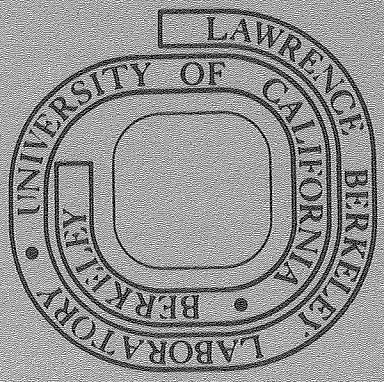
LBL-9056 Rev
UC-66b

NUMERICAL MODEL OF TRANSIENT TWO-PHASE FLOW IN A WELLBORE

Constance W. Miller

October 1979

Prepared for the U. S. Department of Energy
under Contract W-7405-ENG-48



Printed in the United States of America
Available from
National Technical Information Service
U.S. Department of Commerce
5285 Port Royal Road
Springfield, VA 22161
Price Code: A03

LBL-9056 Revised

NUMERICAL MODEL OF TRANSIENT TWO-PHASE FLOW IN A WELLBORE

Constance W. Miller

Earth Sciences Division
Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

Work supported by the Division of Geothermal Energy of the U. S. Department of Energy

TABLE OF CONTENTS

ABSTRACT

Transient two-phase flow in a geothermal well has been modelled with a finite-difference approximation. One-dimensional flow is assumed. The equations of mass, momentum, and energy are solved using a partially implicit method. Terms that would place a severe time restriction on the calculation are solved implicity, while other terms are solved explicitly for computational ease and efficiency. Homogenous flow of one component at thermodynamic equilibrium is assumed initially, but the extension of the model to include slip and a finite rate of condensation or evaporation or a noncondensible gas is given. The wellbore model includes heat and mass transfer and is coupled to a simple reservoir model. Using the model, the transient behavior of a single- or two-phase well during a well test was investigated. Results show that when the reservoir has a relatively large value of kh, as exists in a geothermal field, the slope of the log (pressure) vs. log (time) curve is not necessarily a unit slope when testing a homogenous reservoir. The early-time behavior of this curve is controlled by the interaction of the flow in the reservoir and that in the well, and can be used to determine near bore values of kh. Heat loss in the wellbore is shown to also affect the pressure vs. time plot in a well test. The time to reach the pseudo steady region increases when heat transfer is important, even in a relatively "warm" well, and the slope of the P vs. log t curve is no longer $q\mu/4\pi kh$ as derived in the petroleum literature.

INTRODUCTION

For a complete analysis of a geothermal system, a model of the flow in the wellbore and in the reservoir is necessary. The equations governing the flow in these two cases are different, so reservoir models based on Darcy-type flow cannot be extended to predict the wellbore flow. Several numerical codes have been written to simulate the two phase flow of steam and water in a geothermal well (Sugiura and Faronq, 1979; Gould, 1974; Juprasert and Sanyal, 1977; Ryley, 1964; Elliot, 1975). Most of these models include the slip between the phases, using experimentally determined values of holdup and friction factors for each particular flow regime (such as annular or slug). However, the models reported in the geothermal cases all assume steady state flow. (There are several transient two phase codes, which will be considered below, that were developed for the nuclear energy industry.) The steady state models can be used to approximate downhole conditions from wellhead measurements given an equation of state that includes the effect of the non-dissolved gas and solids.

However, during well testing the steady-state models are not useful because transient changes in the well itself are important. At early times after a flow-rate change has been made in the well, the mass flow rate into the well does not equal the mass flow rate out because of wellbore storage. Under these circumstances, a steady-state model, which naturally assumes $\dot{m}_{in} = \dot{m}_{out}$, is not appropriate. Nevertheless, one might think that once wellbore storage is over, the steady state flow model could be used to determine downhole pressure changes, given the wellhead pressure measurements. At this point, the changes in pressure with time are usually small, so slight errors

in friction factor, slip, or heat transfer can produce relatively large errors in dP/dt. Also, many of the slip correlations used are based on water/air or oil/gas flow, whereas the flow of steam/water has interacting phases that should result in different effective values of holdup. Because of the unknown in these values, a two-phase flow model that includes transient effects is necessary for well-test analysis. Because the flow between the reservoir and the well is a function of the reservoir properties, a transient two-phase code could be used to determine near-bore values of kh/μ, even when wellbore storage is important. During this early time, the error in the experimentally determined values of slip, friction factor, and heat-transfer coefficient are less important than at other times because dP/dt is large.

I have developed a transient two-phase numerical code for one-dimensional flow in the wellbore. The code has been coupled with a reservoir model of simple, one-phase, radial flow in a porous media. (Only flashing in the wellbore is being considered initially.) At early times, the flow in the reservoir is close to radial, so such a code could be used to predict the drawdown pressure curve for single-phase flow and for fluid that is flashing in the wellbore. I have obtained very interesting results with the model, which will be illustrated below. (Additional details are given in Miller, 1979.)

The model developed includes heat and mass transfer and has been written in a way so that very few, if any, iterations are needed at each time step. The effect of noncondensible gas can be included easily at a later stage by introducing a second continuity equation.

As mentioned above, transient multiphase codes have been developed for the nuclear energy industry (Liles and Reed, 1978: Harlow and Amsden,

1975; and Hirt and Romero, 1975). However, these codes usually require several iterations or the inversion of a nonsimple matrix, making them difficult to use efficiently. Some of the models are not designed to include the effect of more than one component. The model developed here solves transient homogeneous equilibrium flow. However, the basic solution procedure can be extended to include slip as well as nonequilbrium effects with few changes.

A description of the method is given below along with example calculations. This report is divided into three sections. The first section is a description of the working model, the second section describes how the model could be extended to include slip and nonequilibrium or a noncondensible gas, and the last section reports calculations of the flow in the wellbore.

NUMERICAL MODEL

The problem is to solve the equations of mass, momentum, and energy for one-dimensional flow. The basic difference between the equations that model the wellbore and those that model the reservoir is that the Navior Stokes equation of momentum is used for the wellbore case instead of assuming Darcy-type flow. The nature of the flow changes when transients are important. For the fluid in the wellbore, the flow can be shown to be governed by a wave equation; in the reservoir, it is governed by a diffusion-like equation. In models developed for flow in porous media for a two-phase system, relative permeability curves are used to describe the flow of the gas and the liquid phases. The relative permeability curves take into account the interaction of the fluid with the rock and the interference of the phases with one another. However, for flow in a tube, the drag of one phase on the other is an important contribution to the flow, that is, the liquid phase can be

carried along by the gas. Only a negative relative permeability could account for this situation. In summary, using a porous media model to describe flow in a wellbore ignores the physics of fluid flow.

