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Is the β phase maximal ?

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The current experimental determination of the absolute values of the CKM elements indicates that $2|V_{ub}/V_{cb}V_{us}| = (1 - z)$, with z given by $z = 0.19 \pm 0.14$. This fact implies that irrespective of the form of the quark Yukawa matrices, the measured value of the SM CP phase β is approximately the maximum allowed by the measured absolute values of the CKM elements. This is $\beta = (\pi/6 - z/\sqrt{3})$ for $\gamma = (\pi/3 + z/\sqrt{3})$, which implies $\alpha = \pi/2$. Alternatively, assuming that β is exactly maximal and using the experimental measurement $\sin(2\beta) = 0.726 \pm 0.037$, the phase γ is predicted to be $\gamma = (\pi/2 - \beta) = 66.3^\circ \pm 1.7^\circ$. The maximality of β , if confirmed by near-future experiments, may give us some clues as to the origin of CP violation.

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I. INTRODUCTION

Over the past few years, our knowledge of CP violation in particle physics has improved significantly. Experiments have confirmed the so-called CKM paradigm, in that all the CP-violation processes measured with good precision in particle physics to date seem to be described by the CP-violating phase present in the CKM matrix. CP-violating phases are usually parametrized in the (α, β, γ) phase convention. We have two precise B-factory measurements of β extracted from the CP asymmetries in $B \rightarrow \psi K_s$ [1]:

$$\sin(2\beta)_{\text{Belle}} = 0.728 \pm 0.056 \pm 0.023, \quad (1)$$

$$\sin(2\beta)_{\text{BaBar}} = 0.722 \pm 0.040 \pm 0.020. \quad (2)$$

The current world average is $\sin 2\beta = 0.726 \pm 0.037$, which translates to a value for β of,

$$\beta_{\text{exp}} = 23.3^\circ \pm 1.6^\circ. \quad (3)$$

The phase γ is not so well known. The first experimental result, which has been obtained at Belle, is $\gamma = (68_{-15}^{+14} \pm 13 \pm 11)^\circ$, while BaBar's preliminary result is $\gamma = (88 \pm 41 \pm 19 \pm 10)^\circ$ [3]. Both experiments have directly measured γ by studying the time-independent CP asymmetries in B decays to $D^0 K$ and $\bar{D}^0 K$ [2]. The direct determination carries a large combined uncertainty (statistical+systematic+model dependent) of about 60% to 80% [3]. There are other methods to extract γ [4]. Recently a new method has been proposed that makes use of an SCET analysis of the $B \rightarrow \pi\pi$ decays. While this method reduces the theoretical uncertainty of the final result, the experimental error remains large [5]. These measurements are compatible with the indirect determination through CKM global fits [6] and earlier constraints developed in Ref. [7], which provide approximately the following value for the phase γ at 95% C.L.,

$$\gamma_{\text{fit}} \approx 61^\circ \pm 11^\circ. \quad (4)$$

Regarding the phase $\alpha = (\pi - \beta - \gamma)$, although none of the decays $B \rightarrow (\pi\pi, \rho\rho, \rho\pi)$ used for the determination of α [4, 8] gives a precise number, the combination of the three modes [9, 10] gives a result that is in agreement with the value obtained from CKM global fits [6]. The latter, combining statistical and systematic uncertainties, is approximately,

