to be in conformity with these laws.

This interpretation has been gaining popularity recently, and rightly so: the texts that discuss generality itself clearly point us in this direction. But there is something odd about the way it is usually endorsed. Commentators often state the *Normative Interpretation*, but then immediately make additions to the basic claim

ness of language" rather than the absence of something sharp that distinguishes the logical laws from the others.

[23]Quine 1986a, 399. The Quinean origin of Ricketts' idea comes out in the way Ricketts puts the claim in terms of "vocabulary" rather than the way Frege himself would, in terms of the special *concepts and objects* of logic. As we would expect, the fact that the logical vocabulary can be used to state that there are a certain number of non-logical objects is sometimes pointed out to Quine as a problem for his way of demarcating logic: see, for example, footnote 35 of Parsons 1986.

[24]MacFarlane 2002, 37.

that these laws are special because they prescribe for *all* thinking. For example, they add that the logical laws are general in that it is *constitutive* of all thinking *that* it ought to proceed in conformity with the laws of logic,[25] which adds the metaphysical claim that *what makes the activity of thinking what it is* is that these laws prescribe for it. One author adds that Frege's notion of generality has "an even more profound dimension: to count as a thinker at all one must *acknowledge* the categorical normative authority that logic has."[26] Neither addition is implied by Frege's basic claim that the laws of logic are special because they tell us "what holds with the utmost generality for all thinking, whatever its subject matter." Perhaps Frege accepts these additional claims, and perhaps not—but either way, why do commentators always make some addition or other to Frege's basic claim?

We will find the answer to this question in a moment. For now, let us stick with the basic *Normative Interpretation* of generality. It will identify a distinctive feature of logical laws only if we ought to think in conformity with the logical laws in *all* of our thinking, whereas for every non-logical law, there is some thinking that is permitted *not* to be in conformity with it. Whether or not that is so depends on what exactly Frege means by "thinking," and "conformity."

Though Frege sometimes identifies thinking with "the grasp of a thought,"[27] MacFarlane thinks we should "take Frege's talk of norms for thinking as talk of norms for judging." I agree, and I see three reasons to do so.[28]

1. Frege tells us to read it this way. He offers (in passing) a correction to his tendency to talk about these laws as applying to "thinking" and "thought": "if we call them laws of thought, *or, better, laws of judgement…*"[29] What he is saying would be better put using the word "judgement."

---

[25]Some of MacFarlane's own statements express this additional constitutive element: "The generality of logic…is a normative generality: logic is general in the sense that it provides constitutive norms for thought as such, regardless of its subject matter." MacFarlane 2002, 37 Another example is Taschek 2008, 383: "what is distinctive about logical laws…is…that in an important sense they issue in, while the laws of physics do not, constitutive norms of thinking as such."

[26]Taschek 2008, 384. This additional idea is tentatively endorsed in Steinberger 2017, 152.

[27]e.g. Frege 1918-1919a, 355.

[28]The question here is not whether Frege thinks the laws of logic issue prescriptions for other mental activities in addition to judging, such as intending. The question is whether, in the particular passages where Frege distinguishes logical laws from the others by claiming that they prescribe for all "thinking," by "thinking" he is referring only to judging, or to other mental activities too.

[29]Frege 1897a, 145. (My italics.) The original reads: "Wenn man sie also Denkgesetze oder besser Urteilsgesetze nennen will, so muss man nicht vergessen, dass es sich dabei um Gesetze handelt, die wie die Sittengesetze oder Staatsgesetze vorschreiben, wie gehandelt werden soll…"

2. In the relevant sections, Frege identifies the goal of "thinking" as *truth*.[30] But it is only *judging*—"the acknowledgement of the truth of a thought"[31]—that has truth as its goal.[32] So "thinking" here must be judging.[33]

3. In these sections, Frege switches between "think" and "judge" without distinguishing between the two. For example, while the claims about generality are in terms of "all thinking," he also says that the laws issue "prescriptions for making judgements," and "prescriptions to which our judgements must conform…if they are to remain in agreement with the truth," and that "we must comply with them in our judgements if we are not to fail of the truth."[34] The lack of any distinction between the use of "thinking" and "judging," suggests that they refer to the same thing.

By "thinking" in conformity with a law, then, Frege probably means "judging" in conformity with it. I will call this interpretive suggestion "*Thinking=Judging*."

It is harder to know what Frege means by "conformity." Frege says that our thinking must be in "conformity" with the logical laws if it is "not to fail of the truth," so conformity must be a relation such that, if our thinking *fails* to bear it to logical laws, we will not be thinking something true. One obvious candidate for this relation is *consistency*. This looks to be what MacFarlane has in mind: his examples are prescriptions not to judge what is inconsistent with logical laws.[35] On this interpretative suggestion, which I will call "*Conformity=Consistency*," to think in conformity with a law is to think thoughts that are not *collectively inconsistent* with it.[36]

On the *Normative Interpretation*, supplemented with both *Thinking=Judging* and *Conformity=Consistency*, Frege is saying that the laws of logic are those with

---

[30]e.g. Frege 1897a, 128: "How must I think in order to reach the goal, truth? We expect logic to give us the answer to this question…"

[31]Frege 1918-1919a, 356; also Frege 1892, 34.

[32]Truth is not the goal of *grasping* thoughts, because sometimes "knowledge of the truth is attained precisely through our grasping a false thought" as in a proof by *reductio*. (Frege 1918-1919b, 375)

[33]MacFarlane's reason in favour of his interpretation is very close to this one, but his formulation depends on assumptions about "conformity" that we should keep separate.

[34]Frege 1897a, 145.

[35]i.e.: "One ought not believe both a proposition and its negation." A perhaps more general statement of what conformity is comes a bit later: "a thinker *ought not* make judgements that are incompatible with [the laws]." (MacFarlane 2002, 36.)

[36]This is stronger than holding that *each* thought that one thinks should be consistent with the law, which allows one both to think that P and think that not-P, so long as one does not think that P and not-P. The stronger one is more likely to be what Frege has in mind, because one who thinks that P and thinks that not-P "fails of the truth" just as surely as one who thinks that P and not-P.

which all of our judgements ought to be consistent. But there is a problem: this feature does not distinguish the laws of logic from those of other sciences. The following argument shows that all judging ought to be consistent with every law of every science.

- S) Suppose that I am judging, and L is a law of some science.
- P1) All judging ought not proceed in a way that fails to reach the goal of judging.[37]
- C1) Therefore, I ought not judge in a way that fails to reach the goal of judging. (S and P1)
- P2) To judge things that are collectively inconsistent with L is to fail to judge only true things.[38]
- P3) The goal of all judging is to judge only true things.[39]
- C2) Therefore, judging things that are collectively inconsistent with L is to judge in a way that fails to reach the goal of judging. (P2 and P3)
- C3) Therefore, I ought not to judge things that are collectively inconsistent with L. (C1 and C2)

It looks like the *Normative Interpretation* cannot demarcate the logical laws any more than the *Required Vocabulary Interpretation* can. This, I suggest, is why commentators do not rest for long with the *Normative Interpretation* itself. They do not really believe that it identifies a distinctive feature of the logical laws at all. To find a distinctive feature, they think we must go beyond Frege's normative claim, to a metaphysical claim about what makes thinking thinking, or a claim about what all thinkers must acknowledge. This would give Frege the chance to say: even though all laws prescribe for all thinking, only in the case of the *logical* laws is it *constitutive of all thinking* that they prescribe for it—so they are special. Or the chance to say: even though all laws prescribe for all thinking, only in the case of the *logical* laws is it *impossible to be a thinker* while failing to *acknowledge* their authority—so they are special. This is to abandon Frege's claim that the logical laws are special because they say "what holds with the utmost generality for all thinking, whatever its subject matter," and "prescribe universally the way in which one ought to think if one is to think at all." It is to admit that this *normative* feature of logical laws is not special after all.

   I think this line of thought is a mistake. Frege clearly thinks that the logical

---

[37]Though a principle forbidding one to proceed in a way that makes the goal of one's activity unattainable might be dubious in complete generality, it is because Frege accepts this instance of it that he thinks that the prescriptions of the laws of thought apply to our thinking at all. We ask: "How must I think in order to reach the goal, truth?" and the prescriptions that the laws generate give the answer. (Frege 1897a, 128.)

[38]This is because laws are truths, and what is inconsistent with a truth is false.

[39]The quotes above identify "truth" and "not to fail of the truth" as the goal of judging.

laws are special because they prescribe for all thinking.[40] The key to understanding why he thinks this is to see that "conformity," does not just mean "consistency."

3.4 Systematic Conformity

So: according to Frege it is distinctive of logical laws that all thinking ought to be in conformity with them, and by "thinking," he almost certainly means judging. But if this is to be a distinguishing feature of these laws, conformity must be some relation such that if any thinking fails to bear it to the *logical* laws, that thinking will "fail of the truth", whereas for each law of the other sciences, there is some thinking that need not bear it to that law in order to "remain in agreement with the truth." The argument above shows that *consistency* is not such a relation. In fact, it is not easy even to think of such a relation. (Give it a try!) But if we recall some features of the way Frege thinks about truth, this relation comes into view.

In several places, Frege makes clear that when he talks about "truth," he means only what we might call "scientific truth."[41] He says that "in logic, we are concerned with truth in the strictest sense of the word,"[42] which he calls "truth in the scientific sense"[43] and "that sort of truth which it is the goal of science to discern."[44] There claims express a substantive assumption: that all truths figure in the goal of science. This is what allows Frege to argue that certain words cannot be used to express truths on the grounds that they "are illegitimate in science,"[45] or unsuitable "for conducting proofs."[46]

---

[40]Whether or not Frege accepts those other claims, then, is a side-issue, so I will not try to settle it here—but I will say what I think. Frege does *not* accept that all thinkers must acknowledge the normative authority of the logical laws. It is central to his discussion of the so-called "logical alien" that though we may be unable to bring *ourselves* to reject (or even doubt) the normative authority of a law of logic, this "hinders us not at all in supposing beings who do reject it." (Frege 1983, Introduction.) To "reject" something requires thinking, so there are possible thinkers who reject the authority of the logical laws. (The only way to escape this point is to endorse the heroic attempt in Conant 1991 to read Frege as ultimately forced to retract many of the claims he makes in his discussion of the logical alien, as "a ladder which one climbs up and then throws away.") On the other hand, I think that while Frege *does* accept that it is constitutive of thinking that it ought to be consistent with the laws of logic, he also thinks it is constitutive of thinking that it ought to be consistent with the laws of physics. This is simply because these laws are true, and it is constitutive of thinking to have truth as its goal. This feature, then, is not distinctive of the logical laws.

[41]For a fuller discussion of Frege's notion of truth, see Chapter Two of this dissertation.

[42]Frege 1895, 226.

[43]Frege 1906a, 186; Frege 1914, 232.

[44]Frege 1918-1919a, 352.

[45]Frege 1895, 226-228.

[46]Frege 1896, 115.

The centrality of proof for science—and so, for scientific truth—is a consequence of the *systematic conception of science* which Frege shares with most 19th century figures.[47] He officially introduces the term "proof," and the related term "primitive truth," in connection with these systems: to have a scientific system is to see how all the truths belonging to a science can be *proved* from the *primitive truths* of that science.[48] According to Frege, it is systems that allow us to achieve the central purposes of science: to grasp the *nature* or *essence* of the subject in question,[49] to have *command* or *mastery* over its domain,[50] and to *understand* and *explain* its phenomena.[51] This is one reason why Frege is so concerned to police the boundaries between sciences: if we wrongly include a certain truth in the system of a certain science, we fail to grasp its nature.[52]

When Frege says that the goal of judgement is truth, then, he means something that has a place in a scientific system. This suggests a notion of "conformity" that is much stronger than mere consistency: one that picks up on the relationship to laws that truths have in the proof-structure of the scientific systems in which they appear. Suppose, for example, that I judge that all whales are mammals, which is a truth that has a place in the science of zoology, but not that of geology, or any other science. In that case, this biological law's status as a scientific truth depends on its having a place in the scientific system of zoology—it depends on the definite relationship is bears in the proof-structure of that system to the other laws of that science. By contrast, it has no such relationship to the geological law of superposition. I suggest that this relation is what Frege means by "conformity": when I judge the law that whales are mammals, I am judging in conformity with the rest of the laws in the science of zoology, but not in conformity with the geological law of superposition. More generally: to judge in conformity with a particular law is to judge something that *only has a place in a scientific system that includes that law.* Each truth bears this relation to all and only the laws on which its status as a truth depends. I will call this interpretive suggestion *Conformity=Systematicity.*[53] Let us

---

[47]For a fuller discussion of Frege's conception of science, see Chapter One of this dissertation.

[48]Frege 1914, 204-205.

[49]Frege 1914, 204-205.

[50]Frege 1880-1881, 39.

[51]Frege 1880-1881, 36.

[52]For examples of such policing, see See Frege 1879-1891, 5 and Frege 1914, 203: logic does not include psychological truths, physics does not include chemical truths, physics does include geometrical truths, jurisprudence does include both historical and psychological truths, and so on.

[53]There would be a potentially problematic circularity in appealing to conformity to characterize logical laws, if we had to appeal to a notion of logical truths to characterize scientific systems. But there is no need to do so. For example, though Frege ultimately argues that it is a bad idea, it is consistent with the systematic conception of science to allow non-logical proofs, such as that from "x is a whale" directly to "x is a mammal."

see what the generality of the logical laws looks like when we read "conformity" this way.

3.5 The Systematic Generality of the Logical Laws

Frege says that the laws of logic are special because of their generality. If we understand this in terms of the *Normative Interpretation* of generality, supplemented by *Thinking=Judging*, and *Conformity=Systematicity*, what he means is this: all our judging ought to be of what only has a place in a scientific system that includes the laws of logic. I will call the feature that this claim attributes to the laws of logic "systematic generality."

So far, I have identified two criteria of adequacy for an interpretation of Frege's notion of generality. In discussing the *Required Vocabulary Interpretation*, I argued that an acceptable interpretation must identify generality with a feature that, at least plausibly, distinguishes the logical laws from the laws of the other sciences. In discussing the *Normative Interpretation*, I argued that a good interpretation must identify this distinctive feature as a normative one: what is special about these laws is that they prescribe for all thinking. I will now argue that my interpretation satisfies both criteria of adequacy. If so, that makes it the best interpretation that we have.

Does Systematic Generality Pick Out the Right Laws?

Is it *true* that the logical laws are the only ones such that all of our judging ought to be of what has a place only in systems containing them? This is not quite the right question to ask—at least, not yet. The criterion of adequacy is not that our interpretation must make Frege *correct* about what is distinctive of logical laws, or even correct that all and only the laws he himself picks out as logical have the feature that he claims is distinctive of logical laws. Frege could have made a mistake about what is distinctive of the logical laws, and he could have made a mistake in thinking that a particular law has or lacks a feature. The criterion of adequacy is rather that our interpretation must make it at least *plausible,* given the rest of what Frege thinks, that all and only the laws Frege sees as logical have the feature he identifies as distinctive of logical laws.

Another way to put what is special about laws that exhibit systematic generality is that they appear in *every* scientific system, while non-logical laws do not.[54]

---

[54]In other passages, Frege accepts this claim, and even uses it rather than the explicitly normative claim to distinguish the laws of logic from the laws of other sciences—a point that corroborates this interpretation. For example, he characterizes the primitive logical laws as those laws "on which

What makes it plausible that Frege's logical laws have this feature is his view of how the proofs in every scientific system will proceed. To carry out a proof in any science, he thinks we will first prove various logical theorems from the axioms of logic, then instantiate those theorems for the relevant concepts and objects of the science in question, and then use one of a few modes of inference along with truths belonging to the science in question to reach the desired truth.[55] Because all sciences depend on such proofs, we can see why the laws of logic might be needed in every science. But are there any laws of other sciences that are also plausibly needed in this way? And is it really plausible that *all* of Frege's logical laws will be needed in *all* systems?

Given Frege's views about the boundaries of sciences, it is unlikely that any of the laws that Frege counts as non-logical will be needed in every scientific system. Frege thinks some sciences, such as geometry, are wholly *a priori*. No truths that can be learned only by experience will qualify as logical, then, because no such truths have a place in the science of geometry. As for geometry itself, though its truths appear in many other scientific systems, they will not appear in every science. Frege thinks that alongside our spatial "source of knowledge" which gives rise to the science of space, there is a temporal source of knowledge, that presumably gives rise to a science of time—and just as claims about time do not appear in the science of space, so claims about space will not appear in the science of time.[56] In any case, since Frege is Kantian enough to suppose that the source of geometrical knowledge is the pure intuition of space,[57] he is presumably Kantian enough to suppose that there could be creatures with a different form of outer sense. Such creatures could have systematic sciences, but those sciences would not concern space.[58]

---

all knowledge rests." (Frege 1879, Preface.) The distinction is one of generality: *all* knowledge rests on these laws, while only *some* knowledge rests on the others. By "knowledge," he means what a scientific system provides, and the metaphor of "resting" ("*beruhen*") is usually used in reference to the relationship between theorems and primitive truths in a scientific system: for example, when he claims to be "looking for the fundamental principles or axioms upon which the whole of mathematics rests," (Frege 1897b, 235) and when he claims that it should be "demanded that all propositions appealed to without proof are explicitly declared as such, so that it can be clearly recognized on what the whole structure rests." (Frege 1893, vi.) What is special about the primitive truths of logic, then, is that they are primitive truths in *all* sciences, which means that the laws of logic appear in all scientific systems.

[55]See 34-35 of Goldfarb 2001 for this point and a worked example.

[56]Frege 1924/5, 274 mentions the temporal source of knowledge. Frege 1880 claims that geometry makes no appeal to truths about time.

[57]See, e.g., Frege 1874 56-7, Frege 1884 §13 and §89, and Frege 1903b, 273

[58]What about claims like "There are at least five non-logical objects?" Frege 1884, §3 claims that all empirical sciences involve claims about particular objects, since what it is for a claim to be empirical is for its proof to require such a claim. If every *a priori* sciences involved claims

Frege's views about the boundaries of sciences will also at least rule out *obvious* reasons why some of his logical laws will be unnecessary in some sciences. There would be easy counterexamples if any arbitrary set of truths made up a science. In that case, there would be no difficulty finding sciences in which one of Frege's logical axioms do not appear. But we have seen that a genuine science must have a *nature* or *essence* to be revealed. It is not obvious what Frege thinks makes for an essence for a science, but he seems to be guided by at least two considerations.[59] First, taking the science of geometry as a model, the boundaries of a science are sometimes connected with the *source of knowledge* of its distinctive primitive truths. One reason, then, that we could not count half of the primitive truths of geometry and their consequences as a science in its own right may be that the other half of the primitive truths flow from the same source of knowledge. Second, taking the science of logic as a model, the boundaries of a science are determined by a substantive answer to the question: what do all of these laws have in common? Whatever it is that Frege has in mind here, then, it will rule out easy counterexamples to the claim that the laws he considers logical will be needed in every science.

