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CLASSROOM NOTE

The Botetano arithmetic method: introduction and early evidence*

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ABSTRACT

In Peru, national assessments repeatedly rank Indigenous mathematics students as the lowest performing across the entirety of Latin America and South America. Whereas lack of financial resources often predicts low measures, the history of educational practice teaches us that students' poverty need not predict their educational outcomes – creative instructional approaches may turn the tables. Here we report on an innovative, body-based arithmetic technique, the Botetano Method, that has been enabling poverty rural children from remote mountainous regions of Peru to match and even greatly surpass their urban peers on comparable test items. The article explains the method's guiding humanistic and cognitive principles and then reports on findings from explorative action research that implemented and evaluated the method. Using observational methodologies, we argue that the students developed in their conceptual understanding of the content as well as in their attraction to the discipline, their professional identity, their personal pride in their achievement, and their general epistemic capacity for concentration and self-regulation. Throughout, we emphasize the methodological limitations of this grassroots proof-of-concept action research, which threaten the validity of the assertions. We speculate on early extensions of the method to literacy studies.

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1. Introduction

Let's begin by pointing out that there are several ways to learn, be it mathematics or language. One way, which we might call *traditional*, is characterized by repetition and memorization and is centred on symbolic notations. Children who study mathematics in traditional curricula learn to enact algorithms, such as inverting and multiplying a fraction divisor, but they are liable to develop little to no conceptual understanding of these technical procedures. Similarly, children studying language arts may become 'functional

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*Unless otherwise stated explicitly, the pedagogical resources reported in this article, including material and electronic artifacts, diagrammatic formats, and illustrative images as well as oral routines, interaction techniques, and activities, all remain the intellectual property of the first author, Cesar Botetano. Mr. Botetano wishes to thank the students who participated in the evaluation studies as well as Teachers Gala Chura, Carlota Huamán, and Ena Botetano. Dor Abrahamson compiled and edited Mr. Botetano's texts and figures for publication and offered perspectives from the learning sciences.

illiterates', that is, they know how to read, but they do not understand what they read – they are unable to explain in their own words what they have read and yet less to summarize the contents adequately. In addition to low conceptual gains, traditional instruction may lead to anxiety, low engagement, little interest, and lack of disciplinary identity (Freire, 1968/1973; Illich, 1971; Lockhart, 2009; Thompson, 2013; Turkle & Papert, 1991). What is common to these examples from mathematics and literacy is an early, *perhaps premature*, introduction of formal symbolic systems into the pedagogical regimen. Is there another way?

In Indigenous methodology, routines involving quantitative reckoning are tightly tethered to the practices they serve – counting and calculation are situated in the objects they treat, so much so that the Quechua people use different sets of words to enumerate sheep and corn leaves (Urton, 1997). In industrialized techno-scientific society, on the other hand, mathematical activity quickly abstracts away from the situations it serves, even to constitute an intact self-referential practice, where worldly matters at best serve as application contexts.

The pragmatic advantages of a pan-situational formal mathematics couched in decontextualized semiotic systems are colossally evident: dealing with symbolic notations, one operates in pristine logical edifices unmarred by their quotidian analogs; one may thus arrive at universal timeless truisms that could obtain in the case of llamas, beans, or rockets. Surely such generative power is a marvel of civilization. At the same time, though, the inherent epistemic and engineering potency of formalisms need not bear hasty pedagogical implications – mathematics education may not ideally begin from these formalisms (Nathan, 2012). Instead, we believe, there may be cognitive-developmental advantages in sustaining mathematics as situated, material, and embodied for quite a while longer along the developmental process, before moving into symbols.

