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Ultrahigh-speed on-chip all-optical amplitude modulation
via monolayer graphene saturable absorption

A thesis submitted in partial satisfaction
of the requirements for the degree Master of Science
in Electrical Engineering

by

Michael Thomas Hoff

2016

ABSTRACT OF THE THESIS

Ultrahigh-speed on-chip all-optical amplitude modulation
via monolayer graphene saturable absorption

by

Michael Thomas Hoff

Master of Science in Electrical Engineering

University of California, Los Angeles, 2016

Professor Chee Wei Wong, Chair

As global internet traffic continues to climb at an uncurbed rate, the development of correspondingly fast physical communications technology is as crucial as ever. Key milestones have already been reached with graphene-integrated waveguide structures in the field of electro-optic modulators, bolstering hope that practical ultrahigh-speed graphene optical modulators could indeed be actualized, with the potential to yield revolutionary results. But while the electrical gate-tunability of these devices has been soundly demonstrated, there remains the question of modulation speed, limited primarily by the parasitic RC-time intrinsic to such electrical control. Recent advancements have already shown graphene's robust performance as a broadband saturable absorber, due to its massless Dirac fermions and linear dispersion relation near the Dirac point. By capitalizing on these material properties and moving away from electrical control into the all-optical

regime, the only limitation on device operation speed is due to the intrinsic material properties of graphene – namely, excitation, thermalization, and recombination times of the excited fermionic carriers. In this work, an on-chip microring resonator amplitude modulator is constructed, and a theoretical analysis of the intrinsic and extrinsic speed limitations on its all-optical operation is developed. Laboratory data is presented, and a computational model is built to further enhance an understanding of the modulator’s operation and potential limitations. It is found that laboratory data confirms material response times corresponding to modulation speeds exceeding 50 GHz – although limited by the experimental instrumentation rather than the device itself – while the numerical model reveals a higher performance threshold and suggests optimizations to achieve faster speeds in the near future.

The thesis of Michael Thomas Hoff is approved.

Benjamin S. Williams

Kang Lung Wang

Chee Wei Wong, Committee Chair

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2016

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1. INTRODUCTION

Having recently surpassed 3 billion individuals, the expansion of the world's internet-using population seems to show little sign of slowing [1]. In fact, at the current growth rate, it is projected that by 2020 the internet will connect and serve in excess of 4 billion users across the globe, representing an astounding 194.4 exabytes of IP traffic per month – up 168% from the monthly figure of 72.5 exabytes in 2015 [2]. In order to support and promote this expanding development into the coming decade, it is, needless to say, imperative that the physical hardware technologies underpinning the internet's communications infrastructure keep pace with these tremendous demands for bandwidth. Discovered only within the past decade, the highly-conductive material known as graphene immediately became the subject for a wide scope of experimental investigation for electrical applications, with the hope that its conductive speed could play a key role in the evolution of electrically-based processing and related communicational operations [3–5]. Indeed, this continues to be an active field of exploration.

But perhaps even more remarkable are the optical properties of graphene, which naturally suggest applications in ultrafast fiber optic communications. Indeed, it has been remarked that it is in this field of material optics that graphene has shown its “true colors” [6], and accordingly a wide array of electro-optic applications have been probed both in academia and in industry [7], including a number of electro-optic amplitude modulation systems. But as will be discussed in the succeeding section, while these electro-optic modulators have indeed achieved remarkably fast operation speeds, the relevant RC time constant introduces an intrinsic and fundamental limitation on the speed of such devices' operation. Therefore, means by which to circumvent the electrical control within a graphene optical modulator are highly desirable to discover. Most advantageous is the

potential transition to all-optical amplitude modulation, a platform which holds the potential to revolutionize the world of optical communications and help ensure the global demand for bandwidth is met.

This work presents an on-chip platform for ultrafast all-optical amplitude modulation. This development relies heavily on the unique properties of graphene, which will be discussed accordingly, along with prior developments upon which this work is founded and the important physical principles thereby utilized (Section 2). The physical design and qualitative operation of the modulator chip will then be described (Section 3), with an in-depth theoretical analysis of its operation presented thereafter (Section 4). Experimental procedures and the laboratory data thereby attained will be detailed (Section 5), and the measured findings will be compared with numerical simulations to characterize the system’s performance limits (Section 6). Finally, suggestions for future work and improvement will be given (Section 7).

2. BACKGROUND AND PRIOR WORK

2.1. Optical Properties of Graphene

First successfully synthesized in 2004 [8], graphene is a material of carbon atoms arranged in a single-atom-thick hexagonal lattice [9], which boasts many uniquely interesting properties. Among these are its carrier properties – massless Dirac fermions of constant Fermi velocity $v_F \simeq 10^6 \frac{m}{s}$ [9–11] – as well as gapless band structure and linear dispersion relation for both electrons and holes near the Dirac point (Fig. 1) [9, 10, 12, 13]. These material attributes conspire to make graphene a strong broadband optical absorber, as it is guaranteed to have a resonant transition at virtually any incident photon energy. As such, graphene has demonstrated a constant, wavelength-independent normal-incidence

absorption coefficient of $\pi\alpha_f \simeq 2.3\%$, where α_f is the fine-structure constant given by $\alpha_f = \frac{e^2}{\hbar c}$ (in the centimeter-gram-seconds system of units) [9, 12–14]. This calculated spectrally-invariant optical absorption of $\simeq 2.3\%$ has been confirmed experimentally by optical reflectivity and transmission measurements taken on bulk SiO_2 substrates with incident photons of energy between 0.2 and 1.2 eV [13]. These measurements have confirmed the predicted sheet conductivity of $\frac{\pi e^2}{2h}$. However, this simple behavior does not hold as well for lower photon energies near and below 0.2 eV, in which case the effects of finite temperature and, especially, the background-doping-induced shift of the sheet potential away from the Dirac (charge-neutral) point. While the common phenomenon of small p-doping in both epitaxially-grown and chemical-vapor-deposited graphene is important to consider, it is at the same time worth noting that an optical field of single-photon energy at or below 0.2 eV would fall into the mid-infrared region of the electromagnetic spectrum, with a wavelength at or above 6 μm – considerably larger than the 1.7 μm ceiling of the ultra-long wavelength U-band utilized in optical communications technology.

In addition to its strong broadband absorption, graphene's natural ability to function as a robust *saturable* absorber, via a phenomenon known as Pauli blocking (Fig. 2), is of immense interest. When exposed to an optical field of photon-energy $\hbar\omega$, a resonant interband transition is excited, resulting in a number of carrier electrons at $+\frac{\hbar\omega}{2}$ in the conduction band and carrier holes at $-\frac{\hbar\omega}{2}$ in the valence band. Within 10-150 fs, these carriers will thermalize to form a hot Fermi-Dirac distribution [12], thereby blocking some of the formerly-possible interband transitions of energy less than or equal to $\hbar\omega$. This “spreading-out” process of the excited carriers across their relevant energy bands causes the absorption rate to be altered not only for photons exactly at this same frequency, but for lower frequencies as well. This is because, on a certain level, these excited carriers

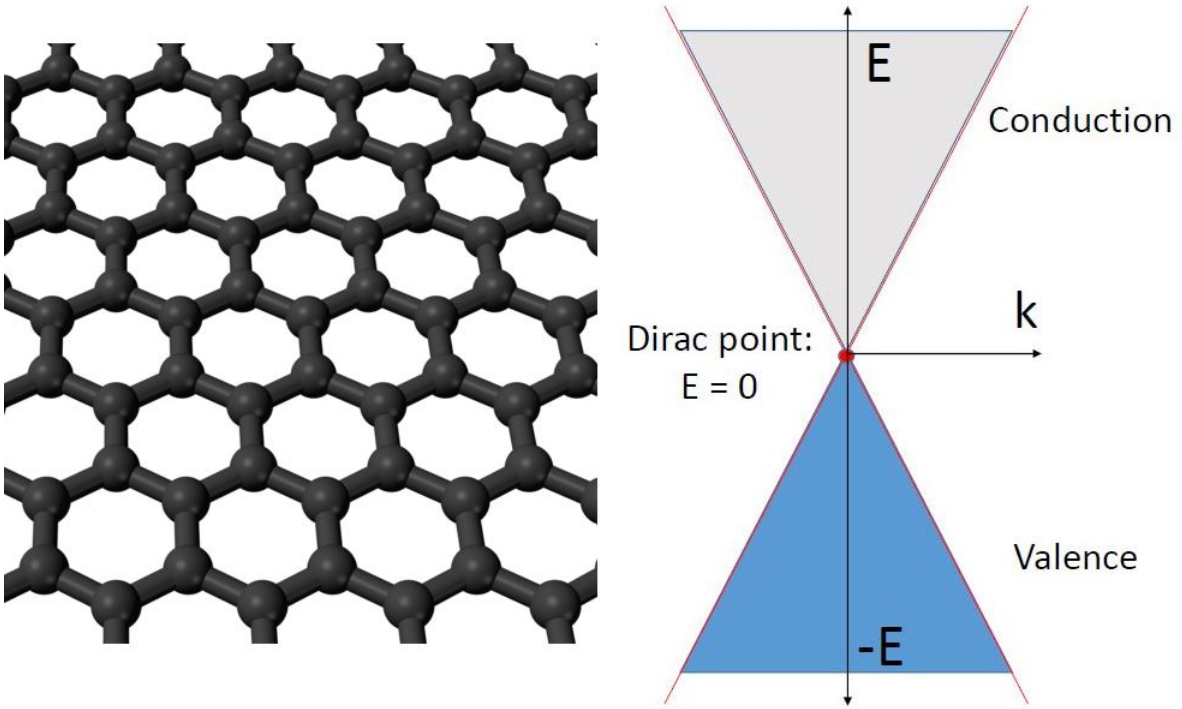


FIG. 1: A diagram of the the hexagonal lattice structure (left) and unique band structure of graphene. The carbon atoms exhibit strong sp^2 hybridization, with an inter-atomic spacing of $\simeq 0.142$ nm [9]. Features of the band structure include a zero-bandgap and linear dispersion relation given by $E(\vec{k}) = s\hbar v_F |\vec{k}|$ [15], where $s = 1$ in the conduction band and $s = -1$ in the valence band. In the intrinsic case, the Fermi energy is located at the Dirac point.

can be considered to cause a temporary Fermi energy up-shift in the conduction band and down-shift in the valence band, such that a transient pseudo-bandgap is introduced: the Fermi-Dirac distributions for carriers in both bands are thereby shifted, reducing the probability that a valence electron will be found above $-\frac{\hbar\omega}{2}$ or that a conduction hole will be found beneath $+\frac{\hbar\omega}{2}$. Subsequently, further absorption of an incident optical field whose photonic energy is less than or equal to the magnitude of this pseudo-bandgap will be reduced accordingly. Of course, depending on how many photons comprise the incident optical field, enough carriers may be excited such that the probability of finding

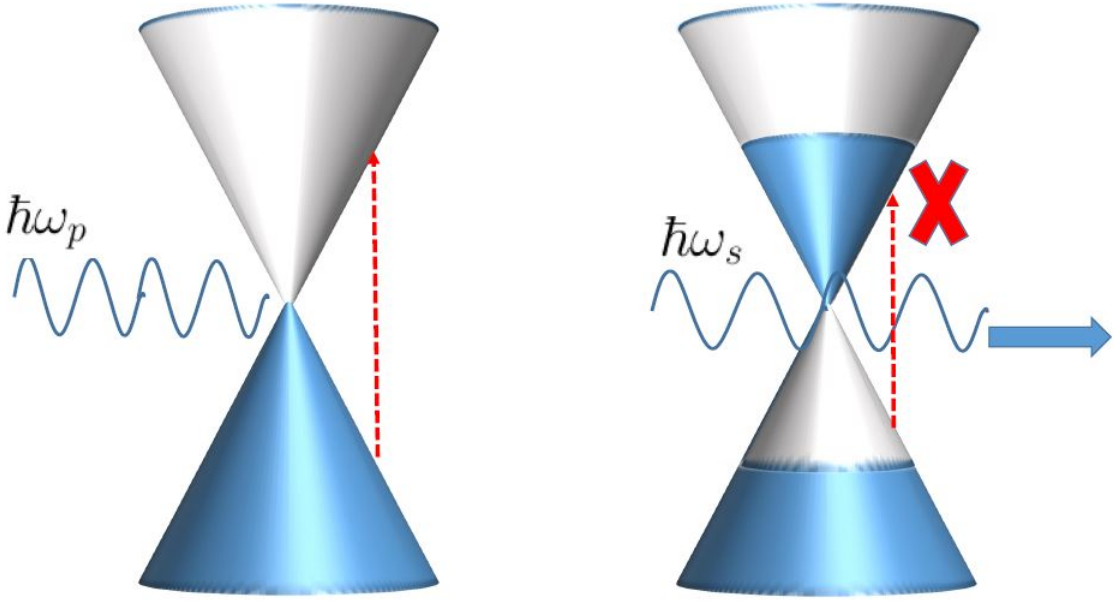


FIG. 2: A schematic outlining the Pauli blocking process in graphene. As a result of an exciting field of photonic energy $\hbar\omega_p$, electrons are excited from the valence to the conduction band. As the higher-energy conduction states are filled, less are available to support subsequent excitation and therefore photo absorption weakens. In the extreme case, all states in the relevant energy range are filled and any signal beam of photonic energy $\hbar\omega_s \leq \hbar\omega_p$ will pass through the material completely transparently.

any additional excitable carriers within the induced bandgap goes to zero. Therefore, for a sufficiently intense incident optical field – that is to say, one which contains a sufficiently large number of exciting photons – all possible excited states in the relevant energy range can be quickly filled, thereby allowing subsequent photons at or below this frequency to pass through the material transparently. The hot Fermi-Dirac distribution, together with the transparency it induces, persists through the following ≈ 1 picosecond after excitation, during which time the thermalized carriers are further cooled by intraband phonon scattering, before finally undergoing interband recombination until the original equilibrium distribution is restored [11, 12, 16, 17].

Similarly, rather than by the use of a saturating optical field, graphene's optical transmission may instead be manipulated via direct control of the material's Fermi level through doping, gating, or some combination thereof, means for which have been previously demonstrated [18–21] and will be discussed in the following subsection.

