Title
ABSORPTION CORRECTIONS AND MULTIPLE SCATTERING IN THE K-MATRIX FORMALISM

Authors
Joachain, Charles J.
Risk, Clif-ford.

Publication Date
1970-10-01

Submitted to Nuclear Physics B

UCRL-20114
Preprint

ABSORPTION CORRECTIONS AND MULTIPLE SCATTERING IN THE K-MATRIX FORMALISM

Charles J. Joachain and Clifford Risk

October 1970

AEC Contract No. W-7405-eng-48

LAWRENCE RADIATION LABORATORY
UNIVERSITY of CALIFORNIA BERKELEY

ABSORPTION CORRECTIONS AND MULTIPLE SCATTERING IN THE K-MATRIX FORMALISM

Charles J. Joachain*

Department of Physics
University of California
Berkeley, California 94720

and

Clifford Risk†

Lawrence Radiation Laboratory
University of California
Berkeley, California 94720

ABSTRACT

The K-matrix formalism is used to derive the absorption model and to reconcile absorption and rescattering corrections in high-energy hadronic two-body collisions. Detailed calculations are made in the case of elastic proton-proton scattering, for which we compare the K-matrix approach to the eikonal calculation of Frautschi and Margolis.

---

* On leave of absence from Physique Théorique et Mathématique, Faculté des Sciences, Université Libre de Bruxelles, Belgium. Research supported by the Air Force Office of Scientific Research, Office of Aerospace Research, U. S. Air Force, under grant No. F44620-C-70-0028.

† Supported by the United States Atomic Energy Commission.

1. Introduction

Since its original introduction by Heitler[1] to analyze radiation damping phenomena, the reaction or K-matrix concept has been widely used to investigate a variety of quantum collision processes. Of particular importance is the case of nuclear reactions where the reaction matrix theory was developed by Wigner[2]. The advantages of the K-matrix method are that it remains valid if the interaction cannot be described by a potential and that it satisfies the unitarity requirements in a particularly simple way.

These features of the K-matrix approach have encouraged a number of authors[3-9] to use this formalism in the case of high-energy hadronic collisions, where it provides a natural way to incorporate unitarity corrections to the Born diagrams. Another, frequently used unitarization procedure is the eikonal formalism, and the two approaches are of course not unrelated. We shall actually show in Sec. 2 that the results obtained within the eikonal framework can also be derived in the context of the K-matrix formalism for both elastic and inelastic two-body processes. In the former case, we recover the multiple scattering interpretation of Arnold[6], whereas in the latter one the basic formulae of the absorption model[10-15] are obtained. We also show that the K-matrix method provides a simple way of resolving the conflict[16] between "absorption" and "rescattering" corrections, without invoking[17] special properties of multiparticle production amplitudes.

In Sec. 3 we carry out a detailed comparison of the eikonal and K-matrix methods for high-energy elastic proton-proton scattering. This process has been analyzed in the eikonal formalism by Frautschi and

Margolis[18] (hereafter referred to as FM) who identified the contribution to the scattering amplitude arising from the (linear) Pomeranchuk trajectory--having a slope $\alpha' = 0.82\ (\mathrm{GeV})^{-2}$--with the Born term and obtained a differential cross section showing diffraction minima* beyond

* A similar diffraction pattern was obtained previously by Chou and Yang[19].

the forward peak. Using the same Born term as an ansatz for the K matrix, we find that the resulting differential cross section exhibits only breaks (no dips), consistent with present experimental evidence[20]. We also analyze the phase of the scattering amplitude and the total cross sections.

2. The K-matrix Approach to Absorption Corrections

Let us consider a two-body collision process. We denote by T the transition matrix and by K the reaction matrix. In terms of these operators the S matrix is given by

$$S = I - 2iT \tag{2.1}$$

and

$$S = \frac{I - iK}{I + iK} \tag{2.2}$$

where I is the unit matrix. The T and K matrices are related by the Heitler equations

$$\begin{aligned} T &= K - iKT \\ &= K - iTK\,. \end{aligned} \tag{2.3}$$

In terms of matrix elements, the Heitler equations for an inelastic transition $a \to b$ read

$$\begin{aligned} T &= K_{ba} - i \sum_n K_{bn}\, T_{na} \\ &= K_{ba} - i \sum_n T_{bn}\, K_{na} \end{aligned} \tag{2.4}$$

where the summation runs over all open channels, and multiplication implies an integration over phase space. Alternatively, we may interpret the Heitler equations (2.4) in terms of partial wave amplitudes, in which case they simply reduce to a set of algebraic equations[21].

We now specialize to high-energy two-body processes. A natural way to proceed in this case is to identify the K-matrix element K_{ba} with the Born term R_{ba} furnished by the peripheral or the Regge model. Before we do this, however, we note that since the Heitler equations automatically lead to a unitary S matrix, the summation on the right of Eqs. (2.4) provides the desired unitarity correction to the ansatz we have chosen for K_{ba}. Since we expect the initial and final states--containing diagonal matrix elements--to dominate[5,22-23] the sums in Eqs. (2.4), we retain only the contributions from these states[7] and write the first of Eqs. (2.4) as

$$T_{ba} = K_{ba} - iK_{ba} T_{aa} - iK_{bb} T_{ba}. \qquad (2.5)$$

Similarly, the second of Eqs. (2.4) yields

$$T_{ba} = K_{ba} - iT_{ba} K_{aa} - iT_{bb} K_{ba}. \qquad (2.6)$$

Combining these two equations, we have

$$T_{ba} = K_{ba} \frac{1 - \frac{i}{2}(T_{aa} + T_{bb})}{1 + \frac{i}{2}(K_{aa} + K_{bb})}. \qquad (2.7)$$

Before we proceed with Eq. (2.7), let us now consider the case of <u>elastic</u> scattering. The Heitler equations then read

$$\begin{aligned} T_{aa} &= K_{aa} - i \sum_n K_{an} T_{na} \\ &= K_{aa} - i \sum_n T_{an} K_{na}. \end{aligned} \qquad (2.8)$$