The thesis of this article is that an embodied instructional approach is better suited for grounding mathematical knowledge in cognitively ergonomic actions (Abrahamson, 2009, 2014, 2019). Specifically, children who initially encounter mathematical notions as non-symbolic perceptuomotor routines are eventually able to reason in forms that appear to be better adapted for the multimodal perceptual and cognitive capacity of our species' neural architecture (cf. Fischer et al., 2011; Gigerenzer et al., 1988; Glenberg et al., 2012; Möhring et al., 2016). By way of analogy, our embodied mind is better suited for solving the Rubik's Cube manually on the cube itself than formally through diagrams and symbols on paper, which refer to the cube's elements and transformations (cf. Kirsh, 2013). Granted, early education generally launches mathematics instruction with the body, such as through finger counting. Yet we speaking about continuing with the body and material tokens for quite a while, before introducing symbols (Abrahamson, *in press*). That is the phylogenetic evolution of mathematics (Ifrah, 1981/2000), and so ontogenetic development might need more time for children to reconstruct this pre-historical process.

This modest research paper is about an alternative to the traditional approach. This approach, the *Botetano Method*, is designed to implement two defining maxims: (1) maximizing children's mathematical learning; and (2) fostering children's enjoyment of, confidence in, and attraction to mathematics. The Botetano Method offers efficient techniques for bringing primary-school students to mastery of the four basic arithmetic operations (addition, subtraction, multiplication, and division). As such, the method saves prodigiously on teacher-hours. For example, children learn the 20-by-20 multiplications

in less than 10 days. Moreover, applying the method does not require any sophisticated or expensive educational materials – not even paper-and-pencil or chalk-and-blackboard – and can be implemented in humble environments, even sitting on the floor. In sum, the Botetano Method aspires to offer a fast, practical, cheap, effective, and efficient way of achieving a set of universally desired pedagogical and institutional goals.

The Botetano Method aspires to meet the following set of principles:

- (1) Mathematics activities are experienced as playful.
- (2) The child is never punished or penalized.
- (3) No memorization is required: you learn not the facts but how to get them.
- (4) Once a child learns the method, they will never forget how to use it.
- (5) All work is mental – no writing, no symbols.
- (6) Students are constantly engaged in cognitive work. With time, their concentration and retention improve. These abilities transfer to other mathematics activities and even to other subjects or courses.
- (7) Once they have learned the method, children greatly increase their self-esteem, because they realize that they are capable of surpassing even high-school children in mental calculation.
- (8) A new dynamic is generated between parents and children. Parents feel happy and proud of their children's achievements over a very short time, and children, in turn, are stimulated by the admiration they arouse among their parents and elders.

The goal of this article is to explain the method and results from an early empirical evaluation. It was not a classical empirical study let alone an experimental study. The author is by no means an academician but, rather, an educator and innovator. One could think of our investigations as modest forms of action research.¹

2. General design problem: the enduring achievement gap between indigenous and non-indigenous mathematics students

There are over than 4 million Indigenous persons in Peru (83% Quechua, 11% Aymara, 1.67% Ashaninka, and 4.31% other Amazonian Indigenous peoples). According to the OECD 2010, 73.5% of Peruvian children tested at, or below, the lowest level for math proficiency in the 2009 PISA learning assessments of 15-year-old children in 62 countries. For Indigenous children, the predicament is worse: their test scores are among the lowest in all of Latin and South America (including Central America and Mexico).

Numerous reasons have been put forth to explain the low scores of Indigenous Peruvian children on these comparison tests: (1) parental low education; (2) poor quality of instructional practices in their schools; (3) lack of instructional resources in their schools; (4) linguistic challenges – Indigenous children who are non-Spanish speaking must study in Spanish; (5) students' compromised health due to malnutrition; (6) students' socioeconomic factors; and (6) students' genetic endowment [*sic*]. All these structural discriminations and racist diagnoses have led to a popular perception of Indigenous students as endemically symptomatic of low scholastic performance.

