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Authors
Kitchen, S. W.
Schelberg, A. D.

Publication Date
1954-05-17

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UCRL-1947 Rev.

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Radiation Laboratory

Contract No. W-7405-eng-48

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S. W. Kitchen and A. D. Schelberg

Berkeley, California

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S. W. Kitchen and A. B. Schelberg *

Radiation Laboratory, Department of Physics,
University of California, Berkeley, California

ABSTRACT

The application of perturbation techniques to the quantitative measurement of both relative electric and relative magnetic fields in resonant cavities is described. The apparatus, procedures, advantages, and limitations are discussed, and the experimental results are compared with the calculable field distributions of a coaxial resonator. The theoretical shunt impedance and Q obtained empirically agree respectively within one and four percent of the calculated values.

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S. W. Kitchen and A. D. Schelberg*

Radiation Laboratory, Department of Physics
University of California, Berkeley, California

Although the field distribution in a resonant cavity can be, and has been, calculated for many geometries, [1] frequently the field distribution of a cavity with nonstandard geometry is desired. A well-known technique is measurement with probes. A more recent technique has been the use of small perturbations which cause a frequency shift related in a known way to the local field. [2] A comprehensive examination of these relationships between the fractional frequency shift, the local relative field strength, and the shape of the perturbing body has recently been published by Mullet, [3] who considers dielectric as well as metallic bodies. The application of the technique, however, has apparently been confined to determinations of electric field. The report describes the application of the technique in the 200-400 Mc range to the measurement of both electric and magnetic local fields in resonant cavities with an attainable precision of better than one percent. This apparatus, procedures, advantages, and limitations of the method are discussed, and experimental results are compared with the calculable field distributions of a coaxial resonator.

The general expression relating the frequency shift due to a sphere of permeability μ and dielectric constant ϵ to the local field (E+H) is

$$\frac{\delta\omega}{\omega} = -\frac{3}{4}\frac{\delta v}{W}\left[\frac{\epsilon-1}{\epsilon+2}\epsilon_o E^2 + \frac{\mu-1}{\mu+2}\mu_o H^2\right] \qquad (1)$$

where δv is the volume of the sphere and W is the stored energy. For a dielectric sphere, this equation may be simplified to:

$$\left(\frac{\delta\omega}{\omega}\right)_D = -\frac{3}{4}\,\delta v_D \left(\frac{\epsilon-1}{\epsilon+2}\right)\epsilon_o \frac{E^2}{W} \qquad (2)$$

Consequently, a dielectric sphere may be used to determine the relative magnitude of electric field even in the presence of a magnetic field. In the known absence of magnetic field, a conducting sphere will produce a fractional frequency shift:

$$\left(\frac{\delta\omega}{\omega}\right)_M = -\frac{3}{4}\,\frac{\delta v_M}{W}\,\epsilon_o\,E^2. \qquad (3)$$

The ratio of Eq. (3) to Eq. (2) at the same point in the cavity is

$$\frac{\left(\frac{\delta\omega}{\omega}\right)_M}{\left(\frac{\delta\omega}{\omega}\right)_D} = \frac{\epsilon+2}{\epsilon-1}\,\frac{\delta v_M}{\delta v_D} = \xi \qquad (4)$$

for a given pair of metal and dielectric spheres. The value of ξ can be determined empirically and used to determine the magnetic field distribution. In the presence of both magnetic and electric fields

$$\left(\frac{\delta\omega}{\omega}\right)_M = -\frac{3}{4}\,\frac{\delta v_M}{W}\left[\epsilon_o E^2 - \frac{1}{2}\,\mu_o H^2\right]$$

and

$$\frac{H^2}{W} = \frac{8}{3\mu_o \delta v_M}\left[\left(\frac{\delta\omega}{\omega}\right)_M - \xi\left(\frac{\delta\omega}{\omega}\right)_D\right] \qquad (5)$$

With the aid of these relations all the theoretical constants of a cavity, such as shunt resistance (Z_s), transit time factor (T), or (Q) can be determined with an accuracy dependent principally on the measurement of $\left(\frac{\delta\omega}{\omega}\right)$. For example, Z_s, Q and T may be expressed as:

$$Z_s = \frac{\frac{\mu_o}{\epsilon_o}\,\sigma\Delta\left[\int (\delta\gamma)^{1/2}\,ds\right]^2}{2\int\left[\xi(\delta\gamma)_D - (\delta\gamma)_M\right]dA} \text{ ohms}$$

$$\frac{1}{Q} = \frac{2\Delta}{3\delta v_H}\int\left[\left(\frac{\delta\omega}{\omega}\right)_M - \xi\left(\frac{\delta\omega}{\omega}\right)_D\right]dA \qquad (6)$$

$$T = \frac{\int (\delta\gamma(s))^{1/2}\cos(\omega t+\phi)\,ds}{\int (\delta\gamma(s))^{1/2}\,ds}$$

where $\int$ Fds is a line integral, $\int$ FdA is a surface integral, Δ is the skin depth, γ is the frequency, and all units are rationalized MKS units. The difference terms in Z_s and Q limit the accuracy of these quantities as obtained from the perturbation technique, even though the apparatus described below permits measurement of $\frac{\delta\omega}{\omega}$ to 10^{-8}.

