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DESIGN OF MORTGAGE INSTRUMENTS

BY

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AND

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**INTEREST RATE RISK AND
OPTIMAL DESIGN OF MORTGAGE INSTRUMENTS**

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Introduction

Recent years of rapidly changing inflation rates and generally uncertain economic conditions have been accompanied by fluctuating interest rates. These volatile conditions have changed the financial environment in the real estate market. The problems of residential mortgage finance caused by changes in inflation should be viewed in the broader context of optimal mortgage design. Even in a disinflationary or deflationary world, the standard periodic fully-amortized mortgage with constant payments would likely be less than optimal because it would allocate risks of interest rate changes between borrower and lender in fixed proportions (0 percent and 100 percent, respectively), and would not allow any tailoring of the borrower's expected payment stream to his expected income stream. Changes in inflation raise the social costs of these rigidities, thereby creating incentives for developing alternatives to the standard mortgage. The danger in the search process for alternative mortgage instruments is that they, too, may be suited for special circumstances, and may not be based on sufficiently general considerations. Clearly, a less pragmatic, more theoretical approach is needed. The existing literature of new lending instruments neglects two fundamental areas. First, there is a scant supply of micro economic analyses for mortgage instrument choice and mortgage instrument pricing.^{1,2} Second, there are surprisingly few empirical studies of mortgage choice behavior.³ This paper will concern itself with the first of these alleged "omissions." In Section I, we develop and analyze a simple expected utility model for interest rate risk and mortgage choice. In Section II, we suggest several extensions for future research.

I. Micro-Foundations for Mortgage Design

A. *The Household-Borrower: Basic Model*

This sub-section analyzes the effects of interest rate risk on household decision-making. It will show that the optimal mortgage for a utility-maximizing borrower is one with varying periodic payments that reflects changes in interest rates, rather than the conventional periodic fixed-payment fully-amortized mortgage. The former mortgage will be referred to as a variable-payment mortgage (VPM) and the latter as a fixed-payment mortgage (FPM).

We will determine the household's desired optimal interest rate risk exposure in its mortgage contract. We assume that the household-borrower is an expected utility-maximizer satisfying the Von Neumann-Morgenstern postulates. For analytical convenience, we separate the mortgage contract interest rate into an independent expectational portion, r , which is assumed to be unchanging over time; and an independent pure interest rate risk portion, \tilde{s} , which is assumed to be random over time with a subjectively known distribution. Conceptually, r is the required rate of return for the given level of prepayment, default and illiquidity risk inherent in the mortgage loan, while \tilde{s} is the required rate of return for inflation risk.⁴ For a given mortgage loan size and term to maturity, the periodic payments could be subdivided into two corresponding portions: $M(r)$, the constant intrinsic risk periodic mortgage payment portion, and $X(\tilde{s})$, the random interest risk periodic mortgage payment portion depending on the realized value of \tilde{s} .

Under an FPM, abstracting from differences in administrative and servicing costs, the periodic payment by the borrower over the life of the mortgage is constant and equal to $M + P > 0$, where $P > 0$ is a "risk loading" charge by the lender for bearing all interest rate risk. Usually, the risk premium would be greater than the expected value of the risk because of the lender's assumed risk aversion; that is, $P > E(X)$. Of course, $E(X) \lesseqgtr 0$, depending on the distribution of X .

Under an unrestricted VPM, the periodic payments would change precisely with interest rate changes. We assume that $M + X \geq 0$. Note that $-K \leq X < \infty$; since r must be non-negative, the value of $-K$ is derived from the minimum possible mortgage payment per period, $M(0) = M(r) - K$. In principle, abstracting from changes in prepayment and default risks, the lender does not charge a risk premium under an unrestricted VPM because he does not bear interest rate risk.

Let T be the total periodic mortgage payment which is a function of the mortgage contract. In our analysis,

$$T = M + X + P - R,$$

where R is the implicit contingent net effective amount paid by the lender to the borrower because of interest rate changes. R represents the risk-sharing clause between

borrower and lender in the mortgage contract. It is related to the amount of interest rate risk not borne by the household (i.e., the amount of risk borne by the lender). Under an FPM, $R = X$; and, for an unrestricted VPM, $P = 0$ and $R = 0$.