Peruvian governmental officials recognize the importance of addressing the educational gap between its Indigenous and Non-Indigenous children. However, a syndrome

of political and bureaucratic obstacles has delimited the government's ability to institute policy changes that would reduce the achievement gap. In the 1970s, the Ministry of Education initiated a National Policy of Bilingual Education and created the National Division of Intercultural and Bilingual Education (DINEBI), charged with finding better ways to educate Indigenous people. Since then, few meaningful advances have been made, largely because the DINEBI has had to compete for resources with other national priorities and with the agendas of the larger and more powerful Ministry of Education and the Vice Ministries of Educational Management and Institutional Management. For example, the DINEBI still does not have the capacity to hire new teachers or to approve a bilingual curriculum for the future (Castillo, 2008). Peru has had some success creating intercultural bilingual schools, but according to the Peruvian Ministry of Education, in 2008 only 35% of the rural bilingual population attended an intercultural bilingual school.

Thus, whereas Peru has undergone several educational reforms over the past 80 years, these have not been effective in changing our international rankings. One can only wryly muse that more effective measures would have been taken if the gap were reversed and Non-Indigenous students were performing lower than Indigenous students. Is that even possible?

3. Proposed design solution: practicing the Botetano Method

The Botetano Method is premised on a visionary rethinking of mathematics didactics. What if we abandoned the parameters of traditional mathematics teaching, which is based on repetition, memorization, and the absence of reasoning? We are proposing a method based on reasoning, play, and intensive use of the senses and the body, in particular the fingers. In this method, we depart from paper- and symbol-based algorithms and rote memorization of 'number facts', instead turning to body-based and, initially, symbol-less procedures with no fact memorization. Our pedagogy lays mathematical foundations first through manual, mental, and oral work and only later consolidates these foundations upon symbolic notation that is first perceived visually and only later inscribed. Studying mathematics in this alternative pedagogical regimen, Indigenous children have harnessed their potential and thrived.

Our method is divided into stages. It is inspired and structurally modelled on schematic conceptions of human phylogeny and ontogeny. Given prevalent cultural practices, it is highly plausible to assume that our human species' primitive ancestors counted and calculated using their hands, their fingers (Bender & Beller, 2012; Saxe, 1981). This paper will discuss elements of our curriculum corresponding to that evolutionary stage.

When we teach, at first we do not use symbolic notations at all, only concrete representations of cardinal sets (e.g. coloured bottlecaps, see Figure 1) followed by iconic representations (e.g. dice). Only once students have mastered the curriculum's arithmetic objectives using these materials, do we introduce symbolic representations (e.g. cards). Still then, the symbolic representations act as prompts only – students do not inscribe these symbols. Thus, the entire process is oral and mental. Nothing is written. That practice comes later.

Before transitioning from counting to arithmetic, students work with a display we have devised (see Figure 2). This diagrammatic-symbolic board offers a visual-spatial representation of numbers from 1 to 1000. The representation is designed to highlight the

Figure 1. Available artifacts serve as counting tokens.

Figure 2. Counting by 1s, 10s, 100s, and 1000s. To perform addition, students place tokens, such as buttons or bottle caps, on the board, to represent the addends, and then they ‘step’ the tokens clockwise to perform addition operations, as one would walk a pawn in a board game. Tokens arriving at a 9×10^x ellipse hop to the $1 \times 10^{x+1}$ wedge.

uniform logical structure of the place-value system whereby the counting ‘poem’ at any order of magnitude is identical – one_, two_, three_, etc. – only that we enumerate sets that grow tenfold. The display features four identical diagrammatic structures for the ones (bottom-right), tens (bottom-left), hundreds (top-left), and thousands (top-right). In each structure, the count begins at the same location (top-left) and then steps clockwise, with eight numbers in the dial and the last one in the centre, foregrounding 9×10^1 , 9×10^2 , 9×10^3 , 9×10^4 .