Keeping only the term $n = a$ in the sum, we now have

$$T_{aa} = K_{aa} - iK_{aa} T_{aa} \tag{2.9}$$

so that

$$T_{aa} = \frac{K_{aa}}{1 + iK_{aa}} . \tag{2.10}$$

If we solve this equation by iteration, we have

$$T_{aa} = K_{aa} - iK_{aa} K_{aa} + \cdots . \tag{2.11}$$

We now return to Eq. (2.7), use the first approximation $T_{aa} \simeq K_{aa}$ and $T_{bb} \simeq K_{bb}$ and expand the right-hand side to obtain

$$T_{ba} = K_{ba}[1 - i(T_{aa} + T_{bb}) + \cdots]. \tag{2.12}$$

This last equation may be rewritten as

$$T_{ba} = S_{bb}^{\frac{1}{2}} K_{ba} S_{aa}^{\frac{1}{2}} \tag{2.13}$$

where we have expanded

$$S_{aa}^{\frac{1}{2}} = (1 - 2i\ T_{aa})^{\frac{1}{2}} = 1 - i\ T_{aa} + \cdots . \tag{2.14}$$

It is interesting to note that a result similar to Eq. (2.13) has been obtained by Ball and Frazer[24)] who studied absorption corrections to the peripheral model from dispersion theory.

We now consider the question of finding a suitable ansatz for the quantities K_{aa} and K_{ba} appearing in Eqs. (2.10) and (2.12). As mentioned above, a natural way to proceed is to identify these quantities with the Born term R_{ba} given by the peripheral or the Regge model. Adopting this point of view, we find that Eq. (2.9) becomes

$$T_{aa} = R_{aa} - iR_{aa} R_{aa} + \cdots \tag{2.15}$$

while Eq. (2.13) yields

$$T_{ba} = S_{bb}^{\frac{1}{2}} R_{ba} S_{aa}^{\frac{1}{2}}. \tag{2.16}$$

We note that the connection between the elastic and inelastic processes is easily achieved within the K-matrix formalism. Equation (2.14) reproduces--up to second-order terms--the prescription given by Arnold[6)] to incorporate absorption corrections to elastic processes within the eikonal approximation and leads to a straightforward multiple scattering interpretation of the elastic scattering (see Fig. 1). Equation (2.15) is the standard absorption model formula for inelastic processes.

It is important to note that several of the approximations necessary to derive the formulae (2.15) and (2.15) are not required per se within the K-matrix formalism. For example, one could use directly Eq. (2.9) to study elastic scattering processes. We shall illustrate this point in the next section, where high-energy elastic proton-proton scattering will be analyzed in this way. It is also worth emphasizing that the K-matrix approach, while preserving unitarity, leads very naturally to the results of the absorption model, and therefore avoids the conflict between "rescattering" and "absorption" corrections pointed out by Finkelstein and Jacob[16)]. Of particular interest is the fact that his conflict is resolved without recourse to absorptive corrections to multiparticle production amplitudes[17)], whose existence is not firmly established.

3. The K Matrix Applied to Elastic Scattering

As an illustration of the methods of Sec. 2, we now apply the K-matrix formalism to study high-energy elastic proton-proton scattering. This process has been considered by FM in the eikonal formalism. We shall use their parameters for the single scattering term in order to compare the K-matrix result directly with the eikonal result.

A main difficulty of any multiple scattering approach to elastic scattering is identifying the Born or "driving" term. That is, should it be a pole or a cut, and fixed or moving? One has some assistance from the shrinking of the diffraction peak, but ambiguities still remain. (In the absorption approach to two-body-inelastic processes, this problem does not arise because all elastic scattering corrections to the inelastic exchange term are grouped into the two blobs displayed in Fig. 2.)

We begin by defining the scattering amplitude in terms of the S matrix by

$$S = 1 + iA/[k(s)^{\frac{1}{2}}], \qquad (3.1)$$

where $A = A(s,t)$ is the full nonflip helicity amplitude, and k is the c.m. momentum; A is normalized in terms of the total cross section by

$$\text{Im } A(s,0) = \frac{k(s)^{\frac{1}{2}}}{4\pi} \sigma_T(s). \qquad (3.2)$$

We pass to the impact parameter representation via

$$A(s,t) = \int \frac{d^2b}{2\pi} A(s,b) \exp(i\underset{\sim}{b}\cdot\underset{\sim}{q}) = \int_0^\infty bdb\, A(s,b)\, J_0(qb), \qquad (3.3)$$

where b is the impact parameter and $\underset{\sim}{q}$ the three-momentum transfer $(t = -\underset{\sim}{q}^2)$. The inverse relation is

$$A(s,b) = \int_0^\infty q\, dq\, A(s,t)\, J_0(qb). \qquad (3.4)$$

From (2.2) and (3.1) we obtain

$$A(s,b) = 2ik(s)^{\frac{1}{2}} \cdot \frac{iK}{1 + iK} = -2ik(s)^{\frac{1}{2}} \sum_{n=1}^{\infty} (-iK)^n. \qquad (3.5)$$

We now identify the first term of (3.5) with a single scattering term defined by a Regge pole

$$A_1(s,b) = -2ik(s)^{\frac{1}{2}} (-iK) = \int_0^\infty q\, dq\, c(\tfrac{s}{1})^{\alpha_P(t)}\, J_0(bq), \qquad (3.5)$$

where $\alpha_P(t) = 1 + t\alpha'$, and c is a constant. This gives

$$-iK(s,b) = -\frac{\xi}{2\mu} e^{-b^2/4\alpha'\mu} \qquad (3.7)$$

where $\xi = -c(s)^{\frac{1}{2}}/2k\alpha' s_0$ and $\mu = \ell n(s/i)$, and then

$$A(s,t) = 2i\alpha' k(s)^{\frac{1}{2}}\, \xi \sum_{n=1}^{\infty} \frac{1}{n}(-\xi/2\mu)^{n-1} \exp(\frac{t\alpha'\mu}{n}). \qquad (3.8)$$

