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Abstract

In the context of a linear chiral σ-model coupled to quarks we argue the single Baryon and high density ground state occurs in a topological chiral symmetry (TCB) broken phase and not the usual space uniform (UCB) chiral symmetry broken phase. We expect this to transit directly to a QCD restored phase at higher density. New, but plausible features of dense matter in the phase are highlighted and known features of nuclear matter are recovered.

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The Skyrme[1] Topological model for the nucleon enjoys a belated but, now, substantial, standing. A feature of topological models, e.g., vortices in Superfluid ^4He, where the mapping is from the boundary of space to the field or order parameter space, is that the energy goes as the second power of the topological charge, n, $E \sim n^2$. This induces a hard core which results in a periodic array of vortices as the ground state above a certain critical rotation.

Though, the Skyrme model is a mapping of the whole of compactified space into the field SU_2 space such a feature persists, though not quite exactly. This is responsible for a repulsive "core" as two inifnitely separated skyrmions are made coincident. This informs us that in a many skyrmion system with high density of skyrmions we may see a phenomenon of a periodic lattice in analogy to superfluid ^4He. It is thus expected that such a periodic array of skyrmions is likely to be the ground state of such matter at high density. Such a model has been, following Skyrme[2] suggested by Kutschera, Pethick and Ravenhall[2]. A feature of Skyrme lagrangian, extrapolated to high densities, is that the energy density goes as the 4/3 power of the density i.e., like the QCD based quark model at high densities. This is indeed noteworthy. However, there are two differences between the Skyrme model and the QCD quark model (1) the Skyrme model based on the nonlinear σ-model, has spontaneous chiral symmetry breaking (for all densities) unlike the QCD quark model ii) The Skyrme model is based on a low energy effective lagrangian which is not expected to be correct at high densities when the QCD quark gluon degrees of freedom are explicit. Yet, the equation of state is very similar at high density!

Recently, a linear chiral model for quark meson (pion) interactions was proposed by the author, Ripka and Kahana[3] and by Birse and Banerjee[4] with no quartic Skyrme like term). In this model the gluon degrees of freedom are completely ignored except as the driving force for π,σ fields. A similar lagrangian was considered by Rajeev and Bhattacharya[5], where they integrated out the quark degrees of freedom to obtain the anomaly and the quartic Skyrme term. This suggests that the Skyrme Lagrangian is an effective lagrangian for the model considered in Refs. 3 and 4, though, probably QCD will introduce some, hopefully minor, modificaitons at short distances. Also, the quark pion lagrangian is an intermediate scale lagrangian between Skyrme and QCD where the quark degree of freedom is still retained i.e., within the confinement scale but pions are still not broken down to $q\bar{q}$ and glue. This would be a more reasonable lagrangian to work with then the skyrme model at smaller distances. Let us review the main features of this model.

The model lagrangian density is given by $(SU_2)_L \times (SU_2)_R$ chiral σ-model

$$L = \frac{1}{2} (\partial\mu\sigma)^2 + \frac{1}{2} (\partial\mu\pi^2)^2 + \frac{\lambda}{4} (\sigma^2 + (\pi^2)^2 - f_\pi^2)^2 + + \sum_{\text{color}} \bar{\psi}_q (\not\partial + g(\sigma + i\gamma_5\pi^a \tau^a)) \psi_q \tag{1}$$

with $f_\pi = 93$ MeV, the pion decay constant. For our present purpose we can as well approximate* this by a non-linear model[3,6] i.e.,

$$\frac{\sigma^2}{f_\pi^2} + \frac{\pi^2}{f_\pi^2} = 1$$

If we then choose $\sigma/f\pi$ = cosθ and $\pi^a/f\pi = (\hat{e}_r)^a$ Sinθ we have (i) for $\theta = 0$ everywhere the usual space uniform spontaneously broken chiral symmetry

*Calculations are done in this approximation as it is found the energy changes only marginally even if $\lambda = 10$. For what follows, however, keep in mind that actually we have a linear σ-model.