$$\alpha_{\text{fit}} \approx 100^\circ \pm 25^\circ.$$

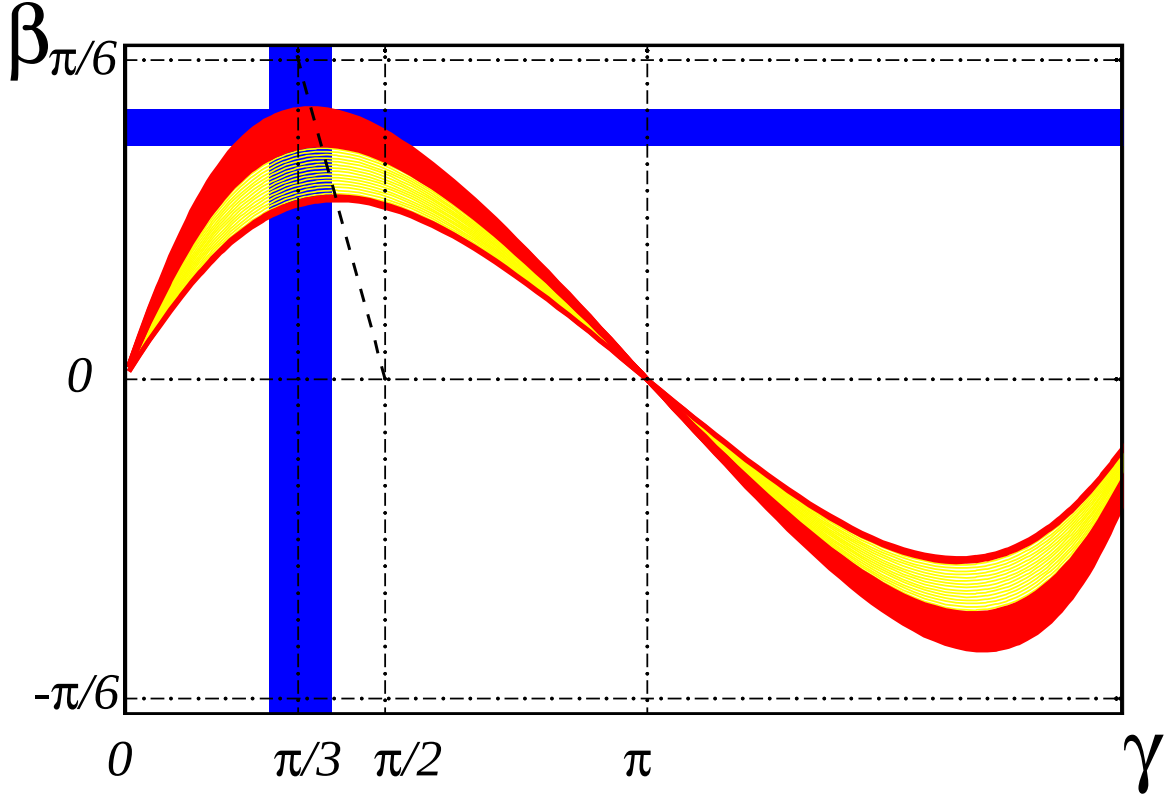


FIG. 1: This figure shows the relation between the CP-violating phases β and γ constrained by the measured absolute values of the CKM elements. The blue horizontal strip corresponds to the measurement $\sin(2\beta)_{\text{exp}} = 0.726 \pm 0.037$. The blue vertical strip corresponds to the $1\text{-}\sigma$ global fit for the angle γ , $\gamma_{\text{fit}} = 61^\circ \pm 11^\circ$. The sinusoidal red area corresponds to the leading-order relation between β and γ determined by unitarity (see Eq. 10). The sinusoidal yellow-hatched area corresponds to the same relation, additionally constrained by the measurement of $|V_{ub}|/|V_{cb}|$. The diagonal dotted line is the line of maximum, $\alpha = \pi/2$.

It is the main goal of any theory of flavor to explain the measured CP phases, as well as the CKM elements and fermion mass ratios. It may be useful when searching for a fundamental theory of flavor to know *a priori* if there is something special about the measured values of α , β and γ that could give us some clue regarding the origin of CP violation. Let us suppose that we had experimental information on the absolute values of the CKM elements but not on the CP-violating phases. We could then expect that the hierarchies observed in the absolute values of the CKM elements would constrain the range of possible values for the CP phases. The question is the following: are the measured values of β and γ a special pair out of all the possible values compatible with the measured absolute values of the CKM elements? It is the main purpose of this short paper to argue that this may be the case. We will show that the measured value of β

is approximately the maximum allowed by the measured absolute values of the CKM elements. In Sec. II we will introduce an alternative parametrization of the CKM matrix, which helps to make our point more clear. In Sec. III we will give a simple formula for the maximum value of β and compare the numerical results with the measurements. In Sec. IV we will briefly comment on near-future prospects for testing the maximality of β .