For a detailed theoretical description of the two-phase separated flow, six equations are necessary: two for mass, two for momentum, and two for energy. Additional relationships among the thermodynamic variables complete the set of equations. However, to solve this set of equations, interface interaction terms must be included, which are not well known. It is possible to approximate the fluid flow with fewer equations, while retaining the important characteristics of the flow. I have used empirical correlations to replace some of the equations. For the initial development of the model, I further reduced the equations to solve two phase homegeneous equilibrium flow.

The equations solved are:

$$\text{continuity,}\quad \frac{\partial}{\partial t}(\rho) + \frac{\partial}{\partial x}(\rho v) = 0 \tag{1}$$

$$\text{momentum,}\quad \frac{\partial}{\partial t}(\rho v) + \frac{\partial}{\partial x}(\rho v^2) + \frac{\partial P}{\partial x} + \rho g + \frac{f\rho v^2}{2D} = 0 \tag{2}$$

$$\text{and energy,}\quad \frac{\partial}{\partial t}(\rho e) + \frac{\partial}{\partial x}(\rho v e) + P\frac{\partial v}{\partial x} + 2H\frac{T_r - T_w}{r_w} = 0 \tag{3}$$

where ρ and e are the mass-averaged values of density and energy, respectively, in the two-phase fluid. Only three equations are solved here instead of the original six. One momentum equation has been replaced with an experimentally determined correlation of the relative velocity, v_r, in terms of the other parameters in the flow. The correlation used above was $v_r = 0$, or that there is no slip. One continuity equation was eliminated by assuming the average density in the fluid can be expressed as a function of the average energy

and pressure or $\rho = fn(e,P)$. (An extension to nonequilibrium flow or to include a noncondensible gas will require a second continuity equation.) The assumption of thermodynamic equilibrium also implies that the temperature in the liquid and vapor are equal, thus eliminating one energy equation. The unknowns are ρ, e, P, and v; the four equations are mass, momentum, energy, and the equation of state.

The viscous term has been written as a friction factor times $\frac{1}{2}\rho v^2/D$. Such a term accounts for the frictional losses with the wall of the wellbore. When slip is taken into account, it also includes the frictional losses between the two phases.

My approach is to solve the equations using a finite-difference approximation with a partially implicit method. Terms that would impose very restrictive time steps, such as $\Delta t < \Delta x/a$, where a is the sound speed, are evaluated implicitly while the other terms are evaluated explicitly for ease and computational efficiency. The finite-differenced equations are:

$$\text{continuity,}\quad \frac{\rho_i^{\ell+1} - \rho_i^{\ell}}{\Delta t} + \frac{(\rho v)_{i+\frac{1}{2}}^{\ell+1} - (\rho v)_{i-\frac{1}{2}}^{\ell+1}}{\Delta x} = 0 \tag{4}$$

$$\text{momentum,}\quad \frac{(\rho v)_{i+\frac{1}{2}}^{\ell+1} - (\rho v)_{i+\frac{1}{2}}^{\ell}}{\Delta t} + \frac{\left[(\rho v^2)_{i+\frac{1}{2}} - (\rho v^2)_{i-\frac{1}{2}}\right]^{\ell}}{\Delta x} +$$

$$\frac{P_{i+1}^{\ell+1} - P_i^{\ell+1}}{\Delta x} + \rho_{i+\frac{1}{2}}^{\ell} g + \left[\frac{f(\rho v)^2_{i+\frac{1}{2}}}{2D}\right]^{\ell} = 0 \tag{5}$$

and energy,

$$\rho_i \frac{\left(e_i^{\ell+1} - e_i^{\ell+1}\right)}{\Delta t} + \rho v \frac{\left(e_i^{\ell} - e_{i-1}^{\ell}\right)}{\Delta x} + P_i^{\ell+1} \frac{\left(v_{i+\frac{1}{2}} - v_{i-\frac{1}{2}}\right)^{\ell}}{\Delta x}$$

$$+ 2H \frac{\left(T_r - T_w\right)^{\ell}}{r_w} = 0 \tag{6}$$

The energy equation has been written in a nonconserving form. The reason for this will be apparent in the solution procedure. The thermodynamic variables are computed at the nodal points (i, i+1, etc.), and the velocity is computed at the half-nodal points (i+½, etc.).

The solution procedure is to combine the four equations to give one expression for the new pressure. To use this method, the equation of state is used in the form of:

$$d\rho = \left(\frac{\partial \rho}{\partial P}\right)_e dP + \left(\frac{\partial \rho}{\partial e}\right)_P de$$

instead of $\rho = fn(P,e)$. The density change in time is written as:

$$\rho_i^{\ell+1} - \rho_i^{\ell} = \left(\frac{\partial \rho}{\partial P}\right)_i^{\ell} (P_i^{\ell+1} - P_i^{\ell}) + \left(\frac{\partial \rho}{\partial e}\right)_i^{\ell} (e_i^{\ell+1} - e_i^{\ell}) \quad , \tag{7}$$

the partial derivatives being evaluated at the old time levels. However, to eliminate large changes in the derivates, equation 7 is rewritten as:

$$\rho_i^{\ell+1} - \rho_i^{\ell} = \left(\frac{\partial \rho}{\partial P}\right)_i^{\ell} \left(P_i^{\ell+1} - P_i^{\ell}\right) + \left(\frac{1}{\rho}\frac{\partial \rho}{\partial e}\right)_i^{\ell} (\rho_i)^{\ell} \left(e_i^{\ell+1} - e_i^{\ell}\right) \quad . \tag{8}$$

The expression $(1/\rho)(\partial\rho/\partial e)$ varies approximately linearly while $(\partial\rho/\partial e)$ changes value abruptly. In addition, $\rho_i^{\ell} (e_i^{\ell+1} - e_i^{\ell})$ is solved for directly with the energy equation. Also at each calculation, the new value of $\rho_i^{\ell+1}$ is compared with the value of $\rho^{\ell+1}$, calculated from $\rho = fn(P^{\ell+1}, e^{\ell+1})$. If the difference is more than some specified percent, an iteration is necessary. The new pressure profile is recalculated with the partial derivatives in equation 7 as an average of the old and new values. The best average is to emphasize the value of $\partial\rho/\partial P$ at level $\ell+1$ because the system can self-correct any errors created when crossing the saturation line. When the fluid flashes, the derivative $\partial\rho/\partial P$ increases abruptly and then decreases slowly. If the

calculation provides an overexpansion of the fluid parcel when crossing the saturation line, then an average of the old and new derivatives at the next calculation can compensate for the overexpansion because of the decreasing value of $\partial\rho/\partial P$. If the fluid parcel underexpands when crossing the saturation line, no amount of averaging of the new and old derivatives can compensate. When condensing across the saturation line it is important not to over-condense, because the derivative $\partial\rho/\partial P$ is approximately constant. However, a slight error when crossing a saturation line for one particular fluid parcel produces little error in the net flow.

