From Phase Locking to Phase Slips: A Mechanism for a Quiescent H mode

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(Received 17 December 2014; published 6 April 2015)

We demonstrate that $E \times B$ shear, $V'_{E\times B}$, governs the dynamics of the cross phase of the peeling-ballooning-(PB-)mode-driven heat flux, and so determines the evolution from the edge-localized (ELMy) H mode to the quiescent (Q) H mode. A physics-based scaling of the critical $E \times B$ shearing rate ($V'_{E\times B,cr}$) for accessing the QH mode is predicted. The ELMy H mode to the QH-mode evolution is shown to follow from the conversion from a phase locked state to a phase slip state. In the phase locked state, PB modes are pumped continuously, so bursts occur. In the slip state, the PB activity is a coherent oscillation. Stronger $E \times B$ shearing implies a higher phase slip frequency. This finding predicts a new state of cross phase dynamics and shows a new way to understand the physics mechanism for ELMy to the QH-mode evolution.

DOI: 10.1103/PhysRevLett.114.145002 PACS numbers: 52.35.Mw, 52.25.Fi, 52.55.Dy, 52.55.Fa

Understanding relaxation mechanisms in far from equilibrium systems is an outstanding goal in many fields, e.g., fluid dynamics, solar physics, space physics, and laboratory plasma. Sometimes the relaxation exhibits violent behavior, such as flares in the solar corona [1], magnetic substorms in the magnetosphere [2], and disruptions in tokamaks [3]. A common feature of these events is that there is an extended period of free energy accumulation prior to a sudden eruption and energy release. Sometimes the relaxation is gradual, and occurs via excitations of waves and turbulence, such as Alfvén waves in the solar wind, drift waves in tokamaks, etc. In this scenario, the energy is released in a much "softer" way. In the H-mode edge of tokamaks, giant type-I ELMs and the QH mode are examples of violent and soft energy release processes, respectively. In this Letter, we relate violent and soft energy release to phase evolution dynamics.

The ELMy H mode and QH mode are two principal favorable operating scenarios of future burning plasma devices, e.g., the ITER. In the ELMy H mode, the thermal energy is released in a highly transient, episodic way, and the induced heat load may erode plasma facing components and degrade performance. ELM physics, especially the presumed underlying instability mechanism (i.e., the PB mode), has been studied extensively [4]. PB modes are ideal MHD instabilities which couple magnetic curvature (i.e., ballooning) drive and current gradient drive (i.e., kink). PB modes are thus hybrids of surface kinks (i.e., "peeling") and ballooning modes. The pressure gradient ultimately is the source for both of these effects. For ballooning, it enters directly. For peeling, it enters via the bootstrap current, driven by pressure gradient [5]. Recently, the nonlinear dynamics of an ELM crash was addressed by employing random phase scattering concepts. A nonlinear criterion for when the ELM crash actually occurred was also given [6]. In experiments, much effort has been devoted to reducing the size of ELMs to an acceptable level [7–9]. In contrast to the large crash of the pressure profile in the ELMy H mode, the edge pressure profile finds a steady weak oscillatory state in the QH mode, so impurities are expelled effectively and the plasma facing components are not eroded [10]. Thus, the QH mode is an attractive scenario for a fusion reactor. There is well-documented experimental evidence [11] that $E \times B$ shear ($V'_{E\times B}$—driven by radial electrostatic field shear) is a central ingredient in determining which type of H mode the plasma system stays in. The most obvious aspect of accessing the QH mode is that it requires $V'_{E\times B}$ to exceed a critical value [11]. The critical role of $V'_{E\times B}$ for the $L \to H$ transition has been extensively studied both by experiment and theory. However, a precise understanding of the role of $V'_{E\times B}$ in the ELMy H mode to the QH mode (ELMy $\to Q$) evolution remains elusive [12]. In other words, how does the critical $E \times B$ shear control access to the QH mode? Note that the QH mode is not a state of pure linear stability, as the edge harmonic oscillation is observed [10].