2.2. Graphene-Integrated Waveguide Structures

The strength of graphene's interaction is, however, severely limited by an atomically-short interaction length in the case of (single-layer) normal incidence. To overcome this limitation, the technique of integrating graphene along the surface of an optical waveguide has been developed. By incorporating the graphene layer along the waveguide surface, the interaction length of the optical field with the graphene layer is significantly extended. Although the interaction per unit length is weaker in this orientation, the total length of the interaction is amplified by many orders of magnitude: tens of microns for waveguide-integrated structures, versus a few-hundred picometers in the case of single-sheet normal incidence. It should be noted that, in contrast to the case of normal incidence, this tangential interaction is dependent on the spatial distribution of the optical field and therefore on the distance between the waveguide surface and graphene layer. As discussed in [22], this leads to a linear absorption coefficient described by

$$\alpha = \frac{1}{P(z)} \frac{\partial P(z)}{\partial z} = \frac{\sigma_0}{2P(z)} \int_L |\vec{E}_t(x, y_0)|^2 dx, \quad (1)$$

where x is the transverse direction, z is the propagation direction, y_0 is the distance between the top of the waveguide and the graphene layer ($y_0 = 0$ for maximum absorption strength), σ_0 is the intraband optical conductivity, $P(z)$ is the optical power, $\vec{E}_t(x, y_0)$ is the optical field along the interface, and L is the length of the graphene layer. Addition-

ally, because the graphene layer in this orientation will only interact with the in-plane (tangential) component of the optical field [23], it is polarization-sensitive; in this study, the polarization is controlled to maximize the strength of the interaction.

This nascent practice of incorporating graphene along on-chip waveguide structures has already shown great promise for the efficient exploitation of graphene's unique optical properties. Thus far, the focus in graphene-waveguide research has been primarily to construct electro-optic modulators [22–26], with the Fermi level and therefore absorption of graphene controlled directly by a gate voltage (Fig. 3.a). In order to boost the power efficiency of these devices, the graphene layer atop the waveguide is incorporated into a capacitor structure, with a dielectric layer separating the bottom graphene sheet – which serves as the bottom capacitor plate – from the top capacitor plate, which in many instances takes the form of a second graphene layer. A gate voltage is then applied to the top plate, and a carrier buildup of $n = \frac{1}{e}CV_G$ accumulates on the graphene layer, where e is the elementary charge, V_G is the applied gating voltage, and C is the capacitance per unit area (taking into account both the geometric and quantum capacitances of the microstructure). The progress made in the field of electro-optic modulators has proven quite promising, with gigahertz modulation bandwidths being notably achieved via this platform. However, for ultra-high-speed applications, the electrical control hampers performance due to the resistor-capacitor (RC) response, which has thus far limited device operation bandwidth to a maximum of ≈ 30 GHz, to the author's knowledge [26]. Unfortunately, in the context of the waveguide structures currently under investigation, the obstacle presented by the RC response limitation is inherently impossible to overcome without a significant sacrifice. Because absorption decreases with carriers n , the efficiency is improved by maximizing C , such that minimal voltage is required to achieve a large

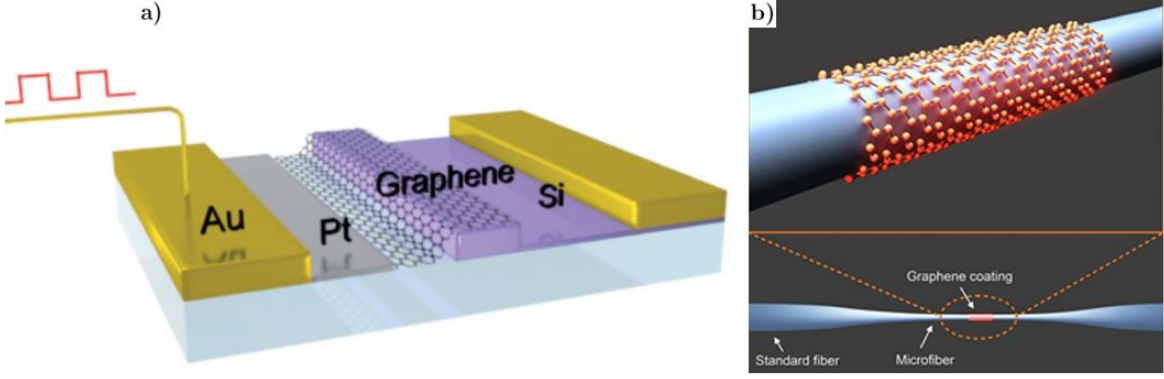


FIG. 3: Schematic of prior graphene-integrated optical modulators. **a)** A graphene-on-waveguide electro-optic modulator, adapted from [23]. An electrical signal is applied to the gold electrode on the left, which modulates the electrical potential (and corresponding Fermi level) of the graphene layer via the platinum contact. Optical transmission through the underlying waveguide is therefore controlled directly. **b)** A graphene-on-fiber all-optical modulator, adapted from [27]. A low-intensity CW signal beam propagating through the waveguide will experience significant absorption at the graphene interface. When a high-intensity pulse is transmitted down the same fiber, however, the carrier excitation is enough to enter the Pauli blocking regime, and the absorption of the signal beam will be decreased. In this way, optical control of the fiber's transmittance is realized.

modulation depth. However, the *time* associated with this charge buildup is of course dependent on the familiar resistor-capacitor time constant of $\tau_{RC} = \frac{1}{RC}$, such that ultrafast operation requires minimum capacitance. In order to reach meaningfully beyond their current speeds, the capacitance of such an electro-optic system would therefore have to be reduced to such levels as would render it prohibitively inefficient to operate.

2.3. Saturable Absorption for All-Optical Modulation

The natural answer to this electrically-imposed speed bottleneck is to transition to an all-optical method of operation, and indeed this approach has already been implemented in the context of a convenient fiber-optic platform [27]. Exploiting its innate material flexibility, a sheet of monolayer graphene is wrapped around a tapered, single-mode microfiber (Fig. 3.b), and acts as an absorber in much the same way as the waveguide-integrated graphene layers already mentioned. However, in the absence of direct electrical control of the Fermi level, modulation is accomplished by using a high-intensity optical pump to saturate the graphene layer's absorption, thereby acting as a “switch” to modulate a lower-intensity signal beam: when the pump is inactive the signal beam experiences a high rate of absorption at the graphene interface, but when the pump is switched on the graphene's absorption saturates and the signal beam is able to pass through the fiber much more transparently. In this prior work, a modulation depth of 38% is achieved, with a response time limited to ≈ 2.2 ps by the relaxation and recombination time of the intrinsic carriers. Based on this response time, it is estimated that a maximum speed of ≈ 200 GHz may be achieved by this graphene-clad-fiber all-optical system – a nearly seven-fold improvement over the best electro-optic modulation results. This promising milestone indicates that graphene-integrated all-optical amplitude modulation systems with RF bandwidths extending to terahertz speeds may be on the horizon.

3. DEVICE DESIGN, STRUCTURE, AND OPERATION

3.1. Design and Fabrication

Figure 4 shows the fabrication process of the graphene-based semiconductor microresonator chip used in this work. In the purchased waveguide chips, the Si_3N_4 microresonator ring is separated by a distance of 600 nm from a bus waveguide of the same material and buried beneath a silica cladding. Using buffer oxide etching (BOE) and lithography, a window of width 350 μm is etched above the ring, reducing the cladding thickness above the core to <20 nm, and a sheet of monolayer CVD graphene measuring 100 by 40 μm is deposited within this window above the ring.

In principle, once these preceding steps are completed, the chip is fully ready to function as an all-optical modulator. However, due to the shared nature of this chip between multiple projects, a capacitive structure is additionally formed above the resonator to provide an electro-optic control mechanism. To serve as the source and drain electrodes, two titanium-gold (20/50 nm) contact pads are deposited using electron beam evaporation. These pads have a surface area measuring 80 by 60 μm . Next, a 40 nm layer of Al_2O_3 is deposited using atomic layer deposition. A second graphene layer is placed on top of the Al_2O_3 to complete the capacitive structure, and a final titanium-gold contact is deposited on this top graphene layer to serve as the gate electrode. An electric probe may then be brought into contact with this last Ti/Au pad to provide a gating voltage. Even though these additional features of the chip are not directly relevant to its all-optical functionality, they do provide a source for additional information regarding the resonator's characteristic properties, as will be encountered toward the end of Section 5 of this work when the performance of the device's all-optical operation is contrasted against

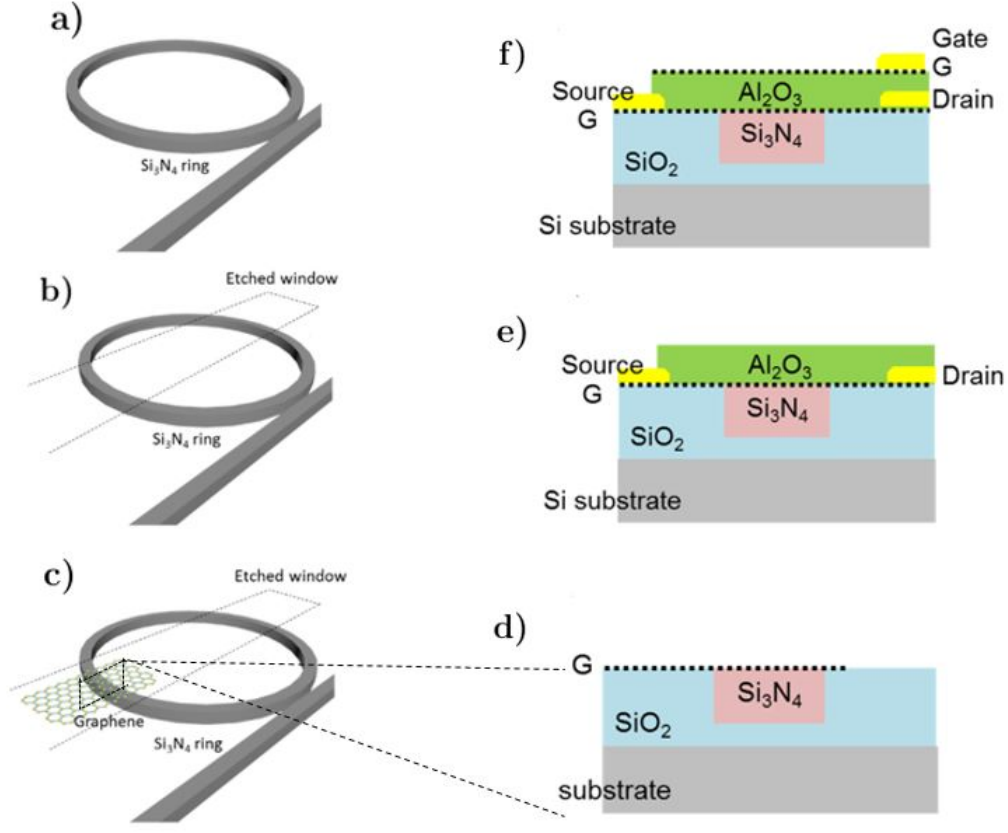


FIG. 4: Fabrication process of the resonator chip, with individual steps shown counterclockwise from top left. **a)** An off-the-shelf waveguide chip, in which the silicon nitride ring resonator is initially buried in a silica cladding. **b)** Buffer-oxide etching is used to etch a window of width $350\ \mu\text{m}$ above the microring. **c)** A single layer of CVD graphene is deposited within the etched window atop the microring. **d)** Cross-section of the resonator, showing the CVD graphene layer deposited atop the silicon nitride core. **e-f)** Titanium-gold contacts and a dielectric aluminum-oxide layer are deposited to form a capacitive structure for electro-optic modulation. Note that these components are not relevant to the all-optical modulation performed in this work (see text), but are included in this discussion for the sake of completeness.

that of its electro-optic functionality. It is also important to note that the introduction of a capacitive structure above the ring resonator will not result in an RC-slowdown of the optical modulation of the graphene layer; this is because the optical excitation of the

graphene electronic states, unlike electric control of the Fermi level, does not introduce any charge or create any current flow, but merely produces a quantum excitation of the particles already present. Additionally, being removed from the waveguide surface by a considerable distance of up to 60 nm (Al_2O_3 layer+ thickness of remaining cladding), the top-layer graphene is considered to have a negligibly-small absorptive effect on the optical field within the waveguide. As can be easily calculated from equation (1), the bottom-layer graphene will have a much more significant effect, as it is deposited much closer to the waveguide surface and can therefore be considered to solely dominate the structure’s absorption.

3.2. Description of Operation

With this design, it is intended to realize an on-chip platform for ultrafast all-optical amplitude modulation applications by exploiting the effects of saturable absorption and critical coupling. When a low-power (“signal”) beam propagates through the bus waveguide, it will be of insufficient power to efficiently overcome the absorption of the graphene layer extending along the designated arc of the ring resonator. Because of this high-loss condition in the microring, the signal beam’s transmission through the bus waveguide will be weak, resulting in an “off” state. To switch the signal to the “on” state, a second beam – a higher-powered (“switching”) pump – may be used to saturate the graphene layer’s absorption, such that the signal beam will be able to pass through the now low-loss cavity more transparently, and experience an increased transmission through the bus waveguide. The detailed mathematics of the critical coupling mechanism, by which a given change in graphene’s absorption achieves a greater modulation depth than could be realized if the graphene layer were to be incorporated directly over the bus waveguide itself [28], is

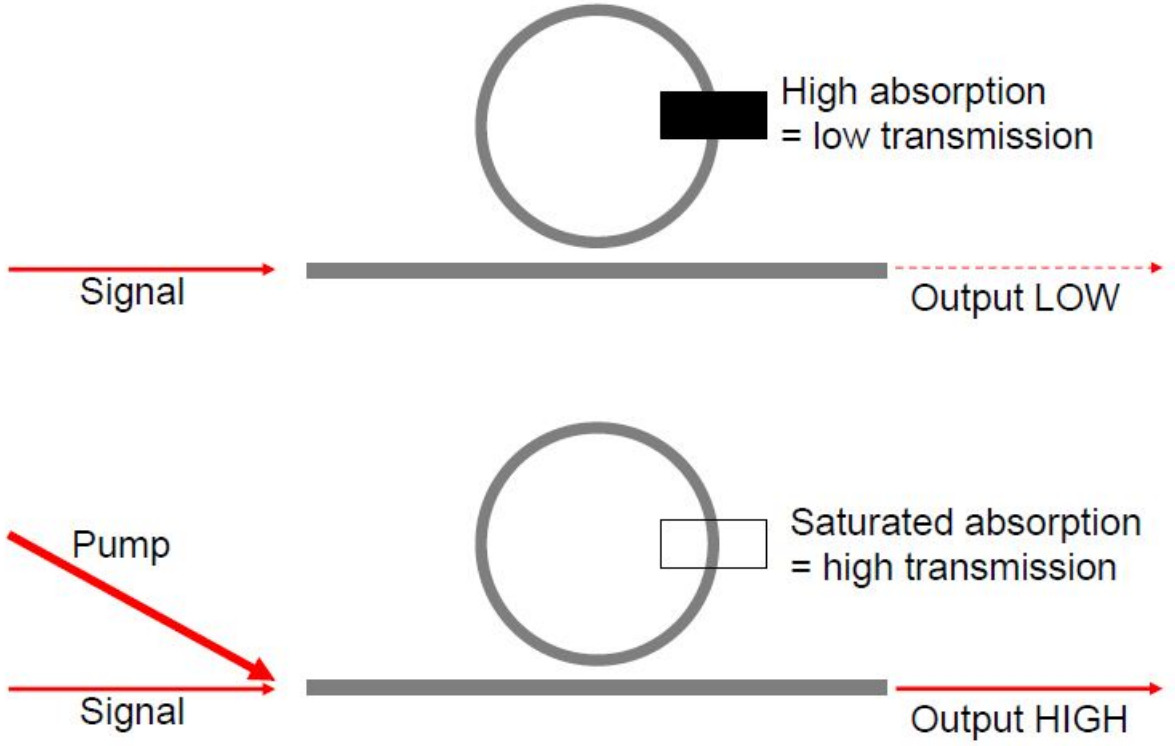


FIG. 5: Diagram showing operation of the on-chip microring amplitude modulation system. When the low-power signal beam propagates alone in the waveguide, the optical field will not be of sufficient strength to begin saturating the absorption of the graphene layer and will therefore experience significant loss, resulting in a low output state. When a high-power pump is introduced, the graphene layer may enter the absorption saturation regime, allowing the signal beam to pass through with reduced absorption and experience greater transmission through the chip.

explored in the following section of this work.

