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Author
Dreiner, Herbi K.

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Herbi K. Dreiner, Marc Thormeier

Physikalisches Institut der Universitat Bonn,
Nußallee 12, 53115 Bonn, Germany

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Herbi K. Dreiner[*], Marc Thormeier[†]

Physikalisches Institut der Universität Bonn,
Nußallee 12, 53115 Bonn, Germany

Abstract

We systematically investigate the embedding of $U(1)_X$ Froggatt-Nielsen models in (four-dimensional) local supersymmetry. We restrict ourselves to models with a single flavon field. We do not impose a discrete symmetry by hand, *e.g.* R-parity, baryon-parity or lepton-parity. Thus we determine the order of magnitude of the baryon- and/or lepton violating coupling constants through the Froggatt-Nielsen scenario. We then scrutinize whether the predicted coupling constants are in accord with weak or GUT scale constraints. Many models turn out to be incompatible.

1 Introduction

The problem of the fermionic mass spectrum is unresolved. Within the Standard Model (*SM*), the masses are due to the Yukawa interactions of the Higgs scalar. A theory of fermion masses is thus a theory of the origin of the Yukawa couplings, a problem not dealt with by the *SM*. This persists when extending the *SM* to supersymmetry. In fact the problem becomes worse. Including all terms in the superpotential which are allowed by the gauge group, there are 45 unknown *extra* R-parity violating ($\not{R}_p$) Yukawa couplings beyond the 27 of the *SM*. A theory addressing the fermionic mass spectrum should explain all parameters in the superpotential, *i.e.* the $\not{R}_p$ coupling constants as well as the bilinear superpotential terms (the μ-problem) and possible higher-dimensional operators. There is extensive experimental data on the conservation of baryon- and lepton-number and thus bounds on the $\not{R}_p$ coupling constants. It is the purpose of this paper to show

[*]dreiner@th.physik.uni-bonn.de
[†]New address as of March 10^{th}, 2003: *Theoretical Physics Group, Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA.* E-mail: mhthormeier@lbl.gov

how this can be utilized to constrain the supersymmetric flavour theory, possibly pointing towards a solution of the fermionic mass problem.

The problem of the fermionic mass spectrum has been extensively discussed in the literature. In Refs. [1-16] and Refs. [17-21], a certain class of (mostly) supersymmetric Froggatt-Nielsen models [22] is presented. The models in Refs. [1-16] all feature:

1. A *single* local $U(1)_X$ symmetry. The gauge charges, X, of the superfields are generation-dependent.

2. This $U(1)_X$ gets broken spontaneously by a single scalar flavon field.

3. The mixed $U(1)_X$ anomalies are canceled by the Green-Schwarz mechanism [23].

4. The fermionic mass spectrum at the grand unified (GUT)-scale satisfies [24, 25],

$$m_e : m_\mu : m_\tau \sim \epsilon^{4 \ or \ 5} : \epsilon^2 : 1, \tag{1.1}$$

$$m_\tau : m_b \sim 1, \tag{1.2}$$

$$m_d : m_s : m_b \sim \epsilon^4 : \epsilon^2 : 1, \tag{1.3}$$

$$m_b : m_t \sim \epsilon^{0,1,2 \ or \ 3} \cot\beta, \tag{1.4}$$

$$m_u : m_c : m_t \sim \epsilon^8 : \epsilon^4 : 1, \tag{1.5}$$

$$m_t \sim \langle H^{\mathcal{U}} \rangle, \tag{1.6}$$

$$\boldsymbol{U^{CKM}} \sim \begin{pmatrix} 1 & \epsilon & \epsilon^3 \\ \epsilon & 1 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} 1 & \epsilon^2 & \epsilon^4 \\ \epsilon^2 & 1 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & 1 \end{pmatrix}. \tag{1.7}$$

Here $\tan\beta$ is the ratio of the vacuum expectation values (VEVs) of the two neutral Higgs scalars, $\tan\beta \equiv \langle H^{\mathcal{U}} \rangle / \langle H^{\mathcal{D}} \rangle$; $\boldsymbol{U^{CKM}}$ is the Cabibbo-Kobayashi-Maskawa matrix and $\epsilon \approx 0.22$ is the Wolfenstein parameter, *i.e.* the sine of the Cabibbo angle. Since the expansion parameter is rather large (ϵ and ϵ^2 are of the same order of magnitude) the powers of ϵ are only approximate.

5. The models are all in four-dimensional space-time.

The models in Refs. [1-9] do not contain right-handed neutrinos, the models in Refs. [10-16] do. We have tried to present an exhaustive list of the existing models fulfilling the four points stated above. One model in Ref. [16] and the models in Refs. [17-21] fulfill the first three points but deviate in their fermionic mass spectrum.[1]

In this paper, we present a systematic embedding of Froggatt-Nielsen models in supersymmetry, including a detailed discussion of the Kählerpotential. We then investigate whether the $U(1)_X$ charge assignments of the aforesaid models give rise to an acceptable low-energy phenomenology. We hereby make the following assumptions:

[1] We do neither consider a gauged $U(1)$ family-dependent R-symmetry as in Ref. [26] nor a discrete gauge symmetry as in Ref. [27].

1. The models are supersymmetric.

2. Baryon- [28] and lepton-parity (the latter one being anomalous) are not imposed by hand, so *a priori* all superpotential terms that are allowed by the symmetries are assumed to indeed exist; we shall hence include $R\!\!\!/_p$ (for an introduction see *e.g.* Refs. [29, 30, 31, 32]) and also higher dimensional operators.[2]

3. Supersymmetry is local and the breaking of supersymmetry is mediated by gravity [36, 37, 38, 39]; hence the gravitino mass $M_{3/2}$ is of the same order as the masses of the squarks and sleptons, and the Kim-Nilles [40] or the Giudice-Masiero [41] mechanism solves the μ-problem [this is consistent with the existence of $U(1)_X$, see Eq. (2.17)].

When the $U(1)_X$ is broken (*c.f.* Section 2), the R-parity violating minimal supersymmetric standard model is generated. In general, we shall allow for three right-chiral neutrino superfields and we denote the model by $R\!\!\!/_p$-*MSSM*+$\overline{N^i}$. Its most general renormalizable superpotential is given by

$$
\begin{aligned}
\mathcal{W} \;=\;& \varepsilon^{ab}\ \delta^{xy}\ G^{(U)}{}_{ij}\ Q^i{}_{xa}\ H^{\mathcal{U}}{}_b\ \overline{U^j{}_y} \\
&+ \varepsilon^{ab}\ \delta^{xy}\ G^{(D)}{}_{ij}\ Q^i{}_{xa}\ H^{\mathcal{D}}{}_b\ \overline{D^j{}_y} \\
&+ \varepsilon^{ab}\ G^{(E)}{}_{ij}\ L^i{}_a\ H^{\mathcal{D}}{}_b\ \overline{E^j} \quad + \quad \varepsilon^{ab}\ G^{(N)}{}_{ij}\ L^i{}_a\ H^{\mathcal{U}}{}_b\ \overline{N^j} \\
&+ \varepsilon^{ab}\ \mu\ H^{\mathcal{D}}{}_a\ H^{\mathcal{U}}{}_b \quad + \quad \Gamma_{ij}\ \overline{N^i}\ \overline{N^j} \\
&+ \frac{1}{2}\ \varepsilon^{ab}\ \Lambda_{ijk}\ L^i{}_a\ L^j{}_b\ \overline{E^k} \\
&+ \varepsilon^{ab}\ \delta^{xy}\ \Lambda'_{ijk}\ Q^i{}_{xa}\ L^j{}_b\ \overline{D^k{}_y} \quad + \quad \Xi_i\ \overline{N^i} \\
&+ \frac{1}{2}\ \varepsilon^{xyz}\ \Lambda''_{ijk}\ \overline{U^i{}_x}\ \overline{D^j{}_y}\ \overline{D^k{}_z} \quad + \quad \varepsilon^{ab}\ \Upsilon_i\ \overline{N^i}\ H^{\mathcal{D}}{}_a\, H^{\mathcal{U}}{}_b \\
&+ \varepsilon^{ab}\ K_i\ L^i{}_a\ H^{\mathcal{U}}{}_b \quad + \quad \Lambda'''_{ijk}\ \overline{N^i}\ \overline{N^j}\ \overline{N^k}.
\end{aligned}
\qquad (1.8)
$$

We use the standard notation, which is explained in the footnote below.[3] The upper two blocks in Eq. (1.8) are R_p conserving, the lower blocks violate R_p; the left blocks

[2] In $SO(10)$ grand unified theories, the GUT symmetry can be broken by a Higgs field in the spinorial **16**-representation or in the **126**-representation. If we exclusively use the latter, then R-parity is conserved. If we include the former, then R-parity will be violated after $SO(10)$ breaking, see Refs. [33, 34, 35]. From the low-energy point of view, the choice of breaking scheme, however, still appears arbitrary.

[3] H, Q, L represent the left-chiral $SU(2)_W$-doublet superfields of the Higgses, the quarks and leptons; U, D, E, N represent the right-chiral superfields of the u-type quarks, d-type quarks, electron-type and neutrino-type leptons, respectively; an overbar denotes charge conjugation; a, b, c and x, y, z are $SU(2)_W$- and $SU(3)_C$-indices, respectively, i, j, k, l are generational indices, summation over all repeated indices is implied; δ^{xy} is the Kronecker symbol, $\varepsilon^{\cdots}$ symbolizes any tensor that is totally antisymmetric with respect to the exchange of any two indices, with $\varepsilon^{12\cdots} = 1$. All other symbols are coupling constants, *a*

do not contain right-handed neutrinos, the right blocks do (hence they are absent in the models of Refs. [1-9]). Thus the superpotential of the *MSSM* is contained in the upper left block. We also consider non-renormalizable F-term operators of dimensionality five; having dropped all gauge and flavour indices they read

$$\begin{array}{rl} LH^{\mathcal{U}}LH^{\mathcal{U}},\ \ QQ\overline{U}\overline{D},\ \ QL\overline{U}\overline{E}, & \overline{N}LL\overline{E},\ \ \overline{N}QL\overline{D},\ \ \overline{N}\overline{U}\overline{D}\overline{D}, \\ QQQL,\ \ H^{\mathcal{D}}H^{\mathcal{U}}H^{\mathcal{D}}H^{\mathcal{U}},\ \ \overline{U}\overline{U}\overline{D}\overline{E}, & \overline{N}\overline{N}H^{\mathcal{D}}H^{\mathcal{U}},\ \ \overline{N}\overline{N}\overline{N}\overline{N}, \\ & \\ LH^{\mathcal{U}}H^{\mathcal{D}}H^{\mathcal{U}},\ \ QH^{\mathcal{D}}\overline{U}\overline{E},\ \ QQQH^{\mathcal{D}}, & \overline{N}QH^{\mathcal{U}}\overline{U},\ \ \overline{N}QH^{\mathcal{D}}\overline{D},\ \ \overline{N}LH^{\mathcal{D}}\overline{E}, \\ & \overline{N}\overline{N}LH^{\mathcal{U}}, \end{array} \tag{1.9}$$

grouped according to the same scheme as in Eq. (1.8). Note that there are operators which conserve R_p but violate baryon- and lepton-parity unlike the case with renormalizable operators. We shall focus on those operators that can lead to rapid proton decay,

$$\frac{\mathbf{\Psi}}{M_s}QQQL,\ \frac{\mathbf{\Psi'}}{M_s}QQQH^{\mathcal{D}}; \tag{1.10}$$

M_s is the string scale ($\approx 10^{18}$GeV), $\mathbf{\Psi}, \mathbf{\Psi'}$ are arrays of dimensionless numbers. We do not consider the operators $\overline{U}\overline{U}\overline{D}\overline{E}$, because they contribute to proton decay only via the unknown quark-squark mixing matrix for the right-handed up-quarks, see for example Refs. [42, 43, 44].

This text is structured as follows. In Section 2, we review the idea of Froggatt and Nielsen within supersymmetry and consider some of its implications. In Section 3, we discuss the phenomenological constraints on the charges of the $U(1)_X$ symmetry. Then we parameterize and list the most general set of X-charges, consistent with anomaly cancellation and Eqs. (1.1-1.7). In Section 4, we explain the procedure with which we get from the string scale where the model is naturally formulated down to the weak scale, as well as which approximations we utilize. In Section 5 together with Appendix A, we present and discuss our results. In Section 6 we offer a short summary. In Appendix A the models of Refs. [1-16] are discussed and listed, as well as the X-charge assignments of the other models. In Appendix B we demonstrate the validity of the mass matrices we use in this paper.

2 The Scenario of Froggatt and Nielsen

In the models we consider, the superpotential and the Kählerpotential are expected to originate from a ten-dimensional heterotic superstring, see *e.g.* Ref. [45]: After compactification to four dimensions and below the string cut-off M_s such a theory may exhibit

priori they are arrays of arbitrary complex numbers (except $\Lambda_{ijk}/\Lambda''_{ijk}$ being antisymmetric with respect to the exchange of the first two/last two indices, Γ_{ij} and Λ'''_{ijk} being totally symmetric with respect to any exchange of indices).

at most[4] one anomalous local $U(1)_X$ (see *e.g.* Refs. [47, 48] and references therein). We assume this anomalous local $U(1)_X$ indeed exists and has generation-dependent charges. For example, X_{Q^i}, $i=1,2,3$, denote the $U(1)_X$ charges of the quark doublet superfields Q^i and correspondingly for the other superfields. In addition to $U(1)_X$, we assume an X-charged *SM*-singlet left-chiral superfield A, the so-called flavon superfield. For simplicity we choose the convention that $X_A=-1$.

Close to the string scale where $U(1)_X$ is unbroken, we have to replace the superpotential by a $U(1)_X$ gauge invariant extension, *e.g.* instead of $G^{(U)}{}_{ij}\, Q^i\, H^{\mathcal{U}}\, \overline{U^j}$ we have the non-renormalizable operators

$$g^{(U)}{}_{ij}\, Q^i\, H^{\mathcal{U}}\, \overline{U^j} \left(\frac{A}{M_s}\right)^{X_{Q^i}+X_{H^{\mathcal{U}}}+X_{\overline{U^j}}} \widetilde{\Theta}\Big[X_{Q^i}+X_{H^{\mathcal{U}}}+X_{\overline{U^j}}\Big]. \tag{2.11}$$

The complex couplings $g^{(U)}{}_{ij}$ are assumed to be of $\mathcal{O}(1)$, *i.e.*

$$\frac{1}{\sqrt{10}} \lesssim \big|g^{(U)}{}_{ij}\big| \lesssim \sqrt{10}\,. \tag{2.12}$$

Furthermore $\widetilde{\Theta}[x]$ is defined as

$$\widetilde{\Theta}[x] \equiv \begin{cases} 1 & \text{for } x \text{ being a non-negative integer}\,, \\ & \\ 0 & \text{else (supersymmetric zero)}\,. \end{cases} \tag{2.13}$$

$\widetilde{\Theta}[...]$ appears in Eq. (2.11) due to: *(i)* Holomorphicity of the superpotential: negative exponents of the superfield A are not allowed. The resulting texture is denoted a *supersymmetric zero* [49]. *(ii)* Fractional exponents are also forbidden, somewhat imprecisely the corresponding zeros shall be referred to as supersymmetric zeros as well. If an exponent of $\frac{A}{M_s}$ is fractional and/or negative it shall be referred to as a *naive exponent*.[5] The superpotential terms $\mu H^{\mathcal{D}} H^{\mathcal{U}}$ and $K_i L^i H^{\mathcal{U}}$ are treated in exactly the same manner:

$$\begin{aligned} &M^{(\mu)}\quad H^{\mathcal{D}}\; H^{\mathcal{U}}\; \left(\frac{A}{M_s}\right)^{X_{H^{\mathcal{D}}}+X_{H^{\mathcal{U}}}} \quad \widetilde{\Theta}\Big[X_{H^{\mathcal{D}}}+X_{H^{\mathcal{U}}}\Big], \\ &M^{(\mu)}\quad \kappa_i\quad L^i\; H^{\mathcal{U}}\; \left(\frac{A}{M_s}\right)^{X_{L^i}+X_{H^{\mathcal{U}}}} \quad \widetilde{\Theta}\Big[X_{L^i}+X_{H^{\mathcal{U}}}\Big]. \end{aligned} \tag{2.14}$$

$M^{(\mu)}$ is a dimensionful parameter, the κ_i are dimensionless and of $\mathcal{O}(1)$. Note that the same mass scale is assumed in both expressions above. Likewise for the superpotential terms $\Xi_i\, \overline{N^i}$ and $\Gamma_{ij}\, \overline{N^i}\; \overline{N^j}$. We again assume a common mass scale, in general different

[4] If there IS more than one anomalous $U(1)$, one can re-express the charges such that only one $U(1)$ is anomalous. For an explicit example see e.g. Ref. [46] and references therein.

[5] However, as stated in Ref. [51] one could "envisage fractional powers of the [super]field [A], stemming from non-perturbative effects".

from $M^{(\mu)}$:

$$\left(M^{(\overline{N})}\right)^2 \xi_i\, \overline{N^i} \left(\frac{A}{M_s}\right)^{X_{\overline{N^i}}} \widetilde{\Theta}\Big[X_{\overline{N^i}}\Big],$$
$$M^{(\overline{N})}\, \gamma_{ij}\, \overline{N^i}\, \overline{N^j} \left(\frac{A}{M_s}\right)^{X_{\overline{N^i}}+X_{\overline{N^j}}} \widetilde{\Theta}\Big[X_{\overline{N^i}} + X_{\overline{N^j}}\Big]. \tag{2.15}$$

$M^{(\overline{N})}$ is a dimensionful parameter presumably of $\mathcal{O}(M_{\mathrm{GUT}})$ or $\mathcal{O}(M_s)$, the ξ_i, γ_{ij} are dimensionless and of $\mathcal{O}(1)$. If one is not willing to introduce yet another mass scale $M^{(\overline{N})}$ in addition to $M_{3/2}$ and M_s (and possibly $M^{(\mu)}$), one can set $M^{(\overline{N})} = M_s$ and sufficiently suppress the righthanded neutrino masses by powers of ϵ. We will explicitly deal with this point in Ref. [50]. For a demonstration see the remarks about Ref. [10] in Appendix A, in particular Eq. (A.1).