The Botetano Method number structure, which the author believes to be unique, is an innovative didactic response to the pragmatic need of supporting students’ cognitive transition to the standard place-value system, and this transition transpires through performing addition operations. For example, when performing $9 + 7$, ‘9’ is the last possible

number in the first diagram (1-through-9), whereas the 10 is already in the next diagram (10-through-90). This way, the 10 is not duplicated but, instead, is a step into the next order of magnitude. Thus, when $9 + 7$ is reconstituted as $10 + 6$, the '10' appears in the second diagrammatic structure of 10-through-90. Later, this '10' will be equivalent to a '1' in '16'. Again, the sequence of diagrams forges epistemic bridges between oral counting and the place-value system. In the standard place-value system, the value of a digit is inferred by its left/right location relative to the other digits. In the Botetano didactic transition table, we still keep the zeros but we represent the next order of magnitude with an identical diagrammatic-symbolic template at a different spatial location relative to the other structures. All these operations are performed in a game-like context, by placing tokens on the board to represent the addends and then 'walking' them step-by-step, as one would walk a pawn on a board game, to determine the sum.

The number board thus serves students in understanding the place-value system. The board is important, because the finger formations that the students will soon learn to perform as their means of conducting arithmetic operations tend to be complex three-dimensional bimanual morphologies that are difficult to comprehend or pin down as an inter-related numerical system. The board 'grounds' these finger shapes and actions into a patterned perceptual structure, so that students can appreciate the comprehensive and systematic logic of the arithmetic operations they are learning to carry out. Through practice, our observations suggest, students come to visualize their finger counting and arithmetic as mentally projected onto this board. In time, the actual board fades out of students' working space, like the Cheshire Cat's smile, to remain a shared imagined object of multimodal discursive reference in their embodied space – a realistic array for plotting the choreography of thinking into the immediate gestured peripersonal space (Fuson & Abrahamson, 2005). That is, the imagined board lives in front of the students, on an imaginary 'canvas' in their mind's eye (visuospatial sketchpad, Baddeley, 1992), so that the students can continue referring to this present-absent diagrammatic display even without the actual material board in front of them. The board thus functions as what cognitive scientists call an *epistemic form* (Collins & Ferguson, 1993) or *material anchor for a conceptual blend* (Hutchins, 2005); what sociocultural theorists call a *cultural tool* – 'embodiments of certain cultural practices, as crystalized templates of action and schematized representations of ways of doing things as discovered in the history of human civilization' (Stetsenko, 2002, p. 129). The board is an enacted interface between embodied action and inscriptional forms.

Finally, students engage in intensive practice of addition and multiplication, using numbered cards (see Figure 3) and dice. Subtraction is introduced as a form of addition, that is, as adding up from the smaller number to reach the larger number. Analogically, division is introduced as the complement of multiplication, where one determines the quotient by searching for the multiplier of the divisor that would yield the dividend. Eventually, children no longer require the numbered cards, so that carrying out the addition and subtraction operations is completely imaginarily. Still, as they transition to imaginary actions, students are welcome to revert to the cards whenever they require support.

Whereas the Botetano Method encompasses all four basic arithmetic operations, this article will focus on multiplication, by way of demonstration. In this method, students first master the 12-by-12 table, then continue to 20-by-20. Once a child is able to multiply up to 20-by-20, extending through to 100-by-100 is only a question of further mental concentration and intensive practice. We will now overview of the method itself.

Figure 3. Sample cards for prompting arithmetic operations.

4. A detailed description of the Botetano Method for learning addition and multiplication

MULTIPLICATION WITH PHALANX- TABLE FROM 2 TO 10

- (1) Note the phalanges. Each phalanx is worth 1. (We will consider the thumb as having three phalanges, even though anatomically there are only two)
- (2) There are 15 phalanges in one hand and 15 in the other, for a total of 30.