4. THEORETICAL ANALYSIS

In this section, the physical principles governing the operation of the all-optical modulator chip will be examined. In general, for a single particle, the transition probability

per unit time from a state n to m under a perturbation H' is given by Fermi's Golden Rule:

$$\Gamma_{n \rightarrow m} = \frac{2\pi}{\hbar} |\langle n | H' | m \rangle|^2. \quad (2)$$

To account for the case of multiple upper states at m , the above is to be multiplied by the density of states D at m :

$$\Gamma_{n \rightarrow m} = \frac{2\pi}{\hbar} |\langle n | H' | m \rangle|^2 D. \quad (3)$$

For the present case of interaction between Dirac fermions and light of instantaneous electric field $\vec{E}_t$, the Hamiltonian matrix element is given by [29]

$$H' = v_F \vec{\sigma} \frac{e}{i\omega} \vec{E}_t, \quad (4)$$

where v_F is the Fermi velocity and $\vec{\sigma}$ are the standard Pauli matrices. From these expressions it is seen that, in the instance of an optical field of photon energy $\hbar\omega$ exciting an initially unlimited supply of fermions originally in state n to a range of unoccupied states at m , the instantaneous energy density gained (absorbed) by the fermionic system per unit time is described by

$$W_t^a = \frac{2\pi}{\hbar} |\langle n | v_F \vec{\sigma} \frac{e}{i\omega} \vec{E}_t | m \rangle|^2 D(\frac{\hbar\omega}{2}) \hbar\omega. \quad (5)$$

However, the supply of particles in state n (as well as the open states at m) will not remain indefinitely limitless, and therefore the time-dependent occupancy probability of the lower state, $f_t(-\frac{\hbar\omega}{2})$, as well as the vacancy probability of the upper state, $1 - f_t(\frac{\hbar\omega}{2})$,

are finally included:

$$W_t^a = \frac{2\pi}{\hbar} |\langle n | v_F \vec{\sigma} \frac{e}{i\omega} \vec{E}_t | m \rangle|^2 D\left(\frac{\hbar\omega}{2}\right) \left[f_t\left(-\frac{\hbar\omega}{2}\right) \left[1 - f_t\left(\frac{\hbar\omega}{2}\right) \right] \right] \hbar\omega. \quad (6)$$

In the case of graphene, D is a linear function of energy and is given by [9, 30]

$$D\left(\frac{\pm\hbar\omega}{2}\right) = \frac{\hbar\omega}{\pi\hbar^2 v_F^2}, \quad (7)$$

and the transition matrix element in (4) may be solved to give [31]

$$|M_t|^2 = |\langle n | v_F \vec{\sigma} \frac{e}{i\omega} \vec{E}_t | m \rangle|^2 = \frac{1}{8} e^2 v_F^2 \frac{|E_t|^2}{\omega^2}. \quad (8)$$

With (7) and (8), the expression in (6) simply reduces to

$$W_t^a = \frac{e^2}{4\hbar} |\vec{E}_t|^2 \left[f_t\left(-\frac{\hbar\omega}{2}\right) \left[1 - f_t\left(\frac{\hbar\omega}{2}\right) \right] \right]. \quad (9)$$

Therefore, the instantaneous energy absorbed by the graphene layer is a linear function of the instantaneous field intensity $I_t \propto |\vec{E}_t|^2$, provided the occupation for the upper and lower energy states remain low and high, respectively. At lower optical intensities this is exactly the case, seeing as the photon-count of a low-intensity optical field will be insufficient to excite enough carriers to saturate the transition. In this scenario,

$$f_t(-\hbar\omega/2) \simeq [1 - f_t(\hbar\omega/2)] \simeq 1, \quad (10)$$

such that that $W_t^a = \frac{e^2}{4\hbar} I_t$. With the instantaneous *total* incident energy per unit area given by $W_t^i = \frac{c}{4\pi} I_t$, the absorption rate is seen to be

$$A = \frac{W_t^a}{W_t^i} = \pi \frac{e^2}{\hbar c} = \pi \alpha_f \simeq 2.3\%, \quad (11)$$

where α_f is the fine-structure constant. This low-power result is, of course, consistent with the analytical and experimental findings previously discussed herein.

However, as the intensity of the incident field increases, so does the population at the excited energy, and the previously-discussed Pauli-blocking effect becomes important. To understand the saturation dynamics of graphene, the time-dependence of the upper- and lower-state occupancy probabilities at $\pm\epsilon = \pm\frac{\hbar\omega}{2}$ are determined to be [29]

$$\frac{\partial f_t(\epsilon)}{\partial t} = \frac{\pi\alpha_f}{D(\epsilon)} \frac{1}{\hbar\omega} I_t [f_t(-\epsilon)[1 - f_t(\epsilon)]] - \frac{f_t(\epsilon)}{\tau_1} \quad (12)$$

and

$$\frac{\partial f_t(-\epsilon)}{\partial t} = -\frac{\pi\alpha_f}{D(\epsilon)} \frac{1}{\hbar\omega} I_t [f_t(-\epsilon)[1 - f_t(\epsilon)]] + \frac{1-f_t(-\epsilon)}{\tau_1}, \quad (13)$$

where the respective first terms in (12) and (13) give the change in occupation probability due to the exciting field (being simply the absorbed power divided by appropriate density of states and per-photon energy), and the second terms reflect the occupancy-proportional thermalization rate at which the excited electrons (holes) cool to form a hot Fermi-Dirac distribution within the conduction (valence) band. The time constant τ_1 associated with this thermalization process has been reported to be on the timescale of tens to hundreds of femtoseconds [16, 17].

Because these excited carriers thermalize much more quickly than they recombine, it is most efficient to use a signal beam of lower per-photon energy ($\hbar\omega_s$) than the pump ($\hbar\omega_p$),

such that its absorption rate will be affected more dramatically. Therefore, it is desirable to give consideration to the occupation dynamics of energy states lower than ϵ in the conduction band and above $-\epsilon$ in the valence band for $\epsilon \equiv \frac{\hbar\omega_p}{2}$. As the excited electrons (holes) thermalize into energy states beneath (above) ϵ in the conduction (valence) band, the transient Fermi-Dirac distribution may be calculated by regarding the thermalized carriers as shifting the intrinsic Fermi level of the material [10, 32, 33]:

$$E_{Fc} = \hbar v_F \sqrt{\pi n_c} \quad (14)$$

and

$$E_{Fv} = -\hbar v_F \sqrt{\pi p_c}, \quad (15)$$

where only the thermalized (“cooled”) carriers n_c and p_c are accounted for in this calculation. Note that, in intrinsic photo-excited graphene layers, the conduction-band Fermi energy E_{Fc} is equal to the negative of the valence-band Fermi energy E_{Fv} . The time-dependence of these carriers is given by

$$\frac{\partial n_c}{\partial t} = \frac{f_t(\epsilon)D(\epsilon)}{\tau_1} - \frac{n_c^2 - n_o^2}{\tau_2} \quad (16)$$

and

$$\frac{\partial p_c}{\partial t} = \frac{[1 - f_t(-\epsilon)]D(\epsilon)}{\tau_1} - \frac{p_c^2 - p_o^2}{\tau_2}, \quad (17)$$

with the first terms in (16) and (17) representing the increase in carriers thermalizing from energy $\pm\epsilon$ and the second terms giving the rate of recombination experienced by the thermalized carriers, where τ_2 is the recombination-rate time constant (reported to be on the order of a few picoseconds [16, 17]) and n_o and p_o are the equilibrium carrier concentrations. As reflected in the above expressions, the carrier recombination rate

has been observed to correspond to the difference of squares between the excited carrier populations and their intrinsic values [17]. With this non-equilibrium carrier concentration temporarily establishing altered Fermi levels E_{Fc} and E_{Fv} , the transient Fermi-Dirac distribution is easily determined from the familiar calculation

$$f_{Dc}(\epsilon) = \frac{1}{e^{(\epsilon - E_{Fc})/k_B T} + 1} \quad (18)$$

in the conduction band and

$$f_{Dv}(\epsilon) = \frac{1}{e^{(\epsilon - E_{Fv})/k_B T} + 1} \quad (19)$$

in the valence band, reducing to

$$f_D(\pm\epsilon) = \frac{1}{e^{(\pm\epsilon \mp E_F)/k_B T} + 1} \quad (20)$$

in the case that $E_{Fv} = -E_{Fc}$. With this distribution informing all occupancies between $-\epsilon$ and $+\epsilon$, the time-dependent power absorption of a signal beam of $\frac{\hbar\omega_s}{2} < \epsilon$ incident on the graphene layer may then be determined from (9) by substituting f_{Dc} and f_{Dv} for $f_t(+\frac{\hbar\omega}{2})$ and $f_t(-\frac{\hbar\omega}{2})$, respectively.

Within the current microresonator chip, the critical coupling condition between the ring cavity and bus waveguide magnifies this optical modulation effect to achieve even more dramatic results than could be realized if the graphene layer were to be integrated along the bus waveguide directly. As shown in Figure 6, power exchange between the resonator and bus waveguides occurs in a coupling region (indicated by the dashed box). Assuming the the coupling is lossless and that no reflection takes place, the field strength emerging from this region (b_1 and b_2) can be described relative to the incident field strength (a_1

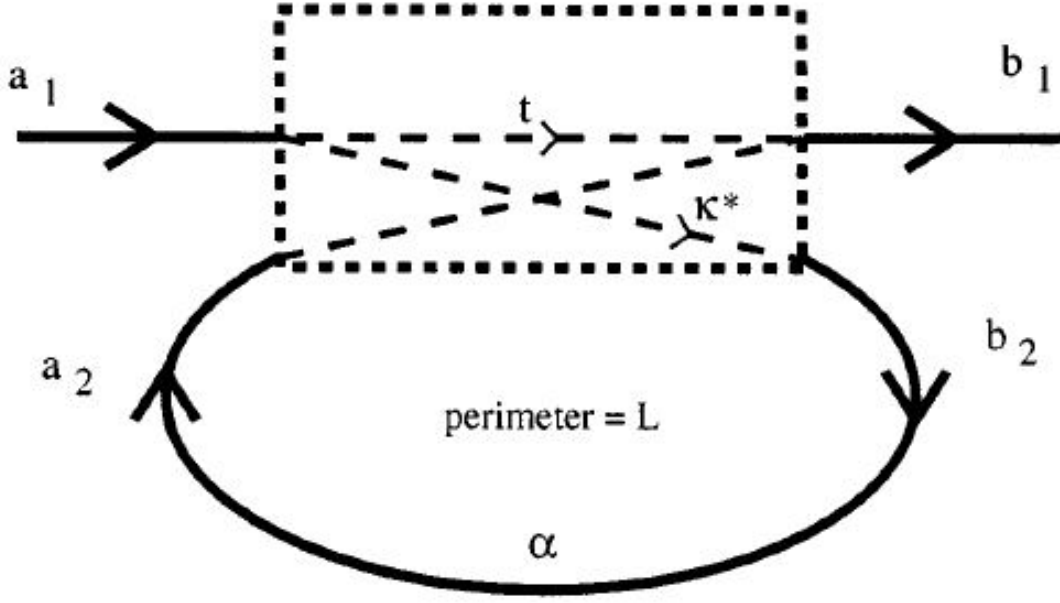


FIG. 6: A sketch of a ring resonator coupled to a bus waveguide, adapted from [28]. The dashed box indicates the coupling region. The parameters a_1 and a_2 denote the strength of the optical field entering the coupling region from, respectively, the bus and the ring, while b_1 and b_2 denote the strength of the optical field exiting the coupling region via, again, the bus and the ring. The parameter $|t|^2$ gives the percentage of a_1 that continues directly to b_1 without coupling to the ring, while $|\kappa|^2 = 1 - |t|^2$ gives the percentage that enters the resonator. It should be noted that here α is not the absorption per unit length, but total field absorption as a dimensionless percentage.

and a_2) by means of a unitary scattering matrix equation [28]