(UCB) model with the quark spectrum being entirely continum beginning at $m = gf_\pi$. (ii) For $\Theta(r = 0) = -\pi$ at the origin and $\Theta(r = \infty) = 0$ we have the skyrme background field with a space varying and topologically non-trivial classical meson field configuration. We shall call this topological chiral symmetry breaking (TCB).

In such a background a soluble model that approximates[3,6] the spectrum with fair accuracy is

$$\begin{aligned} \Theta &= r\pi/R \quad , \quad r \le R \\ \Theta &= \pi \quad , \quad r > R \end{aligned} \tag{2}$$

where R is size parameter, determined by minimizing the energy with respect to it. A valence quark bound state solution is found for case (ii). These are eigenstates of $\vec{I} = (\vec{J} + \vec{\tau})$, with $\vec{I}|\text{Bound State}\rangle = 0$, giving only a color degeneracy of 3.

Further, the conserved topological[1] number of the familiar $S^3 \to SU_2$ mapping is given by $T = (\Theta - \sin 2\Theta/2)\big|_0^\infty$ and can be identified with the fermiom number[(7)] of the vacuum state* for each color species of quark which implies, for QCD quarks, a baryon number equal to 1 for the new vacuum.

The total energy of the color singlet, occupied, 3 quark bound state configuration is composed of quark orbital energy and meson gradient energy minimised with respect to R (eqn. 2).

It is found (Refs. 3,4, and 6) that the TCB (ii) configuration exists with an energy obviously <u>lower</u> than the 3 plane wave quark UCB (i) color singlet configuraton of $E = 3m$.

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*The vacuum shall designate the state in which all negative energy states are occupied.

Occupation of the TCB bound state of 3 quarks forming a color singlet adds a unit of baryon number to the vacuum. Since the Skyrme vacuum already carries a unit of baryon number this state was expected to carry B = 2 as maintained in Refs. (3 and 6). However, the identification of T with the baryon number of the vacuum is valid only in the slowly varying approximation. This is equivalent in taking the parameter $R \to \infty$ in the parameterization of Θ (eqn. 2). One finds (Refs. 3 and 6) that in this limit the valence quark bound state recedes to negative energy. The new vacuum in $R \to \infty$ limit, then, has this state occupied and has baryon No. B = T = 1. As R is reduced this same state pops out of the Dirac sea into a positive energy, valence, quark bound state. Thus, the baryon number of the occupied bound state 3 quark solution above is actually 1 and not 2 as suggested in Refs. 3 and 6.*

The connection between the Skyrme model is now most transparent. At R_{MIN} the bound state sits in the "negative" energy sea and we have a Skyrmion. As the Skyrmion is squeezed the quarks pop out and the solution acquires a quark model structure. Consider now the generic expression for the energy of the (3 quark) singlet or B = 1 state in the TCB model[22]

$$\frac{E^B}{gf_\pi} = 3\left\{\frac{a}{X} - B\right\} + \frac{27}{g^2}X \qquad (3)$$

Above, the term in parenthesis is the quark orbital energy for a profile given as in (eqn. 2) with $X = gf_\pi R$ a dimensionless size parameter. The other term is the meson field gradient energy. For the single nucleon (B = 1) solution this expression must be minimized W.rt.X

*A correction to this effect is now in preparation.

$$X_{MIN} = g\,\frac{\sqrt{a}}{3} \tag{4}$$

This immediately provides a new feature of <u>universality</u> associated with this model, $R = X_{MIN}/gf_\pi = (\sqrt{a}/3)(1/f_\pi)$, is <u>independent</u> of g.

