Title
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Publication Date
1966-03-28

UCRL-16781

University of California

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Submitted to Physical Review for publication

UCRL-16781

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AEC Contract No. W-7405-eng-48

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ABSTRACT

In this note we assume that matrix elements of currents are dominated by vector meson intermediate states. This assumption is then used to relate pseudoscalar meson photoproduction to the (strong interaction) production of vector mesons. Numerical comparison is hindered by the lack of "overlapping" data for the above two reactions, but predictions appear to be in rough agreement with extrapolated data for photoproduction of K's and π's. If valid, this vector dominance assumption could be a useful tool for high energy photoproduction prediction.

INTRODUCTION

In this note we shall study some consequences of the assumption that matrix elements of a vector current (such as the electromagnetic current) are dominated by vector meson intermediate states. This assumption[1] will provide a relation between electromagnetic and strong-interaction matrix elements that has been previously derived from current-commutation relations.[2] The main concern of this paper, however, will be to relate photoproduction of 0^- mesons to production of vector mesons by 0^- mesons (via strong interactions). Using the idea of vector dominance, we are not evidently restricted to peripheral models of photoproduction,[3] but hopefully can estimate photoproduction from experimental data on vector meson production. Our aim here is therefore to indicate in some detail how the comparison between the two above reactions is to be carried out, and to examine briefly some preliminary data. We feel that this type of comparison should be more meaningful than a theoretically unfounded comparison of photoproduction with elastic pion-nucleon scattering. It is to be hoped that future photoproduction analysts will continue the suggested comparison as a test of the vector dominance approximation.

II. VECTOR DOMINANCE

We first consider a photon emission matrix element;[4] vector dominance is illustrated in Fig. 1a and Eq.(1a):

$$T(A \to B + \gamma) = \sum_i T(A \to B + V_i) \times \frac{e}{f_i} \quad . \tag{1a}$$

The vector mesons here have zero (4-momentum)^2 but nonzero 4-momentum; i.e. the vector meson amplitude is "off-mass-shell." The photon vector-meson coupling[5] for the ith meson is taken to be

$$\frac{eM_i^2}{f_i} \epsilon^\gamma \cdot \epsilon^{V_i} \equiv \langle V_i | J_\gamma | 0 \rangle \tag{2}$$

where ϵ are polarization 4-vectors.

Similarly, an amplitude for P-wave $\pi - \pi$ emission will be approximated as in Figure 1b and Eq. 1b :

$$T(A \to B + [\pi\pi] \text{ P - wave}) = T(A \to B + \rho) \frac{g_{\rho\,\pi\pi}}{M_\rho^2 - s_{\pi\pi}} \quad . \tag{1b}$$

Here $s_{\pi\pi} = (k_{\pi_1} + k_{\pi_2})^2$, i.e., the square of the "effective mass" of the ρ . If we extrapolate to $s_{\pi\pi} = 0$ in Eq. 1b, then the above equations imply

$$T(A \to B + [\pi\pi] \text{ P-wave, zero mass}) = \frac{g_{\rho\,\pi\pi}\, f_\rho}{eM_\rho^2} T(A \to B + \gamma_{\text{ISOVECTOR}}) \quad . \tag{3}$$

This is precisely the result of Ref. 2 , assuming $g_{\rho\pi\pi} = f_\rho$ and appropriately evaluating certain constants appearing in current commutation relations as in Reference 2.

Approximations equivalent to the above equations were used by Gell-Mann et al.[5] to estimate the rates $\omega \to 3\pi$ and $\omega \to \pi\gamma$ in terms of $\rho\,\pi\pi$ couplings--their results can be considered a particular case of Eq. 3 with $A = \omega(1^-, 785\ \mathrm{MeV})$ and $B = \pi$. For example, we have the relation

$$f_{\rho\pi\gamma} = \frac{e}{f\omega} f_{\rho\pi\omega} , \tag{4}$$

with f's defined as in Reference 2. Relations such as Eq. 4 have also been used by Berman and Drell[3] to estimate ρ photoproduction in peripheral models of the reaction.[1] It would be interesting and useful to be able to estimate photoproduction rates without the restriction to peripheral reaction models; we will use vector dominance in this context in the next section.