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} t & \kappa \\ \kappa^* & -t^* \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \quad (21)$$

where t and κ are (complex) constant parameters such that

$$|t|^2 + |\kappa|^2 = 1. \quad (22)$$

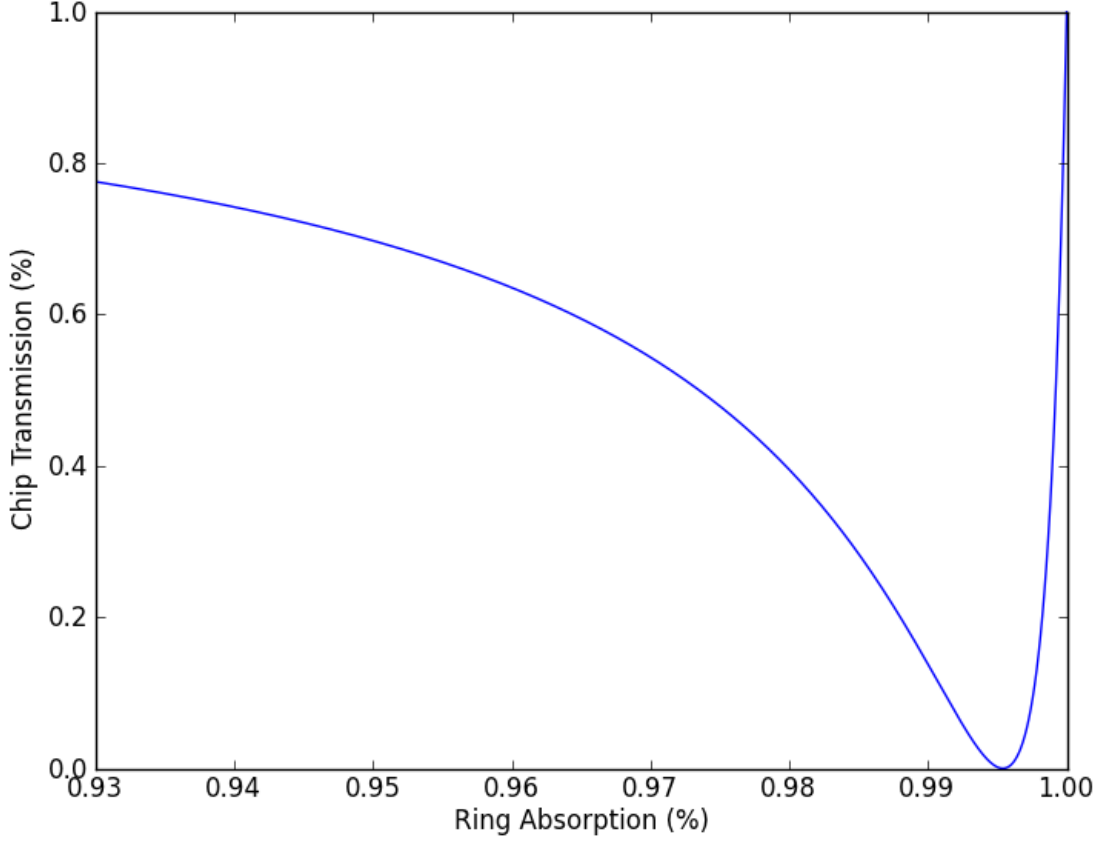


FIG. 7: Transmission curve based on equation (25), with a coupling coefficient $t_c \approx 0.9954$. Note the steep change in chip transmission for α near t_c .

Additionally, the circulation in the ring itself is governed by

$$a_2 = b_2 \alpha e^{i\theta}, \quad (23)$$

where, as usual, α is the cavity loss and θ gives the per-circulation phase shift of the optical field. Taken together, these equations can be solved to yield the power transmission from the bus output:

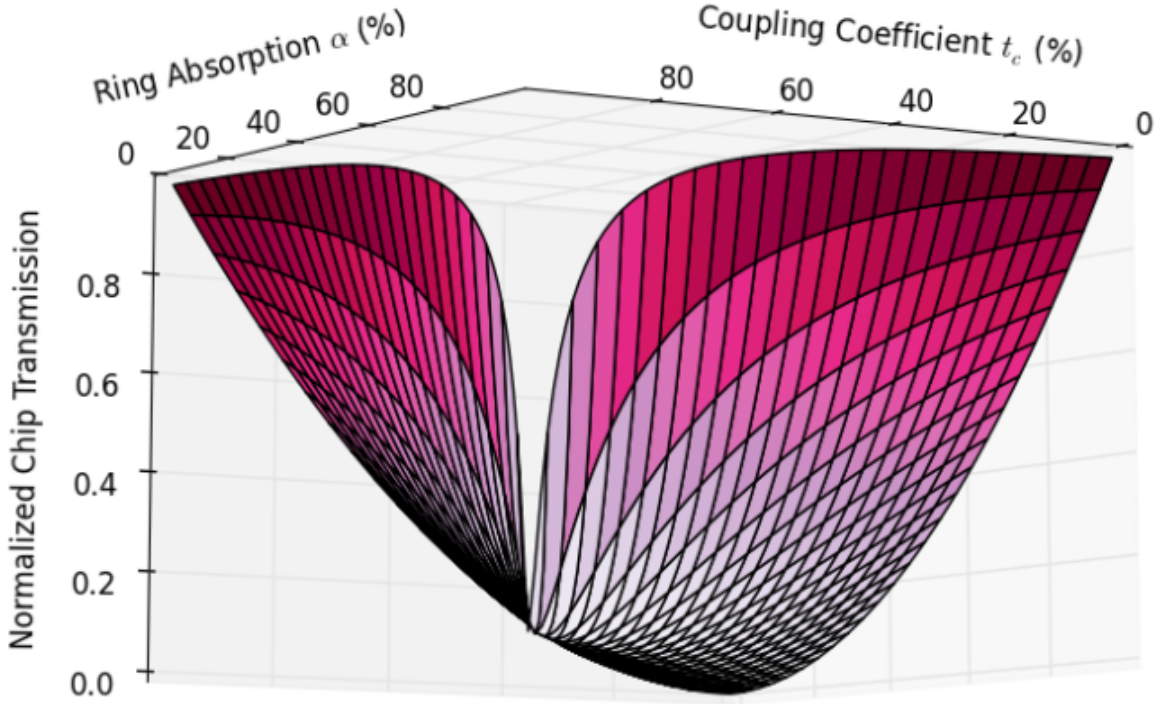


FIG. 8: Three-dimensional surface showing the normalized ring transmission in the space of the coupling coefficient t_c and ring absorption α .

$$T = \left(\frac{b_1}{a_1}\right)^2 = \frac{\alpha^2 + |t|^2 - 2\alpha|t|\cos\theta}{1 + \alpha^2 t^2 - 2\alpha t \cos\theta}. \quad (24)$$

Without loss of generality, a_1 can be taken as unity, such that at resonance ($\theta = 2\pi m$) the transmitted power reduces to

$$T = \frac{(\alpha - t_c)^2}{(1 - \alpha t_c)^2}, \quad (25)$$

with subscript c now appended to the coupling coefficient, $t \rightarrow t_c$, thereby to be better distinguished from time t hereforward.

Equation (25) results in a subtle but important feature: when the absorption experi-

enced by the optical field in the microring α is near to the $a_1 \rightarrow b_1$ coupling factor t_c , the transmission T will be sharply reduced, and in the case that $\alpha = t_c$ the transmission will go to zero identically (Figs. 7 and 8). This phenomenon has already been used to enhance the modulation depth of an electro-optic microring modulator [26], and here stands to improve the transmission modulation experienced by the signal beam based on the pump-induced change in α .

5. METHODS AND EXPERIMENTAL RESULTS

Throughout the measurements taken in this work, the setup of the laboratory equipment is as follows (Fig. 9). The waveguide chip is placed on a spatially-adjustable stand in free space, between two similarly adjustable lenses. The pump beam is provided by a PolarOnyx Uranus-Series high-power 1532 nm pulsed fiber laser of pulsewidth 2.2 ps, repetition rate ≈ 40 MHz, and 500 W peak power (20 mW average), while the signal beam is from a Santec TSL-710 CW laser of wavelength 1600 nm and average power 10 mW. The two beams are coupled together via a C-band/L-band wavelength-division multiplexer (WDM), and transmitted to a single-mode fiber (SMF). The SMF passes through a series of three polarization controllers, and then is emitted to a free space optical path via an optical collimator. Once in free space, the beam enters the lens and couples to the on-chip waveguide. Upon exiting the waveguide at the opposite end, the beam is then channeled back into fiber, and the optical signal is passed to a balanced photodetector (BPD). The resultant electrical signal is sent to an Agilent Technologies CXA Signal Analyzer oscilloscope for analysis.

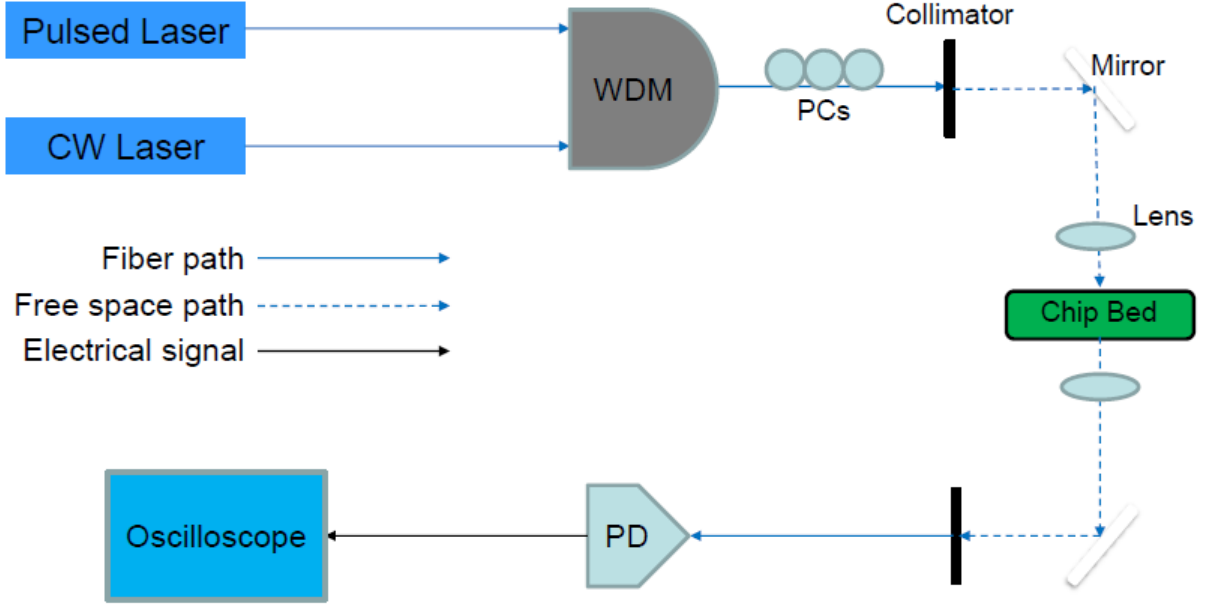


FIG. 9: Experimental setup used in this work. After being combined at the WDM and passing through a series of polarization controllers (PCs), the signal/pump beam enters free space and is coupled to the waveguide chip. Upon exiting the chip at the opposite side, the beams are returned to fiber confinement and sent, via a balanced photodetector (BPD), to the oscilloscope for analysis.

5.1. Determination of I_s

The first procedure is to determine the (static-case) saturation intensity I_s of graphene in this ring-coupled configuration. To do so, a single pump beam of slowly-increasing intensity is input to the system, and the total transmitted power is recorded via the BPD/oscilloscope. By normalizing the output intensity by the corresponding input intensity, the dependence of transmission percentage can then be easily determined for the relevant range of input intensities I_o (Fig. 12). It is seen from the data that the microring resonator chip exhibits a maximum modulation depth of $\approx 35\%$, comparable to that reported previously [27].

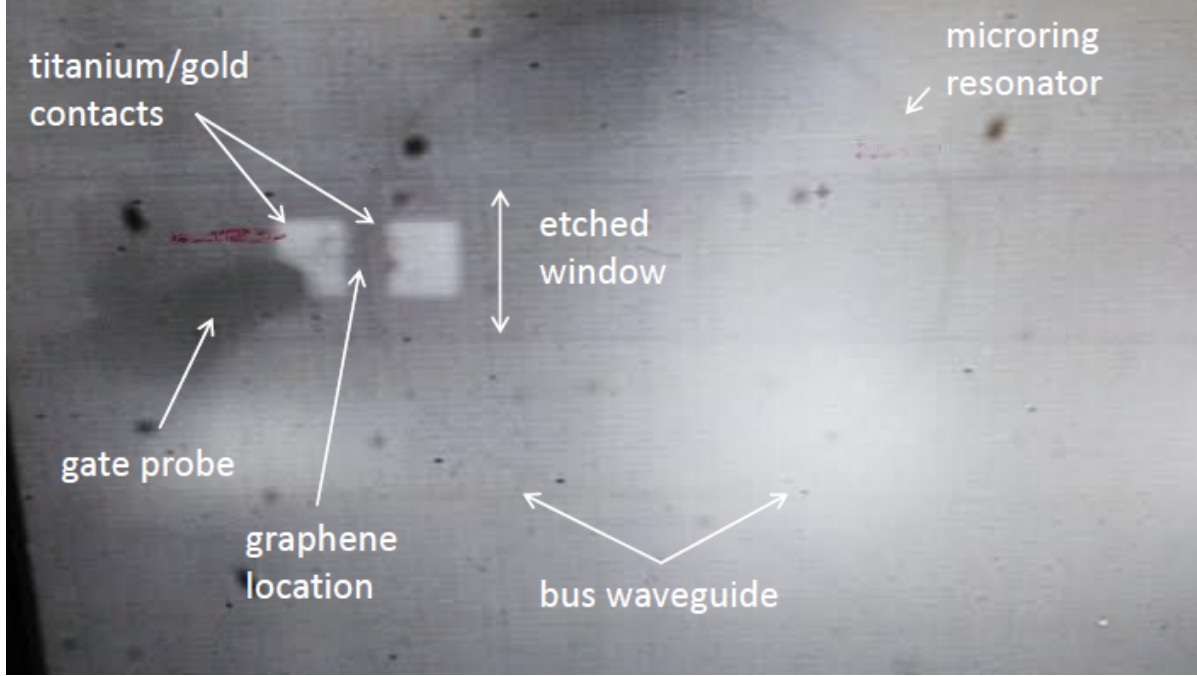


FIG. 10: A microscopic image of the resonator chip, with key features indicated. Although the titanium-gold contacts are not used in this work, they do provide a helpful visual aid in identifying the location of the graphene layer.

To determine the “effective” saturation intensity of the system, the curve is fit via the familiar equation

$$\frac{dI}{dz} = -\frac{\alpha_o I}{1+I/I_s}, \quad (26)$$

where α_o is the low-power (unsaturated) absorption coefficient, I is the instantaneous intensity, I_s is the saturation intensity, and z is the direction of propagation. It is determined from this data that the low-intensity attenuation constant of the graphene layer α_o is approximately 0.11 dB, consistent with prior work [22, 24]. It is also found that $I_s \approx 5 \text{ MW/cm}^2$, once again similar to values previously observed [34].