For simplicity, we assume that the household owns its home and incurs a mortgage payment, T , as its only cost of housing services. For a given, fixed expected stream of gross income, Y , once the household chooses its housing consumption and mortgage payment scheme, T , its utility depends upon the household's expected *net* income after mortgage payment, I .⁵ The household's ordinal utility index is $U(I)$, where $I = Y - T$.

The household negotiates a mortgage, explicitly deciding on the proportion of the interest rate risk it will be responsible to pay, conditional on interest rates. The borrower obtains reimbursement, R , for each X in the following way:

$$R = \lambda X \quad \text{for } 0 \leq \lambda \leq 1, \quad (1-a)$$

where λ is the proportion of the interest rate mortgage payment risk borne by the lender; and $(1 - \lambda)$ is the risk portion endured by the borrower.

The lender's fee for bearing the interest rate risk is

$$P = \lambda[E(X) + aM], \quad (1-b)$$

where $a\lambda M$, with $a > 0$, is the loading charge by the lender above the fair actuarial value of the interest rate risk.

B. Optimality Conditions

The household maximizes expected utility, $E(U(I))$, subject to conditions (1-a), (1-b) and $0 \leq \lambda \leq 1$. The first two derivatives are:

$$\frac{dE(U(I))}{d\lambda} = E[U'(I)(X - aM - E(X))], \quad (2)$$

$$\frac{d^2 E(U(I))}{d\lambda^2} = E[U''(I)(X - aM - E(X))^2]. \quad (3)$$

Equation (3), the second derivative of the expected utility function, is negative because the borrower is risk averse, which assures a unique maximum over the domain of λ .

We explore conditions for taking *full* protection against interest rate risk (i.e., $\lambda = 1$). It is necessary and sufficient that the first derivative of the expected utility function, equation (2), at $\lambda = 1$ be non-negative, as shown in equation (4):

$$\left. \frac{dE(U(I))}{d\lambda} \right|_{\lambda=1} = (-aM)U'[Y - (1+a)M - E(X)] \geq 0. \quad (4)$$

Since $U' > 0$ and $M > 0$, the inequality in condition (4) will hold if and only if $a \leq 0$; the risk loading offered by lenders needs to be *at least* actuarially fair. If the risk payment were actuarially unfavorable, it would never be optimal for the household to take full protection against interest rate risk. Given that lending institutions are likely to be risk-averse, the lender's interest rate risk charges are not likely to be actuarially fair.

The conditions for not taking interest rate risk protection at all (i.e., $\lambda = 0$) would occur if the value of equation (2) were non-positive for all λ . Unfortunately, this condition does not present any clear interpretation. Therefore, we assume utility maximization for an interior solution; equation (2) equals zero for $0 < \lambda < 1$. In such circumstances, the rearrangement of equation (2) yields

$$E[U'(I)X] - E[U'(I)(aM + E(X))] = 0. \quad (5)$$

Equation (5) signifies that the household equilibrium occurs when the loss in marginal expected utility from an increased interest rate risk protection payment is balanced against the gain in marginal expected utility from additional protection against interest rate risk. $[aM + E(X) - X]$ is the net payment received by the lender for additional protection against interest rate risk. The loss of marginal expected utility of this amount relates to the expected payout by the household to the lender. Therefore, protection against interest rate risk is optimized when the net marginal expected utility gain is zero. Following this interpretation, it is plausible to conclude that full protection against interest rate risks, as a normative guideline, is likely to be sub-optimal for the household.

