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THREE DIMENSIONAL IMAGE RECONSTRUCTION USING PINHOLE ARRAYS*

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Summary

From multiple single-pinhole exposures of a three dimensional object, taken with a position-sensitive gamma-ray detector, tomographic images are formed with the aid of a computer. The effects of obscuring, off-plane activity are removed for a number of transverse planes through the object in a three dimensional image reconstruction method which uses the tomographic images as input. Lateral resolution, detector counting rate, total exposure times, and statistical accuracy of the final image are comparable to those of the single pinhole camera. The mathematical technique of the reconstruction method is applicable to images produced by a number of tomographic imaging devices.

Introduction

Nuclear Medicine imaging of radioactive organs can be considered as a branch of optics but it is a branch characterized by low intensity, low object contrast, low resolution, and pinhole optics. Exposures are often less than 400 gamma-ray events per picture element and the effects of quantum noise must be considered. The standard detector in this field, the scintillation camera, uses a pinhole aperture, or its variant, the parallel-hole collimator, and has about 5 mm resolution. This gives about 64x64 picture elements in its field of view which makes computer processing feasible.

Conventional pinhole and parallel-hole collimators give an image which is only a projection of the gamma-emitting source distribution. Surrounding activity masks the effect of smaller lesions and also makes difficult quantification of radioactivity in volumes of interest. A number of different instruments and techniques have been devised which provide depth information, such as the multiplane tomographic scanner with focussing collimator, the Tomocamera with a rotating collimator, the coincidence positron camera, and coded aperture imaging.[1] These methods have in common that they give tomographic images of the object, that is, that images of a given object plane have that plane in focus with a superimposed background of the blurred images of the other object planes. Various methods differ in how the out-of-focus images are blurred and considerable work has been done to determine blurring patterns which produce the fewest artifacts. Light microscopes also give tomographic images but the objects studied in microscopy are generally of high contrast, and the blurred contributions of the out-of-focus planes are not disturbing. In Nuclear Medicine imaging the objects are of relatively low contrast and it is frequently not possible to distinguish the object plane in focus from the underlying and overlying planes.

We present here a specific method for producing transverse planes of tomographic images and a general three dimensional image reconstruction method which removes the effects of off-plane activity from these planes. Although we use multiple single-pinhole images to make the tomograms, the reconstruction method is applicable to other types of tomographic images such as those from a scanner with focussing collimator or a positron camera.

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Three Dimensional Imaging Reconstruction Method

A tomographic image of a plane section through an object has a finite width slab of the object in focus. The thickness of this slab is the depth of field which depends on geometry and detector resolution. Also present in this tomographic image is an out-of-focus background contributed by the rest of the object. If this background is removed from a collection of these planes which are separated by the system depth of field, then the three dimensional object is known to an accuracy determined by the longitudinal and lateral resolutions of the system.

Formation of the Tomographic Images

To produce the tomograms needed, a number of separate exposures of the object are obtained with a pinhole aperture and a planar gamma ray detector, the pinhole being moved successively to different points of an N_h-point array (Fig. 1a). In our work N_h was usually between 9 and 24. The known location of the pinhole and the event positions recorded by the detector determine the direction of emission for each detected gamma-ray. Using these N_h single pinhole images tomographic images on N_p transverse planes through the