III. PHOTOMESON PRODUCTION AND PRODUCTION OF VECTOR MESONS

In this section we use Eq. 1a to relate single pion photoproduction to π + nucleon $\rightarrow \rho$ + nucleon; we therefore take Eq. 1a in the following form:

$$T(\gamma\ P \rightarrow \pi_\alpha\ B_\beta) \approx \sum_{\rho,\omega,\phi} \frac{e}{f_i}\ T(\pi_\alpha\ B_\beta \rightarrow V_i\ P) \quad . \tag{5}$$

Here α, β are the charge (or SU_3) indices of 0^- meson π_α and baryon B_β. In Eq. 5 we have an amplitude for an "off-mass-shell" vector meson with zero "mass". We assume that we may instead use an on-mass-shell amplitude (square of 4-momentum = actual vector rest mass) without incurring any serious error. This assumption appears plausible when all energies are large compared with the vector rest mass; we further justify this assumption in Appendix A.

Empirically, we note strong suppression of the $\phi(1^-, 1020$ MeV) production amplitudes (both in π and K reactions) and therefore drop the term in ϕ. For π photoproduction, it seems likely that the ω term will be small, for we know that:

(a) From[5] SU_3, $f_\omega \approx f_\rho\ \sqrt{3}/\sin\theta$ where the[6] ω - ϕ mixing angle $\theta = \sin^{-1}(0.6)$.

(b) The rate π + nucleon $\rightarrow \omega$ + nucleon seems slightly smaller than ($\rightarrow \rho$ + nucleon).

If the ω and ρ amplitudes interfere constructively, we might expect perhaps a 40% contribution to cross-sections from the interference; we shall bear this in mind, but for convenience we will now drop the ω term for π photoproduction. From all these assumptions we obtain

$$\frac{d\sigma}{d\Omega}(\gamma P \to \pi_\alpha B_\beta) \approx r \frac{e^2}{f_\rho^2} \frac{d\sigma}{d\Omega}(\pi_\alpha B_\beta \to \rho^0 P) , \qquad (7)$$

where

$$r = \frac{(\pi P) \text{ phase space}}{(\rho P) \text{ phase space}} \times \frac{1}{2} \frac{\sum \text{transverse } \rho \text{ helicity } |T|^2}{\sum \text{all } \rho \text{ helicity states } |T|^2} .$$

The last factor of r is due to the purely transverse nature of the photon; the 1/2 comes from an average over incident photon spins. In terms of the ρ production density matrix X in the ρ helicity representation in c.m. we have

$$r = \frac{1}{2} (\text{phase space ratio, } \approx 1)(1 - X_{00}) . \qquad (8)$$

In practice, although experiment furnishes our X, experimentalists usually present their data via a density matrix in the "magnetic quantum number" representation in the vector meson rest frame, with the quantization axis parallel the direction of the incident π momentum as seen in the ρ rest frame. The simple transformation to a helicity representation is given in Appendix B. From the data, we have X_{00} between 0.5 and 0.7 near 0° production angle, and approximately 0.5 at 60° c.m. for 4 GeV/c lab π momentum. With $X_{00} \doteq 1/2$, and the estimate[5] $f_\rho^2 \approx 25$, we obtain

$$\frac{d\sigma}{d\Omega}(\gamma P \to \pi_\alpha B_\beta) \approx 10^{-3} \frac{d\sigma}{d\Omega}(\pi_\alpha B_\beta \to \rho^o P) \quad . \qquad (9)$$

We first consider π^o production; experimentally, one can ascertain relatively cleanly that a measured π^o did not result from a photoproduced ρ. Since $\pi^o P$ is not experimentally accessible, we use isospin conservation (see Appendix C) to obtain

$$d\sigma(\pi^o p \to \rho^o P) = \frac{1}{2}\left[d\sigma(\pi^- p \to \rho^- p) + d\sigma(\pi^+ p \to \rho^+ p) - d\sigma(\pi^- p \to \rho^o n)\right] \quad . \qquad (10)$$

If we wish to study $\gamma p \to \pi^+ n$, we use the relations C2 to obtain

$$\frac{d\sigma}{d\Omega}(\gamma p \to \pi^+ n) = \left[2(1 - X_{00}) \text{ assumed} \approx 1 \text{ here}\right] \times 10^{-3} \frac{d\sigma}{d\Omega}(\pi^- p \to \rho^o n) \quad . \qquad (11)$$