Equations (3.5), (3.7), (3.8) are to be compared with the corresponding equations in the eikonal approach. Instead of (3.5), one uses (3.1) to get

$$A(s,b) = ik(s)^{\frac{1}{2}}(1 - S) = ik(s)^{\frac{1}{2}}\Big(1 - \exp[2i\delta(b)]\Big)$$
$$= -ik(s)^{\frac{1}{2}} \sum_{n=1}^{\infty} [2i\delta(b)]^n/n! \;. \qquad (3.5')$$

This gives

$$2i\delta(b) = -\frac{\xi}{\mu}\exp[-b^2/4\alpha'\mu] \qquad (3.7')$$

and hence

$$A(s,t) = 2i\alpha' k(s)^{\frac{1}{2}} \sum_{n=1}^{\infty} \frac{1}{nn!}\left(-\frac{\xi}{\mu}\right)^{n-1} \exp\left(\frac{t\alpha'\mu}{n}\right) \qquad (3.8')$$

Using the values of FM, $\xi = 7$, $\alpha' = 0.82(\text{GeV})^{-2}$, we display in Fig. 3 the predictions for $d\sigma/dt$. For small t the two methods agree because the first two terms of (3.8) are identical with the first two terms of (3.8'). We note, however, that for $t \simeq -1$ the dip that occurs in the eikonal case is replaced by a break in the K-matrix calculation, even at 1600 GeV. To understand this, we recall that the dip occurs in the eikonal case because of a net cancellation between the positive first term and negative second term in (3.8'); the positive third term is small (see Fig. 4). In the K-matrix, the third term is larger, and the net result is that the dip is filled in.

One can also examine the phase $\theta = \tan^{-1}\{\text{Re}[A(s,t)]/\text{Im}[A(s,t)]\}$, as a function of t at fixed s. We do this in Fig. 5, where we show a polar plot of $\Theta(t)$ at 30 and 1600 GeV for the two models. The results are essentially the same.

We compare total cross sections in Fig. 6a. For the given α',c, the K matrix gives a larger σ_T because of the larger third term of Eq. (3.8) (see Fig. 6b). The overall rise in σ_T as a function of p is maintained, however. This rise, although suggested by the 1969 Serpukhov data, is not confirmed by the 1970 Serpukhov data. The discrepancy of this new data with Fig. 6a can, of course, be accounted for by introducing appropriate secondary poles.

Finally, one can also examine the larger angle behavior analytically by converting the sum of (3.8) to an integral on n and performing a steepest descent analysis[27)] on n, or by performing a steepest descent analysis directly on the variable b using (3.3), (3.5), and (3.7) (see Ref. 28 for this type of analysis for the eikonal case). We comment briefly on the first approach. One converts (3.8) to

$$A(s,t) \propto s \int \frac{dn}{\sin \pi n} \exp(-na - \tau\mu/n) \tag{3.9}$$

where $\tau = -\alpha' t > 0$, $a = -\ell n(\xi/\mu)$, and μ is sufficiently large so that $|\xi/\mu| < 1$ and $|\mu| \gg \tau$. The saddle points are at

$$0 = -a + \frac{\tau\mu}{n^2} - \frac{1}{n} - \frac{\pi \cos \pi n}{\sin \pi n} \tag{3.10}$$

or, for large n,

$$n = \left(\frac{\tau\mu}{\pi}\right)^{\frac{1}{2}} (\sin \phi)^{\frac{1}{2}} e^{\pm \frac{i\phi}{2}}, \quad \tan \phi = \frac{\pi}{a} . \tag{3.11}$$

For large enough s, this reduces to the formula of Ref. 27, and eventually to an Orear-type expression

$$A(s,t) \propto i \exp\{[-(2\pi\tau\mu \text{ cotan } \frac{\phi}{2})^{\frac{1}{2}}]\}. \qquad (3.12)$$

The case $\tau \gg |\mu|$ can be dealt with in a similar way.

REFERENCES

1. W. Heitler, Proc. Cambridge Phil. Soc. 37 (1941) 291; The Quantum Theory of Radiation, Third Ed. (Oxford University Press, 1954)
2. E. P. Wigner, Phys. Rev. 70 (1946) 15; for a review of the reaction matrix theory of nuclear collisions, see for example A. M. Lane and R. G. Thomas, Rev. Mod. Phys. 30 (1958) 257
3. R. H. Dalitz, Rev. Mod. Phys. 33 (1961) 471; Strange Particles and Strong Interactions (Oxford University Press, 1962)
4. A. Bialas, T. W. Ruijgrok, and L. Van Hove, Nuovo Cimento 37 (1965) 608; A. Bialas and L. Van Hove, Nuovo Cimento 38 (1965) 1385
5. K. Dietz and H. Pilkuhn, Nuovo Cimento 37 (1965) 1561
6. R. C. Arnold, Phys. Rev. 136 (1964) B1388
7. H. D. D. Watson, Phys. Letters 17 (1965) 72
8. P. J. O'Donovan, Phys. Rev. 185 (1969) 1902
9. W. Dreschler, Phys. Rev. 2D (1970) 364
10. N. J. Sopkovich, Nuovo Cimento 26 (1962) 186
11. L. Durand and Y. T. Chiu, Phys. Rev. Letters 12 (1964) 399, E13, (1964) 45; Phys. Rev. 137, (1965) B1530; Phys. Rev. 137 (1965) B646
12. K. Gottfried and J. D. Jackson, Nuovo Cimento 34 (1964) 735
13. J. D. Jackson, Rev. Mod. Phys. 37 (1965) 484
14. R. C. Arnold, Phys. Rev. 153 (1967) 1523; R. C. Arnold and M. L. Blackmon, Phys. Rev. 176 (1968) 2082
15. F. Henyey, G. L. Kane, J. Pumplin, and M. H. Ross, Phys. Rev. 182, (1969) 1579
16. J. Finkelstein and M. Jacob, Nuovo Cimento 56A (1968) 681