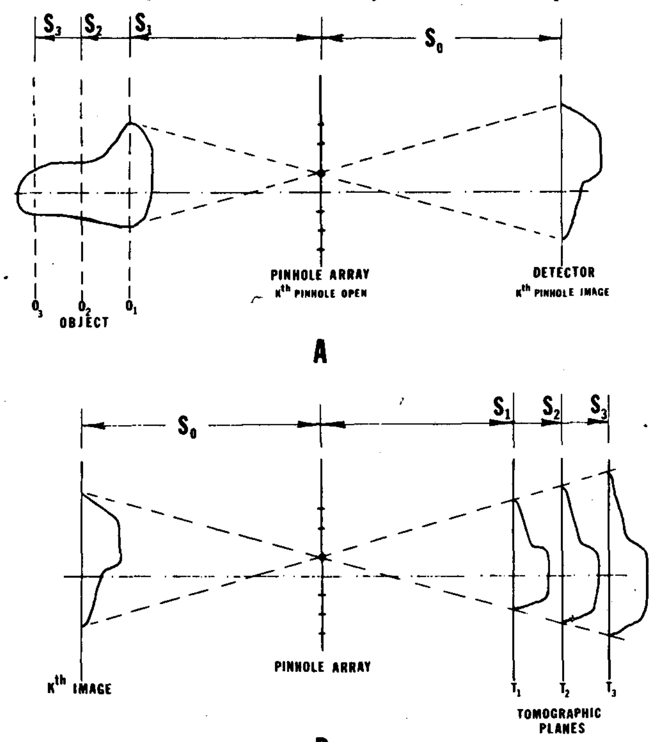


Fig. 1 Making the tomographic images. (a) Exposure of the k^{th} image using the k^{th} pinhole of the array. (b) The contribution of the k^{th} single pinhole image to the N_p = 3 tomographic planes. The final images on t_1, t_2, t_3 are the sum of the contributions from each of the N_h pinholes.

object are built up with a computer in the following manner (Fig. 1b). The k^{th} pinhole image is projected back through the k^{th} pinhole onto each of the N_p tomographic planes. This is done for all the pinhole images and the final tomographic images t_j, $j=1,\ldots N_p$, are the sum of the contributions from each of the N_h single-pinhole images. These are true tomographic images, the image t_1, for example, having in focus all the points of the corresponding object plane o_1 with all other planes contributing out-of-focus backgrounds.

Object Reconstruction from Tomographic Images

If the three dimensional object can be approximated by intensity distributions $o_i(\underline{r})$ on a finite number of planes, $i=1,\ldots N_p$, then, to reconstruct these o_i's from the tomographic images, one needs to consider the equations which describe the formation of the tomograms. We assume here and in the following that the pinholes in the array are small (delta functions). The effect of finite width pinholes will be discussed later.

We can find the tomographic images $t_j(\underline{r})$ produced by the single pinhole images by considering the response $h_{ij}(\underline{r},\underline{r}')$ of the j^{th} tomographic plane to a point source at $\underline{r}'$ located in object plane i (Fig. 2).

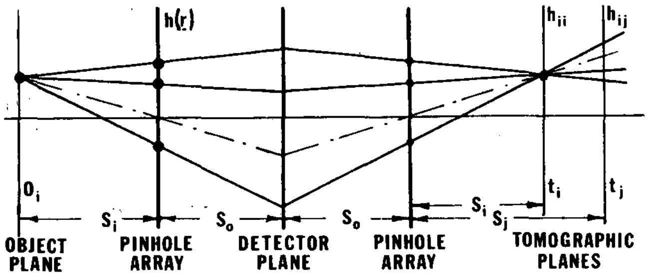


Fig. 2 Response of tomographic planes t_i,t_j to a point source in object plane o_i.

The figure shows that h_{ii} is a delta function of intensity N_h and that $h_{ij}(\underline{r},\underline{r}')$ is just the pattern h of the N_h hole array used, but displaced and with a size dependent on geometry. If $m_{ij} \equiv (S_i-S_j)/S_i$ is the size parameter and the pinholes in h are located at positions $\underline{r}_k$, $k=1,\ldots N_h$, we have

$$h_{ij}(\underline{r},\underline{r}') = h(\underline{r}-\underline{r}'S_j/S_i,\ m_{ij}) \qquad i,j=1,\ldots N_p \qquad (1)$$

$$= \sum_{k=1}^{N_h} \delta(\underline{r} - (\underline{r}'S_j/S_i + m_{ij}\underline{r}_k))$$

The contribution of an object distribution o_i to tomographic plane j is just the convolution o_i*h_{ij} and t_j is the sum of these contributions over the N_p object planes.

$$t_j(\underline{r}) = \sum_{i=1}^{N_p}\int o_i(\underline{r}')\ h_{ij}(\underline{r},\underline{r}')\ d^2\underline{r}' \qquad j=1,\ldots N_p$$

$$= \sum (S_i/S_j)^2 \int o_i(\underline{r}''S_i/S_j)\ h(\underline{r}-\underline{r}'',m_{ij})\ d^2\underline{r}''$$

$$= \sum_{i=1}^{N_p} (S_i/S_j)^2\ o_i(\underline{r}S_i/S_j) * h(\underline{r},m_{ij}) \qquad (2)$$

For a given value of $\underline{r}$, position relative to the optic axis, equation (2) is a set of N_p equations in the N_p variables o_i. The t_j's are combinations of single pinhole image data and $h(\underline{r},m_{ij})$ depends only on the pinhole locations in the array and the placement of the reconstruction planes.