There is a great deal more to be said about this, but this is enough to see that my interpretation meets our criterion of adequacy: it is at least plausible, given the rest of what Frege thinks, that his logical laws are distinguished from the laws of other sciences by their systematic generality.[60]

---

about particular non-logical objects, then this truth would qualify as logical on this interpretation. However, I know of no reason to suppose that Frege thinks that every a priori science involves a commitment to particular non-logical objects. Frege sometimes goes out of his way to show that truths about concepts are not about the objects, if any, that fall under them, which allows them to remain true even if the concepts are empty. (e.g., Frege 1884, §47.) This independent of truths about concepts from the existence of objects is, presumably not coincidentally, a major emphasis of Lotze's famous endorsement of Platonism in Book III, Chapter II of Lotze 1874. (One might worry that every *a priori* science other than logic depends, like geometry, on *a priori* intition, and further worry that *a priori* intuition must automatically commit us to claims about particular objects. But even if the first claim is true, §13 of Frege 1884 explicitly denies that pure intuition yields claims about genuinely particular objects: "One geometrical point...cannot be at all distinguished from any other; the same holds for lines and planes...the points or lines or planes which we intuit are not particular at all, which is why they can count as representatives of their whole kind."

[59]As we have seen, listing the concepts with which the science deals does not qualify as identifying an essence, because there are truths that can be stated using only the concepts of a science that do not themselves belong to that science (for example: using only logical concepts, we can state that there are at least five non-logical objects.)

[60]We might also want to know what positive *reasons* Frege has for putting forward this or that law as logical: as one that will appear in every science. I cannot settle this question here, but I will say a bit about it. One reason Frege has for thinking that particular laws are logical is that they are necessary to the science of arithmetic. (See, for example, the claim about Basic Law V in Appendix 2 of Frege 1893: "I do not see how arithmetic can be founded scientifically...if it is

Is Systematic Generality a Normative Interpretation?

In discussing the previous criterion of adequacy, I noted that we can put what is distinctive of logical laws by saying that they will appear in every scientific system. This might make us worry: wasn't the second criterion of adequacy that what is special about the logical laws be a normative feature, namely that they prescribe for all judgement? Haven't we now identified a merely descriptive feature that is distinctive of those laws?

But there is no conflict here. The criterion of adequacy is that our interpretation must make it plausible that the laws of logic are the only ones that prescribe how *all* our judging ought to be. On my reading, this is so: all our judging ought to be of what only has a place in scientific systems that include the laws of logic, and this is not true of any other laws. It is not a criterion of adequacy that this be the *only* way to put what is special about the laws of logic, or the only thing that is special about them.

But there is also a deeper response available. I think that Frege appeals to an explicitly prescription to distinguish the laws of logic because to state what is special about logic in terms of scientific systems is to give an *implicitly* prescriptive characterization of it. That Frege sees *truth* as implicitly prescriptive is suggested from the way he counts it among those paradigmatic loci of normativity, values. He says that the word "true" belongs with the words "beautiful" and "good,"[61] and he calls the True and the False truth-*values*.[62] The connection between values and prescriptions is so close that to know what the values are is to know certain prescriptions: that one ought to *judge* the True, *feel* the Beautiful, and *will* the Good.[63] Since a scientific system is a system of scientific truths, and a scientific

---

not...permissible to pass from a concept to its extension." This reason for identifying particular laws as logical goes *by way of* his logicism about arithmetic, which (on my reading) amounts to the claim that the laws of *arithmetic* will be needed in every systematic science. This view is sometimes called Frege's "Pythagoreanism." (See, for example, Beaney 1996: "Pythagoreanism...had an epistemological dimension—that we can have no [scientific] *knowledge* of anything without ascribing it a number; and this fundamental Pythagorean belief was certainly endorsed by Frege...") It might seem implausible to maintain Pythagoreanism about as broad a range of sciences as Frege means to include: does jurisprudence *really* require a whole scientific arithmetic? But the view that it does is right in line with that expressed in the most widely-read logic book of the day, Lotze 1874. Lotze claims that all sciences require laws, and that all laws—even in jurisprudence—exhibit the kind of order that we must express using the full resources of arithmetic. As Thomas 1921 (6) puts it, Lotze thinks that this order is to be "expressed as a mathematical proportion between its terms. Speaking of law in the judicial sphere, [Lotze] says that the aim of law is to establish a graduation in penalties that shall correspond to a graduation in offences, this graduation proceeding on a quantitative or mathematical basis."

[61] e.g. Frege 1918-1919a, 351.

[62] See Section 2.4 for more on this point.

[63] As Windelband puts it, "thinking has the goal of being True, willing the goal of being Good,

truth is one that has a place in a scientific system, it will be the same for systems: one cannot understand what a scientific system is without understanding that all our judging ought to be of what has a place in one.

Let me put this point another way. Someone might object to what I have been saying as follows. Sure, maybe all of our judging ought to be of what only has a place in a scientific system including the logical laws, and maybe that is unique to those laws, and maybe that is what Frege means by "generality". But, the objection goes, this cannot be the whole story. There must be some explanation of *why* these laws are needed in every science. And here, we must appeal to a prior fact about them—a semantic, metaphysical, or even a psychological fact. Since Frege would have seen the need for this explanation, he must have not yet been satisfied with an account of logical laws in terms of this generality. The normative claim is just a place-holder, until a deeper characterization of logicality can be found.[64]

I think there is indeed more to be said about why these particular laws appear in every science—as we saw above, for example, more can be said about the boundaries of sciences—but what more there is to say will be normative "all the way down." The logical laws are what they are because of the basic contours of sciences, and these contours are not determined by anything more basic than the facts about our most ambitious cognitive goals. That is why Frege's basic characterizations of logic are always normative. Even if there were a special metaphysical or semantic feature that the logical laws share, that would not be why the laws are logical—and that feature itself might be explained in terms of these normative features.

3.6 Should We Endorse Frege's View?

The main task of this paper, to identify how Frege understands the generality that distinguishes the laws of logic from those of other sciences, is now complete. But the views on the normativity and the nature of logic that I have ascribed to Frege are very different from those that appear in contemporary discussions of these topics,[65] and readers might be disappointed to learn that he holds them. It might seem that despite his status as the founder of modern logic, Frege's views about the philosophy of logic are unavailable to logicians these days, as a consequence of being bound up with views about truth and science that are dubious and outdated.

---

and feeling the goal of grasping the Beautiful." (Windelband 1883, 328.)

[64]Thanks especially to John Campbell and Richard Heck for making it clear how this question can still seem unanswered. As Campbell memorably put it, unless we say more, it seems that a logical law is something that just *shows up* everywhere in science—"like a stalker!"

[65]See, for example, the absence of anything like Frege's view from the survey of positions on the normativity of logic in Steinberger 2016.

I want to conclude by making a preliminary case that this is not so. Though Frege's philosophy of logic may indeed be different from the popular views today, I think that it deserves to be taken seriously, and that it is a major shortcoming of contemporary discussions if they ignore it. I will not make this case by defending Frege's views of truth and science, but by showing that we can abstract from the specifics of those views to discover a core conception of the normativity and the nature of logic that is very promising, even if accepting it may involve some re-orientation of our logical investigations. I will begin with the normative role that Frege claims logical laws play, and then discuss the idea that playing this normative role is the distinctive feature of logical laws.

On the Normativity of Logic

Frege's interest in scientific truths and scientific systems derives partly from his conviction that it is only by having such systems that we can understand the world and explain its phenomena—important cognitive goals whose status as such yields prescriptions for our thinking. His core idea about the normativity of logic is that because of their role in proof, the logical laws play a central role whenever we achieve these goals. Abstracting from details, we can identify a *broadly Fregean* view of the normativity of logic as one on which our highest cognitive goals provide prescriptions for cognitive activity, and on which our recognition of the logical laws play an essential role whenever we meet those goals.

Leading contemporary accounts of how we reach our highest cognitive goals find a similar role for logical laws, partly because of a continued focus on interconnected systems of information. To take one example: the leading general theory of *explanation* is probably the "unificationist" theory defended by Philip Kitcher,[66] according to which an explanation is an argument that is an instance of one of the few "argument patterns" that best "systematize" the "set of statements endorsed by the scientific community." Kitcher defends the "deductive chauvinist" view that the only explanatory argument-patterns are deductively valid arguments, which provides a special role for logical laws in all explanations. To take another example: there is much disagreement in the expanding literature on *understanding*[67] but "if there is a common idea here it seems to be that understanding is directed at a complex...with parts or elements that depend upon, and relate to, one another."[68]

---

[66]Quotes here are from Kitcher 1989, which develops the ideas in Friedman 1974, according to whom an explanation is something that "gives us understanding of the world." While counterfactual or interventionist accounts are also popular, they have difficulty accounting for explanations in sciences whose truths are necessary, like mathematics.

[67]See, for example, Zagzebski 2001, Kvanvig 2003, Elgin 2006, and Pritchard 2009.

[68]Grimm 2012, 105.

These dependencies and relationships paradigmatically include logical relations.[69]

No matter what we say about science and truth, then, a broadly Fregean view of the normativity of logic follows from leading accounts of our highest cognitive goals, assuming that those goals provide prescriptions for our cognitive activity. There will, of course, be substantive disagreements between proponents of broadly Fregean views. For example: some of the theorists just mentioned give the laws of *probability* the same normative role as the logical laws. Frege would deny this, because he denies that the laws of probability must appear in every science. He does agree, however, that the laws of probability will appear in all sciences involving empirical laws, since such laws will be proved on the basis of induction, and "induction must be based on the theory of probability."[70] One way to convince Frege that the laws of probability have this normative role, then, would be to convince him of the empiricist thesis that genuine understanding requires a connection with empirical laws—so that, for example, there could be no *a priori* science of space that provides genuine understanding of it.

On The Nature of Logic

A broadly Fregean view of the normativity of logic makes space for a broadly Fregean view of the *nature* of logic: we can hold that what is special about the logical laws is that they have the normative role just described. Whether or not that would be a good account depends on why it matters which laws count as logical in the first place. I think there are two reasons it matters, which we can call "looking after the future" and "looking after the past."[71]

First, consider a 20th-century philosopher who announces that what is distinctive of philosophical truths is that they are true in virtue of the meanings of the words involved, so that the proper method for philosophy is linguistic analysis. This pronouncement might be revolutionary in spirit: regardless of what

---

[69]For example, according to Kvanvig 2003, "understanding requires...an internal grasping or appreciation of how the various elements in a body of information are related to each other in terms of explanatory, logical, probabilistic, and other kinds of relations." (192-193.)

[70]Frege 1884, §10

[71]Neither of these desiderata involve satisfying "pre-theoretical intuitions." This goal of philosophy is unavailable here, because there do not seem to be any pre-theoretical intuitions about logical laws. As MacFarlane puts it, "Students beginning an introductory logic class typically have inferential intuitions, but they can be brought to distinguish logically valid inferences from materially valid ones only by instruction. All of our intuitions about logicality bear the stamp of theory." MacFarlane 2000, section 1.4. (Anecdotally: I once bought Aristotle's *Politics* from a book store whose owner told me that he liked reading Aristotle, but ultimately found "Aristotelian logic" inadequate. I asked what he thought was wrong with the theory of the syllogism, but received a puzzled look in return. Eventually it became clear that he had only read the *Politics,* and what he meant was only that that book was wrong about a lot of things.)

those called "philosophers" have hitherto been doing, its point might be to orient our future investigations toward important truths that can henceforth be productively investigated using a definite method. An account of the nature of logical laws that *looks after the future* would, in this way, identify a group of laws that are important and can be investigated using a definite method. In giving such an account, we must be in principle prepared for revolution: we may need to deny that the questions and methods of those hitherto called "logicians" have anything to do with what logic really is.

Second, we want an account of logic to do something that Hartry Field has recently emphasized: to tell us "what people who disagree in logic are disagreeing about."[72] More generally, an account of logic that *looks after the past* lets us see similarities and differences among what prominent thinkers say about logic as not merely verbal, but *genuine* agreements and disagreements about some topic, pursued in a reasonable way. For example: if Kant denies that arithmetical laws are logical, and Frege holds that they are logical, we want an account of what logic is to let us see this as a *disagreement* about something, in which Frege and Kant say things in support of their views that are (at least plausibly) genuine reasons that count in favour of those views. In giving such an account, we must be in principle prepared for logic to become a historical curiosity: a faithful account of what people have been agreeing and disagreeing about sometimes involves recognizing that they were not disagreeing about anything that matters very much at all.

Ideally, we would not have to choose: our account of what logic is would look after both the future and the past. It would show us that prominent thinkers have been having reasonable agreements and disagreements about important questions that can be pursued by some relatively definite method. By this standard, I think the broadly Fregean account of the nature of logic is very promising. It looks after logic's future because, as discussed above, it remains a major focus of productive work in philosophy to study cognitive goals, achieving which always involves a certain a set of laws. To make a start on showing that it looks after the past as well, I will indicate a few ways in which it helps us see the right kind of disagreements between Frege and four other important writers on logic: Aristotle, Kant, Carnap, and Quine.

- **Frege and Aristotle**: A well-known difference between Frege's logic and the syllogistic logic of the *Analytics* is that the latter does not allow for proofs involving multiple generality: from the premise that there there is somebody whom everyone loves, we cannot prove that everybody loves at least one person. On the broadly Fregean view, this is a failing of Aristotle's logic

---

[72]Field 2015, 35.

only if such proofs are needed whenever we reach our highest cognitive goals. Frege would first show some scientifically relevant proofs involve multiple generality, including those involving the continuity of a function, limits, and the notion of following in a series. To complete the argument, Frege must show that some such proofs are needed every time we achieve our highest cognitive goal—perhaps because scientific *arithmetic* requires proofs involving the notion of following in a series, and scientific arithmetic is required in every scientific system.

- **Frege and Kant:** On the broadly Fregean view, to deny that arithmetical laws are logical is to deny that we need them whenever we reach our highest cognitive goal. It might seem that there is no disagreement between Frege and Kant here, since Kant identifies that goal as "proper science", and claims that "a doctrine of nature will contain only as much proper science as there is mathematics capable of application there." The disagreement is located in Kant's reasons for saying "mathematics" rather than "arithmetic": his argument only establishes that proper science must include *some* mathematics.[73] Kant would deny that proper science always requires *arithmetic*, because all creatures who share our faculties of reason and the understanding share that cognitive goal, but such creatures need not share the form of sensibility upon which arithmetic depends, time. Frege would respond that arithmetic does not depend on the form of our sensibility.[74]

- **Frege and Carnap:** Carnap is famous for his "principle of tolerance," according to which there is no unique set of logical laws: we can choose one as a matter of "convention," guided by "pragmatic" considerations such as "expediency and fruitfulness," "elegance," and "convenience."[75] On the

---

[73]The discussion is in the Introduction to Kant 1786. Kant's argument is that any science needs to have an a priori part "lying at the basis of the empirical part," but to cognize determinate natural things a priori requires "that [an] intuition...be given a priori," and any knowledge based in this way on "the presentation of the object in a priori intuition, is called mathematics."

[74]Of the many things Kant calls "logic," the broadly Fregean view implies that we should compare Frege's logic not only with "pure general" logic (which has no content at all) but with aspects of "transcendental" logic, which presupposes that the conditions on having the content appropriate for science are met in one way or another without assuming that they have been met by our sensibility: what we study "distinguishes itself not merely from all that is empirical but completely also from all sensibility." (Kant 1781/1787, A65/B90.) MacFarlane 2002 argues that the dispute focusses only on what Kant calls "pure general logic," since he thinks only this has the general normative role in terms of which Frege understands logic. I am convinced by Tolley 2012, however, that transcendental logic is not a special logic, and that the difference between transcendental logic and pure general logic is only one of aspect, not normative generality.

[75]Carnap 1950, section 4

broadly Fregean view, this is logical *nihilism*, as Carnap probably recognizes: while Frege and others had identified logic as the "ethics of thought," Carnap announces that "in logic, there are no morals."[76] The substantive issue is whether or not there is a *single* set of laws that is in every case needed to reach our highest cognitive goal.[77] Frege would argue that many of the considerations Carnap might think of as "pragmatic"—such as maximal simplicity—are actually mandated by the goal itself.[78]

- **Frege and Quine:** Quine accepts that it is a special fact about logic that its "truths and techniques are consequential in all branches of science."[79] The main disagreement is how we are to *identify* which laws have this feature. Quine thinks we must wait and see how well the completed sciences can be systematized using them. Since some sciences are empirical, it is partly an empirical question what the logical truths are. By contrast, Frege would argue that even *before* we can survey the completed empirical sciences, we can recognize that the best systems of every one of the as-yet uncompleted sciences will require a certain set of laws.[80]

3.7   Conclusion

I hope to have made a start toward showing the promise of a broadly Fregean view of the normativity and nature of logic. But more centrally, I hope to have shown that Frege, at least, thinks that the nature of logic is to be found in its connection with the ambitious cognitive goal of science. The key to understanding Frege's claims is to recall that that the only "sort of truth" that he ever writes about is

---

[76]Carnap 1937, §17.

[77]Frege accepts (in Frege 1914, for example) that there can be multiple equally good systems of logic, in that it can be a matter of indifference which logical laws are taken as *axioms* and which are proved as *theorems*, but it is not a matter of indifference which truths are logical laws.

[78]See, for example, Frege 1880-1881, 39 for the importance of simplicity for scientific systems. Quine made just this kind of reply to Carnap: "What seemed to smack of convention....was 'deliberate choice, set forth unaccompanied by any attempt at justification other than in terms of elegance and convenience'...[but] surely the justification of any theoretical hypothesis can...consist in no more than the elegance or convenience which the hypothesis brings." Quine 1960, section VI.

[79]Quine 1986a, 399

[80]Quine is prepared to meet Frege half-way in this, noting that even before the empirical sciences are complete, we can recognize that "The classical logic of truth functions and quantification is...a paragon of clarity, elegance, and efficiency." Quine 1986b, 85-86. Frege, for his part, sounds rather Quinean when he claims that "scientific workshops are the true field of study for logic." (Frege 1880-1881, 33) Though I cannot elaborate here, this is one of the indications that Frege's epistemology of logic is a subtler one than traditional-sounding claims of "self-evidence" make it seem.

itself essentially connected to that same goal. Only by bearing this in mind can we understand why Frege claims that it is distinctive of the laws of logic that they prescribe for all judging.

Part Two

Chapter 4 Frege's Critical Arguments for Axioms

Abstract

There is a substantive philosophical question about how we can be justified in accepting the most basic truths, including logical axioms. We can uncover a neglected answer by resolving an outstanding interpretive puzzle about Frege's approach to logical axioms. The puzzle is: why does he insist that logical axioms are "self-evident," to be recognized as true "independently of other truths," and then offer *arguments* for those very axioms? The answer is that he is following the prevailing Neo-Kantian approach to justifying logical axioms. This approach allows for *epistemically amplifying* arguments: arguments that give us justification for accepting their conclusions even if we lack justification for accepting the premises. Arguments are thought to be able to do this when they derive the conclusions from a goal that is presupposed: the arguments serve to show what else must be true, if the goal can be reached. This is what Frege's arguments for logical axioms do.

4.1   The Puzzle about Frege's Arguments

There is something puzzling about Frege's introduction of his "Basic Laws" or "axioms" of logic. On the one hand, he thinks that "it is part of the concept of an axiom that it can be recognized as true independently of other truths,"[1] and he offers a regress argument to show that there must be truths that have that feature:

> The grounds which justify the recognition of a truth often reside in other truths which have already been recognized. But if there are any truths recognized by us at all, this cannot be the only form that justification takes. There must be judgements whose justification rests on something else, if they stand in need of justification at all.[2]

---

[1]Frege 1899-1906, 168.
[2]Frege 1879-1891, 3.