Just like the superpotential, the Kählerpotential also has to be replaced, *e.g.* instead of $H^{(Q)}{}_{ij}\ \overline{Q^i}\, Q^j$ we have

$$h^{(Q)}{}_{ij}\, \overline{Q^i}\, Q^j \times \left\{ \left(\frac{A}{M_s}\right)^{X_{Q^j}-X_{Q^i}} \widetilde{\Theta}\Big[X_{Q^j} - X_{Q^i}\Big] + \left(\frac{\overline{A}}{M_s}\right)^{X_{Q^i}-X_{Q^j}} \widetilde{\Theta}\Big[X_{Q^i} - X_{Q^j}\Big] \right\}. \tag{2.16}$$

The complex couplings $h^{(Q)}{}_{ij}$ are of $\mathcal{O}(1)$ and form a positive-definite Hermitian matrix. For simplicity the gauge connection matrix is not written out as it is of no importance here. Note that these terms are not of the canonical form of the kinetic energy, which is important later on (*c.f.* Step 2 in Section 4).

In addition to the expression above, we also obtain Kählerpotential terms of the form

$$C_{\overline{N^i}}\, \overline{Z}\, \overline{N^i} \left\{ \left(\frac{\overline{A}}{M_s}\right)^{-X_{\overline{N^i}}} \widetilde{\Theta}\Big[-X_{\overline{N^i}}\Big] + \left(\frac{A}{M_s}\right)^{X_{\overline{N^i}}} \widetilde{\Theta}\Big[X_{\overline{N^i}}\Big] \right\},$$
$$C_{H^{\mathcal{D}}H^{\mathcal{U}}}\, \frac{\overline{Z}}{M_s} H^{\mathcal{D}}\, H^{\mathcal{U}} \left\{ \left(\frac{\overline{A}}{M_s}\right)^{-X_{H^{\mathcal{D}}}-X_{H^{\mathcal{U}}}} \widetilde{\Theta}\Big[-X_{H^{\mathcal{D}}} - X_{H^{\mathcal{U}}}\Big] + \ \ldots \right\}, \tag{2.17}$$
$$C_{L^iH^{\mathcal{U}}}\, \frac{\overline{Z}}{M_s} L^i\ H^{\mathcal{U}} \left\{ \left(\frac{\overline{A}}{M_s}\right)^{-X_{L^i}-X_{H^{\mathcal{U}}}} \widetilde{\Theta}\Big[-X_{L^i} - X_{H^{\mathcal{U}}}\Big] + \ \ldots \right\},$$
$$C_{Q^iH^{\mathcal{U}}\overline{U^j}}\, \frac{\overline{Z}}{M_s{}^2}\, Q^i\ H^{\mathcal{U}}\ \overline{U^j} \left\{ \left(\frac{\overline{A}}{M_s}\right)^{-X_{Q^i}-X_{H^{\mathcal{U}}}-X_{\overline{U^j}}} \widetilde{\Theta}\Big[-X_{Q^i} - X_{H^{\mathcal{U}}} - X_{\overline{U^j}}\Big] + \ \ldots \right\},$$

together with the corresponding complex conjugate terms. The $C_{\ldots}$ symbolize the dimensionless $\mathcal{O}(1)$ coupling constants. Z is an X-uncharged left-chiral superfield of the hidden sector, its F-term breaks supersymmetry.

Just as with the $U(1)_X$ itself, the *breaking* of the $U(1)_X$ is also embedded in string theory. This can be traced back to the necessity to get rid of the mixed chiral anomalies of $U(1)_X$ with the *SM* gauge group. They are canceled by the four-dimensional field theory remnant of the Green-Schwarz mechanism [23], relying on two assumptions:

1. The left-chiral scalar complex dilaton superfield S shifts under a $U(1)_X$ gauge transformation, namely

$$S \;\rightarrow\; S \;+\; \frac{i}{2}\,\delta_{\rm GS}\,\Lambda_X, \tag{2.18}$$

while the other superfields transform the usual way, *e.g.* $H^{\mathcal{U}} \rightarrow \exp(iX_{H^{\mathcal{U}}}\Lambda_X)H^{\mathcal{U}}$, $V_X \;\rightarrow\; V_X + \frac{i}{2}(\Lambda_X - \overline{\Lambda_X})$. Here, V_X is the $U(1)_X$ gauge vector superfield, Λ_X is the left-chiral superfield parameterizing the extended $U(1)_X$ gauge transformations and $\delta_{\rm GS}$ is a real number.

2. The coefficient of the gravity-gravity-$U(1)_X$ anomaly, $\mathcal{A}_{\rm GGX}$, as well as the coefficients for the mixed gauge anomalies $SU(3)_C$-$SU(3)_C$-$U(1)_X$, $SU(2)_W$-$SU(2)_W$-$U(1)_X$, $U(1)_Y$-$U(1)_Y$-$U(1)_X$ and $U(1)_X$-$U(1)_X$-$U(1)_X$ should fulfill, see *e.g.* Ref. [53],

$$\frac{\mathcal{A}_{\rm CCX}}{k_C} \overset{!}{=} \frac{\mathcal{A}_{\rm WWX}}{k_W} \overset{!}{=} \frac{\mathcal{A}_{\rm YYX}}{k_Y} \overset{!}{=} \frac{\mathcal{A}_{\rm XXX}}{3\;k_X} \overset{!}{=} \frac{\mathcal{A}_{\rm GGX}}{12} \overset{!}{=} 2\pi^2\;\delta_{\rm GS}\,. \tag{2.19}$$

The $k_{...}$ are the affine or Kač-Moody levels of the corresponding symmetry. The most easily constructed string models and thus so far most string models to date have[6]

$$k_C = k_W, \quad k_Y = 5/3\;k_W\,. \tag{2.20}$$

The anomaly coefficients are given in terms of the X-charges as

$$\mathcal{A}_{\rm CCX} = \sum_{i=1}^{3}\Big(2\;X_{Q^i} + X_{\overline{U^i}} + X_{\overline{D^i}}\Big), \tag{2.21}$$

$$\mathcal{A}_{\rm WWX} = X_{H^{\mathcal{U}}} + X_{H^{\mathcal{D}}} + \sum_{i=1}^{3}\Big(3\;X_{Q^i} + X_{L^i}\Big), \tag{2.22}$$

$$\mathcal{A}_{\rm YYX} = X_{H^{\mathcal{U}}} + X_{H^{\mathcal{D}}} + \frac{1}{3}\sum_{i=1}^{3}\Big(X_{Q^i} + 8\;X_{\overline{U^i}} + 2\;X_{\overline{D^i}} + 3\;X_{L^i} + 6X_{\overline{E^i}}\Big). \tag{2.23}$$

Due to the possible existence of X-charged *SM*-singlets other than A and $\overline{N^i}$, the anomaly coefficients $\mathcal{A}_{\rm XXX} = {\rm Tr}X^3$ and $\mathcal{A}_{\rm GGX} = {\rm Tr}X$ are not helpful to find constraints on the X-charges of the *SM*, so they are not listed.

These two assumptions together ensure that the shift of the dilaton eliminates the anomalies of the X-charged current, because the dilaton couples universally to the topological terms $\epsilon^{\mu\nu\alpha\beta}\;k_{...}\;F^{...}_{\mu\nu}\;F^{...}_{\alpha\beta}$ of the various gauge groups. For more details see *e.g.* Ref. [53, 54].

[6]The $k_{...}$ for non-Abelian groups have to be positive integers. For gauge theories arising from string theories $k_{...}\;{g_{...}}^2 = 2{g_s}^2$; furthermore heterotic string theory always — no matter whether one has a GUT or not — gives rise to unification of the coupling constants, see Ref. [52]. Hence at high energies $g_Y = \sqrt{3/5}\cdot g_W$, and thus $k_Y = 5/3\;k_W$.

To be entirely anomaly-free, the mixed anomaly $U(1)_Y$-$U(1)_X$-$U(1)_X$ must vanish on its own,[7]

$$\mathcal{A}_{\mathrm{YXX}} = X_{H^{\mathcal{U}}}{}^2 - X_{H^{\mathcal{D}}}{}^2 + \sum_{i=1}^{3}\left(X_{Q^i}{}^2 - 2\ X_{\overline{U^i}}{}^2 + X_{\overline{D^i}}{}^2 - X_{L^i}{}^2 + X_{\overline{E^i}}{}^2\right) \stackrel{!}{=} 0\,. \quad (2.24)$$

It is crucial that Assumption 1 makes it necessary to have a modified Kählerpotential for S, in order to have $U(1)_X$ gauge invariance: instead of being a function of $S+\overline{S}$ it has to be a function of

$$S\ +\ \overline{S}\ -\ \delta_{GS}\ V_X\,. \quad (2.25)$$

This "new" Kählerpotential inevitably leads to string radiative corrections generating a finite $U(1)_X$ Fayet-Iliopoulos term [55, 56, 57, 58], its coefficient reads

$$\xi^{FI}{}_X\ =\ \frac{g_s{}^2}{192\ \pi^2}\ M_s{}^2\ \mathcal{A}_{\mathrm{GGX}}\,. \quad (2.26)$$

$\xi^{FI}{}_X{}^{\text{tree level}}$ has to vanish in local supersymmetry, see Ref. [59]. Despite $\xi^{FI}{}_X \neq 0$ we demand that *(i)* $SU(3)_C{\times}SU(2)_W{\times}U(1)_Y$ and *(ii)* supersymmetry both remain unbroken at this scale. *(i)* is the case if the scalar components of all *SM*-superfields have vanishing VEVs. Taking this into account, *(ii)* is given if at least one of the X-charges of the remaining superfields whose scalar component gets a VEV has the opposite sign of $\xi^{FI}{}_X$. This one superfield is the flavon superfield A, its scalar component acquiring the VEV (thereby breaking $U(1)_X$) denoted by $\upsilon \propto \sqrt{\xi^{FI}{}_X}$, see also footnote 18. Thus the string radiative correction is essential for the breaking of $U(1)_X$, hence occurring near the string scale. This is why $\upsilon/M_s \propto \sqrt{\mathcal{A}_{\mathrm{GGX}}/192\pi^2}$ may not be a *very* small number.[8] On the other hand it must be ensured that υ/M_s is small enough so that we can identify

$$\frac{\upsilon}{M_s} \equiv \epsilon = 0.22\,. \quad (2.27)$$

Thus the X-charges should be moderately valued, otherwise $\mathcal{A}_{\mathrm{GGX}}$ would be so large (assuming that there is no fine-tuned cancellation among the X-charges) that it cannot be adequately suppressed by $192\pi^2$.[9]

After the breaking of $U(1)_X$ we get from Eqs. (1.8) and (2.11) that

$$G^{(U)}{}_{ij}\ =\ g^{(U)}{}_{ij}\ \ \epsilon^{X_{Q^i}+X_{H^{\mathcal{U}}}+X_{\overline{U^j}}}\ \ \widetilde{\Theta}\left[X_{Q^i}+X_{H^{\mathcal{U}}}+X_{\overline{U^j}}\right]. \quad (2.28)$$

Similarly, Eq. (2.16) leads to

$$H^{(Q)}{}_{ij}\ =\ h^{(Q)}{}_{ij}\ \ \epsilon^{|X_{Q^i}-X_{Q^j}|}\ \ \widetilde{\Theta}\left[|X_{Q^i}-X_{Q^j}|\right], \quad (2.29)$$

[7]Note that if the *SM* gauge group derives from an $SU(5)$-GUT, *i.e.* $X_{Q^i}=X_{\overline{U^i}}=X_{\overline{E^i}}$, $X_{L^i}=X_{\overline{D^i}}$, and furthermore $X_{H^{\mathcal{D}}}=-X_{H^{\mathcal{U}}}$, one automatically has $\mathcal{A}_{\mathrm{CCX}}=\mathcal{A}_{\mathrm{WWX}}=\frac{3}{5}\mathcal{A}_{\mathrm{YYX}}$ and $\mathcal{A}_{\mathrm{YXX}}=0$.

[8]As can be seen from Eq. (2.26), if $U(1)_X$ is non-anomalous (thus $\mathcal{A}_{\mathrm{GGX}}=0$) the string radiative correction vanishes; furthermore if in this case one chooses $\xi^{FI}{}_X{}^{\text{tree level}}$ appropriately one can break $U(1)_X$ at a much lower scale; such a model is sketched in Ref. [60].

[9]As was shown in Ref. [61] (see also Ref. [62]), with Eqs. (2.19, 3.58) and using $k_{\ldots}\ g_{\ldots}{}^2 = g_s{}^2$, ϵ is indeed of the correct order of magnitude. See also Ref. [50].

and likewise for the other coupling constants. Depending on the $U(1)_X$ charges, *e.g.* the sum $X_{Q^i} + X_{H^{\mathcal{U}}} + X_{\overline{U^j}}$, we get generation-dependent exponents of ϵ and thus generation-dependent suppressions of the $G^{(U)}{}_{ij}$. We thus have generation-dependent hierarchical coupling constants. It should be emphasized that the idea of Froggatt and Nielsen does not reduce the number of parameters, but only explains their hierarchy. As ϵ is rather large, instead of the constraint in Eq. (2.12) we shall restrict ourselves to

$$\sqrt{\epsilon} \quad \lesssim \quad \left|g^{(U)}{}_{ij}\right| \quad \lesssim \quad \frac{1}{\sqrt{\epsilon}} \,. \tag{2.30}$$

Due to the Giudice-Masiero mechanism, Kählerpotential terms as in Eq. (2.17) also contribute to the effective superpotential,[10] giving

$$\begin{array}{lllll}
M_s\ M_{3/2} & C_{\overline{N^i}} & \epsilon^{|X_{\overline{N^i}}|} & \widetilde{\Theta}\Big[|X_{\overline{N^i}}|\Big] & \overline{N^i}\,, \\
M_{3/2} & C_{H^{\mathcal{D}}H^{\mathcal{U}}} & \epsilon^{|X_{H^{\mathcal{D}}}+X_{H^{\mathcal{U}}}|} & \widetilde{\Theta}\Big[|X_{H^{\mathcal{D}}}+X_{H^{\mathcal{U}}}|\Big] & H^{\mathcal{D}}\ H^{\mathcal{U}}\,, \\
M_{3/2} & C_{L^iH^{\mathcal{U}}} & \epsilon^{|X_{L^i}+X_{H^{\mathcal{U}}}|} & \widetilde{\Theta}\Big[|X_{L^i}+X_{H^{\mathcal{U}}}|\Big] & L^i\ H^{\mathcal{U}}\,, \\
M_{3/2} & C_{\overline{N^i}\,\overline{N^j}} & \epsilon^{|X_{\overline{N^i}}+X_{\overline{N^j}}|} & \widetilde{\Theta}\Big[|X_{\overline{N^i}}+X_{\overline{N^j}}|\Big] & \overline{N^i}\ \overline{N^j}\,, \\
\dfrac{M_{3/2}}{M_s} & C_{Q^iH^{\mathcal{U}}\overline{U^j}} & \epsilon^{|X_{Q^i}+X_{H^{\mathcal{U}}}+X_{\overline{U^j}}|} & \widetilde{\Theta}\Big[|X_{Q^i}+X_{H^{\mathcal{U}}}+X_{\overline{U^j}}|\Big] & Q^i\ H^{\mathcal{U}}\ \overline{U^j}\,.
\end{array} \tag{2.31}$$

It is safe to ignore the contributions to the trilinear superpotential terms, as they are strongly suppressed by at least a factor of $\mathcal{O}(10^{-15})$,[11] so that coupling constants as in Eq. (2.28) remain unchanged. This is not the case for the linear and bilinear terms: using the expressions above and Eqs. (2.14, 2.15) we obtain

$$\Xi_i \;=\; \left(M^{(\overline{N})}\right)^2 \xi_i\, \epsilon^{X_{\overline{N^i}}}\, \widetilde{\Theta}\Big[X_{\overline{N^i}}\Big] + M_{3/2}\, M_s\, C_{\overline{N^i}}\, \epsilon^{|X_{\overline{N^i}}|}\, \widetilde{\Theta}\Big[|X_{\overline{N^i}}|\Big]\,, \tag{2.32}$$

$$\begin{aligned}
\Gamma_{ij} \;=\; & M^{(\overline{N})}\ \gamma_{ij}\ \epsilon^{X_{\overline{N^i}}+X_{\overline{N^j}}}\ \widetilde{\Theta}\Big[X_{\overline{N^i}}+X_{\overline{N^j}}\Big] \\
& +\ M_{3/2}\ C_{\overline{N^i}\,\overline{N^i}}\ \epsilon^{|X_{\overline{N^i}}+X_{\overline{N^j}}|}\ \widetilde{\Theta}\Big[|X_{\overline{N^i}}+X_{\overline{N^j}}|\Big],
\end{aligned} \tag{2.33}$$

$$\begin{aligned}
\mu \;=\; & M^{(\mu)}\ \epsilon^{X_{H^{\mathcal{D}}}+X_{H^{\mathcal{U}}}}\ \widetilde{\Theta}\Big[X_{H^{\mathcal{D}}}+X_{H^{\mathcal{U}}}\Big] \\
& +\ M_{3/2}\ C_{H^{\mathcal{D}}H^{\mathcal{U}}}\ \epsilon^{|X_{H^{\mathcal{D}}}+X_{H^{\mathcal{U}}}|}\ \widetilde{\Theta}\Big[|X_{H^{\mathcal{D}}}+X_{H^{\mathcal{U}}}|\Big],
\end{aligned} \tag{2.34}$$

$$\begin{aligned}
K_i \;=\; & M^{(\mu)}\, \epsilon^{X_{L^i}+X_{H^{\mathcal{U}}}}\, \widetilde{\Theta}\Big[X_{L^i}+X_{H^{\mathcal{U}}}\Big] \\
& + M_{3/2}\, C_{L^iH^{\mathcal{U}}}\, \epsilon^{|X_{L^i}+X_{H^{\mathcal{U}}}|}\, \widetilde{\Theta}\Big[|X_{L^i}+X_{H^{\mathcal{U}}}|\Big].
\end{aligned} \tag{2.35}$$

[10] Supersymmetry is broken by the VEV of F_Z, where F_Z is the auxiliary field of Z. Thus one gets $\langle F_Z \rangle\ \delta^2(\bar\theta) = M_{3/2}\ M_s\ \delta^2(\bar\theta)$. The δ-function turns the original Kählerpotential term into a superpotential term.

[11] See however Refs. [63, 64]: They use the Giudice-Masiero mechanism to get small neutrino masses.