Any precise understanding of ELMy $\to Q$ evolution requires treating the relaxation physics of the H mode in a proper framework. The ELMy $\to Q$ evolution phenomenon clearly is beyond any linear theory or quasilinear theory. The H mode is a state where the pressure profile is near marginal to PB instability, and the amplitude of the ambient turbulence is weak. Therefore, the nonlinear processes associated with the amplitude of the PB modes are restricted. It has been shown that phase dynamics is a useful concept for describing the multiscale dynamics of a marginally stable system [13,14]. The phase determines the macroscopic state of the system, and so the phase dynamics of the PB modes is crucial in determining the macroscopic state of the H-mode pedestal. In this work, we investigate the ELMy $\to Q$ evolution mechanism by means of phase dynamics concepts, i.e., by calculating the cross phase dynamics of the PB driven heat flux in the presence of an $E \times B$ shear flow. This methodology is in distinct contrast to the conventionally employed eigenmode and quasilinear analysis, which implicitly take the cross phase as fixed. For the first time, we find that if $|V'_{E\times B}| < |V'_{E\times B,cr}|$, the cross phase will lock to a fixed value, for which the PB modes are continually pumped and can reach a large amplitude. In that case, the thermal energy tends to be released in a burst, so the pressure profile collapses rapidly. Therefore, the phase locked state corresponds to the ELMy H mode. If $|V'_{E\times B}| > |V'_{E\times B,cr}|$, we show that the cross phase selects a value that leaves the pressure and velocity components of the PB modes out of phase, except for "phase slips" of short duration. Since the phase slip is short and occurs periodically, the PB modes are weakly and periodically pumped. The stronger the $E \times B$ shearing, the higher the phase slip frequency will be, so the ELM asymptotes to a continuous oscillation. The efficiency of impurity expulsion in the QH mode is enhanced. The phase slip provides a means for regulating thermal energy release, and hence keeps the H mode in a more quiescent state. This model gives a new, general way to understand the ELMy $\rightarrow Q$ evolution mechanism.

The PB instabilities in the edge of a confined plasma are driven by the edge pressure gradient [4]. The PB modes are excited via phase coupling among the PB pressure and velocity perturbations, which in turn produce the PB heat flux. Generally, the evolution of the edge pressure (P) can be written in the following form:

$$\frac{\partial}{\partial t}P + \vec{V}\cdot\nabla P = D\nabla^2 P + s_{\rm DW} + S_{\rm in}, \qquad (1)$$

where the total pressure $P = \langle P\rangle + \delta P$ is composed of a mean and perturbed component with $\langle\cdots\rangle$ the poloidal average. The convection velocity is $\vec{V} = \vec{V}_{E\times B} + \delta\vec{V}_{\rm PB}$ with $\vec{V}_{E\times B}$ the $E \times B$ shear flow driven by the radial electrostatic field and $\delta\vec{V}_{\rm PB}$ is velocity perturbation associated with the PB mode. $s_{\rm DW}$ is the noise associated with the ambient small scale drift wave turbulence (e.g., ion-temperature-gradient turbulence). $D\nabla^2 P$ accounts for the dissipation of the pressure, with D a diffusion coefficient. $S_{\rm in}$ is associated with the heat flux from the core of the Tokamak, so Eq. (1) is a flux-driven system. The evolution equation for the mean pressure follows as

$$\frac{\partial}{\partial t}\langle P\rangle = -\partial_x\langle\delta V_{{\rm PB},x}\delta P\rangle + (D + D_T)\nabla^2\langle P\rangle + S_{\rm in}, \quad (2)$$

where the noise impacts the evolution of $\langle P\rangle$ via a turbulent diffusion process, and hence one has $\langle s_{\rm DW}\rangle = D_T\nabla^2\langle P\rangle$ with D_T the effective diffusion coefficient. A quasisteady state of the mean pressure profile can be sustained by the "fueling" term $S_{\rm in}$ (Fig. 1). Since the ambient turbulence is strongly quenched in the H mode, D_T cannot reach a significant level. $\langle\delta V_{\rm PB,x}\delta P\rangle$ is the heat flux driven by PB modes. To excite PB modes, a finite cross correlation

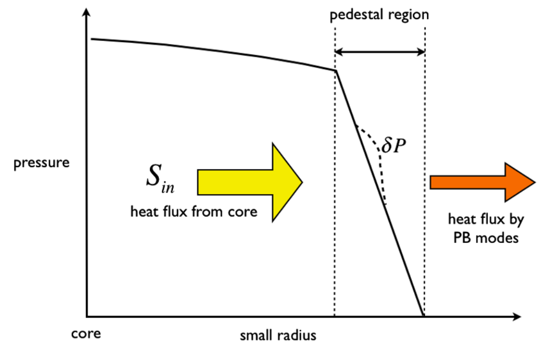

FIG. 1 (color online). Setup of the analysis.