Axioms, whose justification rests on this "something else," are called "self-evident" and "unproveable."[3]

On the other hand, in the *Basic Laws of Arithmetic*, Frege introduces his axioms by offering arguments for them. For example, Basic Law I is introduced as follows:

Γ→(Δ→Γ) would be the False only if Γ and Δ were the True while Γ was not the True. This is impossible; therefore a→(b→a).[4]

What follows the "therefore" is Basic Law I itself. To appreciate this argument seems to require us to accept other truths as true, such as that Γ→(Δ→Γ) would be the False only if Γ were the True, Δ were the True, and Γ were not the True. (It does not matter for now what these other claims actually mean, since the justification of axioms must not rest on *any* other already-recognized truths.)[5]

So: according to Frege, our being justified in accepting anything at all depends on our being justified in accepting axioms independently of other truths. Yet he offers arguments that invite us to accept his axioms on the basis of other truths. The question is: why does he do that? One natural answer is that he is not offering the arguments to *justify* us in accepting the axioms, but for some other reason. Natural as it is, I think this answer is wrong.

4.2   Some other reason?

If Frege is offering these arguments for some other reason, what might that reason be? In a well-known paper, Tyler Burge has suggested that when Frege says that the axioms are "self-evident," this means that "understanding the content of an axiom suffices to warrant one in believing it."[6] Once we fully and clearly understand the sentences that express the axioms, we are, just on the basis of that understanding, justified in accepting those axioms. This suggests a role for the

---

[3]See, for example, Frege 1884, §5.

[4]Frege 1893, §18. (Translating "also" as the standard "therefore" rather than Ebert and Rossberg's "accordingly.")

[5]But for the curious: I replace Frege's notation with the usual symbols for the usual logical functions. "a" and "b" are Frege's Roman letters, which mark maximum-scope generality without requiring a generality-sign, and the Greek letters mark generality in the same way. The conclusion, "a→(b→a)" says something like: "For all objects x and y, if x is the True, then if y is the True then x is the True."

[6]Burge 1998, 338. I focus on Burge's presentation because it is the most well-known, but many readers have hit upon the same idea. Here are two more clear statements. Jeshion 2001, 944: "clearly grasping them is a sufficient and compelling basis for recognizing their truth." Weiner 2005, 341: "all we need, in order to see that the primitive laws…are true, is to understand the *Begriffsschrift* statements of these laws."

arguments: "The point of the arguments is to articulate the content of the axioms and to elicit a firm understanding of them."[7] The arguments can do this because "Frege famously realized that understanding a thought requires understanding its inferential connections to other thoughts." Hence, "Frege's arguments 'for' his axioms elicit understanding of the axioms by bringing out these connections."[8] On this reading, the arguments show how the axioms follow from other truths, which helps us understand the axioms because part of understanding a truth is understanding its inferential relations to other truths.

This proposal seems to gain plausibility from the observation that since Frege knew that the notation and connectives employed in his artificial language *Begriffsschrift* would be unfamiliar to most readers, he might well have wanted to offer some extra help in understanding the axioms.[9] But this support is undermined by the fact that at the point in the *Basic Laws* when the axioms appear, the notation and connectives would no longer be unfamiliar—the *Begriffsschrift* conditional, for example, had been heavily employed on every one of the nineteen pages preceding the discussion of the axioms. Frege would not have expected someone who was not comfortable understanding these formulae to make it to the axioms. Moreover, Frege's standard method for helping us understand what his formulae mean, which he uses especially when he has recently introduced a new symbol, is a much more obvious one: he gives straightforward translations of the formulae into natural language.[10]

---

[7]Burge 1998, 338. Here are two more clear statements. Ricketts 1998, 144: "These arguments...are to bring an audience who is just learning *Begriffsschrift* to grasp and recognize as true the thoughts expressed by the axioms." Weiner 2002, 168: "Frege is...enabling his readers to understand what is meant by these expressions." Sometimes this is put in Fregean terms by saying that the arguments help us "grasp the sense" of the sentences stating the laws, as in Bar-Elli 2010, who thinks that the arguments are "explicating the modes of presentation of...the things [the axioms] are about," and "presenting [the axioms] as expressing features of the *Sinne* [senses]—the ways their objects are given to us."

[8]Burge 1998, 337. This is the general interpretation that Burge endorses, but he also tentatively suggests that some of these arguments might have been intended to help us understand simply by stating the axioms in different ways: they may not be "moving from one truth to another, but simply thinking through and expressing in different ways, via ordinary language, the character of the axiom which is the apparent conclusion of the reasoning." He does not put this interpretation forward very strongly, claiming only that "there is some circumstantial evidence for thinking that this might have been Frege's view of those arguments," and that "it is certainly not obvious that it was not Frege's view." Burge 1998, 335.

[9]Though Burge does not focus on the difficulty of the unfamiliar language, others do. (See Ricketts and Weiner quotes in footnote 7 above.)

[10]Throughout §12 and §13, he follows up several *Begriffsschrift* formulae with translations "in words," (e.g.: "In words: -1 is a square root of 1 and fourth root of 1,") and in §20 he follows up Basic Law IIa by saying: "we might render this law in words this way: What holds for all objects, holds also for any."

Still, Frege's arguments might be there to help us understand the axioms. But it is hard to find any positive reason for reading him this way. Burge himself does not identify any passages in which Frege *says* that this is the purpose of the arguments, nor does he identify any historical tradition to which Frege belongs in which arguments are used to help readers understand axioms. The best he does to motivate his reading is to point to a parallel between the *Basic Laws* and an earlier work of Frege's, in which a version of the argument considered above for the axiom a→(b→a) is sandwiched between two translations of that axiom from symbols into natural language. According to Frege, the axiom

> says 'The case in which *a* is denied, *b* is affirmed, and *a* is affirmed is excluded.' This is evident, since *a* cannot at the same time be denied and affirmed. We can also express the judgment in words thus, 'If a proposition *a* holds, then it also holds in case an arbitrary proposition *b* holds.'[11]

After quoting this passage, Burge immediately states his conclusion: "Frege clearly regards his argument as an elaboration of what is contained as evident in the axiom itself. It is an elaboration of an understanding of the thought...He sees himself as articulating in argument form what is contained in the very content of the basic truth he is arguing for," which makes it likely that in the *Basic Laws*, "the basic procedure is the same."[12]

But what reason is there to read the argument as serving this purpose in this earlier passage either? Frege seems to appeal, in support of his axiom, to the claim that *a* cannot be at the same time affirmed and denied. This claim is distinct from the axiom itself, which (among other things) has *b* in it. Once again, he does not *say* that he gives this argument to help us understand the axiom. Aside from the shift from talk of "the True" and "the False" to "affirmation" and "denial," the main difference between the *Basic Laws* passage and the earlier one is the presence of the two translations of the axiom into ordinary language. The point of these translations probably *is* to help us understand the axiom—but on the face of it, it is just as likely that the argument serves a *different* purpose from the translations as it is that its purpose is also to help us to understand the axiom, in a different way. This is made especially clear by the fact that in the following pages, Frege sometimes accompanies the official *proof* of a theorem with two translations along the same lines.[13] Of course the role of the *proof* is not to help us understand the

---

[11]Frege 1879, §14. (Following the translation Burge uses.)

[12]Burge 1998, 328-329.

[13]For example, Frege gives the following two translations of proposition 36: "The case in which *b* is denied, ¬*a* is affirmed, and *a* is affirmed does not occur. We can express this as follows: 'If

79

content of the formula, even if that is what the translations are for. In the end, I think there is not much reason to think that the arguments serve the purpose Burge suggests.

Burge's is not the only suggestion for some alternative reason that Frege might have given these arguments. Perhaps Frege is using the arguments to show us something *about* the axioms—for example, *that* they are logical, or *why* they are useful in special sciences.[14] But there is a a very general reason not to look for alternative reasons at all. This is that Frege seems to *tell* us that the purpose of the arguments is to justify the axioms. Here is how he distinguishes in the *Basic Laws* between his own "significant" arithmetic and "formal" arithmetic, which treats the modes of inference and axioms of an arithmetical system like the rules in a game:

> "in Formal arithmetic we spare ourselves any account of why we lay down the rules in exactly this way...we do not extract these rules from the referents of the signs; rather we lay them down on our own absolute authority, reserving complete freedom in principle and acknowledging no necessity to justify these rules."[15]

Given the intended contrast, Frege is saying that in significant arithmetic, we do acknowledge a necessity to justify our laying down axioms and modes of inference

---

a occurs, then one of the two, *a* or *b*, takes place'." These translations accompany the proof of proposition 36 from proposition 1 (an axiom) and proposition 34.

[14]Richard Heck suggests that when Frege gives an argument for an axiom, the purpose is "to uncover the source of its truth, to demonstrate that it is indeed a law of logic," by showing that it has the kind of "source" in semantic facts that laws of logic have. (Heck 2012, section 2.1.) Jason Stanley thinks that only by providing such arguments "can the arithmetician take himself as accounting for the application of his formal theory of arithmetic to the special sciences." (Stanley 1996, 61.) Stanley reads the arguments as making meta-linguistic claims, but this is not required: even if what the arguments *say* must be understood at the object-level, it could be that giving an object-level argument serves to show that important meta-linguistic facts hold. I think the chapter "Metatheory" in Blanchette 2012 should also be read along these lines.

[15]Frege 1903a, §§89-94. In discussing formal arithmetic, Frege refers to both axioms and modes of inference as "rules;" axioms would be rules for what we may write down anytime. (See Heck 2012, 42 (fn36) for this point.) Because of this same passage, Heck and Stanley call the arguments "justifications," but hasten to add that this must be a special "sense" of justification. (E.g., Stanley 1996, 60 takes the question "in what sense can [arguments like Frege's] serve to justify these laws?" to be "one of the most difficult and important questions of contemporary philosophy of logic.") They add this because they do not see how else it could be consistent with the rest of the things Frege says about the justification of axioms for arguments to justify them in the usual sense. There is no indication in the passage itself that Frege is talking about anything but the ordinary sense of justification: when we lay down an axiom in *significant* arithmetic, it is not an arbitrary "rule" in a game, but a judgement that we are to make in the context of a structured science. Such judgements require justification (in the ordinary sense), and what Frege says here shows that the form he thinks that justification takes is that of an account that extracts them from the referents of the signs.

by providing an account that extracts them from the referents of the signs. But the only passages in the *Basic Laws* that could provide such a justifying account are the arguments themselves. It seems, then, that the arguments are meant to justify us in accepting the axioms, because if Frege's is to be a significant arithmetic, such justification is needed.

If this is right, then we should not look for some other reason why Frege offers arguments. Instead, the key to understanding them is to see how what he says about justifying axioms allows that this justification comes from argument.[16]

4.3   Epistemically Amplifying Arguments

In his regress argument, Frege says: "the grounds which justify the recognition of a truth often reside in other truths which have already been recognized. But if there are any truths recognized by us at all, this cannot be the only form that justification takes."[17] Frege's conclusion is about what is required if we are to "recognize" ("*erkennen*" or "*anerkennen*") any truths at all. By this term, he must mean not just *accepting* things as true, but being *justified* in doing so—otherwise, how could a regress of justification threaten our ability to "recognize" truths? Frege usually uses "recognize" in this epistemically loaded way. For example, when he says that axioms "can be recognized as true independently of other truths," he does not mean merely that we can accept axioms as true, but that we can be justified in doing so.

---

[16]It is worth mentioning that Frege had *better* not rule out offering arguments to justify us in accepting axioms in general, since he wants to leave space for Kant's account of geometry. For Kant, axioms are claims that are "immediately certain," and "indemonstrable," as well as "general." (Kant 1781/1787, A164/B205.) The question is whether he thought that "immediately certain" and "indemonstrable" claims were to be argued for. As Hintikka points out, for Kant "'immediate' and 'indemonstrable' did not serve to distinguish immediate perception from an articulated argument, but to distinguish a certain sub-class of particularly straightforward arguments from other kinds of proofs." (Hintikka 1967, Section 7.) These "particularly straightforward" arguments are ones that involve every stage of a traditional Euclidean proof *except* for the *apodeixis*, in which appeal is made to previously recognized truths (axioms or theorems). Frege endorses (or at least takes seriously) the Kantian account of the justification of geometrical axioms at Frege 1874 56-7, Frege 1884 §13 and §89, and Frege 1903b, 273—though he refrains from discussing it in any detail. (E.g. Frege 1903b, 273: "Here we shall not go into the question of what might justify our taking these axioms to be true.")

[17]The original reads: "Die Gründe nun, welche die Anerkennung einer Wahrheit rechtfertigen, liegen oft in anderen schon anerkannten Wahrheiten. Wenn aber überhaupt Wahrheiten von uns erkannt werden, so kann dies nicht die einzige Art der Rechtfertigung sein. Es muss Urteile geben, deren Rechtfertigung auf etwas anderem beruht, wenn sie überhaupt einer solchen bedürfen." (Frege 1969, 3)

This means that when Frege says that "this" cannot be the only form that justification takes, he is talking about cases in which our justification for accepting a truth is found in other truths that we are already *justified* in accepting. His conclusion is that our justification for accepting axioms must not depend on our *recognition*—justified acceptance—of other truths.[18] This leaves open that this justification could come from other claims that we accept *without justification.* Arguments could then justify us in accepting axioms, so long as we can become justified in accepting a conclusion on the basis of an argument whose premises have a *lesser* epistemic status than justification. Let us call an argument that can do this "epistemically amplifying."[19]

If Frege thinks that the way in which logical axioms are to be justified is by epistemically amplifying arguments, and that his own arguments are of this kind, this would explain why he offers these arguments to justify his axioms. This is the only way I know of to understand what his arguments are doing in the light of all he says about the justification of axioms. Moreover, this reading of Frege proves to be strongly supported by historical evidence. For the foremost philosophers in Frege's philosophical milieu, a certain kind of epistemically amplifying argument was seen as the *only* viable approach to the justification of "self-evident" logical axioms. In the following section, I will explain what this general approach looks like. In the final section, I will argue that Frege's arguments exemplify it.

4.4 The Critical Method

It is often observed that there are close philosophical connections between Frege and philosophers in the Neo-Kantian movement.[20] These philosophers endorsed a strategy for the justification of logical axioms that came to be called the "critical method." We find it in Hermann Lotze's *Logic*, which had a tremendous influence

---

[18]And so, when Frege says that axioms "can be recognized as true independently of other truths," he means independently of other *recognized* truths. (Of course he could not mean all other truths simply *being* true, so *some* cognitive term is being elided here.) When he says they are "unproveable," he means that they are not justified by deducing them from previously "known theorems axioms, postulates, and definitions." (See Frege 1914, 204-206 for this technical use of "proof.") When he says they are self-evident, he means that their evidence does not depend on the evidence of any other truths.

[19]Note that an argument that is "epistemically amplifying" is different from an argument that is "ampliative": an ampliative argument is one whose conclusion contains more *information* than the premises; an epistemically amplifying argument is one whose conclusion has a higher epistemic *status* than the premises.

[20]See, for example, Chapter Two of this dissertation, as well as Sluga 1984, Sullivan 1991, Gabriel 2002, Schlotter and Wehmeier 2013, and Glock 2015.

in Germany and which Frege himself read.[21] Lotze thinks that axioms can be justified in two different ways, both of which lend the axioms "immediate certainty." The first way sounds somewhat psychological: we are justified in accepting claims which "we feel immediately to be necessary, and the opposite we feel with equal conviction to be impossible in thought." The second is quite different: we are justified in accepting claims which "must be considered...as an assumption which serves the purposes of thought." If reflection on the "purposes of thought" leads us to see that a certain claim had *better* be true if those purposes are to be attained, Lotze thinks this justifies us in accepting that claim.[22]

This latter idea is enthusiastically endorsed as the only way to justify axioms by Wilhelm Windelband, who (along with Hermann Cohen) was the central figure in German philosophy when Frege was writing, and one of whom Frege was almost certainly aware.[23] His paper "Critical Or Genetic Method?" is his most sustained treatment of the question how axioms are to be justified—a topic so important that he sees the justification of axioms as "the task of all philosophical investigations." Given that "it belongs to the concept of an axiom...to be unprovable,"[24] Windelband sees only one option. Crediting Lotze with insight into the problem, he develops a version of Lotze's "purposes of thought" approach, and dubs it the "critical method."

This method centers around a "purpose," "goal," or "ideal" for our thinking: "The recognition of axioms is always conditioned through a goal, that as the ideal...must be presupposed."[25] We derive the axiom from this goal, showing that the truth of the axiom is a condition on reaching the goal. We accept the truth of the axiom on the basis of this derivation, thus presupposing that the goal can be reached. In the case of logic, whose goal is truth, such arguments show how "Logic can say to everyone: you want truth? Then remember, you must recognize the validity of these [axioms], if your desire is ever to be fulfilled."[26] Which axioms can be justified in this way depends on how the goal is identified: the more ambitious the goal, the stronger the axioms on which reaching it will depend. Windelband himself has a quite ambitious conception of what the goal, "truth," involves. According

---

[21]The discussion is in Lotze 1874, §65, which was "perhaps the most widely read logic text in Germany during Frege's early career," (Heis 2013, 122.) Dummett 1981a was the first to show that Frege had read and taken notes on the *Logic* sometime between 1876 and 1891. Additional work by Sluga 1984 and Hovens 1997 shows that he was probably reading the 1880 edition in 1882.

[22]He thinks it also matters that we find our assumption again and again confirmed "by the concentrated impression of all experience," but this kind of (inductive) confirmation is not enough to justify the law without the connection with purpose.

[23]See Chapter Two for more on the Windelband-Frege connection.

[24]Windelband 1883, 327.

[25]Windelband 1883, 331

[26]Windelband 1883, 330

to him, reaching the truth is possible only through the construction of a certain kind of systematic science, so it depends on a variety of important axioms.[27]

In this method, the starting claims about the goal—the claims from which the axioms are derived—are described as *assumptions* or *presuppositions* precisely because they are not justified. As Windelband sets up the problem, all justification depends on the justification we have for accepting the logical axioms, so there can be no justified claims prior to the logical axioms from which we can derive them. But if arguments that derive axioms from unjustified claims nonetheless justify us in accepting those axioms, they are epistemically amplifying arguments. For Windelband as well as for Lotze, that an axiom must be justified in this way is consistent with calling it "unprovable," "self-evident," and "immediately evident."[28]

The basic idea behind this critical method admits of elaboration and precisification in different ways, and I will argue that Frege's own arguments provide us with one example of how to implement this basic idea: they are *critical arguments*. But first, let me say something about this approach to axioms more generally.

4.5   Could Critical Arguments Yield Justification?

The basic point of Frege's regress argument is that not all of our acquired justification comes from other claims that we are already justified in accepting. This point is familiar and widely accepted. Put in terms of knowledge, it is the point of Meno's paradox, as considered by Aristotle:[29] if all acquired knowledge *did* depend on knowledge we already had, we would have to accept (with skeptics) that it is impossible to acquire knowledge, or (with Plato) that some of our knowledge is innate and did not have to be acquired.