To summarize this section, the supergravity-embedded models which we consider are determined by the superpotential given in Eqs. (1.8) and (1.10) together with the Kählerpotential. The two potentials are generated close to the string scale by the scalar component of the flavon superfield acquiring a VEV. After $U(1)_X$ symmetry breaking, non-renormalizable operators generate renormalizable operators which are suppressed by powers of ϵ times an unknown $\mathcal{O}(1)$ coupling constant. The power of ϵ is fixed by the generation-dependent quantum numbers of the superfields under the $U(1)_X$ family symmetry. One also has to take into account that via the Giudice-Masiero mechanism [41] terms of the Kählerpotential can contribute to the superpotential after $U(1)_X$ symmetry breaking. Numerically the effects are only relevant for the linear and bilinear terms.

3 Phenomenological Aspects

We would next like to derive a conclusion from the previous section (see also Ref. [65]). Consider Eq. (2.19), it leads to

$$\frac{1}{2}\left(\mathcal{A}_{\mathrm{YYX}}+\mathcal{A}_{\mathrm{WWX}}-\frac{8}{3}\,\mathcal{A}_{\mathrm{CCX}}\right) \;=\; \frac{\mathcal{A}_{\mathrm{GGX}}}{24}\left(k_Y+k_W-\frac{8}{3}\,k_C\right). \tag{3.36}$$

Due to Eq. (2.20) the RHS equals zero, and with Eqs. (2.21), (2.22) and (2.23) we obtain the following condition on the X-charges

$$X_{H^{\mathcal{U}}}+X_{H^{\mathcal{D}}}+\sum_{i=1}^{3}\left(X_{L^i}+X_{\overline{E^i}}-X_{Q^i}-X_{\overline{D^i}}\right)=0\,. \tag{3.37}$$

Now the low energy mass matrix of the d-quarks, $\boldsymbol{M^{(D)}}$, is $\langle H^{\mathcal{D}}\rangle\,\boldsymbol{Y^{(D)}}$, and the low energy Yukawa coupling matrix $\boldsymbol{Y^{(D)}}$ is the renormalization group evolved $\boldsymbol{G^{(D)}}$. Disobeying virtually all of the steps which are outlined in the next section (to get from the string scale to the weak scale) we approximate here

$$\det\boldsymbol{Y^{(D)}}=\det\widetilde{\boldsymbol{g}}^{\boldsymbol{(D)}}\;\cdot\;\epsilon^{3X_{H^{\mathcal{D}}}+\sum_i(X_{Q^i}+X_{\overline{D^i}})}\,, \tag{3.38}$$

with $\widetilde{g}^{(D)}{}_{ij}\equiv g^{(D)}{}_{ij}\;\widetilde{\Theta}[X_{Q^i}+X_{H^{\mathcal{D}}}+X_{\overline{D^j}}]$.[12] The same holds for the mass matrices of the charged leptons. So from Eq. (3.37) we obtain

$$\epsilon^{X_{H^{\mathcal{U}}}+X_{H^{\mathcal{D}}}}\;=\;\left[\frac{\det\boldsymbol{M^{(D)}}}{\det\boldsymbol{M^{(E)}}}\cdot\frac{\det\widetilde{\boldsymbol{g}}^{\boldsymbol{(E)}}}{\det\widetilde{\boldsymbol{g}}^{\boldsymbol{(D)}}}\right]. \tag{3.39}$$

Assuming $\det\widetilde{\boldsymbol{g}}^{\boldsymbol{(D)}}\approx\det\widetilde{\boldsymbol{g}}^{\boldsymbol{(E)}}$ and using

$$m_d\cdot m_s\cdot m_b\;=\;\det\boldsymbol{M^{(D)}},\quad m_e\cdot m_\mu\cdot m_\tau\;=\;\det\boldsymbol{M^{(E)}}\,, \tag{3.40}$$

[12] Thus in order to have $m_d\neq 0$ one needs $\det\widetilde{\boldsymbol{g}}^{\boldsymbol{(D)}}\neq 0$. Using *e.g.* the simplest Ansatz for the $\widetilde{g}^{(D)}{}_{ij}$, namely all of them being equal to unity, is not an option. In Ref. [60] det $\widetilde{\boldsymbol{g}}^{\boldsymbol{(U)}}=0$ was used to get a massless u-quark, see also footnote 8.

Eq. (3.39) can be rewritten as

$$\epsilon^{X_{H^{\mathcal{U}}}+X_{H^{\mathcal{D}}}} \;=\; \frac{m_d \cdot m_s \cdot m_b}{m_e \cdot m_\mu \cdot m_\tau}\;. \qquad (3.41)$$

With Eqs. (1.1), (1.2), and (1.3) we get

$$X_{H^{\mathcal{U}}} + X_{H^{\mathcal{D}}} \;=\; 0 \quad \text{or} \; -1\;, \qquad (3.42)$$

a result that is reproduced later on in a somewhat different way, see Table 1. Looking at Eq. (2.34) we find that for $X_{H^{\mathcal{U}}} + X_{H^{\mathcal{D}}} = 0$ we have to have $|M^{(\mu)}| \ll M_s$, thus the hierarchy problem remains. On the other hand if $X_{H^{\mathcal{U}}} + X_{H^{\mathcal{D}}} = -1$ one has that $\mu = \epsilon \cdot M_{3/2}$. Hence $M^{(\mu)}$ (which is also the mass scale for the K_i) is then naturally chosen to be M_s. As explained in more detail below, the K_i can be rotated away which however changes the μ (*c.f.* Step 3 of the next section). So if $M^{(\mu)} \sim M_s$, one thus needs $X_{L^i} + X_{H^{\mathcal{U}}}$ to be either ≥ 24 (so that $M^{(\mu)} \sim M_s$ is adequately suppressed) or < 0 and/or fractional so that the entry is forbidden. In that way the K_i do not generate an effective μ which is too large, see Eq. (2.35).

A similar kind of calculation was presented in Refs. [51, 25]: The conditions for Green-Schwarz anomaly cancellation, Eq. (2.19), give

$$\frac{1}{5}\,\mathcal{A}_{\mathrm{YYX}} \;=\; \frac{1}{2}\,\mathcal{A}_{\mathrm{YYX}} \;+\; \frac{1}{2}\,\mathcal{A}_{\mathrm{WWX}} \;-\; \mathcal{A}_{\mathrm{CCX}}\,. \qquad (3.43)$$

Expressing the RHS in terms of X-charges gives

$$\begin{aligned}\frac{1}{5}\,\mathcal{A}_{\mathrm{YYX}} \;&=\; X_{H^{\mathcal{D}}} + X_{H^{\mathcal{U}}} + \sum_i \big(X_{L^i} + X_{\overline{E^i}} - \frac{1}{3}X_{Q^i} + \frac{1}{3}X_{\overline{U^i}} - \frac{2}{3}X_{\overline{D^i}}\big)\\ &=\; 3X_{H^{\mathcal{D}}} + \sum_i \big(X_{L^i} + X_{\overline{E^i}}\big) \;+\; \frac{1}{3}\Big(\,3\,X_{H^{\mathcal{U}}} + \sum_i \big(X_{Q^i} + X_{\overline{U^i}}\big)\,\Big)\\ &\qquad -\;\frac{2}{3}\Big(\,3\,X_{H^{\mathcal{D}}} + \sum_i \big(X_{Q^i} + X_{\overline{D^i}}\,\big)\,\Big). \end{aligned} \qquad (3.44)$$

Similarly to the previous calculation, we get

$$\epsilon^{\frac{\mathcal{A}_{\mathrm{YYX}}}{5}} \;=\; \frac{\dfrac{m_e\cdot m_\mu\cdot m_\tau}{\langle H^{\mathcal{D}}\rangle^3}\left(\dfrac{m_u\cdot m_c\cdot m_t}{\langle H^{\mathcal{U}}\rangle^3}\right)^{\frac{1}{3}}}{\left(\dfrac{m_d\cdot m_s\cdot m_b}{\langle H^{\mathcal{D}}\rangle^3}\right)^{\frac{2}{3}}}\,. \qquad (3.45)$$

Using Eqs. (1.1-1.5) and $\cot\beta = \langle H^{\mathcal{D}}\rangle/\langle H^{\mathcal{U}}\rangle$ this reads

$$\epsilon^{\frac{\mathcal{A}_{\mathrm{YYX}}}{5}} \;=\; \frac{{m_t}^2\;\epsilon^{6+(0,1,2\;or\;3)+(0\;or\;1)}}{\langle H^{\mathcal{U}}\rangle^2}\,. \qquad (3.46)$$

With $\;\langle H^{\mathcal{U}}\rangle^2 + \langle H^{\mathcal{D}}\rangle^2 \;=\; (246\ \mathrm{GeV})^2\;$ one arrives at

$$\epsilon^{\frac{\mathcal{A}_{\mathrm{YYX}}}{5}} \;=\; \epsilon^{6+(0,1,2\;or\;3)+(0\;or\;1)}\;\frac{1+\tan^2\beta}{\tan^2\beta}\left(\frac{m_t}{246\ \mathrm{GeV}}\right)^2\,. \qquad (3.47)$$

Experimentally, the parameters are restricted by [66, 67]

$$2.4 \leq \tan\beta \leq 50 \quad \text{and} \quad m_t = (174.3 \pm 5.1)\ \text{GeV}, \tag{3.48}$$

and therefore $\mathcal{A}_{\text{YYX}}/5 \sim 6 + (0, 1, 2 \textit{ or } 3) + (0 \textit{ or } 1)$. It follows that $\mathcal{A}_{\text{YYX}} = 0$ is impossible, and hence (*c.f.* Eq. (2.19)) $\mathcal{A}_{\text{WWX}} = \mathcal{A}_{\text{CCX}} = 0$ are also not possible. So $U(1)_X$ has to be anomalous, and we need the Green-Schwarz mechanism, which fixes the breaking of $U(1)_X$ to be close to the string scale.

Further phenomenological constraints on the X-charges arise more systematically from requiring the quark mass matrices to reproduce Eqs. (1.3), (1.4), (1.5), (1.6) and (1.7) at high energies. There are four possible pairs of $\boldsymbol{G^{(U)}}$ and $\boldsymbol{G^{(D)}}$ (in Appendix B the validity of these mass matrices is illustrated):

$$\boldsymbol{G^{(U)}} \propto \begin{pmatrix} \epsilon^8 & \epsilon^5 & \epsilon^3 \\ \epsilon^7 & \epsilon^4 & \epsilon^2 \\ \epsilon^5 & \epsilon^2 & 1 \end{pmatrix}, \qquad \boldsymbol{G^{(D)}} \propto \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^3 \\ \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon & 1 & 1 \end{pmatrix}, \tag{3.49}$$

$$\boldsymbol{G^{(U)}} \propto \begin{pmatrix} \epsilon^8 & \epsilon^5 & \epsilon^3 \\ \epsilon^{13} & \epsilon^4 & \epsilon^2 \\ \epsilon^{11} & \epsilon^2 & 1 \end{pmatrix}, \qquad \boldsymbol{G^{(D)}} \propto \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^3 \\ \epsilon^9 & \epsilon^2 & \epsilon^2 \\ \epsilon^7 & 1 & 1 \end{pmatrix}, \tag{3.50}$$

$$\boldsymbol{G^{(U)}} \propto \begin{pmatrix} \epsilon^8 & \epsilon^6 & \epsilon^4 \\ \epsilon^6 & \epsilon^4 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & 1 \end{pmatrix}, \qquad \boldsymbol{G^{(D)}} \propto \begin{pmatrix} \epsilon^4 & \epsilon^4 & \epsilon^4 \\ \epsilon^2 & \epsilon^2 & \epsilon^2 \\ 1 & 1 & 1 \end{pmatrix}, \tag{3.51}$$

$$\boldsymbol{G^{(U)}} \propto \begin{pmatrix} \epsilon^8 & \epsilon^6 & \epsilon^4 \\ \epsilon^{14} & \epsilon^4 & \epsilon^2 \\ \epsilon^{12} & \epsilon^2 & 1 \end{pmatrix}, \qquad \boldsymbol{G^{(D)}} \propto \begin{pmatrix} \epsilon^4 & \epsilon^4 & \epsilon^4 \\ \epsilon^{10} & \epsilon^2 & \epsilon^2 \\ \epsilon^8 & 1 & 1 \end{pmatrix}. \tag{3.52}$$

The matrices in Eqs. (3.49, 3.50) were suggested in Refs. [1, 2] and more rigorously derived in Ref. [3].[13] They are in accord with the *lefthand* choice of Eq. (1.7). Ref. [68] pointed out that the *righthand* choice of Eq. (1.7) can be achieved from the mass matrices in Eq. (3.51).[14] For completeness' sake, in analogy to Eq. (3.50) we found here that the matrices in Eq. (3.52) are also in accord with the righthand choice of Eq. (1.7).[15]

It is convenient to parameterize the X-charges in terms of the entries of the matrices

[13]Utilizing Eq. (4.11), the matrices in Eq. (3.50) can be generated from naive exponents $X_{Q^i} + X_{H^{\mathcal{U}}} + X_{\overline{U^j}}$ and $X_{Q^i} + X_{H^{\mathcal{D}}} + X_{\overline{D^j}}$ given as follows, respectively:

$$\begin{pmatrix} 8 & -1 & -3 \\ 13 & 4 & 2 \\ 11 & 2 & 0 \end{pmatrix}_{ij}, \quad \begin{pmatrix} 4 & -3 & -3 \\ 9 & 2 & 2 \\ 7 & 0 & 0 \end{pmatrix}_{ij}.$$

The negative exponents give textures which are filled up in the process of canonicalization of the Kählerpotential, see Step 2 of the next section.

[14]Note that $\boldsymbol{G^{(U)}}$ in Eq. (3.51) is the only matrix that is compatible with an $SU(5)$-GUT: $\boldsymbol{G^{(U)}}$ is symmetric, being in accord with the $SU(5)$-requirement $X_{\overline{U^i}} = X_{Q^i}$. To have furthermore $X_{L^i} = X_{\overline{D^i}}$, $X_{\overline{E^i}} = X_{Q^i}$ one needs $X_{L^1} = X_{L^2} = X_{L^3}$, $y = 1$, $z = 0$, in the notation below.

[15]Utilizing Eq. (4.11), the matrices in Eq. (3.52) can be generated from naive exponents $X_{Q^i} + X_{H^{\mathcal{U}}} +$

in Eqs. (3.49-3.52):

$$X_{Q^i} + X_{H^{\mathcal{U}}} + X_{\overline{U^j}} = \begin{pmatrix} 8 & 5+y & 3+y \\ 7-y & 4 & 2 \\ 5-y & 2 & 0 \end{pmatrix}_{ij} , \tag{3.53}$$

$$X_{Q^i} + X_{H^{\mathcal{D}}} + X_{\overline{D^j}} = \begin{pmatrix} 4+x & 3+y+x & 3+y+x \\ 3-y+x & 2+x & 2+x \\ 1-y+x & x & x \end{pmatrix}_{ij} , \tag{3.54}$$

with

$$x = 0, 1, 2, 3 \quad \text{if} \quad y = -7, 0, 1 \quad \text{and} \quad x = 0, 1, 2 \quad \text{if} \quad y = -6\,. \tag{3.55}$$

A negative y causes some naive exponents in $\boldsymbol{G^{(U)}}, \boldsymbol{G^{(D)}}$ to be negative as well (see footnotes 13 and 15) and is thus forbidden; these textures are then filled up in the process of canonicalizing the Kählerpotential. In addition to Eqs. (3.53, 3.54), Eq. (1.1) motivates the parameterization

$$X_{L^i} + X_{H^{\mathcal{D}}} + X_{\overline{E^i}} = \begin{pmatrix} 4+z+x \\ 2+x \\ x \end{pmatrix}_{i} , \tag{3.56}$$

with $z = 0, 1$. Finally, Eq. (1.2) leads to

$$X_{L^3} + X_{\overline{E^3}} = X_{Q^3} + X_{\overline{D^3}}\,. \tag{3.57}$$

Solving Eqs. (3.53-3.57) and the conditions for Green-Schwarz anomaly cancellation, *i.e.* Eqs. (2.19, 2.21-2.24) for the X-charges leads to the expressions given in Table 1; the free parameters chosen here are $x, y, z, X_{L^1}, X_{L^2}, X_{L^3}$.[16,17] In the Table in Appendix A we

---

$X_{\overline{U^j}}$ and $X_{Q^i} + X_{H^{\mathcal{D}}} + X_{\overline{D^j}}$ given as follows, respectively:

$$\begin{pmatrix} 8 & -2 & -4 \\ 14 & 4 & 2 \\ 12 & 2 & 0 \end{pmatrix}_{ij} , \quad \begin{pmatrix} 4 & -4 & -4 \\ 10 & 2 & 2 \\ 8 & 0 & 0 \end{pmatrix}_{ij} .$$

As in footnote 13, the negative exponents give textures which are filled up in the process of canonicalization of the Kählerpotential, see Step 2 of the next section.