17. L. Caneschi, Phys. Rev. Letters 23 (1969) 254

18. S. Frautschi and B. Margolis, Nuovo Cimento 56A, (1968) 1155 (hereafter referred to as FM); see also K. S. Kölbig and B. Margolis, Phys. Letters 31B (1970) 20

19. T. T. Chou and C. N. Yang, Phys. Rev. Letters 20 (1968) 1213

20. J. V. Allaby, et al., Phys. Letters 28B (1968) 67

21. M. L. Goldberger and K. M. Watson, Collision Theory (J. Wiley and Sons, New York, 1964) Chap. 6

22. A. Dar and W. Tobocman, Phys. Rev. Letters 12 (1964) 511; ibid. 12 (1964) 511; ibid. 13 (1964) 91

23 E. J. Squires, Nuovo Cimento 34 (1964) 1328; Phys. Letters 26B (1968) 461

24. J. S. Ball and W. R. Frazer, Phys. Rev. Letters 14 (1965) 746

25. J. V. Allaby, et al., Phys. Letters 30B (1969) 500

26. Serpukhov preprint (Kiev Conference, 1970)

27. A. A. Ansel'm and I. T. Dyatlov, Yad. Fiz. 6 (1967) 591, 603 [Sov. J. Nucl. Phys. 6 (1968) 430, 439]

28. K. G. Boreskov and Yu. F. Pirogov, Yad. Fiz. 11 (1970) 210 [Sov. J. Nucl. Phys. 11 (1970) 118]

FIGURE CAPTIONS

Fig. 1. Elastic scattering viewed as a multiple scattering series. The wavy line corresponds to the Born driving term.

Fig. 2. Inelastic two-body scattering with absorption corrections in the initial and final states. The wavy line corresponds to the exchanged particle. The shaded blobs represent elastic scattering.

Fig. 3. Differential cross sections in the two models at 30 GeV/c and 1600 GeV/c. Solid lines: K matrix; dashed lines: eikonal.

Fig. 4. The first three terms of the sums appearing in Eqs. (3.8) and (3.8'); (a) real parts; (b) imaginary parts. Number denotes the corresponding term in the series, 3 refers to the eikonal and 3' to the K matrix.

Fig. 5. The phase of the amplitude as a function of t (eikonal) or T (K matrix) for (a) 30 GeV/c and (b) 1600 GeV/c.

Fig. 6. The total cross section in the two models (a) σ_T vs p_{lab}; (b) individual terms of the real part of the sums appearing in Eqs. (3.8) and (3.8').

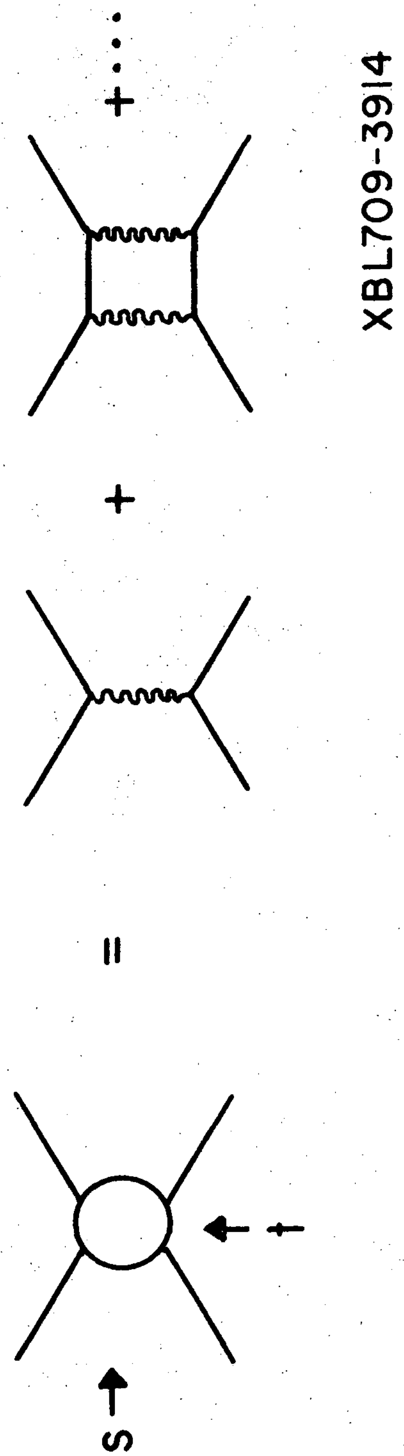


Fig. 1.

