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INTRODUCTION

The giant E1 photo-nuclear resonance has now been studied for a large number of nuclides. In particular, there appears to be experimental indication that the giant E1 resonance exhibits a split peak in strongly deformed nuclei.[1] Danos[2] and Okamoto[3] explained this effect qualitatively in terms of a hydrodynamic model with different characteristic frequencies along the major and minor axis of the nuclear spheroid. This effect was then also calculated on the basis of the independent-particle picture by Wilkinson[4] and by Mottelson and Nilsson[5] using the single-particle wave functions of an anisotropic harmonic oscillator.

Such a simple description of the E1 giant resonance in terms of pure unperturbed single-particle excitations now appears to be contradicted by the empirical fact that the characteristic resonance energy is of the order of a factor of 1.5 larger than the spacing between two oscillator shells, $\hbar\omega$.[6] Already several years ago Elliott and Flowers[7] were able to explain the photo-excitation spectrum of O^{16} by the perturbation of the simple shell-model states by a residual two-body force of finite range and containing exchange mixture. The resulting mixing of configurations is associated with the pushing of the 1^- states with T=1 generally above $\hbar\omega$ and the T=0 states below $\hbar\omega$ on the average. In particular, the two highest-lying E1 states were found to absorb almost all of the E1 oscillator strength and thus together constitute a true giant state.

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** On leave of absence from the University of Lund, Sweden.

Recently, Brown and Bolsterli[8] proposed a very schematic but suggestive picture of the underlying mechanism of shell-model configuration mixing causing the El resonance to occur. In a representation of single-particle states where all El single-particle matrix elements are of equal sign (furthermore provided the matrix elements are of roughly equal magnitude) a coherent excited state, collecting the main El oscillator strength, is obviously the linear combination of single-particle excitations with the amplitudes being roughly equal and all of one relative sign. Such an excited state of one-particle character is indeed also the highest-lying 1^- state provided all matrix elements of the residual interaction are of the same magnitude and sign in this representation. Brown and Bolsterli treated the case of a residual Wigner force of zero range. However, the specific isobaric spin character of the El state as well as the antisymmetrisation of the nuclear wave function are neglected in the first of the papers of Ref. 8. A conjectured change of the over-all sign of the interaction matrix elements qualitatively compensates for the mentioned effects.

Recently the approach based on the Random-Phase-Approximation has been applied with success to the study of certain types of collective states of nuclei. The employed formalism was first developed in the early papers by Sawada[9] and his collaborators in order to treat the electron gas problem. The corresponding methods were introduced into nuclear physics independently by Takagi,[10] Fallieros,[11] Mottelson,[12] and others. This latter type of theory has the distinction relative to the shell-model calculations that it accounts approximately for the effects of correlations in the ground state. These effects are sometimes discussed in terms of the so-called backward-going graphs which means that single-particle deexcitations are considered in addition to the excitations, i.e. the lifting of a particle from below the Fermi surface to above it, as considered in shell-model calculations.

In the present paper we shall employ the density matrix formulation[13,14] which is especially suitable for the study of higher order nonlinearity effects.

As the specific T=1 character of the giant resonance state appears to be of primary importance, it is of particular interest to study nuclei with such a low Z- value that the isobaric spin is a good constant of the motion. Therefore our first choice has been Mg^{24} which is a prolate nucleus and for which the adiabatic coupling scheme seems well established. In addition, as an example of a possibly oblate nucleus, we have considered C^{12}.

THE RANDOM-PHASE-APPROXIMATION TREATMENT OF THE RESIDUAL INTERACTIONS

We consider a Hamiltonian of the form

$$H = H_o + \Delta V \tag{1}$$

where

$$H_o = \sum_{\mu} \epsilon_{\mu} a^{+}_{\mu} a_{\mu} \tag{2}$$

is the single-particle part of the Hamiltonian, including the shell-model field, and where

$$\Delta V = 1/2 \sum_{\alpha\alpha'\beta\beta'} \langle \alpha\beta | V | \alpha'\beta' \rangle \, a^{+}_{\alpha} a^{+}_{\beta} a_{\beta'} a_{\alpha'} \tag{3}$$

is the residual interaction, i.e. the part of the interaction that is not already included in the field.

The two-body interaction V refers to space, spin and isobaric-spin quantum states of both particles involved in the interaction

$$V = V(\vec{r}(1), \vec{\sigma}(1) \vec{\tau}(1), \vec{r}(2) \vec{\sigma}(2) \vec{\tau}(2)). \tag{4}$$

In the absence of a residual interaction, ΔV, the ground state is an eigenvector of H_o corresponding to a sharp Fermi surface in terms of the shell-model wave functions. In the presence of a residual interaction the ground state may contain correlations. As seen from (3), ΔV scatters particles out of the Fermi sea. Obviously such hole-particle pair excitations have to comply with parity and angular-momentum conservation of the ground state.

Our main interest is, however, not the ground state but the excited giant El state. As the ground state appears to be a complicated linear combination of various one-two- etc. particle excitations, also the El state may have a complicated structure. However, we are concerned only with its relation to the ground state. We will see that the simple Random-Phase-Approximation is largely equivalent to the contention that, relative to the correlated ground state, denoted $\|0\rangle$, the "collective state" $|E\rangle$ is of the simple type

$$|E\rangle = (\mathcal{B}^E)^+ \|0\rangle = \sum_{\nu\nu'} c^E_{\nu\nu'} \hat{\rho}_{\nu\nu'} \|0\rangle \tag{5}$$

where $c^E_{\nu\nu'}$ should be considered as variational parameters and where

$$\hat{\rho}_{\nu\nu'} = a^+_{\nu'} a_\nu \tag{6}$$

is the density matrix operator connected with the promotion of a particle from the state ν to another state ν'.

The "collective" states thus correspond to a linear combination of one-particle excitations relative to the ground state. The assumption of a state of this particular character to approximate the physical state in a way already employs some empirical knowledge about the collective state. As this state (or narrow group of a few states) may be reached from the ground state by an El excitation that to a large extent exhausts the total sum rule strength, and as the electromagnetic interaction can excite only one particle at a time

to lowest order in $e^2/\hbar c$, the character of the collective excitation is empirically rather well specified as being dominantly of the type expressed by Eq. (5).

The problem of finding the coefficients $c^E_{\nu\nu'}$ is equivalent to the problem of finding the matrix elements of $\hat{\rho}_{\nu\nu'}$ between the ground state $\|0\rangle$ and the "collective" state $|E\rangle$.

The operators $\hat{\rho}_{\nu\nu'}$ fulfill the equations of motion

$$i\hbar \frac{\partial \hat{\rho}_{\nu\nu'}}{\partial t} \equiv \left[\hat{\rho}_{\nu\nu'}, H\right] = \left[\hat{\rho}_{\nu\nu'}, H_o\right] + \left[\hat{\rho}_{\nu\nu'}, \Delta V\right] \tag{7}$$

The first commutator on the right hand side is easily evaluated to be

$$\left[\hat{\rho}_{\nu\nu'}, H_o\right] = \left(\epsilon_\nu - \epsilon_{\nu'}\right) \hat{\rho}_{\nu\nu'} \tag{8}$$

The details of the evaluation of the second commutator may be found in Ref. 13.

Equation (7) multiplied by $\langle E|$ from the left and by $\|0\rangle$ from the right takes the form

$$\left(\epsilon_{\nu'} - \epsilon_\nu - E\right) \langle E|\hat{\rho}_{\nu\nu'}\|0\rangle = \sum_{K K' s} \Big(\langle \nu K'|V|sK\rangle \langle E|\hat{\rho}_{KK'}\hat{\rho}_{s\nu'}\|0\rangle - \langle sK'|V|\nu' K\rangle \langle E|\hat{\rho}_{\nu s}\hat{\rho}_{KK'}\|0\rangle \Big) \tag{9}$$

where

$$\langle \nu K'|V|sK\rangle = \int \psi^*_\nu(1)\, \psi^*_{K'}(2)\, V\, \psi_s(1)\, \psi_K(2)\, d\tau_1 d\tau_2 \tag{10}$$

This equation thus relates the matrix elements of one ρ to those of a product of two ρ's. By exactly the same procedure an analogous equation can be formed relating two-ρ aggregates to three-ρ aggregates, etc. In the simple Random-Phase-Approximation (RPA) one confines oneself to Eq. (9), which is then linearized according to a certain prescription. In this way Eq. (13) is obtained (see below). Thus quadratic terms of matrix elements of $\hat{\rho}_{\nu\nu'}$ with

$\nu' \neq \nu$ are neglected as being small of a higher order. Although there are many of these terms, they are assumed to be negligible as they occur incoherently, i.e., with random phases.* Furthermore the Fermi surface is considered as nearly sharp and fluctuations in occupation numbers are neglected.