[16] The reasons we have chosen to work with $X_{L^1}, X_{L^2}, X_{L^3}$ are as follows: *(i)* The quark sector is fairly well known, so Froggatt-Nielsen model-building does not need an X-charge of a quark to be an adjustable parameters. *(ii)* If $\boldsymbol{G^{(E)}}, \boldsymbol{G^{(N)}}$ and $\boldsymbol{\Gamma}$ are without supersymmetric zeroes, one finds that the Maki-Nagakawa-Sakata matrix [69] is approximately given by

$$U^{MNS}{}_{ij} \sim \epsilon^{|X_{L^i} - X_{L^j}|}, \quad \text{just like} \quad U^{CKM}{}_{ij} \sim \epsilon^{|X_{Q^i} - X_{Q^j}|};$$

furthermore (see Ref. [2, 13])

$$\boldsymbol{G^{(N)}} \cdot \boldsymbol{\Gamma}^{-1} \cdot \boldsymbol{G^{(N)}}^T \sim \frac{\epsilon^{X_{L^i} + X_{L^j} + 2X_{H^{\mathcal{U}}}}}{M^{(\overline{N})}}\,.$$

[17] If one believes that the actual expansion parameter is ϵ^2 rather than ϵ, one has to work with $x = 0, 2$, $y = 1, -7$, $z = 0$.

$$
\begin{array}{rcl}
X_{H^{\mathcal{D}}} & = & -\,\frac{1}{54+9x+6z}\big[18+X_{L^1}(18+2x+5z)+X_{L^2}(12+2x+2z) \\
 & & \qquad +X_{L^3}(6+2x+2z)-x(36+6x+5z)-18y-2z^2\big] \\
X_{H^{\mathcal{U}}} & = & -X_{H^{\mathcal{D}}}-z \\
X_{Q^1} & = & \frac{1}{9}\big[30-(X_{L^1}+X_{L^2}+X_{L^3})+3x+6y+4z\big] \\
X_{Q^2} & = & X_{Q^1}-1-y \\
X_{Q^3} & = & X_{Q^1}-3-y \\
X_{\overline{U^1}} & = & X_{H^{\mathcal{D}}}-X_{Q^1}+8+z \\
X_{\overline{U^2}} & = & X_{\overline{U^1}}-3+y \\
X_{\overline{U^3}} & = & X_{\overline{U^1}}-5+y \\
X_{\overline{D^1}} & = & -X_{H^{\mathcal{D}}}-X_{Q^1}+4+x \\
X_{\overline{D^2}} & = & X_{\overline{D^1}}-1+y \\
X_{\overline{D^3}} & = & X_{\overline{D^1}}-1+y \\
X_{\overline{E^1}} & = & -X_{H^{\mathcal{D}}}+4-X_{L^1}+x+z \\
X_{\overline{E^2}} & = & -X_{H^{\mathcal{D}}}+2-X_{L^2}+x \\
X_{\overline{E^3}} & = & -X_{H^{\mathcal{D}}}\qquad -X_{L^3}+x
\end{array}
$$

Table 1: The constrained X-charges.

have translated all the models given in Refs. [1-16] into the (standard) notation above. This should allow for an easier comparison. Note that one has

$$\mathcal{A}_{\mathrm{CCX}} = 3\,(6 + x + z)\;, \tag{3.58}$$

as was actually already apparent in Eq. (3.46), and

$$X_{L^i} + X_{H^{\mathcal{D}}} + X_{\overline{E^j}} \;=\; \left(\begin{array}{ccc} 4+x+z & 2+x & x \\ 4+x+z & 2+x & x \\ 4+x+z & 2+x & x \end{array}\right)_{ij} \;+\; X_{L^i} - X_{L^j}\,. \tag{3.59}$$

As an example, consider the first model of Ref. [2]. The charges are determined by $x=2,\ y=0,\ z=0,\ X_{\overline{L^1}}=-12,\ X_{\overline{L^2}}=-13,\ X_{\overline{L^3}}=55$. Thus one has, using Table 1

$$\boxed{X_{H^{\mathcal{D}}} = 0, \quad X_{H^{\mathcal{U}}} = 0}$$

and the quark and lepton charges

Generation $\boldsymbol{i}$	$\boldsymbol{X_{Q^i}}$	$\boldsymbol{X_{\overline{D^i}}}$	$\boldsymbol{X_{\overline{U^i}}}$	$\boldsymbol{X_{L^i}}$	$\boldsymbol{X_{\overline{E^i}}}$
1	$\frac{2}{3}$	$\frac{16}{3}$	$\frac{22}{3}$	-12	18
2	$-\frac{1}{3}$	$\frac{13}{3}$	$\frac{13}{3}$	-13	17
3	$-\frac{7}{3}$	$\frac{13}{3}$	$\frac{7}{3}$	55	-53

.

This results in the quark mass matrices which are given in Eq. (3.49), and (taking into account supersymmetric zeros) the leptonic mass matrix

$$\boldsymbol{G^{(E)}} \sim \left(\begin{array}{ccc} \epsilon^6 & \epsilon^5 & 0 \\ \epsilon^5 & \epsilon^4 & 0 \\ \epsilon^{73} & \epsilon^{72} & \epsilon^2 \end{array}\right)\,. \tag{3.60}$$

4 The Procedure

Taking a top-down approach we start off at or close to the string scale with a family symmetry, *i.e.* with a certain X-charge assignment for the left-chiral superfields. We determine the exponents of ϵ of all allowed interactions, including the baryon- and lepton-number violating interactions. We thus also determine the supersymmetric zeroes. We then translate this into a low-energy model in order to compare with the measured fermion masses, their mixings and especially the experimental bounds on the barity- and lepton-parity violating coupling constants. To do so, we perform the following steps:

1. In models that contain right-handed neutrinos, the corresponding scalar superpartners can acquire VEVs which are determined by the minima of the scalar potential. The scalar potential is non-negative (apart from supergravity effects) and supersymmetry has not been broken yet (since we are just below the $U(1)_X$ breaking scale, thus quite close to the string scale), so the absolute minima of the scalar potential are given by the roots of the scalar potential. These roots in turn are given by the roots of the superpotential.[18] Finding the roots of $\mathcal{W}$ is equivalent to finding the linear constant shifts

$$\overline{N^i}(x,\theta) \longrightarrow \overline{N^i}(x,\theta) + \Delta^i, \tag{4.1}$$

such that the tree-level tadpole term $\Xi_i \; \overline{N^i}$ in the superpotential vanishes[19] [note that the Giudice-Masiero mechanism might produce, $\Xi_i \sim M_{3/2} M_s$, *c.f.* Eq. (2.32)]. This shift results in constant and thus harmless terms (ignoring the problem of the cosmological constant) in the Kähler- and superpotential, as well as the redefinitions of the superpotential coupling constants:

$$\begin{aligned}
K_i &\longrightarrow K_i \;+\; G^{(N)}{}_{ij}\;\Delta^j, \\
\mu &\longrightarrow \mu \;+\; \Upsilon_i\;\Delta^i, \\
\Xi_i &\longrightarrow \Xi_i \;+\; 2\,\Gamma_{ij}\;\Delta^j \;+\; 3\,\Lambda'''{}_{ijk}\;\Delta^j\;\Delta^k \;\stackrel{!}{=}\; 0, \\
\Gamma_{ij} &\longrightarrow \Gamma_{ij} \;+\; 3\,\Lambda'''{}_{ijk}\;\Delta^k.
\end{aligned} \tag{4.2}$$

It is important to notice that the equations above lead to a mixing among the coupling constants.

2. The breaking of $U(1)_X$ generates a Kählerpotential which does not have the canonical form [49]. Thus we must perform a transformation of the relevant superfields to the canonical basis [71]. For example, for the quark doublets we obtain for the relevant Kählerpotential term (where the $H^{(Q)}{}_{ij}$ are Kählerpotential coupling constants which are to be canonicalized)

$$\overline{Q^i}\;H^{(Q)}{}_{ij}\;Q^j \;\;=\;\; \overline{\left[\sqrt{\boldsymbol{D_{H^{(Q)}}}}\;\boldsymbol{U_{H^{(Q)}}}\;\boldsymbol{Q}\right]^i}\;\delta_{ij}\;\left[\sqrt{\boldsymbol{D_{H^{(Q)}}}}\;\boldsymbol{U_{H^{(Q)}}}\;\boldsymbol{Q}\right]^j. \tag{4.3}$$

$\boldsymbol{D_{H^{(Q)}}}$ is a diagonal matrix, its entries are the eigenvalues of the Hermitian matrix $\boldsymbol{H^{(Q)}}$; the unitary matrix $\boldsymbol{U_{H^{(Q)}}}$ performs the diagonalization. We define the matrix

$$\boldsymbol{C^{(Q)}} \equiv \sqrt{\boldsymbol{D_{H^{(Q)}}}}\;\boldsymbol{U_{H^{(Q)}}}\;. \tag{4.4}$$

[18] Of course this consideration neglects the part of the scalar potential which arises from the D-terms. However, this is justified by our starting point: We are working with *one* flavon field, *i.e.* $U(1)_X$ gets broken predominantly by the scalar component of A acquiring a VEV, rather than the scalar components of the $\overline{N^i}$ acquiring VEVs. This is equivalent to $\upsilon \gg \Delta^i$, which is justified with hindsight by the Δ^i being functions of Ξ_i, Γ_{ij}.

[19] Ξ_i is not regenerated by the RG-flow after having been shifted away, see the expression in Ref. [70] for the beta-function of a general linear term of a superpotential.

The transformation to the canonical basis can then be written as

$$Q^i \to {C^{(Q)}}^i{}_j \; Q^j. \tag{4.5}$$

The redefinition also affects the superpotential, *e.g.* the u-type quark Yukawa couplings

$$\boldsymbol{G^{(U)}} \longrightarrow \frac{1}{\sqrt{H^{(H^{\mathcal{U}})}}} \; \boldsymbol{C^{(Q)}}^{-1^T} \; \boldsymbol{G^{(U)}} \; \boldsymbol{C^{(\overline{U})}}^{-1}. \tag{4.6}$$

In this way the canonicalization of the Kählerpotential "fills up" possible supersymmetric zeroes (which are due to holomorphy and fractional X-charges [49]),[20] in this particular case in the mass matrix $\boldsymbol{G^{(U)}}$.

Since L^i, $H^{\mathcal{D}}$ have the same gauge quantum numbers under $SU(3)_C \times SU(2)_W \times U(1)_Y$, they can be mixed by kinetic terms. So the canonicalization of the Kählerpotential causes mixing of ${G^{(E)}}_{ij}$ with Λ_{ikj}, ${G^{(D)}}_{ij}$ with ${\Lambda'}_{ikj}$ (an example is given at the end of this section), μ with K_i, ${G^{(N)}}_{ij}$ with Υ_i, and Ψ_{ijkl} with ${\Psi'}_{ijk}$.

To maintain the canonicalized form, all further field redefinitions have to be unitary.[21] To demonstrate an exact canonicalization, consider the quark doublets with the $U(1)_X$ flavour charges $X_{Q^1} = 4,\ X_{Q^2} = 2,\ X_{Q^3} = 0$ and setting ${h^{(Q)}}_{ij} = 1$ (*c.f.* Eq. (2.16)). For the Kählerpotential matrix of the Q^i-superfields we obtain

$$\boldsymbol{H^{(Q)}} = \begin{pmatrix} 1 & \epsilon^2 & \epsilon^4 \\ \epsilon^2 & 1 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & 1 \end{pmatrix}. \tag{4.7}$$

This is canonicalized by the matrix (*c.f.* Eq. (4.4)),

$$\boldsymbol{C^{(Q)}}^{-1^\dagger} = \frac{1}{\sqrt{1-\epsilon^4}} \begin{pmatrix} -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{w_+}{2} & \frac{w_+ f_-}{2} & \frac{w_+}{2} \\ \frac{w_-}{2} & \frac{w_- f_+}{2} & \frac{w_-}{2} \end{pmatrix}, \tag{4.8}$$

with $w_\pm = \sqrt{1 \pm \frac{3\epsilon^2}{\sqrt{8+\epsilon^4}}}$, $f_\pm = \frac{3\epsilon^2 \pm \sqrt{8+\epsilon^4}}{2+\epsilon^4 \pm \epsilon^2\sqrt{8+\epsilon^4}}$. As ϵ is a small quantity, the expression above can be approximated to leading order in ϵ as

$$\boldsymbol{C^{(Q)}}^{-1^\dagger} \approx \begin{pmatrix} -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{2} + \frac{3\,\epsilon^2}{8\sqrt{2}} & -\frac{1}{\sqrt{2}} - \frac{5\,\epsilon^2}{8} & \frac{1}{2} + \frac{3\,\epsilon^2}{8\sqrt{2}} \\ \frac{1}{2} - \frac{3\,\epsilon^2}{8\sqrt{2}} & \frac{1}{\sqrt{2}} - \frac{5\,\epsilon^2}{8} & \frac{1}{2} - \frac{3\,\epsilon^2}{8\sqrt{2}} \end{pmatrix}. \tag{4.9}$$

[20] There are exceptions though: If one works with certain X-charge assignments that involve fractional charges (*e.g.* the one stemming from $X_{\overline{D^3}} = \frac{1}{4}$ and $X_{H^{\mathcal{D}}} = \frac{1}{2}$) the four zeroes of $\boldsymbol{G^{(E)}}$ are not filled up. This seems to have first been mentioned in Ref. [4].

[21] Actually the canonicalization is affected by loop-diagrams, but these effects can be neglected because they are small: *e.g.* the graphs contributing to the propagators of the neutrinos are similar to the graphs that radiatively generate tiny neutrino masses.

However, while this article was reviewed, a preprint [73] came out, stating and nicely demonstrating that in many cases one can utilize a further unitary transformation to bring $\boldsymbol{C^{(Q)}}^{-1\dagger}$ to its "standard" form, see Eq. (4.11).

3. Next we perform a unitary field redefinition of the fields $\left(H^{\mathcal{D}}, L^i\right)$ such that the term $K_i L^i H^{\mathcal{U}}$ vanishes. This was first realized in Ref. [72], however, the rotation matrix was approximate and in addition only valid for real μ and K_i, with $K_i/\mu \ll 1$. We have implemented the full transformation for *arbitrary complex* K_i, μ. With $K \equiv \sqrt{K_i{}^* K^i}$ and $\mathcal{M} \equiv \sqrt{\mu^*\mu + K^2}$ (summation over repeated indices implied) the exact transformation is given by

$$\underbrace{\frac{|\mu|}{\mathcal{M}}\begin{pmatrix} 1 & \left(\frac{K_j}{\mu}\right)^* \\ -\frac{K^i}{\mu} & \frac{K_j{}^*\ K^i}{K^2}\left(1-\frac{\mathcal{M}}{|\mu|}\right)+\frac{\mathcal{M}}{|\mu|}\ \delta^i{}_j \end{pmatrix}}_{\equiv\ \boldsymbol{U}_{\not{\boldsymbol{K}}_{\boldsymbol{i}}}{}^*} \cdot \begin{pmatrix} \mu \\ K^j \end{pmatrix} = \begin{pmatrix} \frac{\mu}{|\mu|}\mathcal{M} \\ 0 \end{pmatrix}. \qquad (4.10)$$

The $SU(4)$ matrix acting on the fields $\left(H^{\mathcal{D}}, L^i\right)$ is given by $\boldsymbol{U}_{\not{\boldsymbol{K}}_{\boldsymbol{i}}}{}^T$ (we shall refer to it as a rotation matrix, since for real coupling constants it is an $SO(4)$ matrix). The rotation gives further mixing of the coupling constants: $G^{(E)}{}_{ij} \leftrightarrow \Lambda_{ikj}$, $G^{(D)}{}_{ij} \leftrightarrow \Lambda'_{ikj}$, $G^{(N)}{}_{ij} \leftrightarrow \Upsilon_i$ and $\Psi_{ijkl} \leftrightarrow \Psi'_{ijk}$. As explained in Section 2, if $m_e/m_\mu \approx \epsilon^5$, one naturally has $M^{(\mu)} \sim M_s$. So one obtains the phenomenologically unacceptable relation that $\frac{\mu}{|\mu|}\mathcal{M}$ is much larger than the weak scale, unless the K_i are suppressed by at least $\epsilon^{X_{L^i}+X_{H^{\mathcal{U}}}} \sim \epsilon^{24}$ ($\epsilon^{24} M_s \sim$ weak scale), see the text below Eq. (3.42).

4. In the next step, the coupling constants are evolved according to their renormalization group equations (RGEs) down from the $U(1)_X$-breaking scale to the weak scale. The one-loop RGEs of the $\not{R}_p$ coupling constants are given in Ref. [74], the two-loop RGEs of the gauge coupling constants and the coupling constants of the superpotential of the $\not{R}_p$-*MSSM* were derived in Ref. [75], the one-loop RGEs of the soft supersymmetry-breaking coupling constants of the $\not{R}_p$-*MSSM* are given in Ref. [35], see also Ref. [76] for the third generation only. The RG-flow once again causes $G^{(E)}{}_{ij} \leftrightarrow \Lambda_{ikj}$, $G^{(D)}{}_{ij} \leftrightarrow \Lambda'_{ikj}$ and $G^{(N)}{}_{ij} \leftrightarrow \Upsilon_i$ mixing and regenerates K_i [77]. Likewise for the corresponding soft coupling constants.

5. The superfields L^i and $H^{\mathcal{D}}$ are rotated unitarily such that only the scalar neutral CP-even components of $H^{\mathcal{D}}$ and $H^{\mathcal{U}}$ acquire VEVs. The relevant matrix is given by Eq. (4.10), with the appropriate replacements. The rotation again mixes the regenerated K^i with μ, and again causes $G^{(E)}{}_{ij} \leftrightarrow \Lambda_{ikj}$, $G^{(D)}{}_{ij} \leftrightarrow \Lambda'_{ikj}$, $G^{(N)}{}_{ij} \leftrightarrow \Upsilon_i$ and $\Psi_{ijkl} \leftrightarrow \Psi'_{ijk}$ mixing.

6. In order to compare with experiment, we must transform into the mass basis of the quarks, which once again skews the coupling constants. As we will assume massless neutrinos here, we do not have to be concerned about the mass basis of the leptons. The inclusion of neutrino masses will be discussed elsewhere.[22]

7. Finally, in the last step, the tree and loop contributions to the mass matrix of the neutral fermions are determined. The neutralinos, and the regenerated operators $K_i L^i H^{\mathcal{U}}$ can cause non-zero neutrino masses. The $R\!\!\!/_p$ operators allow loop graphs that radiatively generate masses for the neutrinos, see *e.g.* Refs. [79, 80, 81, 82].[23] In this paper we have not taken into account phenomenological constraints on the X-charges that arise from the neutrino sector.

Note that if *all* of the (highly undesired) couplings Λ''_{ijk} are initially forbidden due to naive fractional and/or negative X-charges, none of the above steps can regenerate them.

Of course, performing all these steps entirely satisfactory requires the precise knowledge of the unknown *a priori* arbitrary $\mathcal{O}(1)$ coupling constants $g^{(U)}{}_{ij}, h^{(U)}{}_{ij}, C_{H^{\mathcal{D}}H^{\mathcal{U}}}$, etc. Thus the only thing we can do is assume that there is no fine-tuning leading to accidental cancellations and to perform order-of-magnitude estimates. We would now like to point out several caveats and also comment on how our procedure differs from relevant previous work:

1. Step 1 has been neglected in the past; we have taken it into account whenever $X_{\overline{N^1}}, X_{\overline{N^2}}, X_{\overline{N^3}}$ were given (this is not the case in the model presented in Ref. [10], the models presented in Ref. [12] and the model in Ref. [16]). The right-handed neutrino charges did however not change whether the model under consideration is compatible or not: the $X_{\overline{N^i}}$ in model [13] are fractional and thus no $\Xi_i \overline{N^i}$ is generated, the models [14] and [15] are already ruled out even if the explicitly given $X_{\overline{N^i}}$ are not considered, likewise for the models [17]-[21].