TABLE OF 2

Example 1: Multiply 3×2 or 2×3

Explanation:

Fingers (multiplier)	Phalanxes (multiplicand)
3	2

To solve the operation:

- (1) Use 3 fingers, on each finger we use 2 phalanxes.
- (2) Mark with a pen a dot on each of these phalanxes.
- (3) Count the marked points marked: It gives us 6. Thus, $3 \times 2 = 6$

Example 2: Multiply 5×2 or 2×5

Explanation:

Fingers	Phalanxes
5	2

- (1) Use 5 fingers, where on each finger we use 2 phalanxes.
- (2) Count the marked points: It gives us 10. Thus, $5 \times 2 = 10$

TABLE OF 3

Example 1: Multiply 5×3 or 3×5

Explanation:

Fingers	Phalanxes
5	3

- (1) Use 5 fingers, on each finger we use 3 phalanxes.
- (2) Count the marked points: It gives us 15. Thus, $5 \times 3 = 15$

Example 2: Multiply 3×8 or 8×3

Explanation:

Fingers	Phalanxes
8	3

- (1) Use 8 fingers. Note we now use the second hand. On each finger, mark 3 phalanxes.
- (2) Count the marked points: It gives us 24. Thus, $8 \times 3 = 24$

TABLE OF 4

Example 1: Multiply 4×4

Explanation:

Fingers	Phalanxes
4	4

- (1) Use 4 fingers, on each finger we use 4 phalanxes. (Since we only have 3 phalanxes per finger, we will add one more using the pen)
- (2) Count the marked points: It gives us 16. Thus, $4 \times 4 = 16$

Example 2: Multiply 9×4 or 4×9

Explanation:

Fingers	Phalanxes
9	4

- (1) Use 9 fingers, on each finger we use 4 phalanxes.
- (2) Count the marked points: It gives us 36. Thus, $9 \times 4 = 36$

TABLE OF 6–10

Note: The following techniques extend a pre-existing finger-arithmetic technique that we adopted and elaborated.

Example 1: Multiply 6×6

- (1) For these multiplications, fingers serve double duty as both multiplier/multiplicand and 'tens' cumulative elements of the product.
- (2) Here the pinkies represent the multiplier 6 and the multiplicand 6, so that 6×6 is represented by linking the pinkies (see, in the image below, red lines marking the pinkies, and horizontal lines linking them to each other).
- (3) But as elements of the *product*, the pinkies are each worth 10. Together, the two pinkies represent 20
- (4) The fingers that are on top are each worth 1 (see, in the image below, the eight fingers signified by the blue lines). We multiply the number of remaining up-top fingers on each hand, for $4 \times 4 = 16$
- (5) Now we add the 20 from the pinkies to the 16, for $20 + 16 = 36$

- (1) Now it is the ring fingers that represent the multiplier 7 and the multiplicand 7, so that 7×7 is represented by linking the ring fingers (see, in the image below, the horizontal line linking the ring fingers).
- (2) But as elements of the *product*, the ring fingers *as well as the pinkies* are each worth 10 (see, in the image below, the four fingers marked by red lines). Together, the two ring fingers and two pinkies represent 40.
- (3) Again, the fingers on top are each worth 1 (see the six fingers marked by blue lines). We multiply the number of remaining up-top fingers on each hand, for $3 \times 3 = 9$
- (4) Now we add the 40 from the ring finger & pinkies to the 9, for $40 + 9 = 49$

Example 3: Multiply 6×9 or 9×6

- (1) Now it is the left-hand index fingers that represent the multiplier 9 and the right-hand pinky representing the multiplicand 6, so that 9×6 is represented by linking those two fingers (see, in the image below, the black line).
- (2) Again, these linked fingers as well as the fingers below them (marked by the red lines) are each worth 10. Altogether, these five fingers add up to 50.
- (3) And again, the fingers on top are each worth 1. We multiply the number of remaining up-top fingers on each hand, for $1 \times 4 = 4$
- (4) Now we add the 50 and 4, so $50 + 4 = 54$, so that $9 \times 6 = 54$

The reader is invited to apply this system to perform 9×9 , as illustrated below.

TABLE OF 11–15

For these multiplications, we tell the children that we are ‘going up to a second floor’. Now the fingers take on new values that are 5 more than before.