To identify the B = 1 state above with the nucleon we set

$$\frac{E^B}{gf_\pi} = 18\,\frac{\sqrt{a}}{g} - 3B = \frac{1000}{93g}$$

where the energy is fitted to M_B = 1 GeV. In the orginal model (Ref. 3), for quarks, the parameters a and B were obtained by fitting the single bound state orbital which crosses the zero energy as X is increased to be, a = 3.12, B = 0.94. These will be obviously modified on inclusion of the Dirac sea effects which includes the other[8] negative energy bound state and the new negative energy continuum. In this work, our considerations shall not take these effects directly into account. Instead, it is easy for a given "a" to derive a bound on B by invoking that the bound state solution (TCB) be the lowest one i.e., the energy per baryon must be less than <u>$3gf_\pi$</u>, the (<u>U</u>CB) <u>one</u>.

$$\text{(i)}\qquad 18\,\frac{\sqrt{a}}{g} - 3B = \frac{1000}{93g} \tag{5}$$

$$\text{(ii)}\qquad 18\,\frac{\sqrt{a}}{g} - 3B \leq 3 \tag{6}$$

to get $B \leq 0.093 \times 18\sqrt{a} - 1$.

This provides a maximum value for B. For the value of a = 3.12, given above, this is ~2. We shall look at two cases:

(a) a = 3.12, B = 0.94* which gives g = 7.5, R_{min} = 1.28F,

(b) a = 1.53, B = 0.94 which gives g = 4.1, R_{MIN} = 0.89F.

*Actually, B is expected to be 1 (to be explained in later work).

In (a) the numbers are those from fitting the single bound state[3] (b) is fitted to reproduce an $R_{MIN} = (X_{MIN}/gf_\pi)$ such that $(1/2R_{MIN})^3 = n_B = 0.18\ F^{-3}$, i.e. where the size of the soliton (not necessarily quark density) R_{MIN} corresponds $n_B = 0.18\ F^{-3}$.

At this point we are ready to consider the anticipated ground state in this model just as for the pure Skyrme model we find an even more substantial energy barrier when two B = 1 objects are made coincident from an infinite separation. Clearly, making a giant Skrymion with many cycles in θ centered at some point is a most unfavorable/unstable configuration. The expectation is that a periodic lattice, the first example of a relativistic solid, will be the favored ground state at high density just as for superfluid ^{4}He.

1) We shall consider the simplest of such ground states for $X < X_{MIN}$ we consider a cubic lattice of our B = 1 configurations with lattice spacing 2R(2X).* The energy per baryon is at high density is then simply given by (in dimensionless units)[22]

$$\frac{E^B}{gf_n} = 3\left(\frac{a}{X} - B\right) + \frac{27}{g^2} X \tag{7}$$

where a and B are taken given and g is determined by fitting the minimum E^B to a nucleon mass, 1 GeV. The qualitative and quantitative limitations of the above will be considered in the discussion.

The baryon density is given by $n_B = [(1/2R)]^3\ F^{-3} = 1/8X^3$ in our units.

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*This is equivalent to having a spherical Wigner-Sietz construction in each cubic unit cell with θ going from $-\pi$ at centre to 1 at the boundary.

Clearly, in this model, for low density, the TCB state will be favored over the UCB state of the σ model, since the quarks now appear is bound states; the energy per quark, then, being obviously less than gf_π, the energy of a UCB quark.

2) Let us now consider a possible model for UCB which has been considered by many authors[9] in the context of "nucleon" σ-model. This is the mean field σ model in which the quark mass is dynamically generated from the vacuum expectation value of the σ field. In this model we have $\langle\sigma\rangle = \sigma_0$ and $\langle\pi\rangle = 0$. (Charged pion condensation in the σ model admits the possibility of $\langle\pi\rangle$ = (space uniform) a and $\langle\sigma\rangle$ = b with $\sigma^2 + \pi^2 = f_\pi^2$, but will not be considered here as it has been dealt with mainly as a non-relativistic model, see Ref. 11).