One difficulty encountered in measuring $\frac{\delta\omega}{\omega}$ for substitution in the above formulas lies in the fact that the perturbation formula (1) does not apply to spheres that are close to a conducting surface, for then image effects come into play. In practice, $\frac{\delta\omega}{\omega}$ observed when the sphere is less than a diameter away from a metal surface is discarded, although it is possible to calculate a proximity correction by the following expressions for the measured fields in terms of the actual fields.

$$E = E_a \left[1 + \frac{1}{4} \frac{\epsilon - 1}{\epsilon + 2} \left(\frac{r}{d}\right)^3\right] \text{ (for metal } \epsilon \to \infty$$

$$H = H_a \left[1 + \frac{1}{16} \left(\frac{r}{d}\right)^3\right] \qquad (7)$$

where r is the radius and d is the distance from the center of the sphere to the surface. These expressions were derived by considering the dipole image effects.

Where reasonably accurate extrapolations to the surface are not possible, another technique may be used. It can be shown that a hemisphere on a plane surface will produce half the frequency shift produced by a sphere of the same radius in the same field. [6] The difficulty encountered in sliding a hemisphere along a surface is that the surface is rarely plane. If the hemispherical diameter is small compared to the radius of curvature of the surface, however, satisfactory data can be obtained by calibrating the ratio of spherical to hemispherical shift in a cavity of standard field distribution, even though the ratio for metal BB's will have one value for the shift due to the magnetic field and another for the shift due to the electric field.

A block diagram of the frequency measuring system is shown in Fig. 1. The frequency of the reference system [7] is adjusted so that it is lower than the cavity frequency by about 1500 cps. The cavity itself is driven by a grid dip oscillator. Choice of a grid dip type of oscillator was dictated by the fact that the grid dip frequency is controlled over the range of $\delta\gamma$ by the cavity. The perturbing sphere (or hemisphere) is introduced into the cavity on a thread, $\delta\gamma$ being observed on an EPUT. [8]

The accuracy of this method of measuring relative field distribution is determined by several factors;

(1) the stability of the referency frequency systems,

(2) mechanical stability of the cavity,

(3) the magnitude of perturbances other than the desired one during the measurement,

(4) errors in positioning the sphere,

(5) "following" of the cavity resonant frequency by the cavity oscillator.

The requirements were much more severe for the measurements of magnetic field than those of electric field.

The time required to make one measurement of $\delta\gamma$ is about ten seconds. The reference frequency system is stable to less than ten cycles per second over a ten-second period at worst, and frequently drifts no more than ten cycles over a period of several minutes. Possible error due to this drift is negligible compared to the other factors.

Another source of error is a change in the resonant frequency of the cavity due to thermal expansion; drifts as high as 100 cps per second have been observed. For precise measurements, therefore, it was found necessary to control the temperature of the room where the cavity was under observations to $\pm 0.5^{\circ}F$ as recorded with a thermocouple thermometer. Although the magnitude of the effect was never determined, the drift was further reduced by running the grid dip oscillator with dc on the filaments and leaving all oscillators on continuously.

Mechanical vibration of a cavity during a measurement was another source of error. Stiffening ribs around the precision cavity's periphery were found to be advisable even though the wall was 1/4 inch thick. In addition, the test cavities were mounted on Lord Shock mounts as further protection against vibration. The sensitivity of the frequency measuring system, however, was such as to respond even to thumb pressure against the cavity. As a result, the guide pulleys had to be mounted in such a way as to prevent stress on the cavity when positioning the sphere.

The thread supporting the sphere had two requirements to meet: (1) that its nonuniformity be sufficiently low that frequency shifts due to the thread alone could be kept much less than those due to the sphere, and (2) that its modulus of elasticity be high. Of all available types of thread--cotton, silk, linen, glass, and nylon; twisted, braided, and nonofilament;

some impregnated and some plastic coated--by far the most uniform were nylon monofilament and an impregnated silk suture with the trade name of Dermal. The least uniform was linen. So far as stretch is concerned, however, glass and linen were the best and nylon was by far the worst. The principal drawback to the glass thread was its inability to withstand stress around sharp corners. Dermal, when boiled and prestretched, was entirely satisfactory in all respects. For less precise measurements, braided silk would be adequate. Satisfactory measurements of magnetic field in the presence of electric field require, however, that the metal and dielectric spheres be at the same point in the cavity, particularly when the field strengths are a rapid function of position.