II. AN ALTERNATIVE PARAMETRIZATION OF THE CKM MATRIX

It is convenient for the subsequent discussion to adopt the following alternative parametrization of the CKM matrix. The unitary matrix $\mathcal{V}_{\text{CKM}}$ can be expressed to leading order as a function of three real parameters λ, a, ζ and a complex phase γ ,

$$\mathcal{V}_{\text{CKM}} = \begin{bmatrix} 1 - \frac{\lambda^2}{2} & -\lambda & e^{-i\gamma}\zeta a\lambda \\ \lambda & 1 - \frac{\lambda^2}{2} - \frac{a^2}{2} & -a \\ (1 - \zeta e^{i\gamma})a\lambda & a & 1 - \frac{a^2}{2} \end{bmatrix}. \quad (5)$$

The parameter a is given to leading order by $a = |V_{cb}|$. We note that based on the experimental value for $|V_{cb}|$, the parameter a is approximately $a \approx \lambda^2$. Accordingly, in the previous parametrization the CKM matrix is unitary to order λ^2 , *i.e.* $\mathcal{V}_{\text{CKM}}^\dagger \mathcal{V}_{\text{CKM}} = \mathcal{I} + \mathcal{O}(\lambda^3)$. The parameter ζ can be determined from the absolute values of the CKM elements. We find it convenient for the rest of the discussion to introduce a new parameter $z = (1 - 2\zeta)$, which can be expressed as,

$$z = 1 - \frac{2|V_{ub}|}{|V_{cb}||V_{us}|}. \quad (6)$$

It is easy to obtain the relation between the new parameters a and ζ and the parameters A , ρ and η of the usual Wolfenstein parametrization [11],

$$A = a/\lambda^2, \quad \zeta = (\rho^2 + \eta^2)^{1/2}, \quad (7)$$

$$\gamma = \tan^{-1}(\eta/\rho). \quad (8)$$

The parameter z can be determined from the measured absolute values of the CKM elements. Using the values given by the PDG collaboration, $|V_{us}| = 0.220 \pm 0.0026$, $|V_{ub}| = 0.00367 \pm 0.00047$ and $|V_{cb}| = 0.0413 \pm 0.0015$ [12], we obtain,

$$z = 0.19 \pm 0.14. \quad (9)$$

Using the newer and more precise value, $|V_{ub}|/|V_{cb}| = 0.086 \pm 0.008$ [13], we obtain $z = 0.22 \pm 0.08$.

III. THE MAXIMAL β PHASE

It is a trivial check to prove that the angle γ introduced in the parametrization of the CKM matrix given in Eq. 5 coincides with the standard definition, $\gamma = \text{Arg} [-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*]$. Using our parametrization for the CKM matrix, we obtain a simple expression for the leading-order relation between the angles β and γ ,

$$\beta = \text{Arg} \left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*} \right] = \text{Arg} [1 - \zeta e^{-i\gamma}] \quad (10)$$