The measured data is additionally compared against the system’s simulated behavior,

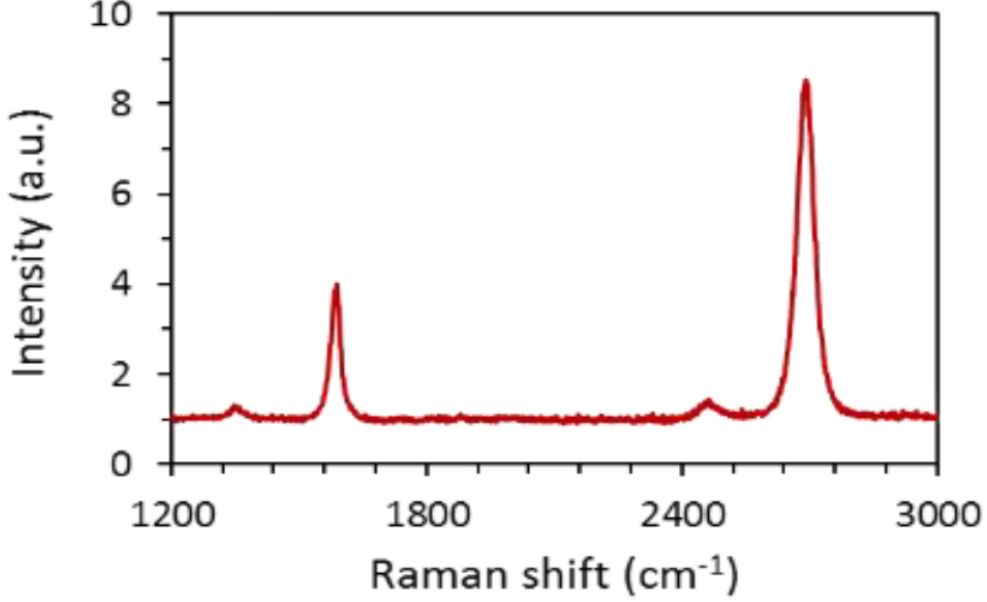


FIG. 11: Raman shift of the graphene layer, courtesy of the author’s research colleague Baicheng Yao, presenting that it is of high quality.

the equations for which have been developed in Section 4. The blue curve in Figure 12 shows the results of the relevant numerical calculations. Section 6 further explicates the numerical study conducted in this work.

5.2. All-Optical Amplitude Modulation

The characteristics of the pump-modulated optical signal beam are now analyzed. The CW signal beam is turned on, and is modulated by an optical pump of repetition rate ≈ 40 MHz. The time-dependent intensity of the output signal under this operation is measured by the oscilloscope. Figure 13 shows the output signal intensity in the time domain, showcasing the well-defined 40 MHz modulation of the pump, as well as the similar noise profile with and without the pump beam. Figure 15 shows the intensity of signal output as a function of RF frequency, and Figure 16 provides an enlarged view

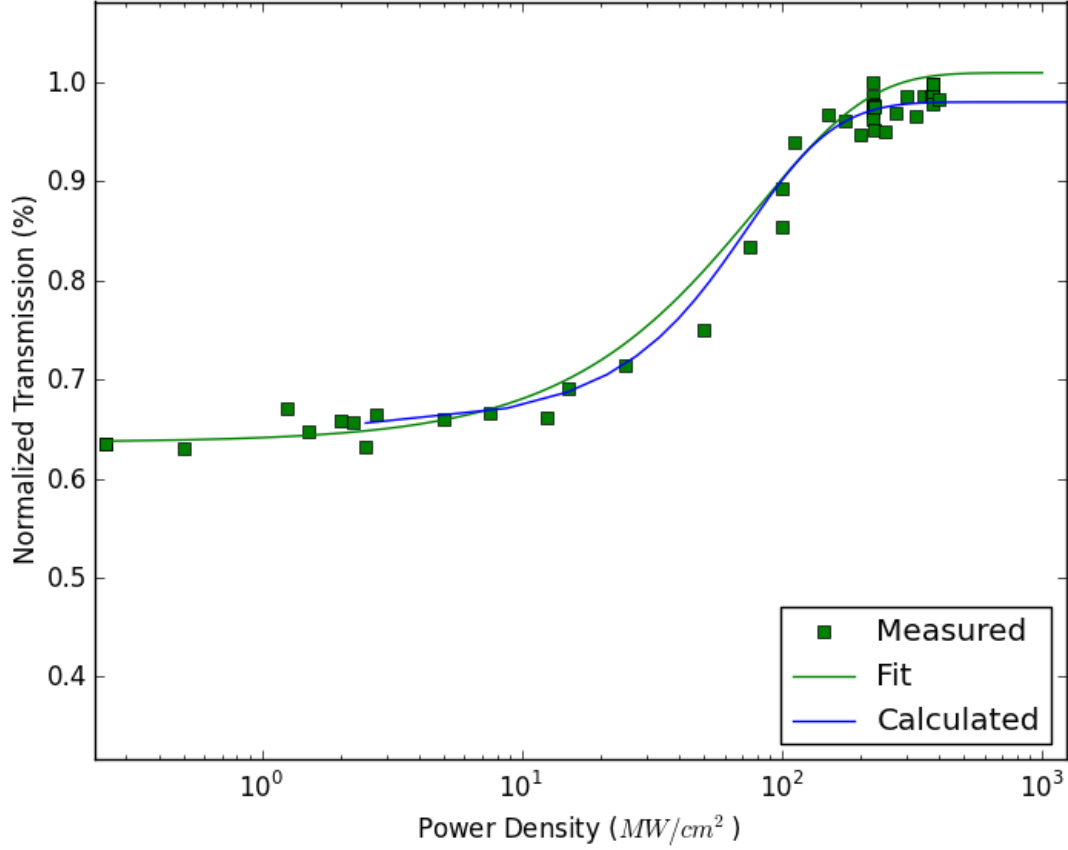


FIG. 12: Determination of the system absorption's dependence on incident intensity. The green squares represent measured data, and the solid green line is the classical saturation-intensity fit (see text). Additionally, the solid blue line represents the behavior predicted by this work's numerical analysis of the system (Sections 4 and 6).

of this same data, again reflecting the 40 MHz response as well as a minimum 30 dBm signal-to-noise ratio (SNR) of the signal output.

5.3. Pump-Probe Spectroscopy and Ultrafast Modulation

Finally, the potential for ultrafast operation of the all-optical modulator is explored. The temporal profile of a single modulated pulse is characterized via pump-probe spec-

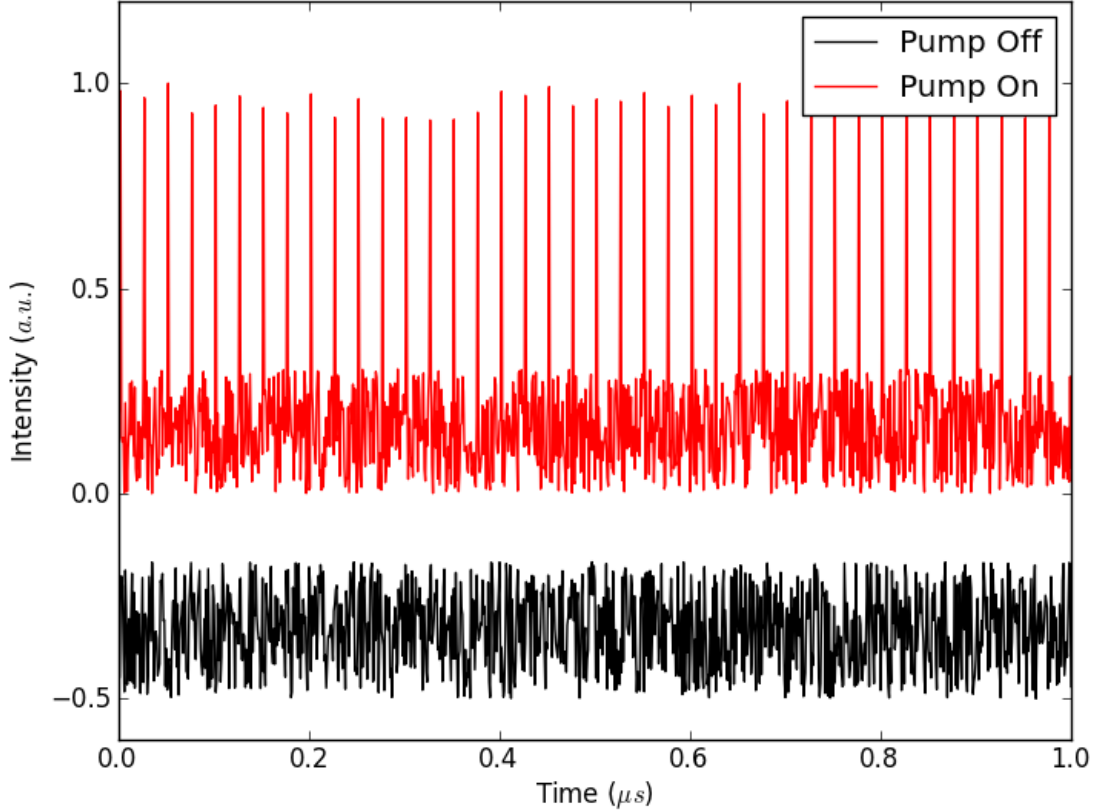


FIG. 13: Temporal output intensity of the signal beam. The red curve shows the output under the influence of the 40 MHz pulsed pump, while the black curve (offset by -0.5 a.u.) gives the output when the pump is turned off. It is worth noting that the noise profile during operation is almost indistinguishable from that observed without the modulating pump's presence. The variation of the transmitted peaks are attributed primarily to lasing instability.

troscopy. By determining the decay time of a pump-enabled signal pulse (dependent on the recombination time of the excited graphene carriers), the modulation bandwidth may then be estimated. As can be seen in Figure 17, the pulse's full width at half its maximum value (FWHM) is <20 ps. However, the fastest sampling rate of the oscilloscope available in this work is ≈ 7 ps. From the examination of the data, it is evident that the FWHM

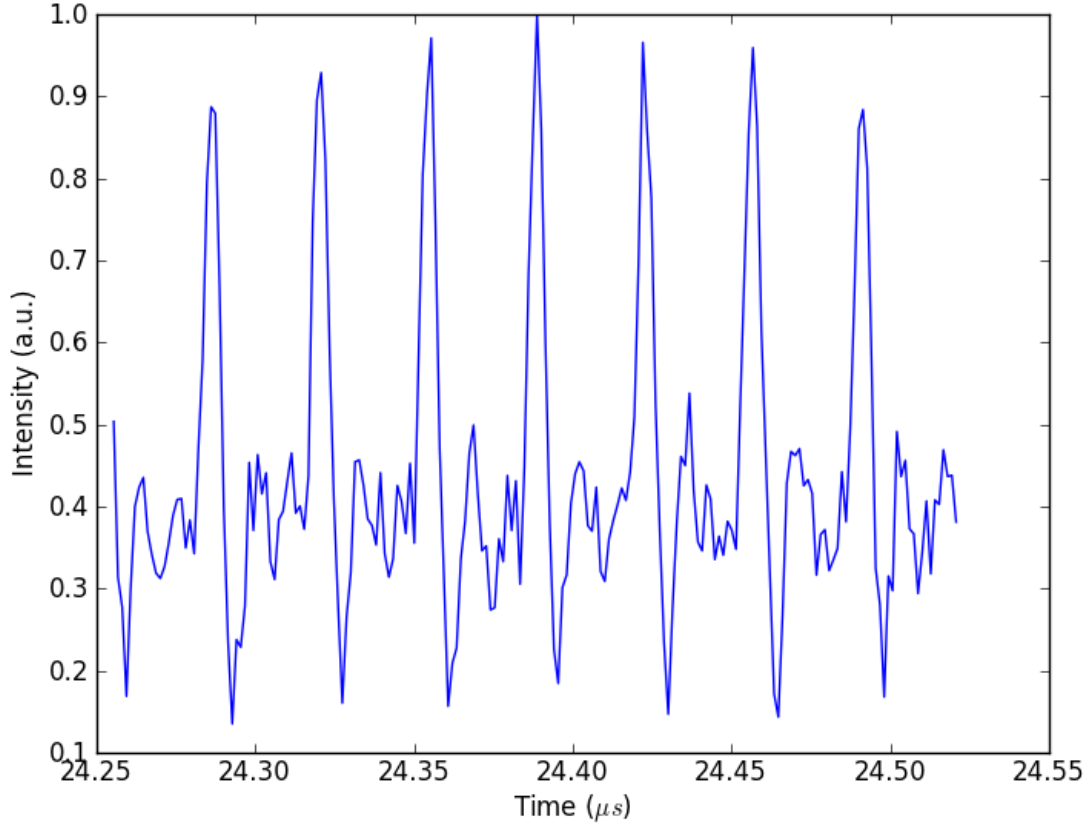


FIG. 14: An enlarged perspective of the modulated signal's transmission profile in the time domain.

pulsewidth may be significantly narrower than herein confirmed, as the two measured points nearest the pulse's peak (Fig. 17) leave open the possibility that the true peak could be non-negligibly higher than either of these two points; Figure 18 shows a linear extrapolation of the pulse edges to obtain an approximation of the pulse's true peak value. (Note that a more accurate extrapolation could even rise and fall exponentially, further heightening the expected maximum.) Given the pulse's steep edges, the true FWHM could be significantly reduced with only a small increase in peak height.