between $\delta V_{\rm PB,x}$ and δP is required. This is in turn determined by their cross phase. If the cross phase is $\pi/2$, $\delta V_{{\rm PB},x}$ and δP will be out of phase, and the pumping of the PB modes will stop. In contrast to eigenmode or quasilinear analysis (where the cross phase is taken fixed), in this model the cross phase is evolving dynamically. The framework of phase dynamics aims to capture this. A direct way to obtain the evolution equation for the cross phase is via the evolution equation of δP

$$\frac{\partial}{\partial t}\delta P + \delta\vec{V}_{\rm PB}\cdot\nabla\langle P\rangle + \vec{V}_{E\times B}\cdot\nabla\delta P = \tilde{s} + D\nabla^2\delta P, \quad (3)$$

where $\tilde{s} = \tilde{s}_{\rm PB} + \tilde{s}_{\rm DW}$ with $\tilde{s}_{\rm PB} = -[\nabla\cdot(\delta\vec{V}_{\rm PB}\delta P) - \partial_x\langle\delta V_{{\rm PB},x}\delta P\rangle]$. $\tilde{s}_{\rm PB}$ accounts for the random phase scattering induced by PB mode couplings, which is relevant to the nonlinear criterion for ELM crash [6]. The noise $\tilde{s}$ can trigger stochastic avalanches of δP, which serves as a mechanism for generating pressure perturbations [15]. After Fourier transformation for δP and $\delta\vec{V}_{\rm PB}$, one has $\delta P \rightarrow |\delta P_k|e^{i\vec{k}\cdot\vec{r}+i\Theta_k}$ and $\delta\vec{V}_{\rm PB} \rightarrow |\delta\vec{V}_{\rm PB,k}|e^{i\vec{k}\cdot\vec{r}+i\alpha_k}$, where Θ_k and α_k are the phases of δP_k and $\delta V_{{\rm PB},k}$. Then the real part of the Fourier component of the cross correlation can be written as $\langle\delta V_{{\rm PB},x}\delta P\rangle_k = |\delta V_{{\rm PB},kx}||\delta P_k|\cos(\Theta_k - \alpha_k)$. The phase difference, $\Theta_k - \alpha_k$, is just the cross phase between $\delta\vec{V}_{\rm PB,k}$ and δP_k. For kinetic velocity fields, α_k acts as a reference phase, so that without loss of generality, we can take $\alpha_k = 0$. Then the cross phase dynamics is determined solely by Θ_k. To obtain a compact form for its evolution, we use the approximations (i) the intensities of δP_k and $\delta V_{{\rm PB},k}$ vary slowly in time and space, i.e., $|\partial_x \ln|\delta P_k||$, $|\partial_x \ln|\delta V_{{\rm PB},k}|| \ll |k|$; (ii) the rate of *spatial* variation of the cross phase is much smaller than $|k|$, i.e., $|\nabla\Theta_k| \ll |k|$; (iii) the poloidal component of $\vec{V}_{E\times B} \gg$ toroidal component. Approximations (i) and (ii) are proper for the H-mode state, where the inhomogeneities in $|\delta P_k|$, $|\delta V_{{\rm PB},k}|$ and Θ_k originate in the pedestal structure. Approximation (iii) applies to toroidally confined plasmas. Since $V_{E\times B}$ is differential rotation, we can reexpress the 3rd term on the left-hand side of Eq. (3) as $\vec{V}_{E\times B}\cdot\nabla\delta P = V'_{E\times B,y}\Delta x\partial_y\delta P$, where Δx measures the distance from the center of the envelope of δP. Δx can be estimated as the radial extent of δP. Substituting the Fourier representations of δP and $\delta V_{\rm PB}$ into Eq. (3) and using the approximations above yields the evolution equation for the cross phase Θ_k:

$$\frac{d}{dt}\Theta_k = k_y V'_{E\times B}\Delta x - \frac{|\delta V_{{\rm PB},kx}|}{|\delta P_k|}\langle P\rangle' \sin\Theta_k + \tilde{s}_k^{\Theta}, \qquad (4)$$

where $\tilde{s}_k^{\Theta}$ is the random phase scattering induced by the noise $\tilde{s}_k$. Equation (4) is just the Adler equation [16], and is also the mean field form of the Kuramoto model—the most representative model describing synchronization phenomena in populations of coupled oscillators [17]. The 1st term on the right-hand side of Eq. (4) is the winding effect due to shearing, which tends to modulate the cross phase between δP_k and $\delta V_{{\rm PB},k}$. The 2nd term acts as a pinning force. It is a nonlinear term and attracts the cross phase to a fixed value. $|\delta V_{{\rm PB},kx}|/|\delta P_k|$ is determined by the response function of the relevant mode (here, the PB mode). This factor is, in turn, determined by the structure of the PB mode, and the dependence upon the linear growth rate, $E\times B$ shearing, and the nonlinear saturation mechanism. Equation (4) provides a simple, straightforward way to capture the essence of the cross phase dynamics. Equation (4) is also a general equation for describing phase dynamics in systems with convective interaction, and so has broad applicability.

