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Commodity Money Equilibrium in a Walrasian Trading Post Model:
An Elementary Example

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Commodity Money Equilibrium in a Walrasian Trading Post Model: An Example

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PRELIMINARY: NOT FOR QUOTATION

Abstract

This paper posits an example of Walrasian general competitive equilibrium in an exchange economy with commodity-pairwise trading posts and transaction costs. Budget balance is enforced for each transaction at each trading post separately. Commodity-denominated bid and ask prices at each post allow the post to cover transaction costs through the bid/ask spread. In the absence of double coincidence of wants, the lowest transaction-cost commodity (with the narrowest bid/ask spread) becomes the common medium of exchange, commodity money. Selection of the monetary commodity and adoption of a monetary pattern of trade results from price-guided equilibrium without central direction, fiat, or government.

JEL Classification: C62, E40

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1 A Price Theory of Money

1.1 Commodity Money as the Most Liquid Good

This paper presents a simple example augmenting the Arrow-Debreu general equilibrium model sufficiently to allow monetary structure to appear as a result. It is well known that the Arrow-Debreu model cannot support money; for households it uses a single budget constraint and for firms a single profit expression, both summarizing all buying and selling transactions in a single equation. The most elementary function of money — the medium of exchange — is as a carrier of value held between successive transactions. In order to model the function of a carrier of value between transactions the model will need to distinguish transactions individually. That notion is formalized below as a trading post model, where the budget constraint is fulfilled at each trading post transaction separately. This paper will demonstrate that the trading post model can derive, from elementary initial conditions, a commodity money equilibrium with a unique common medium of exchange.

The history of this notion goes back over a century. Carl Menger (1892) wrote: "[Call] goods ... more or less saleable, according to the ... facility with which they can be disposed of ... at current purchasing prices or with less or more diminution... Men ... exchange goods ... for other goods ... more saleable...[which] become generally acceptable media of exchange." ¹

Menger says here that every traded good has a bid (wholesale) price and an ask (retail) price. "Saleability" is liquidity, characterized by how small is the bid/ask spread. A commodity that acts as a medium of exchange is necessarily repeatedly bought (accepted in trade) and sold (delivered in trade). Therefore a good with a narrow spread between bid and ask price (a narrow wholesale/retail margin) is priced to encourage households to use it as a carrier of value between trades, as a medium of exchange with relatively low cost. This paper formalizes Menger's remark in a simple example, to demonstrate that a commodity money equilibrium is sustained by a structure of bid and ask prices and transaction costs in a trading post model.

The starting point is to set up a trading system with many separate transactions — with a budget constraint enforced at each transaction, so that there is a role for a carrier of value between them. The model presented here does

¹See Radford (1945) on the evolution of a cigarette currency and Newhouse (2004) on convergence to monetary equilibrium in a 3-commodity model.

that in commodity pairwise trading posts. Walras (1874) forms the picture this way (assuming m distinct commodities): "we shall imagine that the place which serves as a market for the exchange of all the commodities (A), (B), (C), (D) ... for one another is divided into as many sectors as there are pairs of commodities exchanged. We should then have $\frac{m(m-1)}{2}$ special markets each identified by a signboard indicating the names of the two commodities exchanged there as well as their ... rates of exchange..."²

The pattern of trade across trading posts will be determined endogenously. If most trading posts are active in equilibrium, hosting direct trade of each good in exchange for most other goods, then the equilibrium will be described as *barter*. If most trading posts are inactive in equilibrium, active trade concentrating on the small number of posts trading a single good against all others, then the equilibrium will be described as *monetary*, with the commonly traded good as *commodity money*.

1.2 Microeconomic Foundations of Money

Bringing price theory and monetary theory together — ideally, so that they are mutually reinforcing, at least so that they are consistent with one another — is a long-standing problem in monetary theory, Hicks (1935), Tobin (1961). Price theory is the most fundamental part of economic theory. It should be possible to derive the foundations of monetary theory from principles of price theory.³

Frank Hahn (1982) described the impasse: "The most serious challenge that the existence of money poses to the theorist is this: the best developed model of the economy cannot find room for it. The best developed model is, of course, the Arrow-Debreu version of a Walrasian general equilibrium. A first, and...difficult...task is to find an alternative construction without...sacrificing the clarity and logical coherence ... of Arrow-Debreu."