The four equations (4, 5, 6, 8) are combined in the following manner. The continuity equation is solved, using the expression for the new value of $(\rho v)^{\ell+1}$, which is given by the momentum equation. Equation 4 can be written in finite difference form:

$$\rho_i^{\ell+1} - \rho_i^{\ell} = -\frac{\Delta t}{\Delta x}\left[(\rho v)_{i+\frac{1}{2}}^{\ell+1} - (\rho v)_{i-\frac{1}{2}}^{\ell+1}\right] \quad .$$

Then equation 5 is used to express $(\rho v)^{\ell+1}$ in terms of P:

$$\begin{aligned}\rho_i^{\ell+1} - \rho_i^{\ell} = -\frac{\Delta t}{\Delta x}\Bigg\{\Bigg[&(\rho v)_{i+\frac{1}{2}}^{\ell} - \frac{\Delta t}{\Delta x}(\rho v^2_{i+\frac{1}{2}} - \rho v^2_{i-\frac{1}{2}})^{\ell} \\ &- \frac{\Delta t}{\Delta x}(P_{i+1}^{\ell+1} - P_i^{\ell+1}) - \rho_{i+\frac{1}{2}}^{\ell} g\Delta t - \frac{f(\rho v^2)_{i+\frac{1}{2}}^{\ell}}{2D}\Delta t\Bigg] \\ -\Bigg[&(\rho v)_{i-\frac{1}{2}}^{\ell} - \frac{\Delta t}{\Delta x}(\rho v^2_{i-\frac{1}{2}} - \rho v^2_{i-3/2})^{\ell} \\ &- \frac{\Delta t}{\Delta x}(P_i^{\ell+1} - P_{i-1}^{\ell+1}) - \rho_{i-\frac{1}{2}} g\Delta t - \frac{f(\rho v^2)_{i-\frac{1}{2}}^{\ell}}{2D}\Delta t\Bigg]\Bigg\} \quad .\end{aligned} \tag{9}$$

Equation 8 is used to write $\rho^{\ell+1}$ in terms of $P^{\ell+1}$ and the above equation is regrouped to give

$$\left[-2 - \frac{\Delta x}{\Delta t}^2 \left(\frac{\partial \rho}{\partial P}\right)_e\right] P_i^{k+1} + P_{i+1}^{k+1} + P_{i-1}^{k+1} = -\left(\frac{\Delta x}{\Delta t}\right)^2 \left(\frac{\partial \rho}{\partial P}\right)_e P_i^k$$

$$+ \frac{\Delta x}{\Delta t}\left[(\rho v)_{i+\frac{1}{2}} - (\rho v)_{i-\frac{1}{2}}\right]^k - (\rho_{i+\frac{1}{2}} - \rho_{i-\frac{1}{2}})^k g \Delta x \qquad (10)$$

$$- \frac{f\Delta x}{4R}\left[(\rho v^2)_{i+\frac{1}{2}} - (\rho v)^2_{i-\frac{1}{2}}\right]^k + \frac{1}{\rho}\left(\frac{\partial \rho}{\partial e}\right)^k \rho(e_i^{k+1} - e_i^k)\left(\frac{\Delta x}{\Delta t}\right)^2$$

$$- (\rho v^2_{i+\frac{1}{2}} - 2\rho v^2_{i-\frac{1}{2}} + \rho v^2_{i-\frac{3}{2}})^k.$$

The difference $\rho(e_i^{k+1} - e_i^k)$ is given by equation 6. Equation 10 can be written as AP = x, where A is a tridiagional matrix. The solution is straight-forward if the boundary conditions are specified.

The boundary conditions considered were: (1) specification of pressure and mass flow rate or velocity at either the wellhead or downhole; or (2) specification of pressure at both wellhead and downhole. The pressure must be specified at one of the boundaries. If the flow rate is measured, then $\partial(\rho v)/\partial t$ is known, and the pressure can be determined from the momentum equation:

$$\left(\frac{\partial P}{\partial x}\right)_b^{k+1} = -\frac{(\rho v^{k+1} - \rho v^k)_b}{\Delta t} - \frac{(\rho v_b^2 - \rho v_{b-1}^2)^k}{\Delta x}$$

$$- \rho_{b-1} g - \frac{(f\rho v^2)_b}{2D}$$

where b stands for the boundary.

Equation 10 is used to determine the new pressure profile. Once P^{k+1} and e^{k+1} are known, the density is calculated using equation 7, because this was the equation used to express ρ in terms of P and e when the four equations

were combined. If one calculated the density at the new time level with $\rho = fn(P,e)$ instead of $d\rho = fn(dP, de)$, then two different expressions would be used and mass would not be conserved identically. However, as mentioned above, the two ρ's are compared and an iteration is done if they are significantly different. This situation occurs when crossing the saturation line.

Once the new value of ρ is known, the velocity is determined from either the continuity or the momentum equation. If the mass flow rate is being specified as a function of time, the velocity is best calculated by the continuity equation. Given $(\rho v)^{\ell+1}$, the velocity at position ℓ is

$$v_i^{\ell+1} = \left[(\rho v)_{i+1}^{\ell+1} + \frac{\Delta x}{\Delta t} \quad (\rho^{\ell+1} - \rho^{\ell}) \right] / \rho^{\ell+1}$$

because ρv is known at the wellhead, the velocity can be calculated successively down the wellbore. If the pressure is given as a function of time at the top, it is easier to calculate the new velocity using the momentum equation, at least at the first node.

The wellbore model was connected to a reservoir model so that the drawdown pressure was consistent with the variable flow velocity into the well. The reservoir flow was assumed to be radial, homogeneous, and single phase. The fluid was allowed to flow into the wellbore over a finite length, h. (The effect of flashing in the well only was investigated initially. The intent was not to spend a great deal of effort on the reservoir itself but to at least use a model that would give a drawdown pressure consistent with the flow into the well.) When the initial transient changes take place in the well, the flow around the wellbore is almost radial. At a later stage, the simple reservoir model will be expanded to include flashing in the reservoir.

The equation solved in the reservoir was

$$\frac{\partial P}{\partial t} = \frac{k}{\mu c \phi} \left(\frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial P}{\partial r}\right) \quad . \tag{11}$$

Equation 11 was solved at several different heights in the reservoir to allow flow into the wellbore over a finite height, making it possible to consider a layered reservoir system interacting with the well. Far from the wellbore, a constant pressure was assumed. At the well/formation boundary, mass had to be conserved.