The energy per nucleon composed of 3 chiral QCD quarks, is for neutron matter $n_B = n_u(= k_f^{u3}/\pi^2) = 1/2\ n_D(= k_f^{D3}/\pi^2)$

$$E^B(\langle\sigma\rangle, k_f^i) = \frac{E_\rho}{n_B} = \left[\sum_{\substack{i\\u,d}} m^4 \frac{3}{8\pi^2}\left\{x^i\left(1 + x^{i^2}\right)^{1/2}\left(2\,x^{i^2} + 1\right) - \ln(x^i + \sqrt{1 + x^{i^2}})\right\}\right.$$

$$\left. + \frac{\lambda}{4}\left(\langle\sigma\rangle^2 - f_\pi^2\right)^2 + O(m_\pi^2)\right]/n_B \qquad (8)$$

with

$$m = g\langle\sigma\rangle \quad , \quad x^i = \frac{k_f^i}{m} \quad \text{and} \quad \lambda = 2g^2$$

$$n_B = \frac{(k_f^u)^2}{\pi^2} = \frac{1}{(2R)^3}$$

In our units

$$gf_\pi = 1 \quad , \quad R = X \quad ; \quad n_B = \frac{1}{8X^3} \quad , \quad k_f^u = \frac{(\pi^2)^{1/3}}{2X}$$

Substiting n_B gk_f^u, k_f^D in terms of X the energy per baryon is obtained by minimizing $E^B(\sigma,k_f^i)$ with respect to $<\sigma>$ for each density.

The results[10] are plotted for the two values of $g = 7.5$ and 4.1 and as expected yield a higher energy per baryon than our model (topological Chiral Breaking) up to well above nuclear density (Figs. 1 and 2).

Note that in our model, the TCB phase, by construction Chiral symmetry is spontaneously broken for all density.

The model (UCB) has symmetry breaking at lower density but goes to a restored phase[11] at high density at which the quarks begin to resemble a massless QCD quark gas.

3) Finally, we must compare with QCD quark matter model, or rather, to a simple realization of this in terms of a large bag of massless quarks; for this is what we expect for the state of very dense matter.

For a quark gas derived from neutron matter we have the same constraints

$$n_B = n_u = \frac{1}{2}\, n_D = \frac{(k_f^u)^3}{\pi^2} \quad .$$

To order $\alpha = g_{eff}^2/4\pi$ (the QCD fine structure constant) in gluon interactions, for massless QCD quarks, the energy per baryon simplifies[12]

$$E_p/n_B = \sum_{\substack{i \\ u,D}} \frac{3}{4}\,(k_f^i)^4 \left(1 + \frac{2\alpha(\rho)}{3\pi}\right)/n_B + B/n_B$$

$$= \frac{3}{4}\,(1 + 2^{4/3})(\pi^2)\; k_f^u\left(1 + \frac{2\alpha}{3\pi}\right) + B/n_B$$

In terms of our units $gf\pi = 1$, $n_B = 1/8X^3$ as before. Thus

$$\frac{E^B}{gfn} = C \times \frac{2.846}{X} + b\, X^3 \qquad (10)$$

where

$b = 0.015$ for $g = 7.5$

$b = 0.167$ for $g - 4.1$

the second term follows from the bag pressure $B = 55$ MeV.F^3 which has been recast in our units. We take $\alpha = 2.2$ the MIT has value. The constant C is then 1.45. If all interactions are dropped, C = 1. The actual situation for quark matter likely to be C = 1.45 (curve A) at around nuclear density ($0.16F^{-3}$) going to C = 1 at very high density (curve B). Such a curve may be termed Interpolating* QCD(3)

Results

i) As expected, we find that for low density i.e., close to and above nucleons the (TCB) model provides the ground state over the UCB model. At higher density depending on the value of "g" there is cross-over to the quark model/UCB model. At high densities, however, $\langle\sigma\rangle$ is close to zero and model (2) and (3) (with $\sigma \rightarrow$ small) are indistinguishable.

ii) For g = 7.5, there is small density discontinuity between model (1) and interpolating (3). The tangent construction would give a mean coexistence density $\gtrsim 2$ times nuclear matter density.