Taking the Fourier transform of Eq.(2) and using the similarity theorem for Fourier transforms gives

$$T_j(\underline{u}) = \sum_{i=1}^{N_p} O_i(\underline{u}S_j/S_i)\ H_{ij}(\underline{u}) \qquad j=1,\ldots N_p \qquad (3)$$

where the quantities T_j,O_i,H_{ij}, Fourier transforms of the corresponding quantities of Eq.(2), are functions of the spatial frequency $\underline{u}$. To eliminate the j-dependence of the quantities Oi we let $\underline{u} = \underline{u}'/S_j$

$$T_j(\underline{u}'/S_j) = \sum O_i(\underline{u}'/S_i)\ H_{ij}(\underline{u}'/S_j) \quad j=1,\ldots N_p \quad (4)$$

For those (angular) spatial frequencies $\underline{u}'$ for which the determinant $D(\underline{u}') \equiv |H_{ij}(\underline{u}'/S_j)|$ is not zero, Eqs. (4) can be solved for $O_i(\underline{u}'/S_i)$ and inverse Fourier transforms give the desired background-free images $o_i(\underline{r})$.

The Determinant of the Reconstruction

For the determinant $D(\underline{u}')$ to be zero for some angular spatial frequency $\underline{u}'$ means that the reconstructed transform images $O_i(\underline{u})$ are not determined at the spatial frequency $\underline{u}'/S_i$. From Eq. (1) we can get

$$H_{ij}(\underline{u}'/S_j) = \sum_{k=1}^{N_h} e^{-i2\pi\underline{u}'\cdot\underline{r}_k[(S_i-S_j)/S_iS_j]} \qquad i,j=1,\ldots N_p$$

We find that the $N_p x N_p$ determinant $D(\underline{u}')$ formed from these quantities is always zero for $\underline{u}' = o$, since $H_{ij}(\underline{o}) = N_h$. This means that our reconstructions $o_i(\underline{r})$ are indefinite by an additive constant. This is not a problem if this is the only zero since this constant can be determined by some subsidiary condition, for instance, that $o_i(\underline{r})$ has no negative value.

The general solution for the zeros of the determinant $D(\underline{u}')$ is complicated and has not been done. The physical reason for zeros in the binocular vision case (N_h=2) is as follows. Spatial frequencies on each of two planes can be such as to give the same spatial frequency on the detector plane for each of the two single-pinhole exposures. On reconstruction one is uncertain how much of the original frequencies to ascribe to each plane. A regularly spaced array only increases the number of zeros. However, numerical calculations of $D(\underline{u}')$ for non-regular arrays of 9 to 24 pinholes for a three-plane geometry showed no zeros to exist, other than $\underline{u}=\underline{0}$. It would appear that zeros of the determinant can easily be avoided.

This method of three dimensional reconstruction was an outgrowth of our work with multiple pinhole arrays used as a coded aperture to obtain tomographic images. It is interesting to note that tomographic images produced with coded apertures do not contain enough information to allow object reconstructions by our method. In coded aperture imaging a single shadowgram is made by exposing the object simultaneously, for example, with all pinholes of our array $h(\underline{r})$.[2] This shadowgram is given by $s = \Sigma o_i * h_i$ where the subscript on $h(\underline{r})$ indicates the dependence of array scale on source plane location. A tomographic image is then obtained by using the original aperture to project the shadowgram image onto the desired plane. Thus we have for the tomogram $t_j = s*h_j = \Sigma o_i*(h_i*h_j)$. The determinant associated with h_i*h_j is identically zero.

Finite Width Pinholes

To investigate the effect of using an aperture array which has finite width pinholes of diameter d we consider the geometry of Fig. 2. A point source in plane i casts a shadow of a given pinhole onto the detector plane which has a diameter $d_i' = d(S_o+S_i)/S_i$. The diameter of this spot cast on the tomographic plane j through the corresponding zero-width pinhole on the right of the figure (zero-width because this is a mathematical operation not a physical process) is $d_{ij} = d_i'\ S_j/S_o = [d(S_o+S_i)/S_o](S_j/S_i)$. The real point response function that occurs in Eq. (2) is $\tilde{h}_{ij}(\underline{r}) = h_{ij} * w(d_{ij})$ where $w(d_{ij})$ represents a pinhole of width d_{ij}. In the approximation where the separation between planes is small compared to their distances from the aperture array we have $S_i/S_j \approx 1$ and $d_{ij} \approx d(S_o+S_i)/S_o \equiv d_i$. In this approximation we can write Eq. (2) as $t_j = \Sigma\ o_i * w(d_i) * h_{ij}$. This equation shows that the solution we obtain for an object plane, $o_i * w(d_i)$, using tomograms produced with finite-width pinholes, t_j, and the delta function point response function, h_{ij}, is a picture with the resolution that would be obtained with a single pinhole in the same geometry if only that object plane were present.