If we were to assume that at large angles, vector exchange dominates (as is believed to be the case in K^* production), then only ω exchange is relevant, which implies that

$$d\sigma(\pi^- p \to \rho^o n) \approx 0 \qquad (12a)$$

and

$$d\sigma(\pi^o p \to \rho^o p) \approx d\sigma(\pi^- p \to \rho^- p) \quad . \qquad (12b)$$

In fact, at small momentum transfer, $\pi \to \rho^o$ is the largest observed rate.[7] Furthermore, if we accept available π^+ data as genuine single photoproduction, then the observed rates at 90°cm are approximately equal for π^{+} and π^{0}. This last observation, together with Eqs. 11 and 12, would seem to rule out vector exchange dominance at large angles, which is in agreement with tentative ρ production data. If we accept the rough equality of $\gamma \to \pi^{+}$ and $\gamma \to \pi^{0}$ at $\approx 90^{\circ}$c.m., then we also have implied the relation

$$d\sigma(\pi^{-} \to \rho^{-}) + d\sigma(\pi^{+} \to \rho^{+}) \approx 3 \quad d\sigma(\pi^{-} \to \rho^{0})$$

at large angles, which would be worth checking in the future.

We will now turn to numerical comparison, using the above equations.

IV. NUMERICAL COMPARISON: π PRODUCTION

Data: We shall refer the reader to references 9 and 10 for relevant photoproduction data and present only a few pertinent numbers here. It is known that at $\approx$ 4 GeV/c

$$\frac{d\sigma}{d\Omega}\left(\gamma p \to \begin{matrix}\pi^+ n & (75\text{-}85^\circ) \\ \pi^0 p & (90^\circ)\end{matrix}\right) \approx 2 \times 10^{-3} \ \mu b/sr \ . \qquad (14a)$$

Unfortunately, ρ production data do not yet exist anywhere near such large momentum transfer. We also know at 4 GeV/c:

$$\frac{d\sigma}{d\Omega}\left(\gamma p \to \pi^0 p(60^\circ)\right) \approx 0.04 \ \mu b/sr, \qquad (14b)$$

$$\frac{d\sigma}{d\Omega}\left(\gamma p \to \pi^+ n(55^\circ)\right) \approx 0.07(?) \ \mu b/sr \ . \qquad (14c)$$

The latter figure is extrapolated from data of Reference 10 where data are presented averaged over either energy or angle intervals over which $d\sigma$ may vary by a factor of 3. A similar extrapolation gives the estimate

$$\frac{d\sigma}{d\Omega}\left(\gamma p \to \pi^+ n \ (30^\circ, \ 4 \ GeV/c)\right) \approx 0.4 \ \mu b/sr \ .$$

The production data for $\pi p \to \rho n$ are also rather sparse at large momentum transfers. For 4 GeV/c,

$$\frac{d\sigma}{d\Omega}\left(\pi^- p \to \rho^- n\right) \approx 1.7 \ mb/sr \ \text{near} \ t = 0 \ . \qquad (15a)$$

Based on the shape of the forward peak for this reaction (as seen in unpublished[11a] 3.2-GeV/c data) we estimate that for[@] 4 GeV and 60° c.m.,

$$\frac{d\sigma}{d\Omega}(\pi^- p \to \rho n) \approx \frac{1.7}{40} \text{ mb/sr} \quad . \tag{15b}$$

<u>Predictions:</u> At small angles it is known for 4-GeV/c data that $d\sigma(\pi^- \to \rho^0) \geqslant d\sigma(\pi^- \to \rho^-)$, so that from 15a we have the prediction

$$\frac{d\sigma}{d\Omega}(\gamma p \to \pi^+ n \;;\; 4 \text{ GeV/c})$$

$$\geqslant 2 \ \mu\text{b/sr} \quad \text{at} \quad t = 0 \tag{16a}$$

$$\text{and} \approx 0.04 \ \mu\text{b/sr} \quad \text{at} \quad 60^\circ \quad . \tag{16b}$$

The latter number is to be compared with Eq. 14c. If we now guess[7] that in Equation 10 the right-hand side is roughly $1/2 \ d\sigma(\pi^- \to \rho^-)$, than we predict that

$$\frac{d\sigma}{d\Omega}(\gamma p \to \pi^0 p \;;\; 60^\circ,\ 4 \text{ GeV/c}) \approx 0.02 \ \mu\text{b}, \tag{17}$$

even without any additional contribution from the ω intermediate state. This number is to be compared with Eq. 14b. We thus see agreement to within a factor 2 with crudely extrapolated data.