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*Interpolating (3) is one in which α goes from 2.2 around 0.1 Fermi^{-3} gradually changing[12b] to 0.36 at 13 Fermi^{-3}.

iii) $g = 4.1$. There is a density discontinuity between model (1) and (2) which then merges into interpolating (3). The targent construction gives a mean coexistence density of ~2 times nuclear density, between (1) and (2) (see following remarks).

Discussion

The scenario that emerges from the foregoing considerations is the following. Below nuclear matter density the Skyrmions behave as a nucleon gas on quantization(12) the n–n or Skyrmion-Skyrmion force can be calculated(14,15) and it is found to give rather good agreement with the observed n–n potential including π, ρ, w, etc. Thus, the equation of state at low density which follows from quantized Skyrmions is in close agreement with the usual nuclear physics equation of state. At densities close to and above $\rho_{nuc.}$ there should be a tendency, due to topological repulsion, to form an ordered structure e.g., lattice/liquid crystal. We have made a most rudimentary estimate of this.

As is well known (see Baym Ref. 12a) the σ model in the (UCB) phase goes from a chiral breaking phase to the "abnormal" or chirally restored phase at high density, with an unnatural saturation and possible lowest energy minimum (E^B vs n_B) in the latter phase. Many efforts have attempted to variously correct this by means of a hard core, quantum corrections, etc. In our model there is a smooth interpolation from the TCB phase at nuclear density to a UCB chirally restored phase at high density with saturation around nuclear density ($g = 4.1$). It is this new feature of the TCB phase, introduced in this paper, that clear up all the irksome irregularities of the UCB σ-model equation of state.

Actually, the above calculations (Ref. 11b) have been carried out in the nucleonic σ-model. This model does not even have the complication of QCD gluons. If such an exercise is carried out for the nucleonic σ-model (Ref. 6), again all the problems of the previous UCB formulation are obviated.

All three models have $E_\rho \sim n_B^{4/3}$ and are distinguishable only in the coefficient of the 1/X term in $E^b/gf\pi$. Moreover, the coefficient for model (2) and (3) is identical. It is expected and consistent with chiral symmetry restoration that the coefficient of 1/X in our model (1), 3a, be less than 2.846 for (2) and (3). This is the case for g = 7.5 and 4.1 when lower energy bound state solutions exist.

ii) The main imponderable and handicap of our model langrangian is that it neglects the effect of the direct gluon coupling to quarks. It also does not provide a clue to confinement. Such effects are under consideration.(16) Since this model does integrate out to the Skyrme model one is led to believe that at the length scales (small) considered, only the chiral symmetry order parameter, $<\pi>$ and $<\sigma>$, which involves gluons indirectly, is important. It is the topological pion (σ) condensate which controls the dynamics. The gluon quark interactions are through the condensate of π, and σ fields. The gluon condensate itself is presumably important at only larger length scales, e.g., through the bag pressure B. This clearly distinguishes this model from others e.g., MIT and other bags.

iii) The presence of the TCB pion condensate is the underpinning of this model and therefore it is small wonder that the much sought UCB charged pion condensate is not seen. In many respects the TCB condensate is similar to the π° condensate which is also real and periodic and imposes a lattice structure.(17) Of course, a difference still exists--topology.

iv) One may ask when will the confinement mechanism be important. On examining the bag pressure term one finds that it ceases to become important at just above nuclear density and thus at higher density. The formation of the topological background field lattice will result in relativistic band structure (Bloch structure) arising in the quark wavefunction.* This[22,23] is under investigation. This is qualitatively important in finding the correct ground state and would also lower the energy pushing up the phase transition in density. Also, the spreading of the quark wavefunction may provide a natural explanation for the EMC effect[18] (in a highly distorted pre-lattice).