Although Eq. (9) is an exact equation that is valid for a more general state $|E\rangle$ than that assumed in Eq. (5), the assumption of the appropriateness of the linearization procedure is equivalent to the assumption of a more simple collective state $|E\rangle$ in accordance with Eq. (5).

Obviously Eq. (9) simplifies for the amplitudes $\langle E|\hat{\rho}_{\nu\nu'}\|0\rangle$ with $|E\rangle$ defined from Eq. (5), as nonnegligible matrix elements on the right hand side of Eq. (9) for an approximately sharp Fermi surface

$$\langle E|\hat{\rho}_{\kappa\kappa'}\hat{\rho}_{s\nu'}\|0\rangle \equiv \langle E|a^{+}_{\kappa'}a_{\kappa}a^{+}_{\nu'}a_{s}\|0\rangle \tag{11}$$

can occur only if one of the indices of the destruction operators equals one of the indices of the creation operators. In such a case apparently two of the Fermi operators together represent an occupation number operator for a particular single-particle state.

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* The effect of such two-pair excitations neglected in the RPA could be examined, e.g. by an extension of the Random-Phase-Approximation - the "Higher RPA" - consisting of a closed system of equations of motion also connecting the two- and three-operator products where the three-ρ aggregates are in turn linearized. In general there are many more two-ρ aggregates than one-ρ. However, as the components most important in the El problem are connected with the basic energy $\hbar\omega$, the two-pair transitions will mostly correspond to a basic energy $3\hbar\omega$. Therefore on the average the latter are discriminated against energetically. Furthermore, as pointed out above, it is generally conjectured that they occur with random phases, which would ensure their rather small net contribution.

In the further approximation that the Fermi surface is nearly sharp and the "true" ground state $\|0\rangle$ can be considered an eigenvector of the occupation operator, the amplitude (11) with e.g., $s=\nu'$ reduces to

$$\langle E|\hat{\rho}_{KK'}\hat{\rho}_{\nu'\nu'}\|0\rangle \cong n_{\nu'}\ \langle E|\hat{\rho}_{KK'}\|0\rangle \tag{12}$$

where $n_{\nu'}$ is the occupation number of the single-particle state ν'. (For the second term on the right hand side of Eq. (11) we furthermore assume that also $|E\rangle$ behaves approximately as a sharp Fermi surface when acted upon by the occupation number operator $\hat{\rho}_{\nu\nu}$.)

The fact that an occupation number operator $a^{+}_{\nu'}a_{\nu'}$ can occur in (11), alternatively if the summation index s equals ν' or if the summation index κ equals ν', gives rise to the usual exchange matrix elements of V, well-known from shell-model calculations. In the usual procedure for linearizing the bilinear Eq. (11) to obtain Eq. (13) a factorization of this two-ρ product that takes the Pauli principle properly into account is referred to as a Hartree-Fock factorization.

The complete linearized equation can thus be written in terms of the amplitudes $\langle E|\hat{\rho}_{\nu\nu'}\|0\rangle$, which we will denote $\rho_{\nu\nu'}$ for the sake of typographical simplicity

$$(\epsilon_{\nu'} - \epsilon_{\nu} - E)\,\rho_{\nu\nu'} = (n_{\nu'} - n_{\nu}) \sum_{KK'} \langle \nu K'|V(1-P^{12})|\nu' K\rangle \rho_{KK'} +$$

$$+ \sum_{K} n_{K} \left[\sum_{K\neq\nu'} \langle \nu K|V(1-P^{12})|K'K\rangle \rho_{K'\nu'} - \sum_{K'\neq\nu} \langle K'K|V(1-P^{12})|\nu' K\rangle \rho_{\nu K'} \right] \tag{13}$$

The terms on the second line of Eq. (13) are self-energy terms corresponding to the contribution from the self-energy part of the interaction Hamiltonian

$$\hat{\Delta V}^{self} = 1/2 \sum_{Ks} \langle sK|V|sK\rangle \, a_s^+ a_K^+ a_K a_s \tag{14}$$

It appears reasonable to assume that terms of such origin are already effectively included in the shell-model field from which the single-particle energies $\{\epsilon_\nu\}$ have been determined, although we have obviously not attempted to relate the self-energy parts of V to the single-particle energies through a Hartree-Fock procedure or through any other self-consistent method.

After the exclusion of the self-energy terms, Eq. (13) takes the simple form

$$(\epsilon_{\nu'} - \epsilon_\nu - E)\rho_{\nu\nu'} = (n_{\nu'} - n_\nu) \sum_{KK'} \langle \nu K'|V(1-P^{12})|\nu' K\rangle \rho_{KK'} \tag{15}$$

Another question also related to the arbitrariness in the problem of how the self-consistent field is defined concerns the RPA elements of Eq. (15) that are diagonal in the particle hole-pairs, namely $\langle \nu\nu'|V(1-P^{12})|\nu'\nu\rangle$. Those obviously correspond to elastic scattering of particle-hole pairs $(\nu\nu')$, and the problem arises whether they also should be thought of as being already included in the single-particle potential. Ordinarily such terms are not included in a Hartree-Fock calculation based on the Brueckner reaction matrix. Unless one employs another type of reaction matrix (see e.g. Sawada[15]) in finding ϵ_ν, such terms should be retained in the corresponding RPA (calculation). However, in the present calculations we have assumed a "phenomenological" shell-model field to supply $\{\epsilon_\nu\}$. Probably the mentioned terms of

the elastic-scattering type are included in this field on the average. However, individual fluctuations relative to this average may be significant and displaying effects of correlations. For most of our calculations we have chosen to include these diagonal terms, but for some cases they have been excluded. Probably the wave functions and oscillator strength values corresponding to the former case are to be considered as somewhat more plausible.. All the energies should, however, probably be lowered by an average 2-3 Mev in the former case, corresponding to a subtraction of the average diagonal particle-hole-pair interaction energy.

It should finally be emphasized that for the case of an uncorrelated ground state (i.e. when deexcitations, or annihilations of hole-particle pairs are excluded) Eq. (15) leads exactly to the eigenvalue problem occurring in the shell-model calculations. The latter type of calculations are obviously limited to including only the very simplest hole-particle pair graphs (one hole-particle pair being exchanged for another hole-particle pair in the interaction process).

As our occupation factors n_ν and $n_{\nu'}$, in the approximation employed, have the values 1 and 0, corresponding to a sharp Fermi surface, we also confine ourselves to excited states that, relative to the ground state, represent transitions of a particle across the Fermi surface. Thereby obviously a transition of a particle originally being above the Fermi surface in the correlated ground state to another state above the Fermi surface is excluded. The same thing holds true for a hole transition below the Fermi surface. However, this formalism allows for both creation and destruction of hole-particle pairs relative to the correlated ground state.