2. For Step 2, in the past the approximation
$$C^{(Q)}{}^{-1}{}_{ij} \sim \epsilon^{|X_{Q^i}-X_{Q^j}|} \tag{4.11}$$
was utilized (likewise for the other fields), see Ref. [2].[24] For example, for the choice of quark charges $X_{Q^1}=4$, $X_{Q^2}=2$ and $X_{Q^3}=0$, the transformation matrix is given

[22]The neutrino mass matrix pattern required for the solar and atmospheric neutrino problems can also be obtained via the idea of Froggatt and Nielsen, see *e.g.* Ref. [78].

[23]The corrections to the neutrino masses are totally negligible if the model contains right-handed neutrinos, in the same way as the corresponding corrections are unimportant for the masses of the charged fermions.

[24]This result can be reached as follows: Ignoring the fact that $|X_{Q^i} - X_{Q^j}|$ could be fractional,
$$\sum_{i,j} \overline{Q^i}\ H^{(Q)}{}_{ij}\ Q^j \approx \sum_{i,j} \overline{Q^i}\ \epsilon^{|X_{Q^i}-X_{Q^j}|}\ Q^j\ ,$$
see Eq. (4.7). Now $|X_{Q^i} - X_{Q^j}| \leq |X_{Q^j} - X_{Q^k}| + |X_{Q^k} - X_{Q^i}|$, from which follows that $\epsilon^{|X_{Q^i}-X_{Q^j}|} \geq \epsilon^{|X_{Q^j}-X_{Q^k}|}\ \epsilon^{|X_{Q^k}-X_{Q^i}|}$, and hence, neglecting prefactors of $\mathcal{O}(1)$, $\epsilon^{|X_{Q^i}-X_{Q^j}|} \approx$

by

$$\boldsymbol{C^{(Q)}}^{-1} = \begin{pmatrix} 1 & \epsilon^2 & \epsilon^4 \\ \epsilon^2 & 1 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & 1 \end{pmatrix} . \tag{4.12}$$

This is in gross disagreement with the exact result in Eq. (4.9). Of course, Eq. (4.9) relied on a fine-tuning (all $h^{(Q)}{}_{ij}$=1), however it illustrates that Eq. (4.11) might be a very poor approximation. The correct matrix should be found somewhere between these two extreme cases (see however the text below Eq. (4.9) and Ref. [73]). Keeping in mind that the canonicalization of the Kählerpotential is very important for filling up supersymmetric zeroes, one sees that Eq. (4.9) skews the coupling constants much more than Eq. (4.11) and thus leads to a much stronger unwanted attenuation of possible hierarchies. In this paper we shall utilize Eq. (4.11), being the more conservative approach.

3. Step 3 has been explicitly mentioned in the past only by Refs. [7, 8]. We utilize an approximation to leading order in ϵ.

4. For the time being, we shall use the most up-to-date GUT-scale bounds on the single $\not{R}_p$ coupling constants given in Ref. [83]. For products of two $\not{R}_p$ Yukawa coupling constants we use the weak scale bounds[25] of Refs. [83, 84, 85, 86] (instead of using some of the bounds of Ref. [84] one might wish to use the bounds of Ref. [87]: they are stricter, however less conservative). The bounds are compared in units of $(\tilde{m}/100 \text{ GeV})^2$, $\tilde{m}$ being the mass scale of the scalar particles with $R_p = -1$. For the bounds on higher-dimensional operators we followed Ref. [2, 44]. They state $\Psi_{112i} \le 10^{-8}\ \tilde{m}/100$ GeV, deriving from the experimental bounds on proton decay. To take into account the large uncertainties in determining this bound (see *e.g.* Ref. [88]), we worked with the more conservative

$$\Psi_{112i} \ \le\ 10^{-7}\ \frac{\tilde{m}}{100 \text{ GeV}} . \tag{4.13}$$

In addition, we always included a factor of tolerance $1/\sqrt{\epsilon} \sim 2$ to take into account numerical prefactors $\mathcal{O}(1)$. We initially set $\tilde{m} = 100\,\text{GeV}$ for all bounds and then

$\sum_k \epsilon^{|X_{Q^j} - X_{Q^k}|}\ \epsilon^{|X_{Q^k} - X_{Q^i}|}$. Thus

$$\sum_{i,j} \overline{Q^i}\ \epsilon^{|X_{Q^i} - X_{Q^j}|} Q^j \approx \ \sum_{i,j,k} \overline{Q^i}\ \epsilon^{|X_{Q^j} - X_{Q^k}|} \epsilon^{|X_{Q^k} - X_{Q^i}|}\ Q^j ,$$

and hence, comparing with Eq. (4.3), $\epsilon^{|X_{Q^i} - X_{Q^j}|} \approx C^{(Q)}{}_{ij}$. Furthermore, the $\epsilon^{|X_{Q^i} - X_{Q^j}|}$ form a matrix that is approximately equal to its square and thus approximately equal to its cube, so that this matrix is approximately equal to its own inverse, so that

$$\left[C^{(Q)^{-1}}\right]_{ij} \approx \epsilon^{|X_{Q^i} - X_{Q^j}|} = \epsilon^{|X_{Q^j} - X_{Q^i}|} \approx \left[C^{(Q)^{-1^T}}\right]_{ij} = \left[C^{(Q)^{-1^\dagger}}\right]_{ij} .$$

[25]This approach is conservative, as the bounds get tighter when going to higher scales.

indicate whether a model is not ruled out if we allow $100\,\text{GeV} < \tilde{m} \leq 500$ GeV. Heavier sfermion masses lead to weaker bounds.

5. Concerning Step 5, we shall work with the approximation that the VEVs of the left-handed sneutrinos vanish as has always been done in the past (except in Ref. [5]). This is motivated from low-energy phenomenology: the coupling constant of the soft supersymmetry breaking bilinear term with a scalar L^i and a scalar $H^{\mathcal{U}}$ has to be very small compared to the Higgs-VEV, and the soft supersymmetry breaking 4×4 mass matrix of the scalar $(H^{\mathcal{D}}, L^i)$ has to be nearly diagonal. Hence the VEVs of the left-handed sneutrinos have to be very small, see Ref. [79] and also Ref. [35]. When considering the neutrino masses generated through the K_i in order to be consistent we must then include the VEVs for the scalar neutrinos.

6. The matrices that biunitarily diagonalize the quark mass matrices, namely $\boldsymbol{U^{(U_L)}}$, $\boldsymbol{U^{(U_R)}}$, $\boldsymbol{U^{(D_L)}}$ and $\boldsymbol{U^{(D_R)}}$ are given in Ref. [89] as a function of the entries of the quark mass matrices, as well as $\boldsymbol{U^{CKM}} = \boldsymbol{U^{(U_L)}}^\dagger \boldsymbol{U^{(D_L)}}$. We shall not state the corresponding expressions here, because they are long and complicated, being a function of the entries of the quark mass matrices. Since we do not know the $\mathcal{O}(1)$ coefficients and are hence only performing order-of-magnitude estimates, we set all coefficients equal to 1; however, to avoid accidental cancellations we thus have to replace all minus-signs in the expressions in Ref. [89] (this is also the procedure adopted in previous work). We have checked that the corresponding results are in good agreement with the exact results, having chosen a randomly generated set of $\mathcal{O}(1)$ coefficients (see Appendix 2).

7. Due to the uncertainties on masses and mixings in the neutrino sector we shall neither use radiative corrections to neutrino masses (based on $\not{R}_p$ coupling constants) nor Dirac- and Majorana-masses (when considering right-handed neutrinos) to constrain the X-charges.

It is important to note that all simplifications which we have adopted here lead to a more conservative approach than calculating exactly (assuming that there are no accidental cancellations).

Now the various transformations (apart from renormalization group flow) in Steps 1-7 are all unitary or at least almost unitary. Thus the determinant of the mass matrices should not change substantially, so that apart from a few accidental cancellations and "accidental adding-ups" (and apart from supersymmetric zeroes being filled up) the entries of the mass matrices (and also the entries of the other Yukawa coupling constants) *on the average* keep their order of magnitude. In order to estimate the possible "accidental adding-ups" in our conservative approach, we have calculated the effects of Steps 1-7 on the coupling constants to higher order in ϵ (which can contribute quite a lot as ϵ is not a very small number). We used the approximate expressions of the various transformations as for example Eq. (4.11) but kept all powers of ϵ in the result. Consider *e.g.* the charge assignment of Ref. [2] which is displayed at the end of the last section. $\boldsymbol{G^{(U)}}$ is given by Eq. (3.49), and $\boldsymbol{C^{(Q)}}^{-1}$, $\boldsymbol{C^{(\overline{U})}}^{-1}$ are given by (*c.f.* Eq. (4.11))

$$\begin{pmatrix} 1 & \epsilon & \epsilon^3 \\ \epsilon & 1 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix}, \begin{pmatrix} 1 & \epsilon^3 & \epsilon^5 \\ \epsilon^3 & 1 & \epsilon^2 \\ \epsilon^5 & \epsilon^2 & 1 \end{pmatrix}, \tag{4.14}$$

$\boldsymbol{G^{(U)}}$ is then (*c.f.* Eq. (4.6))

$$\begin{pmatrix} 9\ \epsilon^8 & 6\ \epsilon^5 + 3\ \epsilon^{11} & 3\ \epsilon^3 + 3\ \epsilon^7\ + 3\ \epsilon^{13} \\ 6\ \epsilon^7 + 3\ \epsilon^9 & 4\epsilon^4 + 2\epsilon^6 + 2\epsilon^{10} + \epsilon^{12} & 2\epsilon^2 + \epsilon^4 + 2\epsilon^6 + \epsilon^8 + 2\epsilon^{12} + \epsilon^{14} \\ 3\epsilon^5 + 3\ \epsilon^9 + 3\ \epsilon^{11} & 2\epsilon^2 + 2\epsilon^6 + 3\epsilon^8 + \epsilon^{12} + \epsilon^{14} & 1 + 2\epsilon^4 + \epsilon^6 + \epsilon^8 + 2\epsilon^{10} + \epsilon^{14} + \epsilon^{16} \end{pmatrix}.$$

In the past (and in our conservative approach as well) all numerical prefactors and all higher orders in ϵ were neglected. If they are taken into account one gets with $\epsilon \sim 0.22$ that

$$\boldsymbol{G^{(U)}} \sim \begin{pmatrix} \epsilon^{6.5} & \epsilon^{3.8} & \epsilon^{2.3} \\ \epsilon^{5.8} & \epsilon^{3.1} & \epsilon^{1.5} \\ \epsilon^{4.3} & \epsilon^{1.5} & 1 \end{pmatrix}, \tag{4.15}$$

differing drastically from the conservative result, which is the same as in Eq. (3.49). This is not meant as a demonstration that the conservative approach produces the wrong mass matrix, it is just an indication that *some* of its entries *potentially* can get quite large due to “accidental adding-ups”.

Using the same X-charge assignment as before, we give an example which illustrates how important it is to take into account the $L^i \leftrightarrow H^{\mathcal{D}}$ mixing when canonicalizing the Kählerpotential which has been ignored in the past. For the $R\!\!\!/_p$ coupling constants we obtain

$$\Lambda'_{i1k} = 0, \quad \Lambda'_{i2k} = 0, \quad \Lambda'_{i3k} \sim \epsilon^{55}\ G^{(D)}{}_{ik}\,. \tag{4.16}$$

The model has no right-handed neutrinos so that we do not have to perform Step 1 which is outlined above. To canonicalize the Kählerpotential (Step 2) the basis transformation for the $H^{\mathcal{D}}$ and L^i fields is given by

$$\boldsymbol{C^{(L,H^{\mathcal{D}})}}^{-1} = \begin{pmatrix} 1 & \epsilon^{12} & \epsilon^{13} & \epsilon^{55} \\ \epsilon^{12} & 1 & \epsilon & \epsilon^{67} \\ \epsilon^{13} & \epsilon & 1 & \epsilon^{68} \\ \epsilon^{55} & \epsilon^{67} & \epsilon^{68} & 1 \end{pmatrix}. \tag{4.17}$$

This matrix mixes $\boldsymbol{G^{(D)}}$ with the $\boldsymbol{\Lambda'}$ given in Eq. (4.16):

$$\boldsymbol{C^{(L,H^{\mathcal{D}})}}^{-1} \cdot \begin{pmatrix} \boldsymbol{G^{(D)}} \\ 0 \\ 0 \\ \epsilon^{55}\ \boldsymbol{G^{(D)}} \end{pmatrix} = \begin{pmatrix} \boldsymbol{G^{(D)}} + \epsilon^{10}\boldsymbol{G^{(D)}} \\ \epsilon^{12}\ \boldsymbol{G^{(D)}} + \epsilon^{122}\boldsymbol{G^{(D)}} \\ \epsilon^{13}\ \boldsymbol{G^{(D)}} + \epsilon^{123}\boldsymbol{G^{(D)}} \\ 2\ \epsilon^{55}\ \boldsymbol{G^{(D)}} \end{pmatrix}. \tag{4.18}$$

Dropping higher order terms in ϵ and neglecting $\mathcal{O}(1)$ prefactors this gives

$$\begin{pmatrix} \boldsymbol{G^{(D)}} \\ \epsilon^{12}\ \boldsymbol{G^{(D)}} \\ \epsilon^{13}\ \boldsymbol{G^{(D)}} \\ \epsilon^{55}\ \boldsymbol{G^{(D)}} \end{pmatrix}.$$

With $\epsilon = 0.22$ we conclude that the largest Λ'_{ijk} is of $\mathcal{O}(10^{-8})$. This leads to rapid proton decay, because it turns out that the largest Λ''_{ijk} is of $\mathcal{O}(10^{-7})$. So the model has to be regarded as incompatible with broken R-parity, which is why it is labeled as "*no*" in the Table in Appendix A. Had we not taken into account the $L^i \leftrightarrow H^{\mathcal{D}}$ mixing (hence $\boldsymbol{G^{(D)}}$ not mixing with Λ'_{ijk}) we would have obtained

$$\begin{pmatrix} \boldsymbol{G^{(D)}} \\ \epsilon^{122}\ \boldsymbol{G^{(D)}} \\ \epsilon^{123}\ \boldsymbol{G^{(D)}} \\ \epsilon^{55}\ \boldsymbol{G^{(D)}} \end{pmatrix}$$

instead, giving $\Lambda'_{ijk} \leq \mathcal{O}(10^{-38})$, which is in agreement with current experimental bounds on proton decay. The next step in the canonicalization of the Kählerpotential would be to replace $\boldsymbol{G^{(D)}}$ by $\boldsymbol{C^{(Q)}}^{-1^T} \cdot \boldsymbol{G^{(D)}} \cdot \boldsymbol{C^{(\overline{D})}}^{-1}$, and likewise Λ'_{ijk}, but we shall not go into the details anymore.

5 Results

We have taken the existing Froggatt-Nielsen models given in Refs. [1-16] and embedded them into our general framework. As explicitly given in Table 1, after taking into account all the constraints from the quark masses and mixings, and the lepton masses, as well as the Green-Schwarz anomaly cancellation conditions, all $U(1)_X$ charges and thus all models can be expressed in terms of three integer parameters and three $U(1)_X$ charges which we choose to be (*c.f.* footnote [16])

$$x,\ y,\ z\ \in\ \mathbb{Z}, \qquad \text{and} \qquad X_{L^1},\ X_{L^2},\ X_{L^3}\,. \tag{5.1}$$

The constraints on the integer parameters are given in Eq. (3.55), as well as $z = 0, 1$. The specific choice fixes the quark mass matrices given in Eqs. (3.49)-(3.52) as well as the charged lepton mass matrix. As a first step, in columns 2-7 of the Table A in Appendix A we determine the parameters x, y, z and the charges X_{L^i} for all models in the literature. This makes it possible to easily compare them. In Appendix A, we give a detailed discussion of the models and how the charges were reconstructed out of the given information. In Table A, we have models with small integer charges (< 10) as in model [1], the first model in [4], the models in [5], [7], [14], and [15]. Models with fractional charges constitute the rest. Note that $X_{L^1}, X_{L^2}, X_{L^3}$ being integer is only a necessary but not sufficient condition for all X-charges of *MSSM* particles to be integer. In order to see this, it is important to go back to Table 1 and observe that only a special combination of x, y, z and the X_{L^i}-charges guarantees that X_{Q^1} and $X_{H^{\mathcal{D}}}$ are integer. This is why *e.g.* the third to last model in Ref. [3] does not have purely integer X-charges, although $X_{L^1} = 6, X_{L^2} = 1, X_{L^3} = 4$.

Having defined a unified formalism for all models, we developed a numerical code which implements the steps outlined in Section 4 for an arbitrary model defined by the choice in Eq. (5.1). We thus determine for each model the order of magnitude of the R-parity violating Yukawa couplings and the dimension-five operators at the GUT scale. After running the renormalization group equations one then obtains the corresponding weak scale predictions. Of course this occurs under the assumption that there is no additional discrete symmetry such as R-parity or baryon-parity which forbids a subset of terms. We can then compare these predictions with the existing experimental bounds on the $\not{R}_p$-couplings and the dimension-five operators and thus either allow or exclude a given model of fermion masses.

For single R-parity violating couplings the weak scale bounds have been converted to GUT scale bounds in Ref. [83] using the RGEs of Ref. [75]. We thus compare directly to these. However, we necessarily predict more than one non-zero R-parity violating coupling at the weak- or the GUT-scale and we must take into account the bounds on products of couplings. These can often be much stricter than bounds on single couplings due to extra flavour changing neutral current effects [83, 84, 85, 92] and of course the strict bounds from proton decay [86]. In column 8 of Table A in Appendix A ("Compatible with Exp.") we list whether the model satisfies the experimental constraints on baryon- and lepton-number violation or not.

- "*yes*" means that the model is compatible with the experimental constraints after performing the procedure outlined in Sect. 4;
- "*(no)*$_{h.o.}$" means that the model is compatible if one calculates conservatively, but not compatible if one takes into account the summation of higher orders in ϵ (see the example in the previous section). A moderate amount of fine-tuning such that higher order terms cancel would make the model compatible.[26]
- "*no*" means that the model is incompatible even to lowest order in ϵ.
- "*yes/no*" means that the model is compatible if one uses the freedom to choose X_{L^3} correspondingly, but the model is not compatible if the particular value of X_{L^3} is used which is given in the table.
- The models denoted by * change their status when the relevant sfermion mass is increased. The details are given in the table caption.