v) Since we must have charge neutrality and $Q = \vec{\tau}_3/2 + B/2$ we have $\tau_3 = -B$. If all the baryons were to be an $\vec{I} = (\vec{\tau} + \vec{S}) = 0$ state one might expect a ferromagnetic structure (π° condensation) and a possible connection with neutron star magnetic fields. However, the quantization of the Skyrmion and the calculation of n-n potential indicates[19] $n\uparrow n\uparrow (n\downarrow n\downarrow)$ is more repulsive than $n\uparrow n\downarrow (n\downarrow n\uparrow)$ and thus an antiferromagnetic structure may be favored; the effect in (iv) which gives rise to a fermi sea would also favor this. The question of quantization of this model needs a detailed investigation to get an answer for this problem.

vi) The parametrization for θ we have used will obviously yield only an upper bound to E^B. This must be calculated self consistently. Also the "Wigner-sietz" construction and cubic lattice certainly will not select the lowest energy state. All these effects will move the energy per baryon down in this model, consequently moving the cross over to the chiral restored phase up in density.

*Actually, this is more subtle. In our crude estimate we neglected the spillover of the quark wavefunction outside of the background solution size X for small X.

vii) This should give us a rough idea of the pion size as it is expected the pion will dissociate around the restoration density which gives a lower bound on the pion size. An upper bound on the pion size is available from the 1 Baryon TCB configuration. If the pion is a "$q\bar{q}$" cooper pair, then for a pion condensate to exist, its size must be $< R_{MIN}$. The pion, incidentally, the lowest pseudo scalar (goldstone boson) of QCD with fermions and not just a $\bar{q}q$ pair). For example in (ii), $g = 4.1$, the lower bound is 0.7 F and the upper bound 0.78 F.

viii) The transition to quark matter in softened in this model due energy density going as $(n_B)^{4/3}$ in both cases

ix) We have not considered an SU_3 flavor extension where the situation could be rather different with, $B = S = 2$, H^{20} particle or $B = S = 4$ particle[21] replacing the $B = 1$ as a lattice site.

The results seem to indicate a chiral restoration density rather too small $\sim 2\rho_{nuc}$. This will be modified by gluon corrections. At least, once chiral symmetry is restored ($m_q = 0$) in (2) we should add 1 gluon exchange as in the quark model, which drives the transition density up to ($5\sim 8$ ρ_{nuc}). Also (vi) will raise the chiral restoration density.

If, even then, the restoration density is low, the lattice will <u>not</u> last into high density. Lattice formation is then far more delicate and less conclusive.

Also, low chiral restoration* density will give an early transition to quark matter indicated possibly by copious production of the H particle.

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*In this model the transition is weakly first order; it may even become second order in an exact calculation.

This model with quarks included would be a much closer approximation to QCD than the purely mesonic Skyrme model. The crude picture just presented already seems to produce a telling plausibility and clears up most of the erstwhile problems at the σ-model.

Details will appear separately.

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Figure Caption

Figs. 1 and 2. E^B, the energy per baryon plotted against X (n_B, baryon density) for models (1), (ours TCB), (2) (UCB) and (3) A and 3 (B), quark model. The dashed line connecting A and B is interpolating (3). The cut in (2) is the point of chiral restoration. The markers on the bottom line indicate the coexistence from the tangent construction. Fig. 1: Quark matter (3) and (1) TCB; Fig. 2: UCB and (1) TCB.