The mass flow rate out of the reservoir and into the well must be equal. If the reservoir and the wellbore equations are combined, the matrix multiplying the pressure vector will no longer be tridiagonal, increasing the calculation time for each step. Instead, each set of equations was solved separately. The boundary condition at the reservoir/well interface was

$$(\rho v) = \frac{kh}{\mu} \rho \frac{2}{r_w} \left[\frac{P_r^{\ell}(1) - P_w^{\ell+1}}{r(1) - r_w}\right] \quad .$$

The new pressure in the wellbore is solved first using the old value of the reservoir pressure. Then the new pressure in the reservoir is calculated by determining the fluid that flowed from the reservoir into the well over that time that is, the same derivative of $(\partial P/\partial r)$ was used in both calculations. Because this boundary condition for the reservoir flow is solved explicitly, there is a stability limit. The stability limit in terms of the radial spacing is (Zerzan, 1979, private communication.):

$$\left| \frac{(\Delta r_o + \Delta r_1) r_1 \Delta r_o + 2\alpha\Delta t \left[\frac{\Delta r_o}{\Delta r_1} r_{3/2} \pm \left(r_{1/2} + \frac{\Delta r_o}{\Delta r_1} r_{3/2}\right)\right]}{(\Delta r_o + \Delta r_1) r_1 \Delta r_o - 2\alpha\Delta t \, r_{1/2}} \right| \geq 1$$

where $\Delta r_o = r_1 - r_o$, $\Delta r_1 = r_2 - r_1$, and $r_{1/2} = (r_o + r_1)/2$.

To account for the initial large changes near the bore, and the changes far away at later times, a variable radial grid was used. The finite differenced equation solved was:

$$\left(\frac{p^{\ell+1}_{r_i} - p^{\ell}_{r_i}}{\Delta t}\right) = \frac{\frac{k}{\mu c \phi}\left[r_{i+\frac{1}{2}}\left(\frac{p^{\ell+1}_{r_{i+1}} - p^{\ell+1}_{r_i}}{r_{i+1} - r_i}\right) - r_{i-\frac{1}{2}}\left(\frac{p^{\ell+1}_{r_i} - p^{\ell+1}_{r_{i-1}}}{r_i - r_{i-1}}\right)\right]}{\frac{1}{2}\left(r_{i+1} - r_{i-1}\right)}$$

The variable grid was calculated by using a logarithmic transformation

$$r\ (i) = \frac{\left(A^{N_i/\Delta N_o} - 1\right)}{\left(A^{1/\Delta N_o} - 1\right)} B + r(1)\ .$$

The quantity N_i is just $i(\Delta N)$, and B, A, and ΔN_o are adjustable constants.

Once the new pressure is determined in the well, the change in the reservoir pressure is calculated as a function of r and at several different heights. The temperature of the fluid in the producing zone is assumed to be specified. However, the energy loss of the fluid in the wellbore itself must also be included.

One important difference between the well testing in a geothermal well and that done by the petroleum industry is the high temperature found in the geothermal reservoir. Because of this, it is important to include the heat transfer out of (or into) the wellbore from the surrounding rock in numerical models. Ramey (1962) estimated the heat loss from the wellbore as a function of time. This method is less valid when the flow rate is continually changing. To account for the heat transfer, I solved the conduction equation for the temperature in the rock. Again, I used a variable grid to allow for large changes near the wellbore and smaller changes further away. However, the grid variation is closer to the wellbore in this case, because the temperature

changes will be over a smaller spatial region than the pressure changes. The conduction equation solved for the temperature changes is similar to that of the pressure in the reservoir. However, the new temperature is solved for implicitly to avoid time step limitations imposed by the small grid spacing near the wellbore. The temperature far from the well was assumed to be the initial geothermal gradient. At the rock/wellbore boundary, the heat transfer was matched. When the energy equation in the fluid is solved, the heat transfer at the wall is

$$q = 2\pi r_w \Delta x \; H \; (T_r^{\ell} - T_w^{\ell}) \quad .$$

When the temperature in the reservoir is calculated, the boundary condition at the rock/well interface is:

$$k_r \frac{\partial T_r}{\partial r} = H \; (T_r^{\ell+1} - T_w^{\ell+1}) \quad .$$

The heat transfer is matched throughout the calculation except for the first time step for the energy in the fluid that is, the heat transfer into the reservoir for the calculation of the reservoir temperature at time $\ell+1$ is just the heat that will leave the fluid during the calculation for time $\ell+2$. The only heat transfer not matched is for the first calculation of the fluid energy, but usually $T_r = T_w$ for this case, so q will be zero. No stability problems were encountered because the temperature in the reservoir is solved implicitly, even at the boundary.

The model then solves for the transient flow in the wellbore, including heat loss to the surrounding rock. The simple reservoir model provides that the fluid flow into the well is consistent with the drawdown pressure. The assumption of no slip and thermodynamic equlibrium was made. However, the

extension of the model to nonslip and nonequlibrium flow, or to include non-condensible gases can be made.

EXTENSION OF NUMERICAL METHOD

To develop the initial numerical method, two-phase homogenous equlibrium flow was assumed. However, the method can be extended. First, to include slip in the flow, additional terms must be added to both the momentum and the energy equations. These terms account for the fluid accelerating or decelerating because of evaporation or condensation. For the momentum equation, the term that must be added is

$$\frac{\partial}{\partial x} \frac{(1-\alpha)\alpha\rho_\ell\rho_g v_r^2}{\rho_m}$$

where α is the volumetric quality and $\rho_m = (1-\alpha)\rho_\ell + \alpha\rho_g$. The average velocity is defined as

$$v_m = \frac{(1-\alpha)\rho_\ell v_\ell + (\alpha)\rho_g v_g}{\rho_m} .$$

The convection terms were evaluated explicitly in the basic model, so the addition of this term will not involve any modification of the method itself. The relative velocity is given usually by experimental measurements at present. It is specified as a function of void fraction and total flow rate. The main problem in adding the term is in choosing which correlations to use, since the scatter in different correlations can be large.

For the energy equation, the terms that must be included are:

$$\frac{\partial}{\partial x} \frac{(1-\alpha)\rho_\ell\alpha\rho_g v_r(e_g - e_\ell)}{\rho_m}$$

because of the fluid changing velocity and

$$P \frac{\partial}{\partial x} \frac{\alpha(1-\alpha)\, v_r(\rho_\ell - \rho_g)}{\rho_m}$$

because of the volumetric expansion work when the fluid changes phase. Again the convection terms are evaluated explicity so they can be added very easily.

The more difficult problem is the extension of the method to allow for nonequilibrium flow or to include a noncondensible gas. The method in either case is similar so only the former case will be considered in detail. Condensation and evaporation do not take place instantaneously, but require a finite time. If this time is as long or longer than the pressure transients in the flow, nonequilibrium can be very important. Many well tests with flashing in the bore have pressure responses that seem to indicate a two-layer reservoir. These responses may be due to nonequilibrium effects in the wellbore and it is important in modelling these cases to understand if nonequilibrium could account for these pressure changes.

