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Tracking State Estimation in Distribution Networks Using Distribution-level Synchrophasor Data

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***Abstract*—State estimation is often a challenging task in distribution systems due to numerous nodes and limited number of measurement sources. As a result, distribution system state estimation is often not real-time; instead, it is done once every 5 to 15 minutes, e.g., when smart meter measurements are available. Accordingly, this paper proposes a novel method for *tracking state estimation* in distribution systems to update the system states following an event, e.g., a sudden load change, that occurs in between the 5 to 15 minutes intervals of a typical distribution system state estimation cycle, *without* the need to rerun the whole state estimation. This method is of great interest for real-time monitoring and control applications. The proposed method uses measurements from as few as *only two* distribution-level phasor measurement units, a.k.a., micro-PMUs, that are installed at the substation and at the end of the main feeder or laterals. The method is developed based on the *compensation theorem* in circuit theory to generate an equivalent circuit according to the pre-event and post-event feeder data in order to update the state estimation results. The effectiveness of the proposed method is demonstrated through simulating the IEEE 33-bus test system.**

***Keywords*: Tracking state estimation, micro-PMUs, compensation theorem, data-driven, distribution system events.**

I. INTRODUCTION

Real-time monitoring and control play a crucial role in power distribution networks to achieve optimal and reliable operation. Distribution System State Estimation (DSSE) is a cornerstone tool in advance distribution system operation. It uses the measurements, e.g., smart meters or asset sensors, to best approximate the states of the system which best fit the available measurements. As the system scales up, the computational burden of DSSE grows to the extent that the computational time may exceed the rate at which new measurements are obtained. This is particularly challenging when a subset of measurements come from the emerging advanced sensor devices, such as distribution-level phasor measurement units, a.k.a, micro-PMUs [1]. However, the states of the system *do* change on a continuous basis; thus, updating the DSSE results *is* necessary to enhance *situational awareness* at distribution level, as well as to carry out more efficient control and operation.

A technique called Tracking State Estimation (TSE) can address the above issues. TSE does not require running a full state estimation. It rather only fine tunes the states of the system at occasions during the standard state estimation cycles.

The concept of TSE was first introduced by F. C. Schweppe in 1980 [2] for transmission system state estimation. The usage of TSE in distribution system is developed to incorporate unsynchronized measurements within DSSE. The synergy of smart meter and supervisory control and data acquisition (SCADA) data for DSSE is proposed by [3], where the archived customer meter data and the SCADA data with the same time stamp are used off-line to estimate the states at feeder buses. The results are then used in the on-line quasi dynamic state estimations. In [4], the authors investigate the effect of smart meter data in real-time state estimation. To do so, they treat smart meter data as pseudo-measurement to incorporate them into the real time state estimation, and consider the effect of delay in the collected smart meter data on real time DSSE. In addition, the incorporation of different real-time metering and measurement instruments was scrutinized in [5], [6].

Recently, various applications have been reported for micro-PMUs in distribution systems, e.g., see [1], [7]–[9]. Considering the applications that are relevant to DSSE, in [10], the authors proposed an efficient branch-current-based DSSE, which integrates synchronized phasor measurements provided by PMUs. Haughton *et al* [11] developed a linearized, three-phase, distribution class state estimation algorithm for real-time applications in smart distribution systems. The practical use of micro-PMUs in standard DSSE algorithms is reported in [12]. The integration of micro-PMU data, together with remote terminal unit data and smart meter data, into the DSSE is discussed in [13]. To the best of our knowledge, no prior study has addressed the application of micro-PMU data in TSE.

In this paper, we propose a novel TSE method to be added to the standard DSSE, so as to make use of voltage and current synchrophasor data in order to update the system states following a change in a system element. The changes in the system can be detected by the voltage and current phasors recorded through micro-PMUs. The essence of the proposed method is based on the analysis of the equivalent-circuit for the distribution feeder, by making use of *compensation theorem* from circuit theory [14], according to the pre-event and post-event micro-PMUs measurement. Our approach is highly practical because it requires using as few as only two micro-PMUs. The micro-PMUs are proposed to be installed at the substation and at the end of the main feeder or laterals.