Tobin (1980) argued that the research program Hahn implicitly recommended would necessarily be unsuccessful: "Social institutions like money are

²Cournot (1838) and Shapley and Shubik (1977) also treat the trading post model.

³ Recent contributions on this issue include the overlapping generations model, Wallace (1980) and a vast literature following, the search and matching model, Jones (1976) and Kiyotaki and Wright (1989) with a similarly vast literature, Walrasian general equilibrium with transaction cost models with a smaller literature including Foley (1970), Hahn (1971), Heller and Starr (1976), Howitt (2005), Kurz (1974), Newhouse (2004), Starrett (1973), and Starr (2003a, 2003b).

public goods. Models of general equilibrium — competitive markets and individually optimizing agents — are not well adapted to explaining the existence and quantity of public goods... General equilibrium theory is not going to explain the institution of a monetary ... common means of payment.”⁴ The example of this paper is intended to show that Tobin’s forecast was excessively pessimistic. In the example below, barter is possible, but monetary trade is the competitive general equilibrium outcome. Consistent with Menger’s argument, the good with the narrowest bid/ask spread in equilibrium becomes the common medium of exchange.

2 Households, Trading Posts, Transaction Costs

Consider a pure exchange trading post economy with ten commodities denoted 1, 2, 3, ..., and 10.

2.1 Households

Let $[i,j]$ denote a household endowed with good i who prefers good j ; $i \neq j$, $i,j = 1, 2, \dots, 10$. Household $[i,j]$ ’s endowment is 1 unit of commodity i . Denote the endowment of $[i,j]$ as $r_i^{[i,j]} = 1$. $[i,j]$ ’s utility function is $u^{[i,j]}(x_1, x_2, x_3, \dots, x_{10}) = \sum_{k \neq j} x_k + Ax_j$, $A \gg 1$. That is, household $[i,j]$ values goods 1, 2, 3, ..., 10 as linear substitutes, with good j being many times more desirable than any other.

Consider a population denoted Λ of households including several households endowed with each good and each household desiring a good different from its endowment. Thus, there are four households endowed with good 1, preferring respectively, goods 2, 3, 4, and 5: $[1,2]$, $[1,3]$, $[1,4]$, $[1,5]$. There are four households endowed with good 2, preferring respectively goods 3, 4, 5, 6: $[2,3]$, $[2,4]$, $[2,5]$, $[2,6]$. The roll call of households proceeds so forth, through $[9, 10]$, $[9,1]$, $[9, 2]$, $[9, 3]$ and finally $[10, 1]$, $[10, 2]$, $[10, 3]$, and $[10,4]$.

⁴ Tobin’s focus here is apparently on a unique common medium of exchange and principal intertemporal store of value, ‘monetary’ in the sense of being a fiat money or fiduciary instrument without intrinsic value. Thus it may not be completely fair to infer that Tobin views the topic of this paper as impossible.

Population Λ displays absence of double coincidence of wants ⁵. For each household endowed with good i and desiring good j , $[i,j]$, there is no precise mirror image, $[j,i]$. Nevertheless, there are four households endowed with one unit of commodity 1, and four households strongly preferring commodity 1 to all others. That is true for each good. Thus gross supplies equal gross demands, though there is no immediate opportunity for any two households to make a mutually advantageous trade. Jevons (1875) tells us that this is precisely the setting where money is suitable to facilitate trade.

2.2 Trading posts

For each pair of distinct commodities, there is a trading post where those two are traded for one another. The notation $\{i,j\}$ represents the trading post where good i is traded for good j and (vice versa) good j is traded for i . Operating the trading post is a resource-using activity. For ease of notation, costs will be compensated unit-for-unit by goods traded at the post. Denote this cost of operating trading post $\{i,j\}$ as $C^{\{i,j\}}$. In actual economies these costs include the inputs of trading firms such as brokers, retailers, shippers, etc. and the non-marketed resources of households and firms used in the transactions process. The current representation, pricing transaction costs in the bid/ask spread, is unrealistic but convenient and effective.

The notion of a trading post for each good in exchange for each alternative sets up many more specialized trading firms than we expect actually to see active in any economy, but it is a convenient formalization. With N commodities, there are $N(N-1)/2$ trading posts (45 trading posts when $N = 10$). Most will be inactive but priced in a monetary equilibrium. Using this model as a basis for deriving the use of a common medium of exchange represents: (i) that a meaningful discussion of means of payment, depends on the notion that goods do not trade for all other goods in a single transaction; segmentation of the market is part of monetization, Alchian(1977); (ii) monetary trade is an equilibrium outcome based on individual optimization and market clearing, where barter could be chosen as an alternative.