To include nonequilibrium, a second continuity equation must be used. The evaporation (or condensation) will be given by some rate term. However, inclusion of non-equilibrium will require some modification of the current method. The continuity equation needed is

$$\frac{\partial}{\partial t} \alpha\rho_g + \frac{\partial}{\partial x} \alpha\rho_g v_g = I$$

where I is the rate of evaporation or condensation. Now instead of $\rho_m = fn(e_m, P)$, the expression for the densities will be determined separately:

$$\rho_\ell = fn(e_\ell, P), \text{ and } \rho_g = fn(e_g, P)$$

where e_ℓ and e_g are functions of temperature only. It will still be assumed

that $T_\ell = T_g$. Such an assumption means that a parcel of fluid can either condense or flash, but can not do both at the same time. Since the temperature of the two phases are equal, the mixture as a whole can be either above or below the saturation temperature. In a completely nonequilibrium situation, the temperature of the phases would differ. Although the approach used here assumes $T_\ell = T_g$, it does allow for the finite rate of flashing or condensation. This model is similar to that suggested by Liles and Reed (1978).

The individual equations of state can be combined to give

$$\rho_m = fn(T,P,\alpha).$$

Again the differential form of the equation of state will be used:

$$d\rho_m = \left.\frac{\partial \rho_m}{\partial T}\right|_{P,\alpha} dT + \left.\frac{\partial \rho_m}{\partial P}\right|_{T,\alpha} dP + \left.\frac{\partial \rho_m}{\partial \alpha}\right|_{T,P} d\alpha$$

The detail of writing $\partial\rho_m/\partial t$ in terms of $\partial P/\partial t$ is given in Appendix A. However, the general method is: (1) write $d\rho_\ell$ and $d\rho_g$ in terms of (dT, dP); (2) use the equation of energy to determine $\partial T/\partial t$ in terms of $\partial P/\partial t$ and $\partial\alpha/\partial t$; and (3) use the continuity equation to write $\partial\alpha/\partial t$ in terms of $\partial P/\partial t$. Using the equations for $\partial P/\partial t$, we can invert the tridiagonal matrix to solve for the new pressure.

Once the new pressure is known, the new void fraction, temperature, density, and average energy can be calculated from the expressions used to develop the equation

$$A\ P = B \quad .$$

To consider the effect of noncondensible gases instead of nonequilibrium effects, the density is written as a function of P, T, S, where S is the saturation of the gas phase (noncondensible gas and steam). The average

density is

$$S\rho_g + (1 - S)\rho_\ell$$

where ρ_g, ρ_ℓ are the densities of the gas and liquid phases respectively. To determine S, a continuity equation conserving the noncondensible gas can be used. Then

$$\frac{\partial}{\partial t} Sx_g\rho_g = -\frac{\partial}{\partial x} Sx_g\rho_g v_g$$

where x_g is the mass fraction of say CO_2 in the gas phase. The steam is assumed to be at the saturation density corresponding to the temperature of the phase. Then $\rho_g = \rho_{lg}/(1-x_g)$ where ρ_{lg} is the density of the steam and is specified as a function of temperature only.

To solve the set of equations, it is necessary to specify an equation of state for the mixture. One set that is suggested by Iglesias (1979) is that

$$x_g = 1 - \frac{P_{lg}}{P}$$

and that x_ℓ can be determined by specifying the boiling curves of an H_2O - CO_2 mixture. By specifying the equation of state, a complete set of equations will again exist, and the solution procedure is as specified above. The model is presently being updated to include these details.

EXAMPLE CALCULATIONS

The numerical model has been used here to determine the early-time behavior of the wellbore flow for both single-phase and two-phase flow. Examples of the calculations are given below. Figure 1 is a plot of the pressure changes that propagate down the wellbore after a stepwise change in flow rate at the wellhead. In this figure, the calculations were done for a liquid-

filled well flowing under a positive head. The well is flowing steadily at one rate and then the rate is increased. At early times after the change, the increase in produced fluid is obtained from the well instead of from the reservoir. A pressure drop propagates down the well. After a certain amount of time, depending on the compressiblility of the fluid, the pressure pulse interacts with the formation/well boundary. In the particular case plotted, the reservoir has a large value of kh/μ, and it is capable of supplying more fluid for this pressure drop than the well could. This case results in a reverse pressure pulse, which propagates back up the well, cancelling part of the initial pressure drop. The pressure pulse oscillates until it is finally damped out by the interaction with the boundaries.

Figure 2 shows the same calculations for a flashed system. Again, the fluid is flowing slowly and then the flow rate was suddenly increased. The pressure pulse propagates down the well. However in this case, there is a brine/two-phase boundary. The dashed line in Figure 2 shows the approximate location of the flash point. (Obviously, as the flow rate is increased, the flash level drops.) When the pressure pulse reaches this boundary, the pulse is partly reflected and partly transmitted. The reflected pulse propagates back toward the surface. In the single-phase region, the propagation of the signal is much faster; oscillations are mainly in the two-phase region.

One can use the program to determine the pressure drawdown during the early time of a well test. It has been shown (Miller, 1979) that the initial slope of a log-log plot of pressure vs. time in well testing is not necessarily unity, as derived in the petroleum literature. As seen in the figures above there is a time delay until the downhole pressure registers the change made at the wellhead. Wellbore storage curves are derived assuming the fluid in