Equation (15) is an eigenvalue equation in terms of the matrix elements of the density matrix $\rho_{\nu\nu'}$. The corresponding transposed matrix equation holds for the coefficients $c^{E}_{\nu\nu'}$, which are the components of the "collective" eigenvector $|E\rangle$ defined in Eq. (5). This equation may be written in the form[18]

$$\begin{pmatrix} \underset{\sim}{A} & -\underset{\sim}{B} \\ \underset{\sim}{B} & -\underset{\sim}{A} \end{pmatrix} \begin{pmatrix} \underset{\sim}{u} \\ \underset{\sim}{v} \end{pmatrix} = E \begin{pmatrix} \underset{\sim}{u} \\ \underset{\sim}{v} \end{pmatrix} \tag{16}$$

where $\underset{\sim}{u}$ corresponds to creation of a hole-particle pair relative to the ground state while $\underset{\sim}{v}$ corresponds to the annihilation of such a pair relative to the ground state. Thus in a pure shell model calculation $\underset{\sim}{v} = 0$. The matrix $\underset{\sim}{B}$ is obviously associated with the coupling between the excitations and the de-excitations. While both of the matrices $\underset{\sim}{A}$ and $\underset{\sim}{B}$ are Hermitian, the total matrix is non-Hermitian.

The reality of E depends on the actual strength of interaction and is for the present case always very well ensured. It is easy to verify that Eq. (16) has the simple property that to each positive eigenvalue E there corresponds a negative and unphysical eigenvalue - E, which may be ruled out by imposing the extra condition on the correlated ground state

$$\mathcal{B}^{E}|0\rangle = 0. \tag{17}$$

Also, as the typical matrix elements of $\underset{\sim}{B}$ are of the order of, or less than, 1 Mev for the light nuclei considered compared to a separation of the roots of $\underset{\sim}{A}$ and $-\underset{\sim}{A}$ by a magnitude of the order of $2\hbar\omega$, the amount of mixture of de-excitations in the eigenfunctions of (16) is expected to be rather small in the treated cases. This is also born out in the explicit calculation (see below).*

* In view of this fact the effects of the neglected interactions of particle-particle and hole-hole pair type neglected in the RPA approximation may appear equally worthy of a special investigation.

As a result of the non-hermiticity of the secular problem the corresponding eigenvectors satisfy the orthonormality and completeness relations characteristic of an indefinite metric:

$$\sum_{\nu\nu'} \left[\left(c^{E_n}_{\nu\nu'} \right)^* c^{E_m}_{\nu\nu'} - \left(c^{E_n}_{\nu'\nu} \right)^* c^{E_m}_{\nu'\nu} \right] = \frac{E_n}{|E_n|} \delta_{mn} \tag{18}$$

$$\sum_{E_n, E_m \neq 0} \left(c^{E_n}_{\nu\nu'} \right)^* c^{E_m}_{K K'} \frac{E_n}{|E_n|} = \delta_{\nu K} \delta_{\nu' K'} \tag{19}$$

where the sum over ν in (18) runs over all single-particle states below the Fermi surface and the sum over ν' includes all such states above it.

DETAILED CALCULATIONS

In our calculations we have considered the self-conjugate nuclei Mg^{24} and C^{12} having T=0 ground states. Excitations relative to the ground state of E1 character have thus the isobaric spin T=1 and are generated by the operators*

$$\hat{\Phi}_{\nu\nu'} = \frac{1}{\sqrt{2}} \left(\hat{\rho}^p_{\nu\nu'} - \hat{\rho}^n_{\nu\nu'} \right) \tag{20}$$

where, from here on, ν and ν' label only the space and spin coordinates but not the charge of the state. If we rewrite Eq. (15) in terms of $\Phi_{\nu\nu'}$, it takes the form:

$$(\epsilon_{\nu'} - \epsilon_{\nu} - E)\, \Phi_{\nu\nu'} = (n_{\nu'} - n_{\nu}) \sum_{KK'} \langle \nu K' | U | \nu' K \rangle\, \Phi_{KK'} \tag{21}$$

where $\langle \nu K' | U | \nu' K \rangle$ is the matrix element of $V(1-P^{12})$ in terms of the T=1 wave functions (see Eq. (20)).

* That only T=1 states are excited is associated with the fact that for such self-conjugate nuclei considered here the effective charges of the proton and neutron are e/2 and -e/2, respectively. Physically the T=1 state, as seen from Eq. (20) corresponds to neutrons and protons moving 180° out of phase relative to each other, a picture which retains an essential feature of the original Goldhaber and Teller[16] two-fluid model.

Restricting ourselves to an interaction of the form

$$V = J(r_{12})(w + bP^{\sigma} + mP^{r} + hP^{r}P^{\sigma}) \tag{22}$$

and introducing the integrals

$$W = \int \psi^{*}_{\nu}(1)\ \psi^{*}_{K'}(2)\ J\ \psi_{\nu'}(1)\ \psi_{K}(2)\ d\tau_1\, d\tau_2 \tag{23a}$$

$$B = \int \psi^{*}_{\nu}(1)\ \psi^{*}_{K'}(2)\ JP^{\sigma}\ \psi_{\nu'}(1)\ \psi_{K}(2)\ d\tau_1\, d\tau_2 \tag{23b}$$

$$M = \int \psi^{*}_{\nu}(1)\ \psi^{*}_{K'}(2)\ JP^{r}\ \psi_{\nu'}(1)\ \psi_{K}(2)\ d\tau_1\, d\tau_2 \tag{23c}$$

$$H = \int \psi^{*}_{\nu}(1)\ \psi^{*}_{K'}(2)\ JP^{r}P^{\sigma}\ \psi_{\nu'}(1)\ \psi_{K}(2)\ d\tau_1\, d\tau_2 \tag{23d}$$

we may write

$$\langle \nu K' | U | \nu' K \rangle = - wH - bM - mB - hW. \tag{24}$$

The calculation of the interaction matrix elements as well as the E1 transition matrix elements is particularly simple if one uses the "asymptotic" wave functions,[5] valid in the limit that the quadrupole part of the shell model field is large relative to the spin-orbit coupling. In this limit appropriate at large deformations only the diagonal parts of the spin-orbit interaction is retained. In the Appendix we present the results of such calculations applied to the C^{12} nucleus. Unfortunately the applicability of not only the adiabatic coupling scheme but in particular the asymptotic wave functions to the nucleus is very uncertain, and the results obtained are only of interest as illucidating the general mechanism behind the calculations that employ the more detailed wave functions of Ref. 17. They should thus not be directly compared with the empirical data.

In the coupling scheme appropriate to deformed nuclei, there are two separate modes of excitations corresponding to K=0 and K=1. These are basically characterized by the two different oscillator frequencies ω_z and ω_1

where $\omega_z < \omega_\perp$ for prolate nuclei as Mg^{24} and $\omega_z > \omega_\perp$ for oblate nuclei, of which we have assumed C^{12} to be an example. Within each of the K=0 and K=1 groups of states a giant E1 excitation is then formed.

Furthermore in the case of deformed nuclei, an E1 transition from the ground state populates only the lowest member of a K=1 rotational band and only the I=1 member of a K=0 rotational band. There is thus in this case no sharing of the intrinsic E1 strength on different rotational states. The transition probability is therefore given exclusively in terms of the intrinsic wave functions.[17]

Most of the degeneracy of the spherical problem is now removed by the distortion of the nuclear field. The remaining degeneracy associated with the time reversal degeneracy of the single-particle orbitals may be exploited to reduce the secular matrix in the K=0 case by the introduction of the new state vectors generated by

$$\Phi_I = \frac{1}{\sqrt{2}} (\Phi_{ab} + \Phi_{-a-b}) \tag{25}$$

$$\Phi_{II} = \frac{1}{\sqrt{2}} (\Phi_{ab} - \Phi_{-a-b}) \tag{26}$$

It is easily seen that the matrix elements of V vanish between states of those two different types. Furthermore it is also easy to verify that the second group of states are associated with a vanishing E1 matrix element with the ground state and therefore can be left out of the discussion.

In calculating the strength of the K=1 transition we will have to remember to double their relative strength corresponding to the additional degeneracy of the K=1 states (the angular momentum component may be +1 or -1).

RESIDUAL INTERACTION EMPLOYED

In these calculations we have limited ourselves to considering a phenomenological interaction V simulating the "actual" nuclear force. We have chosen the "empirical" force employed by Ferrell and Visscher, having a Gaussian radial shape and a particular exchange mixture.

