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A Hint of Abstraction in Mathematical Proof

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Problem-solvers create mathematical proofs with different strategies depending on the way in which a problem is posed to them. Claims to be proven can be existential ones or universal ones. Existential claims assert that there is at least one instance for which some statement is true or false (e.g., "There is a set of three consecutive odd numbers whose sum is a perfect square"). Universal claims are those which assert that something is true or false for all members of a set (e.g., "For all numbers, the following is true: Add one to a number. Then, double that result, and take two away from it. That gives you the same result as doubling the original number"). In order to adequately prove an existential claim, a problem-solver just needs to present one example which supports the claim (e.g., "1, 3, and 5 are three consecutive odd numbers whose sum is 9, the square of 3"). In contrast, a single example does not suffice for a universal claim. The proof must include some manner of abstraction--usually a representation of the problem using variables (e.g., "Let x be a number. Then (x+1)2 - 2 = 2x + 2 - 2 = 2x. So that process doubles the initial number").

When problem solvers see different types of hints, they can be alerted to diffferent levels of abstraction necessary in their proofs. Participants pay careful attention to cues in problem wording which may guide them toward using test cases (specific examples) or toward generalizing through the use of variables. Telling people to use a "general" representation elicits use of variables, but not in all contexts. Within a difficult problem-solving context (demonstrating associativity of, and lack of commutativity of matrix multiplication), the word "general" prompts greater application of abstraction through the use of variables than it does in a simpler context (demonstrating the equivalence of two equations).