II. PROPOSED METHOD

In this paper, we seek to develop a TSE method to update the results in distribution system state estimation after an *event* occurs. Here, distribution system events are defined as any change in system parameters which does *not* interrupt the

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normal operation of the system, such as a load changing in a certain location. Such events are detected by monitoring the voltage and current measurements that are recorded by micro-PMUs. For example, by applying the method in [7], one can not only detect the distribution system events but also identify their root causes, i.e., to indicate whether each event occurred on the distribution feeder of interest; or whether it occurred in some other parts of the network, e.g., at transmission or sub transmission systems. Accordingly, for the rest of this paper, we assume that a method such as the one in [7] is already applied to the micro-PMU measurements to narrow down our focus in TSE to only those events that have root causes in the distribution system and thus they require updating the distribution system state estimation results.

A. Equivalent Circuit

An event in a circuit, such as a change in an element, can change all or a subset of nodal voltages and branch currents along the circuit. According to the *compensation theorem* in circuit theorem [14, pp. 177], the amount of such changes in the nodal voltages and branch currents can be obtained through an *equivalent circuit*, in which the changed element is replaced with a *current source* that injects current at a level equal to the amount of change in the current going through the element; and all sources are replaced with their *internal impedances*.

Consider a segment in a power distribution network, such as the one shown in Fig. 1. Assume that two micro-PMUs are installed[1] at the upstream and downstream of this segment; thus distinguishing this part of the network from the rest of the power system . Suppose n denotes the number of buses between these two micro-PMUs, enumerated from 1 to n.

Suppose an event occurs at bus k, where $k \in \{1, ...n\}$. From compensation theorem, an equivalent circuit can be constructed for this power system in which a current source is placed with current ΔI_k at bus k, and the upstream and downstream networks are replaced with their equivalent impedances. The equivalent impedance for the upstream network is obtained as:

$$y^u \triangleq \frac{\Delta I^u}{\Delta V^u}, \tag{1}$$

where ΔV^u and ΔI^u indicate the changes in voltage and current before and after the event that are captured by the micro-PMU installed at the upstream. Similarly, the equivalent impedance for the downstream network is obtained as:

$$y^d \triangleq \frac{\Delta I^d}{\Delta V^d}, \tag{2}$$

where ΔV^d and ΔI^d indicate the changes in voltage and current captured by the micro-PMU installed at the downstream.

The nodal voltages and branch currents of the buses equipped with micro-PMUs in the presence of this current source are equal to the changes in nodal voltages and branch currents obtained from subtracting pre-event and post-event voltage and current phasors recorded by the micro-PMUs. Therefore, in the equivalent circuit, the voltages of the buses equipped with micro-PMUs are known, i.e., we have:

$$\Delta V_1 = \Delta V^u \tag{3}$$

and

$$\Delta V_n = \Delta V^d, \tag{4}$$

where ΔV_1 and ΔV_n indicate the *difference voltage phasors* of bus 1 and bus n which are home to the two micro-PMUs.

The proposed application of the compensation theorem is to help estimating the location and injection of the current source associated with the event to best fit the the changes captured via micro-PMUs. The importance of the compensation theorem is to allow analyzing an event through examining such equivalent circuit, which makes the analysis easier and faster.