Probably the most common response to this problem, especially in the Cartesian tradition, is to hold that we are justified in accepting mathematical axioms

---

[27]Windelband 1883 makes it clear that reaching the truth depends on *proof.* According to Windelband 1907, 11, it is *only* through the "methodical work of science" that the confused contents of our ordinary mental lives can come to have the "conceptual clarity and distinctness" needed to "have a claim to necessary validity and general recognition," which is his characterization of what truth has. In addition to more standard principles of logic, the kind of science that Windelband has in mind always depends on the uniformity of nature, which means that we can use the critical method and the goal of truth to justify claims that make induction legitimate. This links Windelband with another philosopher who pursues the critical method: Hans Reichenbach. Reichenbach's "pragmatic" justification of induction is based on an argument that the truth of principles of induction is a necessary condition on the possibility of reaching our goal of predicting the unobserved (Reichenbach 1938, §40).

[28]Windelband calls the kind of reasoning by which we connect the axioms with our goals "argument" ("Argumentation") but never "proof" ("Beweis") (Windelband 1883, 326, 328, and 330).

[29]See *Posterior Analytics*, 99b15-30.

and other fundamental truths by a kind of *rational intuition:* having a particularly clear idea, or understanding some claim particularly clearly, justifies us in affirming it. The usual interpretation of Frege's approach to axioms reads him this way. Rational intuition has no justifying role for arguments, which tend to be seen by its proponents as, at best, *preserving* the epistemic value of the premises.[30] An alternative response is to affirm the possibility of epistemically amplifying arguments: our justification for accepting fundamental truths comes from our unjustified acceptance of other truths. This approach is also well-represented in philosophy: Aristotle's own response, for example, depends on it.[31] Most recently, this approach is endorsed by those who hold that acquiring justification depends on appealing to "hinges" or "entitlements" for which we have no justification.[32]

There are many good questions to ask about this general approach, as well as about its individual variants, but let us focus here on one particular worry about the distinctive claim of the critical method. This is that the premises of an epistemically argument have the special epistemic status that allows the argument to justify us in accepting the conclusion because they are assumptions or presuppositions that introduce a *goal* or *ideal.* The worry is: why would the fact that something is a goal or ideal have anything to do with what is actually the case? Presumably it must, if it is to make any contribution to justification. Windelband himself presses this kind of objection as the basic problem with a different strategy for justifying axioms, which focuses on trying to show that an axiom has the status Barry Stroud calls "indispensability": everyone who thinks anything at all necessarily accepts it.[33] As Stroud puts the same basic problem, "even if there is something that everyone thinks, it does not follow that it is true. Nor does the truth of something follow from the fact that everyone has to think it, or cannot

---

[30]Leibniz worries that this value is leached away in the course of an argument: what we prove is "not so clear as the [premises], as the image reflected in several mirrors from one to another grows more and more faint with each reflection..." (Leibniz 1765, Book IV, Chapter 2, §2).

[31]He thinks we can arrive at knowledge of first principles by arguments that begin from things we perceive or remember, but do not know. In his discussion, he emphasizes (indeed exaggerates) the difficulty of the problem by claiming that it is "impossible" to get a piece of knowledge "from no pre-existing knowledge." (Posterior Analytics, 99b27-32) In these terms, his solution is to make a distinction between true "knowledge" and lesser forms of "knowledge", like perception. His point is that no *true* knowledge is immediate in the way perception is—all true knowledge requires accepting other truths with a lower epistemic status and reasoning from them.

[32]For example, Annalisa Coliva thinks that "justifications are indeed possible...but only thanks to a system of unwarrantable assumptions," (Coliva 2015, Introduction), while Crispin Wright thinks that if we are to be justified, "Independent warrant for [certain other assumptions] is indeed required, but...it is conferred by epistemic entitlement" rather than justification (Wright 2014, Section 11.1). See also Wright 2004.

[33]This approach is that of a "modest transcendental argument." See, e.g., Stern 2007 and Brueckner 1996.

avoid thinking that it is true."[34] To claim otherwise would be "the familiar but hollow arrogance of idealism: things must be a certain way because we thinkers must think things are that way." But if a claim's indispensability has nothing to do with its truth, how could it contribute to justification? This point seems to apply just as strongly to the critical method: to assume that things must be a certain way because reaching our goals requires that they be that way looks like a kind of arrogance, and to require a kind of idealism.

But there is a difference in the kind of idealism required. The critical method depends on an "idealism" of the prosaic kind that is also called "optimism"—it presupposes that our ideals can be achieved. This is to presuppose that the world is, in one respect at least, as good as any possible world could be. When a *cognitive* ideal—like Windelband's ambitious notion of "truth"—can be achieved, the relevant way in which the world is good is that it meets a certain standard of intelligibility. This claim does not immediately require any emphasis on what is "in us," nor any claims of mind-dependence or the metaphysical priority of cognitive or normative notions over the general notion of reality.[35] The philosophical interest of the critical method is tied up with the philosophical defensibility of this kind of idealism.[36] If that defense extends only to certain kinds of cognitive goals, then this will be one thing that restricts the range of goals that can yield justification in connection with the critical method.

In any case, it is not that by accepting this kind of idealism, though, we become *justified* in accepting the premises. That would transform the unjustified assumptions or presuppositions that characterize the critical method into justified claims, and obscure an important epistemological distinction. Accepting it is rather part of an attempt to explain *why* arguments connecting presuppositions about goals to axioms justify us in accepting those axioms. Someone *becoming* justified by giving such an argument depends neither on that person recognizing *that* this is so,

---

[34]Stroud's discussion is in the chapter "Indispensability" in Stroud 2011. Windelband claims that "The plurality can go wrong just as easily as one person can," (Windelband 1883, 337) Windelband also argues that given the diversity of human life, there will not be anything that everyone accepts, let alone necessarily accepts.

[35]Though it does not *immediately* imply such priority, this kind of claim has often led philosophers to accept that aspects of the world are mind-dependent. Any view on which the world looks very *good* for us *qua* thinkers also makes us look very *lucky*, and can thus seem "too good to be true." Recently, the route for Sharon Street away from "realism" to "constructivism" in meta-ethics is based on an unwillingness to to accept such a thing: she rejects "a fluke of luck that's not only extremely unlikely…but also astoundingly convenient to the realist." See Street 2006.

[36]See Della Rocca 2010 and Dasgupta 2016 for recent defenses of versions of the Principle of Sufficient Reason, which, stating as it does that the world meets a certain standard of intelligibility, puts forward one form of this kind of idealism. Philosophers whose work was central to the Neo-Kantian movement sought a of endorsing a principle like this without violating basically Kantian commitments—see, for example, the chapter on Kuno Fischer in Beiser 2014.

nor *why* it is so. Being justified is one thing, while identifying what justification requires, and recognizing that one is justified, is another. The division of labour here is the same one Kant describes in connection with geometry, which "follows its secure course…without having to beg philosophy for any certification…"[37] Geometers do not need any help from philosophy to be justified in accepting the claims of geometry. The procedure they follow is sufficient for justification on its own. What philosophy asks for is an explanation of *why* this is so, which is what Kant tries to provide. Similarly, any logician would be able to become justified in accepting an axiom just by working through the argument, from the assumptions about the goal to the axiom.

4.6 Frege's Critical Arguments for Logical Axioms

We began by seeing how Frege's claims about axioms seem to leave us with an interpretive question: since Frege thinks arguments cannot justify us in accepting axioms, what *other* purpose are his arguments intended to serve? I have argued that we should reject this question. First, there is strong textual evidence that the point of the arguments is precisely to justify us in accepting the axioms. Second, there is no good reason to think that this is not their intended purpose: Frege's regress argument does not conclude that axioms must be justified without argumentative appeal to other truths, and his claim that axioms are "self-evident" does not have that implication either, at least as this claim is understood by the most prominent figures who wrote on the subject at the time. On the contrary, those figures thought that an argument deriving those axioms from a goal would provide such justification.

In the light of these observations, our question ought to be: do Frege's arguments conform to the prevailing theory about justifying axioms in his philosophical milieu: the critical method? If Frege's arguments serve to derive the axioms from unjustified presuppositions about a cognitive goal, it would neatly explain why he proceeds the way he does. In the remainder of this paper, I argue that this is just what they do. I first explain what the premises of Frege's arguments actually are, and then explain what they have to do with a cognitive goal.[38]

---

[37]Kant 1781/1787, A87/B120. This is also a natural way of seeing why the Cartesian Circle is not a problem for Descartes: the "truth-rule" is not something we must *already* recognize in order to gain knowledge from clear and distinct perceptions, though appreciation of *why* clear and distinct perceptions count as knowledge depends on recognizing it. (See van Cleve 1979.)

[38]Since the point is to say what connects the premises of Frege's arguments, as he understood them, with the cognitive goal, as he saw it, the discussion will necessarily involve idiosyncratic aspects of Frege's thinking at the time of the *Basic Laws*. But even if some of the claims that Frege makes in his arguments would be available for use in critical arguments only to one who shares

What are the premises of Frege's arguments?

Consider again Frege's argument for Basic Law I:

> $\Gamma \to (\Delta \to \Gamma)$ would be the False only if Γ and Δ were the True while Γ was not the True. This is impossible; therefore $a \to (b \to a)$.

In this argument, Frege uses (not mentions) *Begriffsschrift* terms alongside those of natural language. We can understand these vocabulary-mixing sentences with no more difficulty than we have in understanding someone who throws French words into English sentences. So long as we know what the words mean, we can understand what is being said *sans difficulté*.[39] With that in mind, let us examine the premises of this argument.

This argument involves claims about what is and is not the True and the False. This is not just an odd way of saying that something—a sentence, proposition, or whatever—is true or false. Frege thinks that the True and the False are objects—called "truth-values"—and when he says that something "is the True," he means that it is identical with that object. The relationship between sentences and these truth-values is that of reference: a true sentence refers to the True.

This argument also involves claims about the conditional concept to which "$\to$" refers. For Frege, this concept is a two-argument function whose values are the truth-values and whose arguments are any objects whatsoever. For the truth-values as arguments, this function gives the values we expect: its value is the True when the first argument is the False or the second argument is the True.[40]

Finally, this argument uses letters, like "a" and "Δ." As Frege explains, "letters do not, as a rule, have a reference; they do not designate anything, but only indicate in order to lend generality to the thought."[41] Frege uses the "Roman" letters

---

the particulars of his view, closely analogous claims may be available to those who think of the goal in a different way. (He himself would have had to give somewhat different arguments at the time of the *Begriffsschrift*, since he had not yet identified the True and the False as objects.)

[39]Burge and and others point out that we can always translate these premises into a single language: "All of the arguments could be carried out within the language of the logic of Begriffsschrift (by avoiding the modal terminology)." (Burge 1998, 330.) Frege makes the usual suggestions for translating natural-language connectives like "and" and quantifiers using *Begriffsschrift's* conditional, negation, and generality-sign. To say that something "is the False" or "is impossible" is to apply the negation function to it—Frege claims that the only thing a modal term does is offer a "hint" about the kinds of grounds one has for the claim. (Frege 1879, §4.)

[40]The conditional function is defined for other arguments too: when its first argument is the moon and its second argument is the sun, for example, its value is the True, since officially, the conditional function yields the True unless the first argument is the True and the second argument is not the True.

[41]Frege 1903b, 274 footnote.

("a" and "b") without a scope-symbol, since he stipulates that they always have maximum scope.[42] By contrast, "Gothic" letters (which do not appear in this argument) are used together with another symbol to indicate scope, much as we use "x" and "y" while indicating scope with "∀.") Though Greek letters do not officially belong to the *Begriffsschrift,* Frege's general statement that "letters" invoke generality presumably applies to them.[43]

The premises of the argument for Basic Law I, then, are general claims about the truth-values and logical functions, and so too are the premises of the other arguments. Many of these premises are straightforward statements of the *truth-or falsity-conditions* of logical concepts.[44] It is in terms of these truth-and-falsity conditions that the logical concepts are introduced in the first place, a few sections before the arguments for the axioms.[45] Others appeal to the self-identity of the truth-values or to their non-identity with each other.[46] We affirm these claims about identity and difference in the course of taking the truth-values to be distinct objects in the first place.[47] In general, then, we can say that the premises of these

---

[42]"The *scope* of a *Roman letter* is to include everything that occurs in the proposition apart from the judgement-stroke." Frege 1893, §17. We might write Basic Law I, "a→(b→a)" as "∀x∀y(x→(y→x))".

[43]This reading of the Greek letters is shared by Weiner: "The generalization in the statements in which [the Greek letters] appear is generalization over all objects." (Weiner 1996, 334.) There are good questions about Frege's use of letters that I cannot resolve here. For example: If Greek, Roman, and Gothic letters are basically the same, why does he need all three? And why does he treat Greek and Roman letters as if they were *names* in reasoning, performing (for example) *modus tollens* without first instantiating them using a free variable? The answers depend on an extended discussion of how Frege sees the relation between inference and notation.

[44]Greimann 2008 has remarked on this. For example, the truth-conditions of the primitive concept of generality are given in the argument for Basic Law IIa, "∀xΦx is the True only if the value of the corresponding function Φ(ξ) is the True for every argument." (Frege 1893, §20.) The falsity conditions of a complex concept ξ→(ζ→ξ) are given in the argument for Basic Law I: "Γ→(Δ→Γ) could be the False only if both Γ and Δ were the True while Γ was not the True." A premise in the argument for Basic Law IV (Frege 1893, §18) simply notes that under any conditions, its value is the true *or* the false: "HΓ is...always a truth-value…" (Frege's horizontal concept, referred to by "H" in my notation, is a one-place function whose value is the True if its argument is the True, and the False otherwise.)

[45]For example, Frege 1893, §12: "I introduce the function with two arguments ζ→ξ by means of the specification that its value shall be the False if the True is taken as the ζ-argument, while any object that is not the True is taken as ξ-argument; that in all other cases the value of the function shall be the True."

[46]Two examples appear in the arguments for Basic Laws I and IV (Frege 1893, §18). The first: "This [i.e. both Γ and Δ being the True while Γ is not the True] is impossible." The second: "HΔ and ¬Δ are always different..." (This latter might seem to depend only on the truth-conditions of the horizontal and the negation function, but they are always *different* because the True is not the False.)

[47]See Frege 1884, §56 and §62 for the way that making claims about identity and difference

89

arguments are very basic claims about Frege's logical functions and objects, which echo the way they are introduced in the *Basic Laws*.

How do these premises relate to a cognitive goal?

How does Frege choose these objects and functions, and why does he introduce them in the way he does? He tells us that his motivation for introducing the truth values is "how much simpler and sharper everything becomes"[48] by their inclusion in his logical system. He gives a similar defense of each of his primitive logical *functions*. For example, He introduces the concept of generality because its truth-conditions allow one to make a claim that "comprises many—infinitely many—particular facts as special cases."[49] His case for both the concept of negation[50] and of the conditional[51] centers on what is required for "an economy of logical primitives." In each case, Frege's reason is that the inclusion of the relevant function contributes to simplicity.

The kind of simplicity that Frege is concerned with is always the minimization of the number of axioms in the system.[52] This kind of simplicity is very important to Frege. It is "in itself a goal worth striving for"; that it ought to be pursued is a "basic principle of science."[53] This means that the logician's goal is the construction of a *maximally simple* logical system. Assuming Frege is right about which objects and concepts will be under discussion in a maximally simple logical system, this means the logician's goal is the construction of a system of claims about *these* objects and functions in particular.

Considerations about simplicity may serve to identify the logician's goal; but they do not provide any reason to think that it is possible to *reach* that goal. Rec-

---

matters for thinking of something as an object. (That passage follows Lotze 1874, §9, which argues that to think of something as an object, it must be "thought as identical with itself" and "as different from others.")

[48]Frege 1893, Introduction. Such reasons sometimes disappoint commentators: Dummett thinks it is a "gratuitous blunder," to acquire a "great simplification…at the price of a highly implausible analysis of language," (Dummett 1973, 183-184) which has led some commentators to seek a deeper motivation than those Frege offers in these passages. I do not want to rule out that there are other motivations as well, but as we will see, the simplicity of the logical system is extremely important to Frege.

[49]Frege 1923-1925, 258.

[50]Frege 1918-1919b, 384-385.

[51]Frege 1880-1881, 35-37.

[52]It is clear how making a single claim that comprises infinitely many helps with this; having few primitive logical *functions* contributes to simplicity in this sense because as Frege 1880-1881, 36 argues, the more primitive functions you have, "the more axioms you need."

[53]Frege 1880-1881, 36-39; Frege 1884, §2. For more on simplicity, see Chapter One of this dissertation.

ognizing, for example, that a maximally simple logical system will involve claims about the truth-values does not provide justification in any ordinary way for thinking that the True and the False really exist, are distinct, and so on. To nonetheless claim, solely on the basis of our thinking about what the goal is, that the True exists and is not the False, and to draw conclusions from that claim, is to *presuppose* that this goal can be reached.[54]

This is how we should understand the premises of Frege's arguments. The premises put forward basic claims about the objects and functions of logic. These claims attribute to them the features in virtue of which they make their contribution to the simplicity of the system, even though Frege's has no grounds for putting them forward except for his thinking about what the goal, a maximally simple logical system, involves. In putting these claims forward as true, Frege is presupposing that the goal can be reached, and drawing the axioms as conclusions from those claims depends on this presupposition. This is just what the critical method claims is sufficient for justification. Frege's arguments, then, conform to the critical method.[55]

Logical systems and the goal of judgement

I have argued that Frege's arguments serve to derive his axioms from a cognitive goal we have *qua* logicians: the construction of a maximally simple logical system. But one might see an important disanalogy between this goal and the one that Windelband described, which was *truth itself*, conceived in an especially weighty way. The goal of truth seems to be special, in many ways. For example,

---

[54]Compare: "The man I want to marry is wise, kind, and an art-lover. His keen aesthetic sensibility leads him to see the supreme beauty of classical music. That's why I look for him at concert-halls." The first two sentences describe an aspect of a goal, but their descriptive form introduces the presupposition that the goal can be reached: that there really is such a person. The third sentence shows that the presupposition is indeed being made, since concrete action is based on the description.

[55]This is not the only way to implement the critical method. One could make the relation to the goal part of the content of the premises of the argument, beginning not with claims like: "The True is not the False," but with something like: "Our goal is a logical system which includes claims about an object, the True, to the effect that it is different from another object, the False." Such an argument might first conclude: "Therefore, the goal is a system such that P," and only from there conclude: "therefore, P." This alternative implementation seems equally good to me, though not significantly different. It does not, for example, avoid the necessity of an epistemically amplifying argument. At least as Windelband and Frege see things, the problem posed by logical axioms is that there can be *no* justified claims prior to logic from which we can derive them, *including* claims about ourselves and our goals. In any case, if epistemically amplifying arguments on the model of the critical method are objectionable, then so is the last stage of this alternative argument, which moves from what our goal is to what is so.

it is plausibly not an *optional* goal for rational creatures like us, and is plausibly involved in any epistemic goal.[56] Since we noted in the previous section that there may be constraints on which goals can yield justification via the critical method, this disanalogy might make us worry that only goals that have the special features just mentioned will be acceptable—and that our goal *qua* logicians is not among those goals.[57]

One way to alleviate this worry on Frege's behalf would be to determine just which kinds of goals can yield justification via the critical method, and to argue that the construction of a maximally simple logical system is of that kind. Here, I will offer a more modest response, which is that Frege's approach is closer to Windelband's than it seems: he sees himself as deriving these axioms not just from our goal *qua* logicians, but from our goal *qua* judgers, truth.