If we consider β as a function of γ , we can determine the value of γ that maximizes β , or in other words solve the equation $d\beta/d\gamma = 0$. We find that at the maximum,

$$\beta_{\max} = \sin^{-1}(\zeta), \quad (11)$$

$$\gamma_{\max} = \cos^{-1}(\zeta), \quad (12)$$

where $\gamma_{\max}$ must be interpreted as the value of the phase γ that maximizes β , and not the maximum value of γ . This is equivalent to the condition,

$$\beta + \gamma = \frac{\pi}{2}. \quad (13)$$

Thus, the maximality condition predicts not β or γ but α , $\alpha = \pi/2$. Furthermore, the measured absolute values of the CKM elements indicate that $z \ll 1$. We can therefore expand the maximal β solution in powers of z around $z = 0$ and obtain the following expression for the maximum,

$$\beta_{\max}^{\text{LO}} = \frac{\pi}{6} - \frac{z}{\sqrt{3}} + \mathcal{O}(z^2), \quad (14)$$

$$\gamma_{\max}^{\text{LO}} = \frac{\pi}{3} + \frac{z}{\sqrt{3}} + \mathcal{O}(z^2). \quad (15)$$

Using the numerical value of z as calculated in Eq. 9 from the absolute values of the CKM elements, we obtain the following numerical values for the maximum,

$$\beta_{\max}^{\text{LO}} = 23.6^\circ \pm 4.7^\circ \quad (16)$$

$$\gamma_{\max}^{\text{LO}} = 66.3^\circ \mp 4.7^\circ. \quad (17)$$

These values are close to the central measured value for β and the global fit for γ given in Eqs. 3 and 4 respectively. They can be observed graphically in figure 1 as the intersection between the diagonal line of maximum and the red sinusoidal strip. If we use the value of z determined using the measured ratio for $|V_{ub}|/|V_{cb}|$, we obtain a narrower strip that is shown as the hatched-yellow area in Fig. 1.

A. Next-to-leading-order corrections

In some applications, the next-to-leading-order (NLO) corrections to the CKM matrix play an important role. We are interested in calculating the effect that these corrections may have on the location of the maximum β phase, as they may be crucial for testing the maximality of β with precision. Requiring unitarity to order $\mathcal{O}(\lambda^3)$, *i.e.* $\mathcal{V}_{\text{CKM}}^\dagger \mathcal{V}_{\text{CKM}} = \mathcal{I} + \mathcal{O}(\lambda^4)$, we obtain,

$$\begin{bmatrix} 1 - \frac{\lambda^2}{2} & -\lambda(1 + 2a) & e^{-i\gamma}\zeta a\lambda \\ \lambda(1 + 2a) & 1 - \frac{\lambda^2}{2} - \frac{a^2}{2} & -a(1 + \frac{a}{2}) \\ (1 + \frac{5a}{2} - \zeta e^{i\gamma})a\lambda & a(1 + \frac{a}{2} + \lambda^2(\zeta e^{i\gamma} - \frac{1}{2})) & 1 - \frac{a^2}{2} \end{bmatrix}. \quad (18)$$

We note that the parameter a is determined from $|V_{cb}|$ to NLO from $|V_{cb}| = a(1 + \frac{a}{2})$. We can use the general definition of the β phase given in Eq. 10 and expand in powers of z and a . We obtain the following expression for the maximal β phase at NLO in the CKM matrix,

$$\beta_{\text{max}}^{\text{NLO}} = \frac{\pi}{6} - \frac{\eta}{\sqrt{3}} + \mathcal{O}(\eta^3), \quad (19)$$

$$\gamma_{\text{max}}^{\text{NLO}} = \frac{\pi}{3} + \frac{\eta}{\sqrt{3}} + \mathcal{O}(\eta^3), \quad (20)$$

where the parameter η is related to z and a by,

$$\eta = (z + 5a/2)(1 - \frac{(z + 5a/2)}{6}). \quad (21)$$

Using the measured values of the CKM elements, we obtain $\eta = 0.28 \pm 0.14$. This translates to the following numerical values for the maximum at NLO,

$$\beta_{\text{max}}^{\text{NLO}} = 20.7^\circ \pm 4.7^\circ, \quad (22)$$

$$\gamma_{\text{max}}^{\text{NLO}} = 69.3^\circ \mp 4.7^\circ. \quad (23)$$

Alternatively, assuming that β is exactly maximal and using the experimental measurement $\sin(2\beta) = 0.726 \pm 0.037$, the phase γ is predicted to be $\gamma = (\pi/2 - \beta) = 66.3^\circ \pm 1.7^\circ$. We would like to note that the approximate maximality of the β phase was pointed out in the context of a particular Yukawa ansatz in Ref. [14], where the generality of this observation was not emphasized.