Vector dominance also predicts that $\gamma \to \pi$ exhibits a forward peak shaped like the forward peak in $\pi \to \rho$. The situation with regard to forward peaks is as follows:

(a) $d\sigma(\gamma \to \pi)$ is approximately[10] given by

$$d\sigma \approx \exp[3t]$$

(b) $d\sigma(\pi \to \pi) \approx \exp[+ 9t]$

(See the bibliography of Reference 10).

(c) $d\sigma(\pi \to \rho)$: the result of the European collaboration[11b] was, for $\pi^- \to \rho^0$, $d\sigma \approx \exp[9t]$; however, this exponential begins to fall appreciably below data around $t \approx 0.4\ GeV^2$. The same group also found a considerably broader peak for $\pi^+ \to \rho^+$. We feel that $\pi \to \rho$ data exhibit a definitely broader forward peak than $\pi \to \pi$, but clearly more data are necessary to clarify this point.

V. K PHOTOPRODUCTION

We shall now discuss K photoproduction in a similar manner to π photoproduction. For K production, the ω contribution turns out to be important, so we write Eq. 1b in the form:

$$T(\gamma p \to K^+\Lambda) \approx \frac{e}{f_\rho} T(\rho^0 p \to K^+\Lambda) + \frac{e}{f_\omega} T(\omega p \to K^+\Lambda) \quad . \tag{18}$$

If we furthermore assume that the reactions involved are peripheral, in the sense that the amplitudes have unique t-channel quantum numbers, but no unique s-channel isospin,[12] then we can rewrite Eq. 18 :

$$T(\gamma p \to K^+\Lambda) \approx \frac{e}{f_\rho} T(K^- p \to \rho\Lambda) + \frac{e}{f_\omega} T(K^- p \to \omega\Lambda) \ . \tag{19}$$

Motivated by SU_3 predictions for relative signs of coupling constants for meson exchanges (and unable to experimentally determine relative phases of ω and ρ reactions), we tentatively assume that both amplitudes in 19 have the same phase.

The only statistically reliable data available for the relevant reactions are at 2.45-GeV/c (p_K lab), where[13]

$$\begin{aligned} \frac{d\sigma}{d\Omega}\left(K^- p \to \Lambda\omega\right) &\approx 0.07 \ \text{mb/sr}, \\ \frac{d\sigma}{d\Omega}\left(K^- p \to \Lambda\rho^0\right) &\approx 0.04 \ \text{mb/sr} \end{aligned} \tag{20}$$

at forward angles.

From these data we infer that $T_\omega / T_\rho \approx 4/3$, and therefore, using our earlier assumptions about f_ω and f_ρ, we shall approximate 18 by

$$T(\gamma p \to K^+\Lambda) \approx \frac{e}{f_\rho} \quad 1.1\, T(K^- p \to \Lambda\omega) \quad . \tag{19'}$$

Assuming $X_{00} \approx 1/2$ for both reactions (a reasonable approximation to the tentative data), we have

$$\begin{aligned} \frac{d\sigma}{d\Omega}(\gamma p \to K^+\Lambda) &= 10^{-3} \times 1.2 \left[\frac{d\sigma}{d\Omega}(K^- p \to \Lambda\omega) \approx 0.07 \text{ mb} \right] \\ &\approx 0.085 \ \mu\text{b/sr} \quad \text{at} \quad 0^\circ \ . \end{aligned} \tag{21}$$

The data on $K^- p \to \omega\Lambda)$ imply that

$$d\sigma)_{45^\circ} \approx \frac{4}{7}\, d\sigma)_{0^\circ} \ ,$$

from which we predict that at 2.45 GeV/c and 45° c.m. production angle

$$\frac{d\sigma}{d\Omega}(\gamma p \to K^+\Lambda) \approx 0.05 \ \mu\text{b/sr} \quad . \tag{22}$$