[26]Strictly speaking taking higher orders into account also rules out the mass matrices from which one starts out, see Eq. (4.15)

5.1 Discussion

We find that only three models from Refs. [1-9] (the models without right-handed neutrinos) pass with flying colors, *i.e.* produce a "*yes*". *(i)* One model of Ref. [2] is effectively R-parity conserving due to the very high value of X_{L^3}. *(ii)* The model in Ref. [6] is compatible due to complicated fractions for the X-charges. *(iii)* The model [8] also survives due to complicated fractional charges.

Many models, like most of the models in Ref. [7], are incompatible solely due to the bounds on higher dimensional operators. As an example, consider the first model of Ref [7]: $x = y = z = 0$, $X_{L^1} = X_{L^3} = -6$, and $X_{L^2} = -3$ give $X_{Q^1} = 5$, $X_{Q^2} = 4$, so that the operator $Q^1Q^1Q^2L^3$ is suppressed by ϵ^8, which is a factor ~ 55 too large.

It is important to note that including the exact transformation matrix to the canonical basis for the kinetic terms, $\boldsymbol{C}^{\cdots}$, as well as the exact matrix for rotating away the $LH^{\mathcal{U}}$-term makes it even harder for models to survive.

In the case without right-handed neutrinos, we see that a flavour dependent $U(1)_X$ cannot solve the problem of baryon- and lepton-number violation if we demand small integer or simple fractional X-charges. We have thus also considered the case of an additional discrete symmetry: R-parity, baryon-parity, or lepton-parity [28]. In the last column of Table A, we show which set of couplings must vanish for the model to survive. Thus for example the second set of models in Ref. [1] survive (for certain n) if $\Psi = \Lambda'' = 0$, which can be guaranteed by baryon-parity. Similarly, the third model in Ref. [3] is compatible with the experimental constraints if $\Lambda = \Lambda' = 0$, which can be achieved by lepton-parity (anomalous). The first model in Ref. [7] becomes compatible if $\Psi = \Lambda' = 0$, which is also obtained by lepton-parity. In this manner, we can obtain models with small integer charges, but we also lose a central part of our motivation. If there is no entry in the last column the model is either compatible or beyond repair.

When we allow for right-handed neutrinos, Refs. [10-16], we find a significant number of allowed models. Here the X-charges of the right-handed neutrinos present extra degrees of freedom, making the charge assignments more flexible to avoid bounds on the $R\!\!\!/_p$ coupling constants. (If one does not have right-handed neutrinos one is forced to have non-vanishing Λ_{ijk} and/or Λ'_{ijk} in order to get neutrino masses.) However, again we find that the models with integer X-charges ([14] and [15]) are not compatible. The models [11,12] automatically have conserved R-parity and no $QQQL$ operator, *i.e.* $\Psi = 0$, due to the fractional charges (although $X_{L^1}, X_{L^2}, X_{L^3}$ are integers). On the other hand, while the models in [10,2] certainly can fulfill all constraints if one chooses an appropriate X_{L^3}, certain choices of X_{L^3} providing special realistic cases are in conflict with the constraints. The charges are not "fractional enough", that is, they still produce positive-integer exponents of ϵ for certain $R\!\!\!/_p$ coupling constants, thus not forbidding $R\!\!\!/_p$ by fractional X-charges as is the case in Refs. [11,12].

For completeness in Table B we also investigated and listed the Froggatt-Nielsen models (one in Ref. [16], the rest in Refs. [17-21]) which exhibit a slightly different fermionic mass spectrum than the models in Refs. [1-16].

6 Summary

We have presented a systematic embedding of four-dimensional Froggatt-Nielsen models with a single Froggatt-Nielsen field into supersymmetry. We have developed a simple notation into which we have translated all models [fulfilling Eqs. (1.1-1.7) and the Green-Schwarz conditions] which we found in the literature. We have then extended (where necessary) the models to include their predictions for baryon- and lepton-number violation, including the dimension-five operators. Throughout, we have taken into account the modification of the fermion mass matrices due to canonicalization of the Kähler potential, and the rotation of the $H^{\mathcal{D}}$ and L_i fields. We also consider the case of summing sub-leading powers of ϵ, which can lead to significant effects. We have outlined in detail how the models are translated from high-energy textures to low-energy predictions. We have discussed the required steps and pointed out where we have improved on the existing literature. In our Table in Appendix A, we show how many of the existing models fair when compared to the most recent bounds on B- and L-parity violating coupling constants. Only very few models survive unscathed. In the future we hope to improve on our procedure as mentioned in the steps. We also wish to extend our discussion to neutrino masses which we have not yet taken into account.

7 Acknowledgments

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A The Catalogue of Froggatt-Nielsen Models

The first nine references deal with models with no right-handed neutrinos, so no $\Xi_i\ \overline{N}^i$ term has to be shifted away. The values for the six parameters (x, y, z, X_{L^i}) were obtained as follows (the references are listed chronologically according to their appearance on the arXive):

- [1] explicitly states four sets of X-charges, three of these sets fit the class of model considered in this paper (in the third model there is a typo in the X-charge assignments: instead of $e_1 = -1 + q,\ e_2 = 5 + q$ it should read $e_1 = 5 + q,\ e_2 = -1 + q$). The construction of these models did not take into account the Giudice-Masiero mechanism, higher-dimensional operators and $\not{R}_p$. It turns out that the models are in conflict with the bounds on Ψ independent of the value chosen for X_{L^3}. For low values of $|X_{L^3}|$, the models also disagree with bounds on $\not{R}_p$.

- [2] states two complete sets of X-charges. The authors take into account higher-dimensional and $\not{R}_p$-operators as well as the Giudice-Masiero mechanism. The second model is in accord with all constraints due to the large value of $|X_{L^3}|$. The first model was treated as an example in Section 4. The paper presents a further model with right-handed neutrinos that is also discussed in Ref. [10], see below.

- [3] considers the choice $y = -6, 0$ (in our notation). The corresponding quark mass matrices must be combined with four different explicitly stated charged lepton mass matrices. These are obtained through four different choices for the charge differences $X_{L^1} - X_{L^3}$ and $X_{L^2} - X_{L^3}$. The first model is given (in our notation) by $x = 0$, the second one has $y = -6$ with $x = 0, 1, 2$. The third model is required to have $x = 0$, and for the last one $x = 0$, and $y = -6$ is demanded. In addition, the authors work with $\ell_0 \equiv X_{L^3} + X_{Q^3} + X_{\overline{D}^3}$ to be 4 or 5, -4 or -3, 7 or 8, and -7 or -6, respectively. They thus have $2 \cdot (2 + 3 + 2 + 1) = 16$ X-charge assignments. All are such that no dangerous higher-dimensional operators arise. But the authors do not take into account the Giudice-Masiero mechanism, the $LH^{\mathcal{U}}$-term, or the mixing of L^i and $H^{\mathcal{D}}$. They do consider $\not{R}_p$ (explicitly needed to generate neutrino masses), however twelve models disagree with the experimental bounds at lowest order in ϵ and the four other models disagree when including higher order terms.

- [4] explicitly states the three sets of X-charges. The authors take into account higher-dimensional operators as well as $L^i \leftrightarrow H^{\mathcal{D}}$ mixing and thus mixing of $QQQL$ and $QQQH^{\mathcal{D}}$. The first two models are in accord with the bounds to lowest order in ϵ, however not so to higher order. The third model is ruled out because it does not agree with $\not{R}_p$: $X_{L^1} + X_{L^3} + X_{E^1} = 0$ and thus $\Lambda_{131} = \mathcal{O}(1)$.

- [5] explicitly states an incomplete but sufficient set of X-charges. They disregard higher-dimensional operators, and the corresponding constraints are not met.

- [6] explicitly states two sets of X-charge assignments which however are not compatible with the Green-Schwarz mechanism. But from their requirements (translated to our notation) of $y = 0$, $z = 1$, $X_{L^1} - 2 = X_{L^2} = X_{L^3}$ and $7 \leq X_{L^3} - X_{H^{\mathcal{D}}} \leq 9$ and $\tan\beta \approx 50 \;\Rightarrow\; x = 0$ one can extract one X-charge assignment which is compatible with the bounds.

- [7] states 66 (!) X-charge assignments, many of them however referring to quark mass matrices that deviate slightly from the ones considered here: The naive exponents in $\boldsymbol{G^{(U)}}$ and $\boldsymbol{G^{(D)}}$ are given as follows

$$\begin{pmatrix} 8 & 5 & 4 \\ 7 & 4 & 3 \\ 4 & 1 & 0 \end{pmatrix}, \qquad \begin{pmatrix} 5 & 3 & 4 \\ 4 & 2 & 3 \\ 1 & -1 & 0 \end{pmatrix},$$
$$\begin{pmatrix} 8 & 5 & 4 \\ 7 & 4 & 3 \\ 4 & 1 & 0 \end{pmatrix}, \qquad \begin{pmatrix} 3 & 3 & 4 \\ 2 & 2 & 3 \\ -1 & -1 & 0 \end{pmatrix},$$
$$\begin{pmatrix} 8 & 5 & -2 \\ 7 & 4 & -3 \\ 10 & 7 & 0 \end{pmatrix}, \qquad \begin{pmatrix} 4 & 3 & -2 \\ 3 & 2 & -3 \\ 6 & 5 & 0 \end{pmatrix}.$$

We shall not consider these charge assignments. Focusing on Eq. (3.49), we are left with 26 X-charge assignments. Note that their 2^{nd}, 12^{th} and 15^{th} model are special cases of the 1^{st}, 2^{nd} and 2^{nd} model of Ref. [1], respectively. Their 8^{th} model equals the one in Ref. [5]. The authors did not take into account higher-dimensional operators; the corresponding bounds rule out many models. Although the authors do consider $\not{R}_p$, some of the models are ruled out due to disagreement with more recent bounds on $\not{R}_p$ coupling constants. In the end, none of the models survives even to lowest order.

- [8] explicitly states a set of X-charges. All constraints are carefully obeyed, leading to a valid model. The price to be paid are highly fractional charges.

- [9] explicitly states a set of X-charges. The author does not take into account higher-dimensional operators, as the corresponding bounds rule out the model.

- [10] and [2] (the first reference is a talk discussing one of the models – this time with right-handed neutrinos – of the second reference) demand $X_{L^1}-3 = X_{L^2}-1 = X_{L^3}$. This still leaves X_{L^3} undetermined. Combining this with the fact that the authors work with $y = -6, 0$, we get with $z = 0, 1$ that there are $2 \times (3+4) = 14$ charge assignments with X_{L^3} being a free parameter. Although explicitly including right-handed neutrinos, no corresponding X-charges are stated. Of course one might argue that it is always possible to choose a value for X_{L^3} such that R-parity is exactly conserved or at least broken in a way that experimental limits are obeyed. However, to consider a special realistic case, the authors demand the Dirac and Majorana neutrino mass matrices to be without supersymmetric zeroes and the (3,3)-component to dominate, *i.e.* one needs $X_{\overline{N^i}}+X_{H^{\mathcal{U}}}+X_{L^j} \geq X_{\overline{N^3}}+X_{H^{\mathcal{U}}}+X_{L^3} \geq 0$ and $X_{\overline{N^i}} + X_{\overline{N^j}} \geq X_{\overline{N^3}} + X_{\overline{N^3}} \geq 0$. From Eq. (14) of Ref. [10], we thus find

$$m_{\nu_\tau} \sim \frac{\left\langle H^{\mathcal{U}} \right\rangle^2 \ \epsilon^{2(X_{H^{\mathcal{U}}} \, + \, X_{L^3})}}{M} \,. \tag{A.1}$$

With $y = 0, -6$ the X-charge assignments are not GUT compatible, one thus assumes $M = M_s \sim 10^{18}$ GeV (and not $M = M_{\mathrm{GUT}} \sim 10^{16}$ GeV). The authors demand $m_{\nu_\tau} = 0.1$ eV. Combining this with $\epsilon \sim 0.22$, $\langle H^{\mathcal{U}} \rangle \sim 100$ GeV, we get that $X_{H^{\mathcal{U}}} + X_{L^3} \sim -3$; we can thus fix X_{L^3}.

- [11] and [12] (the first reference is a talk discussing one of the models of the second reference) present a model (aimed at atmospheric neutrinos) that is based on the quark sector in Ref. [2] (Ref. [12] is a kind of continuation of Ref. [2]. Ref. [11] also reviews the model of Refs. [2, 10]), so $y = -6, 0$. The authors demand $\frac{3}{2} \leq X_{L^2} = -X_{L^3} \leq \frac{5}{2}$, $X_{L^1} - X_{L^2} \geq 3$, which they specify to $X_{L^1} = 5, X_{L^2} = 2$, giving $X_{\overline{E^1}} - X_{\overline{E^2}} = z-1$. As they furthermore suppose $X_{\overline{E^1}} = X_{\overline{E^2}}$, the authors get $z = 1$. This leads to $m_e/m_\tau \sim \epsilon^5$, as is correctly stated in Ref. [12]. It should however be mentioned that both Refs. [11] and [12] explicitly demand $X_{H^{\mathcal{U}}} = X_{H^{\mathcal{D}}} = 0$, which is in contradiction to $z = 1$. Furthermore, in Ref. [12] it is assumed that $2 \leq X_{\overline{E^3}} \leq 4$, which requires $0 < 143 + 29x + 3x^2 - 18y < 120 + 18x$, which is not possible. Therefore the model is actually self-contradicting. However, Ref. [11] has not stated this constraint on $X_{\overline{E^3}}$ and it shall hence be ignored. This gives 4+3=7 different models. The constraints on the X-charges of the right-handed neutrinos are $0 \leq X_{\overline{N^1}} < -X_{\overline{N^3}} \leq X_{L^2} \leq X_{\overline{N^2}}$.

- [12] furthermore considers a model that was not dealt with in Ref. [11], aimed at the adiabatic Mikheev-Smirnow-Wolfenstein effect [90, 91] for the solar neutrinos. The authors demand $0 < X_{L^2} < -X_{L^3} = X_{L^1}$ and the following constraints on the X-charges of the right-handed neutrinos: $0 \leq X_{\overline{N^1}}$, $X_{L^2} < -X_{\overline{N^3}} \leq X_{L^1} \leq X_{\overline{N^2}}$. Specifically they use $X_{L^1} = \frac{9}{2}, X_{L^2} = \frac{3}{2}, X_{\overline{E^1}} = X_{\overline{E^3}} - 4$. This results in $z = 1$ and $X_{\overline{E^1}} = X_{\overline{E^2}}$; furthermore the authors demand $\frac{9}{2} \leq X_{\overline{E^3}} \leq \frac{17}{2}$, which is fulfilled. So there are 4+3=7 models.

- [13] indirectly states that $x = 3$, $y = 0$, $z = 0$ and gives an explicit charged lepton mass matrix and a right-handed neutrino Majorana mass matrix. The latter depends on a parameter labeled $X^{[\bar{N}]}$, which is given as $-\frac{9}{2}$, in order to accomodate the non-adiabatic MSW effect. This completely fixes the X_{L^i}, and one has $X_{\overline{N^1}} = \frac{13}{2}, X_{\overline{N^2}} = \frac{11}{2}, X_{\overline{N^3}} = \frac{3}{2}$.

- [14] states an explicit X-charge assignment, with right-handed neutrino charges $X_{\overline{N^1}} = X_{\overline{N^2}} = X_{\overline{N^3}} = 0$. Note that totally uncharged fields may give rise to dangerous tadpoles, *e.g.* see Refs. [93, 94, 95, 96].

- [15] states an explicit X-charge assignment, with right-handed neutrino charges $X_{\overline{N^1}} = 8, X_{\overline{N^2}} = 4, X_{\overline{N^3}} = 0$. These X-charges are the only difference to the model in Ref. [14]. This model and the previous one are the only ones that have the quark mass matrices of Eq. (3.51).

- [16] explicitly demands conserved R_p, however without checking whether this is a consequence of the given X-charge assignments. The authors also do not constrain the X-charges by anomaly cancellation, which gives them more freedom. We reconsider their model taking the Green-Schwarz mechanism into account. The authors indirectly state that $z = 1, y = 0$ and give several X-charges parameterized by the integers m, n, p, r. Then these parameters are fixed to be $p = r = 5,\ n = 0$ (so-called "best-fit assignment"). One then distinguishes $m = 0$ or 1 ("anarchical type" or "lopsided type", respectively). x and X_{L^3} are left undetermined. The authors thus have $2 \times 4 = 8$ models with X_{L^3} unconstrained, so one should always be able to choose X_{L^3} such that R_p is conserved. In the end of the paper the authors explicitly emphasize that $\tan\beta \sim 3 \;\;\Rightarrow\;\; x = 3$, together with $2X_{Q^3} - X_{\overline{U^3}} - X_{\overline{E^3}} \stackrel{!}{=} 1$. This seems to give a way to fix X_{L^3}; however with Table 1 one gets $2X_{Q^3} - X_{\overline{U^3}} - X_{\overline{E^3}} = \frac{4-m}{3}$, independent of X_{L^3} (note that one actually needs to have $m = 1$, otherwise the model is inconsistent). So we had to leave X_{L^3} unfixed. Although explicitly including right-handed neutrinos, no corresponding X-charges are stated.

by	x	y	z	X_{L^1}	X_{L^2}	X_{L^3}	Compatible with Exp.	O.K. if ...
[1]	0	0	0	X_{L^3}	X_{L^3}	$1-3$n, n$\in \mathbb{Z}$	*no* if n$\neq -3,\ldots,1$ *no* if n$=-3,-2,-1$ *no* if n$=0,1$	$\Psi=0$ $\Lambda'',\Psi=0$
	2	0	0	$X_{L^3}+1$	$X_{L^3}-1$	$3-3$n, n$\in \mathbb{Z}$	*no** if n$\neq -2,\ldots,2$ *no* if n$=-2,-1$ *no* if n$=0,1,2$	$\Psi=0$ $\Lambda'',\Psi=0$
	2	0	0	$X_{L^3}+1$	$X_{L^3}+5$	$1-3$n, n$\in \mathbb{Z}$	*no* if n$\neq -2,\ldots,2$ *no* if n$=-2,-1$ *no* if n$=0,1,2$	$\Psi=0$ $\Lambda'',\Psi=0$
[2]	2	0	0	-12	-13	55	*no*	$\Lambda''=0$
	2	0	0	23	22	-78	*yes*	
[3]	0	0	0	$\frac{32}{5}$	$\frac{22}{5}$	$\frac{7}{5}$	$(no)_{h.o.}$	$\Lambda'=0$
	0	-6	0	$\frac{26}{5}$	$\frac{16}{5}$	$\frac{1}{5}$	*no*	$\Lambda'=0$