C. Optimal Choice as a Function of Income and Loan Size

For a given level of housing services, if the household has decreasing Pratt–Arrow absolute risk aversion, the optimal value of λ will decrease as income increases. By differentiating the household equilibrium condition in equation (5), we have

$$\frac{\partial \lambda}{\partial I} = \frac{-E[U''(I)(X - aM - E(X))]}{E[U''(I)(X - aM - E(X))^2]} < 0. \quad (6)$$

Since the denominator of equation (6) is always negative, the effect of income on λ (the proportion of interest rate risk protection) will depend upon the sign of the expected value term in the numerator. This is negative, proving our claim $\frac{\partial \lambda}{\partial I} < 0$. To see this, rewrite net income as

$$I = Y - M - E(X) - aM - (1 - \lambda)(X - aM - E(X)).$$

For $\lambda = 1$, when the household has an FPM, net income will be

$$I_{\lambda=1} = Y - M - E(X) - aM.$$

There are two possibilities for $[X - aM - E(X)]$. First, we assume

$$[X - aM - E(X)] > 0,$$

and

$$0 < \lambda < 1.$$

Then, it follows that

$$I_{\lambda=1} \geq I_{\lambda \neq 1} = I.$$

By the definition of risk aversion, we have

$$\frac{-U''(I)}{U'(I)} \geq \frac{-U''(I|\lambda = 1)}{U'(I|\lambda = 1)}. \quad (7)$$

If we multiply both sides of equation (7) by equation (8),

$$-U'(I)(X - aM - E(X)) \leq 0, \quad (8)$$

we will derive equation (9):

$$U''(I)(X - aM - E(X)) \leq \frac{U''(I|\lambda = 1)}{U'(I|\lambda = 1)}U'(I)(X - aM - E(X)). \quad (9)$$

Second, the inequality in equation (9) does not change if we were to assume that $[X - aM - E(X)] < 0$, because both inequalities in conditions (7) and (8) reverse to offset each other. Applying the expected value operator to equation (9) will produce the following condition:

$$E[U''(I)(X - aM - E(X))] \leq \frac{U''(I|\lambda = 1)}{U'(I|\lambda = 1)}E[U'(I)(X - aM - E(X))].$$

The left-hand side is the numerator of equation (6). $E[U'(I)(X - aM - E(X))]$ will be zero for a household in equilibrium, equation (5), in which case the numerator in equation (6) will be negative for $0 < \lambda < 1$, proving our claim that λ decreases with increasing income.

We consider how λ changes as the loan size, M , increases, with gross income Y held constant. By differentiating the household equilibrium condition, equation (5), we find that

$$\frac{\partial \lambda}{\partial M} = \frac{-(1 + \lambda)E[U''(I)(X - aM - E(X))] - aE[U'(I)]}{-E[U''(I)(X - aM - E(X))^2]}.$$

The denominator is positive because $U'' < 0$. The numerator's sign depends upon the balancing of the first term, which, as has just been demonstrated, is positive, and the second term, which is negative. Thus, the sign of the effect of M on λ is *a priori* indeterminate. Since a is the risk premium loading charge, the second term in the numerator

represents the loss in expected utility for the household by purchasing additional risk protection (i.e., increasing λ). This is balanced against the change in the marginal utility for a risk-averse household that is reducing risk by purchasing risk protection. Alternatively, the numerator can be rearranged as follows:

$$E \left[(1 + \lambda a) \frac{[-U''(I)(X - aM - E(X))]}{E[U'(I)]} - a \right] \bullet E[U'(I)].$$

For a given gross income, by examining the first expectation operator, λ will increase as M increases if the household's risk aversion multiplied by the change in expected costs of increasing its mortgage risk protection ($1 + \lambda a$) is greater than the risk premium, a , charged by lenders. In general, this would seem to be true. As would be expected, the incentive to protect against interest rate risks will be greater for households with relatively large mortgage burdens for their income level.