The Arrow-Debreu model includes delivering goods and services to a single

⁵ "Double coincidence of wants", Jevons (1875) posits that barter is the outcome in the rare event where traders can directly, without an intermediary good, arrange pairwise mutually improving trades. An exchange of good i for good j then includes one trader with an excess supply of i and an excess demand for j , and a second trader with the opposite unsatisfied supply and demand.

(centralized) market, receiving an accounting credit for the delivery, and withdrawing goods and services of equal value. The trading post model further decentralizes this process, allowing sellers to decide separately which of $N-1$ other goods they will accept in exchange for the good they supply.

Budgets must balance at each trading post — that is, you pay for what you get not only over the course of all trade (as in the Arrow-Debreu model) but at each trading post separately. This is a note of realism; that is how budget constraints apply in actual transactions. A household delivers good i to trading post $\{i,j\}$ and the delivery is evaluated at the post's bid price determining how much good j the household receives. Budget balance requires that the values be equal.

3 Transaction Costs and Trading Post Prices

3.1 Transaction Costs

Consider trading posts with a linear transaction cost structure. The trading post buys goods from households delivering goods to the post and resells them or retains them to cover transaction costs. Thus, let the cost structure of trading post $\{i,j\}$, $i,j = 1, 2, \dots, 9, i \neq j$, be:

$$C^{\{i,j\}} = .1 \times (\text{volume of goods } i \text{ and } j \text{ purchased by the post})$$

Marginal cost of trading i for j is 0.1 times the gross quantity traded. The trading post expects to cover its transaction costs through the bid/ask spread.

Trading good 10 is assumed to be costless. Thus,

$$C^{\{10,j\}} = .1 \times (\text{volume of good } j \text{ purchased by the post}), \text{ for } j = 1, 2, \dots, 9.$$

3.2 Bid and ask prices

Trading post $\{1,2\}$ accepts good 1 in exchange for good 2 and accepts good 2 in exchange for good 1. Prices are expressed as a rate of exchange between goods 1 and 2. That is, good 1 is priced in units of good 2 and good 2 is priced in units of good 1. In order to cover the post's operating costs, the prices at which the public buys (ask or retail prices) are higher than those at which the public sells (bid or wholesale prices). The difference between buying

and selling prices covers operating costs. Marginal cost pricing leads to initial bid prices depicted in Table 1, below. Prices will eventually adjust to market clearing levels.

Table 1: Marginal Cost Pricing Initial Bid Prices at Trading Posts

selling:	1	2	3	4	5	6	7	8	9	10
buying:	1	X	.9	.9	.9	.9	.9	.9	.9	1
	2	.9	X	.9	.9	.9	.9	.9	.9	1
	3	.9	.9	X	.9	.9	.9	.9	.9	1
	4	.9	.9	.9	X	.9	.9	.9	.9	1
	5	.9	.9	.9	.9	X	.9	.9	.9	1
	6	.9	.9	.9	.9	.9	X	.9	.9	1
	7	.9	.9	.9	.9	.9	.9	X	.9	1
	8	.9	.9	.9	.9	.9	.9	.9	X	1
	9	.9	.9	.9	.9	.9	.9	.9	.9	X
	10	.9	.9	.9	.9	.9	.9	.9	.9	X

Each entry in Table 1 represents the bid price (denominated in units of the row good) for delivery of a unit of the column good. Thus, the diagonal is blank — no good is bought or sold for itself. The prices in this example show that selling one unit of good 1 for good 2 pays 0.9 units of 2. Conversely, selling one unit of good 2 for good 1 pays 0.9 units of 1. Reflecting marginal costs, the bid price of good 10 is unity. Consider trade at the $\{1,2\}$ trading post. Suppose one trader delivers one unit of 1 and a second trader delivers one unit of 2. The post pays out .9 good 2 to the first and .9 good 1 to the second. Trade at the post clears. The remainder, .1 good 1 and .1 good 2, stays with the trading post covering its operating costs.