In addition to the true height of the pulse, the true width of the pulse is similarly

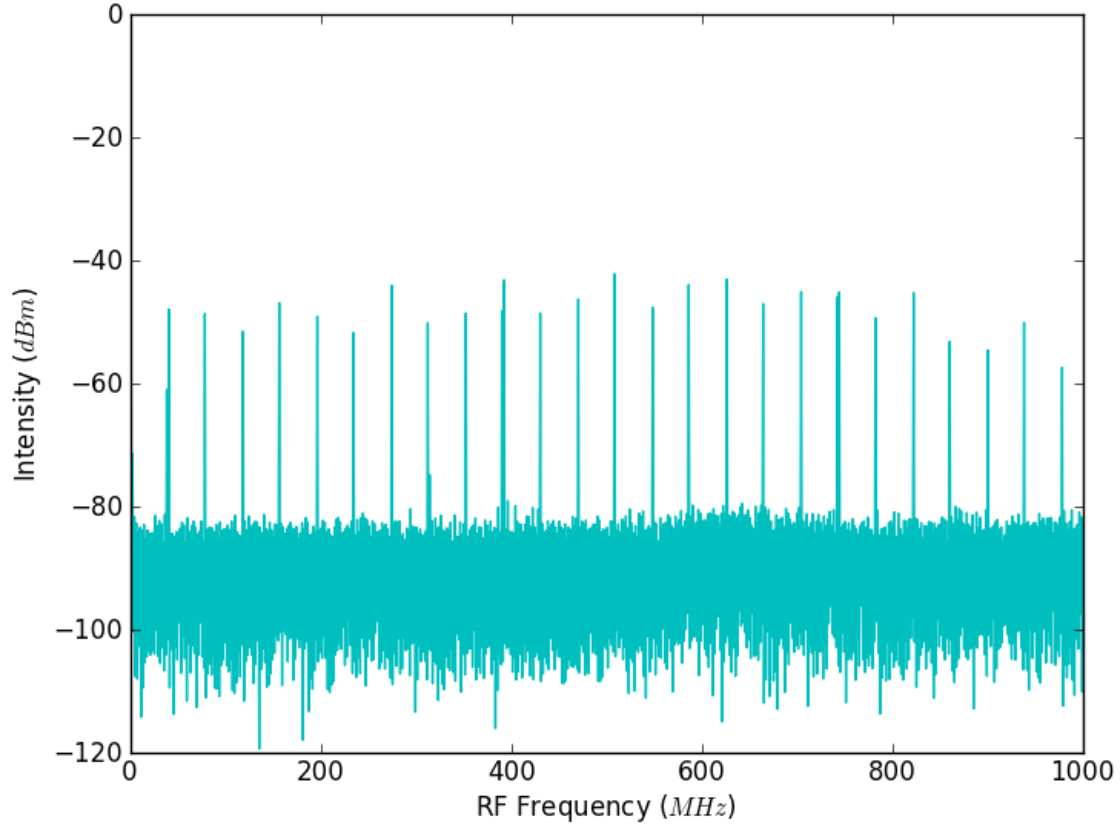


FIG. 15: The transmitted signal intensity as a function of RF Frequency, demonstrating a signal-to-noise-ratio (SNR) of ≥ 30 dBm.

obscured by equipment limitations: the BPD used to collect the transmitted optical signal is characterized by a slow recovery time, on the order of tens of picoseconds. Therefore, the trailing edge of the measured temporal pulse is likely limited not by the response time of the microring chip transmitting the signal, but by the BPD collecting and communicating it to the oscilloscope.

However, even in spite of these equipment limitations, the confirmed pulsewidth of ≤ 20 ps corresponds to an approximate modulation bandwidth of ≥ 50 GHz, which would represent a 50x improvement over the measured *electro*-optic 3-dB modulation ceiling of

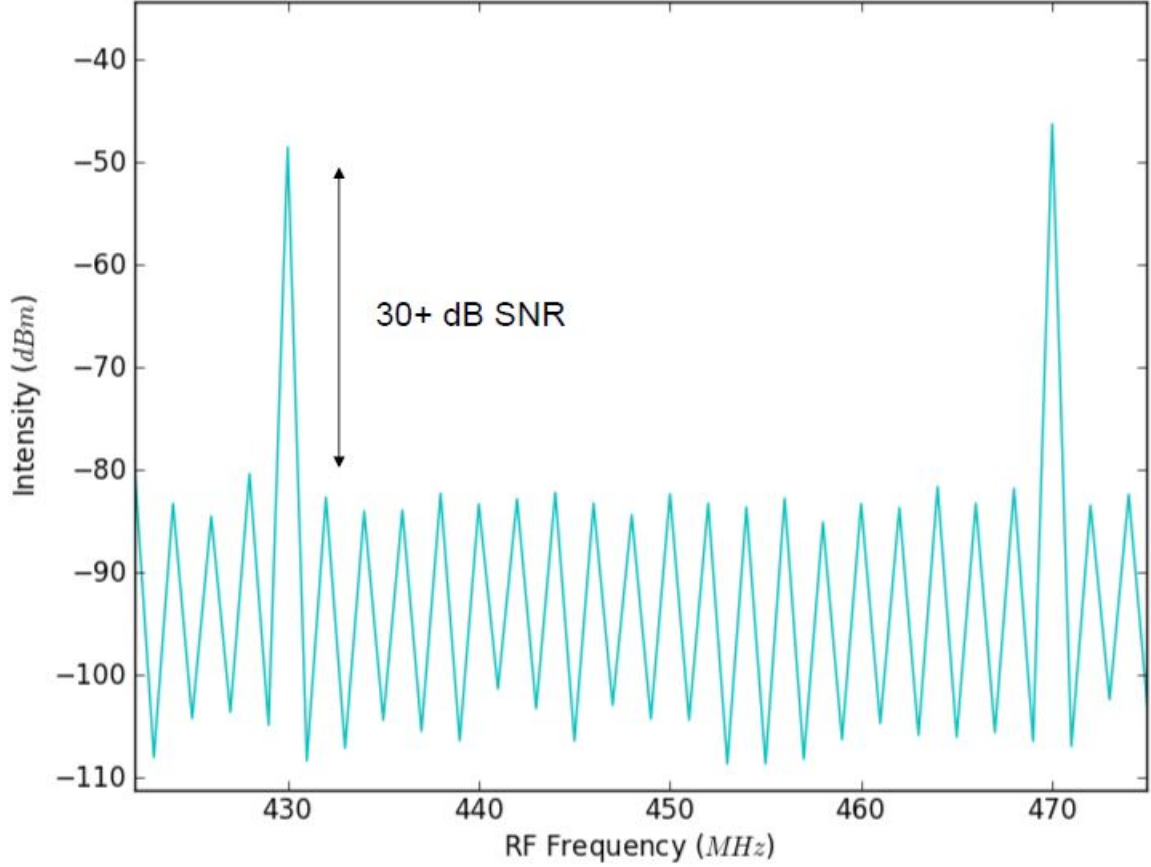


FIG. 16: A closer view of the intensity-RF frequency relationship, showing more clearly the system's ≥ 30 dBm SNR and 40 MHz response.

this same chip, bottlenecked by the large RC time-constant as previously discussed in Section 2.2.

6. DISCUSSION AND NUMERICAL RESULTS

To the author's best knowledge, the experimentally-confirmed signal pulsewidth and projected corresponding modulation bandwidth of ≥ 50 GHz establishes the highest all-optical amplitude modulation speed thus far achieved within the context of an on-chip absorptive platform. However, as alluded to in the prior section, a visual inspection of the

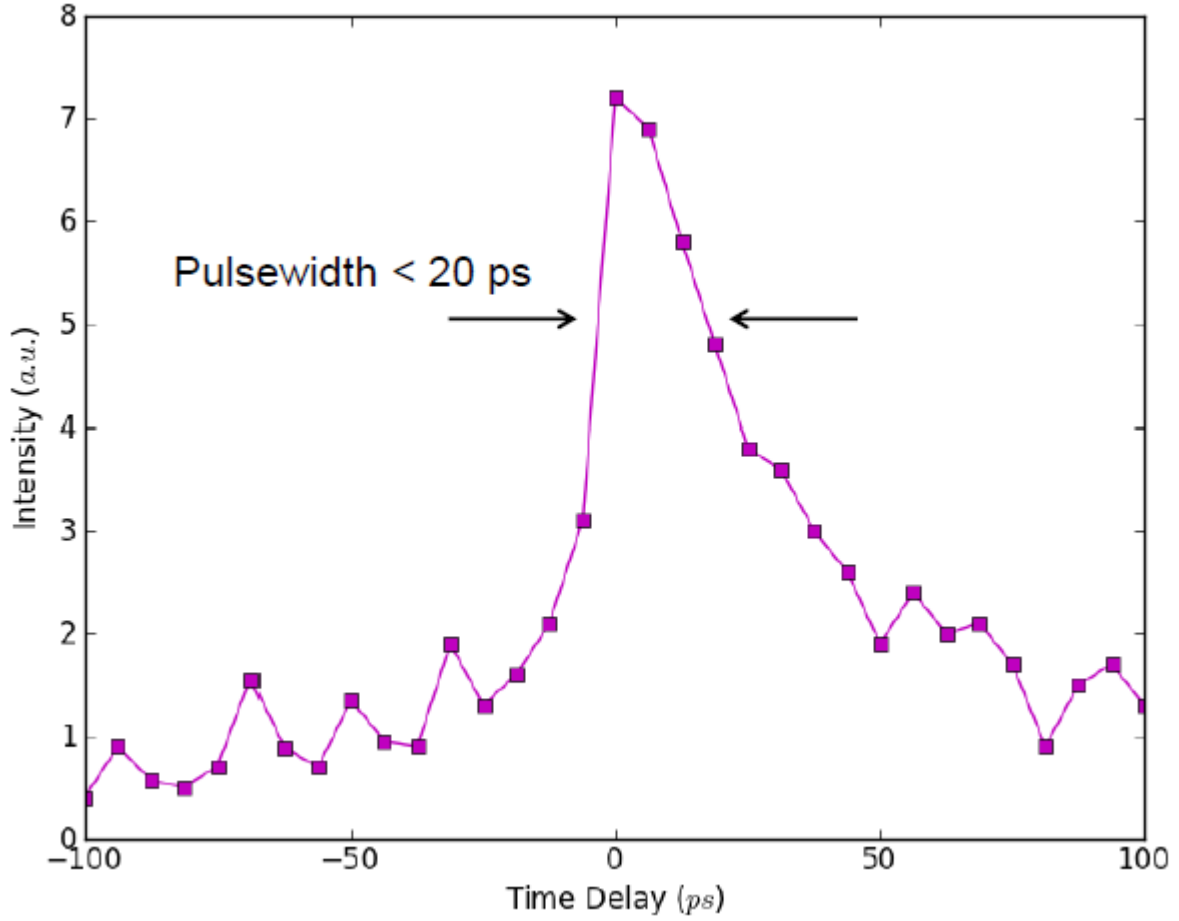


FIG. 17: Temporal profile of a single transmitted signal pulse. Note that the true pulse peak falls between the two highlighted data points, and an oscilloscope with a faster sampling rate could the true peak to be non-negligibly higher than either of these measured points. Additionally, the BPD used in this work is expected to severely broaden the measurement of the signal pulse’s falling edge.

measured temporal pulse data suggests that this may underestimate the modulator’s true performance ceiling (Fig. 18): if the upward- and downward-sloping edges of the single signal pulse were continued upward at their observed slopes, the true peak-intersection of these slopes could be approximately 8 a.u. (in the scale of the figure shown), translating to a FWHM pulsewidth of 12-15 ps – with a corresponding modulation ceiling of ≈ 70 -80

GHz. These speeds may in fact already be realized within the context of this work’s modulator chip, but the aforementioned limitations in measurement resolution render it impossible to confirm unequivocally. It is important to note that the 70-80 GHz figure comes from an analysis of the limitation imposed by the oscilloscope’s sampling rate alone, and that a faster BPD could reveal the device’s performance as being even faster still. However, the effect of cavity dispersion in temporally broadening both the incident pump and transmitted signal pulses obscures the degree to which the BPD increases the transmitted signal’s pulsewidth, as it is unclear how much of the observed broadening is due to dispersion and how much is due to the slow response time of the BPD. It is expected that the broadening caused by cavity dispersion is small compared to that imposed by the BPD’s measurement capacity.

Although a hard empirical limit on the modulation bandwidth of the all-optical amplitude modulator could not be firmly established experimentally, the confirmation that its bandwidth meets and almost certainly exceeds 50 GHz is already a milestone. However, to reach a deeper understanding of the chip’s operation and true limitations, it is informative to compare the chip’s measured operation with the results of an in-depth numerical analysis. To this end, a Runge-Kutta fourth-order (RK4) algorithm is developed (see Appendix), built on the foundational equations and physical insights already detailed in Section 4.

Figure 19 shows the single-pulse dynamics computed via the RK4 algorithm. The leading edge (left) rises very quickly due to the strong influence of the pump beam and limited only by the quantum expectation value for the time associated with the excitation, while the trailing edge (right) falls less sharply due to its dependence on the thermalization and, especially, recombination time of the pump-excited carriers. A FWHM pulsewidth

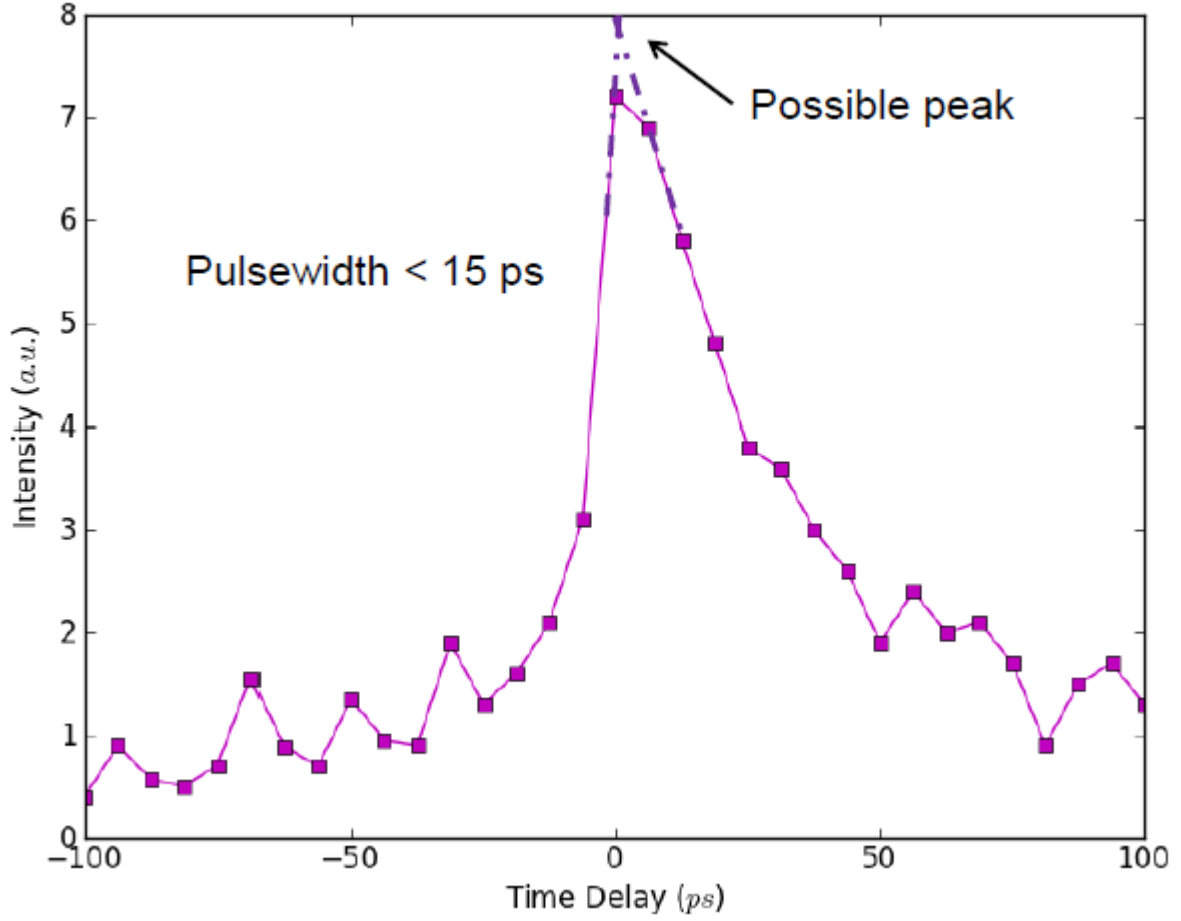


FIG. 18: Temporal profile of a single transmitted signal pulse, with a linear fit extrapolated from the pulse’s leading and trailing edges. Based on this simple extrapolation, the pulse’s true peak could be $\approx 15\%$ higher than the equipment used herein can verify. With this peak, it is estimated that the corresponding time-domain FWHM could be less than 12-15 picoseconds.