Again, we find no photoproduction data at this energy.[9] At 3.6-GeV/c,

$$\frac{d\sigma}{d\Omega}(\gamma p \to K^+\Lambda)_{45^\circ} \approx 0.05 \ \mu\text{b/sr} \quad . \tag{23}$$

Now, at 60° the π° photoproduction cross section increases by 2 in going from 3.6 to 2.5 GeV/c , which leads to an estimate that extrapolated data imply [14]

$$\frac{d\sigma}{d\Omega}\,(\gamma p \to K^{+}\Lambda \ ;\ 2.5\ \mathrm{GeV/c}\ ,\ 45^{\circ}) \approx 0.1\ \ \mu b/sr\ . \qquad (24)$$

Thus our prediction is again within a factor of 2 of crudely extrapolated data. This situation we feel merits further analysis when more data are available; it is nontrivial to be within a factor of 2 when differential cross sections exhibit diffraction structure.

VI. LEPTON PAIR PRODUCTION--ANOTHER APPLICATION

Lepton pairs may be assumed to arise from an off-mass-shell virtual photon. Here we adopt the approximation that the virtual photon comes from a vector meson state; in effect we assume that lepton pairs come mainly from vector meson decays, at least when the invariant mass of the lepton pair is close to the mass of a typical vector meson. With this assumption (actually, we must really include interference effects again) we obtain, for example,

$$\frac{d\sigma}{d\Omega}\,(\pi N \rightarrow e^+e^-\, N'\ ;\ \text{averaged over } d\Omega e^+) = \frac{\Gamma_i(e^+e^-)}{\Gamma_i(\text{total})}\,\frac{d\sigma}{d\Omega}\,(\pi N \rightarrow V_i + N')\,. \tag{25}$$

For more detailed discussions of resultant lepton distributions, the reader is referred to References 15 and 16. Another process closely related to lepton pair production is electroproduction. For the latter reaction we would assume that a virtual photon comes from the incident lepton, and essentially transforms to a vector meson state, which then scatters off the target. It is not our intention, however, to further discuss these lepton reactions here.

VII. CONCLUSION

We acknowledge that these ideas are very simple; they have undoubtedly occurred to many people, but the author does not recall seeing such comparisons as suggested here in the past literature.[1] The content of these photoproduction predictions is not trivial, as both momentum-transfer and energy dependence are predicted to duplicate vector-production features. A comparison between photoproduction and elastic $\pi - p$ scattering (at fixed angle and varying energy) has appeared,[9] but we feel this comparison to be theoretically unfounded. A comparison of reactions at all momentum transfers will constitute a test of whether $\pi \to \rho$ or $\pi \to \pi$ is more relevant, because the forward diffraction peaks are differently shaped for $\pi \to \rho$ as compared with $\pi \to \pi$ (see Section 3).

Finally, we emphasize that even the approximate validity of the vector-dominance assumption would provide us with a useful tool for preparation and analysis of wide-angle photoproduction experiments in a range of momentum transfer where absorption-corrected peripheral[7] models are least reliable.

APPENDIX A. OFF-MASS SHELL EFFECTS AND GAUGE INVARIANCE

We first motivate our assumption that off-mass-shell extrapolation encounters no rapid variation in matrix elements by considering the transverse polarization amplitudes for one-meson exchange. For π exchange we obtain

$$d\sigma \approx \frac{(-t) \text{ from baryon vertex}}{(t - \mu^2)^2} \times (\text{phase-space}) \times \left[\sum_{\lambda \text{ transverse}} |\epsilon^\lambda \cdot q_\pi|^2 \equiv |q|^2 \sin^2 \theta_{c.m.} \right] . \quad \text{(A1)}$$

It is evident here that there is no strong dependence on the vector mass. At high energies, fixed t and fixed $\theta_{c.m.}$ are very little different, and also the available phase space is insensitive to rest masses. In fact,

$$(\pi p) \text{ phase-space} \,/\, (\rho p) \text{ phase space} \approx 1.07 \text{ at } 5 \text{ GeV/c} \quad \text{(A2)}$$

It is rather the longitudinal helicity amplitudes that are mass-sensitive, since

$$\epsilon_\mu^{\lambda = 0} = \frac{M}{|q|} \delta_{\mu 0} - \frac{q_0}{|q| M} q_\mu . \quad \text{(A3)}$$

One can similarly check that off-mass-shell effects are small for vector exchange. On the basis of these perturbation theory properties, we infer that our off-mass-shell extrapolation is fairly "safe".