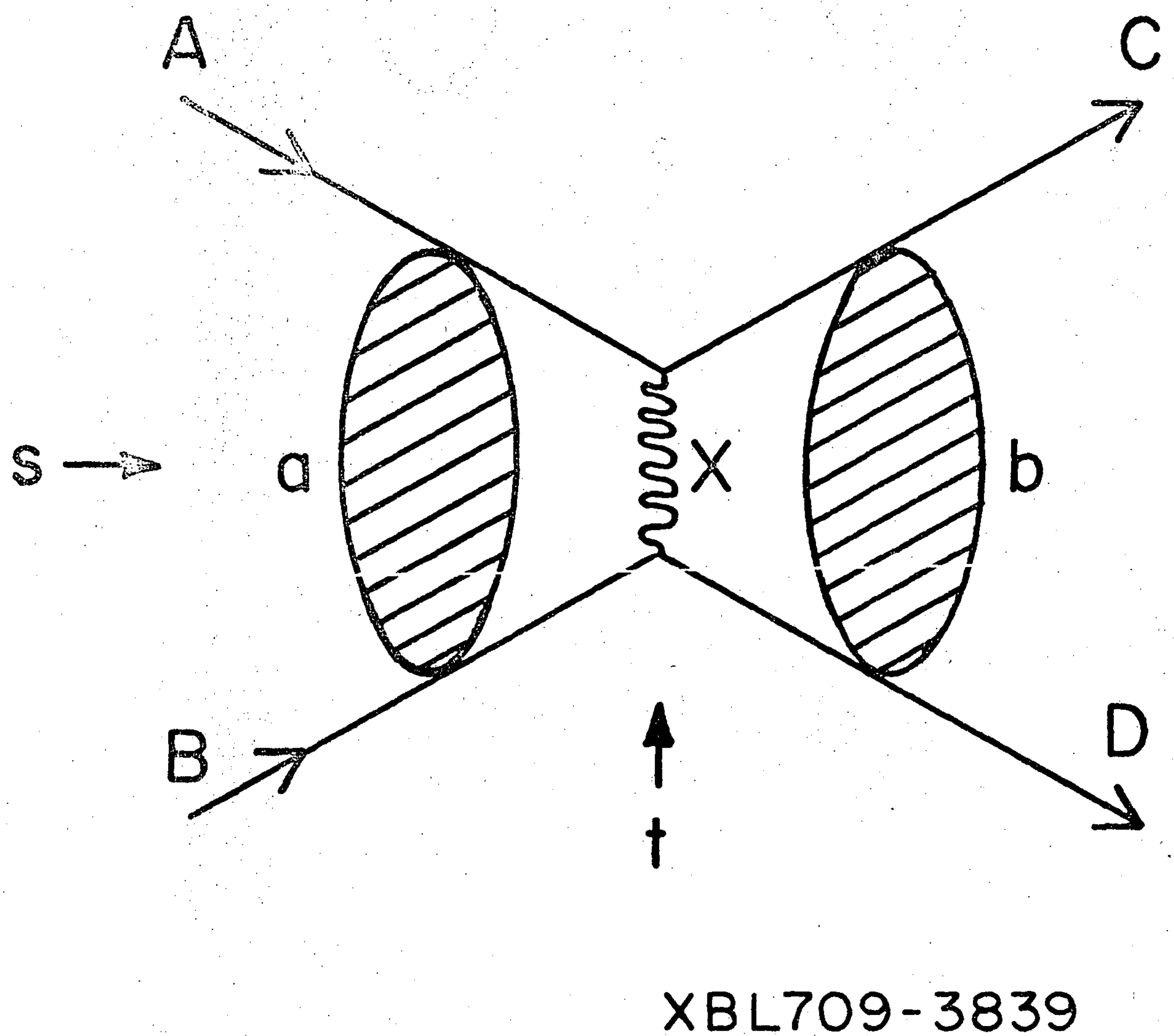


Fig. 2.

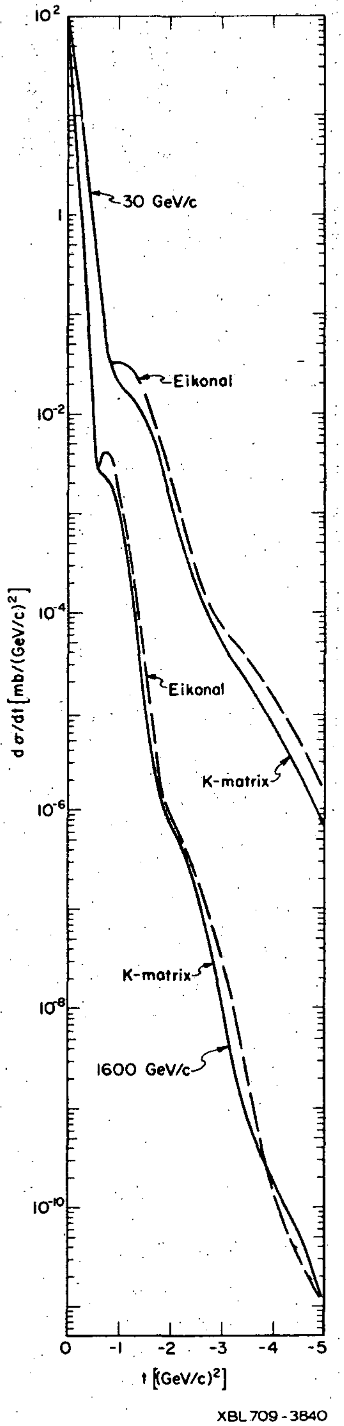

30 GeV/c
Eikonal
Eikonal
K-matrix
K-matrix
1600 GeV/c
dσ/dt [mb/(GeV/c)²]
t [(GeV/c)²]
10²
1
10⁻²
10⁻⁴
10⁻⁶
10⁻⁸
10⁻¹⁰
0
-1
-2
-3
-4
-5
XBL 709-3840

Fig. 3.