In addition we have also considered a force of the same radial dependence but with the exchange parameters of a "Rosenfeld mixture". (We will somewhat inadequately label this potential as "Rosenfeld" in the tables.) The parameters characterizing these forces may be found in Table 1.

Furthermore, to isolate the effects due to exchange mixtures, we have also considered the case of a pure Wigner force in some instances.

RESULTS OF CALCULATIONS

a. Excitation spectra

The effects of the inclusion of the backward-going graphs are, as pointed out, rather small as far as the positions of the roots and the relative distribution of El strength is concerned. Generally the resonances are slightly lowered by some tens or hundreds of kev.

Of more interest is the way in which the sum rules are affected. This will, however, be discussed later in this paragraph.

For Mg^{24}, due to numerical difficulties with the computer program, calculations are so far complete only for the case where B is put equal to zero, i.e. ground state correlations are neglected. The results are exhibited in Table 4. Some incomplete calculations also exist for Mg^{24}, K=0 (where such correlation effects should be the largest) and can be studied in Table 5. For C^{12} calculations are complete and the results, particularly concerning effects of ground state correlations, may be studied in Table 6.

The matrix elements computed numerically on the IBM 709 of Lawrence Radiation Laboratory exhibit many of the features conjectured in, e.g., Ref. 8. Thus, for instance, the effective interaction for T=1 in most cases is repulsive, thereby generally pushing the roots above their single-particle values. The matrix elements are in no way, however, constant and fluctuations in their size is of decisive importance. Instead the single-particle excitations that originally carry most of the E1 oscillator strength (asymptotically unhindered) also have the large interaction matrix elements in between themselves. It is effectively on one or two of the states in this smaller group of states that most of the E1 strength is collected as they are being pushed upwards due to the interaction.

Thus, e.g., for Mg^{24} the single-particle excitations carrying most of the E1 oscillator strength scatter in energy around $\hbar\omega_z$ or say 11-13 Mev for K=0 and around $\hbar\omega_\perp$ or about 15-17 Mev for K=1 (with the well parameters assumed as η=4 and κ=0.08).

The one or two states collecting the giant strength are now pushed upwards by amounts of about 7 Mev for K=1 and about 5-6 Mev for K=0. These figures refer specifically to the Ferrell and Visscher case with diagonal elements retained. Note that, in the total spectrum of states reached by E1 excitations the giant states are by no means the highest-lying as in the spherical case. Thus for the mentioned case of Mg^{24} with K=1 (as exhibited in Table 4b) the highest-lying root is more than 5-Mev above the K=1 giant resonance peak, and for K=0 about 11 Mev above the corresponding giant peak. Indeed in the latter case the giant peak is found in the lower part of the energy spectrum.

One may also note that the interaction mechanism described tries to enlarge the splitting between the two peaks, even relative to the basic energy difference ($\hbar\omega_1 - \hbar\omega_z$) which latter for the employed case of the distortion parameter[17] $\epsilon \cong 0.34$ ($\eta \cong 4$) equals (15.9 - 11.4) = 4.5 Mev. This should be compared with the energy splitting existing after the interaction has been turned on, which is about 6.5 Mev.

A calculation where a pure Wigner force was assumed, failed to give a strong resonance peak for K=1. Instead the strength was distributed over several states several Mev apart. This force in the case of a finite range is obviously of the "wrong specificity". (Cf. Mottelson, Ref. 12.)

Turning the attention to the case of C^{12} (Table 6), where the whole coupling scheme may be very much less appropriate, we notice that the clear separation encountered in Mg^{24} into two peaks of K=0 and K=1, respectively, does not occur here. In analogy with the prolate Mg^{24}, the oblate C^{12} would be expected to exhibit a low-lying K=1 peak and a higher-lying K=0 peak with a splitting of order $\delta\hbar\omega$. In C^{12} there is indeed for K=1 a strong low-lying peak. However, 7 Mev higher there is still another peak for K=1 which may be a remnant of the spherical coupling scheme, from which we are much less distant in C^{12} (with $\eta = -2$) than in Mg^{24} (with $\eta = 4$). For K=0 there is essentially one, high-lying peak in C^{12}.

b. Discussion of sum rules

It is obvious that a pure shell model calculation (ground state correlations not considered, cf. Table 4-6) retains the oscillator strength unchanged. The Thomas-Kuhn-Reiche sum proportional to $\sum_n E_n |M_n|^2$ will in the shell model case exceed the sum rule value $\frac{NZ}{A}$, derived for non-exchange interactions, as the energy values E_n are pushed upwards relative to the independent-particle case. The effect is usually larger with the mixtures

considered that contain exchange components. However, also the Wigner interaction treated in the approximation where backward-going graphs are neglected violates the sum rule, due to the fact that only a particular set of graphs are included in the conventional shell model treatment of this interaction. This is borne out in Table 4, although the diagonal interaction matrix elements are neglected in the calculation; the more interesting case of these being retained would lead to an even larger violation.

Now the inclusion of the backward-going graphs increases the oscillator strength sum by about (10-20)% for C^{12} and by as much as 30% for Mg^{24}, K=0. Although the energies are somewhat lowered when the backward-going graphs are included, still the TKR-sum rule is increased due to the increase in oscillator strength. This is a somewhat surprising result as it has been shown* that the random-phase approximation without inclusion of "direct"** graphs leads to this sum rule being exactly obeyed. The effects of increase in the sum rule have thus to be attributed to these usually neglected "direct" graphs.

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* Private communication from B. R. Mottelson (Cf. also Ref. 19).

** The exclusion of "direct" graphs here implies that only such interaction graphs are included where a hole-particle pair with its spin coupled to 1^- is destroyed or created.

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All numerical results were obtained from the IBM computing facilities of the laboratory.

TABLE LEGENDS

Table 1. Parameters of the shell-model potential appropriate to deformed nuclei. For the parameters see Ref. 17.

Table 2. The exchange mixture parameters of the force $V_o\ (w+bP^\sigma+mP^r+hP^r\ P^\sigma)$ $\exp -\left(\frac{r}{\alpha}\right)^2$ where $V_o = -51.9$ Mev and $\alpha = 1.732$ fm.

Table 3. Elementary single-particle excitations for Mg^{24}; Table 3a referring to K=0 and Table 3b to K=1. The energy values are taken from Ref. 17. In this reference the N=0, 1, 2 shells are calculated with an assumed μ=0 (coefficient of ℓ^2 -part of potential), but μ is assumed =0.35 for N=3. The energy eigenvalues of the N=3 shell are now a posteriori corrected to correspond to μ=0 while the wave functions calculated for μ=0.35 are left as before. This inconsistency as well as the slightly inaccurate energy adjustment does not affect the total oscillator strength. (The table lists the pure single-particle matrix elements of z and $\frac{x+iy}{\sqrt{2}}$ in units of $\sqrt{\frac{\hbar}{M\omega_z}}$ and $\sqrt{\frac{\hbar}{M\omega_\perp}}$ respectively.) However, it affects somewhat the distribution on the different states and is responsible for the fact that the Thomas-Kuhn-Reiche sum rule $\sum_n f_{on} = \frac{NZ}{A}$ (for K=1, $\frac{2NZ}{A}$, as both K=1 and -1 are included) is somewhat exceeded in the independent-particle case. The matrix elements are defined for negative angular-momentum states with a phase that differs from the time-reversal convention by a factor $(-)^{\Omega-1/2}$. By this convention the following relation holds $|-(-\Omega)\rangle = |\Omega\rangle$.