It is also meaningful to question whether the vector dominance approximation is a gauge invariant assumption; this question applies equally well to peripheral models[3] of photoproduction. If (as pointed out in Reference 3) the vector mesons couple to conserved currents (as in the case in the exact SU_3 limit, and is probably the case for the ρ coupling to isospin), then our assumption can be stated

$$T(\gamma + \text{etc}) \approx \epsilon_\mu^{\ \gamma} J_V^{\ \mu}, \tag{A4}$$

where $J_V^{\ \mu}$ is conserved, i.e.

$$k_\mu J_\mu(k^2) = 0 \, , \text{ all } k^2 \, .$$

The current J_V is the same current that couples to the ρ. Equation A4 suffices to ensure gauge and Lorentz invariance of the sum over squares of transverse amplitudes. Incidentally, the Born vector-exchange amplitude is automatically gauge invariant.

If A4 is satisfied, the invariant result obtained is of the form

$$\sum_{\text{transverse spins}} (k^2 = 0) \equiv |J_{\text{transverse}}(k^2 = 0)|^2,$$

$$\sum_{\text{all spins}} (k^2 = M_\rho^{\ 2}) \equiv |J_{\text{transverse}}(k^2 = M_\rho^{\ 2})|^2 + \frac{M^2\rho}{|\vec{k}|^2} |J_0(k^2 = M_\rho^{\ 2})|^2 \, . \tag{A5}$$

Our assumptions are thus that $|J_{\text{transverse}}(k^2)|^2$ is relatively insensitive to k^2.

APPENDIX B. DENSITY MATRIX TRANSFORMATIONS

In Section 2 we described the conventional coordinate system for density matrix analysis of ρ production. We transform back to the c.m. system and a helicity representation in two steps:

(a) Rotate in ρ rest frame to align quantization axis with "direction of motion" of the ρ.

(b) Lorentz-transform along the direction of motion back to the c.m. frame.

As transformation (b) does not mix helicity states, we need only consider (a), which is a simple rotation. If the c.m. ρ 4-momentum is p_μ then we reach the ρ rest frame by a Lorentz boost with $\beta = |p|/p_0$. If q_μ is the c.m. π 4-momentum, and the direction of p is defined by unit vector e, then we have to rotate an angle θ', where

$$\tan \theta' = q'(\perp \text{ to } e) / q'(\parallel \text{ to } e)$$

$$= \frac{\sin \theta_{c.m.}}{\frac{p_0 q_0}{M_\rho q} - \frac{p}{M_\rho} \times \cos \theta_{c.m.}} , \qquad \text{(B1)}$$

where all quantities are in c.m. in the last expression.

APPENDIX C. ISOSPIN RELATIONS

In the following we present some relations between cross sections which follow from isospin conservation. We first observe that[8]

$$
\begin{aligned}
\left|{\pi^+ \atop \rho^+}\, n\right\rangle &= \sqrt{\tfrac{1}{3}}\ \left|I = 3/2\right\rangle + \sqrt{\tfrac{2}{3}}\ \left|I = \tfrac{1}{2}\right\rangle, \\
\left|{\pi^0 \atop \rho^0}\, p\right\rangle &= \sqrt{\tfrac{2}{3}}\ \left|I = 3/2\right\rangle - \sqrt{\tfrac{1}{3}}\ \left|I = \tfrac{1}{2}\right\rangle, \\
\left|{\pi^0 \atop \rho^0}\, n\right\rangle &= \sqrt{\tfrac{2}{3}}\ \left|I = 3/2\right\rangle + \sqrt{\tfrac{1}{3}}\ \left|I = \tfrac{1}{2}\right\rangle, \qquad \text{(C1)} \\
\left|{\pi^- \atop \rho^-}\, p\right\rangle &= \sqrt{\tfrac{1}{3}}\ \left|I = 3/2\right\rangle - \sqrt{\tfrac{2}{3}}\ \left|I = \tfrac{1}{2}\right\rangle, \\
\left|{\pi^+ \atop \rho^+}\, p\right\rangle &= \left|I = 3/2\right\rangle .
\end{aligned}
$$