of ≈ 8.5 ps is observed, which may be about $\approx 40\%$ less than the value estimated from the experimental data. As discussed, this disparity is majorly attributed to experimental limitations, as well as somewhat to the chip’s cavity dispersion, both of which serve to temporally broaden the measured pulse. Correspondingly, Figure 20 displays the extinction ratio of the transmitted signal with increasing modulation speeds, given by the

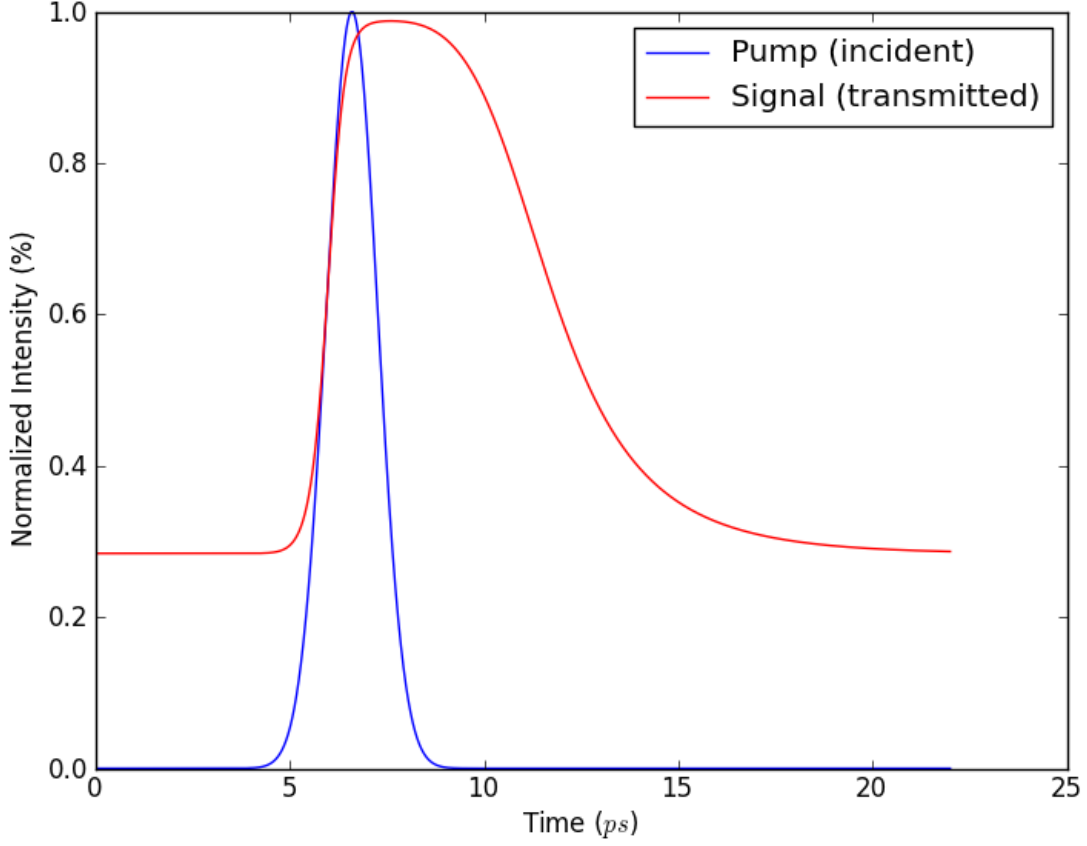


FIG. 19: Temporal profile of a single transmitted signal pulse, as computed by this work's numerical analysis (red). For reference, the 2.2 ps pump pulse is also shown (blue). The computed signal FWHM pulsewidth is found to be ≈ 8.5 ps, limited largely by the recombination time of the excited carriers.

pump beam's repitition rate. As predicted by the single-pulse FWHM analysis, the ceiling modulation speed should be slightly above ≈ 100 GHz. The figure also shows the ceiling modulation speed corresponding to the measured pulsewidth, as well as to the pulsewidth likely to be found with a higher oscilloscope sampling rate; these represent, respectively, ≈ 50 -75% of the value computed numerically. The dashed green and dashed orange lines are drawn to provide context.

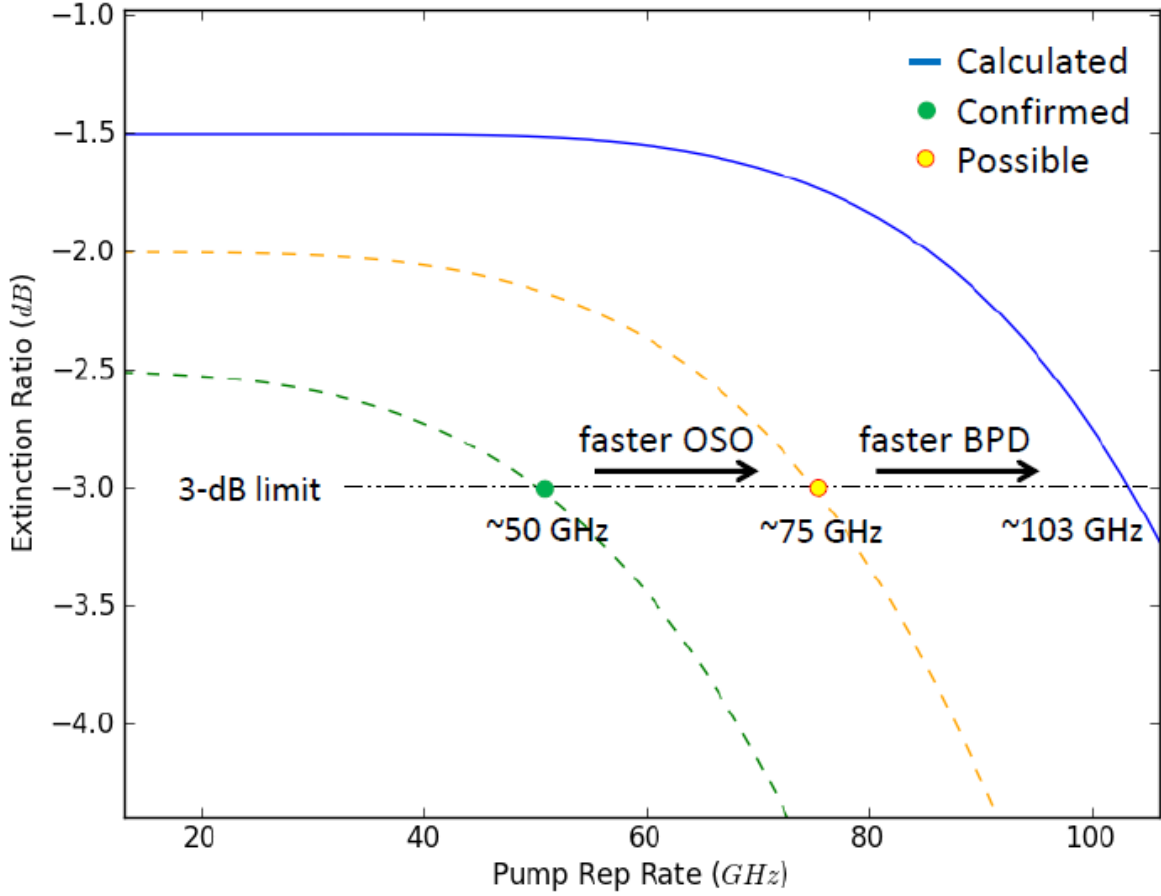


FIG. 20: Results of the RK4 computational analysis, showcasing the extinction ratio of the all-optical modulator with increasing modulation speed (given by the repetition rate of the switching pump). Also shown is the 50-GHz 3-dB limit associated with the experimentally-confirmed signal pulsewidth of 20 ps, as well as the 75-GHz speed that could potentially be observed on the same device with a higher sampling rate. The dashed green and orange curves represent possible curves that could correspond to the 50 and 75 GHz modulation ceilings, and are provided purely to guide the eye.

Establishing a precise description of the cavity's dispersion is an unwieldy proposition, as the resonator's quality factor Q varies sharply over the course of a single pulse due to the dramatic change in absorption. However, from the RK4 analysis, upper and lower bounds on Q may be estimated (Fig. 21). From this treatment, Q is seen to vary from

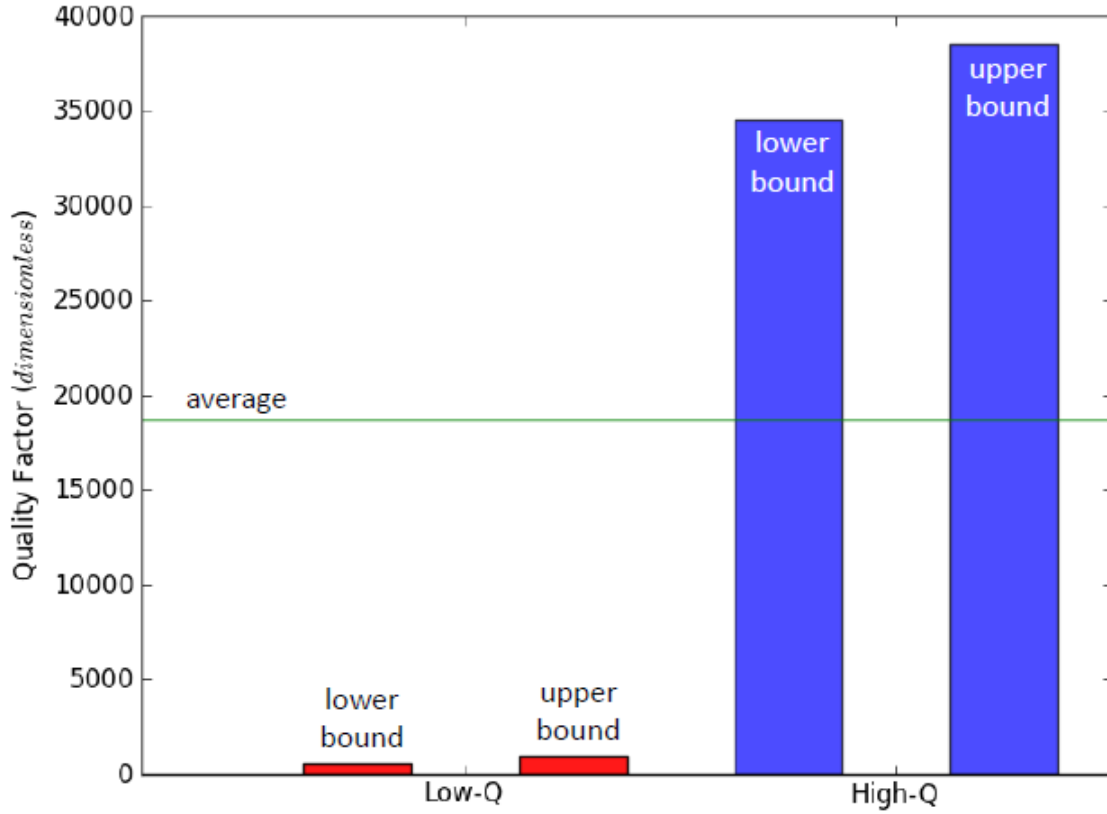


FIG. 21: Approximate bounds on the maximum and minimum Q over the course of a single pump pulse. The maximum resonator Q (*High-Q*) is when the graphene layer is most highly saturated (blue), while the minimum (*Low-Q*) occurs when it is the least saturated (red). In both categories the two bars represent, respectively, calculated upper and lower bounds. The overall average for the entirety of a single pulse duration is also shown (green line).

less than 1,000 to nearly 40,000.

To further explore the all-optical modulator's speed limitations, the peak pump intensity within the numerical analysis is varied. Figure 22 shows the extinction ratios of several pump powers with increasing repetition rate. It is important to note that the higher-powered pump beams perform worse than their lower-powered counterparts; this is because the operation speed is limited by the recovery time of the excited graphene

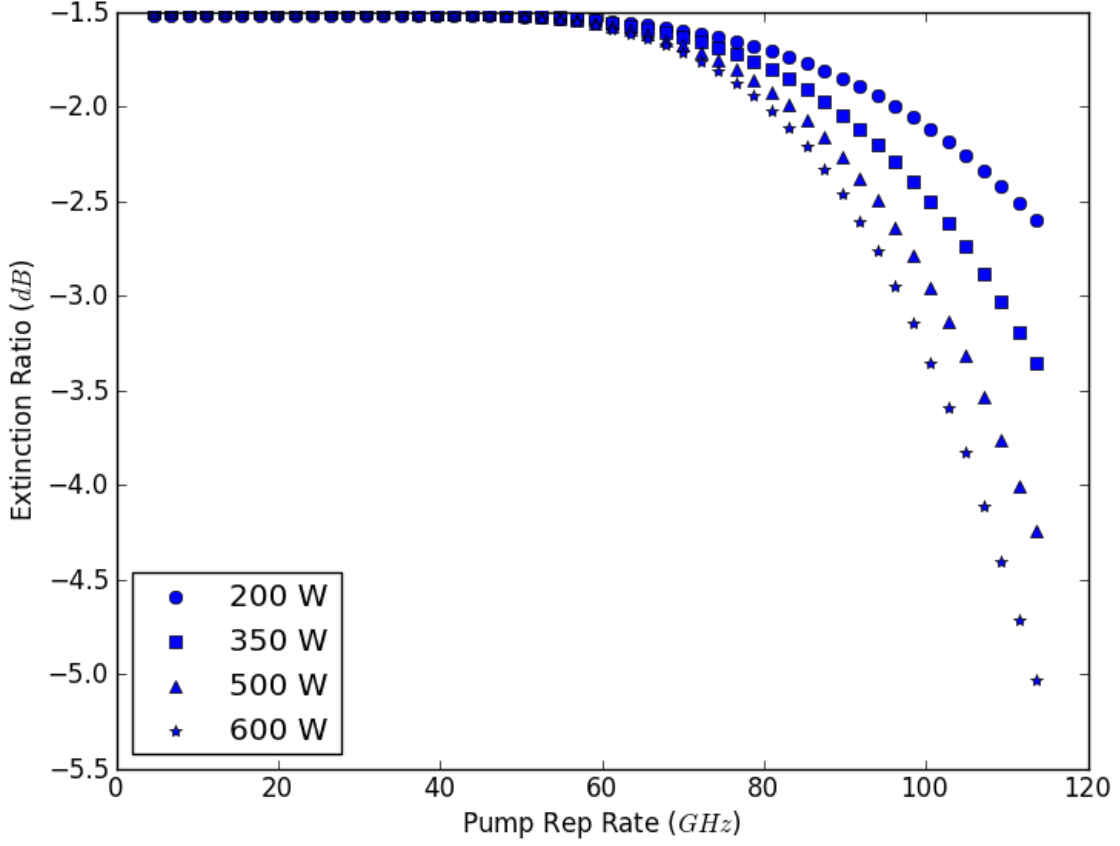


FIG. 22: Simulated extinction ratio curves for several pump pulse peak powers. After the power threshold at which the graphene layer is sufficiently saturated, additional pump power serves to hamper the device’s highspeed operation, as the limiting speed factor is the recombination rate of the excited graphene carriers; a higher pump pulse power corresponds to a longer recovery time of the graphene absorber.

layer, a timescale which increases as more carriers are promoted to a higher energy state. Therefore, provided the pump is of sufficient power to excite enough carriers such that the graphene layer’s absorption will appreciably saturate, further increasing the pump’s power beyond this threshold will be detrimental to the speed of the modulator’s operation.