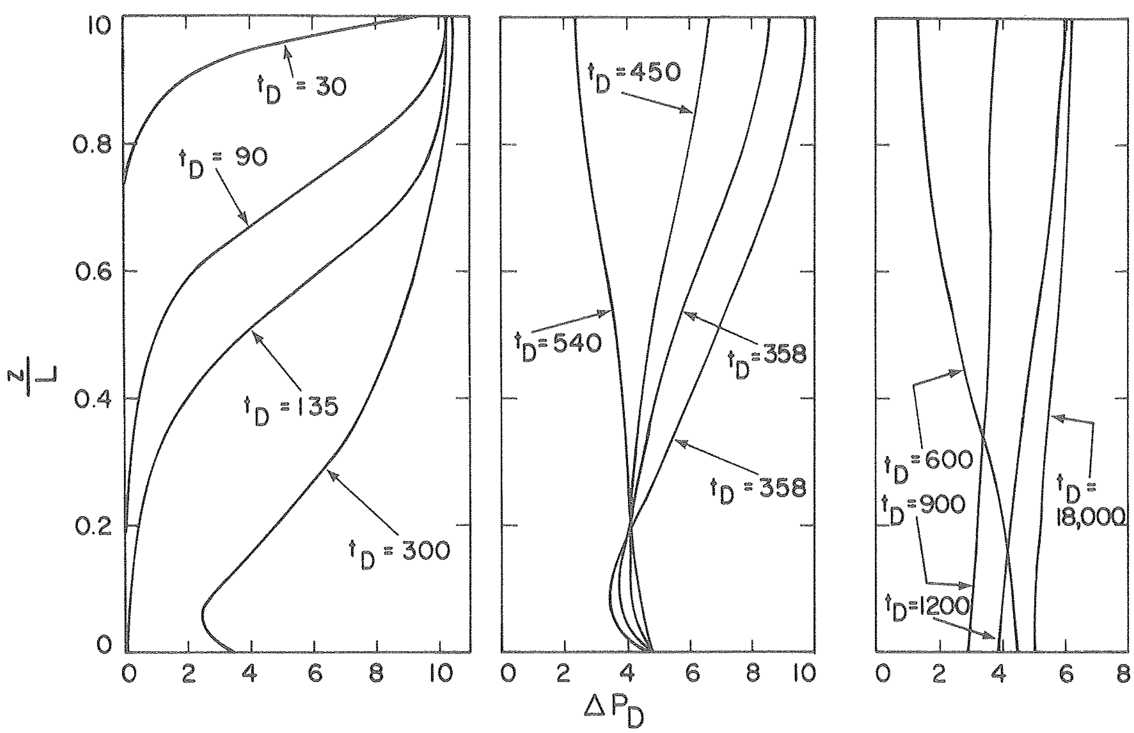


Figure 1. Propagation of pressure pulse down a wellbore as a function of time for a liquid-filled well. XBL 794-7404

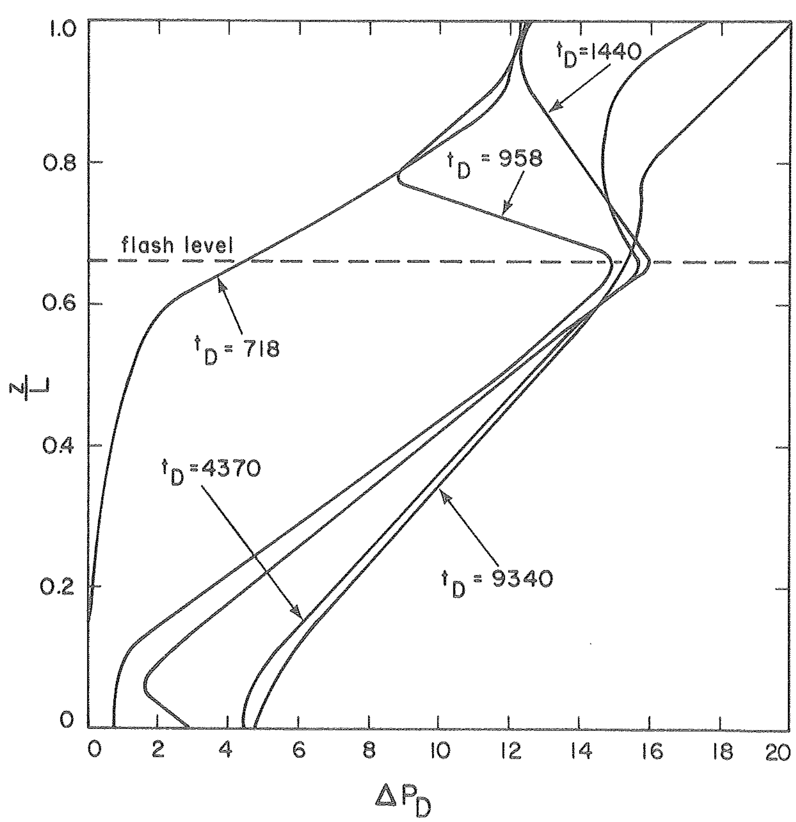


Figure 2. Propagation of pressure pulse down a wellbore in a two-phase well as a function of time. XBL 794-7403

the well responds as a well-mixed fluid. By being able to model the transient flow in the wellbore, it has been possible to calculate the expected drawdown in the well, taking into account the nonuniformities in the well. The results show that another nondimensional time t_{RW} must also be determined in addition to the average wellbore storage coefficient C_D (see Fig. 3). The plot shows calculations for flashed and unflashed wells. The parameter t_{RW} is defined as

$$\left(\frac{\mu}{kh}\right) \frac{D^2}{8} \frac{1}{\rho} \left(\frac{\partial \rho}{\partial P}\right)_s^{\frac{1}{2}} .$$

As kh/μ decreases, t_{RW} increases, and the early-time behavior of the log P vs. log t approaches a one-to-one plot. As kh/μ increases, t_{RW} decreases, and the slope of the log P vs. log t curve is steeper than unity.

The numerical model can also be used to determine the effect of heat loss to the rock surrounding the wellbore during a well test. The calculations shown are done for a well that has been flowing and is reasonably "warm". The assumed temperature profile is shown by the inset in Figure 4. The well has been flowing steadily, then the flow rate is decreased. Figure 4 compares the buildup curve and subsequent drawdown curve with and without heat transfer. The calculations show that even when the well has been flowing for several hours and the rock around the bore has been heated, heat transfer during a well test is still important and must be considered. When the well test data are plotted, the slope of the P vs. log t curve in the pseudo-steady region is significantly affected by the heat transfer. Also the time to reach the pseudo-steady region is longer when heat transfer is important.