With an obvious symbolic notation we now have

$$
\begin{aligned}
d\sigma(\pi^- p \to \rho^- p) &= \tfrac{1}{9}\left[\left|A_{3/2}\right|^2 + 4\,\mathrm{Re}\,A^*_{3/2}\,A_{1/2} + 4\left|A_{1/2}\right|^2\right], \\
d\sigma(\pi^- p \to \rho^0 n) &= \tfrac{2}{9}\left[\left|A_{3/2}\right|^2 - 2\,\mathrm{Re}\,A^*_{3/2}\,A_{1/2} + \left|A_{1/2}\right|^2\right], \\
d\sigma(\pi^+ p \to \rho^+ p) &= \left|A_{3/2}\right|^2 , \qquad \text{(C2)} \\
d\sigma(\pi^0 p \to \rho^0 p) &= \tfrac{1}{9}\left[4\left|A_{3/2}\right|^2 + 4\,\mathrm{Re}\,A^*_{3/2}\,A_{1/2} + \left|A_{1/2}\right|^2\right], \\
d\sigma(\pi^+ n \to \rho^0 p) &= \tfrac{2}{9}\left[\left|A_{3/2}\right|^2 - 2\,\mathrm{Re}\,A^*_{3/2}\,A_{1/2} + \left|A_{1/2}\right|^2\right] .
\end{aligned}
$$

From the first four relations of C2 we immediately obtain Eq. 10.

FOOTNOTES AND REFERENCES

* This work was done under the auspices of the U. S. Atomic Energy Commission.

1. An extension of this idea has been used to obtain mass and magnetic moment relations by P. G. O. Freund, Phys. Rev. Letters 16, 291 (1966). After this work was completed we also received a preprint from Dr. Freund making suggestions of the type considered here with applications to vector meson photoproduction.

2. K. Kawarabayashi and M. Suzuki, Phys. Rev. Letters 16, 255 (1966).

3. S. Berman and S. D. Drell, Phys. Rev. 33, B791 (1964).

4. We write the S matrix

$$S = 1 + i(2\pi)^4\, \delta^4(\ )\ \Pi\ (2Ei)^{-1} \times T\ .$$

5. M. Gell-Mann, D. Sharp, and W. G. Wagner, Phys. Rev. Letters 8, 263 (1962).

6. R. F. Dashen and D. Sharp, Phys. Rev. 133, B1585 (1964).

7. J. D. Jackson et al., Phys. Rev. 139, B428 (1965). The data, as summarized here, does not yet permit any reliable evaluation of Eq. (10). S. D. Drell and M. Jacob, Phys. Rev. 138, B1312 (1965).

8. See any standard table of Clebsch-Gordan coefficients.

9. L. S. Osborne, Springer Tracts in Physics, Vol. 39, 91 (1965).

10. V. B. Elings et al., Phys. Rev. Letters 16, 474 (1966).

11a. L. Jacobs, Lawrence Radiation Laboratory, private communication.

11b. Aachen et al. collaboration; Nuovo Cimento 31, 729; 34, 495 (1964).

12. If we think of t-channel exchange dominating (e.g. K, K^*), then Eq. (19) follows trivially from Eq. (18) except for differences in final-state interactions.

13. S. Flatte, Lawrence Radiation Laboratory, private communication and Dubna Conference (1964) paper.

14. An alternative extrapolation of data starts from the data of C. Peck, Phys. Rev. 135, B830 (1964), where $\frac{d\sigma}{d\Omega}(45^{\circ}, 1.2\mathrm{GeV}, \gamma P \to K + \Lambda) \approx 0.3 \mu b/sr$. Assuming the same energy dependence as wide-angle π^{0} photoproduction, we have $d\sigma \approx 0.06\ \mu b/sr$ at 2.5 GeV/c.

15. D. Beder, Distribution of Lepton Pairs from Vector Meson Decays, California Institute of Technology Report, CALT-68-43 (1965) (unpublished).