D. Interest Rate Risk Deductibility Clauses in Mortgage Contracts: The Payment CAP

It has become commonplace for adjustable rate mortgages to contain a CAP (or, as we characterize it, a deductible). The household-borrower protects itself against interest rate risk by the use of a deductibility clause. Under such a mortgage contract, there is an amount D , the deductible, such that $|X| \leq |D|$ is translated into changes in mortgage payments. X above the deductible, $|X| > |D|$, does not affect further the borrower's mortgage payment obligations. The reimbursement received by the household for a particular X will be

$$R = \begin{cases} X + D & \text{for } X < -D \\ 0 & \text{for } -D \leq X \leq D \\ X - D & \text{for } X > D. \end{cases}$$

As before, R is the net effective amount paid (or received) by the lender because of interest rate changes for each X . The risk premium charged by the lender is assumed to be a function of D such that

$$E(R) = \int_{-K}^{-D} (X + D)f(X)dX + \int_D^{\infty} (X - D)f(X)dX.$$

In this case, the household's expected utility function will be

$$\begin{aligned}
E(U(I)) &= U(Y - M - P(D) + D) \int_{-K}^{-D} f(X) dX \\
&\quad + \int_{-D}^{+D} U(Y - M - P(D) - X) f(X) dX \\
&\quad + U(Y - M - P(D) - D) \int_D^{\infty} f(X) dX,
\end{aligned} \tag{10}$$

where $I = Y - M - P(D) - X + R$, and $P(D)$ is the premium charged as a function of the deductible.

The household's optimum D is found by maximizing equation (10). The first two derivatives of the expected utility function with respect to the deductible are

$$\begin{aligned}
\frac{dE(U(I))}{dD} &= \\
&U'(Y - M - P + D) \int_{-K}^{-D} f(X) dX \left(1 - \int_{-K}^{-D} f(X) dX + \int_D^{\infty} f(X) dX - P'(D)\right) \\
&+ \int_{-D}^{+D} U'(Y - M - P - X) f(X) dX \left(-\int_{-K}^{-D} f(X) dX + \int_D^{\infty} f(X) dX - P'(D)\right) \\
&+ U'(Y - M - P - D) \int_D^{\infty} f(X) dX \left(-1 - \int_{-K}^{-D} f(X) dX - \int_D^{\infty} f(X) dX - P'(D)\right)
\end{aligned} \tag{11}$$

and

$$\begin{aligned}
\frac{d^2 E(U(I))}{dD^2} &= \\
&U''(Y - M - P + D) \int_{-K}^{-D} f(X) dX \left(1 - \int_{-K}^{-D} f(X) dX + \int_D^{\infty} f(X) dX - P'(D)\right)^2 \\
&- U'(Y - M - P + D) \int_{-K}^{-D} f(X) dX (f(D) + P''(D)) \\
&- \int_{-D}^{+D} U'(Y - M - P - X) f(X) dX (f(D) - f(-D) + P''(D)) \\
&+ U''(Y - M - P - D) \int_D^{\infty} f(X) dX \left(-1 - \int_{-K}^{-D} f(X) dX + \int_D^{\infty} f(X) dX - P'(D)\right)^2 \\
&- U'(Y - M - P - D) \int_D^{\infty} f(X) dX (1 - f(D) + P''(D)).
\end{aligned} \tag{12}$$

Solving equation (11) for $D = 0$, the first derivative of the expected utility function becomes

$$\left. \frac{dE(U(I))}{dD} \right|_{D=0} = -U'(Y - M - P(0))P'(0) > 0$$

under the assumption that $P'(D) < 0$ for all D . For the typical household, we conclude that some positive deductible $D > 0$, for any positive loading charge $P(D)$, will be optimal. This result suggests that the optimal design of adjustable rate mortgages should include a CAP provision.

The second derivative, equation (12), of the expected utility function is not necessarily negative everywhere on the domain of D , although it is likely to be negative for our assumptions about P and for $U' \geq 0$, if additionally it is assumed that

$$\int_{-K}^{-D} f(X)dX = \int_D^{\infty} f(X)dX \text{ for all } D.$$

In this case, equation (12) will be negative, thereby assuring second-order conditions for maximization. If the second-order conditions for expected utility maximization were subsumed, the expected utility function, equation (10), has two possible shapes as a function of D . First, its range might be increasing over its entire domain. Then, the optimal D will approach infinity at its upper limit and $-K$ at the lower limit, meaning that no interest rate risk protection is optimal. Second, $E[U(I)]$ might achieve a maximum for some finite D , which is the optimal interest rate change payment deductible. In such a case, the optimal D , $0 < D < \infty$, will engender a unique expected utility maximum.