TABLE LEGENDS (Con't)

Table 4. Shell-model calculations for Mg^{24} K=0(a) and K=1(b). The energies of the calculated 1^- states are given in column one. The squared matrix elements $|M|^2$ of z and $\frac{1}{\sqrt{2}}(x+iy)$ in terms of the basic linear combinations of Eq. (20) and (25) are listed in column two in units of $\hbar/M\omega_z$ and $\hbar/M\omega_1$, respectively. The sum of matrix elements is compared with the single-particle sums of the included single-particle excitations. Column three lists the dipole strength (the quantity occurring in the Thomas-Kuhn-Reiche sum), f_{on}, which for K=0 equals $\left(2M/\hbar^2\right)\left(E_n|M_n|^2\right)\cdot\left(1/4\right)$, where the factor 1/4 comes from the effective charge. The f_{on} sums may be compared with the estimate NZ/A. For K=1, as we have added the contribution from $\frac{1}{\sqrt{2}}(x+iy)$ and $\frac{1}{\sqrt{2}}(x-iy)$, the comparison should be made with the estimate 2 NZ/A. Column four lists the gamma widths $\Gamma_\gamma = 2\left(e^2/\hbar c\right)\left(E_n^{\,2}/Mc^2\right)\cdot f_{on}$. In a separate row in the table the integrated photo-absorption cross section is given in units of Mev-mb. For a discussion of the listed cases of the diagonal hole-pair interaction matrix elements being alternatively excluded and retained, see the main text. The small deviation of the sum of squared matrix elements $|M|^2$ from the single-particle sum is a reflection of the inaccuracy of the wave-functions obtained in the matrix diagonalization.

Table 5. The effect of the inclusion of the "backward-going graphs" for Mg^{24}, K=0. The organization of the table is the same as that of Table 4, where the different entries are explained. In the present table the results with backward-going graphs excluded and retained are exhibited for the case of Mg^{24}, K=0. In this calculation only the

TABLE LEGENDS (Con't)

roughly most important seven of the basic twelve elementary excitations of Table 3a are included (corresponding to only the lowest half of the N=3 shell considered). The calculations still, however, serve well the purpose of illustrating the effects of the backward-going graphs.

Table 6. Results of calculations for C^{12} K=0 and K=1. For a detailed explanation see caption of Table 4. This table also exhibits the inclusion of the effects of the ground state correlation. Only the Ferrell-Visscher force is considered in the case of C^{12}. The number of elementary excitations is not quite complete. Three transitions associated with very weak El matrix elements are left out for K=1 and two for K=0. This is the reason why we fall short of the TKR sum rule by a few percent in the independent-particle case.

Table A-1. The K=0 matrix elements of J(r) with the C^{12} "asymptotic" wave functions expressed in terms of the Talmi integrals I_p.

Table A-2. The T=1, S=0, odd-parity states of the K=0 and K=1 groups and their respective El strengths for the C^{12} nucleus computed with the "asymptotic" wave functions. Results for two different interaction potentials and two values of the deformation parameter, $\eta = -2$ and $\eta = -4$, are compared.

Table 1.

	η	κ	$\hbar\omega_z$ (Mev)	$\hbar\omega$ (Mev)
C^{12}	-2 (-4)	0.10	20.3	16.8
Mg^{24}	4	0.08	11.4	15.9

Table 2.

Exchange mixture	w	b	m	h	Triplet even	Singlet even	Triplet odd	Singlet odd
Ferrell-Visscher	0.317	0.0	0.5	0.183	1.0	0.634	-0.366	0.0
Rosenfeld	-0.13	0.46	0.93	-0.26	1.0	0.6	-0.34	-1.78

Table 3a.

Hole-particle excitation		$\epsilon_{\nu'}-\epsilon_{\nu}$ (Mev)	Single-particle matrix elements $\langle\|z\|\rangle$	Classif. according to asympt. rule	$\langle\|z\|\rangle^2$	f_{on}
Hole state $[Nn_z\Lambda\Omega]$	Particle state $[Nn_z\Lambda\Omega]$					
[110 1/2]	[211 1/2]	17.59	0.1693	h	0.029	0.09
[110 1/2]	[200 1/2]	21.61	0.0181	h	0.000	0.00
[101 3/2]	[202 3/2]	19.94	0.1815	h	0.033	0.12
[101 1/2]	[211 1/2]	11.06	0.6645	u	0.442	0.86
[101 1/2]	[200 1/2]	15.08	0.3017	h	0.091	0.24
[220 1/2]	[330 1/2]	12.34	1.1278	u	1.272	2.76
[211 3/2]	[321 3/2]	11.21	0.9528	u	0.908	1.79
[211 3/2]	[312 3/2]	18.44	0.2215	h	0.049	0.17
[211 3/2]	[301 3/2]	22.43	-0.1008	h	0.010	0.00
[220 1/2]	[321 1/2]	16.92	0.3669	h	0.135	0.40
[220 1/2]	[310 1/2]	21.25	-0.1774	h	0.032	0.12
[220 1/2]	[301 1/2]	28.54	-0.0125	h	0.000	0.00
					Sum =4x3.000	6.55

Table 3b.

Hole-particle excitation		$\epsilon_\nu - \epsilon_\nu$ (Mev)	Single-particle matrix elements $\left\langle \left\| \frac{x+iy}{\sqrt{2}} \right\| \right\rangle$	Classified according to asympt. rule	$\left\langle \left\| \frac{x+iy}{\sqrt{2}} \right\| \right\rangle^2$	f_{on}
Hole state $[Nn_z\Lambda\Omega]$	Particle state $[Nn_z\Lambda\Omega]$					
[110-1/2]	[211 1/2]	17.59	-0.6540	u	0.428	0.95
[110-1/2]	[200 1/2]	21.61	-0.1143	h	0.013	0.04
[101 3/2]	[202 5/2]	14.78	1.0000	u	1.000	1.86
[10-1-3/2]	[21-1-1/2]	13.76	-0.2919	h	0.085	0.15
[10-1-3/2]	[200-1/2]	17.78	0.6427	u	0.413	0.92
[10-1-1/2]	[211 1/2]	11.06	0.1297	h	0.017	0.02
[10-1-1/2]	[200 1/2]	15.08	-0.6947	u	0.483	0.92
[101 1/2]	[202 3/2]	17.24	0.9809	u	0.962	2.09
[220 -1/2]	[330 1/2]	12.34	0.3898	h	0.152	0.23
[220 1/2]	[321 3/2]	14.86	0.7423	u	0.551	1.03
[21-1-3/2]	[330-1/2]	8.70	0.2012	h	0.041	0.04
[211 3/2]	[312 5/2]	17.15	1.0142	u	1.029	2.22
[220-1/2]	[321 1/2]	16.92	-0.5839	u	0.341	0.73
[220-1/2]	[310 1/2]	21.25	-0.0667	h	0.004	0.01
[220-1/2]	[301 1/2]	28.54	0.0671	h	0.005	0.02
[220 1/2]	[312 3/2]	22.08	-0.0534	h	0.003	0.01
[220 1/2]	[301 3/2]	26.07	-0.0772	h	0.006	0.02
[21-1-3/2]	[32-1-1/2]	13.42	-0.2381	h	0.057	0.10
[21-1-3/2]	[310-1/2]	17.60	0.6347	u	0.403	0.89
[21-1-3/2]	[30-1-1/2]	24.90	0.0019	h	0.000	0
[211 3/2]	[303 5/2]	23.94	-0.0667	h	0.004	0.01
					Sum=2 x 5.997	12.26