Focusing on the influence of flow shear on the cross phase dynamics, we first consider the scenario of no noise ($\tilde{s}_k = 0$). In this scenario, one has

$$\frac{d}{dt}\Theta_k = \frac{|\delta V_{{\rm PB},kx}|}{|\delta P_k|}\langle P\rangle'(K - \sin\Theta_k), \qquad (5)$$

with $K = k_y V'_{E\times B}\Delta x|\delta P_k|/(\langle P\rangle'|\delta V_{{\rm PB},kx}|)$. There exist two types of solutions of Eq. (5): one phase locked and the other the phase slip. The phase locked solution is

$$\Theta_k = \arcsin K, \quad \text{for } |K| < 1. \qquad (6)$$

Θ_k is "locked" to a stable fixed value (Fig. 2) and $|\Theta_k| < \pi/2$, so δP_k and $\delta V_{{\rm PB},k}$ stay coherent. Meanwhile, since the mean pressure profile stays in a quasisteady state before the crash, the thermal energy stored in the mean pressure profile is continuously extracted by PB mode-induced heat flux. The phase locked solution provides a robust route for thermal energy release. With locked phase, δP will grow large, leading to collapse of the edge pressure profile and the formation of filamentary structures [18]. This *violent* thermal energy eruption phenomenon corresponds to the so-called ELMy H mode. $V_{E\times B}$ shearing tends to stabilize the ELMy H mode via upshifting the value of $|\Theta_k|$, which in turn reduces the size of the ELM. Another factor impacting the size of the ELM is the spectrum structure of $\delta V_{{\rm PB},k}$. For a broad spectrum, the random scatterings among different PB modes tend to facilitate the formation of a state of PB turbulence, so that size of the induced ELM is reduced [6].

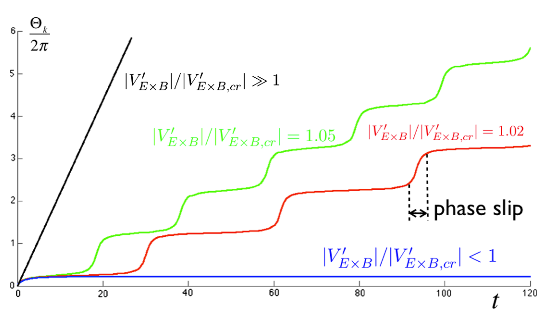

FIG. 2 (color online). Phase locked (blue plot) vs phase slip (green and red plots).

The phase slip solution can be cast in the following form

$$\Theta_k = \omega_k t + h(\omega_k t), \quad \text{for } |K| > 1, \qquad (7)$$

where $\omega_k = (|\delta V_{{\rm PB},kx}|/|\delta P_k|)|\langle P\rangle'|\sqrt{K^2-1}$ and $h(x)$ is a certain periodic function with period 2π, i.e., $h(x+2\pi) = h(x)$. The specific form of $h(x)$ [16] is

$$h(\omega_k t) = 2\tan^{-1}\left[\frac{1}{K} + \frac{\sqrt{K^2-1} - K}{K}\tan\frac{\omega_k}{2}t\right]. \qquad (8)$$

A very interesting property of the phase slip solution is that most of the time, $\Theta_k = 2n\pi + \pi/2$ (here we assume $k_y V'_{E\times B}\Delta x > 0$; n is a positive integer); i.e., δP_k and $\delta V_{{\rm PB},k}$ stay *out* of phase, except for short durations of the phase slip (Fig. 2). During the phase slips, the PB modes are pumped, *impulsively*. Since most time δP_k and $\delta V_{{\rm PB},k}$ are out of phase, the thermal energy tends to be released in small episodes and hence the H mode accesses a more quiescent state. The frequency of the phase slip in Eq. (7) is

$$\Omega_{\rm slip} = k_y V'_{E\times B}\Delta x \frac{\sqrt{K^2-1}}{K}. \qquad (9)$$

In contrast to the phase locked scenario, here $E\times B$ shearing tends to increase the frequency of the phase slips. That is, the phase slip occurs easier when $V'_{E\times B}$ becomes stronger (Fig. 2). The increase of the edge $E\times B$ shearing will improve the effectiveness of the QH mode for impurity control. In the strong shearing limit, $\sqrt{K^2-1}/K \to 1$, one has $\Omega_{\rm slip} \simeq \omega_k$. There, the cross phase evolves so that the QH mode supports a periodic oscillation with no bursts.