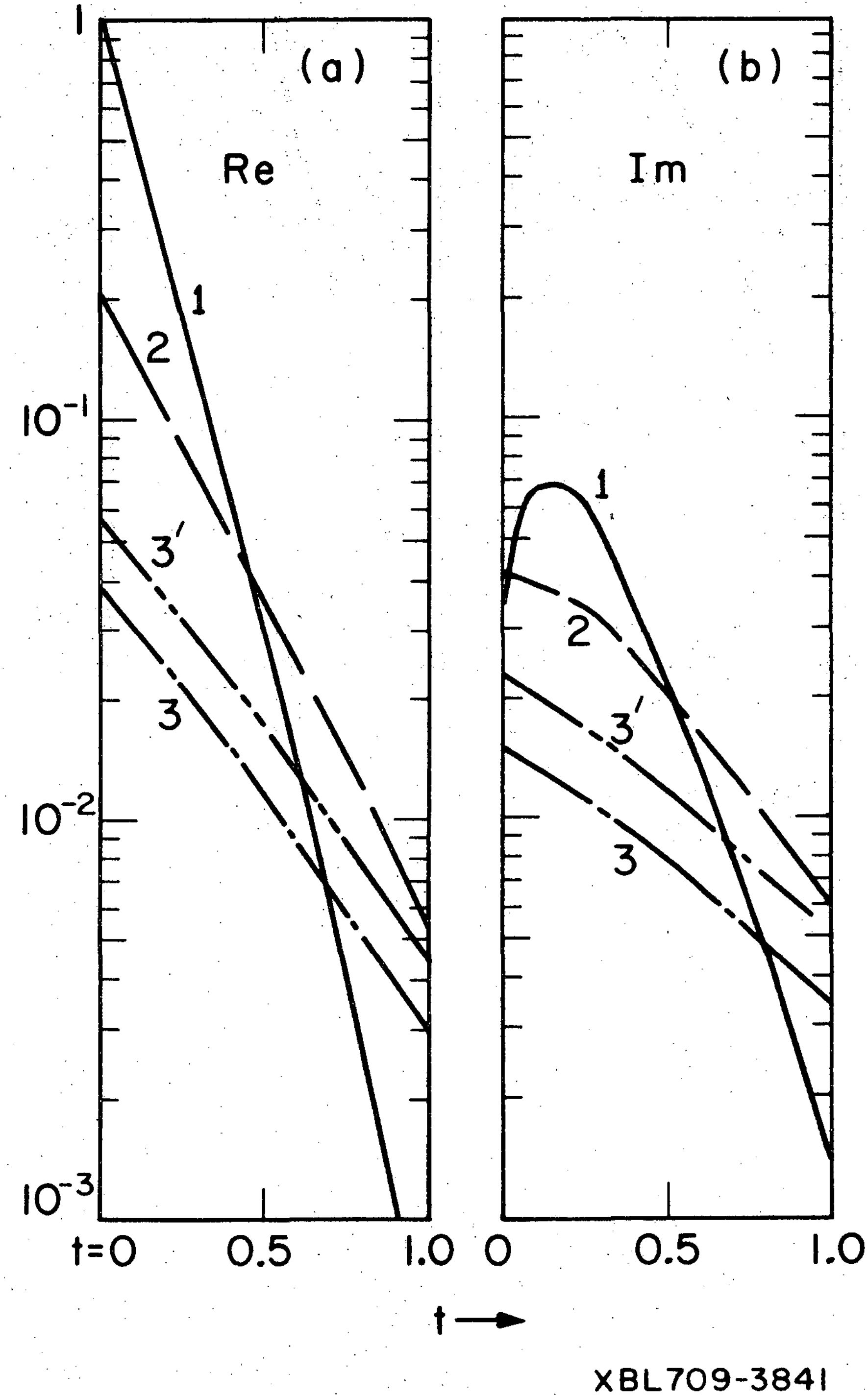
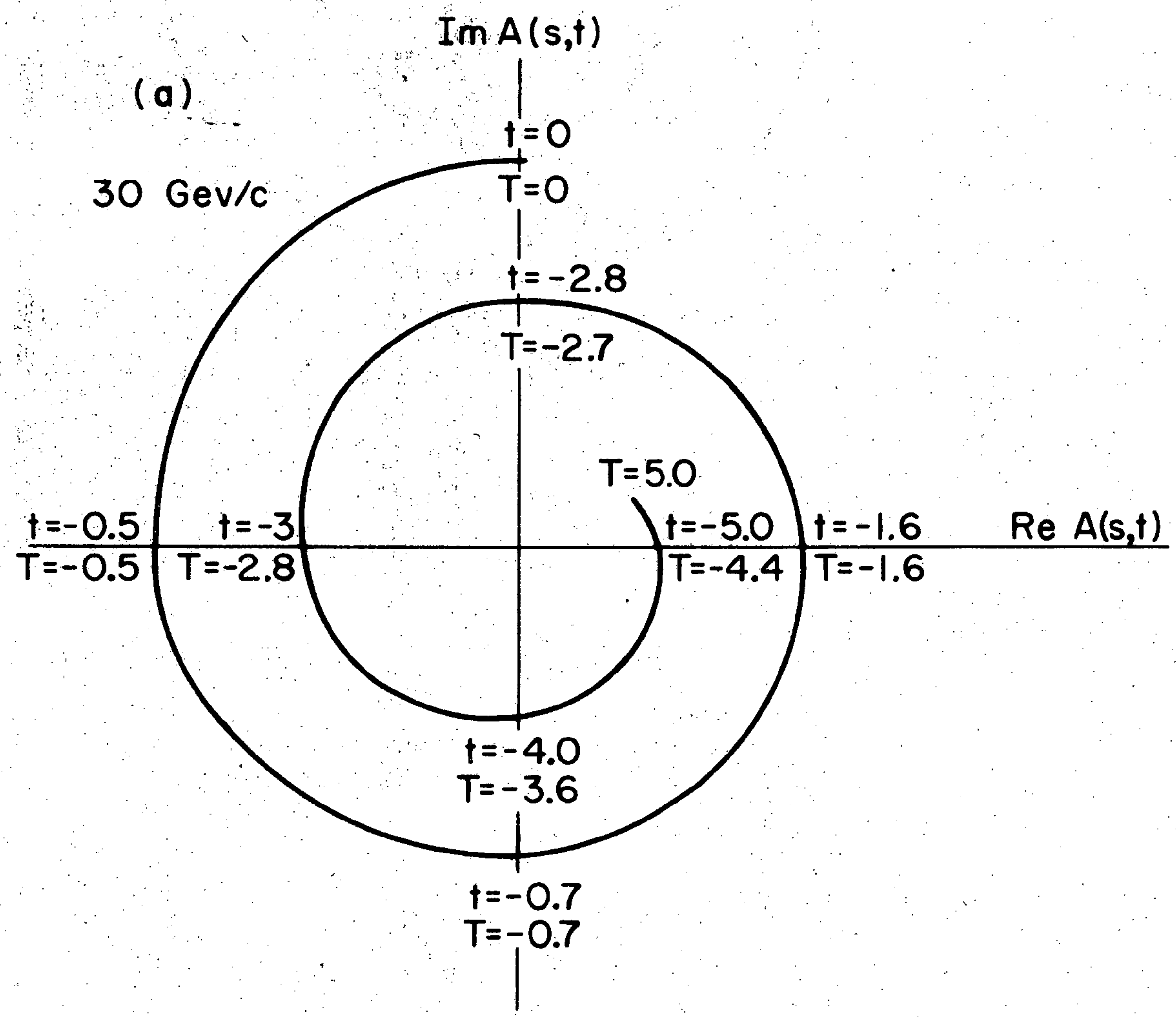