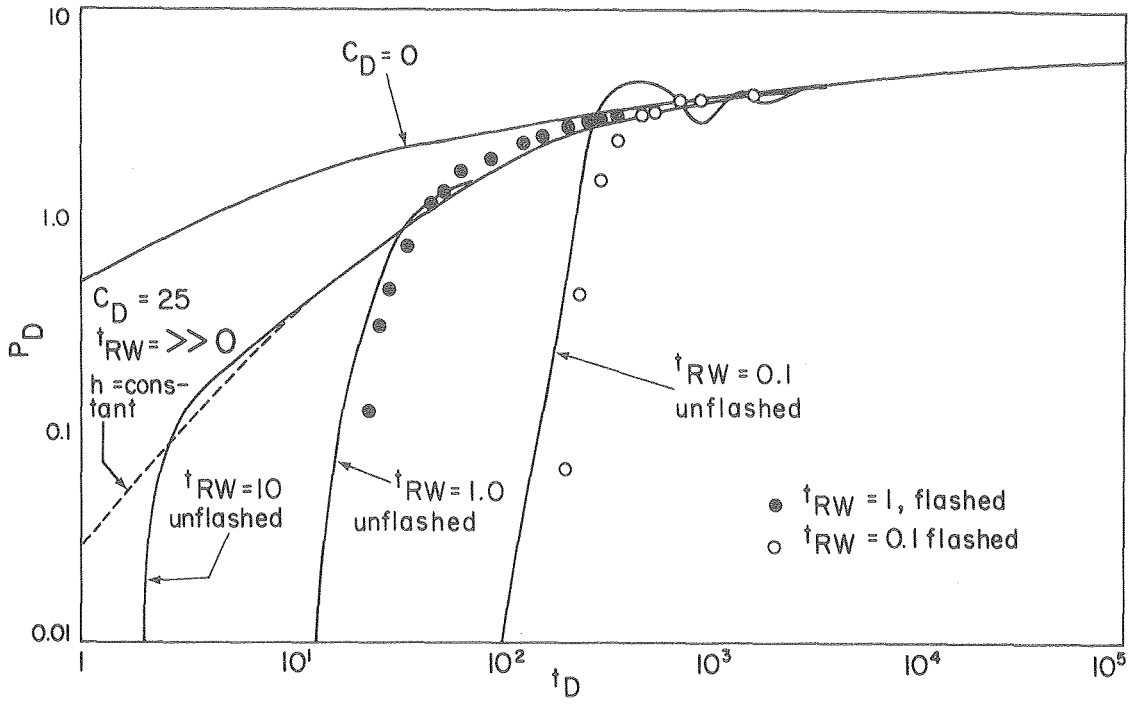


Figure 3. Nondimensional pressure vs. nondimensional time for different values of C_D and different values of the parameter t_{RW}.
XBL 794-7402

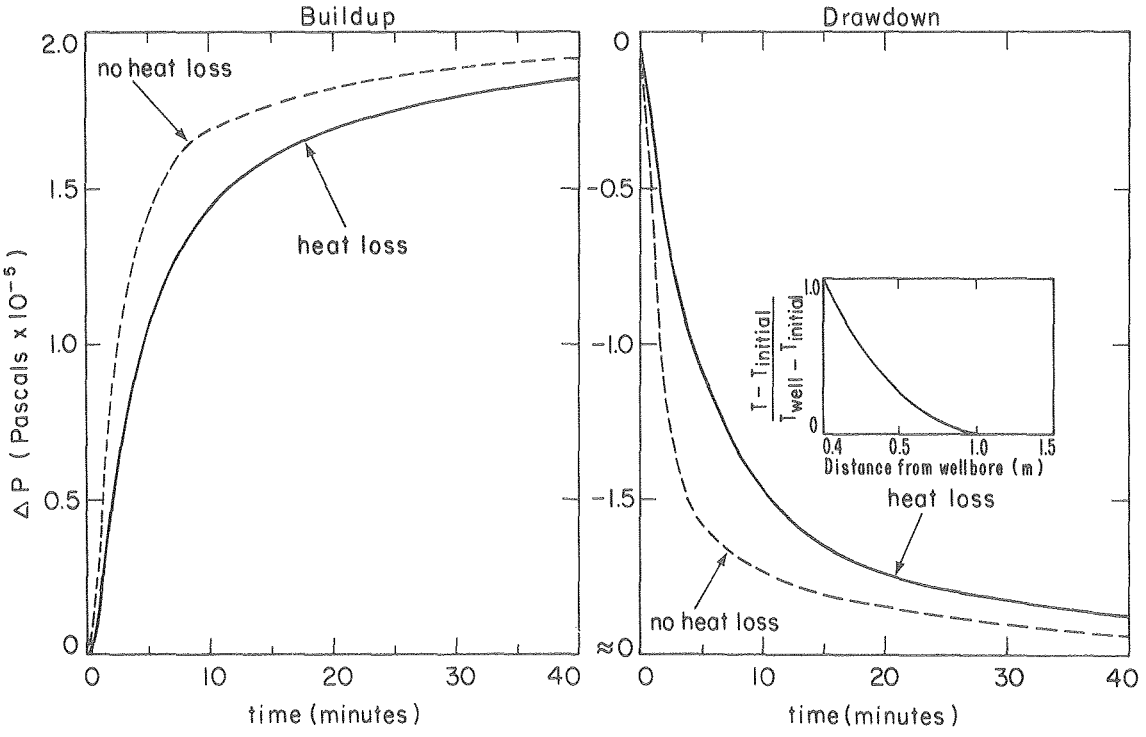


Figure 4. Effect of heat transfer on the buildup and drawdown well test.
XBL 794-7412

NOMENCLATURE

a	Speed of sound
c	Reservoir compressibility
C_D	Wellbore storage coefficient
D	Diameter of well
e	Specific energy
f	Friction factor
g	Gravity
h	Thickness of reservoir
H	Heat transfer coefficient
I	Rate of evaporation or condensation
k	Permeability
k_r	Conductivity of rock
$\mathring{m}$	Mass flow rate
P	Pressure
q	Volume flow rate
r	Radial direction
r_w	Radius of well
S	Saturation
s	Specific entropy
t	Time
T_r	Temperature of rock surrounding well
T_w	Temperature of fluid in well
x	Axial direction
v	Velocity
v_r	Relative velocity ($v_g - v_\ell$)
α	volumetric quality (holdup)
ρ	density
μ	Absolute viscosity
ϕ	Porosity

Subscripts

r	Reservoir
w	Well
ℓ	Liquid
m	Mixture
g	Gas
D	Non-dimensional parameter

REFERENCES

Elliot, D.G., 1975, Comparison of brine production methods and conversion processes for geothermal electrical power generation: Environmental Quality Laboratory, California Institute of Technology, EQL Report #10.

Gould, T. L., 1974, Vertical two phase steam water flow in geothermal wells: Journal of Petroleum Technology, v. 26, p. 883-842.

Harlow, F. H. and Amsden, A. A., 1975, Numerical calculation of multiphase Fluid Flow: Journal of Computational Physics, v. 17, p. 19-52.

Hirt, C. W. and Romero, N. C., 1976, Application of a drift-flux model to flashing in straight pipes: Los Alamos, Los Alamos Scientific Laboratory, Report LA-6005-MA.

Iglesias, E. R., 1979, Approximate properties of H_2O-CO_2 mixture for geothermal reservoir and wellbore simulations: Lawrence Berkeley Laboratory LBL-9991 (in preparation).

Jupraset, S., and Sanyal, S. K., 1977, A numerical simulator for flow in geothermal wellbores: Geothermal Resources Council, Transactions, v.1, p. 159-161.

Liles, P. R., and Reed, W. H., 1978, A semi-implicit method for two-phase fluid dynamics: Journal of Computational Physics, v. 26, p. 390 - 407.

Miller, C. W., 1979, Wellbore storage in geothermal wells: presented at 1979 Annual Meeting of Society Petroleum Engineers, Las Vegas, Nevada, paper SPE 8203.

Ramey, H. J., Jr. 1962, Wellbore heat trasmission: Journal Petroleum Technology, v. 225, p. 427-435.

Ryley, D. J., 1964, Two-phase critical flow in geothermal steam wells: International Journal of Mechanical Science, v. 6, p. 273.