The depth resolution of tomograms produced with our multiple single-pinhole exposures can be obtained by considering, in the geometry of Fig. 2, the intensity produced, along the optic axis in the region near the tomographic plane i, by a point source in object plane i. The depth of field δz, full width at half maximum, is approximately

$$\delta z/S_i = (d/r_m)\,(1+S_i/S_o) \qquad (6)$$

where r_m is the distance in the aperture plane from the optic axis for the median pinhole, that is, for that pinhole for which half the pinholes lies further away.

Comparison with Transverse Axial Tomography

The reconstruction scheme we describe here can be characterized as axial tomography where depth resolution is done longitudinally, along the optic axis The only other method for three-dimensional image reconstruction of radioisotope distributions known to the authors is the extensively studied Transverse Axial Tomography (TAT).[3] This method, highly successful for X-ray transmission measurements of tissue density has only recently been applied to radioisotope imaging.[4] In this emission imaging with TAT, about 36 views are taken with a scintillation camera having a parallel hole collimator, the object being rotated by about 10° about a single axis perpendicular to the camera axis between successive views. Reconstructions are made of object planes parallel to the camera axis.

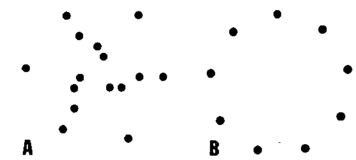
An extended comparison of the two methods cannot be made as of now. Reconstructions which have been done with the two methods have similar lateral resolutions and distances between reconstructed planes. Tissue absorbtion of gamma-rays has a great effect on the quality of image reconstruction with TAT since the absorbtion correction for a picture element can differ widely between two different views. In our axial tomography method the effects of absorbtion should be much smaller since all points on a given plane have approximately the same correction being at about the same distance from camera. Tissue absorbtion with TAT requires the use of the iterative least squares technique of reconstruction which, on a large computer (CDC 7600), requires about 6 seconds for reconstruction of a plane. On the same machine our method has taken about 0.3 sec/plane for a three plane reconstruction.

Results

The 15 pinhole array of Fig. 3a was used in a computer simulation in order to test the reconstruction method. The object (Fig. 4b) was located in the three planes, S_1 = 8 cm, S_2 = 10 cm, S_3 = 12 cm. The computer generated the tomographic images of Fig. 4c in the same three planes. The reconstructions produced, using these tomographic images as input, are given in Fig. 4d.

The results show excellent agreement with the original object. The tomograms were produced, however, with no statistical variation in intensity of the picture elements of the object from one single pinhole exposure to the next. In the more realistic case when the object picture elements vary statistically we find that there occurs a small component of background in the reconstructed images in addition to the expected variation in the intensity of the image elements. This is shown in Fig. 5 where an average of 400 events total from each object picture element has been collected, distributed statistically over the N_h = 15 single pinhole exposures. This gives a 5% statistical fluctuation in object picture element intensities. The

Fig. 3 Pinhole arrays used in making tomograms.