At trading post $\{i,j\}$, the ask price of j (denominated in i per unit j) is the inverse of the bid price of i (denominated in j per unit i). The bid price of goods 1, ..., 9 in Table 1 is 0.9, implying that the ask price is 1.11. Denote the bid price of good i at $\{i,j\}$ as $q_i^{\{i,j\}}$. Then the ask price of j is $[q_i^{\{i,j\}}]^{-1}$. Denote the purchase of i by a typical household h at $\{i,j\}$ as $b_i^{h\{i,j\}}$, sale of j as $s_j^{h\{i,j\}}$. Then the budget constraint facing household h at $\{i,j\}$ is $b_i^{h\{i,j\}} = s_j^{h\{i,j\}} q_j^{\{i,j\}}$. Household h 's consumption of good i then is $x_i^h \equiv r_i^h + \sum_{j=1}^{10} [b_i^{h\{i,j\}} - s_i^{h\{i,j\}}]$.

In an economy of N commodities there are $N(N-1)/2$ trading posts each with two posted prices (bid for one good in terms of a second, and bid price of the second in units of the first) totaling $N(N-1)$ pairwise price ratios. In this paper's example, with 10 commodities, there are 90 posted bid prices in Table 1. Prices are posted at all trading posts — including those without active trade.

3.3 Pricing at inactive trading posts

The price system provides incentives that communicate what goods should be produced. Less conspicuous, but obvious and equally important, prices provide the incentives that determine corner solutions, which goods should not be produced. Goods provided at zero quantity are not produced because the prices at which they would trade make the equilibrium quantity zero.

The price system here must answer the question: which trading posts operate at positive trading volume? In actual economies, most conceivable pairwise commodity trades do not occur. Professors would like to trade lectures for food, but find that the implicit market prices for this exchange are unattractive. Better to trade lectures for money and money for food.

How is this choice formalized with the help of the price system? A trading post becomes unattractive in equilibrium, and will have zero trading volume, when its bid/ask spread is wide enough to discourage trade. This is a corner solution of the trading post model.

A barter equilibrium would be an outcome where most pairwise trading posts operate at a positive trading volume. Conversely, in a monetary equilibrium most of the 45 trading posts posited here would be inactive (have zero trade) in equilibrium; it is the price system in equilibrium that determines their inactivity. In a commodity-money equilibrium, trading activity concen-

trates on the 9 trading posts that deal in a single one of the 10 goods, where the commodity money is traded for the other 9 goods. The money price of a good then is the price at the trading post where it is traded for the commodity money.

4 Marginal cost pricing equilibrium

An array of prices $q_i^{o\{i,j\}}$ and trades $b_i^{oh\{i,j\}}$, $s_j^{oh\{i,j\}}$ for $h \in \Lambda$ is said to be a *marginal cost pricing equilibrium* if each household $h \in \Lambda$ optimizes utility subject to budget at prevailing prices, each trading post clears, and trading posts cover marginal costs through bid/ask spreads at prevailing trading volume. This description leaves unspecified whether marginal costs are recouped through the pricing on i , j , or both.

More formally, a marginal cost pricing equilibrium under the transaction cost function above consists of $q_i^{o\{i,j\}}$, $q_j^{o\{i,j\}}$, $1 \leq i, j \leq 10, i \neq j$, so that:

For each household $h \in \Lambda$, there is a utility optimizing plan $b_n^{oh\{i,j\}}$, $s_n^{oh\{i,j\}}$ so that

$$b_i^{oh\{i,j\}} = s_j^{oh\{i,j\}} q_j^{o\{i,j\}} \text{ (budget balance),}$$

$$\sum_h b_n^{oh\{i,j\}} \leq \sum_h s_n^{oh\{i,j\}}, \quad n = i, j \text{ (market clearing),}$$

For $i = 1, \dots, 9; j = 1, 2, \dots, 9; i \neq j$,

$$\begin{aligned} & 0.1 \times \sum_{h \in \Lambda} [s_i^{oh\{i,j\}} + s_j^{oh\{i,j\}}] \\ &= \sum_{h \in \Lambda} ([s_i^{oh\{i,j\}} - b_i^{oh\{i,j\}}] + [s_j^{oh\{i,j\}} - b_j^{oh\{i,j\}}]) \end{aligned}$$

For $i = 1, \dots, 9; j = 10$,

$$\begin{aligned} & 0.1 \times \sum_{h \in \Lambda} [s_i^{oh\{i,10\}}] \\ &= \sum_{h \in \Lambda} ([s_i^{oh\{i,10\}} - b_i^{oh\{i,10\}}] + [s_{10}^{oh\{i,10\}} - b_{10}^{oh\{i,10\}}]) \end{aligned}$$

(transaction cost coverage).