B. Network Model

Let $\Delta V = [\Delta V_1, \cdots, \Delta V_n]^T$ and $\Delta I = [\Delta I_1, \cdots, \Delta I_n]^T$ denote the voltage and current difference vectors. Even with no knowledge on the location of the event, we can write the Kirchhoff's current law (KCL) for this equivalent circuit as:

$$\Delta I = (Y^{bus} + e_1 e_1^T y^u + e_n e_n^T y^d)\Delta V \tag{5}$$

where, Y^{bus} is the standard nodal admittance matrix of the equivalent circuit. Let $y_{ii'}$ denote admittance line between two nodes i and i'. In this regard, Y^{bus} is an $n \times n$ complex-valued matrix whose off-diagonal elements are $Y^{bus}_{ii'} = -y_{ii'}$ and diagonal elements are $Y^{bus}_{ii} = \sum_{i' \neq i} y_{ii'}$. The equivalent admittance of the upstream and downstream networks are integrated into the model via $e_1 e_1^T y^u$ and $e_n e_n^T y^d$, where e_i is an $n \times 1$ standard basis vector, i.e., $e_i = [0, \cdots, 0, 1, 0, \cdots, 0]^T$.

C. Lease Squares Estimation

Assume that the initial states of the system are given through the *most recent cycle* of the standard DSSE application, i.e., sometime over the past 15 minutes. Therefore, the loading situation at each node in the system is known prior to the occurrence of the first event. Without loss of generality, we assume that all loads are constant-impedance, such that at each bus i, ΔV_i is equal to $Z_i \Delta I_i$, where Z_i indicates the impedance of the load at bus i and is calculated from the most recent state of the system. Other types of loads, namely constant-current and constant-power loads, can also be formulated and similarly integrated into the model, e.g., see [15].

The product of bus current injection and bus impedance is valid at all buses *except for* at bus k in which the event occurs. Recall from the compensation theorem that at the event bus, a current source injects ΔI_k into the equivalent circuit; and hence, the product of bus current and bus impedance is no longer a correct indication of the bus current. Therefore, we can consider the following least squares (LS) problem to estimate vectors ΔV and ΔI that are induced by the event:

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[1] The case with several micro-PMUs is explained in Section II-E.

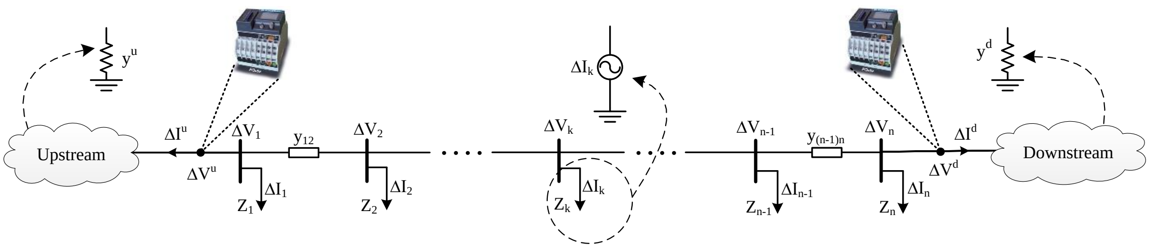

Fig. 1. Representation of a network based on compensation theorem equivalent circuit. The targeted distribution network is restricted via several micro-PMUs.

$$
\begin{aligned}
\underset{\Delta V, \Delta I, b}{\text{Minimize}} \quad & \sum_{i=1}^{n} \| b_i(\Delta V_i - Z_i \Delta I_i) \|^2 \\
\text{Subject to} \quad & \text{Eqs. } (3), (4), (5) \qquad (6) \\
& \sum_{i=1}^{n} b_i = n - 1,
\end{aligned}
$$

where $b = [b_1, \cdots, b_n]^T$ indicates the vector of binary variables, in which b_k serves to check the hypothesis of the event occurring at bus k. If we assume that the event occurs at bus k, then we can conclude that $\Delta V_i = Z_i \Delta I_i$, for any $i \in \{1, \cdots, n\} - \{k\}$. Therefore, the objective function in (6) is minimized once the binary variable associated with the true event bus, i.e., b_k, is zero. This is enforced by the binary summation constraint. In addition, the correct solution should be in line with the network model and the measurements which are directly recorded by the micro-PMUs. To do so, the constraints in (3) and (4) serve to check the validity of the solution with the micro-PMU measurements. Also, constraint in (5) is used for check the consistency of the solution with the equivalent circuit model. Here, we make the practical assumption that the measurements from the micro-PMUs are precise. However, a large value of the residual in the optimal objective value of the problem in (6) could be an indication for bad data measurement.