Sugiura, T., and Farouq, S. M., 1979, A comprehensive wellbore steam-water flow model for steam injection and geothermal applications: paper presented at 1979 California Regional Meeting of Society of Petroleum Engineers, Ventura, Ca., April 1979, SPE-7966.

APPENDIX A

To use the same numerical method that was developed for modelling the equilibrium flow, $\partial\rho_m/\partial t$ must be written in terms of $\partial P/\partial t$ only. In the non-equilibrium case, the number of independent variables has increased by one, and the average energy is no longer convenient to use as one of the independent variables. Now, temperature, pressure, and void fraction will be the independent variables. To be able to write the density change in time as a function of the pressure change only, the vapor continuity and the energy equations must be used to express $\partial\alpha/\partial t$ and $\partial T/\partial t$ in terms of $\partial P/\partial t$ respectively. After the new pressure is evaluated along the wellbore, the other independent and dependent variables can be calculated.

In the overall continuity equation (1), the convective term, $\partial(\rho_m v_m)/\partial x$, is evaluated by using the momentum equation as developed in the first section of this paper. The difficult part is expressing $\partial\rho_m/\partial t$ in terms of the three independent variables. The average density is $(1-\alpha)\rho_\ell + \alpha\rho_g$. The derivatives of ρ_m with respect to T, P, and α are:

$$\left.\frac{\partial\rho_m}{\partial T}\right|_{P,\alpha} = (1-\alpha)\left.\frac{\partial\rho_\ell}{\partial T}\right|_{P,\alpha} + \alpha\left.\frac{\partial\rho_g}{\partial T}\right|_{P,\alpha} ,$$

$$\left.\frac{\partial\rho_m}{\partial P}\right|_{T,\alpha} = (1-\alpha)\left.\frac{\partial\rho_\ell}{\partial P}\right|_{T,\alpha} + \alpha\left.\frac{\partial\rho_g}{\partial P}\right|_{T,\alpha} , \text{ and}$$

$$\left.\frac{\partial\rho_m}{\partial\alpha}\right|_{T,P} = \rho_g - \rho_\ell .$$

Then

$$\frac{\partial\rho_m}{\partial t} = \left[(1-\alpha)\frac{\partial\rho_\ell}{\partial T} + \alpha\frac{\partial\rho_g}{\partial T}\right]\frac{\partial T}{\partial t} + \left[(1-\alpha)\frac{\partial\rho_\ell}{\partial P} + \alpha\frac{\partial\rho_g}{\partial P}\right]\frac{\partial P}{\partial t} + (\rho_g - \rho_\ell)\frac{\partial\alpha}{\partial t} .$$

The continuity equation for the vapor component is rewritten to give $\partial\alpha/\partial t$:

$$\rho_g \frac{\partial \alpha}{\partial t} = -\alpha \left(\frac{\partial \rho_g}{\partial T} \frac{\partial T}{\partial t} + \frac{\partial \rho_g}{\partial P} \frac{\partial P}{\partial t} \right) - \frac{\partial}{\partial x}(\alpha \rho_g v_g) + I \qquad \text{(A1)}$$

where the term in the first parenthesis is just $\partial\rho_g/\partial t$.

The energy equation can be used to relate $\partial T/\partial t$ in terms of $\partial P/\partial t$. The average energy, e_m is

$$\frac{1}{\rho_m} \left[(1-\alpha)\ \rho_\ell e_\ell + \alpha \rho_g e_g \right]$$

and the derivative of $(\rho_m e_m)$ in terms of $\partial T/\partial t$, $\partial P/\partial t$, and $\partial\alpha/\partial t$ is

$$\begin{aligned} \frac{\partial}{\partial t}\ (\rho_m e_m) = (1-\alpha) & \left\{ \left[\frac{\partial}{\partial T}\ (\rho_\ell e_\ell) \right] \frac{\partial T}{\partial t} + \left[\frac{\partial}{\partial P}\ (\rho_\ell e_\ell) \right] \frac{\partial P}{\partial t} \right\} \\ + \alpha & \left\{ \left[\frac{\partial}{\partial T}\ (\rho_g e_g) \right] \frac{\partial T}{\partial t} + \frac{\partial}{\partial P} \left[(\rho_\ell e_\ell) \right] \frac{\partial P}{\partial t} \right\} \\ + (\rho_g e_g - \rho_\ell e_\ell) & \frac{\partial \alpha}{\partial t} \end{aligned} \qquad \text{(A2)}$$

However, the energy equation gives $\partial(\rho_m e_m)/\partial t$, which is:

$$\begin{aligned} & -\frac{\partial}{\partial x}\ (\rho_m e_m v_m) - \frac{\partial}{\partial x} \left[\frac{(1-\alpha)\rho_\ell \alpha \rho_g v_r (e_g - e_\ell)}{\rho_m} \right] - P \frac{\partial v_m}{\partial x} \\ & - P \frac{\partial}{\partial x} \left[\alpha \frac{(1-\alpha) v_r (\rho_\ell - \rho_g)}{\rho_m} \right] - \frac{2H\ (T - T_r)}{r_w} \quad . \end{aligned} \qquad \text{(A3)}$$

All these terms are evaluated explicitly so they can be calculated directly. Then equation A2 can be solved for dT/dt by using equation A1 to express $\partial\alpha/\partial t$ in terms of $\partial P/\partial t$ and $\partial T/\partial t$, and equation A3 is used to determine $\partial(\rho_m e_m)/\partial t$ in terms of known values. The resultant form is

$$A \frac{\partial T}{\partial t} = B \frac{\partial}{\partial t}\ (\rho_m e_m) + C \frac{\partial P}{\partial t} + D \ .$$

where

$$A = \left[(1-\alpha) \frac{\partial}{\partial T}\ (\rho_\ell e_\ell) + \alpha \frac{\partial}{\partial T}\ (\rho_g e_g) - (\rho_g e_g - \rho_\ell e_\ell) \frac{\alpha}{\rho_g} \frac{\partial \rho_g}{\partial T} \right]$$

$$B = 1,$$

$$C = \left[(1-\alpha)\frac{\partial}{\partial P}(\rho_\ell e_\ell) + \alpha\frac{\partial}{\partial P}(\rho_g e_g) - (\rho_g e_g - \rho_\ell e_\ell)\frac{\alpha}{\rho_g}\frac{\partial \rho_g}{\partial P}\right]$$

and

$$D = -(\rho_g e_g - \rho_\ell e_\ell)\left[-\frac{\partial}{\partial x}(\alpha \rho_g v_g) + I\right]$$

Since $\partial\rho_m/\partial t$ has been written in terms of $\partial P/\partial t$, one can evaluate the overall continuity equation for the new pressure. Once the new pressure is determined, the method is similar to that derived in the first section of the report.

U.S.GPO:1980-698-169/F-112

TECHNICAL INFORMATION DEPARTMENT
LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720