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ABSTRACT

A survey is given of an approach to the dynamics of strong interactions that treats both high and low energies for small momentum transfers. It is conjectured that all strong interactions may be "as strong as possible", consistent with requirements of unitarity and analyticity of the S matrix.

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I. INTRODUCTION

In the past, theoretical approaches to strong interactions have differentiated sharply between the high- and low-energy regions. At high energies, where the wavelength is short compared to the interaction distance and many partial waves interact, semi-classical methods have generally been employed--optical models to describe total and elastic cross sections and statistical models for inelastic processes. At low energies, where only a few partial waves are significant, the analyticity of the S matrix has played a dominant role, and many semi-phenomenological dynamic calculations have been made on this basis--with a limited but significant success in correlating experimental facts. Some empirical connection between the two energy regions has been achieved through forward dispersion relations, and very recently analyticity arguments have been applied to give a crude description of high-energy peripheral collisions--as described by Drell.¹ However, the dynamics of high and low energies have not been related heretofore in a direct way. My report deals with the recent efforts of a number of Berkeley theorists in this direction. This work is entirely post-Rochester

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University of California, Berkeley, December 1960.

and still tentative; we are anxious, however, to stimulate discussion of some of the points encountered. The people involved include John Charap, Steve Frautschi, Marcel Froissart, Virendra Singh, and B. M. Udgaonkar. We have benefited very much already from vigorous criticism of our ideas by M. Gell-Mann and S. Mandelstam.

Frautschi and I got started in this direction because of frustration with attempts to use the Mandelstam representation to make self-consistent low-energy dynamical theories that include P resonances. In particular, Mandelstam and I had found in the $\pi\pi$ problem that, although large low-energy S phase shifts could be accommodated without involving the high-energy region, strong elastic P scattering at low energies seemed to require conditions on inelastic cross sections at high energies.² There is as yet no direct evidence for a P-wave $\pi\pi$ resonance, but Frautschi and Walecka³ and, independently, Frazer and Fulco⁴ confirmed the same difficulty in the πN problem with respect to the famous $(3/2, 3/2)$ resonance. Mandelstam had noticed the possibility of trouble in his first paper, but had hoped that the coupling between high and low energies would be weak.⁵ However, by the time of the Rochester Conference last September, it was indicated by a number of detailed calculations that nature is not going to permit a realistic dynamic theory of strong interactions which is self-consistent within the low-energy domain.

As I shall explain, Frautschi and I are proposing⁶ the simplest extension of the original Mandelstam program⁵ we feel can conceivably accommodate low-energy P resonances. We have arrived at a set of equations that may or may not be self-consistent and complete; but quite apart from the validity of our particular approximation, we have been led to a way of

thinking that suggests to us a startling circumstance. If total cross sections are to approach constants systematically at very high energies, then in the low-energy elastic region the forces should be strong enough to produce large S and P phase shifts, but not sufficiently strong to make resonances or bound states for $J \geq 2$. We are encouraged, in fact, to propose a universal definition of strong interactions: That they are always as strong as possible--consistent with the requirements of unitarity and analyticity. It has been plausible for some time that such a definition would correctly lead to constant high-energy total cross sections corresponding to a radius of the order of a pion Compton wavelength. One object of my presentation is to suggest that the same definition may lead to low-energy forces of the order of magnitude observed.

II. THE MANDELSTAM DIAGRAM AND THE STRIP APPROXIMATION

I assume that the audience is familiar with the Mandelstam variables and the Mandelstam representation.⁵ Let me consider for simplicity the case of equal masses (e.g. $\pi\pi$ scattering) in which the three physical regions are as shown in Fig. 1. The three double-spectral functions are nonzero in the shaded domains, and it is the geometry of these regions that is central to the approach I want to describe.

The basic idea behind all successful approximations in strong-interaction theory (in which there are no small coupling constants) is to take accurate account of "nearby" singularities and to approximate or neglect "faraway" singularities. Unitarity restricts the strength of all singularities, so generally speaking the inverse dependence on distance given by the Cauchy formula for an analytic function in terms of its poles and branch cuts makes the "nearby" singularities more important. Now the notion of the "distance"

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Fig. 1. Mandelstam diagram for $\pi\pi$ scattering

of a real physical point from a singularity is straightforward for a function of a single complex variable, particularly when all singularities lie on the real axis, but we are dealing here with a function of two variables. As long as the only large low-energy phase shifts are in the S state, it turns out that the double spectral functions are unimportant in comparison to the single-spectral functions, and so the problem can be made one-dimensional. However, in the interesting case, we must decide what is "near" and what is "far" in the two-dimensional Mandelstam diagram. (For substantial parts of the following argument I am indebted to Froissart.)