16. R. J. Oakes, Muon Pair Production in Strong Interactions, Stanford preprint ITP-198, 1966.

FIGURE CAPTIONS

Fig. 1a: Vector dominance illustrated for the electromagnetic current.

Fig. 2b: Vector dominance illustrated for the $[\pi,\pi]$ (isospin) current.

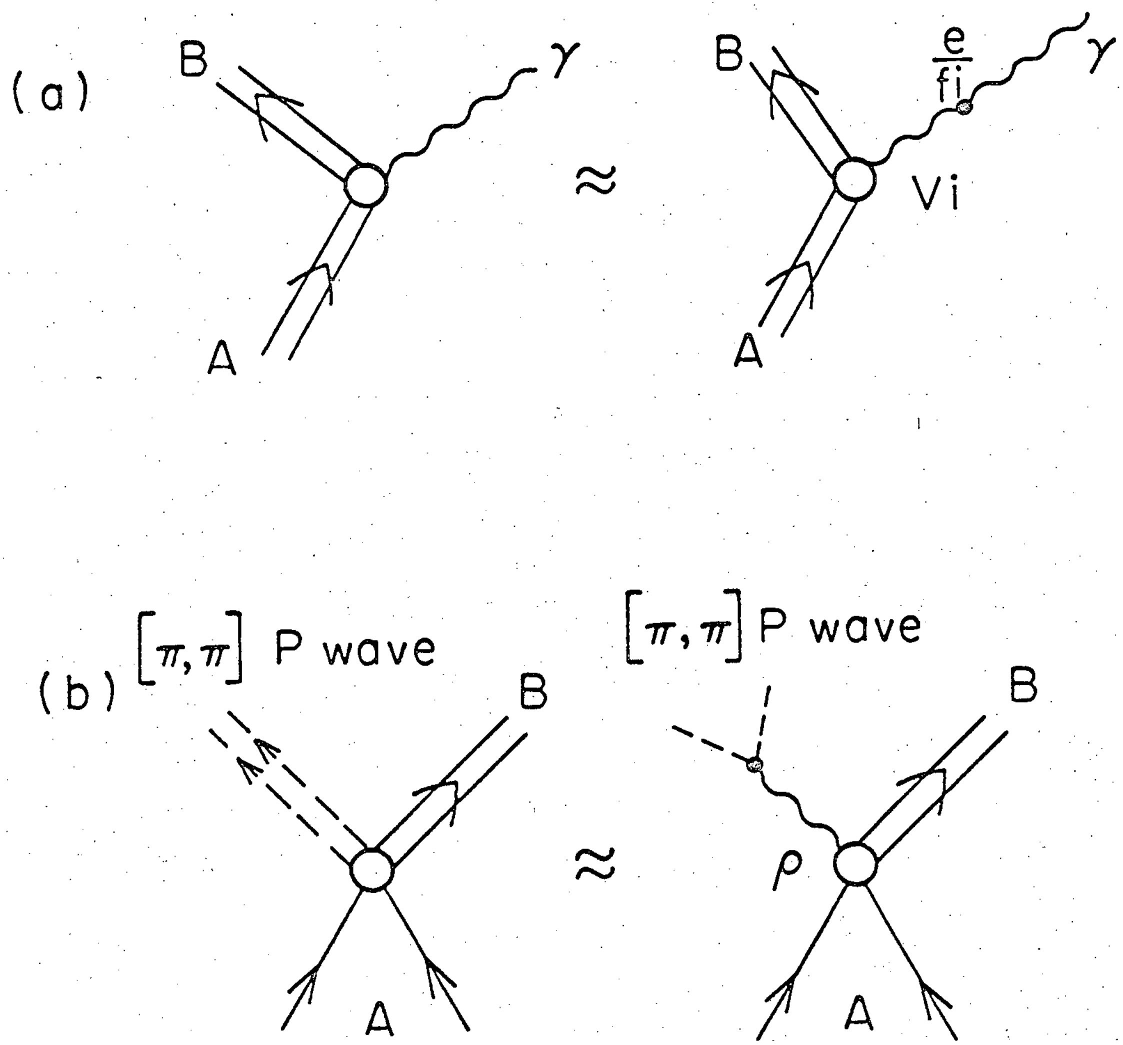

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Fig. 1