Table 4a

Interaction	Ferrell and Visscher								Wigner			
Diagonal matrix elements	Included				Excluded				Excluded			
	E (Mev)	$\|M\|^2$	f_{on}	Γ_γ (kev)	E (Mev)	$\|M\|^2$	f_{on}	Γ_γ (kev)	E (Mev)	$\|M\|^2$	f_{on}	Γ_γ (kev)
	29.41	0.06	0.08	1.1	28.56	0.00	0.00	0.0	28.60	0.00	0.00	0.0
	25.94	0.48	0.55	5.8	23.94	0.17	0.19	1.7	23.99	0.02	0.02	0.2
	24.15	0.08	0.08	0.7	21.93	0.58	0.57	4.2	22.41	0.84	0.84	6.5
	22.94	0.28	0.28	2.3	21.46	0.16	0.16	1.1	21.48	0.00	0.00	0.0
	22.19	0.03	0.03	0.2	20.52	0.13	0.12	0.8	20.08	0.10	0.09	0.6
K=0	20.72	0.02	0.02	0.1	18.20	0.03	0.02	0.1	18.67	0.06	0.05	0.3
	18.81	0.12	0.11	0.6	16.76	0.44	0.32	1.4	16.84	0.36	0.26	1.1
	18.44	3.60	2.91	15.4	16.06	1.63	1.15	4.6	15.37	0.67	0.45	1.7
	17.68	5.24	4.08	19.8	15.08	4.38	2.91	10.3	14.58	0.10	0.07	0.2
	16.57	1.87	1.37	5.8	13.84	4.58	2.79	8.3	12.95	9.55	5.44	14.2
	13.46	0.14	0.08	0.2	10.45	0.00	0.00	0.0	11.02	0.28	0.14	0.3
	12.23	0.07	0.04	0.1	9.60	0.00	0.00	0.0	10.41	0.01	0.00	0.0
Sums		11.99	9.63			12.10	8.23			11.99	7.36	
Indep.p.sums		12.00	6.55			12.00	6.55			12.00	6.55	
$\int \sigma_\gamma dE$		578 (Mev-mb)				494 (Mev-mb)				442 (Mev-mb)		

Table 4b

Interaction	Ferrell and Visscher								Wigner			
Diagonal matrix elements	Included				Excluded				Excluded			
	E (Mev)	$\|M\|^2$	f_{on}	Γ_γ (kev)	E (Mev)	$\|M\|^2$	f_{on}	Γ_γ (kev)	E (Mev)	$\|M\|^2$	f_{on}	Γ_γ (kev)
K=1	29.70	0.04	0.07	1.0	28.78	0.01	0.01	0.2	28.67	0.01	0.01	0.1
	27.28	0.86	1.48	17.1	26.26	0.26	0.44	4.7	26.41	0.03	0.06	0.6
	26.29	0.55	0.91	8.8	25.10	0.12	0.20	2.0	25.14	0.00	0.00	0.0
	25.61	1.10	1.77	18.1	24.27	0.27	0.41	3.8	24.22	0.04	0.07	0.6
	24.97	4.50	7.07	68.6	23.05	2.93	4.25	35.2	22.59	0.25	0.36	2.8
	23.69	3.79	5.65	49.3	21.97	0.89	1.23	9.2	22.11	0.19	0.27	2.0
	23.20	0.15	0.22	1.8	21.35	6.32	8.49	60.2	20.12	1.88	2.38	15.0
	22.14	0.02	0.02	1.7	20.52	0.02	0.02	0.1	19.86	2.83	3.54	21.7
	21.09	0.40	0.54	3.7	18.72	0.59	0.69	3.8	19.22	0.11	0.14	0.8
	19.76	0.02	0.02	0.1	17.40	0.05	0.06	0.3	18.04	1.16	1.31	6.6
	18.95	0.12	0.14	0.8	16.91	0.06	0.06	0.3	17.05	0.27	0.29	1.3
	18.32	0.02	0.03	0.1	15.90	0.00	0.00	0.0	16.53	3.81	3.96	16.8
	18.01	0.02	0.03	0.1	15.35	0.04	0.04	0.1	15.44	0.11	0.10	0.4
	17.53	0.02	0.03	0.1	15.03	0.06	0.05	0.1	15.24	0.08	0.07	0.3
	16.64	0.14	0.14	0.6	14.52	0.00	0.00	0.0	14.84	0.15	0.23	0.8
	16.41	0.00	0.00	0.0	14.32	0.13	0.12	0.4	14.69	0.47	0.44	1.5
	16.12	0.05	0.05	0.2	13.25	0.11	0.10	0.3	12.97	0.38	0.31	0.8
	15.49	0.13	0.13	0.5	13.05	0.04	0.03	0.1	12.52	0.03	0.03	0.1
	13.60	0.01	0.02	0.0	11.39	0.00	0.00	0.0	11.47	0.02	0.01	0.0
	12.69	0.01	0.01	0.0	10.98	0.01	0.01	0.0	10.95	0.00	0.00	0.0
	10.04	0.00	0.00	0.0	8.54	0.00	0.00	0.0	8.57	0.01	0.00	0.0
Sums		11.95	18.33			11.91	16.21			11.83	13.58	
Indep.p.sums		11.99	12.26			11.99	12.26			11.99	12.26	
$\int\sigma_\gamma dE$		1100 (Mev-mb)				973 (Mev-mb)				814 (Mev-mb)		

Table 5

Ferrell and Visscher								"Rosenfeld"			
Shell-model calculations				Correlations included				Shell-model calculations			
E (Mev)	$\|M\|^2$	f_{on}	Γ_γ (kev)	E (Mev)	$\|M\|^2$	f_{on}	Γ_γ (kev)	E (Mev)	$\|M\|^2$	f_{on}	Γ_γ (kev)
21.84	0.22	0.22	1.6	21.71	0.23	0.23	1.6	21.87	0.08	0.08	0.6
20.77	0.12	0.12	0.8	20.51	0.23	0.22	1.4	21.21	0.16	0.15	1.0
17.10	0.02	0.02	0.1	17.06	0.01	0.01	0.0	16.83	0.00	0.00	0.0
15.18	6.14	4.10	14.7	14.80	4.08	2.66	9.1	15.11	5.62	3.74	13.3
13.87	4.56	2.78	8.3	13.45	10.40	6.16	17.3	13.68	5.20	3.12	9.1
10.46	0.00	0.00	0.0	10.37	0.00	0.00	0.0	10.55	0.00	0.00	0.0
9.61	0.02	0.01	0.0	9.59	0.00	0.00	0.0	9.58	0.02	0.01	0.0
Sum	11.08	7.25			14.95	9.28			11.08	7.10	
Indep. p. sum	11.08	5.86			11.08	5.86			11.08	5.86	
$\int \sigma_\gamma dE$		526 (Mev-mb)				557 (Mev-mb)				427(Mev-mb)	

Table 6.

Shell-model calculation									Ground state correlation included							
Diagonal m.e.	included				excluded				included				excluded			
	K = 0															
	E (Mev)	$\|M\|^2$	f_{on}	Γ_γ (kev)	E (Mev)	$\|M\|^2$	f_{on}	Γ_γ (kev)	E (Mev)	$\|M\|^2$	f_{on}	Γ_γ (kev)	E (Mev)	$\|M\|^2$	f_{on}	Γ_γ (kev)
	31.91	3.52	2.77	48.8	27.43	3.46	2.33	27.3	31.26	4.20	3.23	49.2	26.73	4.14	2.73	30.3
	26.31	1.26	0.83	8.9	22.32	2.16	1.20	9.3	26.02	1.62	1.04	10.9	21.90	3.24	1.75	13.0
	23.74	1.08	0.64	5.6	19.31	0.22	0.11	0.6	23.61	1.56	0.91	7.9	19.35	0.20	0.10	0.6
	20.85	0.08	0.04	0.3	17.27	0.12	0.05	0.2	20.73	0.18	0.09	0.6	17.15	0.32	0.13	0.6
	17.82	0.00	0.00	0.0	14.29	0.00	0.00	0.0	17.77	0.02	0.01	0.0	14.27	0.00	0.00	0.0
sums		5.94	4.28			5.96	3.69			7.58	5.28			7.90	4.71	
indep.p sums		5.98	3.00			5.98	3.00			5.98	3.00			5.98	3.00	
$\int \sigma_\gamma dE$		257 (Mev-mb)				222(Mev-mb)				317 (Mev-mb)				283 (Mev-mb)		
	K = 1															
	29.50	1.04	1.82	24.7	25.58	1.10	1.67	17.0	29.22	1.24	2.15	28.6	25.09	1.34	1.99	19.5
	24.77	0.04	0.06	0.6	21.90	0.02	0.03	0.2	24.61	0.00	0.00	0.0	21.84	0.02	0.01	0.1
	24.08	0.00	0.01	0.0	20.50	0.16	0.19	1.3	24.06	0.00	0.00	0.0	20.38	0.10	0.11	0.7
	22.97	1.08	1.48	12.2	20.27	0.06	0.06	0.4	22.88	0.70	0.95	7.7	20.24	0.00	0.01	0.0
	22.21	3.66	4.85	37.2	18.84	4.10	4.60	25.4	22.00	4.20	5.52	41.6	18.30	5.22	5.67	29.5
	19.74	0.08	0.10	0.6	16.48	0.40	0.40	1.7	19.46	0.24	0.29	1.7	16.27	0.86	0.85	3.5
	17.82	0.00	0.00	0.0	14.62	0.02	0.01	0.0	17.68	0.00	0.03	0.2	14.56	0.04	0.03	0.1
	17.12	0.00	0.00	0.0	13.36	0.08	0.01	0.0	16.85	0.02	0.01	0.1	13.28	0.20	0.15	0.4
sums		5.92	8.32			5.92	6.96			6.40	8.95			7.56	8.82	
indep. p sums		5.92	5.92			5.92	5.92			5.92	5.92			5.92	5.92	
$\int \sigma_\gamma dE$		499 (Mev-mb)				418 (Mev-mb)				537 (Mev-mb)				529 (Mev-mb)		