Example 1: Multiply 12×14 or 14×12

- (1) In this table we will always have a constant value that is equal to 100

- (2) Now join the '14' and '12' fingers on their respective hands (see in the image below, the connector line).
- (3) Again, these linked fingers as well as the fingers below them are each worth 10. Altogether, these six fingers (four on the left, two on the right) add up to 60.
- (4) In this case, unlike the Tables from 6 to 10, to find the value of the units we will do the following. For 14×12 we multiply only the units, so $4 \times 2 = 8$. At this point, we have $60 + 8 = 68$.
- (5) Finally we add our constant value of 100. We finally have $68 + 100 = 168$, so that $14 \times 12 = 168$.

Example 1: Multiply 13×14 or 14×13

- (1) Now join the '14' and '13' fingers on their respective hands
- (2) Again, these linked fingers as well as the fingers below them are each worth 10. Altogether, these seven fingers add up to 70.
- (3) Again, for 14×13 we multiply the units, so $4 \times 3 = 12$
- (4) At this point, we have $70 + 12 = 82$. Finally, we add our constant value of 100. We finally have $82 + 100 = 182$, so that $14 \times 13 = 182$.

5. Evaluation research: focus on the Cusco study

Early evaluations of the method were conducted with almost 8,000 Non-Indigenous children from Peru, Bolivia, and Mexico. That formative phase of development led to the focal study reported here below. A caveat is here in place. The study reported herein was never intended to emulate the rigour of scientific methodology. The author is an educator, not a researcher, and therefore the argumentation logic of the project cannot lean on normative standards of empirical inquiry. Moreover, our meagre equipment and limited personnel did not enable us to videotape all sessions and keep detailed records of all events. Yet the arguments put forth may be considered as more than anecdotal, at the least due to the intriguing results of the intervention. The intention would be to inform fellow scholars of an opportunity to engage in the work that would be necessary to better understand the causes for these findings and, hopefully, to promote the pedagogical method that may benefit many students.

Evaluations with Indigenous populations include an intervention with Quechua-speaking children in the town of Cusibamba, Cusco (see Figure 4). These students live in extreme poverty, in one of the poorest places in this region: In this area, more than 57% children under 10 years old suffer chronic malnutrition, and only 20% of the population has access to basic services like water, electricity, and sewer services. These are the students ranked by PISA as the lowest achieving in Latin and South America

Our evaluation group included 120 students aged 7–12 years old. Staff included the author as well as Teacher Ena. Pre-intervention assessments indicated that the students could barely multiply by 3 or 4. After two weeks of using our method 1 h per day, the students were able to multiply by 15 and even 20. In so doing, these Indigenous students surpassed the achievement of their Non-Indigenous peers in Lima, the capital city.

6. Discussion

In attempting to explain the Cusco students' superior achievement, we considered several hypotheses. These poor rural children do not have access to digital media offering games and entertainment, and so perhaps they continued to practice and reflect on our activities in the afterhours. Also, in the absence of expensive media, these children engage in more outdoor play than their urban cohort, and so perhaps their cognitive growth could be partially ascribed to their physical activity. Finally, we have noticed anecdotally that the Cusco children were predisposed to apply one of our method's heuristics for mental math. Namely, when asked to perform a subtraction, they reconfigured the subtraction problem as an addition situation, where they searched for the missing addend 'back up' from the subtrahend to the minuend. For example, they solved $24 - 9$ by mentally determining $9 + ? = 24$. By way of contrast, Lima students solve the same problem by mentally conducting the vertical algorithm of borrowing 10 to make 14, then determining $14 - 9$, and so on. It appears that the addition approach to subtraction is both faster and 'safer', in that it is less error-prone.

Whereas all these explanations may be plausible, we are quite confident that the remarkable learning gains can be ascribed to the pedagogical and didactical method.