Consider the term

$$\int ds' dt' \frac{\rho_{st}(s', t')}{(s' - s)(t' - t)} . \quad (1)$$

The strength of the "source" is ρ_{st} , but the relevant "distance" is the product of the two one-dimensional displacements from "source" point to physical point. Thus the double-spectral regions of constant "distance" from a low-energy physical point are bounded by hyperbolas running more or less parallel to the boundaries of the double-spectral region. The "nearest" region is the heavily shaded strip just inside the boundary. It is this region that we must accurately represent in a consistent theory of low energies.

One should realize that it is really only to the principle value part of formula (1) that the above argument applies. That is to say, for example, in the s physical region the $t' - t$ denominator cannot vanish, but we must interpret $(s' - s)^{-1}$ as

$$P \frac{1}{s' - s} + i \pi \delta(s' - s) . \quad (2)$$

Thus, the imaginary part comes entirely from the single value $s' = s$. At low energies, the real part of the amplitude is generally of the same order of magnitude or larger, than the imaginary part. However, at high energies, there is reason to believe that the imaginary part dominates the forward diffraction peak. Such a dominance is suggested by all classical models and has substantial experimental support. We believe that it will emerge naturally from the Mandelstam approach when the latter is thoroughly understood, but for the moment our faith is of classical origin. Combining formulas (2) and (1), we see that the imaginary part of the amplitude in the s physical region, arising from ρ_{st} , is

$$\int dt' \frac{\rho_{st}(s, t')}{t' - t} \quad (3)$$

Thus the "distance" from source point to physical point is only one-dimensional, and so far as small angles in the physical region are concerned, the "nearest" part of the double-spectral region is the strip on which we have already focused attention in connection with low energies. Thus we have been led to the conclusion: If one can calculate the double-spectral function in a strip adjacent to the boundary, of width $\Delta t \gtrsim 16 m_\pi^2$ (it must be substantially wider than the gap between the double-spectral boundary and the physical region), then one should achieve a theory appropriate to all energies (s) for small momentum transfers (t). "Small" means

$$-\Delta t \lesssim t \lesssim 0. \quad (4)$$

Recall that

$$t = -2q^2(1 - \cos \theta_s)$$

and

$$s = 4(q_s^2 + m_\pi^2) \quad .$$

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For energies such that $s \lesssim \Delta t$, all physical angles are included, but at high energies, only the first diffraction peak is covered by such a theory. However, this first peak comprises most of the observed collisions, and so rather few experimental phenomena are excluded. It is worth noting that the theory also could handle backward scattering in an angular interval corresponding to

$$-\Delta u \lesssim u \lesssim 0, \quad (5)$$

where

$$u = -2q^2(1 + \cos \theta) .$$

Whether backward peaks systematically occur at high energies is a most interesting question.

Some check on our conclusion that the strip regions should be dominant is given by the very existence of the forward-diffraction peak, which in all measured cases has a width corresponding to $\Delta t \sim 10\text{-}20 m_\pi^2$. If in formula (3) there were important contributions from $t' \gg \Delta t$, it would be difficult to understand why the physical amplitude is so small for $t \ll -\Delta t$. Further assurance is given by the circumstance that it is only in the low-energy (resonance) region that total cross sections ever become really large. A glance at expression (3) shows that in the s channel this fact implies the existence of maxima in $\rho_{st}(s, t)$ for low s , while the corresponding argument for the t channel implies maxima for small t . Again, the strip regions are emphasized.

III. ASYMPTOTIC BEHAVIOR IN THE STRIP DIRECTIONS

It is indicated experimentally and very plausible from classical arguments that (to within logarithmic factors) total cross sections approach constant high-energy limits and that the widths (in t) of the corresponding forward-diffraction peaks also approach constants. Now the imaginary part of the amplitude at $t = 0$ (in the s channel) is proportional to s times the total cross section. Thus we conclude from expression (3) that⁷

$$\lim_{\substack{s \rightarrow \infty \\ -\Delta t \leq t < 0}} \int dt' \frac{\rho_{st}(s, t')}{t' - t} \rightarrow s f(t) . \quad (6)$$

We find it hard to believe that if the amplitude goes linearly with s for small negative t , it does not also go roughly linearly with s for small positive t , even beyond the branch point at $t = 4 m_\pi^2$, the physical threshold for the t channel. Mandelstam has repeatedly pointed out to us that such a conclusion is not required if oscillations are important in the double-spectral function, and that in nonrelativistic potential scattering such oscillations necessarily occur. There is no analogue of the limit (6) in potential scattering, however.