Let us begin with a suggestive passage. We have seen that treating the truth-values as objects is motivated by the simplicity that this brings to a logical system. This makes some of the things Frege says about these objects puzzling. The *Basic Laws* refers readers wondering about the truth-values ("which indeed may seem strange at first sight"[58]) to Frege's paper "On Sense and Reference," where he makes the striking claim that "these two objects are recognized, if only implicitly, by everybody who judges at all—who holds something true—and therefore also by skeptics."[59] What is striking about this is that even though few before Frege have ever *explicitly* affirmed that these objects exist, he thinks that nonetheless, every judger has implicitly done so.[60] Given that the reason for supposing that they are objects at all is so closely connected with the simplicity that they bring to the logical system, it is hard to see how Frege's claim could be true unless *all* judgers are—perhaps "implicitly"—committed to simple logical systems. According to Frege, though, the goal of judgement is simply *truth*.[61] I believe that what explains this claim is that Frege thinks that truth *itself*—the goal of judgement—has an essential connection with a maximally simple system of logic. This means that, though it might be only "implicit," everyone who judges is committed to the existence of such a logical system. Thus the premises of Frege's arguments are

---

[56]See Lynch 2009, section 2 for a discussion of "whether we can conceive of someone who has distinct epistemic goals that don't include among them the goal of having true beliefs."

[57]On the view of Enoch and Schechter 2008, certain cognitive projects yield justification in a way that is must like the critical method, but only when those projects are *rationally required.*

[58]Frege 1893, Introduction.

[59]Frege 1892, 34.

[60]Frege's point is not just that anyone who judges must, in some nonspecific sense, be interested in something they call "the truth": in a passage which is sensitive to the oddness of the view that the truth-values are objects, he specifically says that judgers recognize these *objects.*

[61]See Frege 1892, 34 and Frege 1918-1919a, 356 on what judgement is. Frege 1897a, 128 and 145 identify truth as the goal: the "laws of judgement" tell us how to "reach the goal, truth."

presuppositions characterizing our goal not only as logicians, but as judgers. This interpretation follows from two further interpretive claims.

The first is that, as Frege often reminds us, whenever he says "truth," he always means "truth in the scientific sense": "that sort of truth which it is the goal of science to discern."[62] On Frege's conception, which is broadly shared not only by Windelband, but most other prominent figures at the time, truth proves to be quite an ambitious goal. Everything that is true (in this sense) ultimately has a place in a *scientific system*: a structure in which a from a few "primitive" truths belonging to a particular branch of a science, the rest are derived by "proofs." This, in turn, places substantive constraints on what can count as true.[63]

Of course, even if all truths figure in scientific systems, that does not explain what truth as such has to do with *logical* systems. This is where the second interpretive point comes in: for Frege, every scientific system depends on a logical system, because it includes one as a part. This is because the logical laws are always required to carry out the proofs that structure every system.[64]

If these two interpretive claims are correct, then the requirements on a maximally simple logical system are requirements on truth itself. In that case, the assumptions that characterize our goal as logicians also characterize our goal as judgers.

4.7   Conclusion

Now that all the pieces are in place, let me conclude by reviewing what Frege is doing with his arguments and why we should read him that way, and then saying something about why it matters.

Frege gives arguments for his axioms because he thinks we must argue for them if we are to be justified in accepting them. The arguments provide this justification by deriving the axioms from the goal of judgement, presupposing that that goal—truth—can be reached. We are not justified in accepting the premises of the arguments, either by intellectual insight or in any other way. As Windelband puts it, "The recognition of axioms is…conditioned through a goal, that as the ideal…must be presupposed." The point of the arguments is to "say to everyone: you want truth? Then remember, you must recognize the validity of these [axioms], if your desire is ever to be fulfilled."

---

[62]See Frege 1906a, 186, Frege 1914, 232, and Frege 1918-1919a, 352. See Chapter Two of this dissertation for a defense of the claims made in this paragraph.

[63]See Chapter One of this dissertation for a discussion of Frege's conception of science.

[64]See Chapter Three of this dissertation for a defense of the claims made in this paragraph.

Here is the reason for reading Frege this way. Once we have read Lotze and Windelband, we know that Frege's claims that axioms are self-evident underdetermine which account he has in mind of how we are justified in accepting them. His regress argument establishes only that there is a kind of justification for axioms that does not come from truths we are already justified in accepting. Having said all this, Frege could then pursue either the goal-based "critical method," or the method of "rational intuition," according to which understanding the claim is enough to justify us in accepting that it is true. Lotze's statements of the latter approach, however, would have struck Frege as objectionably psychologistic, and it was the critical method that was seen as the most promising approach to axioms for the prominent philosophers of Frege's own day. Accordingly, we should (if anything) expect to see Frege opt for the critical method rather than that of rational intuition. We should not be surprised to find him claiming that in his significant arithmetic, he *justifies* axioms by an account that derives them from the referents of the signs (the objects and functions of logic) and offering arguments that do just that. Since rational intuition has no space for a justifying role for argument, Frege looks to be opting for the critical method. This is confirmed when we notice that the premises of these arguments—these claims about the referents of the signs—are motivated only by Frege's thinking about what our cognitive goal is, presupposing that it can be reached. Frege's treatment of axioms, then, is what the critical method requires.

In addition to our interest in understanding Frege, this discovery matters to us for two reasons. First, these arguments draw our attention to an independently interesting Neo-Kantian approach to justifying basic truths: that of epistemically amplifying arguments whose justifying power derives from the presupposition of a cognitive goal. Second, so understood, Frege's *Basic Laws* provides us with another model of the kind for which we frequently turn to Frege: how to do philosophy in a way that combines the most rigorous technical developments with a deep engagement with fundamental philosophical problems.

Chapter 5 Frege's Early Philosophy of Language

Abstract

Frege's *Foundations of Arithmetic* introduces linguistic doctrines that have shaped thinking about language ever since: for example, his claim that naming an object requires having a criterion of identity for it. I argue that this principle is directly motivated by purely epistemological considerations that we find in Hermann Lotze's *Logic.* But how does an epistemological thesis motivate a claim about language? Lotze himself makes such inferences because he thinks that the essence of language is to be found in the expression of thought. Frege, on the other hand, does so because he has adopted a revisionary, prescriptive perspective on language, of which the *Begriffsschrift* is the model: he thinks we ought to regard language that way, if we are to think as effectively as possible.

5.1   The Recognition Requirement

The Role of the Recognition Requirement

§62 of the *Foundations of Arithmetic* begins: "How, then, is a number to be given to us, if we cannot have any idea or intuition of it? Only in the context of a sentence do words mean anything. It will thus depend on explaining the sense of a sentence in which a number word appears." The first sentence introduces a question using the terms "given," "idea", and "intuition," while the next two sentences start to provide an answer in terms of "the sense of a sentence" and what "words mean." This looks like a change of topic: while the terms of the question are familiar from Kantian epistemology, those of the answer are linguistic. What is the point of this shift? To understand, we first need to understand Frege's starting question.

The question how objects are "given to us" gets its point from Kant's claim that "If knowledge is to have objective reality, that is, to relate to an object...the object must be capable of being in some manner *given*." Kant explains: "That an

object be given...means simply that the representation through which the object is thought relates to actual or possible experience"—to "immediate presentations in intuition." This yields Kant's *intuition requirement*: to count as objective knowledge, "all thought must, directly or indirectly...relate ultimately to intuitions, and therefore, with us, to sensibility."[1] "Sensibility" comes in "with us" because Kant thinks humans can only have sensible intuitions. Kant thus identifies a chain of dependence going from *objective knowledge* to *given objects* to *intuitions*, then (for us) to *sensibility*.

Frege has this discussion in mind when he argues that though numbers cannot, in general, "be given...in intuition...we know various things about them," and on these grounds rejects "the generality of Kant's claim that without sensibility no object would be given to us."[2] He breaks the link in Kant's chain between *given objects* and *intuitions,* which means that for Frege, there must be some other way for objects to be given to us. The substantive issue is whether there is any relationship to objects *aside* from intuition that enables us to "know various things about them."[3]

After posing this question, Frege shifts into linguistic terms, and in those terms offers an (uncharacteristically use-mention confusing) answer: "If we are to use the symbol *a* to signify an object, we must have a criterion for deciding in all cases whether *b* is the same as *a*, even if it is not always in our power to apply this criterion."[4] What we need is "a general criterion for the identity of numbers": "a means of grasping a definite number and recognizing it as the same again." Though these are claims about using symbols to signify objects, Frege takes them to answer the epistemological question of how numbers will be given to us, which is necessary if we are to know things about them.[5] In identifying this *recognition requirement*

---

[1]Quotes are from Kant 1781/1787, A155-6/B194-5 and A19/B33.

[2]Frege 1884, §89. He earlier (§12) identifies the question as that of whether "an intuition...[can] serve as the ground of our knowledge in arithmetic."

[3]There is some terminological confusion between Frege and Kant, and I think that Frege breaks the connection at the point he does partly because he is not faithful to Kant's terminology. Frege thinks Kant uses the term "intuition" ambiguously, and decides to himself use it as definitionally connected with sensibility. (Frege 1884, §12.) Kant himself, however, certainly means to leave open the possibility of non-sensible intuitions (bringing in sensibility only "with us") and the quote above seems to definitionally connect givenness with "immediate presentations in intuition." I expect that while Frege puts the dispute as being about whether objects can only be given by intuition, Kant would describe Frege's view as one on which objective knowledge does not require an object's being given at all.

[4]The generality of this claim shows that Frege takes meeting it to be necessary to name any object. If, say, intuiting objects allows us to name them, it must be because it helps us to meet this requirement.

[5]We know that Frege thinks of himself as answering, not replacing, the question about how objects are given, because when considering different sets of objects than the natural numbers, he

on naming, then, he takes himself to be also stating the relevant requirement on objective knowledge.

This requirement on naming would prove to be central to 20th century philosophy of language,[6] but what was perhaps even more fateful was that though it is taken to answer an epistemological question, the discussion by which Frege arrives at it does not make any obvious appeal to any claims about what objective knowledge is or requires. It looks as if Frege answers the epistemological question *by way of* linguistic considerations—as if he exploits some relationship between language and knowledge to reach conclusions about knowledge from independently identified facts about language. This is the basis of Michael Dummett's classic account of §62, which sees it as "arguably the most pregnant philosophical paragraph ever written." Dummett sees here "the first example of what has become known as the 'linguistic turn' in philosophy…an epistemological problem…is…converted into one about the meanings of sentences…"[7] The "conversion" is possible because "the theory of meaning…is the foundation of all philosophy, and not epistemology as Descartes misled us into believing"; Frege has started "a revolution…as great as the similar revolution previously effected by Descartes."[8] The philosophy of language has *foundational* status, on Dummett's reading, because we are to use the relationship between knowledge and language in one direction only: to draw conclusions about knowledge from claims about language.[9] But is this really what Frege is doing?

---

once again raises the question how they are "given", and once again says that we must "determine the sense of a recognition judgement." (Frege 1884, §104)

  [6]See, e.g., Lowe 1989, Strawson 1976.

  [7]Dummett 1991, Chapter 10.

  [8]Dummett 1973, 669. Presumably, Dummett has in mind passages in which Descartes uses a relationship between epistemology and metaphysics to to provide "strictly" correct answers to metaphysical questions like "what is called 'having a sensory perception'" by thinking about what we can know at a certain stage in our meditation. (i.e.: "I certainly *seem* to see, to hear, and to be warmed. This cannot be false; [therefore] what is called 'having a sensory perception' is strictly just this…" (Descartes 1641/1642, Second Meditation.) One approach characteristic of the linguistic turn, as Dummett understands it, would be Austin-style linguistic philosophy, which aims to infer the answers to questions about various philosophical topics from claims about the ordinary use of words.

  [9]This reading of §62 could also fit with a more reciprocal relationship between language and knowledge, on which we use the relationship between them in both directions, by turns using each to learn about the other. §62 could just be one stage in this virtuous circle, in which the connection is used to go from language to knowledge.

The Epistemological Source of the Recognition Requirement

The reading of §62 that sees Frege draw epistemological conclusions from linguistic premises leaves us with two puzzling questions.

1: *Where does the recognition requirement on naming come from?* Why does Frege think that using a name to refer to an object requires being able to recognize the object?[10] He says that because number words stand for objects, "there is one group of sentences, that must have a sense: the sentences that express recognition." But he does not tell us why the fact that numbers are objects means that those *recognition*-sentences are the ones that need "senses," or why explaining their senses requires a "general criterion for the identity of numbers."[11]

2: *What about Kant's motivations for the intuition requirement?* Frege rejects Kant's intuition requirement on objective knowledge. But Kant motivates that requirement with a fundamental line of thought—that for beings like us, objective knowledge *must* be a compound of the "receptivity" of sensible intuitions, and the "spontaneity" of concepts.[12] Does Frege think Kant is wrong that objective knowledge for us requires an element of "receptivity"? Or does he rather think that meeting the recognition requirement somehow involves such an element?

Frege would have known that both questions would have occurred to his readers.[13] If Frege expects his audience to go along with him, then, not only must

---

[10]Dummett 1991, 162 does little to justify it, characterizing it only as a "profound insight."

[11]The recognition requirement had also been assumed in §56, when Frege rejects possible "definitions" of number-words that do not enable us to recognize what is and is not identical with the numbers—for example, we could not tell whether they are identical with Julius Caesar or not. Even less is said there to justify it. Others have offered arguments for the recognition requirement—for example, Evans 1982 argues that to refer to an object requires having an *ability* that meets certain constraints, which seems to require being able to act on *that* object in particular, which we could not do unless we had a way of distinguishing it from the others. But the question is whether we have any reason to attribute this (or any other) argument to Frege.

[12]See, e.g., Kern 2006 for an account of "Kant's fundamental idea" that "Knowledge...is a 'compound' of receptivity and spontaneity." According to Kern, what matters is that sensible intuitions are caused by their objects, which is the only way for creatures like us to have representations that guarantee the existence of their objects: "However intuitions are further specified, the definition entails that one intuits an object only if the object exists. This is what it means that the representation relates immediately to the object. And precisely because an intuition of an object exists only together with the object of that intuition, Kant can say that an intuition is the kind of representation through which an object is given." (147.)

[13]For example: writing for Neo-Kantian philosophers during the "Back to Kant!" era as he was,

98

he have answers to these questions, but he must suppose that his audience is already aware of those answers. Indeed, there are such answers, but acknowledging them precludes giving the reading of §62 that we just considered. The answers are provided in a passage with which Frege would have expected his readers to be familiar, which offers a fuller justification for the recognition requirement and explicitly addresses the concerns about "receptivity" that lead to Kant's intuition requirement. This passage, however, is not written by Frege. It is written by his onetime teacher, Lotze.[14]

The opening sections of Lotze's *Logic* concern the epistemological question how the "objectification of the subjective" is effected, which is the first step toward thought's goal of objective knowledge.[15] Lotze argues that such objectification requires "giving affirmative position to the object-matter, [and] distinguishing it negatively from all others"[16] by paying attention to that "in virtue of which the [thing] is what it is and differs from everything else."[17] When we do this, we are "emphasizing its difference from others, and saying, not only, it is what it is, but also, it is not what others are." We identify the thing and distinguish it from others in "one inseparable thought...even when we do not expressly attend to the others which are tacitly negated."[18] To do what Lotze is describing here is to have a criterion of identity: we identify the thing and are able to distinguish it from all others. He further claims that *this* aspect of our thinking has "the character of receptivity," contrasting with other aspects that exhibit "spontaneity."[19]

We know that Frege read this part of Lotze's *Logic* carefully, probably while writing the *Foundations.*[20] There is also strong textual evidence for a connection

---

Frege could hardly expect them to dismiss so central a piece of Kantian epistemology without so much as considering its motivations. See Chapters I and II of Sluga 1980 for a characterization of Frege's audience.

[14]It is disputed among Frege interpreters as to whether Frege and Lotze have the same conception of what it is for knowledge to be objective, but we can avoid this debate here. What matters to us is that they place the same requirement on what they each call "objective knowledge," even if their conceptions of what exactly we get when we meet this requirement are different.

[15]Lotze 1874, §3. See the Introduction for a description of this goal. Both Lotze and Frege are focussed here on necessary conditions for reaching that goal, not sufficient ones. Meeting the recognition requirement gets us over one hurdle on the path to our goal, but we can still fall short of it by, for example, saying something false.

[16]Lotze 1874, §19

[17]Lotze 1874, §10

[18]Lotze 1874, §11

[19]Lotze 1874, §19. It exhibits receptivity because it constitutes "the recognition of facts," and "adds no other form to them except this recognition of their existence."

[20]Dummett 1981a was the first to show that Frege had read and took notes on this part of the *Logic* sometime between 1876 and 1891. Additional work (Sluga 1984, fn:8 and Hovens 1997) show that he was probably reading the 1880 edition in 1882. (The *Foundations* was published in 1884.)

between this passage and Frege's discussion: Frege's claim that "If we are to use the symbol *a* to signify an object, we must have a criterion for deciding in all cases whether *b* is the same as *a*" echoes Lotze's claim that "No matter can have a *name* made for it unless it has been thought as identical with itself [and] as different from others," which requires "distinguishing every *a* from every *b*."[21]

The way to make sense of what Frege does in §62 is to see his discussion as depending on the epistemological ideas of the most widely read logic book of the day.[22] Frege's readers already know that we need a criterion of identity in order to be "given" objects, because that is how we meet the need for "receptivity" which Kant thought required sensible intuitions. But this implies that Dummett's reading of §62 is wrong: Frege's question is epistemological, and his answer is provided by Lotze's explicitly epistemological reflections, rather than by any independent thinking about language.

5.2   The Relationship Between Knowledge and Language

This account of §62 leaves us with two questions. If Frege's question is epistemological, and his answer is motivated epistemologically, then why does he bring in language at all by stating the recognition requirement as a requirement on naming? And moreover, what connection between language and knowledge justifies him in doing so? Since the recognition requirement and its linguistic formulation both derive from Lotze's *Logic,* let us begin by asking those same questions about Lotze.

Lotze

Lotze thinks that "nothing is more familiar to us" than the "activity of thought" wherein we meet the recognition requirement. That is because even though the activity itself and the reason it is the "necessary beginning" of objective knowledge must be understood independent of language,[23] this activity in fact accompanies

---

[21]Lotze 1874, §9 and §13. Lotze also emphasizes another part of the aspect of thought "with the character of receptivity" which Frege does not mention: the *comparison* of the distinguished things with one another. Though Lotze clearly thinks this component is very important, it is dropped from some of his discussions. For example, in §8 of "Logic" in Lotze 1885, the relevant claims that we make when we meet this requirement is that the "content is something which holds good of itself, is identical with itself, and different from others...[it is] a whole which in itself belongs together, and as thus belonging together is marked off from every other."

[22]Heis 2013 (Section 3) calls it "perhaps the most widely read logic text in Germany during Frege's early career." See also Gabriel 1989.

