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Title

Unit 28 - Affine and Curvilinear Transformations

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UNIT 28 - AFFINE AND CURVILINEAR TRANSFORMATIONS

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A. INTRODUCTION

- coordinate transformations are required when you need to register different sets of coordinates for objects in the same area that may have come from maps of different (and sometimes unknown) projections
    - will need to transform one or more sets of coordinates so that they are represented in the same coordinate system as other sets

- are two ways to look at coordinate transformations:

    1. move objects on a fixed coordinate system so that the coordinates change

    diagram

    2. hold the objects fixed and move the coordinate system

- this is the more useful way to consider transformations for GIS purposes

     diagram

- (x,y) is the location of the object before transformation (in the old coordinate system)

- (u,v) is the location of the object after transformation (in the new coordinate system)

- are two major groups of transformations:
    - affine transformations are those which keep parallel lines parallel
        - as we will see they are a class of transformations which have 6 coefficients

    - curvilinear transformations are higher order transformations that do not necessarily keep lines straight and parallel
        - these transformations may require more than 6 coefficients

B. AFFINE TRANSFORMATION PRIMITIVES

- affine transformations keep parallel lines parallel

- are four different types (primitives): handout - Affine transformation primitives

1. Translation

- origin is moved, axes do not rotate

     diagram

     $u = x - a \quad v = y - b$

- origin is moved a units parallel to x and b units parallel to y

2. Scaling

- both origin and axes are fixed, scale changes

     diagram

     $u = cx \quad v = dy$

- scaling of x and y may be different
    - if the scaling is different, the shape of the object will change

3. Rotation

- origin fixed, axes move (rotate about origin)

     diagram

     $u = x \cos(a) + y \sin(a) \quad v = -x \sin(a) + y \cos(a)$ (note: a is measured counterclockwise)

4. Reflection

- coordinate system is reversed, objects appear in mirror image

    diagram

- to reverse y, but not x: u = x v = c - y

- this transformation is important for displaying images on video monitors as the default coordinate system has the origin in the upper left corner and coordinates which run across and down

C. COMPLEX AFFINE TRANSFORMATIONS

- usually a combination of these transformations will be needed

- the combined equations are: u = a + bx + cy v = d + ex + fy

- often cannot actually separate the needed transformations into one or more of the primitives defined above as one transformation will cause changes that appear to be caused by another transformation, and order is important
    - e.g. translation followed by scale change is not the same as scale change followed by translation, has different effect
    - exception:
        - reflection has occurred if bf < ce

D. AFFINE TRANSFORMATIONS IN GIS

- frequently, when developing spatial databases for use in GIS, data will be provided on map sheets which use unknown or inaccurate projections

- in order to register two data sets, a set of control points or tics must be identified that can be located on both maps

    - must have at least 3 control points since 3 points provide 6 values which can be used to solve for the 6 unknowns
        - control points must not be on a straight line (not collinear)

Simple Example

- control points are: x y u v

    0 0 1 10 1 0 1 9 0 1 3 10 1 1 3 9

- solution:
    - hint: note that y and u are related, a change in y always produces the same change in u; x and v are similarly related
    - therefore:

$$v = 10 - x \quad u = 1 + 2y$$

- complete equations are:

$$u = 1 + 0x + 2y \quad v = 10 - 1x + 0y$$

- note that $bf = 0$, $ce = -2$
  - therefore, $bf > ce$, there is no reflection involved

City Fire Study Example

handout - City fire study example (2 pages)

Problem:

- given: two sets of spatial data: 1. Census tract boundaries in the city with coordinates given in UTM 2. Fire locations in the city plotted on a crude road map
  - UTM is to be the destination system

- count the number of fires in each census tract and analyze the numbers of fires in relation to characteristics described by census data
  - e.g. is number of fires related to number of houses constructed of wood?

Solution:

- use major street intersections as control points
  - for destination system, determine UTM coordinates of the intersections
  - for the fire map system, use any arbitrary rectangular coordinate system (i.e. digitizer table) with fixed origin and axes

- using the two sets of coordinates for the control points and linear regression techniques, solve for the 6 coefficients in the two affine transformation equations

- examine the residuals to evaluate the accuracy of the analysis
  - the spatial distribution of residuals may indicate weaknesses of the model
  - may show that map has been distorted unevenly
  - magnitude of the residuals gives an estimate of the accuracy of the transformation
  - in this example, the magnitude of residuals indicates an accuracy of 150 m
  - i.e. UTM coordinates of fire locations are +/- 150 m from their locations as indicated on the road map
  - for this analysis, this is sufficient accuracy

E. CURVILINEAR TRANSFORMATIONS

- simple linear affine transformation equations can be extended to higher powers:

$$u = a + bx + cy + gxy \text{ or } u = a + bx + cy + gx2 \text{ or } u = a + bx + cy + gx2 + hy2 + ixy$$

- equations of this form create curved surfaces

> provides rubbersheeting in which points are not transformed evenly over the sheet, transformations are not affine (parallel lines become non-parallel, possibly curved)

- rubber-sheet transformations may also be piecewise
  - map divided into regions, each with its own transformation equations
  - equations must satisfy continuity conditions at the edges of regions

- curvilinear transformations usually give greater accuracy
  - accuracy in the sense that when used to transform the control points or tics, the equations faithfully reproduce the known coordinates in the other system
  - however if error in measurement is present, and it always is to some degree, then greater accuracy may not be desirable
  - a curvilinear transformation may be more accurate for the control points, but less accurate on average

REFERENCES

Goodchild, M.F., 1984. "Geocoding and Geosampling," Spatial Statistics and Models, G.L. Gaile and C.J. Willmott, eds., Reidel Publishing Company, Dordrecht, Holland, pp. 33-53.

DISCUSSION AND EXAM QUESTIONS

1. A map has been digitized using an origin in the lower left corner. The x axis is the lower edge of the map, and the y axis is the left edge. The study area ranges from 0 to 100 in x, and 0 to 50 in y. Calculate the transformations necessary to show this area on a screen containing 640 columns and 480 rows, with the columns and rows numbered from the top left corner, assuming that positions on the screen are referred to by row and column numbers.

2. Describe the transformations between x and y and UTM Northing and UTM Easting given in the text of the unit for the fire study example in terms of (a) translation of the origin, (b) scaling, (c) rotation and (d) reflection.

3. Discuss the factors which contribute to lack of fit between maps of the same area, and failure of affine transformations to register maps perfectly using control points. Discuss the relative importance of each of these factors.

4. Discuss the criteria you would use to select control points, including the number and distribution of such points, in setting up a coordinate transformation.

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