Another source of possible difficulty lies in the method of exciting the cavity. Even though the cavity has a very high Q as compared to the grid dip oscillator, it is still a tuned circuit coupled to another tuned circuit. Consequently, the observed resonant frequency is not exactly the resonant frequency of the cavity. Furthermore, as the cavity resonant frequency changes owing to perturbation, the change in the excitation frequency observed in the cavity is not equal to the change in the cavity resonant frequency, but depends on the setting of the grid dip. It was found that for a setting nearly equal to the maximum needle dip, the ratio of $\delta\omega$ observed to $\delta\omega$ actual would remain constant at about 0.95 for shifts up to 0.05 mc. For values of Z_s and T, the exact value of the ratio is unimportant as long as one knows it remains constant over the range of $\delta\gamma$ for a given setting of the grid dip. Calculation of Q, however, requires a measurement of the ratio using a standard geometry.

The determination of Q requires, in addition to the absolute frequency shift, a knowledge of the shape and diameter of the sphere (or BB). Bronze ball bearings make excellent metal BB's, but no satisfactory dielectric BB's could be procured until the technique of making precise dielectric spheres was developed.[9]

To illustrate the quality of the technique, measurements made on the coaxial cavity of Fig. 5 are shown in Figs. 2, 3 and 4. Note that the agreement between the observed and calculated distributions is excellent on the whole. The only major deviations occur near surfaces or in regions of very low field. Equation (7) indicates that the imaged dipole acts to increase both the electric field and the magnetic field at the sphere; the observed frequency shifts due

to these fields behave in a like manner. This effect was verified by observations taken along the inner conductor with spheres touching the conductor and 1/4 inch off (see Fig. 4-b). The pronounced dip at the upper end of the curve of Fig. 3 seems to be at odds with predicted behavior. These data, however, were taken perpendicular to the surface. Consequently, in this region, the sphere was approaching a 3/8 inch hole in the conducting surface.

When the aberrations due to surface proximity effects were removed by logical extrapolation of the curves as indicated in the figures, the $\delta\omega$ for the spheres was used to calculate the theoretical shunt impedance and theoretical Q of the coaxial cavity with the assumption that the surfaces were of copper whose conductivity was 5.8×10^7 mhos/meter. If the end-wall losses were not included, the Z_s and Q calculated from the data were respectively 6.07×10^5 ohms and 16,000. As calculated from Maxwell's equations, they were 6.02×10^5 ohms and 14,600. The 10 percent discrepancy between the measured and calculated Q values arises principally from the fact mentioned earlier that the $\frac{\delta\omega}{\omega}$ observed is always less than the theoretical value. The measured Q must be corrected by the ratio of the observed $\frac{\delta\omega}{\omega}$ to the theoretical $\frac{\delta\omega}{\omega}$ (equal to 0.95). The Q calculated from the data then becomes 15,200, four percent higher than the predicted value. Even though Q obtained by this method is the least accurate of the quantities obtained, it is still a far better figure for cavities of nonstandard geometry than can be obtained by educated guesses.

In summary, one may say that the perturbation technique may be applied successfully to the quantitative measurement of both magnetic and electric fields within the volume of a resonant cavity. Unlike the probe technique, it does not distort the fields other than locally, and it does not depend on the power level. On the other hand, the accuracy is impaired by small mechanical and thermal changes of the resonant system. It is not the best technique for every circumstance, but it does permit quantitative measurements that were not previously possible.

It is desired at this time to acknowledge the contributions of Dr. Andrew Longacre, who initiated the program, Dr. W. K. H. Panofsky, who suggested the use of the method for magnetic field measurements, and of Messrs. A. J. Schwemin and R. G. Smits, whose instrumentation improvements materially added to the speed and accuracy of the measurements.

REFERENCES

1. R.D. Hill, Sci. Inst. 22, 221 (1945).
Microwave Transmission Design Data, Sperry Gyroscope Co. 1944.
S.A. Schelkunoff, Electromagnetic Waves. D. Van Nostrand Co. 1943.
W.C. Hahn, Jour. Appl. Phys. 12, 62-64 (1941).
S. Ramo and J.R. Whinnery, "Fields and Waves in Modern Radio," Wiley, (1944).

2. W.W. Hansen and R.F. Post, J. Appl. Phys. 19, 1059 (1948).
L.C. Maier, Jr., and J.C. Slater, J. Appl. Phys. 23, 68 (1952).

3. L.B. Mullet, "Perturbation of a Resonator," A.E.R.E. G/R 853 (1952).

4. Note that $\gamma\delta\omega$ measures only the ratio of the local fields relative to the average fields in the cavity, and is therefore indenpendent of the power consumption. Owing to the difficulties of measuring power precisely, this factor is an important advantage of the perturbation technique.

5. Originally derived by M.L. Good of this Laboratory.

6. J.C. Slater, Revs. Modern Phys. 20, 473 (1948).

7. R.G. Smits and A.J. Schwemin, "A Stable Microwave Reference System," University of California Radiation Laboratory Report No. UCRL-1843, June, 1952.

8. Events Per Unit Time Meter, Berkeley Scientific Corp., Richmond, Calif.

9. A.J. Schwemin, "Making Precise Nonmetallic Spheres," University of California Radiation Laboratory Report No. UCRL-1560, November, 1951.