D. Updating System State

Let $\Delta V^\star$ and $\Delta I^\star$,denote the optimal solution of problem (6). The system voltages and currents are updated as:

$$
\begin{aligned}
V^{post} &= V^{pre} + \Delta V^\star \\
I^{post} &= I^{pre} + \Delta I^\star
\end{aligned} \qquad (7)
$$

where V^{pre} and I^{pre} indicate the bus voltages and injection currents prior to the event. It is assumed that the most recent system state is given based on the standard state estimation carried out for each cycle and several continuous updating in the cycle. Therefore, the pre-event system state is known and the system sate is updated following an event occurrence as described in (7). Moreover, for the next round of the updating, the impedance of the bus undergoing the event is modified as:

$$
Z_{k^\star} = \frac{V_{k^\star}^{post}}{I_{k^\star}^{post}}, \qquad (8)
$$

where $k^\star$ is determined based on the binary variable associated with the event bus. Specifically, $k^\star = i$ for bus index i for which $b^\star = 0$. It is worth to mention that the location of the event, i.e., $k^\star$, can be determined also through a method proposed in [7], which we use in this paper as a validation for our proposed method. Algorithm 1 summarized the proposed model for TSE.

Algorithm 1 TSE

```
Get initial system states.
Obtain network model equations using (5).
Solve LSE (6).
return  Updated system states using (7).
return  Updated impedance of event bus using (8).
```

E. Extension for Multiple Micro-PMUs

The method that was developed in Sections II-A to II-D are based on the assumption that the distribution network segment of interest is restricted by two micro-PMUs at its upstream and downstream. Next, suppose the network segment is restricted by several micro-PMUs. Each micro-PMU has a downstream, with the following equivalent impedance of:

$$
y^j \triangleq \frac{\Delta I^j}{\Delta V^j}, \quad \forall j \in \Omega^{PMU}, \qquad (9)
$$

where Ω^{PMU} represent the set of buses which are equipped with micro-PMUs, j is an index. In this regard, the network model in (5) can be modified as:

$$
\Delta I = \left(Y^{bus} + \sum_{j \in \Omega^{PMU}} e_j e_j^T y^j \right) \Delta V. \qquad (10)
$$

The rest of the model would be the same as expressed in (6) for the case of two micro-PMUs. Of course, the constraints with respect to (3) and (4) will also be expanded to include the voltage of the buses equipped with micro-PMUs.

III. CASE STUDY

This section demonstrates the effectiveness of the proposed TSE method on the 33 bus distribution test system. The single line diagram of the under-study feeder is shown in Fig. 2, which includes a main feeder from bus 1 to bus 18 as well as three laterals respectively connected to bus 2, bus 3, and bus 6. The

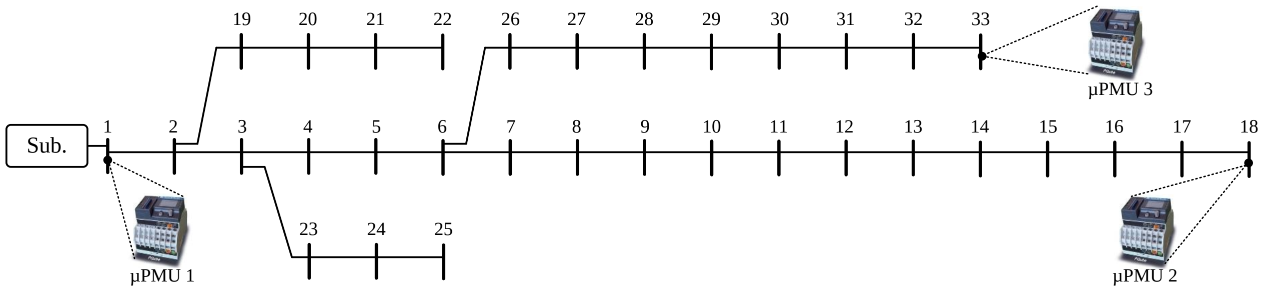

Fig. 2. Under-study feeder.