A linear increase in the modulus of the amplitude for $t > 4 m_\pi^2$ would have potent physical implications for the low-energy behavior in the t channel, where s is to be interpreted as the (negative) square of momentum transfer. It has been shown rigorously by Regge for potential scattering that the asymptotic behavior of the amplitude for large momentum transfer is $\sim s^{\alpha(t)}$, where $\text{Re } \alpha$ is positive for an attractive potential and increases with potential strength.⁸ Regge also showed that partial waves for $l \leq [\text{Re } \alpha]_{\text{max}}$ may have bound states or resonances, while those

for $l > [\text{Re } \alpha]_{\text{max}}$ will necessarily have small phase shifts. Now if one examines the Mandelstam representation with regard to the relative amount of scattering in states of different angular momenta at low energy in the t channel, one finds that a natural way to achieve large and fluctuating phase shifts for $l \leq l_{\text{max}}$, with a smooth variation for $l > l_{\text{max}}$ according to the usual range criterion, is to have an asymptotic behavior $\sim s^{\alpha}$, where $[\text{Re } \alpha]_{\text{max}} \approx l_{\text{max}}$. Thus we believe Regge's criterion to be of general validity, and we conclude that if $[\text{Re } \alpha]_{\text{max}}$ is approximately one for strong interactions, as suggested above, they should be capable of producing large S and P phase shifts in the low-energy elastic region but only weak scattering in states for $J \geq 2$.

Such an argument in the $\pi\pi$ problem, where all three channels refer to the same process, immediately eliminates the so-called "S-dominant" solutions⁹ as being physically uninteresting and focuses attention on solutions where P as well as S phase shifts are large at low energies. Moreover, we expect that no resonances will occur for D or higher waves. For problems such as πN , where spin is present and where the three channels do not all correspond to the same process, the situation is more complicated, but the same basic principles apply. That is, the region of high energy and low momentum transfer in one channel is closely related to low energy and high momentum transfer in another. Unitarity puts on the asymptotic behavior in the former an absolute maximum, which by analytic continuation is carried over to the latter.

The most exciting aspect of such considerations, of course, is the prospect that coupling constants, heretofore regarded as independent, will turn out to be determined by the principle of "maximum strength" for strong interactions. For example, the magnitude of the pion-nucleon constant, $f^2 = 0.08$,

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Fig. 2. Cutkowski diagram for the strips.

corresponds to strong nucleon-nucleon low-energy scattering in states of $J = 0$ and 1 and to strong pion-nucleon scattering in states of $J = 1/2$ and $3/2$. Semiphenomenological analyses already made indicate that if f^2 were much smaller or much larger than its actual value, this situation would not be possible. The pion-pion constant λ is not yet known, and it will be an exciting race to see if theory can predict its value before measurement is made. Whether a prediction is possible, of course, depends on the formulation of a reasonable program for calculating the double spectral function in a strip of sufficient width.

IV. DYNAMICAL EQUATIONS FOR THE STRIPS

Cutkosky has proposed a general recipe for calculating Mandelstam's double spectral functions in which one must consider all four-vertex graphs and insert at the corners the appropriate (complete) scattering amplitudes.¹⁰ In the $\pi\pi$ problem, the outermost-strip contributions arise from graphs of the type shown in Fig. 2. In one channel only two pions occur in intermediate states, but in the other an arbitrary number of particles is allowed. Fig. 2(a) controls the strip for $4m_\pi^2 < t < 16m_\pi^2$ as s approaches infinity, while 2(b) controls the strip for $4m_\pi^2 < s < 16m_\pi^2$ as t approaches infinity. Cutkosky's formula for 2(a) and 2(b) is identical with that given in Mandelstam's first paper⁵ and involves only the absorptive parts of elastic $\pi\pi$ amplitudes. Thus, one can hope to calculate the strip regions without becoming involved with the full fury of a many-body problem.