Our results raise questions for Schild et al. (2020), who, following 50 training sessions over 10 weeks, found no advantages for finger arithmetic as compared to mainstream arithmetic training. In contrast, students participating in 15–20 sessions of the Botetano Method exhibit impressive results. By way of evidence, readers are referred to the following video: <https://www.youtube.com/watch?v=AiIW2X6pN0A>. It features Indigenous Shipibo children, who mastered the 15-by-15 table after only five 45-minute classes. One of these students, who at the time was 5 years old, mastered the 10-by-10 tables after 15 sessions: https://www.youtube.com/watch?v=I_YQWlrUYwE.

The pedagogical methodology presented here looks to create opportunities for students to use their body as a pliable external representation (cf. Kirsh, 2013). The hands become an embodied abacus of sorts, where mathematical reasoning is distributed over material resources that are instrumentalized according to a specified cultural practice. Here the body is not a metaphor for abstracted Platonic ideas – the ten fingers do not 'represent' containment, and enumerating the fingers does not 'refer to' or 'signify' some transcendent sequence of signs. Rather, the body serves a situated practice (Greeno, 1998; Lave & Wenger, 1991; Suchman, 1987). As such, principles of embodiment in this enacted

Figure 4. Top: Teacher Ena with students in a school situated in the county town of Cusco. Below: Students ascending the muddy approach to a school near Cusibamba. The school, located in the general geographic vicinity of Machu Picchu, is a challenging seven-hours walk from the Cusco, which is the nearest internet connection.

Figure 5. The future is bright: A student, who has been tilling his family's field, takes a reading break with a friend.

arithmetic system do not agree with the psycholinguistic cognitive semantics theory of conceptual metaphor proposed by Lakoff and Núñez (2000). Instead, the embodied arithmetic system might be viewed as an ethnomathematical methodology (Barton, 2008). That said, whereas the Botetano method looked to draw on these Indigenous children's epistemic capacity, the method did not sprout authentically from these people's cultural-historical practice and no deliberate attempt was made to forge the calculational routines on the egocentric, non-dualist, and action-based ontological syntax of the students' Indigenous languages (cf. Barton, 2008, pp. 36–39). At most, perhaps fostering student dependence on imaginary forms that they envisage and use as present-at-hand relies on the embodied

epistemic resources of Indigenous referential narrative practices (cf. Núñez & Sweetser, 2006).

Should we think of fingers as symbols? On the one hand, there is some intuitive allure to say that the fingers are standing for something else, if to use the fundamental definition of signs from C. S. Peirce (1839–1914). But, on the other hand, while the method does not utilize fingers for their normal biological function of gripping, perhaps they are not really symbolizing anything at all. Perhaps they are just constituting *ad hoc* epistemic forms or structured slates that afford particular routines of marking and enumeration as embedded in a cultural practice. It would be interesting to pursuing these important questions over the ontological status of embodied semiotic resources. Perhaps my pedagogical method could frame an empirical context for such research studies.

7. Conclusion

Children are marvellous learners. They do not fail. What fails is the method. If we want students to succeed in mathematics, we must make a mathematics curriculum that nurtures their minds. Children's minds are embodied – they draw on manipulation. Fingers are cheap. In fact, they are free. Even the poorest and lowest performing children in the whole of Latin and South America have fingers. Used properly, these digits become mathematical ways of thinking.

Mathematics, as it is constituted by the moving fingers, should only eventually arrive to paper. Beginning from paper teaches children paper math, and yet paper math is not necessarily the best for mental math. Consider the abacus, which experts use with staggering speed to solve formidable multiplication problems, even in the absence of the device itself, just through their imagination (Brooks et al., 2018; Hatano et al., 1977; Stigler 1984). Perhaps we have more to learn from Indigenous practices, so that we can all become better teachers for all children (Figure 5).

Disclosure statement

No potential conflict of interest was reported by the authors.

Note

1. To view online several background informational videos, readers are referred to the following links: <https://www.youtube.com/watch?v=11BLBT8kDKE> <https://www.youtube.com/watch?v=uh3FKKLLKg58>

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