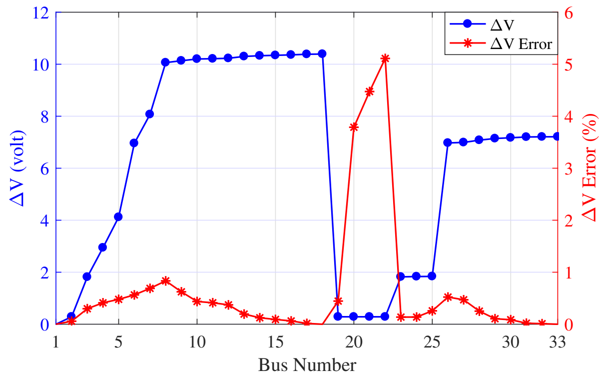

Fig. 3. Estimated buses voltage and related errors with the hypotheses of event at bus 8.

load data and line data corresponding to the under-study test system can be found in [16].

To this case study, MATLAB Power System Simulation Package (MATPOWER 6.0) is utilized to simulate the under-study test system [17]. Also, we assume that three micro-PMUs are deployed at bus 1, 18, and 33, see Fig. 2. Accordingly, voltage difference phasors and current difference phasors corresponding to monitored buses are calculated by conducting pre-event and post-event steady state power flows through MATPOWER. Next, the obtained results for different loads switching will be reported followed by sensitivity analyses to demonstrate the effectiveness of the proposed TSE method against network topology data as well as measurement data quality.

A. *Results*

Suppose a *50kW+20kVAR* load curtailment on the bus *8*, which is about 30% and 20% reduction of nominal loading. Table I shows the results associated with estimated load changing in bus 8. As it can be seen, the load changing is estimated as *49.7kW+21.2kVAR*, which demonstrates the effectiveness of the proposed TSE method, i.e., 1% error in active power curtailment and about 6% error in reactive power curtailment.

In addition, Fig. 3 shows the estimated voltage magnitude changes, due to load curtailment in bus 8, obtained from TSE method in the equivalent circuit of under-study network. Also, errors of estimated voltage magnitude changes are reported in 3. As it can be seen from figure, the maximum errors in estimated ΔV corresponds to bus 22, which is about 5%. It is worth to mentioned that the errors greater than 1% correspond to the buses hosted by first lateral branch. This error can be decrease by installing a micro-PMU at the end of the lateral. Also, from comparing errors of bus 1 to 18 in Fig. 3, we can conclude that by moving far away from the monitored buses, generally the error values increase.

TABLE I
RESULTS ASSOCIATED WITH CHANGES IN BUS 8

Bus #	V^{pre}	ΔV	ΔI	ΔS
8	6880.4 ∠ -0.06	10.3 ∠ 12.85	2.61 ∠ 156.81	49.7+j21.2

TABLE II
RESULTS FOR DIFFERENT LOAD CHANGING IN SEVERAL BUSES

Bus #	Load Switching	Max Error in ΔV (%)	ΔS
4	120+j80	1.3	120.5+j79.7
8	20+j10	5.0	20.1+j10.5
24	200	2.9	199.9+j4.5
30	j200	7.8	12.4+j190.7

Table II shows the results associated with loads curtailment in buses 4, 8, 24, and 30. For an individual event at the buses on the main feeder, i.e., buses 4 and 8, as well as the third lateral, i.e., bus 30, there is only one optimal solution for (6). However, for the event at bus 24, which is hosted by the second lateral, there are four possible optimal solutions for (6), i.e., $b_i = 0, \forall i \in \{3, 23, 24, 25\}$ results the minimum value for objective function. In other words, it can be noticed that the event has occurred in one of the buses hosted by second lateral, but it is not precisely determined. In order to distinguish these buses, another micro-PMU should be installed at the end of the second lateral.