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Table A-1

$\langle 000\ 110 | J(r) | 110\ 000 \rangle = 1/2\ (I_0 - I_1)$

$\langle 000\ 110 | J(r) | 000\ 110 \rangle = 1/2\ (I_0 + I_1)$

$\langle 000\ 211 | J(r) | 110\ 101 \rangle = 1/4\ (I_0 - I_2)$

$\langle 000\ 211 | J(r) | 101\ 110 \rangle = 1/4\ (I_0 - I_2)$

$\langle 000\ 101 | J(r) | 211\ 110 \rangle = 1/4\ (I_0 - 2I_1 + I_2)$

$\langle 101\ 211 | J(r) | 211\ 101 \rangle = 1/4\ (I_0 - I_1 + I_2 - I_3)$

$\langle 101\ 211 | J(r) | 101\ 211 \rangle = 1/4\ (I_0 + I_1 + I_2 + I_3)$

$\langle 101\ 21\text{-}1 | J(r) | 10\text{-}1\ 211 \rangle = 1/4\ (I_0 - I_1 - I_2 + I_3)$

$\langle 101\ 10\text{-}1 | J(r) | 21\text{-}1\ 211 \rangle = 1/4\ (I_0 - 3I_1 + 3I_2 - I_3)$

Table A-2

NUCLIDE	η	FORCE	DIAGONAL MATRIX ELEMENTS	K	$\hbar\omega$ (Mev)	DIPOLE STRENGTH f_{on}	$\epsilon_{\nu'} - \epsilon_{\nu}$ (Mev)
$_{6}C^{12}$	-2	FERRELL and VISSCHER	INCLUDED	0	28.5 21.6	5.31 0.04	$20.3 = \hbar\omega_z$
				1	22.1 18.1	9.68 0.02	$16.8 = \hbar\omega_1$
			EXCLUDED	0	24.3 17.2	4.86 0.00	20.3
				1	18.2 14.8	7.97 0.28	16.8
		ELLIOTT and FLOWERS	INCLUDED	0	28.0 22.4	4.77 0.18	20.3
				1	21.5 18.8	8.75 0.16	16.8
			EXCLUDED	0	23.6 17.9	3.86 0.00	20.3
				1	18.0 15.3	7.48 0.20	16.8
	-4	FERRELL and VISSCHER	INCLUDED	0	31.0 24.0	5.05 0.11	22.7
				1	21.1 17.1	9.94 0.02	15.8

APPENDIX

Calculations with the Asymptotic Wave Functions

In the limit when the deformation energy is large compared to the off-diagonal elements of the spin-orbit coupling we can use the approximation in which the spin motion is decoupled from the orbital motion.[5,14]

In this approximation we shall use the harmonic oscillator wave functions $\psi_{Nn_z\Lambda}(\rho,z,\phi)$ in the representation of the (dimensionless) cylindrical coordinates $\left(\rho=\sqrt{x^2+y^2},\ z,\ \phi;\ x,\ y=\sqrt{\frac{\hbar}{M\omega_\perp}}\,x',y';\ z=\sqrt{\frac{\hbar}{M\omega_z}}\,z'\right)$: where Λ has the meaning of the projection of the orbital angular momentum $\vec{\ell}$ on the body axis z', n_z and $n_\perp=N-n_z$ are the oscillator quantum numbers referring to oscillations parallel to the z'-axis and perpendicular to it, respectively.

Using the above wave functions it is easy to eliminate the degenerate spin substates from the equation of motion (Eq.(15)). For a general collective state of isotopic spin T and spin S and with the two-body interaction $\hat{V}$ of Eq. (22) we obtain the RPA "equation of motion" of the $\rho_{\alpha\alpha'}$'s in the form:

$$(\varepsilon_{\nu'}-\varepsilon_\nu-E)X^{ST}_{\nu\nu'}=(n_{\nu'}-n_\nu)\sum_{\kappa\neq\kappa'}\Big\{\langle\nu\kappa'|f(r)|\nu'\kappa\rangle\left[(w+b-m-h)+(-)^S(w-h)\right.$$
$$\left.+(-)^T(w+b+(-)^S w)\right]-\langle\nu\kappa'|f(r)|\kappa\nu'\rangle\left[(w+b-m-h)+(-)^S(b-m)+\right.$$
$$\left.-(-)^T(m+h+(-)^S m)\right]\Big\}X^{ST}_{\kappa\kappa'} \qquad \text{(A1)}$$

$$X^{ST}_{\alpha\alpha'}\equiv\tfrac{1}{2}\Big(\rho_{\alpha\alpha'}(\tau\tau,\Sigma\Sigma)+(-)^S\rho_{\alpha\alpha'}(\tau\tau,-\Sigma-\Sigma)+(-)^T\rho_{\alpha\alpha'}(-\tau-\tau,\Sigma\Sigma)+ \qquad \text{(A2)}$$
$$+(-)^{S+T}\rho_{\alpha\alpha'}(-\tau-\tau,-\Sigma-\Sigma)\Big)$$

Here $\rho_{\alpha\alpha'}$ $(\tau\tau, \Sigma\Sigma)$ means a $\rho_{\alpha\alpha'}$-component with the (α,α') pair characterized by the charge state τ and spin state Σ. In the giant El problem, i.e., for S = 0, T = 1 we have:

$$(\varepsilon_{\nu'}-\varepsilon_{\nu}-E)\,\chi^{01}_{\nu\nu'} = -(n_{\nu'}-n_{\nu})\sum_{x\neq x'}\left[\langle \nu x'|J(\tau)|\nu' x\rangle(m+2h)+\langle \nu x'|J(\tau)|x\nu'\rangle \times(w+2b)\right]\chi^{01}_{xx'} \tag{A3}$$

Here the matrix elements are expressed in terms of spacial wave functions only, i.e. the subscripts refer specifically to spacial states.