II. A Focus for Extending Mortgage Instrument Design Research

Future research should extend our model in several ways. First, only limits of imagination appear to constrain the types of new alternative mortgages that can be concocted; and there exist a growing family of active alternative mortgage programs that vary in complexity, creativity, and intended beneficiary target groups.⁶ We believe future research should be directed to integrating at least four parameters into a theory of optimal mortgage design. They are

1. The proportion of interest rate changes (or other index) borne by the lender and borrower;

2. The minimum index change required to activate risk-sharing;
3. The maximum risk exposure caused by index changes (the risk exposure limits);
and
4. The amortization-graduation schedule for loan repayments.

Second, an extended model of optimal mortgage design should incorporate mortgage termination features (i.e., prepayments or defaults). The analysis for mortgage termination should take into account that household income and housing market values are stochastic, and are likely to be affected by the same economic forces that determine housing finance conditions. Typically, a household's income is the key variable for determining a household's capability to meet periodic mortgage payments; and changes in property values affect a household's equity and the lender's collateral. Interest rate changes in conjunction with changes in household income and property values affect mortgage terminations, sometimes in complex ways. If interest rate increases tend to accompany falling household incomes and property values, the probabilities of mortgage defaults will tend to increase. In contrast, if decreasing interest rates are associated with rising incomes and property values, earlier than expected mortgage prepayments are likely to occur.

Third, some researchers and policy analysts claim that the U.S. tax system plays consequential role in home ownership and mortgage financing decisions. Extensions of our mortgage choice models should incorporate explicitly tax effects.⁷

Footnotes

1. Noteworthy exceptions to the general lack of theoretical analyses of alternative mortgage designs are Findlay and Capozza (1977), Asay (1977), Van Order (1981), Barney and White (1986), and Smith (1987).
2. See Hendershott (1986), and Hendershott and Van Order (1987). These articles review the burgeoning use of option pricing models to evaluate mortgage instruments.
3. Recent empirical analyses of how mortgage value and choice are affected by mortgage design features include Lea (1985) and Brueckner and Follain (1988).
4. Our analysis is a single-period utility maximization model that abstracts from prepayment and default risks, and focuses upon payment risks. Obviously, interest rate changes affect directly or indirectly all of these risks.
5. Our analysis does not consider the simultaneous effects of a stochastic income stream and stochastic mortgage payments; we also do *not* discuss tax effects.
6. See Brueggeman, Fisher and Stone (1989, Chs. 6 and 9) for a catalog of existing alternative mortgage types.
7. Assume that household income is taxed at a rate τ , and that mortgage interest is totally tax deductible. If ρ is the portion of mortgage debt service, M , that is tax deductible, and P and X are assumed to be treated as tax deductible, then *after tax* income (I) will become

$$\begin{aligned}
 I &= (1-t)Y - (1-t)(1-\rho)M - \rho M - (1-t)X \\
 &\quad - (1-t)\lambda(E(X) + aM) + (1-t)\lambda X.
 \end{aligned}$$

The first-order condition for optimal risk bearing (analogous to equation (2) above) will be

$$\begin{aligned}
 \frac{\partial E(U(I))}{\partial \lambda} &= E [(U'(I))(1-t)(X - E(X) - aM)] \\
 &= 0.
 \end{aligned}$$

Comparing the above equation with equation (2) suggests that the optimal λ under our assumptions will be unaffected by a proportional tax. This is true because the introduction of taxes causes the government to share interest rate risk: If mortgage payments rise, the borrower's increased interest payments will be partially offset by tax saving, and if mortgage payments fall, the loss of interest deductible expenses creates offsetting tax increases. It is, also, a straight-forward application of our analysis to show that, for plausible values of the variables, increases in tax rates will cause borrowers to increase the amount of interest rate risk they will bear

$$\frac{\partial \lambda}{\partial t} = \frac{E[(U''(I))(-Y + (1 - \rho)M + X - \lambda(X - E(X) - aM))]}{[(U''(I))((1 - t)^2(X - E(X) - aM)^2)]} < 0.$$

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