B. *Sensitivity Analysis*

In this section, the effectiveness of proposed TSE method against network topology data as well as measurement accuracy will be studied. To such aim, sensitivity analyses will be conducted by considering line admittance uncertainty, voltage phasor measurement accuracy, and current phasor accuracy. In order to model uncertainties, a Monte Carlo algorithm is utilized to generate scenarios.

Table III shows the results obtained from TSE considering lines admittance uncertainty. As it can be seen, for line admittances error with 5% standard deviation, the load switching bus is precisely located. However, as expected, by increasing

TABLE III
METHOD EFFECTIVENESS VS. LINE ADMITTANCES UNCERTAINTY

Line Admittace Error SD (%)	Correct (%)	Neighbors (%)	ΔV Error SD (%)	P Error SD (%)	Q Error SD (%)
5	100	0	15.6	0.1	0.2
10	81	18	28.4	0.2	0.5
15	61	33	45.8	0.3	0.8
20	37	36	59.1	0.5	1.1

TABLE IV
METHOD EFFECTIVENESS VS. VOLTAGE MEASUREMENTS UNCERTAINTY

Voltage Phasor Error SD (%)	Correct (%)	Neighbors (%)	ΔV Error SD (%)	P Error SD (%)	Q Error SD (%)
5	98	2	5.1	0.1	0.5
10	76.6	21.8	10.2	0.2	0.9
15	61	30.6	15.3	0.3	1.5
20	44.2	36.6	20.9	0.5	2.0
25	35.2	34.4	24.8	0.6	2.3

the lines admittance error, the wrong event location increase, but most of the wrong identifications are related to identifying the neighboring buses as the load switching location, see third column of Table III .Moreover, table III shows that the maximum error in estimated ΔV is highly sensitive to the lines admittances. It is obvious that once the knowledge about the network is not precise, estimation of the network state is accompanied with error. Finally, the last two columns of table III shows the effectiveness of proposed TSE method in estimating load switching active power and reactive power even in presence of lines admittance uncertainties.

Table IV and V show the sensitivity of the proposed TSE method for measurements accuracy. The standard deviation of errors associated with micro-PMUs include both phasor angle and magnitude errors. We can clearly see that the error in voltage phasors has greater effect on the event location accuracy than the error in current phasors. However, the estimated value of load changing is highly sensitive to the current measurements error.

IV. CONCLUSIONS

This paper proposed a novel method for Tracking State Estimation (TSE). The method is built on equivalent circuit generated by compensation theorem. It updates system states following a change in the system measurements recorded by micro-PMUs, without the need to rerun the whole DSSE. Case studies confirm that the proposed method can accurately update the system states. The effectiveness of the method is scrutinized also over system parameters uncertainties. For a reasonable range of error in lines admittance and measurement accuracy, TSE results remain in an acceptable range of error, confirming a reliable estimation with potential application in practice.

TABLE V
METHOD EFFECTIVENESS VS. CURRENT MEASUREMENTS UNCERTAINTY

Current Phasor Error SD (%)	Correct (%)	Neighbors (%)	ΔV Error SD (%)	P Error SD (%)	Q Error SD (%)
5	100	0	4.1	4.3	5.8
10	97.4	2.6	7.6	8.1	10.8
15	79.4	19.2	12.3	13.2	16.3
20	65	26.8	16.2	17.9	22.2
25	49.4	36	21.6	23.0	31.2

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