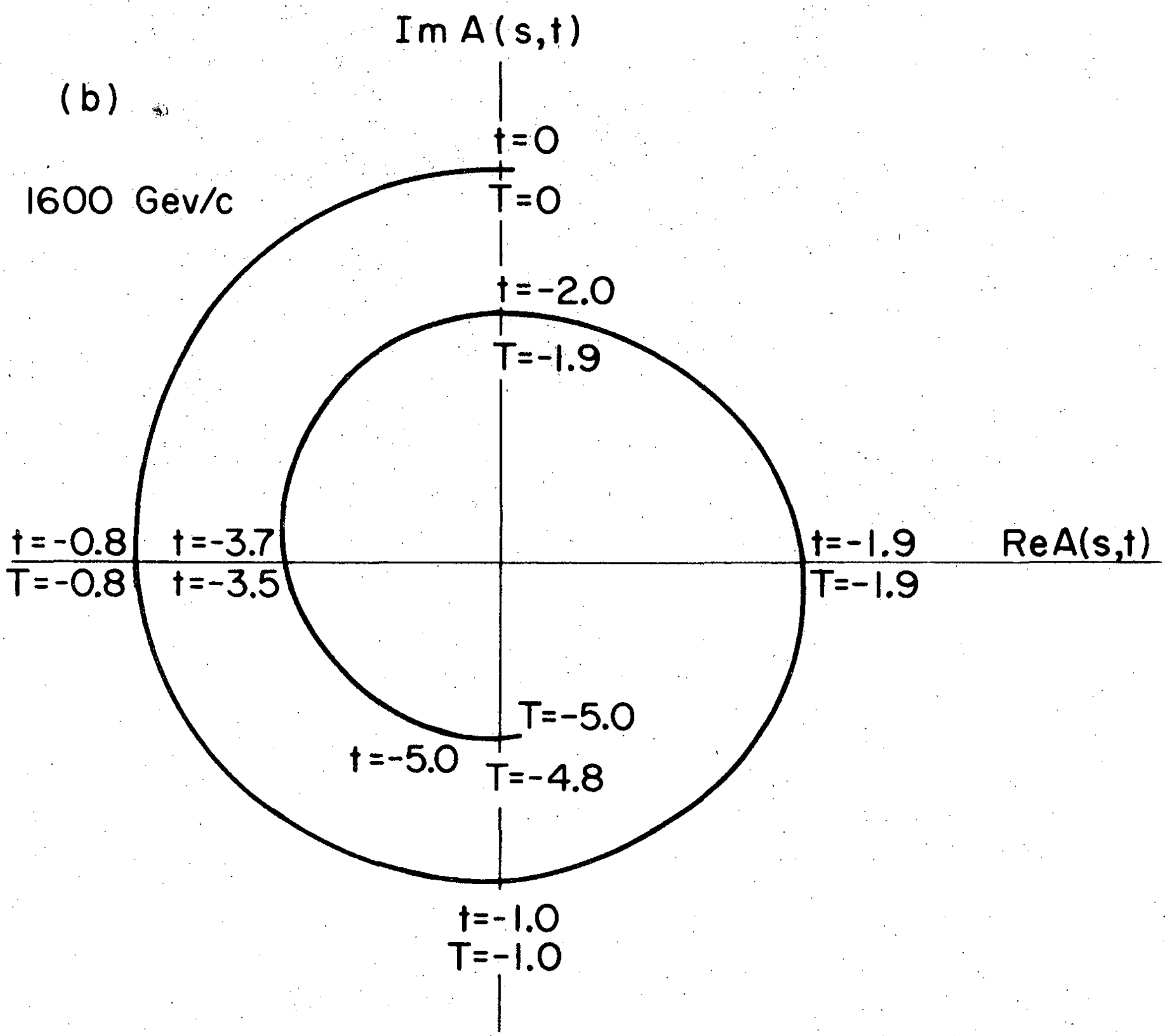