Our wave functions given in cylindrical representation can be expanded in the spherical functions as

$$\Psi_{N n_z \Lambda}(\rho, z, \varphi) = \sum_{l} a_{l\Lambda}\,\Psi_{N l \Lambda}(\vec{r}) \tag{A4}$$

The matrix element $\langle \beta\delta|\,J\,|\alpha\gamma\rangle$ are now obviously

$$\langle \beta\delta|J(\tau)|\alpha\gamma\rangle = \sum_{l_\alpha l_\beta l_\gamma l_\delta} a_{l_\alpha\Lambda_\alpha} a_{l_\beta\Lambda_\beta} a_{l_\gamma\Lambda_\gamma} a_{l_\delta\Lambda_\delta} \langle N_\beta l_\beta \Lambda_\beta, N_\delta l_\delta \Lambda_\delta| \times |J(r_{12})|\, N_\alpha l_\alpha \Lambda_\alpha, N_\gamma l_\gamma \Lambda_\gamma\rangle \tag{A5}$$

Sometimes it is convenient to use the Talmi method of separation of the center of mass and the relative coordinates of the particles "1" and "2". This method is especially useful when the radial shape of $J(r_{12})$ is complicated and it is difficult to employ an electronic computer for the evaulation of $\langle \beta\delta|\,J\,|\alpha\gamma\rangle$ of Eq. (A5). We can expand the product $\psi_{(\alpha)}(\vec{r}_1)\psi_{(\gamma)}(\vec{r}_2)$ as:

$$\Psi_{N_\alpha l_\alpha \Lambda_\alpha}(\vec{r}_1)\,\Psi_{N_\gamma l_\gamma \Lambda_\gamma}(\vec{r}_2) = \sum_{\lambda\mu}(l_\alpha l_\gamma \Lambda_\alpha \Lambda_\gamma|\lambda\mu)\sum_{nl\,NL}\langle nl, NL, \lambda|N_\alpha l_\alpha, N_\gamma l_\gamma, \lambda\rangle \times |nl, NL, \lambda\mu\rangle \tag{A6}$$

where the "transformation brackets" $\langle | \rangle$ are defined and extensively tabulated by Moshinsky and Brody.[20] The relative motion (nl)-and center of mass motion (NL) wave functions and the "ket" $|n\ell, N L, \lambda\mu\rangle$ are as defined in Ref. 20.

Using Eq. (A6) we can now express $\langle \beta\delta | \mathcal{J} | \alpha\gamma \rangle$ as:

$$\langle \beta\delta | \mathcal{J}(r) | \alpha\gamma \rangle = \sum_{l_\alpha l_\beta l_\gamma l_\delta} a_{l_\alpha \Lambda_\alpha} \cdots a_{l_\delta \Lambda_\delta} \sum_{\lambda} \sum_{n n' \ell N L} (l_\alpha l_\gamma \Lambda_\alpha \Lambda_\gamma | \lambda\mu) \times$$
$$\times (l_\beta l_\delta \Lambda_\beta \Lambda_\delta | \lambda\mu) \langle n\ell, NL, \lambda | N_\alpha l_\alpha, N_\gamma l_\gamma, \lambda \rangle \langle n'\ell, NL, \lambda | N_\beta l_\beta, N_\delta l_\delta, \lambda \rangle \times$$
$$\times \langle n'\ell \| \mathcal{J}(r) \| n\ell \rangle \qquad \text{(A7)}$$

where the reduced radial integral $\langle n'\ell \| J(r) \| n\ell \rangle$ can be expressed in terms of the Talmi integrals I_p as:

$$\langle n'\ell \| \mathcal{J}(r) \| n\ell \rangle = \sum_{p} B(n\ell, n'\ell, p) I_p \qquad \text{(A8)}$$

where the coefficients B ($n\ell$, $n'\ell$, p) are tabulated in Ref. 20.

We have performed explicit computations for the problem of the 1^-, T=1, states in C^{12}. We have employed two different interactions:

$$\text{I)} \quad \mathcal{J}(r) = -V_0 e^{-r^2/\alpha^2} \quad ; \quad V_0 = 51.9 \text{ MeV}, \alpha = 1.732 \times 10^{-13} \text{ cm} \qquad \text{(A9a)}$$

w = 0.317, m = 0.5, b = 0, h = 0.183

—identical with the interaction listed in Table 2

$$\text{II)} \quad \mathcal{J}(r) = -V_0 \left[e^{-r/r_0} / r/r_0 \right] ; \quad V_0 = 43 \text{ MeV}, r_0 = 1.4 \times 10^{-13} \text{ cm} \qquad \text{(A9b)}$$

w = -0.13, m = 0.93, b = 0.46, h = -0.26

i.e. the Rosenfeld mixture used by Elliott and Flowers.[7]

In the C^{12} problem we have the "occupied" ($n_\nu = 1$) states $[N n_z \Lambda] = [000]$, $[10\pm1]$ and the "unfilled" ($n_\nu = 0$) states [110], $[20\pm2]$, $[21\pm1]$, and [200], where the $\pm\Lambda$ states are degenerate in energy. In this problem we have also

evaluated the self-energy terms (second line of Eq. (13)). The above states, denoted $|\rangle$, are expressible in terms of spherical states denoted $|\rangle\rangle$ (where the phase convention of the radial wave function is that of Ref. 17, i.e. such that the sign of the term of highest power in the radial polynomial is always positive) as follows:

$$|000\rangle = |N = 0,\ \ell = 0, \Lambda = 0\rangle\rangle,$$

$$|10\pm1\rangle = |N = 1,\ \ell = 1, \Lambda = \pm1\rangle\rangle,$$

$$|110\rangle = |N = 1,\ \ell = 1, \Lambda = 0\rangle\rangle,$$

$$|20\pm2\rangle = |N = 2,\ \ell = 2, \Lambda = \pm2\rangle\rangle,$$

$$|21\pm1\rangle = |N = 2,\ \ell = 2, \Lambda = \pm1\rangle\rangle,$$

$$|200\rangle = -\sqrt{\tfrac{1}{3}}\,|N = 2,\ \ell = 2,\ \Lambda = 0\rangle\rangle + \sqrt{\tfrac{2}{3}}\,|N = 2,\ \ell = 0, \Lambda = 0\rangle\rangle$$

In the RPA the secular matrix for the K = 0 E1 modes is 6x6, and the K = 1 modes have a secular 4x4 matrix.

In Table A1 we give the explicit expressions for the relevant matrix elements for the case K = 0 in terms of the Talmi integrals I_p.* The basic single particle E1 matrix elements $(\alpha'|z|\alpha)$ are in units of $\sqrt{\frac{\hbar}{M\omega_z}}$: $\langle 110|z|000\rangle = \langle 211|z|101\rangle = \langle 21\text{-}1|z|10\text{-}1\rangle = 1/\sqrt{2}$; the $\left\langle \alpha' \left| \frac{x+iy}{\sqrt{2}} \right| \alpha \right\rangle$ elements in units of $\sqrt{\frac{\hbar}{M\omega_\perp}}$: $\left\langle 202 \left| \frac{x+iy}{\sqrt{2}} \right| 101 \right\rangle = \left\langle 10\text{-}1 \left| \frac{x+iy}{\sqrt{2}} \right| 20\text{-}2 \right\rangle = 1$; $\left\langle 200 \left| \frac{x+iy}{\sqrt{2}} \right| 10\text{-}1 \right\rangle = \left\langle 101 \left| \frac{x+iy}{\sqrt{2}} \right| 200 \right\rangle = 1/\sqrt{2}$. The computed oscillator strengths and dipole strengths for K = 1 have to be doubled to take account of the two nuclear spin projections = ± 1. Our final numerical results are displayed in Table A2. The computations were performed for two values of the

* For the range of force considered here I_o is the dominant term, which together with our choice of parameters w, m, b, and h in (A3) ensures that all the occurring matrix elements are of about equal magnitude and have one and the same sign. The condition for I_o to be dominant is that $\alpha \ll \sqrt{\hbar/M\omega_o}$. In the limit of an extremely long range interaction we have instead $I_o = I_1 = I_2$ etc. The interaction matrix elements then all vanish, as seen from Table A1, except the diagonal ones. Thus in this limit the coherence is again destroyed.

oblate deformation $\eta = -2$ and $\eta = -4$. The splitting of the giant E1 states corresponding to K = 0 and K = 1 is roughly in agreement with the more detailed calculations. The second K = 0 giant peak occurring in the latter calculations does not appear in the asymptotic limit and is obviously associated with the fact that the wave functions of Ref. 17 are intermediate between the spherical shell-model wave functions and the asymptotic ones.

Our results for the Rosenfeld mixture of Elliott and Flowers[7] with a Yukawa well (Eq. (A9b)) are quite similar to those for the rather different interaction of Ferrell and Visscher (Eq. (A9a)).

We have also computed the self-energy terms originating from the self-energy hamiltonian Eq. (13). Some of them are very large (of the order of 40 Mev for the interaction of Eq. (A9a)). This indicates that a self-consistency calculation for the single particle energy spectrum, or, conversely, a self-consistent determination of the residual interaction for a given single-particle model is generally important.

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