[23]Lotze 1874, §6: we must "distinguish [the activity of thought] from its linguistic expression...in itself it is not inseparably bound up with the existence of language."

all our speaking.[24] He mentions language in the *Logic* because he thinks we will be most familiar with meeting the recognition requirement from linguistic contexts—so that mentioning language will help us identify what he means.

The reason Lotze thinks we accompany our speaking with mental activity by which we meet this requirement is that he thinks that not only do we "use the language of sounds for the communication of...thoughts,"[25] this is the core linguistic activity. According to Lotze, it is "a peculiar form of thinking" that "first makes the sound into a word"—"the sound first becomes a word through logical accessory thoughts."[26] Other aspects of language are ornaments on the primary purpose of expressing and communicating objective thoughts, which aspire to knowledge: "as much as language is determined to record the subtlest movements of feeling," what makes it *language* is its relation to the "forms of thought."[27] The reason, then, that "no matter can have a *name* made for it unless it has been thought as identical with itself [and] as different from others," is that being accompanied by an activity of thought in which the recognition requirement is met is what makes our sounds into names in the first place.

This core function of language exerts a formative influence on the natural development of ordinary languages. Words come to be differentiated by the way they sound, for example, into types that match with the different components of objective thoughts, so that the resulting language can more effectively express such thoughts.[28] As our languages come to correspond more and more to the thoughts we express in them, they in turn become more and more helpful for reaching the goal of thinking itself—which is why we develop the useful habit of talking silently to ourselves while we think.[29] So Lotze's view is that the epistemologically signif-

---

[24]Lotze 1874, §1-2.

[25]Lotze 1874, §6.

[26]§4 of Lotze 1858, Volume 2, Book 5 (Mind), Chapter 3 (Speech and Thought). Quotes from Lotze 1858 in this section are from this chapter, which provides somewhat fuller discussion of language that illuminates that in Lotze 1874.)

[27]Lotze 1858, §4. An aspect of language that have no relevance for thought is described later in this section as a "superfluous aesthetic addition."

[28]This leads Lotze to place individual languages on a scale of development, based on how well-suited they are for the purpose of expressing thoughts. In some languages "the grammatical form" fails to properly mirror our activities of thought, but "every language that is far both from primitive crudeness and from decomposing erosion will always express the logical distinctions of its stock of words in their audible sound-shapes." In "the progess of a virile [*männlicher*] development," language naturally comes to match thought. (Lotze 1858, §4)

[29]Lotze elucidates the idea of silent speech with the claim that our vocal organs work while we think. He devotes significant effort to defending this habit of ours against charges that it hinders our thinking in Lotze 1858, §5. Some argue that the need to think in language slows our thought down to the speed of the voice box; Lotze argues that we couldn't think much faster anyway. Some argue that language leads us wrongly to think we're making progress in understanding when we

icant activity of thought both "is manifested by the organization of language and conversely is supported by the latter in its undertakings,"[30] and these points are connected: it is because the basic purpose of language is to express thoughts that language develops so that it mirrors thought, and thereby comes to be helpful to thinking itself in the achievement of its goal.

Wittgenstein

I think that Frege does not understand the connection between language and knowledge in the same way as Lotze. But before I explain why, let us pause to note why Lotze's view of that connection may be problematic. It is the very view that Wittgenstein criticizes when he answers the question "What is the relation between name and thing named?" as follows:

> Among many other things, this relation may...consist in the fact that hearing a name calls before our mind the picture of what is named; and sometimes in the name's being written on the thing named or in its being uttered when the thing named is pointed at...we call very different things "names"; the word "name" serves to characterize many different, variously related, kinds of use of a word.[31]

Wittgenstein thinks that there is no general account of what a name is, because there is no central purpose to language at all—and in particular, no purpose to be found in a relationship to *thought.* He diagnoses views like Lotze's as deriving from the philosopher's characteristic "craving for generality" and "preoccupation with the method of science."[32] Together, these tendencies make the kind of language-use that is central to science—the expression of thoughts—look like the essence of language itself. As Charles Taylor elaborates the point,[33] though the tendency to "interpret ourselves in terms of representative primacy" is "obviously justified"

---

merely learn something's name; on the contrary, Lotze argues that learning a name—a real name, in common language—is a reflection of the ultimate aim of thought, the achievement of a completely systematic view of the world, and can be an important step toward achieving it, since it helps us bring our thinking in line with the rest of humanity. Some argue that the pliability of language leads us to hypostasize things, forgetting what is a dependent property and what is a self-standing object; Lotze thinks this is indeed a fruitful source of error, but we need only be careful to avoid it, and it is a false logic itself, with its overemphasis on a structure of genus and species, that more often leads us to this error. (I think that the very helpful Sullivan 1991 overestimates the degree to which Lotze concedes that accompanying thought with language causes problems for thinking.)

[30]Lotze 1858, §4
[31]Wittgenstein 1953, §§37-38.
[32]Wittgenstein 1958, 17-18.
[33]Taylor 1980, 290-291.

as a *norm*, adopted for the scientific purposes that have become central to our culture, among theorists of language "the norm has ended up being forgotten as a norm." When we wrongly see the nature of linguistic activity itself in terms of this single purpose, it both distorts our account of language, and leads us to "utterly fail correctly to conceive what the task is that the norm prescribes. We think in some confused way that we are already really there at the point it calls us to." If our own language, as it is, is *already* especially suited to the expression of thoughts, then there will be no room for improvement along that dimension.

Frege

Frege agrees with Lotze that we have a habit of speaking silently to ourselves while we think,[34] but he usually mentions this fact in order to lament the influence of language on our thinking. Language is not only "deficient...when it comes to protecting thought from error," but what is worse, "errors in our own thinking...have their cause in the imperfection of language."[35] In this connection, Frege talks about "illusions, [Täuschungen] that through the use of language often almost unavoidably arise."[36] He offers *ambiguity* as an example: when language includes an ambiguous word, this word causes us to wrongly think that the two distinct things referred to by it are one and the same.[37] Conflating two things, we fail to reach our goal, objective knowledge.

Just as Lotze's view that language helps us to reach the goal of thought fits closely with his view that the central purpose of language is the expression of thought, so do Frege's doubts on this score fit with the rejection of any such special connection between language and thought. He sums up the situation in a late paper this way: "Language is a human creation, and so man had...the capacity to shape [it] in conformity with the logical disposition alive in him. Certainly the logical disposition of man was at work in the formation of language but equally alongside this many other dispositions - such as the poetic disposition. And so lan-

---

[34]Sometimes this is put relatively weakly ("We are disposed to think in some language or other." Frege 1879-1891, 6.) and sometimes strongly: "We need sensible symbols to think." (Frege 1882, 106.) and "that a thought...is connected in our mind with some sentence or other is for us men necessary." (Frege 1924/1925, 269) He does make clear in making the stronger claim, that this is merely a psychological fact, which "does not lie in the nature of thought but in our own nature...if we...attend...to the true nature of thinking, we shall not be able to equate it with speaking...thinking will then emerge as that which has priority..."

[35]Frege 1882, 106.

[36]Frege 1879, Preface.

[37]Frege 1882, 108. For example: the same words (e.g. "the horse") are used in natural language for both an object and a concept, leading to the error of thinking that some objects are concepts.

guage is not constructed from a logical blueprint."[38] Frege has in mind something like the following. The poetic disposition leads us to engage in fanciful imaginings and the recitation of poetry, and these poetic activities influence the development of language. At the same time, our logical disposition leads us to engage in objective thinking and to express these thoughts, and these logical activities also influence the development of language. But a language shaped by the poetic disposition would tend to be highly ambiguous (which helps reach the poet's goal, which involves drawing surprising connections between things) while a language shaped by thinking would tend to have few ambiguities (which helps reach the thinker's goal, which involves keeping distinct things separate). A single language, then, cannot be wholly formed by both activities, and our combination of dispositions lands our natural language somewhere in between. But the features of our language affect how successfully we can pursue our various activities. To the extent that the features of language are shaped by the poetic in opposition to the logical disposition, our language will hinder our attempts to reach the goal of thought, objective knowledge. This leads to the situation Frege describes: natural language is not, and will never be, very helpful when it comes to thinking.

But this means that Frege cannot share Lotze's reason for treating the requirements on objective knowledge as also requirements on the basic linguistic activity of naming. As a basic linguistic activity, Frege expects names to be used in connection with many activities, so that no conditions on reaching the goal of thought will be conditions on naming itself—which is just what Wittgenstein urges us to recognize. So what entitles Frege to claim that the recognition requirement is a requirement on naming?

The answer is found in Frege's celebrated response to his observation that language has no close connection with thought: his adoption of the Leibnizian project of inventing a language that has one. Frege's artificial language *Begriffsschrift* is not shaped by our various dispositions and activities, but artificially designed to match thought perfectly. If we accompany our thought with *Begriffsschrift* rather than with natural language, it will help us to reach the goal of thought. This language will "break the domination of the word over the human mind, by freeing thought" from influences with which "the condition of ordinary language afflicts it."[39]

The components of Frege's artificial language include a set of new symbols that are written in a two-dimensional structure,[40] new rules for the use of those

---

[38]Frege 1924/1925, 269. Frege holds this view consistently: Frege 1882, 110 claims that it is the very fact that ordinary language has developed in a way that makes it suitable for multiple purposes that explains why it is not particularly useful for thinking in particular.

[39]Frege 1879, Preface.

[40]This is because "The relations between the levels of written symbols on the two-dimensional

symbols (including a ban on introducing ambiguous *Begriffsschrift* terms),[41] the new technical concept *conceptual content* for thinking about the significance of these symbols,[42] and an assignment of particular conceptual contents to particular symbols.

If Frege is correct about the effects of *Begriffsschrift* on our thinking, there is much to recommend implementing these aspects, whether together or separately.[43] But Frege frequently reminds us that *Begriffsschrift* is not an all-around "perfect language," superior to natural language in some general way. Though it may be ideal for reaching the goal of thought, that is not the only important thing that language does for us. Frege says that we ought only adopt the elements of *Begriffsschrift* when we are pursuing "scientific goals,"[44] because for most purposes, ordinary language is better than it, much as using our ordinary eyes is better than going about peering through a microscope all the time.[45] The recommendations that make up *Begriffsschrift,* then, should be understood as *conditional*: it is only *if* and *when* we are interested exclusively in reaching the goal of thought that we ought not use ambiguous names, that we ought to regard the significance of linguistic items in terms of conceptual content, and so on.

Frege has not, then, given undue priority to scientific purposes in his general descriptions of what language is, and he has accordingly not forgotten that it takes work to use language in a way that serves the purposes of thought: if we care about them, we should design new linguistic practices. I suggest that the claims about language in the *Foundations* are made within a perspective set up by this task. In §38 and §39 of the *Foundations,* for example, Frege described it as a linguistic *failure* when the same sign stands for multiple objects.[46] But as we have just seen, there is nothing wrong with ambiguous words in themselves—they

---

writing surface can...be used to express inner relationships...and this makes it easier to detect that to which we want to direct our attention." (Frege 1882, 111.)

   [41]e.g., Frege 1882, 108. Frege himself seems not to follow this rule; what he says about the identity statement seems to imply that all proper names are ambiguous, sometimes standing for an object and sometimes for themselves.

   [42]"I call that part of the content that in both [sentences with the same consequences] is the same, the conceptual content...only this has significance for our *Begriffsschrift*." Frege 1879, §3.

   [43]Presumably our thought will be *maximally* benefitted if we adopt all of these elements together, but the different aspects can also be separately implemented, with more modest beneficial effects. We could introduce a few *Begriffsschrift* symbols into German, or resolve in German to strictly observe the rule banning ambiguous terms, or we could regard the significance of German terms in terms of their conceptual content.

   [44]Frege 1879, Preface.

   [45]The preface to Frege 1879 gives this microscope/eye analogy; Frege 1882 describes the relationship as that between a specific tool and the human hand.

   [46]There are other errors to which doing so would commit arithmetic—such as errors of equivocation in reasoning—but Frege treats the ambiguity *itself* as an failure.

are the perfect sort of word for various purposes which have as much claim to be served by language as scientific ones. It is only when we are interested only in the goal of thought that that we ought not allow ambiguous names, and hence it is only from that perspective that ambiguous names are a kind of failure. Frege's claim about ambiguous names, shows that he has has self-consciously taken up that scientific perspective, devoted wholly to the goal of thought, and is speaking about language from within it.

Speaking from this perspective puts Frege's claims about language in an implicitly conditional, prescriptive frame. Within this frame, prescriptions about language must be understood only to apply when our sole interest is in achieving the goal of thought, and descriptions of language are descriptions of the appropriately reformed language that we desire, which is perfectly suited to thought. The language Frege is describing is, in that very specific sense only, *ideal.*[47]

With this in mind, we can accept that Frege endorses that requirement on objective knowledge for the same epistemic reasons that Lotze does, but that his claim about *language* is made for a different reason. He recognizes is that *if* we are to reach our scientific goal, knowledge, as effectively as possible, we *ought* to treat naming as possible only when the requirements on objective knowledge have been met. The claim that naming requires meeting the recognition requirement is a prescription for how we ought to deploy names when thinking is our only goal. Frege does not issue such prescriptions in order to determining the epistemological facts, but in order to help us do what the correct epistemology shows us we must do, if we are to acquire objective knowledge as effectively as possible.

5.3   The Context Principle

I have argued that linguistic claims in the *Foundations* tell us how we ought to speak in order to most effectively reach the goal of thought. Such claims must, accordingly, always be backed up by claims about what objective knowledge requires. We can confirm that this reading is correct by seeing that it provides the best understanding of the most famous linguistic claim in Frege's book: the context principle that "we must ask after the meaning of words only in the context of a proposition."

---

[47]This reading has many points of contact with Dummett 1981b, as well as Weiner 1996 and Weiner 1997. I will try to make my departures from Dummett, in particular, clear in the conclusion.

The Role of the Context Principle

Though much is disputed about the context principle, everyone agrees that its primary role is to help avoid an *idea theory of meaning*, according to which the meaning of a word is the idea with which it is mentally associated.[48] But it is not obvious how it serves this purpose. Accepting the context principle does not seem to be a necessary condition on rejecting the idea theory, since it seems consistent to hold that words mean objects and properties even when no proposition is supplied. Nor does the context principle on its own obviously constitute any kind of argument against the idea theory. By "proposition," ["Satz"] Frege typically means some words or signs that express the kind of content that can be judged, proved, and so on,[49] but an idea-theorist like Locke would maintain that ideas are central to such propositions, and hence that words mean ideas, even in the context of a proposition.

The context principle, then, needs major supplementation if it is to stand in any important logical relationship with the idea theory. For example, we could add that propositions themselves are not ideas or combinations of ideas, and add a principle saying what the meanings of words have to do with the propositions in whose "context" they appear. Since no such connections are spelled out by Frege, a standard interpretive response is to treat the context principle as elliptical, saying more than it seems to. Readers are often led to find in the context principle not only hidden content sufficient to provide an argument against the idea theory, but also to bolster other parts of Frege's discussion that seem problematic.[50] The central

---

[48]e.g., Milne: "the context principle is important in avoiding psychologism [i.e. an 'ideational theory of meaning']." ("Frege's Context Principle," 491.) Dummett notes that though we might have supposed "that an assignment of a reference to a term consisted in a mental association [via an idea]...once we have grasped the context principle...we recognize that this is quite the wrong way to think about the matter." Dummett 1991, 201. Some also think that the context principle has additional work to do in the *Foundations*: for example, Michael Beaney thinks that "Its role is not just to provide support for the rejection of the supposition...that the meaning of a word is the idea it stands for, but also, more positively, to justify the assignment of meaning to number terms." (*Frege: Making Sense* 97.) Milne thinks that the idea that avoiding psychologism "is its whole purpose...is hard to swallow...the principle is used to legitimate contextual definitions." ("Frege's Context Principle," 491.)

[49]Frege 1902 clarifies to Russell that "German logicians understand by ["Satz"] the expression of a thought, a group of audible or visible signs expressing a thought. You [Russell] evidently mean the thought itself. This is how mathematicians tend to use the word. I prefer to follow the logicians in their usage." He writes this much later than the *Foundations,* but it seems to fit well with his use there. See Frege 2013, xxi-xxii for a recent discussion of some of the difficulties understanding Frege's word "Satz."

[50]See "The Context Principle," in Dummett 1991 for an example of a reading on which the context principle contains the assumption that the meanings of words must *determine* or *fix* the *truth-conditions* that make up propositions: the Context Principle tells us that "what is needed [for

interpretive difficulty in understanding how the context principle is to accomplish its most basic job is what makes that principle seem necessarily cryptic, which is why it is famous for being "puzzling," "fog-bound," and "the most difficult idea in the whole of Frege's philosophy."[51]

But this approach to understanding the context principle makes things more difficult than they need to be. *Just as stated,* the context principle can play its intended role with respect to avoiding the idea theory. The key is to see that the Context Principle helps us avoid the idea theory *without* being part of any argument against it, or standing in any logical relationship with it.[52]

How the Context Principle Works

Frege gives an argument against the idea theory of meaning in §60, but the context principle is not part of it. Frege argues that the ideas we have of the earth, of our distance from the sun, and of microscopic particles have no impact on the "support for our inferences", the "correctness of our calculations," and the rest of that on which our knowledge of these things actually depends. From this claim, he immediately concludes: "That no idea can be formed of the content of a word is therefore no reason for denying it any meaning or excluding it from use." Frege finishes his argument against the idea theory before he mentions the context principle.

---

words to have meaning] is [something] that will fix the truth-conditions of sentences in which [those words] occur." Ricketts 2010 (149-150, 163) claims that "Frege's Context Principle sets forth the connection between logical segmentation and quantificational generality...The Context Principle encapsulates the connection between names and quantificational generality: an expression is a meaningful (designating) name by occurring in true or false sentences that express instances of generalizations expressible by replacing the name with a variable." Wright 1983 (51-52) thinks that only if we read it as asserting "the priority of syntactic over ontological categories...does it become possible to understand Frege's reliance on what otherwise appear to be the signally flimsy considerations on which he rests his [claim that numbers are non-sensible objects] or to understand why that [view] does not impel him in the direction of a mythology of quasi-perceptual cognitive relations with mathematical objects."

[51]The first two are from Wright 1983, 8-10; the last is from Dummett 1995, 3.

[52]One commentator who has tentatively suggested a reading on which the Context Principle's role does not require any logical relation with the idea theory of meaning is Kluge, who writes that "It can be argued with more than merely superficial plausibility that...[the Context Principle] merely amounts to a heuristic piece of advice intended by Frege to indicate what is necessary in order to avoid confusion over the nature of numbers: Namely, if we conceive of numbers as non-mental entities and then ask what numbers are, in isolation, we shall inevitably arrive at a psychologistic and therefore mistaken answer. The only way to avoid this...is to consider the names of numbers only in sentential contexts, for in that case the very context itself will nullify the temptation to consider them as representations and at least part of their true nature will shine forth." I think the argument below has *much* more than merely superficial plausibility.