The critical $E\times B$ shearing rate, governing the evolution from phase locked state to phase slip state (i.e., the ELMy $\to Q$ evolution), is obtained by requiring $|\Theta_k| = \pi/2$, i.e.,

$$|V'_{E\times B,cr}| = \frac{1}{|k_y\Delta x|}\frac{|\delta V_{{\rm PB},kx}|}{|\delta P_k|}|\langle P\rangle'|. \qquad (10)$$

$|V'_{E\times B}| < |V'_{E\times B,cr}|$ corresponds to the phase locked state and $|V'_{E\times B}| > |V'_{E\times B,cr}|$ is the phase slip state. To obtain a more detailed scaling of the critical shearing, one needs to know the structure of $|\delta V_{\mathrm{PB},kx}|/|\delta P_k|$. In the linear approximation, one has $m_i n_i(\gamma_{\mathrm{PB}}\delta V_{\mathrm{PB},kx} + ik_y V'_{E\times B}\Delta x\, \delta V_{\mathrm{PB},kx}) = -j_{\mathrm{BS,k}}B_\theta - ik_x\delta P_k$, where j_{BS} is the perturbed bootstrap current, $j_{\mathrm{BS,k}} = -ik_x\epsilon^{1/2}\delta P_k/B_\theta$ (m_i is the ion's mass, n_i is the ion's density, γ_{PB} the linear growth rate of the PB mode, $\epsilon = a/R$ is the inverse aspect ratio of the tokamak, and B_θ, the strength of poloidal magnetic field) [19]. In the case of strong $E\times B$ shearing ($V'_{E\times B} > \gamma_{\mathrm{PB}}$), one obtains $|\delta V_{\mathrm{PB},kx}|/|\delta P_k| \simeq |(1-\epsilon^{1/2})k_x|/\sqrt{\gamma_{\mathrm{PB}}^2 + k_y^2 V'^2_{E\times B}\Delta x^2} \simeq |(1-\epsilon^{1/2})k_x|/|k_y V'_{E\times B}\Delta x|$. Substituting it into Eq. (10) yields the critical shearing scaling:

$$|V'_{E\times B,cr}| \simeq \tau_A^{-1}(1-\epsilon^{1/2})^{1/2}\frac{\beta^{1/2}}{|k_y\Delta x|}\left(\frac{L_P}{\Delta x}\right)^{1/2}, \qquad (11)$$

where L_P is defined as $L_P \equiv |\langle P\rangle|/|\langle P\rangle'|$ and $\tau_A = V_A/L_P$ is the Alfvén time across the edge with $V_A = B/\sqrt{m_i n_i}$ (B—the strength of total magnetic field). $\beta = 2\langle P\rangle/B^2$ is the plasma beta in the edge region. In deriving Eq. (11), the approximation $|k_x| \simeq 1/\Delta x$ was employed. Using the radial force balance relation for the ions $enE = -V_\phi B_\theta + \partial_x\langle P\rangle$ (V_ϕ the ion's toroidal rotation velocity; we have assumed ion's poloidal rotation in the H-mode pedestal is low [11]), one has $V'_{E\times B} = -V'_\phi B_\theta/B + (\langle P\rangle'/enB)'$. Therefore, there are two ways to facilitate accessing the QH mode: enhancing the steepness of the edge pressure profile (which requires more external power input) and increasing the toroidal rotation shear (which is more feasible in practice, currently).

The noise impacts the phase dynamics by introducing a random source in the phase equation. The cross phase exhibits different responses to the noise in the phase locked and phase slip states. An enlightening way to understand the noise effect is by using the "phase potential" concept [13]. In the phase locked scenario, the potential well has finite depth. To "kick" the cross phase out of the well, it requires the amplitude of the noise to reach a certain level, or else the cross phase only bounces around its fixed value (Fig. 3). In the phase slip scenario, the potential well is flattened, so even weak noise can induce phase slips (Fig. 3).

