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MILLS ON OPTIMUM CITY SIZE

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An average reader of books and papers in Urban Economics today is likely to emerge from the reading with a sense of frustration -- with a feeling of ending up in a cul-de-sac. He reads so many words about the acute urban problems, about the urban crisis, but the words do not seem to lead anywhere. He follows the winding thoughts of the renowned analysts in the field, only to find that they use partial-equilibrium approaches to problems which by all appearances and flavors are general-equilibrium (or general-disequilibrium) in nature.

However, reading Ed Mills is an entirely different story. In a series of ingeniously and skillfully varied, and truly pioneering, papers he has opened up new vistas. Although one cannot properly describe his framework as a general-equilibrium format, Mills has come a very long way in that direction, in comparison with the standard literature on location and urban land use. The main thrust of his present paper (written together with D. de Ferranti) lies in its analysis of congestion cost in relation to city size. The authors show that the elasticity of urban costs (as they define costs) with respect to city size (defined as the number of workers in the CBD) is less than unity for small cities and greater than unity for sufficiently large cities.

Assuming the discussant's role as devil's advocate, I shall now make some critical observations about the Mills-Ferranti paper -- first some specific comments about a few of the assumptions underlying their analysis, and then some more general comments relating to their criterion of optimality.

A weakness of the analysis is that  $N$ , the number of workers employed in the central city, is exogenously determined. This does of course not mean that  $N$  is fixed; it can be varied parametrically. However, the convenience of having  $N$  treated this way, comes at a price. With  $N$  exogenous, it was apparently necessary to make  $\epsilon$ , the radius of CBD, exogenous. In turn, this means that within the central area of the city, transportation and congestion costs are zero, the opposite of a real-world city. Related to this, there is also a slight anomaly in the calculation of city optimum on the last page, where the numerator includes workers in the transportation industry ( $\bar{p}$ ), whereas the denominator ( $N$ ) does not. Further, I question the validity of the form of the basic transportation (and congestion) cost function (1). It would be more meaningful with an explicit lower-bound for the density variable:

$$\frac{T(u)}{L_2(u)} \geq b(N)$$

$$\text{or: } \rho_1 = 0 \text{ for } \frac{T(u)}{L_2(u)} < b(N)$$

As this suggests, the lower-bound value should ideally be a function of city size,  $N$ .

Equation (2) is derived on the premise that "in an optimum system (without congestion distortion) the reduction in transportation cost from an increase in  $L_2(u)$  would just equal land rent  $R(u)$ ." However, this result also seems to imply:

$$\frac{\partial T(u)}{\partial L_2(u)} = 0$$

More importantly, it excludes corner solutions. Is there a deceptive analogy with land use in agriculture lurking behind this kind of reasoning? In principle, movement of goods and people can occur both underground and above ground. To assume that it can only occur on the ground level does seem rather limiting. Looking at the problem from a different point of view, one can also say that in urban analysis (possibly in contrast with agricultural analysis), it is undesirable to use (transport-service and other) production functions,  $f(x)$ , with the property that

$$f(x) = 0 \quad \text{for any } x_i = 0$$

$$x = (x_1, x_2, \dots, x_i, \dots, x_n)$$

Presumably, no other production function having this undesirable property, is better known than the Cobb-Douglas function.

Now, for some more general comments, it should first of all be stressed that the (implied) optimum-city-size criterion used by the authors is that of myopic allocative efficiency (cost minimization). Distributional considerations, which are stressed by both Ellickson and Bateman-Hochman, might lead to smaller optimum city size.

Mills and de Ferranti say that two complementary ideas dominate the literature as to "what cities are for," namely central place theory and (interregional and international) trade theory. Whereas this (unfortunately) may be a fair summary of the literature, it represents nevertheless a serious omission. Central place theory is a highly descriptive approach with very limited explanatory value as a theory. The second strand of ideas, international trade theory, is in all

essentials a static theory; using it to comprehend city phenomena may inadvertently lead us away from another notion which may be just as important in our understanding of why cities exist and grow, namely the dynamic notions of learning and technical change. Indeed, and essentially basing it on this concept, I would venture to hypothesize that optimum city size is greater in the developing countries than in the developed countries. There is in my view yet another notion which is also indispensable in our attempts to understand city phenomena, namely the concept of collective services (or with its better known but somewhat misleading name: "public goods"). Indeed, the ancient cities were public goods: They were centers of assembly and protection. The walls that enclosed the towns were an important collective service for the inhabitants; it is significant that the word "town" originally meant "enclosure." Other components in the bundle of collective services were the temple, the market place and at least the exterior of the palace. As cities grew in density and size, the bundle of collective services also grew. Lack of space precludes me from elaborating further on the significance of notions of learning and technical change and of collective services for our understanding of cities.<sup>1</sup>

The main point I wish to make is this: Whereas Mills and de Ferranti concentrate their analysis to the cost side, in the determination of optimum city size, we cannot meaningfully exclude the benefit or revenue side -- and including it means facing the well-known conceptual difficulties of dynamics and of collective services.

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<sup>1</sup>See R. Artle, Urbanization and Economic Growth in Venezuela, Institute of Urban and Regional Development, U.C., Berkeley, 1970.

Both collective services, or public goods, and public bads (exemplified by congestion in Mills-Ferranti) are manifestations of indivisibilities in consumption. As the authors say in their introduction, there is a strong tendency today to point to the multitude of public bads in the large cities (and to emerging and ominous new ones, such as urban guerilla warfare) in condemning the cities as being excessively large. It behooves one, however, to remind himself that cities were always both public goods and public bads. How exceedingly harsh the urban condition was for the majority of city-dwellers in centuries past! I can think of no better illustration than to cite the mortality rate among infants in Stockholm in the middle of the eighteenth century: An average of 387 babies, per 1,000, died before reaching the age of one. That was almost twice as high as the national average at the time. (Incidentally, it is also about 35 times as high as today's figure for Stockholm.)

What was it now again that Gertrude Stein used to say?  
A city is a public good, is a good, is a good, is a public bad, is a bad, is a bad.