B. RGE invariance

The underlying flavor symmetry is broken at an energy scale much higher than the electroweak scale. Therefore, the issue of the renormalization-scale evolution of the CP phases may be very

relevant if β is approximately maximal. The evolution equations for the entries of the CKM matrix have been known for quite a long time [15]. It was pointed out [16] that using the measured hierarchies of the CKM elements simplifies these expressions considerably. The entries $|\mathcal{V}_{ub}|$, $|\mathcal{V}_{cb}|$, $|\mathcal{V}_{td}|$ and $|\mathcal{V}_{ts}|$ receive an identical and sizeable correction given by,

$$\frac{d|\mathcal{V}_{i\alpha}|}{d\ln\mu} = -\frac{3c}{32\pi^2} |\mathcal{V}_{i\alpha}| (h_t^2 + h_b^2). \quad (24)$$

Here $c = -1$ for the SM, and $c = 2/3$ for the MSSM. This equation can be approximately solved to yield,

$$|\mathcal{V}_{i\alpha}^0| \approx |\mathcal{V}_{i\alpha}| \left(\frac{m_W}{\Lambda} \right)^{\frac{3c}{32\pi^2} (h_t^2 + h_b^2)}. \quad (25)$$

Therefore, $|\mathcal{V}_{i\alpha}|$ can receive a correction of about +20% in the SM and about -85% in the MSSM. On the other hand, the entries $|\mathcal{V}_{us}|$ and $|\mathcal{V}_{cd}|$ receive a tiny identical RGE correction given by,

$$\frac{d|\mathcal{V}_{us}|}{d\ln\mu} = -\frac{3c}{32\pi^2} |\mathcal{V}_{us}| (h_c^2 + h_s^2 + h_t^2 \frac{|\mathcal{V}_{ub}|^2 - |\mathcal{V}_{td}|^2}{|\mathcal{V}_{cd}|^2}). \quad (26)$$

This generates a correction to $|\mathcal{V}_{us}|$ and $|\mathcal{V}_{cd}|$, which in the SM and MSSM is at most of the order,

$$|\mathcal{V}_{us}| \lesssim |\mathcal{V}_{us}^0| \left(\frac{\Lambda}{m_W} \right)^{\frac{9c}{32\pi^2} \frac{m_s^2}{m_b^2}}. \quad (27)$$

This amounts to a correction of about 0.1%, which can be neglected for practical purposes. Therefore $|\mathcal{V}_{us}|$ and $|\mathcal{V}_{cd}|$ can be considered renormalization-scale independent. As a consequence, the parameters λ and ζ are to a very good approximation renormalization-scale independent, and the CKM matrix at the scale Λ , $\mathcal{V}^0$, adopts the same functional form as the electroweak scale CKM matrix, with the replacement $a \rightarrow a(\frac{m_W}{\Lambda})^{\frac{3c}{32\pi^2} (h_t^2 + h_b^2)}$. Since the parameter a does not enter into Eq. 10, and the renormalization-scale dependence of V_{ub} and V_{cb} cancels in the definition of γ , we can claim that the CP phases are to leading order renormalization-scale independent. Therefore, the maximality of β is an approximately renormalization-scale independent fact that should be addressed by the theory of flavor, irrespective of the flavor breaking scale.