The context principle plays a different kind of role. Frege thinks that the idea theory has a way of *appearing* true, even to those who already know it is false. Something that appears true even to one who knows it is false is an *illusion,* which is how Frege describes things: he says that this "illusion [Schein] surely arises because we consider the words in isolation." This diagnosis means that we can remove this appearance—an appearance that we have independently recognized to be illusory—by not doing that. He accordingly prescribes that when we ask about meaning, "we must always have in mind a complete proposition." This is how the context principle helps us avoid the idea theory.[53]

In more detail: the context principle is arrived at in two stages. In the first stage, Frege infers from the fact that our knowledge of some objects does not depend on ideas to the fact that linguistic meaning does not require ideas. This inference is not straightforward. One might reasonably suppose that if a word is to have meaning, it ought to be able to serve all the central purposes to which words are put. For many central purposes, the ideas associated with words matter very much. Frege, for example, points out that to write and appreciate poetry, we must know the ways that different words "act in different ways on the feelings and ideas of the hearer," and that words are often used "to the same end in everyday language."[54] The fact that no idea can be formed of the content of a word, then, might be some reason to deny it a meaning: after all, it cannot figure in some central linguistic purposes. Since Frege does not erroneously assume that scientific purposes are the only central linguistic ones, what licenses his move from these claims about knowledge to claims about linguistic meaning is rather the conditional, prescriptive frame that he has adopted. It is only when our sole interest is in achieving the goal of thought that we should not deny meaning to a word when we cannot attach any idea to it.

---

[53]In the introduction to the *Foundations,* Frege gives a preview of the context principle's role. He claims that we must resolve "never to ask for the meaning of a word in isolation, but only in the context of a proposition," because otherwise, we will be "almost forced [fast genötigt]" to think that an idea is the meaning of the word. But thinking that way leads us to confuse the "objective and the subjective": we will violate our independent understanding of what our objective knowledge depends on. The main-text argument in §60 makes clear just what it is about objective knowledge that conflicts with the idea theory of meaning, and he adds that the kind of "forcing" toward the idea theory that we experience is that of an illusion. When Frege identifies the principle as "fundamental" he does not mean that it has no external justification, but only that we must *always* adhere to it, lest we fall prey to the illusion.

[54]Frege 1897a, 140. We find corroboration of this very reasonable claim from no lower authority than the Supreme Court of the United States, interpreting its Constitution: "[W]ords are often chosen as much for their emotive as their cognitive force. We cannot sanction the view that the Constitution, while solicitous of the cognitive content of individual speech, has little or no regard for that emotive function which, practically speaking, may often be the more important element of the overall message sought to be communicated."(Cohen v. California 1971)

In the second stage, Frege recommends the context principle because it will remove an illusion that the idea theory is true. The talk of illusion should remind us of the "illusions, that through the use of language often almost unavoidably arise." but which *Begriffsschrift* is able to remove. The context principle supports our rejection of the idea theory in just the way that an unambiguous language supports our keeping distinct things distinct, and the way that the two-dimensional notation of *Begriffsschrift* " makes it easier to detect [logical relations] to which we want to direct our attention."[55] When we ask for the meaning of a word without having anything in particular in mind, we will be inclined to try to convey *general* competence with the word, and for the reasons just discussed, this will mean conveying appropriate ideas. What Frege recommends is that we ask for meaning with a particular proposition in mind: that is, we think of the word as part of a group of expressions that express the sort of thing that can be judged, proved, and so on. Asking about meaning with one of those in mind focusses our attention on judging and proving and away from poetry writing. This removes the inclination to talk about ideas. (Of course, the context principle will not do very much for someone who is committed to thinking that judging and proving are *themselves* a matter of ideas, but arguing against such people is not its point.)

To make good sense of the context principle, like the recognition requirement, we must link it up with the general perspective on language described above—that induced by the *Begriffsschrift* project's goal of removing linguistic illusions. This allows us to take what Frege says at face value. In fact, this reading allows us to understand something that sometimes goes unnoticed by those attempting to read the context principle as logically conflicting with the idea theory: its standard form is *prescriptive.* In its first appearance in the book, its first appearance in the main text, and in Frege's concluding summary of the *Foundations*, it appears as a prescription about what we ought to *do* when we ask about meaning: "We must ask after the meaning of a word only in the context of a proposition, not in isolation," "one must always keep in mind a complete proposition," and "the meaning of a word is not to be explained in isolation, rather in the context of a proposition."[56] It is only derivatively that it takes the descriptive form in which it is often discussed: "It is only in [a proposition] that words really have a meaning," and "only in the context of a proposition do words mean anything."[57] If its connection with the idea theory were logical, then its central form to be descriptive, and the textual primacy of the prescriptive formulation would be a problem. On the reading I provide, however, we should expect the prescriptive form to be primary, and the

---

[55]Frege 1882, 111.

[56]Frege 1884, Introduction, §60, and §106, respectively.

[57]Frege 1884, §60 and §62.

110

derivative form can be understood as part of a description of a language that is ideal for scientific purposes: despite its descriptive form, what it tells us is how we ought to speak, if we are to think as effectively as possible. Claims about what is ideal emerge naturally out of claims about values or norms.[58]

5.4 Frege and 20th-Century Philosophy of Language

A language that is ideal in Frege's sense contrasts with different sorts of "ideal" languages that philosophers have been interested in. In particular, it contrasts with a language which is idealized in the same way that other objects of scientific study sometimes are idealized: through selective simplifications and distortions that impose coherence on an unruly phenomenon in order to make possible a satisfying systematic theory of the phenomenon. An ideal language in this sense is the minimal modification of the language we actually speak so that it admits of systematic understanding. On my reading, this is not what Frege is doing.[59] But according to Frege's most active 20th century defender, Michael Dummett, this is just the sort of ideal language that Frege is concerned with.[60]

---

[58]Consider, for example, this passage from Chapter 10 of Pearl 2008, a manual for piano playing, that describes how to play keys that are far apart on the piano: "Maintain a comfortable, balanced hand position as you jump from the starting hand position across the keyboard to your landing destination...the image you want to keep in mind is of a frog jumping from lily pad to lily pad...it starts and lands with its body centered over the lily pad, contracting to the same closed, restful shape. Your hands are two frogs. As they jump from one position to another, they should look the same before and after the jump." This idealized, metaphorical description emerges from, and must be understood in terms of, the prescription.

[59]What Frege is doing also contrasts with a language that is "ideal" in that it avoids distracting and merely apparent philosophical problems. This is one of the criteria of ideality for language in Bergmann 1953 (133): "in such a language the philosophical puzzles disappear, or...they cannot even be stated in it." Many "ordinary language" philosophers can be understood as claiming that ordinary language is *already* ideal in just this sense—as Wittgenstein puts it, "philosophical problems arise when language goes on holiday," and departs from its ordinary business. Frege has none of the positivist philosopher's desire to make philosophical problems go away like this.

[60]Much of what Dummett says on this point sounds like my own interpretation: as when he claims that for Frege, "natural language has many defects which impair its effectiveness as an instrument for the expression of thought; these defects need to be eliminated whenever we use language as a means of arriving at the truth, and especially when we engage in deductive reasoning." (All quotes in this footnote are from Dummett 1981b, 28-35.) Where Dummett and I differ is in our understanding of what ultimately counts as a "defect." On my reading, it is anything that makes language less than perfectly helpful in reaching the goal of thought—which means that most features of ordinary linguistic practices are defects. For Dummett, by contrast, a feature of language is not a defect unless it renders language literally unintelligible: these features are "defects because no fully coherent account of a language exhibiting such features is possible." What Frege wants is "a [linguistic] practice capable of being codified...we therefore need to revise our

When we study an ideal phenomenon in that sense, we stick with how the phenomenon actually is as much as possible, and revise it only when that is strictly necessary to allow for a satisfying theoretical understanding. This means that the language we ordinarily speak is, basically, the one that Frege is describing. Since we know a lot about the language we speak, this means that ordinary speakers can be expected to have a lot of insight into the language Frege is talking about. We are not infallible, since some parts of the language we speak make it unsuitable for systematic understanding and are the parts that get idealized away; but by and large, what we are inclined to say about language is how the language that Frege means to describe is. It was, then, the study of a minimally revised natural language that Dummett's Frege saw as the foundation of the rest of philosophy. It was largely under this interpretation that Frege's claims were evaluated by 20th century philosophy of language. Let me conclude by giving an example of how different a now-classic 20th-century dispute would have looked, had Frege's true view been more clearly in focus.

Consider the arguments for a "causal" rather than a "descriptivist" theory of reference. Whether or not Frege held a descriptivist view, he certainly rejected a causal theory, since he thought that we can refer to numbers and other abstract objects, which do not stand in causal relations. Some arguments for the causal theory were based on the "intuitions" we have as ordinary speakers: our sense that the referent of the name "Gödel" would be Kurt Gödel, even if the things we knew about him were false, our sense that our word "water" would refer to some other substance if we had grown up on another, "Twin Earth" planet with another substance in the lakes and streams, and so on.[61] Dummett, in accordance with his understanding of Frege's project, must take such intuitions seriously; since there is a possible systematic view of language that works in accordance with these intuitions, he agrees that they motivate a revision of Frege's account.[62] But if I am right, these facts have not the slightest bearing on what Frege is saying. These intuitions are powerful appearances, something that "almost forces" us to say one thing or another about language. Frege would give such intuitions no more consideration than he gave to the claim that the meanings of our words

---

practice...so as to make such codifiation possible." Revision is called for only where "no systematic theory of meaning will fit our linguistic practice as it actually is." So for Dummett, fewer aspects of language count as defects. Our disagreement here traces back to a disagreement about just what it is that Frege needs and why; and another about how to understand the goal of thought—but I need not make all of this clear here.

[61]See Kripke 1981, 84, for one such argument, culminating in an intuition-report: "On the [descriptivist] view in question...we, when we talk about 'Godel', are in fact always referring to [someone else]. But it seems to me that we are not. We simply are not." (I do not mean to suggest that such intuition-based arguments are the only ones in favour of the causal theory of reference.)

[62]Dummett 1973, 135.

are ideas. Since ordinary language has little to do with the facts about what our objective knowledge depends on, there is no reason to think that our intuitions are right about the language Frege is discussing, which perfectly matches those epistemological facts. If we find that our intuitions persist, even when we have thus concluded that we ought not think about reference in their light, Frege would devise up some steps that we could take to remove what he would call an "illusion". If Frege had lived further into the 20th century, then, we might expect him to have enunciated another principle, on the model of the context principle: "never to ask after the meaning of a word in the context of a thought experiment about Twin Earth."

5.5   Conclusion

Now that all of the interpretive pieces are in place, let me conclude by summarizing briefly what I have argued, and making a suggestion for future research.

Frege does not think that numbers or our knowledge of them are themselves linguistic matters. So why are there linguistic claims in his book about the foundations of arithmetic? Thinking about language must be meant to help him answer his questions somehow. In key sections, Frege seems to be assuming a strong mirroring relation between language and knowledge, which implies that the conditions on speaking are the same as those on having objective knowledge. But why does he assume this, and how does it help us understand the foundations of arithmetic?

One straightforward idea is that he will use claims about language as premises in arguments for the non-linguistic conclusions of the book—inferences that would be permitted by the mirroring assumption. But close attention to the passages themselves show that linguistic claims do not play this kind of role. Even given the mirroring relationship, what Frege actually says about language does not imply the relevant conclusions, and moreover, the linguistic claims sometimes only appear *after* the relevant non-linguistic conclusions have been reached.

We instead find the answer by recalling the perspective induced by Frege's *Begriffsschrift* project. Frege's assumption that language mirrors knowledge is not a view about how language is, but part of adopting that perspective. When he talks about language, Frege is, most basically, prescribing for the way we should use and think about language; any descriptions he gives are of a language that is "ideal" in the corresponding sense. The point of these prescriptions is to help us reach the goal of thought most effectively. This is both the point of talking about language throughout the book and the justification for treating language as mirroring thought.

Though I have here discussed only Frege's early writings, I expect that his later discussions of language are no different. I expect that reading them in the way I have described will allow us to resolve the outstanding puzzles about Frege's later philosophy of language. Let me conclude with an example of how it may be able to do so.

Consider Frege's later introduction of the notion of the *sense* of a linguistic expression. Interpreters have long been puzzled why Frege needs this notion, and they have noticed that it seems to have a close connection with the (nonlinguistic) *axioms* of his logical system, especially *Basic Law V*. But what is this connection? Once again, the most natural suggestion is that Frege will use claims about the senses of expressions—linguistic claims—to *justify* his choice of those axioms.[63] But such readings have not been widely accepted, largely because Frege does not actually appeal to any claims about sense when justifying his choice of axioms.[64] Many commentators, accordingly, are puzzled about why Frege puts so much time into thinking about sense. On my reading, we can accept that the point of sense is to help us identify Basic Law V as a logical axiom, without supposing that it is part of an argument to justify that identification. Rather, we should expect it to removes certain linguistic illusions that would interfere with that acceptance. It is linguistic illusions connected with identity, I suggest, that are under discussion at the beginning of "On Sense and Reference." But just what those illusions are, how sense is meant to remove them, and what Frege's nonlinguistic argument for accepting his axioms is, must all wait for another time.

---

[63]Sluga 1986, for example, notes that Basic Law V is a generalized identity statement, in each of whose instances the identity sign connects expressions that have the same sense; he accordingly suggests that "apprehension of sameness [of sense]...guarantees the status of the axiom as a logical principle."

[64]Ebert 2016, for example, argues that Frege "never draws on [the relevant claim about sense] as an assumption in his arguments or when he offers...considerations in favour of accepting [the axiom]...he never makes this move."

Bibliography

Abela, Paul. 2006. "The Demands of Systematicity: Rational Judgment and the Structure of Nature." In *A Companion to Kant*, 408-422. Ed. Bird. Oxford: Blackwell. 2006.

Aristotle. *Posterior Analytics.* In *A New Aristotle Reader,* 39-59. Trans. Barnes, ed. Ackrill. Princeton: Princeton University Press, 1987.

Bar-Elli, Gilead. 2010. "Analyticity and Justification in Frege." *Erkenntnis* 73: 165-184

Beaney, Michael. 1996. *Frege: Making Sense.* London: Duckworth.

——— 2016. "The Analytic Revolution." *Royal Institute of Philosophy Supplement* 78: 227-249.

Beiser, Frederick. 2011. *The German Historicist Tradition.* Oxford: Oxford University Press.

——— 2014. *The Genesis of Neo-Kantianism, 1796-1880.* Oxford: Oxford University Press.

Blanchette, Patricia. 2012. *Frege's Conception of Logic.* Oxford: Oxford University Press.

Bolzano, Bernard. 1837. *Wissenschaftslehre.* Sulzbach: Seidel.

Brueckner, Anthony. 1996. "Modest Transcendental Arguments." *Philosophical Perspectives* 10: 265-280.

Burge, Tyler. 1998. "Frege on Knowing the Foundation." In *Truth, Thought, and Reason: Essays on Frege*, 317-356. Oxford: Oxford University Press. 2005.

Carl, Wolfgang. 1994. *Frege's Theory of Sense and Reference: It's Origins and Scope.* Cambridge: Cambridge University Press.

Carnap, Rudolf. 1937. *The Logical Syntax of Language.* Trans Smeaton. London: Routledge. Reprinted 2001.

——— 1950 "Empiricism, Semantics, and Ontology." *Revue Internationale de Philosophie* 4: 20-40.

——— 1962. *Logical Foundations of Probability.* 2nd ed. Chicago: Univeresity of Chicago Press.

Cohen, Hermann. 1871. *Kants Theorie der Erfahrung.* 1st Edition. Berlin: Dümmler.

——— 1883. *Das Princip der Infinitesimal-Methode und seine Geschichte.* Berlin: Dümmler.

——— 1885. *Kants Theorie der Erfahrung.* 2nd Edition. Berlin: Dümmler.

——— 1914. "The Relationship of Logic to Physics." In *The Neo-Kantian Reader,* 117-136. Trans. Patton, ed. Luft. London: Routledge. 2015.

Cohen v. California. 1971. 403 U.S. 15.

Coliva, Analisa. 2015. *Extended Rationality: A Hinge Epistemology.* Oxford: Oxford University Press.

Conant, James. 1991. "The Search for Logically Alien Thought." *Philosophical Topics* 20: 115-180.

Dasgupta, Shamik. 2016. "Metaphysical Rationalism." *Noûs* 50: 379-418.

David, Marian. 2005. "On 'Truth is Good'." *Philosophical Books* 46: 292-301

Della Rocca, Michael. 2010. "PSR." *Philosopher's Imprint* 10: 1-13.

de Jong, Willem. 1996. "Gottlob Frege and the analytic-synthetic distinction within the framework of the aristotelian model of science." Kant-Studien 87: 290-324

de Jong, Willem, and Betti, Arianna. 2010. "The Classical Model of Science: a millennia-old model of scientific rationality." *Synthese* 174: 185-203. 2010.

Descartes, René. 1641/1642. *Meditations on First Philosophy.* In *The Philosophical Writings of Descartes,* vol II, 1-62. Trans. Cottingham, eds. Cottingham, Stoothoff, and Murdoch. Cambridge: Cambridge Univeristy Press. 1984.

Detlefsen, Michael. 1988. "Fregean Hierarchies and Mathematical Explanation." *International Studies in the Philosophy of Science* 3: 97-116.

Dummett, Michael. 1973. *Frege Philosophy of Language.* London: Harper and Row.

——— 1981a. "Frege's 'Kernsätze zur Logik'." *Inquiry* 24: 439-48.

——— 1981b. "Principles of Frege Exegesis." In *The Interpretation of Frege's Philosophy*, 6-35. London: Duckworth. 1981.

——— 1981c. "Frege's Notion of Reference." In *The Interpretation of Frege's Philosophy.* 148-165. London: Duckworth. 1981.

——— 1991. *Frege Philosophy of Mathematics.* London: Duckworth.

——— 1995. "The Context Principle: Centre of Frege's Philosophy." In *Logik und Mathematik: Frege-Kolloquium Jena 1993,* 3-19. Eds. Max and Stelzner. Berlin: de Gruyter. 1995.

Ebert, Philip. 2016. "Frege on Sense Identity, Basic Law V, and Analysis." *Philosophical Mathematica* 24: 9-29.

Elgin, Catherine. 2006. "From Knowledge to Understanding." In *Epistemology Futures,* 199-215. Ed. Hetherington. Oxford: Oxford Univeresity Press. 2006.

Evans, Gareth. 1982. *The Varieties of Reference.* Oxford: Oxford University Press.

Field, Hartry. 2015. "What is Logical Validity?" in *Foundations of Logical Consequence,* 33-70. Eds. Caret and Hjortland. Oxford: Oxford University Press.

Frege, Gottlob. 1874. "Methods of Calculation based on an Extension of the Concept of Quantity." in Frege 1984, 56-94. Trans. Kaal.

——- – 1879-1891. "Logic." in Frege 1979, 1-8. Trans. Long and White.

——— 1879. *Begriffsschrift.* In Frege 1993.

——— 1880-1881. "Boole's logical Calculus and the Concept-script" in Frege 1979, 9-46. Trans. Long and White.

——— 1882. "Über die wissenschaftliche Berechtigung einer Begriffsschrift" in Frege 1993, 106-114.

——— 1884. *Die Grundlagen Der Arithmetik.* Breslau: Wilhelm Koebner Verlag.

——— 1885. "Rezension von: H. Cohen, Das Prinzip der Infinitesimal-Methode und seine Geschichte." *Zeitschrift für Philosophie und philosophische Kritik* 87:324-329

——— 1892. "Über Sinn und Bedeutung." *Zeitschrift für Philosophie und Philosophische Kritik*, 100: 25-50.