Whether the strip bounded by $16m_\pi^2$ is of sufficient width to constitute a reasonable first approximation is another matter. Clearly, Fig. 2 corresponds to considering inelastic processes reached by the exchange of a single pion; this phenomenon has been discussed by Drell, who has

expressed doubt that this mechanism can account quantitatively for a majority of inelastic processes.¹ However, the estimate from which this doubt arose was based on an extension of physical cross sections off the mass shell, an extension that is not necessary in the Mandelstam-Cutkosky approach. To put the situation differently, the one-pion exchange approximation according to Mandelstam-Cutkosky is not really the same as that surveyed by Drell. We continue to hope, therefore, that Fig. 2 does constitute a reasonable first approximation.

Frautschi and I have made a preliminary examination of the iterative solution of the problem defined by keeping only the diagrams of Fig. 2. We conclude that the iteration should converge for the S-dominant type of solution and probably not differ appreciably from the result found earlier⁹ by a method in which the double spectral function was treated much more crudely. K. Wilson at Harvard has independently arrived at exactly the same equations and is actually attempting the numerical iteration.¹¹ However, a straightforward iteration may not converge for the interesting case in which the high-energy total cross section approaches a constant. Whether the equations, without modification to represent some effect from the neglected Cutkosky diagrams, have any consistent solutions of the true strong-interaction type remains to be seen. It is correspondingly uncertain as to whether the $\pi\pi$ constant λ will be required to have a unique value.

V. CONCLUSION

This, then, is the situation at present. The $\pi\pi$, NN , and πN problems are being attacked at Berkeley by the new approach. At the same time Charap is studying nonrelativistic potential scattering by the analogous technique in the hope of discovering general properties of the double spectral function (in particular, the oscillations) that will guide us in the relativistic theory. We are full of hope but at the same time aware from past experience that the strong-interaction problem has a knack of continuously developing new twists so as to defy systematic analysis. It will be surprising to us, in any event, if nothing useful develops from the notion that unitarity puts an upper limit on interaction strengths, at low as well as at high energies, and that nature seems to approach this limit closely.

Gell-Mann often reminds us that the analyticity properties of S-matrix elements on which all our thinking is based, may in fact not be correct at the high energies to which they are now being pushed. There is no answer to this remark except that at the moment we see nothing to be gained by abandoning Mandelstam and Cutkosky. If they go down, there is nothing in sight to save us from the abyss.

Post-Conference Note: In the published Proceedings of the 1960 Conference on High Energy Physics, p. 340, a paper by Gribov presents arguments closely related to the subject discussed here.

FOOTNOTES

1. S. D. Drell, "Peripheral Contributions to High Energy Interaction Processes," invited paper at the Conference on Strong Interactions, Berkeley, 1960.
2. G. F. Chew and S. Mandelstam, "Theory of the Low Energy Pion-Pion Interaction, Part II," Lawrence Radiation Laboratory Report UCRL-9126, March 1960; to be published in *Nuovo cimento*. See also G. F. Chew, Proc. 1960 Annual Conf. on High Energy Physics, Rochester, 1960, p. 273.
3. S. Frautschi and D. Walecka, Phys. Rev. 120, 1486 (1960).
4. W. Frazer and J. Fulco, Phys. Rev. 119, 1420 (1960); see also W. Frazer, Proc. 1960 Annual Conf. on High Energy Physics, Rochester, 1960, p. 282.
5. S. Mandelstam, Phys. Rev. 112, 1344 (1958).
6. G. F. Chew and S. C. Frautschi, Phys. Rev. Letters 5, 580 (1960); also "A Dynamical Theory for Strong Interactions at Low Momentum Transfers but Arbitrary Energies," Lawrence Radiation Laboratory Report UCRL 9510, 1960.
7. All statements here are uncertain with respect to logarithmic factors. Only the powers are intended to be taken seriously.
8. T. Regge, *Nuovo cimento* 14, 951 (1959) and University of Rochester Physics Department preprint NYO-9266, 1960 (unpublished).
9. G. F. Chew, S. Mandelstam and H. P. Noyes, Phys. Rev. 119, 478 (1960).
10. R. Cutkosky, Phys. Rev. Letters 4, 624 (1960) and J. Math. Phys. 1, 429 (1960).
11. K. Wilson, Harvard Physics Department Preprint, 1960 (unpublished).

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