In the H-mode state, the level of the noise is bounded and relatively low, so for the phase locked sate, the cross phase keeps jumping around its locked value (blue plot in Fig. 4), which corresponds to small bursts in the heat flux. The random phase scattering induced by the noise becomes crucial when the ELM approaches its crash threshold, in which any tiny enhancement of the cross correlation may induce an ELM crash. For the phase slip scenario, the noise adds extra random phase slips to the coherent phase slips induced by the mean $E\times B$ shearing. As a result, the periodic phase slips are "smeared" by the noise and the QH

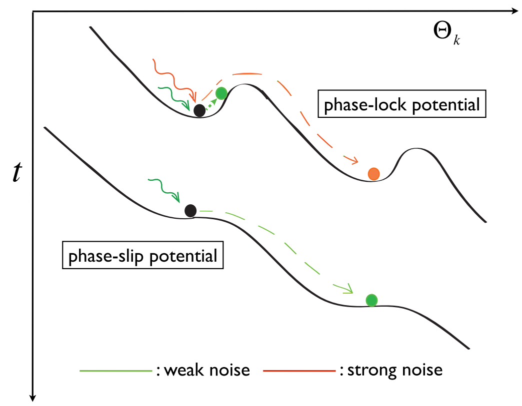

FIG. 3 (color online). Sketch of noise effects.

mode enters a state of weak MHD oscillations with a broad frequency spectrum (Fig. 4).

In summary, the phase dynamics concept is shown to be a useful framework for describing nonlinear MHD relaxation dynamics in the H mode, which is a near marginal, self-organized state. By studying the $E\times B$ shearing effects on the cross phase dynamics, we derive a physics-based scaling of the $E\times B$ shear strength required to access the QH mode. We show that if $|V'_{E\times B}| < |V'_{E\times B,cr}|$, the cross phase is locked to a fixed value and PB modes are continually pumped. There the thermal energy is released in large bursts with collapse of the edge pressure profile, so the so-called ELMy H mode occurs. If $|V'_{E\times B}| > |V'_{E\times B,cr}|$, δP and δV_{PB} are coupled only during periodic, short duration phase slips. The thermal energy is released during short episodes and a QH mode like is induced. The periodic phase slips can be interpreted as the edge harmonic oscillation phenomenon, observed during the QH mode [10]. The noise is benign for H-mode operation. In the phase locked scenario, the noise tends to reduce the coherence between δP and δV_{PB}, and hence reduce the size of the ELM. In the phase slip scenario, the noise can increase the phase slip frequency, and hence make for an attractive quasicontinuous "grassy ELM" state, which efficiently expels impurities. The theoretical framework proposed in this Letter unifies the treatment of several different effects, such as $E\times B$ shearing (relevant to

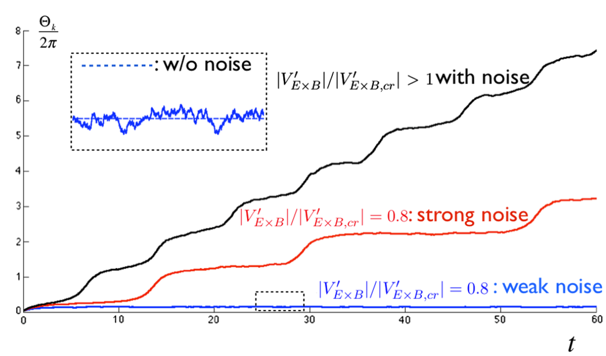

FIG. 4 (color online). Noise effects on cross phase dynamics.

coherent phase dynamics) and noise-mode couplings (relevant to stochastic phase dynamics [6]), on the phase dynamics. In this work, we assumed that, before the crash, the mean pressure profile is in a quasisteady state. For future work, it is important to construct a flux driven system and investigate the feedback dynamics between the edge pressure profile and the cross phase.

We are grateful to P. W. Xi and X. Q. Xu for useful discussions. We thank Peking University and Huazhong University of Science and Technology (HUST), where part of the work was done, for their hospitality. We acknowledge fruitful interactions at the Festival de Théorie, Aix-en-Provence. This work was supported by the World Class Institute (WCI) Program of the National Research Foundation of Korea funded by the Ministry of Education, Science and Technology of Korea [WCI 2009-001], the Department of Energy under Award No. DE-FG02-04ER54738 and CMTFO.

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