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by	$\boldsymbol{x}$	$\boldsymbol{y}$	$\boldsymbol{z}$	$\boldsymbol{X_{L^1}}$	$\boldsymbol{X_{L^2}}$	$\boldsymbol{X_{L^3}}$	**Compatible with Exp.**	**O.K. if ...**
	0	-6	0	$\frac{62}{5}$	$-\frac{48}{5}$	$-\frac{33}{5}$	*no*	$\Lambda, \Lambda' = 0$
	1	-6	0	$\frac{1321}{105}$	$-\frac{989}{105}$	$-\frac{674}{105}$	*no*	$\Lambda, \Lambda' = 0$
	2	-6	0	$\frac{38}{3}$	$-\frac{28}{3}$	$-\frac{19}{3}$	*no*	$\Lambda, \Lambda' = 0$
	0	0	0	$\frac{33}{5}$	$\frac{8}{5}$	$\frac{23}{5}$	$(no)_{h.o.}$	$\Lambda' = 0$
	0	-6	0	$\frac{27}{5}$	$\frac{2}{5}$	$\frac{17}{5}$	*no*	$\Lambda' = 0$
	0	-6	0	$\frac{61}{5}$	$-\frac{34}{5}$	$-\frac{49}{5}$	*no*	$\Lambda' = 0$
	0	0	0	$\frac{29}{5}$	$\frac{19}{5}$	$\frac{4}{5}$	$(no)_{h.o.}$	$\Lambda' = 0$
	0	-6	0	$\frac{23}{5}$	$\frac{13}{5}$	$-\frac{2}{5}$	*no*	$\Lambda' = 0$
	0	-6	0	$\frac{59}{5}$	$-\frac{51}{5}$	$-\frac{36}{5}$	*no*	$\Lambda, \Lambda' = 0$
	1	-6	0	$\frac{1258}{105}$	$-\frac{1052}{105}$	$-\frac{737}{105}$	*no*	$\Lambda, \Lambda' = 0$
	2	-6	0	$\frac{181}{15}$	$-\frac{149}{15}$	$-\frac{104}{15}$	*no*	$\Lambda, \Lambda' = 0$
	0	0	0	6	1	4	$(no)_{h.o.}$	$\Lambda, \Lambda' = 0$
	0	-6	0	$\frac{24}{5}$	$-\frac{1}{5}$	$\frac{14}{5}$	*no*	$\Lambda' = 0$

continued next page

by	x	y	z	X_{L^1}	X_{L^2}	X_{L^3}	Compatible with Exp.	O.K. if ...
	0	-6	0	$\frac{58}{5}$	$-\frac{37}{5}$	$-\frac{52}{5}$	*no*	$\Lambda, \Lambda' = 0$
[4]	3	0	1	-8	-8	-4	$(no)_{h.o.}$	$\Psi = 0$
	0	0	1	-4	-7	$-\frac{9}{2}$	$(no)_{h.o.}$	$\Psi = 0$
	3	0	1	$-\frac{7}{2}$	-4	$-\frac{7}{2}$	*no*	$\Lambda = 0$
[5]	0	0	0	-8	-8	-8	*no*	$\Psi = 0$
[6]	0	0	1	$X_{L^3} + 2$	X_{L^3}	$\frac{358}{105} \leq ... \leq \frac{478}{105}$	*yes*	
[7]	0	0	0	-6	-3	-6	*no*	$\Lambda', \Psi = 0$
	0	0	0	-5	-5	-5	*no*	$\Psi = 0$
	0	0	0	-4	-7	-4	*no*	$\Psi = 0$
	0	0	0	-3	-9	-3	*no*	$\Lambda, \Lambda', \Psi = 0$
	0	0	0	-10	-4	-1	*no*	$\Lambda', \Psi = 0$
	0	0	0	-10	-4	-10	*no*	$\Psi = 0$
	0	0	0	-9	-6	-9	*no*	$\Psi = 0$

continued next page

by	x	y	z	X_{L^1}	X_{L^2}	X_{L^3}	Compatible with Exp.	O.K. if ...
	0	0	0	-8	-8	-8	*no*	$\Psi = 0$
	0	0	0	-7	-10	-7	*no*	$\Psi = 0$
	2	0	0	-7	-3	-8	*no*	$\Psi = 0$
	2	0	0	-6	-5	-7	*no**	$\Psi = 0$
	2	0	0	-5	-7	-6	*no**	$\Psi = 0$
	2	0	0	-4	-9	-5	*no*	$\Psi = 0$
	2	0	0	-9	-8	-10	*no**	$\Psi = 0$
	2	0	0	-8	-10	-9	*no**	$\Psi = 0$
	0	0	1	-6	-3	-2	*no*	$\Lambda, \Lambda', \Psi = 0$
	0	0	1	-4	-6	-10	*no*	$\Psi = 0$
	1	0	1	-3	-5	-9	*no*	$\Psi = 0$
	0	0	1	-10	-1	-9	*no*	$\Lambda, \Psi = 0$
	0	0	1	-8	-6	-6	*no*	$\Psi = 0$

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by	x	y	z	X_{L^1}	X_{L^2}	X_{L^3}	Compatible with Exp.	O.K. if ...
	1	0	1	-8	-4	-5	*no*	$\Lambda, \Psi = 0$
	1	0	1	-6	-9	-2	*no*	$\Psi = 0$
	2	0	1	-8	-2	-4	*no*	$\Lambda, \Psi = 0$
	1	0	1	-10	-7	-9	*no**	$\Psi = 0$
	2	0	1	-10	-5	-8	*no**	$\Psi = 0$
	2	0	1	-8	-10	-5	*no**	$\Psi = 0$
[8]	1	0	1	$-\frac{113}{30}$	$-\frac{113}{30}$	$-\frac{113}{30}$	*yes*	
[9]	0	0	0	$-\frac{19}{5}$	$-\frac{19}{5}$	$-\frac{19}{5}$	*no*	$\Lambda, \Lambda', \Psi = 0$
[10, 2]	0	0	0	$X_{L^3} + 3$	$X_{L^3} + 1$	$\ldots \big/ -\frac{41}{15}$	*yes/no*	$\Lambda, \Lambda' = 0$
	1	0	0	$X_{L^3} + 3$	$X_{L^3} + 1$	$\ldots \big/ -\frac{239}{105}$	*yes/no*	$\Lambda, \Lambda' = 0$
	2	0	0	$X_{L^3} + 3$	$X_{L^3} + 1$	$\ldots \big/ -\frac{11}{6}$	*yes/no*	$\Lambda, \Lambda' = 0$
	3	0	0	$X_{L^3} + 3$	$X_{L^3} + 1$	$\ldots \big/ -\frac{7}{5}$	*yes/no*	$\Lambda, \Lambda' = 0$
	0	0	1	$X_{L^3} + 3$	$X_{L^3} + 1$	$\ldots \big/ -\frac{73}{35}$	*yes/no*	

continued next page

by	x	y	z	X_{L^1}	X_{L^2}	X_{L^3}	Compatible with Exp.	O.K. if ...
	1	0	1	$X_{L^3}+3$	$X_{L^3}+1$	$\ldots\big/ -\frac{33}{20}$	*yes*/*no*	
	2	0	1	$X_{L^3}+3$	$X_{L^3}+1$	$\ldots\big/ -\frac{11}{9}$	*yes*/*no*	
	3	0	1	$X_{L^3}+3$	$X_{L^3}+1$	$\ldots\big/ -\frac{4}{5}$	*yes*/*no*	
	0	-6	0	$X_{L^3}+3$	$X_{L^3}+1$	$\ldots\big/ -\frac{59}{15}$	*yes*/*no*	$\Lambda, \Lambda' = 0$
	1	-6	0	$X_{L^3}+3$	$X_{L^3}+1$	$\ldots\big/ -\frac{347}{105}$	*yes*/*no*	$\Lambda, \Lambda' = 0$
	2	-6	0	$X_{L^3}+3$	$X_{L^3}+1$	$\ldots\big/ -\frac{41}{15}$	*yes*/*no*	$\Lambda, \Lambda' = 0$
	0	-6	1	$X_{L^3}+3$	$X_{L^3}+1$	$\ldots\big/ -\frac{109}{35}$	*yes*/*no*	
	1	-6	1	$X_{L^3}+3$	$X_{L^3}+1$	$\ldots\big/ -\frac{51}{20}$	*yes*/*no*	
	2	-6	1	$X_{L^3}+3$	$X_{L^3}+1$	$\ldots\big/ -\frac{91}{45}$	*yes*/*no*	
[11, 12]	0	0	1	5	2	-2	*yes*	
	1	0	1	5	2	-2	*yes*	
	2	0	1	5	2	-2	*yes*	
	3	0	1	5	2	-2	*yes*	

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by	x	y	z	X_{L^1}	X_{L^2}	X_{L^3}	Compatible with Exp.	O.K. if ...
	0	-6	1	5	2	-2	*yes*	
	1	-6	1	5	2	-2	*yes*	
	2	-6	1	5	2	-2	*yes*	
[12]	0	0	1	$\frac{9}{2}$	$\frac{3}{2}$	$-\frac{9}{2}$	*yes*	
	1	0	1	$\frac{9}{2}$	$\frac{3}{2}$	$-\frac{9}{2}$	*yes*	
	2	0	1	$\frac{9}{2}$	$\frac{3}{2}$	$-\frac{9}{2}$	*yes*	
	3	0	1	$\frac{9}{2}$	$\frac{3}{2}$	$-\frac{9}{2}$	*yes*	
	0	-6	1	$\frac{9}{2}$	$\frac{3}{2}$	$-\frac{9}{2}$	*yes*	
	1	-6	1	$\frac{9}{2}$	$\frac{3}{2}$	$-\frac{9}{2}$	*yes*	
	2	-6	1	$\frac{9}{2}$	$\frac{3}{2}$	$-\frac{9}{2}$	*yes*	
[13]	3	0	0	$\frac{79}{30}$	$-\frac{11}{30}$	$-\frac{11}{30}$	$(no)_{h.o.}$	$\Psi = 0$
[14]	0	1	0	0	0	0	*no*	
[15]	0	1	0	0	0	0	*no*	

continued next page

by	x	y	z	X_{L^1}	X_{L^2}	X_{L^3}	Compatible with Exp.	O.K. if ...
[16]	0	0	1	X_{L^3}	X_{L^3}	...	*yes*	
	1	0	1	X_{L^3}	X_{L^3}	...	*yes*	
	2	0	1	X_{L^3}	X_{L^3}	...	*yes*	
	3	0	1	X_{L^3}	X_{L^3}	...	*yes*	
	0	0	1	$X_{L^3}+1$	X_{L^3}	...	*yes*	
	1	0	1	$X_{L^3}+1$	X_{L^3}	...	*yes*	
	2	0	1	$X_{L^3}+1$	X_{L^3}	...	*yes*	
	3	0	1	$X_{L^3}+1$	X_{L^3}	...	*yes*	

Table A: *In the table above we present the catalogue of Froggatt-Nielsen models of the type defined in Section 1, parametrized by* $x, y, z, X_{L^1}, X_{L^2}, X_{L^3}$*. For the entries "*yes*", "*no*", etc. and* $\Psi = 0$*,* $\Lambda'' = 0$ *etc. see Section 6. Models where the "*no*" is labeled by a* * *are more in accord with the bounds if* $\tilde{m} > 250$ *GeV, one gets a (no)*$_{h.o.}$ *instead.*

by	$X_{H^{\mathcal{D}}}, X_{H^{\mathcal{U}}}$	X_{10^i}	$X_{\overline{5^i}}$	$X_{\overline{N^i}}$	ϵ	Compatible with Constraints?
[16]	$2q_3, -2q_3$	$3+q_3, 2+q_3, q_3$	$1+d_3, d_3, d_3$	–	0.22	*no*
[17]	$0, 0$	$2, 1, 0$	$1, 0, 0$	$0, 1, 2$	$\frac{1}{\sqrt{300}}$	*no*
	$0, 0$	$2, 1, 0$	$1, 0, 0$	$0, 0, 2$	$\frac{1}{\sqrt{300}}$	*no*
	$0, 0$	$2, 1, 0$	$1, 0, 0$	$1, 1, 2$	$\frac{1}{\sqrt{300}}$	*no*
	$0, 0$	$2, 1, 0$	$2, 1, 1$	$0, 0, 1$	$\frac{1}{\sqrt{300}}$	*no*
	$0, 0$	$2, 1, 0$	$1, 0, 0$	$0, 0, 1$	$\frac{1}{\sqrt{300}}$	*no*
[18]	$0, 0$	$3, 2, 0$	$2, 0, 0$	–	0.22	*no*
	$0, 0$	$3, 2, 0$	$3, 1, 1$	–	0.22	*no*
	$0, 0$	$3, 2, 0$	$4, 2, 2$	–	0.22	*no*
[19]	$0, 0$	$3, 2, 0$	$2, 0, 0$	$2, 0, 0$	0.22	*no*
[20]	$0, 0$	$2, 1, 0$	$1, 0, 0$	$2, 1, 0$	0.07	*no*
	$0, 0$	$2, 1, 0$	$2, 1, 1$	$2, 1, 0$	0.07	*no*
	$0, 0$	$2, 1, 0$	$1, 0, 0$	$0, 0, 0$	0.07	*no*
	$0, 0$	$2, 1, 0$	$2, 1, 1$	$0, 0, 0$	0.07	*no*
	$0, 0$	$3, 1, 0$	$2, 1, 1$	$2, 1, 0$	0.07	*no*
	$0, 0$	$3, 1, 0$	$2, 1, 1$	$0, 0, 0$	0.07	*no*
[21]	$0, 0$	$2, 1, 0$	$1, 1, 1$	$0, 0, 0$	0.05	*no*

Table B: *The models in this table do not lead to a fermionic mass spectrum as in Eqs. (1.1-1.7). Note that all models are* $SU(5)$ *invariant.*

B Examining the Mass Matrices

To demonstrate the validity of Eqs. (3.49-3.52), note that the CKM matrix is nearly a unit-matrix, so that one should have $\boldsymbol{U}^{(\boldsymbol{U_L})} \approx \boldsymbol{U}^{(\boldsymbol{D_L})\dagger}$, *i.e.* these two matrices should have "the same Cabibbo structure" (for a more detailed analysis see Ref. [13]). It is thus reasonable that $\boldsymbol{G}^{(\boldsymbol{U})\dagger}\boldsymbol{G}^{(\boldsymbol{U})} \approx \boldsymbol{G}^{(\boldsymbol{D})}\boldsymbol{G}^{(\boldsymbol{D})\dagger}$. This is indeed the case for the matrices presented, *e.g.* Eq. (3.51) gives

$$\begin{pmatrix} \epsilon^8 & \epsilon^6 & \epsilon^4 \\ \epsilon^6 & \epsilon^4 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & 1 \end{pmatrix}^T \cdot \begin{pmatrix} \epsilon^8 & \epsilon^6 & \epsilon^4 \\ \epsilon^6 & \epsilon^4 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & 1 \end{pmatrix} = \frac{1+\epsilon^4+\epsilon^8}{3} \begin{pmatrix} \epsilon^4 & \epsilon^4 & \epsilon^4 \\ \epsilon^2 & \epsilon^2 & \epsilon^2 \\ 1 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} \epsilon^4 & \epsilon^4 & \epsilon^4 \\ \epsilon^2 & \epsilon^2 & \epsilon^2 \\ 1 & 1 & 1 \end{pmatrix}^T . \quad \text{(B.2)}$$

Furthermore the $\boldsymbol{G}^{(\cdots)}$ have the correct eigenvalues[27] and hence reproduce the correct masses. For example, $\boldsymbol{G}^{(\boldsymbol{U})}$ in Eq. (3.49) has the characteristic polynomial

$$- \det \boldsymbol{g}^{(\boldsymbol{U})}\ \epsilon^{12} \ + \ \Big[\Big({g^{(U)}}_{22}\ \ {g^{(U)}}_{33} \ - \ {g^{(U)}}_{23}\ \ {g^{(U)}}_{32} \Big)\ \epsilon^4$$

$$\Big({g^{(U)}}_{11}\ \ {g^{(U)}}_{33} \ - \ {g^{(U)}}_{13}\ \ {g^{(U)}}_{31} \Big)\ \epsilon^8 \ + \ \Big({g^{(U)}}_{11}\ \ {g^{(U)}}_{22} \ - \ {g^{(U)}}_{12}\ \ {g^{(U)}}_{21} \Big)\ \epsilon^{12}\ \Big]\ \lambda$$

$$- \Big({g^{(U)}}_{11}\ \epsilon^8 \ + \ {g^{(U)}}_{22}\ \epsilon^4 \ + \ {g^{(U)}}_{33} \Big)\ \lambda^2 \ + \ \lambda^3 \ = \ 0. \tag{B.3}$$

Assuming that

$$\begin{aligned}
&\det \boldsymbol{g}^{(\boldsymbol{U})} \approx 1\,, \\
&{g^{(U)}}_{22}\ \ {g^{(U)}}_{33} \ - \ {g^{(U)}}_{23}\ \ {g^{(U)}}_{32} \approx 1\,, \\
&{g^{(U)}}_{33}\ \ {g^{(U)}}_{11} \ - \ {g^{(U)}}_{31}\ {g^{(U)}}_{13} \ \approx 1\,, \\
&{g^{(U)}}_{11}\ \ {g^{(U)}}_{22} - {g^{(U)}}_{12}\ \ {g^{(U)}}_{21} \ \approx 1\,, \\
&{g^{(U)}}_{33} \approx {g^{(U)}}_{22} \ \approx {g^{(U)}}_{11} \ \approx 1\,,
\end{aligned} \tag{B.4}$$

this reduces to

$$\lambda^3 - (\epsilon^8 + \epsilon^4 + 1)\ \lambda^2 + (\epsilon^4 + \epsilon^8 + \epsilon^{12})\ \lambda - \epsilon^{12} \approx 0\,, \tag{B.5}$$

which can be factored as

$$(\lambda - 1)(\lambda - \epsilon^4)(\lambda - \epsilon^8) \approx 0\,, \tag{B.6}$$

thus reproducing the correct ratio of the eigenvalues, see Eq. (1.5). Alternatively, in the expression for the characteristic polynomial one can also neglect higher order terms and

[27] As the matrix $\boldsymbol{G}^{(\cdots)} \cdot \boldsymbol{G}^{(\cdots)\dagger}$ is Hermitian, it can be diagonalized with its eigenvalues on the diagonal. Thus the matrix $\boldsymbol{D}^{(\cdots)}$ which one gets when bi-unitarily diagonalizing $\boldsymbol{G}^{(\cdots)}$ contains the square roots of the eigenvalues of $\boldsymbol{G}^{(\cdots)} \cdot \boldsymbol{G}^{(\cdots)\dagger}$ (times a phase) on its diagonal. This should not be confused with the strictly speaking wrong statement that $\boldsymbol{D}^{(\cdots)}$ contains the eigenvalues of $\boldsymbol{G}^{(\cdots)}$. As a demonstration, consider the following matrix, its coefficients having been randomly generated with *Mathematica*©, their absolute values being in the interval $\left[\frac{1}{\sqrt{10}}, \sqrt{10}\right]$:

$$\boldsymbol{\mathcal{M}} \ = \ \begin{pmatrix} 0.8614 - 2.9121\ i & -1.2754 + 0.1818\ i & 0.6656 - 2.2503\ i \\ -1.2074 - 1.3980\ i & 1.2850 - 0.3739\ i & 0.5473 + 0.8524\ i \\ -0.2591 + 0.5662\ i & -0.4475 + 0.0637\ i & -0.6963 + 1.5219\ i \end{pmatrix}$$

The absolute values of the eigenvalues of $\boldsymbol{\mathcal{M}}$ are 2.6456, 2.5975, 1.7199. However, the absolute values of the square roots of the eigenvalues of $\boldsymbol{\mathcal{M}}\boldsymbol{\mathcal{M}}^\dagger$ are 4.3262, 2.4631, 1.1091. On the other hand, if the entries of $\mathcal{M}$ are ϵ-suppressed, e.g. with the exponents

$$\begin{pmatrix} 5 & 3 & 2 \\ 4 & 2 & 1 \\ 3 & 1 & 0 \end{pmatrix},$$

one gets 2.3586, 0.0813, 0.0051 and 2.3701, 0.0819, 0.0050, respectively, almost the same.

only demand

$$
\begin{aligned}
&{g^{(U)}}_{33} \;\approx 1, \\
&{g^{(U)}}_{22}\, {g^{(U)}}_{33} \;-\; {g^{(U)}}_{23}\;\; {g^{(U)}}_{32} \approx 1, \\
&\det \boldsymbol{g^{(U)}} \approx 1,
\end{aligned}
\tag{B.7}
$$

so that

$$
\lambda^3 - \lambda^2 + \epsilon^4 \lambda - \epsilon^{12} = 0\,. \tag{B.8}
$$

Using $\epsilon = 0.22$ we thus obtain $\lambda_1 = \epsilon^{8.00212},\;\lambda_2 = \epsilon^{3.99},\;\lambda_3 = \epsilon^{0.001}$, which is also in good agreement.