Fig. 4.



XBL 709-3843

Fig. 5a.



XBL 709-6647

Fig. 5b.

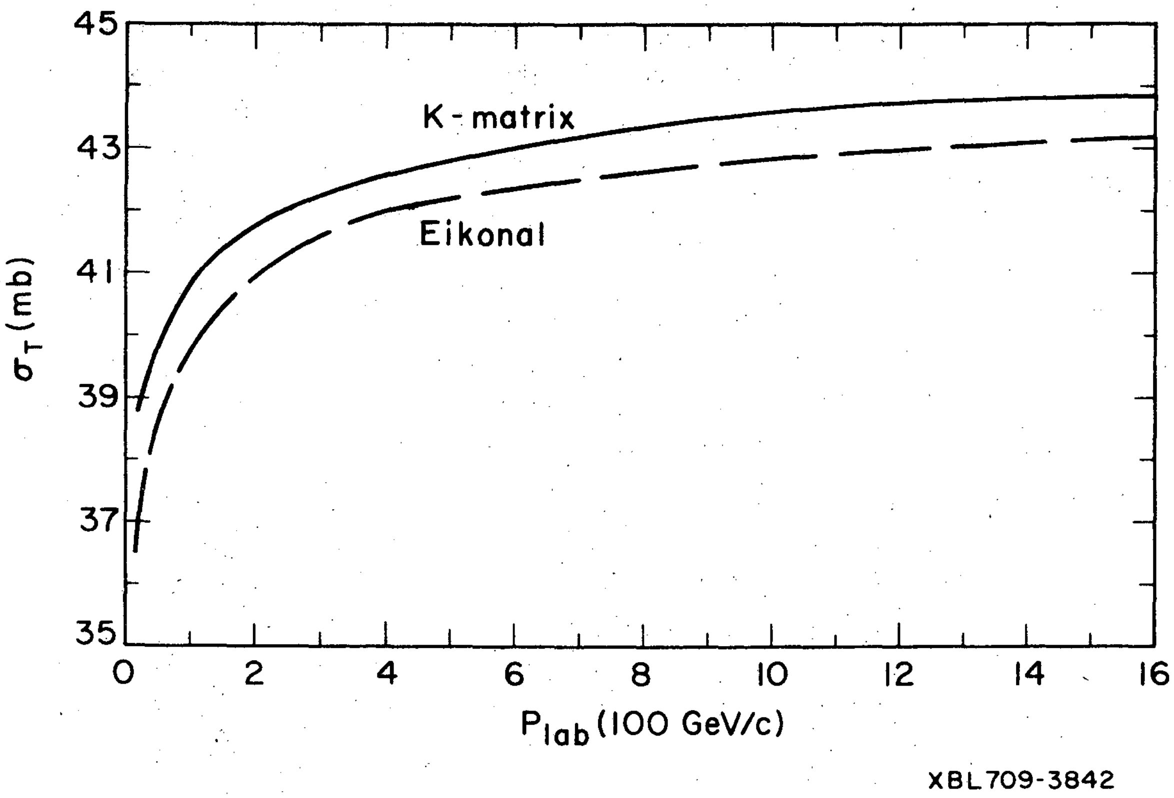

45
43
41
39
37
35
σ_T (mb)
K-matrix
Eikonal
0
2
4
6
8
10
12
14
16
P_{lab} (100 GeV/c)
XBL709-3842

Fig. 6a.

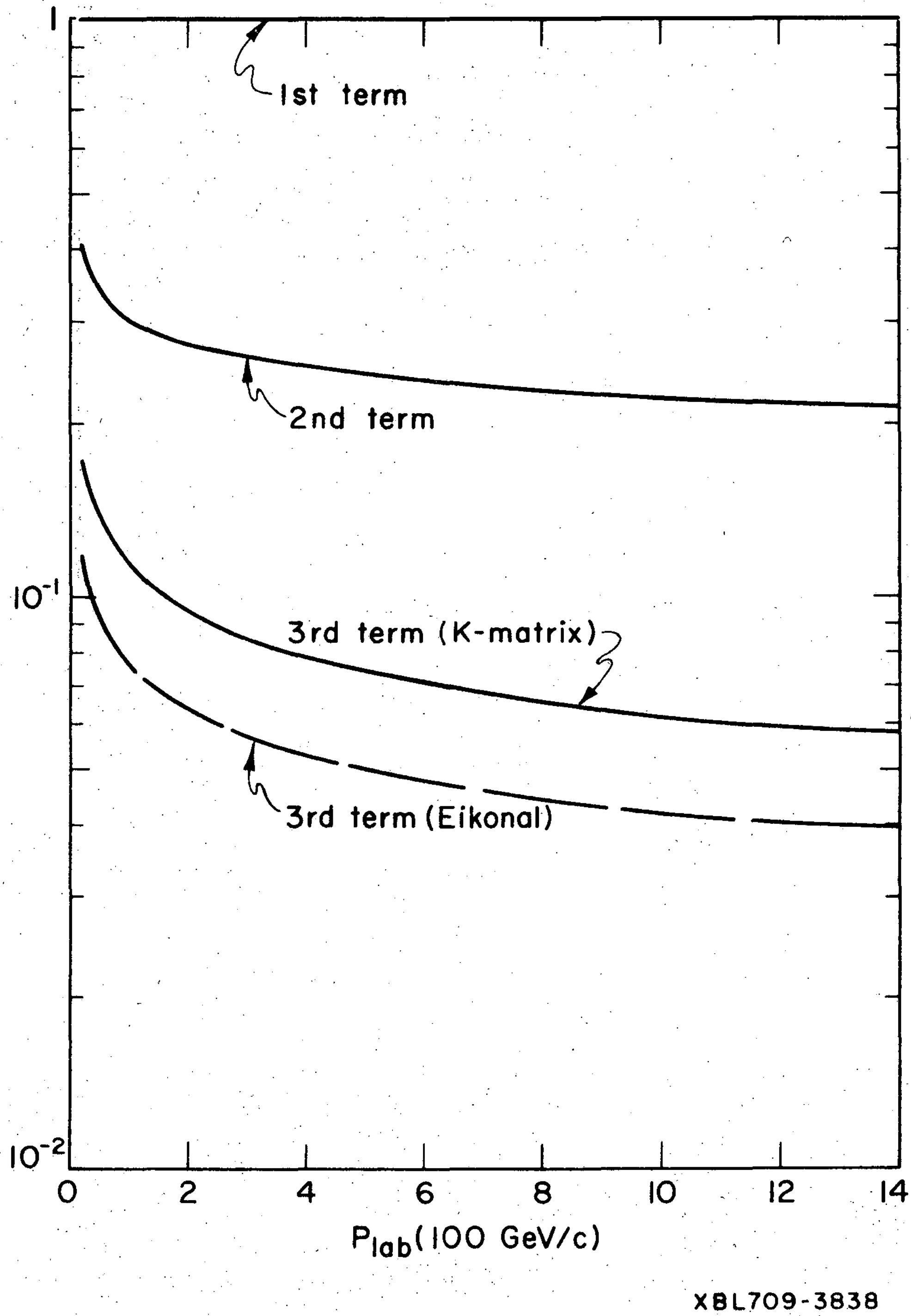

1
1st term
2nd term
10^{-1}
3rd term (K-matrix)
3rd term (Eikonal)
10^{-2}
0
2
4
6
8
10
12
14
P_{lab}(100 GeV/c)
XBL709-3838

Fig. 6b.