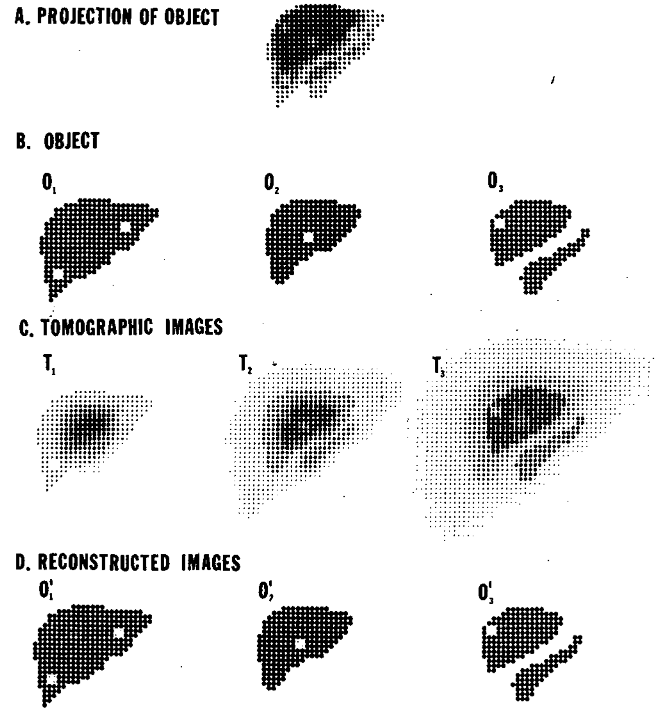


Fig. 4 Image reconstruction with no statistical variation of object picture elements. (a) Projected view of object as it would be seen by a parallel hole collimator. (b) The three dimensional object located in planes S_1 = 8 cm, S_2 = 10 cm, S_3 = 12 cm. (c) Tomographic images using 15 single pinhole exposures in the array of Fig. 3a. (d) The three dimensional image reconstructions of the object planes using the tomographic planes t_1, t_2, t_3.

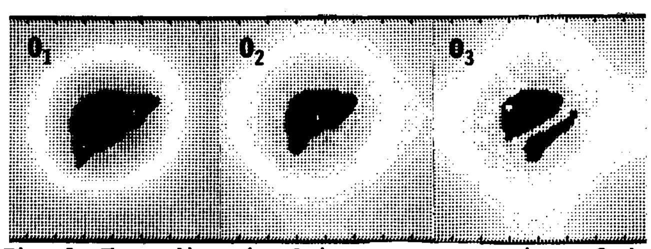


Fig. 5 Three dimensional image reconstructions of the objects of Fig. 4 but with 5% statistical variation of the object picture elements.

corresponding fluctuation measured in the image is 5.5%. Thus, the reconstruction method introduces a small amount of noise. If the total number of events collected is held fixed we find that the amount of introduced noise becomes larger as the number of pinholes is decreased. In addition to the relative insensitivity of the reconstruction to statistical noise, further work indicates that the method is not sensitive to errors in geometry. Reconstructions on planes differing somewhat from the actual object planes give results which are close to the actual object.

A xenon-filled multi-wire proportional chamber was used with a radioactive source to make the pinhole images. The 48x48 cm chamber has 2 mm resolution. Coordinates of detected events are digitized and put

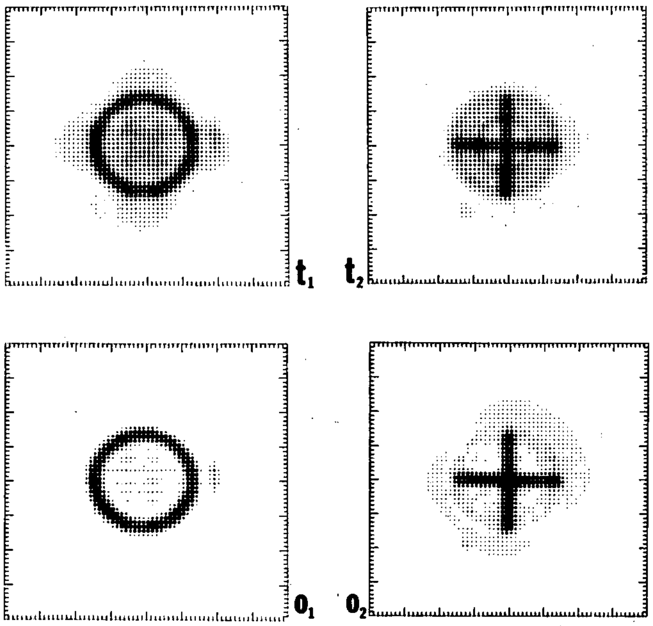


Fig. 6 Tomograms (t_1,t_2) and reconstructions (o_1,o_2) using wire proportional chambers and nine single-pinhole exposures in the array of Fig. 3b.

onto magnetic tape as input to the computer program. The object, a circle and a cross, was located on two planes, S_1 = 25 cm, S_2 = 20 cm. S_o was 39 cm. The nine hole array, Fig. 3b, was chosen to be circular (4 mm holes, 9 cm in diameter) so as to maximize depth resolution and field of view. The tomograms and their reconstructions, the first attempted, are given in Fig.6. There was 100,000 counts in each plane, bin width in the display was 2½ mm, resolution was 6 mm. Although the choice of objects was such as to make their nature readily identifiable from the tomograms alone the reconstruction method has clearly removed artifacts and background successfully from the tomograms.

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