The concluding expressions are marginal cost pricing conditions; at equilibrium trading volume each trading post should cover its costs.

The budget balance requirement applies at each transaction at each trading post. Thus, a household acquiring good j for i at $\{i,j\}$ and retrading j at $\{j, k\}$ is acquiring j at its ask price (in terms of i) at $\{i,j\}$ and delivering j at its bid price at $\{j, k\}$. In that sequence of trades, the trader experiences — and pays — j 's bid/ask spread.

4.1 Starting trades

Start with the population Λ described in Section 2.1, and the transaction costs described in Section 3.1 summarized as the price array in Table 1. Each household decides what trade it wants to make. Household $[1,2]$ goes to trading post $\{1,2\}$ and sells its good 1 for good 2. All the other households behave similarly.

This pattern of trade poses a problem. It is not an equilibrium. At each of the active trading posts, there is an excess demand. Trading post $\{1,2\}$ has a supply of good 1 and a demand for good 2. It needs some trader to come with a complementary supply of good 2 and a demand for good 1. All of the active trading posts have a similar problem.

4.2 Price adjustment

The starting trades above create an excess demand for one good and an excess supply of the other at each post hosting active trade. The price mechanism (aided by trading post managers or the Walrasian auctioneer) adjusts prices as in Table 2. Goods in excess demand have their bid prices increased to unity. Goods in excess supply are discounted to bear the full marginal transaction cost of both sides of their trade.

Table 2: Marginal Cost Pricing - Adjusted Bid Prices at Trading Posts

selling:	1	2	3	4	5	6	7	8	9	10	
buying:	1	X	1	1	1	1	.9	.8	.8	.8	.9
	2	.8	X	1	1	1	1	.9	.8	.8	.9
	3	.8	.8	X	1	1	1	1	.9	.8	.9
	4	.8	.8	.8	X	1	1	1	1	.9	.9
	5	.8	.8	.8	.8	X	1	1	1	1	1
	6	.9	.8	.8	.8	.8	X	1	1	1	1
	7	1	.9	.8	.8	.8	.8	X	1	1	1
	8	1	1	.9	.8	.8	.8	.8	X	1	1
	9	1	1	1	.9	.8	.8	.8	.8	X	1
	10	1	1	1	1	.9	.9	.9	.9	.9	X

5 Monetary Equilibrium

The adjusted prices in Table 2 do not lead to a barter equilibrium. Household [1,2] considers trading 1 for 2 at trading post $\{1,2\}$. But $\{1,2\}$ has $q_1^{\{1,2\}} = 0.8$, representing a discount of 20% on [1,2]'s endowment of good 1. There appears to be a more attractive alternative. Household [1,2] can exchange 1 for 10 without discount at $\{1, 10\}$ and then 10 for 2 with a 10% discount at $\{2, 10\}$, a far better deal. At Table 2's prices, households want to treat good 10 as commodity money.

The pricing array above generates excess demands for goods 5, 6, 7, 8, at $\{10, 5\}$ through $\{10,8\}$. Prices adjust once again. Market clearing prices appear in Table 3. In this array, good 10 — with the narrowest prevailing bid/ask spread — is the most liquid (saleable) good, Menger's candidate for

commodity money.

Table 3: Marginal Cost Pricing - Market Clearing Bid Prices at Trading Posts

selling:	1	2	3	4	5	6	7	8	9	10
buying:	1	X	.9	.9	.9	.9	.8	.8	.8	.9
	2	.8	X	.9	.9	.9	.9	.8	.8	.9
	3	.8	.8	X	.9	.9	.9	.9	.8	.9
	4	.8	.8	.8	X	.9	.9	.9	.9	.9
	5	.8	.8	.8	.8	X	.9	.9	.9	.9
	6	.9	.8	.8	.8	.8	X	.9	.9	.9
	7	.9	.9	.8	.8	.8	.8	X	.9	.9
	8	.9	.9	.9	.8	.8	.8	.8	X	.9
	9	.9	.9	.9	.9	.8	.8	.8	.8	X
	10	1	1	1	1	1	1	1	1	X

The array of equilibrium trades follows:

For $i = 1, 2, 3, 4, \dots, 9; j \neq 10$,

$$s_i^{o[i,j]\{i,10\}} = 1, b_{10}^{o[i,j]\{i,10\}} = 1, s_{10}^{o[i,j]\{j,10\}} = 1, b_j^{o[i,j]\{j,10\}} = .9$$

For $i = 6, 7, 8, 9; j = 10$,

$$s_i^{o[i,j]\{i,10\}} = 1, b_{10}^{o[i,j]\{i,10\}} = 1$$

For $i = 10, j = 1, 2, 3, 4$,

$$s_{10}^{o[10,j]\{10,j\}} = 1, b_j^{o[10,j]\{10,j\}} = .9$$

The arrangement is a market clearing equilibrium with all trade going through good 10. Good 10 acts as medium of exchange, commodity money. The trading posts dealing in good 10, $\{10,1\}, \{10,2\}, \{10,3\}, \dots, \{10,9\}$, cover their operating costs. For each good $n = 1, 2, 3, \dots, 9$, they find four sellers coming to the post delivering one unit of n in exchange for 10, and four buyers

coming to the post, exchanging good 10 for good n . The trading post clears.

Household $[3,4]$, for example, wants to trade good 3 for good 4. He considers trading the goods directly at $\{3,4\}$. Pricing at $\{3,4\}$ means that household $[3,4]$ could deliver good 3 to $\{3, 4\}$ and receive good 4 after incurring a 20% discount covering the bid/ask spread, using direct trade. Alternatively, $[3,4]$ can trade at $\{3,10\}$ and at $\{4,10\}$. He sells 3 at $\{3,10\}$ in exchange for 10 and sells 10 at $\{4,10\}$ in exchange for the 4 he really wants. In this indirect trade, he incurs a 10% discount, saving 10% compared to direct trade, by using monetary trade with good 10 as 'money.' Indirect monetary trade is more attractive because it is less expensive. The lower expense reflects lower resource costs due to the low transaction cost of good 10 and the matching of suppliers and demanders of each good $n = 1, 2, \dots, 9$, at the trading posts $\{10, n\}$ where good 10 is traded. As Jevons (1875) reminds us, the common medium of exchange overcomes the absence of a double coincidence of wants. Thus each household needs to incur the transaction cost on only one side of the monetary trade he enters.

In equilibrium, all trading posts $\{i,j\}$, $i, j \neq 10$, except the those dealing in good 10 become inactive. All trading posts are priced, but trade is transacted only at the nine posts dealing in 10. The trading posts clear. Good 10 has become the common medium of exchange, commodity money.

5.1 Demand for Money

This is a single period flow equilibrium model. Hence this model of commodity money does not imply a demand for money-holding — or for an inventory of any good. There are no stocks to account for. Pure flows clear the markets. At trading post $\{i,j\}$ it is sufficient that inflow of each good be at least equal to outflow. It is sufficient that each seller provide his supplies from endowment or simultaneous purchases elsewhere. There is no time structure and no cash-in-advance (or inventory-in-advance) restriction on trade. Those restrictions would create a demand for inventories — including money stock — but require a time structure and an equilibrium notion that includes both stocks and flows (e.g. Kurz (1974), Heller and Starr (1976)).

5.2 Fiat Money

The obvious interpretation of good 10 in the treatment here is that it is a commodity money. In order to interpret good 10 as fiat money, the model would need to provide an explanation for why households [6,10], [7,10], [8,10], [9,10] desire an unbacked currency. It would be sufficient that they plan to retrade fiat money for some other desired good in a succeeding period and that they expect it to be valuable then, Grandmont (1977), Wallace (1980).

6 Conclusion

There is a surprise here. Tobin (1961, 1980) and Hahn (1982) despaired of getting a general equilibrium model based on elementary price theory to result in a common medium of exchange. But the pricing array in Table 3 leads directly to a monetary equilibrium. Monetary trade is the result of decentralized optimizing decisions of households guided by prices without government direction or fiat. The price system provides all of the co-ordination required to maintain a common medium of exchange. That's the successful co-ordination by prices we expect in an Arrow-Debreu Walrasian general equilibrium model, Debreu (1959). But the logic of the Arrow-Debreu model is framed for a non-monetary economy. The example here demonstrates — as Menger (1892) said — that the logic of the price system can be used to generate a monetary equilibrium with a single common medium of exchange.

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