——— 1893. *Basic Laws of Arithmetic*, volume I. In Frege 2013. Trans. and ed. Ebert and Rossberg.

——— 1895. "A Critical Elucidation of some Points in E. Schröder's *Vorlesungen über die Algebra der Logik.*" In Frege 1984, 210-228. Trans. Geach.

——— 1896. "Frege to Peano 29.9. 1896." In *Philosophical and Mathematical Correspondence*, 112-117. Trans. Kaal, eds. Gabriel, Hermes, Kambartel, Thiel, and Veraart Oxford: Blackwell, 1980.

——— 1897a. "Logic." in Frege 1979, 126-151. Trans. Long and White.

——— 1897b. "On Mr. Peano's Conceptual Notation and My Own." in Frege 1984, 234-248. Trans. Dudman.

——— 1899-1906. "On Euclidean Geometry." in Frege 1979, 167-169. Trans. Long and White.

——— 1902. "Letter to Russell, October 20th 1902." In *Philosophical and Mathematical Correspondence*, 149-150. Trans. Kaal, eds. Gabriel, Hermes, Kambartel, Thiel, and Veraart Oxford: Blackwell, 1980.

——— 1903a. *Basic Laws of Arithmetic*, volume II. In Frege 2013. Trans. and ed. Ebert and Rossberg.

——— 1903b. "On the Foundations of Geometry: First Series." In Frege 1984, 273-284. Trans. Kluge.

——— 1906a. "Introduction to Logic." In Frege 1979, 185-196. Trans. Long and White.

——— 1906b. "On the Foundations of Geometry: Second Series." In Frege 1984, 293-341. Trans. Kluge.

——— 1914. "Logic in Mathematics." In Frege 1979, 203-250. Trans. Long and White.

——— 1918-1919a. "Thoughts." in Frege 1984, 351-372. Trans. Geach and Stoothoff.

——— 1918-1919b. "Negation." in Frege 1984, 373-389. Trans. Geach and Stoothoff.

——— 1918-1919c. "Compound Thoughts." in Frege 1984, 390-406. Trans. Geach and Stoothoff.

——— 1923-1925. "Logical Generality." In Frege 1979, 258-262. Trans. Long and White.

——— 1924/1925. "Sources of Knowledge in Mathematics and the Mathematical Natural Sciences." In Frege 1979, 267-274. Trans. Long and White.

——— 1979. *Posthumous Writings.* Trans. Peter Long and Roger White, ed. Hans Hermes, Friedrich Kambartel, and Friedrich Kaulbach. Oxford: Blackwell.

——— 1984. *Collected Papers.* Trans. Black, Dudman, Geach, Kaal, Kluge, McGuinness, and Stoothoff, ed. McGuinness. Oxford: Blackwell.

——— 1993. *Begriffsschrift und andere Aufsätze.* ed. Angelelli. Hildesheim: George Orms Verlag.

——— 2013. *Basic Laws of Arithmetic.* Trans. and ed. Ebert and Rossberg. Oxford: Oxford University Press.

Friedman, Michael. 1974. "Explanation and Scientific Understanding." *The Journal of Philosophy* 71: 5-19.

Gabriel, Gottfried, 1989. "Lotze und die Entstehung der modernen Logik bei Frege." In Hermann Lotze, *Logik. Erstes Buch,* xi-xliii. Ed. Gabriel. Hamburg: Felix Meiner Verlag. 1989.

——— 2002. "Frege, Lotze, and the Continental Roots of Analytic Philosophy." In *From Frege to Wittgenstein,* 39-51. Ed. Reck. Oxford: Oxford University Press, 2002.

Glock, Hans-Johann. 2015. "Neo-Kantianism and Analytic Philosophy." In *New Approaches to Neo-Kantianism,* 59-81. Eds. Warren and Staiti. Cambridge: Cambridge University Press. 2015.

Goldfarb, Warren. 2001. "Frege's Conception of Logic." In *Future Pasts: The Analytic Tradition in twentieth-century philosophy,* 25-41. Ed. Floyd. Oxford: Oxford University Press. 2001.

Goldman, Alvin. 2001. "The Unity of the Epistemic Virtues." In *Virtue Epistemology*, 30-48. Ed. Fairweather and Zagzebski. Oxford: Oxford University Press. 2001.

Greimann, Dirk. 2008. "Does Frege Use a Truth-Predicate in his 'Justification' of the Laws of Logic? A Comment on Weiner." *Mind* 117: 403-425.

Grimm, Stephen. 2012. "The Value of Understanding." *Philosophy Compass* 7: 103-117.

Heck, Richard. 1995. "The Sense of Communication." *Mind* 104: 79-106.

——— 2012. *Reading Frege's Grundgesetze.* Oxford: Oxford University Press.

Heck, Richard, and May, Robert. Forthcoming. "Truth in Frege." In *The Oxford Handbook of Truth.* Ed. Glanzberg. Oxford: Oxford University Press. Forthcoming.

Heis, Jeremy. 2013. "Frege, Lotze, and Boole." In *The Historical Turn in Analytic Philosophy,* 113-138. Ed. Reck. London: Macmillan. 2013.

Hempel, Carl G., and Oppenheim, Paul. 1948. "Studies in the Logic of Explanation." *Philosophy of Science* 15: 135-175.

Hilbert, David. 1902. "Mathematical Problems." *Bulletin of the American Mathematical Society* 8: 437–479. Trans. Newson.

Hintikka, Jaakko. 1967. "Kant on the Mathematical Method." In *Kant's Philosophy of Mathematics,* 177-220. Ed. Posy, Dordrecht: Kluwer Academic Publishers. 1992.

Horkheimer, Max and Adorno, Theodor. 1947. *Dialectic of Enlightenment.* Trans. Jephcott. Palo Alto: Stanford University Press. 2012.

Hovens, Franz. 1997. "Lotze and Frege: The dating of the 'Kernsätze'." *History and Philosophy of Logic* 18: 17-31.

Huxley, Thomas. 1871. "Mr. Darwin's Critics." In *Collected Essays, Vol. 2,* 120-186. London: Macmillan, 1899.

Jeshion, Robin. 2001. "Frege's Notions of Self-Evidence." *Mind* 110: 937-976.

Jesseph, Douglas. 2010. "*Scientia* in Hobbes." In *Scientia In Early Modern Philosophy,* 117-128. Eds. Sorell, Rogers, and Kraye. Dordrecht: Springer. 2010.

Kant, Immanuel. 1781/1787. *Critique of Pure Reason.* Trans. Kemp Smith. London: Macmillan. 1929.

——— 1786. *Metaphysical Foundations of Natural Science.* Trans. and ed. Friedman. Cambridge: Cambridge University Press. 2004.

Kern, Andrea. 2006. "Spontaneity and Receptivity in Kant's Theory of Knowledge." *Philosophical Topics* 34: 145-162. Trans. Smyth.

Kitcher, Philip. 1989. "Explanatory Unification and the Causal Structure of the World." In *Scientific Explanation,* 410-506. Eds. Kitcher and Salmon. Minneapolis: University of Minnesota Press. 1989.

Korsgaard, Christine. 1996. *The Sources of Normativity.* Cambridge: Cambridge University Press.

Kvanvig, Jonathan. 2003. *The Value of Knowledge and the Pursuit of Understanding.* Cambrdige: Cambridge University Press.

Levine, James. 1996. "Logic and Truth in Frege II." *Proceedings of the Aristotelian Society* 70, 121-175.

Lewis, David. 1978. "Truth in Fiction." *American Philosophical Quarterly* 15: 37-46.

Leibniz, Gottfried Wilhelm. 1680-1684. "First Truths." *Philosophical Papers and Letters,* 267-271. Trans. and ed. Leomker. Dordrecht: Kluwer Academic Publishers. 1992.

——— 1714. *Monadology.* Trans. Schrecker and Schrecker. In *Monadology and other Philosophical Essays.* Indianapolis: Bobbs Merril Company. 1965.

——— 1765. *New Essays Concerning Human Understanding.* Trans. Langley. London: Macmillan. 1896.

Lotze, Hermann. 1856-64. *Microcosmus.* Trans. Hamilton and Jones. Edinburg: T. & T. Clark. 1885.

——— 1858. *Mikrokosmus.* 2nd vol. Leipzig: Hirzel.

——— 1874. *Logic.* Trans. Bosanquet. Oxford: Oxford University Press. 1884.

——— 1879. *Metaphysics, in three books: Ontology, Cosmology, and Psychology.* Trans. Bosanquet. Oxford: Oxford University Press. 2nd Edition, 1887.

——— 1880. "Philosophy in the Last Forty Years. First Article." In *Kleine Schriften,* v. 3, 451-479. ed. Peipers. Leipzig: Hirzel. 1885–91.

——— 1885. *Outlines of Logic and of Encyclopedia of Philosophy.* Trans. Ladd. Boston: Ginn & Company. 1887.

Lowe, E.J. 1989. "What is a Criterion of Identity?" *The Philosophical Quarterly* 39, 1-21.

Lynch, Michael. 2004. *True to Life: Why Truth Matters.* Cambridge: MIT Press.

——— 2009. "Truth, Value, and Epistemic Expressivism." *Philosophy and Phenomenological Research* 79: 76-97.

Macbeth, Danielle. 2005. *Frege's Logic.* Cambridge: Harvard University Press.

Mackie, J.L. 1977. *Ethics: Inventing Right and Wrong.* New York: Penguin.

MacFarlane, John. 2000. *What Does It Mean To Say That Logic is Formal?* Ph.D dissertation, University of Pittsburgh.

——— 2002. "Frege, Kant, and the Logic in Logicism." *Philosophical Review* 111, 25-65.

Makkreel, Rudolf, and Luft, Sebastian. 2010. "Introduction." In *Neo-Kantianism in Contemporary Philosophy,* 1-24. Eds. Makreel and Luft. Bloomington: Indiana University Press. 2010.

Murdoch, Iris. 1970. *The Sovereignty of Good.* London: Routledge.

Newton, Isaac. 1687. *The Mathematical Principles of Natural Philosophy.* Trans. Motte. New York: Daniel Adee. 1846.

Parsons, Charles. 1986. "Quine on the Philosophy of Mathematics." In *The Philosophy of W.V. Quine,* 369-395. Eds. Hahn and Schlipp. Chicago: Open Court. 1986.

Pearl, David. 2008. *Piano Exercises for Dummies.* Hoboken: Wiley.

Phemister, Pauline. 1993. "Locke, Sergeant, and Scientific Method." *The Rise of Modern Philosophy,* 231-252. Ed. Sorell. Oxford: Oxford University Press. 1993.

Plato. *Phaedo.* In *Complete Works,* 49-100. Trans. Grube, ed. Cooper. Indianapolis: Hackett. 1997.

——— *Laws.* Trans. Saunders. In *Complete Works,* 1318-1616. Trans. Grube, ed. Cooper. Indianapolis: Hackett. 1997.

Pritchard, Duncan. 2009. "Knowledge, Understanding, and Epistemic Value." *Royal Institute of Philosophy Supplement* 64: 19-43.

Quine, Willard van Orman. 1960. "Carnap on Logical Truth." *Synthese* 12: 350-374.

——— 1986a. "Reply to Charles Parsons." in *The Philosophy of W.V. Quine*, 396-404. Eds. Hahn and Schlipp. Chicago: Open Court. 1986.

——— 1986b. *Philosophy of Logic.* 2nd Edition. Cambridge: Harvard University Press.

Reichenbach, Hans. 1938. *Experience and Prediction.* Chicago: University of Chicago Press.

Richardson, Alan. 2006. "'The Fact of Science' and Critique of Knowledge: Exact Science as Problem and Resource in Marburg Neo-Kantianism." In *The Kantian Legacy in Nineteenth-Century Science,* 211-227. Eds. Friedman and Nordmann. Cambridge: MIT Press. 2006.

Ricketts, Thomas. 1985. "Frege, the Tractatus, and the Logocentric Predicament." *Noûs* 19: 3-15.

——— 1996. "Logic and Truth in Frege." *Proceedings of the Aristotelian Society, Supplementary Volumes* 70: 121-175.

——— 1998. "Frege's 1906 Foray into Metalogic." *Philosophical Topics* 25: 169-188.

——— 2010. "Concepts, Objects, and the Context Principle." In *The Cambridge Companion to Frege,* 63-85. Eds. Potter and Ricketts. Cambridge: Cambridge University Press. 2010.

Schechter, Joshua, and Enoch, David. 2008. "How are Basic Belief-Forming Methods Justified?" *Philosophy and Phenomenological Research* 76: 547-579.

Schlotter, Sven and Wehmeier, Kai F. 2013. "Gingerbread Nuts and Pebbles: Frege and the Neo-Kantians—Two Recently Discovered Documents." *British Journal for the History of Philosophy* 21: 591-609.

Sluga, Hans. 1980. *Gottlob Frege.* London: Routledge.

——— 1984. "Frege: The Early Years." In *Philosophy in History*, 329-356. Eds. Rorty, Schneewind, and Skinner. Cambridge: Cambridge University Press, 1984.

——— 1986. "Semantic Content and Cognitive Sense." In *Frege Synthesized*, 47-64. Eds. Haaparanta and Hintikka. Dordrecht: Kluwer Academic Publishers. 1986.

——— 1997. "Frege on Meaning." In *The Rise of Analytic Philosophy,* 17-34. Ed. Glock. Oxford: Blackwell. 1997.

Sorell, Tom. 2010. "*Scientia* and the Sciences in Descartes." In *Scientia In Early Modern Philosophy,* 71-82. Eds. Sorell, Rogers, and Kraye. Dordrecht: Springer. 2010.

Sosa, Ernest. 2003. "The Place of Truth in Epistemology." In *Intellectual Virtue,* 155-180. Eds. DePaul and Zagzebski. Oxford: Oxford University Press. 2003.

Stanley, Jason. 1996. "Truth and Metatheory in Frege." *Pacific Philosophical Quarterly* 77: 45-70.

Steinberger, Florian. 2016. "The Normative Status of Logic." *The Stanford Encyclopedia of Philosophy* (Spring 2017 Edition.) Ed. Zalta, URL = <https://plato.stanford.edu/archives/spr2017/entries/logic-normative/>.

——— 2017. "Frege and Carnap on the Normativity of Logic." *Synthese* 194: 143-162.

Stern, Robert. 2007. "Transcendental Arguments: A Plea For Modesty." *Grazer Philosophische Studien* 47: 143–61.

Strawson, P.F. 1976. "Entity and Identity." In *Entity and Identity and other Essays,* 21-51. Oxford: Oxford University Press. 1997.

Street, Sharon. 2006. "A Darwinian Dilemma for Realist Theories of Value." *Philosophical Studies* 127: 109-166.

Stroud, Barry. 2011. *Engagement and Metaphysical Dissatisfaction.* Oxford: Oxford University Press.

Sullivan, David. 1991. "Frege on the Cognition of Objects." *Philosophical Topics* 19: 245-268.

——— 2017. "Hermann Lotze." *The Stanford Encyclopedia of Philosophy* (Winter 2017 Edition.) Ed. Zalta URL = <https://plato.stanford.edu/archives/win2017/entries/hermann-lotze/>.

Tappenden, Jamie. 1997. "Metatheory and Mathematical Practice in Frege." *Philosophical Topics* 25: 213-264.

Taschek, William. 2008. "Truth, Assertion, and the Horizontal." *Mind* 117: 375-401.

Taylor, Charles. 1980. "Theories of Meaning." In *Human Agency and Language: Philosophical Papers 1,* 248-292. Cambridge: Cambridge University Press. 1985.

Textor, Mark. 2011. *Frege on Sense and Reference.* London: Routledge.

Thomas, Reverend Evan Edward. 1921. *Lotze's Theory of Reality.* London: Longmans, Green, and Co.

Tolley, Clinton. 2012. "The Generality of Kant's Trasncendental Logic." *Journal of the History of Philosophy* 50: 417-446.

Understanding Science. 2018. University of California Museum of Paleontology. (Accessed 9 May 2018.) URL = <https://undsci.berkeley.edu/article/truth>.

van Cleve, James. 1979. "Foundationalism, Epistemic Principles, and the Cartesian Circle." *The Philosophical Review* 88: 55-91.

van den Berg, Hein. 2011. "Kant's Conception of Proper Science." *Synthese* 183: 7-26.

van Heijenoort, Jean. 1986. "Frege and Vagueness." In *Frege Synthesized,* 31-46. Eds. Haaparanta and Hintikka. Dordrecht: Kluwer Academic Publishers. 1986.

Weiner, Joan. 1996. "Has Frege a Philosophy of Language?" In *Early Analytic Philosophy: Frege, Russell, Wittgenstein,* 249-272. Ed. Tait. Chicago: Open Court. 1997.

——— 1997. "Frege and the Linguistic Turn." *Philosophical Topics* 25, 265-288.

——— 2002. "Section 31 Revisited: Frege's Elucidations." In *From Frege to Wittgenstein,* 149-182. Ed. Reck. Oxford: Oxford University Press, 2002.

——— 2004. "What was Frege Trying to Prove? A Response to Jeshion." *Mind* 113: 115-129.

——— 2005. "Semantic Descent." *Mind* 114: 321-354.

Wilson, Margaret Dauler. 1967. "Leibniz and Locke on First Truths." in *Ideas and Mechanism,* 353-372. Princeton: Princeton University Press. 1999.

Windelband, Wilhelm. 1882. "Was ist Philosophie?" in *Präludien,* 24-77. Tübingen: Mohr Verlag. 1907.

——— 1883. "Kritische oder genetische Methode?" in *Präludien,* 318-354. Tübingen: Mohr Verlag, 1907.

——— 1892. *Geschichte der Philosphie.* Tübingen: Mohr Verlag.

——— 1894. "Geschichte und Naturwissenschaft." in *Präludien,* 355-379. Tübingen: Mohr Verlag, 1907.

——— 1907. "Über die gegenwärtige Lage und Aufgabe der Philosophie." in *Präludien,* 1-23. Tübingen: Mohr Verlag, 1907.

——— 1909. *Der Wille Zur Wahrheit.* Heidelberg: Universitäts-Buchdruckerei von J. Hörning.

——— 1920. *Einleitung in die Philosophie.* 2nd Edition. Tübingen: Mohr Verlag.

Wittgenstein, Ludwig. 1953. *Philosophical Investigations.* 4th Edition. Trans. Anscombe, Hacker, and Schulte, Eds. Hacker and Schulte. Chichester: Wiley-Blackwell.

——— 1958. *The Blue and Brown Books.* Oxford: Blackwell.

Wright, Crispin. 1983. *Frege's Conception of Numbers as Objects.* Aberdeen: Aberdeen University Press.

——— 2004 "On Epistemic Entitlement: Warrant For Nothing (and Foundations for Free?)" *Proceedings of the Aristotelian Society, Supplementary Volumes* 78: 167-245.

——— 2014. "On Epistemic Entitlement (II): Welfare State Epistemology." In *Scepticism and Perceptual Justification*, 214-247. Eds. Dodd and Zardini. Oxford: Oxford Unviersity Press. 2014.

Zagzebski, Linda. 2001. "Recovering Understanding." In *Knowledge, Truth, and Duty: Essays on Epistemic Justification, Responsibility, and Virtue,* 235-252. Ed. Steup. Oxford: Oxford University Press. 2001.