C. Maximal β and CP-phase convention

The usual (α, β, γ) phase convention is just one particular convention out of a continuum of other possible conventions to parametrize the CP-violating phases in the SM. If we were using a different phase convention $(\alpha', \beta', \gamma')$, it would be a certain combination of these phases that would become maximal, corresponding to the phase β . Although this would make it more difficult to

notice the correlation, it would have the same observable implications. One may wonder whether this fact is physically relevant. Is it just a coincidence that one of the phases, in the convention that became of common use for historical reasons, is maximal? We note that even though all phase conventions are mathematically equivalent, a good choice must display simple relations to measurable quantities, and this may shed light on some important qualitative issues. For instance, in the standard convention, β parametrizes CPV in the mixing of the two B-meson mass eigenstates.

A related topic is the issue of maximal CP violation. It is known that the standard invariant measure of CP violation [17] is given by the Jarlskog parameter, $\mathcal{J} = \text{Im} [V_{il}V_{jm}V_{im}^*V_{jl}^*]$ ($i \neq j$, $l \neq m$). Using the parametrization given in Eq. 5 we obtain,

$$\mathcal{J} = \zeta a^2 \lambda^2 \sin(\gamma) + \mathcal{O}(\lambda^3), \quad (28)$$

The Jarlskog parameter is determined experimentally to be $\mathcal{J} = (3.0 \pm 0.3) \times 10^{-5}$. Substituting a , λ and ζ determined from the measured absolute values of the CKM we obtain $\gamma \approx 59^\circ$ as the central value, which is fully consistent with the value provided by the current global fit. There are several definitions of maximal CP violation. Originally, the term referred to the phase that maximizes $\mathcal{J}$ [18]. This definition, however, is not well-defined, as it is phase-convention dependent. Moreover, it would correspond to $\gamma = \pi/2$, which is in poor agreement with experiment, as was pointed out a long time ago. There have been a few proposals to define phase-convention independent parametrizations of maximal CP violation [19], which if true would predict an amount of CP violation much larger than is observed.

What is relevant is that once the absolute values of the CKM elements have been fixed, the maximal β hypothesis uniquely determines α , β and γ , as we have shown in the previous section. This has distinct observable implications that are phase-convention independent.

IV. FUTURE PROSPECTS

We have used the current measured values of the CKM elements to determine the location of the maximal β phase solution with about 20% uncertainty. This has been shown to be consistent with the current measured value of β , which carries an uncertainty of about 5%. $|V_{us}|$ and $|V_{cb}|$ are known at the 2% and 4% level, respectively. It is expected that the uncertainty in the experimental determination of $|V_{ub}|$ will decrease in the near future from its present value of about 13% to the present theoretical limit of $\sim 5\%$ [20]. Such an improvement in the determination of $|V_{ub}|$ would allow us to calculate the maximal β phase and the corresponding γ phase with much better

precision, about 6% optimistically. The crucial test of the maximal- β hypothesis will no doubt be provided by a precise determination of the phases γ and α . The experimental situation looks favorable. The precise measurement of γ is the next challenge for B factories. It is expected [21, 22] that if the luminosity in the upgraded B factories is high enough, γ could be determined with approximately 5% percent uncertainty; a measurement with an uncertainty at the level of 1% will require a superB factory. A determination of α at less than the 5% level may also have to wait for the superB factories [22].

V. CONCLUSION

Solving a problem sometimes requires that one ask the right question. The question of why $\beta_{\text{exp}} = (23.3 \pm 1.6)^\circ$ or the question of why $\alpha_{\text{exp}} \simeq 90^\circ$ are equally puzzling. These values of α and β are at first sight two seemingly arbitrary numbers. On the other hand, the observation that this is approximately equivalent to assuming that the phase β is the maximum allowed by the measured absolute values of the CKM elements may give us a clue about the mechanism that determines the amount of CP violation measured in nature. If a mechanism exists in the underlying theory of flavor that selects the maximum allowed value of the phase β , we can successfully predict the measured CP violation once the hierarchies in the absolute values of the CKM elements are fixed. The maximality of β , if confirmed with precision by future measurements of γ and α , could be a guiding light in our search for a fundamental theory of flavor.

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