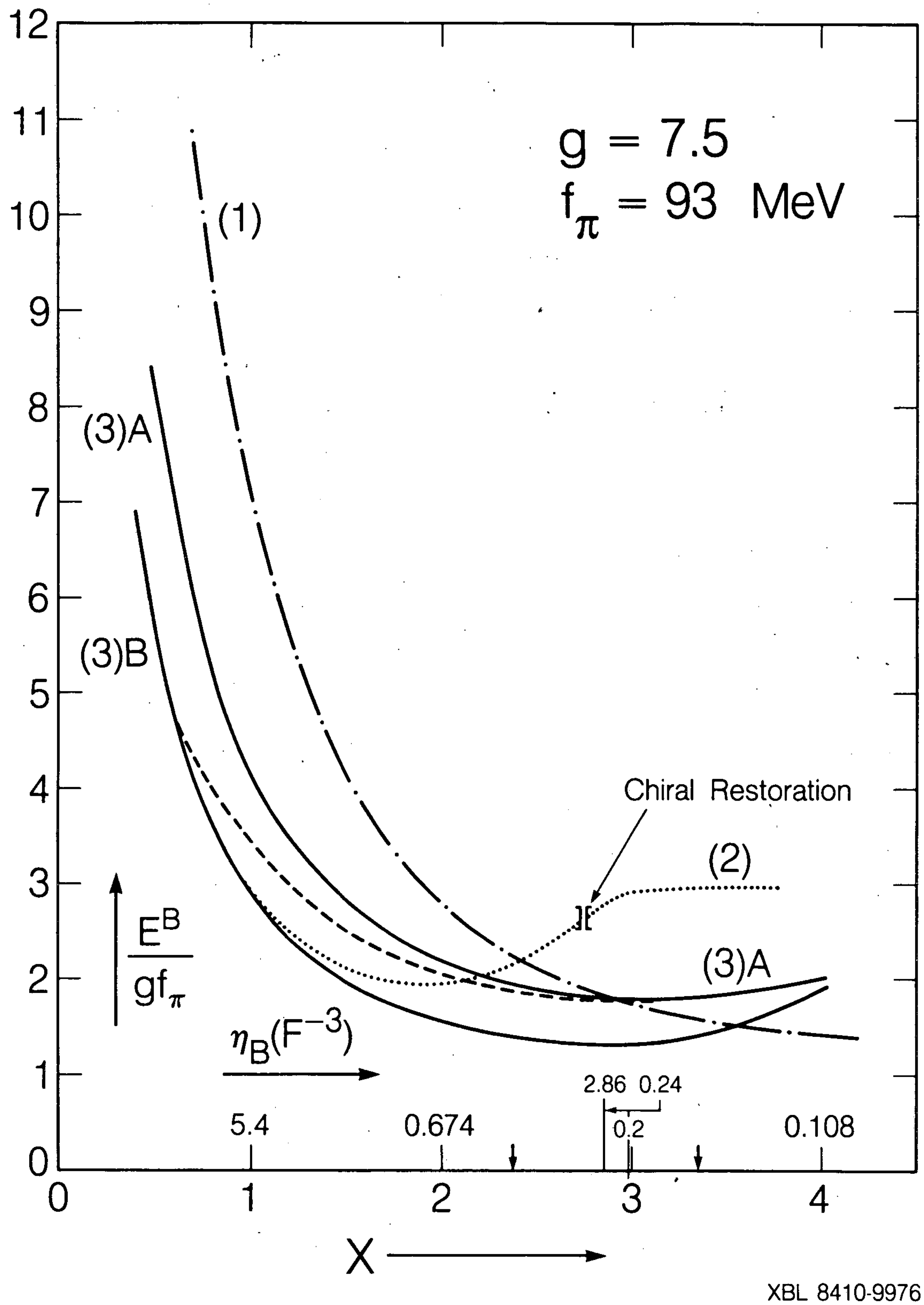
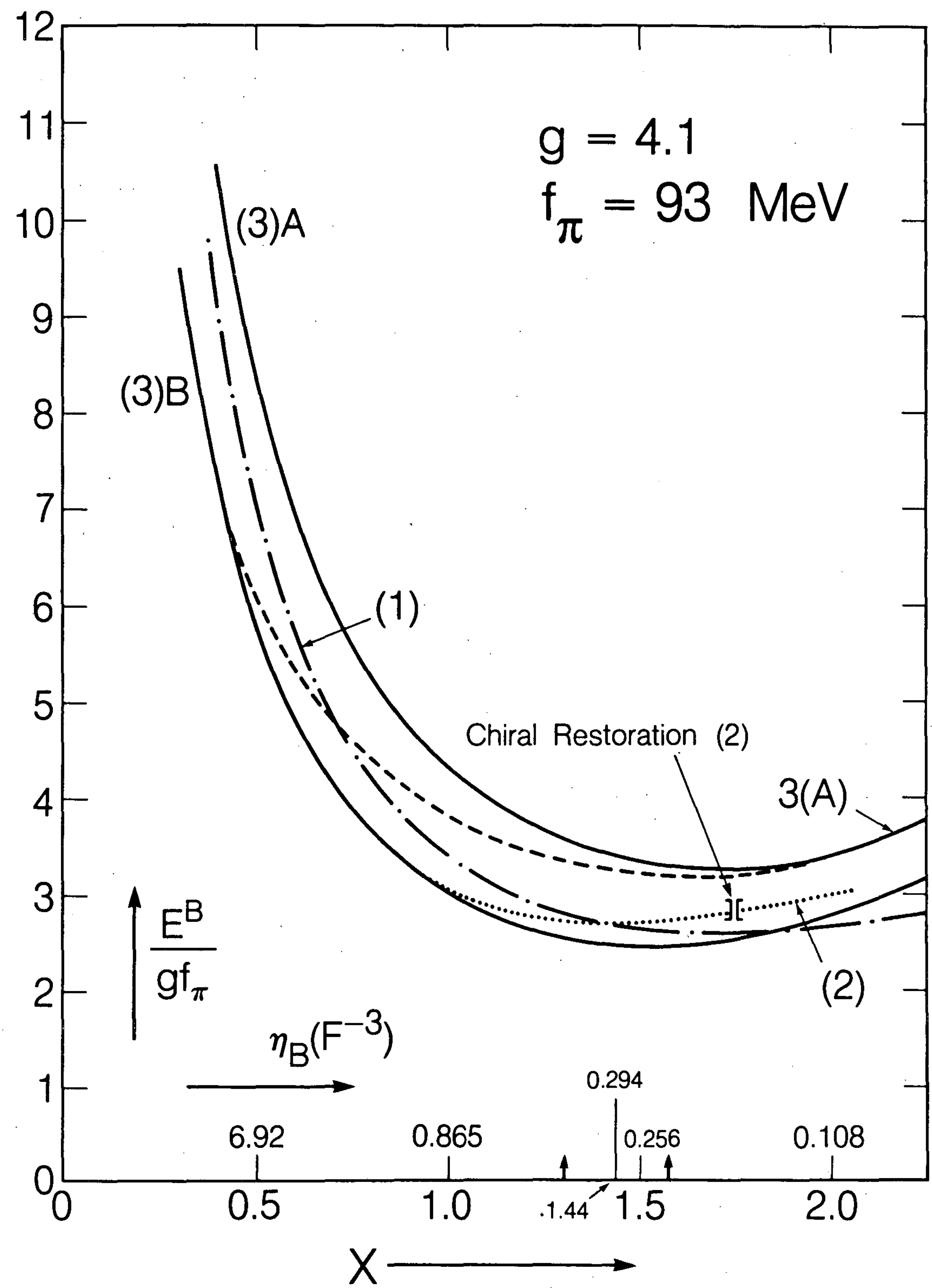

g = 7.5
f_π = 93 MeV
(1)
(3)A
(3)B
(2)
(3)A
Chiral Restoration
$\frac{E^B}{gf_\pi}$
$\eta_B(F^{-3})$
5.4
0.674
2.86
0.24
0.2
0.108
X
XBL 8410-9976

Fig 1



XBL 8410-9977

Fig. 2

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