To numerically demonstrate that $\boldsymbol{G^{(U)}}, \boldsymbol{G^{(D)}}$ of Eq. (3.49) give the correct CKM-matrix, random $\mathcal{O}(1)$ coefficients were generated with *Mathematica*©. With Eq. (3.49) we obtained

$$
\begin{aligned}
\boldsymbol{G^{(U)}} \;&=\; \begin{pmatrix} 1.41\ \epsilon^8 & -0.95\ \epsilon^5 & -1.73\ \epsilon^3 \\ 0.94\ \epsilon^7 & -1.19\ \epsilon^4 & 1.12\ \epsilon^2 \\ 1.35\ \epsilon^5 & 2.10 & 1.52\ \epsilon^2 \end{pmatrix}, \\
\boldsymbol{G^{(D)}} \;&=\; \begin{pmatrix} 1.22\ \epsilon^4 & -1.45\ \epsilon^3 & -1.76\ \epsilon^3 \\ 0.69\ \epsilon^3 & -0.94\ \epsilon^2 & 0.60\ \epsilon^2 \\ -1.02\ \epsilon & 0.68 & -1.97 \end{pmatrix}.
\end{aligned}
\tag{B.9}
$$

Using $\epsilon = 0.22$, this leads to

$$
m_b : m_s : m_d = -1.98 : -0.68\ \epsilon^2 : 1.13\ \epsilon^4\,, \tag{B.10}
$$

$$
m_t : m_c : m_u = -1.51 : 0.33\ \epsilon^4 : 2.56\ \epsilon^8\,, \tag{B.11}
$$

and

$$
\boldsymbol{U^{CKM}} = \begin{pmatrix} -0.98 & -0.62\ \epsilon & 3.66\ \epsilon^3 \\ 0.62\ \epsilon & -0.98 & -0.89\ \epsilon^2 \\ 4.18\ \epsilon^3 & -0.77\ \epsilon^2 & 0.99 \end{pmatrix}, \tag{B.12}
$$

which is the correct order of magnitude.

Finally, for completeness it should be mentioned that Eqs. (3.49) and (3.51) can also be generated from models with a vector-like pair of Froggatt-Nielsen fields A, B, see Ref. [1]. A vector-like pair of Froggatt-Nielsen fields can furthermore generate phenomenologically viable mass matrices which cannot be generated from models with only one Froggatt-Nielsen field: *e.g.* $X_{Q^1} = X_{\overline{U^1}} = -8$, $X_{Q^2} = X_{\overline{U^2}} = 2$, $X_{Q^3} = X_{\overline{U^3}} = X_{H^{\mathcal{U}}} = 0$ and

$\langle A \rangle = \langle B \rangle$, see Ref. [28], give[28]

$$\boldsymbol{G^{(U)}} \propto \begin{pmatrix} \epsilon^{16} & \epsilon^6 & \epsilon^8 \\ \epsilon^6 & \epsilon^4 & \epsilon^2 \\ \epsilon^8 & \epsilon^2 & \epsilon^0 \end{pmatrix}. \tag{B.13}$$

This leads to a characteristic polynomial of approximately the form

$$\lambda^3 - \left(1 + \epsilon^4 + \epsilon^{16}\right)\lambda^2 + \left(\epsilon^4 + \epsilon^{12} + \epsilon^{16} + \epsilon^{20}\right)\lambda - \left(\epsilon^{12} + \epsilon^{16} + \epsilon^{20}\right) = 0, \tag{B.14}$$

with $\epsilon = 0.22$ giving

$$\lambda_1 = 1, \quad \lambda_2 = \epsilon^{4.00156}, \quad \lambda_3 = \epsilon^{7.99689} \tag{B.15}$$

and thus

$$m_u : m_c : m_t = \epsilon^8 : \epsilon^4 : 1, \tag{B.16}$$

as required.

References

[1] E.Dudas, S.Pokorski, C.Savoy, *Phys.Lett.* **356** 45 (1995); *hep-ph/9504292*.

[2] P.Binétruy, S.Lavignac, P.Ramond, *Nucl.Phys.* **B477** 353 (1996); *hep-ph/9601243*.

[3] E.Chun, A.Lukas, *Phys.Lett.* **B387** 99 (1996); *hep-ph/9605377*.

[4] K.Choi, E.Chun, H.Kim, *Phys.Lett.* **B394** 89 (1997); *hep-ph/9611293*.

[5] P.Binétruy, E.Dudas, S.Lavignac, C.Savoy, *Phys.Lett.* **B422** 171 (1998); *hep-ph/9711517*.

[6] K.Choi, E.Chun, K.Hwang, *Phys.Rev.* **D60** 031301 (1999); *hep-ph/9811363*.

[7] A.Joshipura, R.Vaidya, S.Vempati, *Phys.Rev.* **D62** 093020 (2000); *hep-ph/0006138*.

[8] J.Mira, E.Nardi, D.Restrepo, J.Valle, *Phys.Lett.* **B492** 81 (2000); *hep-ph/0007266*.

[9] D.Restrepo (Ph.D.-thesis); *hep-ph/0111198*.

[10] S.Lavignac, *hep-ph/9605355*.

[11] S.Lavignac, *hep-ph/9610257*.

[28] However Froggatt-Nielsen models leading to such a matrix shall not be considered in this paper as it cannot be achieved with only one Froggatt-Nielsen-field A, because

$$\begin{pmatrix} 16 & 6 & 8 \\ 6 & 4 & 2 \\ 8 & 2 & 0 \end{pmatrix}_{ij} = X_{Q^i} + X_{H^{\mathcal{U}}} + X_{\overline{U^j}}$$

does not have a solution.

[12] P.Binétruy, S.Lavignac, S.Petcov, P.Ramond, *Nucl.Phys.* **B496** 3 (1997); *hep-ph/9610481*.

[13] J.Elwood, N.Irges, P.Ramond, *Phys.Rev.Lett.* **81** 5064 (1998); *hep-ph/9807228*.

[14] N.Haba, H.Murayama, *Phys.Rev.* **D63** 053010 (2001); *hep-ph/0009174*.

[15] M.Berger, K.Siyeon, *Phys.Rev.* **D63** 057302 (2001); *hep-ph/0010245*.

[16] M.Kakizaki, M.Yamaguchi, *JHEP* **0206** 032 (2002); *hep-ph/0203192*.

[17] W.Buchmüller, T.Yanagida, *Phys.Lett.* **B445** 399 (1999); *hep-ph/9810308*.

[18] Q.Shafi, Z.Tavartkiladze, *Phys.Lett.* **B473** 272 (2000); *hep-ph/9911264*.

[19] M.Tanimoto, *Phys.Lett.* **B501** 231 (2001); *hep-ph/0010088*.

[20] J.Sato, K.Tobe, *Phys.Rev.* **D63** 116010 (2001); *hep-ph/0012333*.

[21] H.Murayama, *Nucl.Phys.Proc.Suppl.* **111** 136 (2002); *hep-ph/0208005*.

[22] C.Froggatt, H.Nielsen, *Nucl.Phys.* **B147** 277 (1979).

[23] M.Green, J.Schwarz, *Phys.Lett.* **B149** 117 (1984).

[24] P.Ramond, R.Roberts, G.Ross, *Nucl.Phys.* **B406** 19 (1993); *hep-ph/9303320*.

[25] Y.Nir, *Phys.Lett.* **B354** 107 (1995); *hep-ph/9504312*.

[26] A.Chamseddine, H.Dreiner, *Nucl.Phys.* **B458** 65 (1996); *hep-ph/9504337*.

[27] K.Babu, I.Gogoladze, K.Wang, *Nucl.Phys.* **B660** 322 (2003); *hep-ph/0212245*.

[28] L.Ibáñez, G.Ross, *Phys.Lett.* **B332** 100 (1994); *hep-ph/9403338*.

[29] G.Bhattacharyya, *hep-ph/9709395*.

[30] H.Dreiner, *hep-ph/9707435*.

[31] J.Butterworth, H.Dreiner, *Nucl.Phys.* **B397** 3 (1993); *hep-ph/9211204*.

[32] H.Dreiner, G.Ross, *Nucl.Phys.* **B365** 597 (1991).

[33] D.Lee and R.Mohapatra, *Phys.Rev.* **D51** 1353 (1995); *hep-ph/9406328*.

[34] C.Aulakh, B.Bajc, A.Melfo, A.Rasin, G.Senjanovic, *Nucl.Phys.* **B597** 89 (2001); *hep-ph/0004031*.

[35] B.Allanach, A.Dedes, H.Dreiner, *hep-ph/0309196*.

[36] H.-P.Nilles, *Phys.Lett.* **B115** 193 (1982).

[37] A.Chamseddine, R.Arnowitt, P.Nath, *Phys.Rev.Lett.* **49** 870 (1982).

[38] R.Barbieri, S.Ferrara, C.Savoy, *Phys.Lett.* **B119** 343 (1982).

[39] E.Cremmer, P.Fayet, L.Girardello, *Phys.Lett.* **B122** 41 (1983).

[40] J.Kim, H.-P.Nilles, *Mod.Phys.Lett.* **A9** 3575 (1994); *hep-ph/9406296.*

[41] G.Giudice, A.Masiero, *Phys.Lett.* **B206** 480 (1988).

[42] I.Hinchliffe, T.Kaeding, *Phys.Rev.* **D47** 279 (1993).

[43] H.-P.Nilles, *Phys.Rept.* **110** 1 (1984).

[44] V.Ben-Hamo, Y.Nir, *Phys.Lett.* **B339** 77 (1994); *hep-ph/9408315.*

[45] H.-P.Nilles, *hep-ph/0003102.*

[46] A.Faraggi, M.Thormeier, *Nucl.Phys.* **B624** 163 (2002); *hep-ph/0109162.*

[47] T.Kobayashi, H.Nakano, *Nucl.Phys.* **B496** 103 (1997); *hep-th/9612066.*

[48] A.Faraggi, *Nucl.Phys.* **B387** 239 (1992); *hep-th/9208024.*

[49] M.Leurer, Y.Nir, N.Seiberg, *Nucl.Phys.* **B398** 319 (1993); *hep-ph/9212278.*

[50] H.Dreiner, H.Murayama, M.Thormeier, *hep-ph/0312012.*

[51] P.Binetruy, P.Ramond, *Phys.Lett.* **B350** 49 (1995); *hep-ph/9412385.*

[52] P.Ginsparg, *Phys.Lett.* **B197** 139 (1987).

[53] N.Maekawa, *Prog.Theor.Phys.* **106** 401 (2001); *hep-ph/0104200.*

[54] P.Ramond, *hep-ph/9604251.*

[55] M.Dine, N.Seiberg, X.Wen, E.Witten, *Nucl.Phys.* **B278** 769 (1986).

[56] M.Dine, N.Seiberg, X.Wen, E.Witten, *Nucl.Phys.* **B289** 319 (1987).

[57] J.Atick, L.Dixon, A.Sen, *Nucl.Phys.* **B292** 109 (1987).

[58] M.Dine, I.Ichinose, N.Seiberg *Nucl.Phys.* **B293** 253 (1987).

[59] R.Barbieri, S. Ferrara, D.Nanopoulos, K.Stelle, *Phys.Lett.* **B113** 219 (1982).

[60] J.Mira, E.Nardi, D. Restrepo, *Phys.Rev.* **D62** 016002 (2000); *hep-ph/9911212.*

[61] N.Irges, S.Lavignac, P.Ramond, *Phys.Rev.* **D58** 035003 (1998); *hep-ph/9802334.*

[62] K.Babu, T.Enkhbat, I.Gogoladze, *hep-ph/0308093*.

[63] N.Arkani-Hamed, L.Hall, H.Murayama, D.Smith, N.Weiner *Phys.Rev.* **D64** 115011 (2001); *hep-ph/0006312*.

[64] N.Arkani-Hamed, L.Hall, H.Murayama, D.Smith, N.Weiner, *hep-ph/0007001*.

[65] L.Ibáñez, *Phys.Lett.* **B303** 53 (1993); *hep-ph/9205234*.

[66] LEP Higgs Working Group, *hep-ex/0107030*.

[67] Particle Data Group (K.Hagiwara et al.), *Phys.Rev* **D66** 010001(2002).

[68] N.Haba, *Phys.Rev.* **D59** 035011 (1999); *hep-ph/9807552*.

[69] Z.Maki, M.Nagakawa, S.Sakata, *Prog.Theo.Phys.* **28** 247 (1962).

[70] S.Martin, M.Vaughn, *Phys.Rev.* **D50** 2282 (1994); *hep-ph/9311340*.

[71] G.Feinberg, P.Kabir, S.Weinberg, *Phys.Rev.Lett.* **3** 527 (1959).

[72] L.Hall, M.Suzuki, *Nucl.Phys.* **B231** 419 (1984).

[73] I.Jack, D.Jones, R.Wild, *hep-ph/0309165*.

[74] V.Barger, M.Berger, R.Phillips, T.Wöhrmann, *Phys.Rev.* **D53** 6407 (1996); *hep-ph/9511473*.

[75] B.Allanach, A.Dedes, H.Dreiner, *Phys.Rev.* **D60** 056002 (1999); *hep-ph/9902251*.

[76] B.de Carlos, P.White, *Phys.Rev.* **D54** 3427 (1996); *hep-ph/9602381*.

[77] E.Nardi, *Phys.Rev.* **D55** 5772 (1997); *hep-ph/9610540*.

[78] H.Dreiner, G.Leontaris, S.Lola, G.Ross, C.Scheich, *Nucl.Phys.* **B436** 461 (1995); *hep-ph/9409369*.

[79] Y.Grossman, H.Haber, *Phys.Rev.* **D59** 093008 (1999); *hep-ph/9810536*.

[80] S.Davidson, M.Losada, *JHEP* **5** 021 (2000); *hep-ph/0005080*.

[81] S.Davidson M.Losada, *Phys.Rev.* **D65** 075025 (2002); *hep-ph/0010325*.

[82] A.Abada, G.Bhattacharyya, M.Losada, *Phys.Rev.* **D66** 071701 (2002); *hep-ph/0208009*.

[83] B.Allanach, A.Dedes, H.Dreiner, *Phys.Rev.* **D60** 075014 (1999); *hep-ph/9906209*.

[84] H.Dreiner, G.Polesello, M.Thormeier, *Phys.Rev.* **D65** 115006 (2002); *hep-ph/0112228*.

[85] H.Dreiner, G.Polesello, M.Thormeier; *hep-ph/0207160*.

[86] A.Smirnov, F.Vissani, *Phys.Lett.* **B380** 317 (1996); *hep-ph/9601387*.

[87] A.Akeroyd, S.Recksiegel, *Phys.Lett.* **B541** 121 (2002); *hep-ph/0205176*.

[88] S.Weinberg, *The Quantum Theory of Fields, Vol.III, Cambridge University Press, UK 2000*. SPIRES entry

[89] L.Hall, A.Rašin, *Phys.Lett.* **B315** 164 (1993); *hep-ph/9303303*.

[90] L.Wolfenstein, *Phys.Rev.* **D17** 2369 (1978).

[91] S.Mikheev, A.Smirnov, *Yad.Fis* **42** 1441 (1985).

[92] K.Agashe, M.Graesser, *Phys.Rev.* **D54** 4445 (1996); *hep-ph/9510439*.

[93] J.Bagger, E.Poppitz, *Phys.Rev.Lett.* **71** 2380 (1993); *hep-ph/9307317*.

[94] V.Jain, *Phys.Lett.* **B351** 481 (1995); *hep-ph/9407382*.

[95] J.Bagger, E.Poppitz, L.Randall, *Nucl.Phys.* **B455** 59 (1995); *hep-ph/9505244*.

[96] S.Abel, *Nucl.Phys.* **B480** 55 (1996); *hep-ph/9609323*.