Notice that, in Figure 22, the 200 W curve does not reach the 3-dB cutoff within the

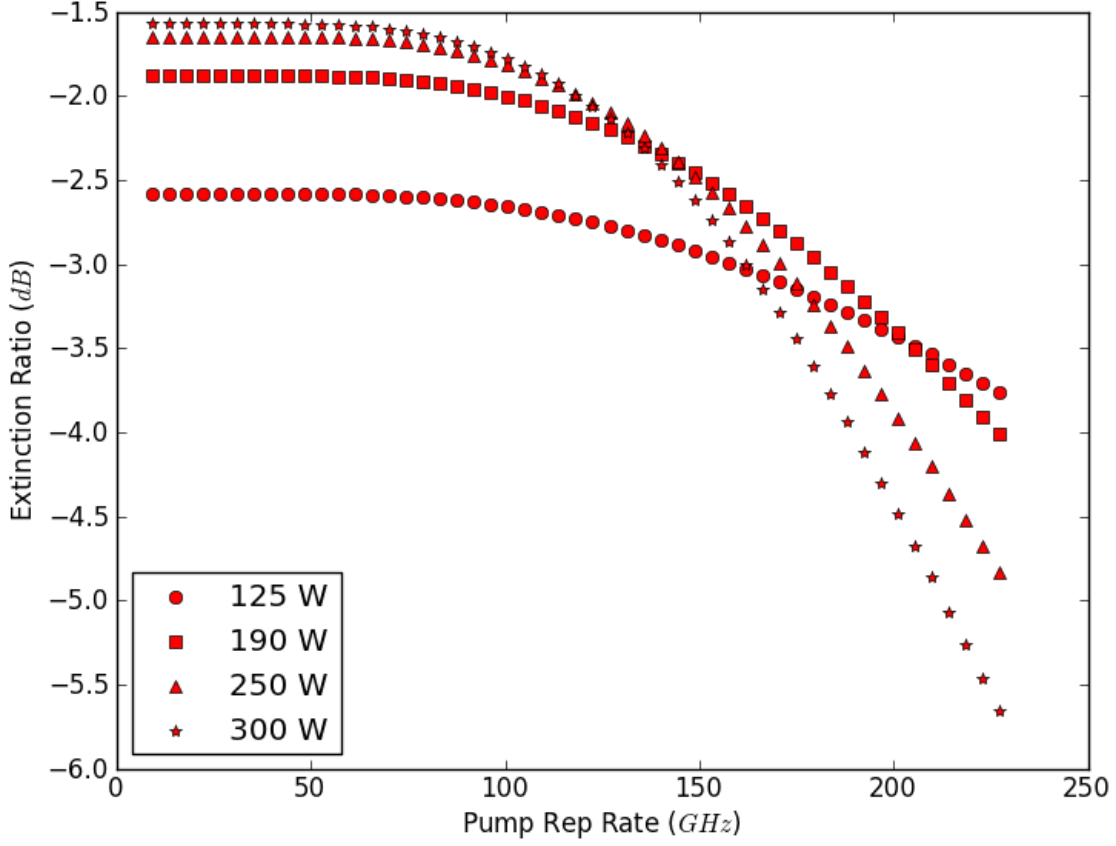


FIG. 23: Simulated extinction ratio curves for several pump pulse peak powers, with a narrower temporal pump pulsewidth of ≈ 1 ps. The peak powers indicated in the legend represent a conservation of power per unit pulse duration.

domain of the graph. Apart from other physical and experimental factors, the repetition rate is itself fundamentally limited by the FWHM of the individual pulses in the train. At a pump FWHM of 2.2 ps, the repetition period rate cannot exceed ≈ 120 GHz, corresponding to a repetition period of about four times the pump's FWHM (assuming the pump intensity returns to zero after each pulse). Therefore, the 200 W curve reaches its maximum possible repetition rate prior to reaching its 3-dB modulation ceiling.

With mode-locked laser systems making rapid progress in the achievement of ever-

shorter pulses in high rep-rate pulse trains, the behavior of this modulator under faster pumping conditions bears investigation. Figure 23 shows the operation of the system with the incorporation of a 1.1 ps pump, representing one half of the FWHM available experimentally in this work. In the relevant computations, the power density of the pump pulses are considered to be conserved from the 2.2 ps case. Incidentally, graphene has recently been used as a saturable absorber in the generation of just such mode-locked femtosecond pulses of high energy [35–37]. As can be seen in the figure, modulation by a femtosecond-pulse train could reach modulation bandwidths in excess of 150 GHz – provided the cavity dispersion is not overwhelmingly large. Even if the $\approx 25\text{--}40\%$ disparity between the computed and likely-achieved modulation ceilings in Figure 20 are considered to be caused purely by a large cavity dispersion and not at all by the slow BPD response, then the predicted 170-180 GHz speeds of Figure 23 could translate to laboratory speeds of $\approx 90\text{--}120$ GHz under comparable dispersion conditions. Of course, the BPD response time is very likely to be a severely limiting factor, and the disparity between computed and measured modulation speeds should be significantly reduced with faster experimental instrumentation.

Further pursuing the relationship between the pump pulse’s temporal width and peak power on device operation speed, Figure 24 shows the relationship between the three in as many dimensions. Again, it cannot be determined from the laboratory data available in this work how much of the disparity between the computed and measured modulation ceiling is due to the cavity dispersion and how much is due to the BPD response time; in the succeeding figures, therefore, the BPD response time is ignored and the primary limitation is attributed to an exaggerated cavity dispersion. As such, the color-coded z-axis of Figure 24 represents a shrewdly conservative estimate of the maximum mod-

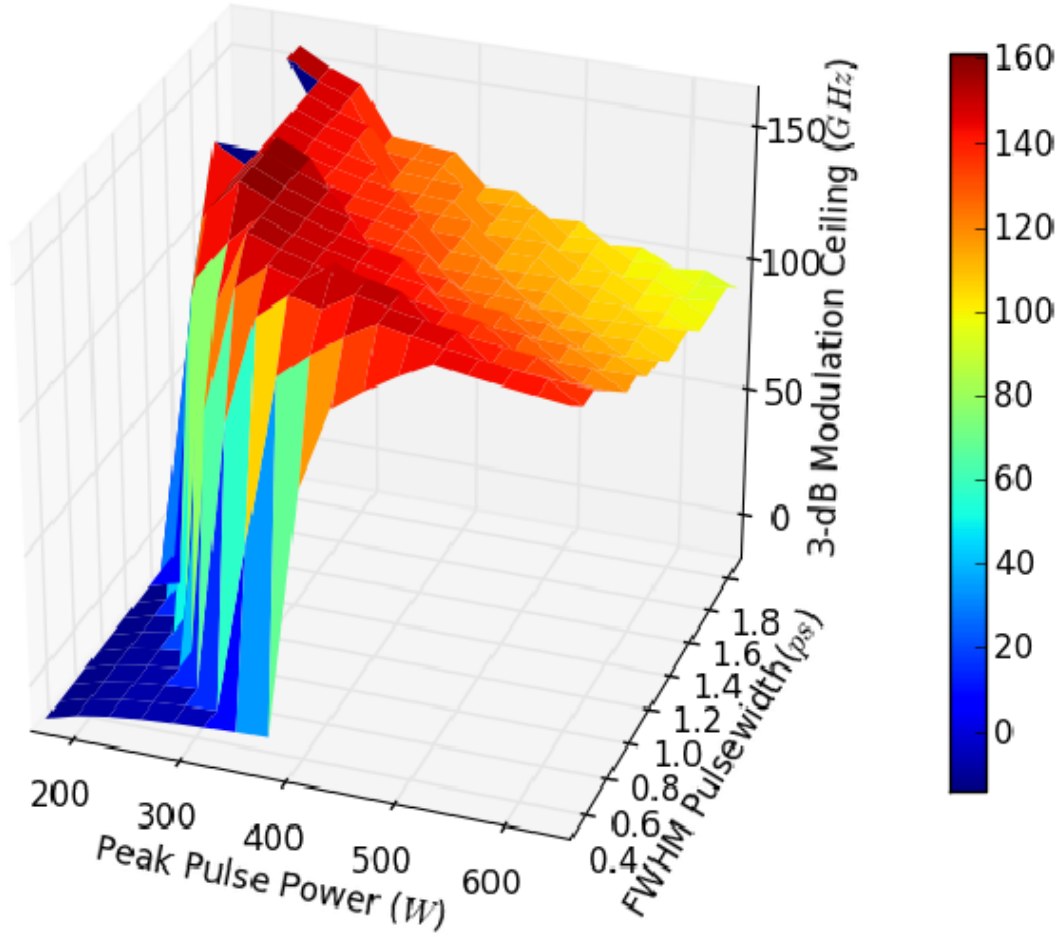


FIG. 24: Maximum modulation frequencies of the device, computed within the space of the pump pulses' peak power and temporal width. Potentially-large cavity dispersions are taken into account, thereby giving a conservative estimate of actual in-practice results. For insufficient pump energy, modulation is not possible (blue valley). With unnecessarily large pump energies, maximum modulation speeds are seen to decrease.

ulation speed achievable while maintaining an extinction ratio of ≥ -3 dB, taking into consideration the exaggerated cavity dispersion in addition to the limitations intrinsic to the graphene material itself. The dark red ridge in the surface visualizes the distinct energy threshold at which the graphene's absorption becomes sufficiently saturated, a

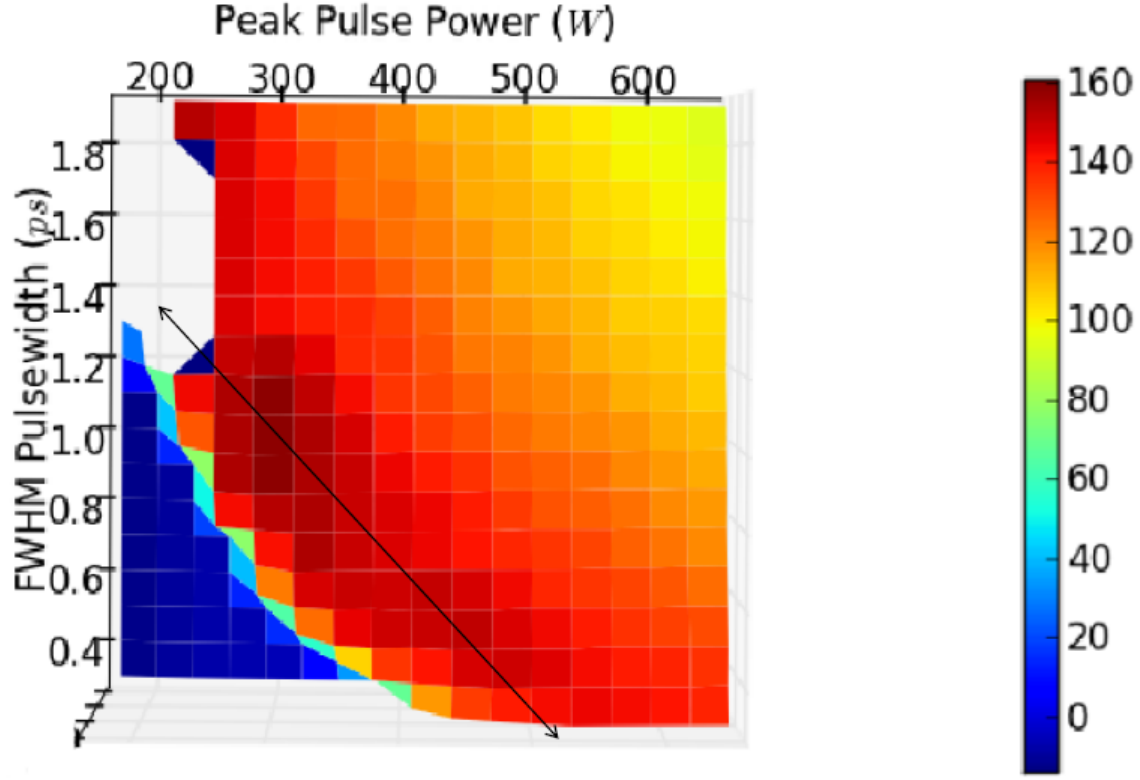


FIG. 25: A top-down view of the surface plotted in Figure 24, highlighting the linear relationship between pump peak power and temporal width along the ridge of fastest operation speeds, in the neighborhood of ≈ 150 -160 GHz after a large cavity dispersion.

point about which the chip's transmission sharply varies – due, in part, to the effects of critical coupling discussed in Section 4. If either the peak energy or temporal FWHM of the pump pulse decreases, a steep dropoff in chip performance occurs (blue valley). Conversely, if either parameter is increased, a slower decline in maximum modulation speed is observed (orange-yellow slope). As noted, this reduced performance with increasing pump energy results from the prolonged time interval required for the excited fermionic carriers to sufficiently recombine.

Moreover, note the linear relationship between the pump pulse's temporal FWHM and peak power along the ridge of maximum operation speed (Fig. 25). Within the range of

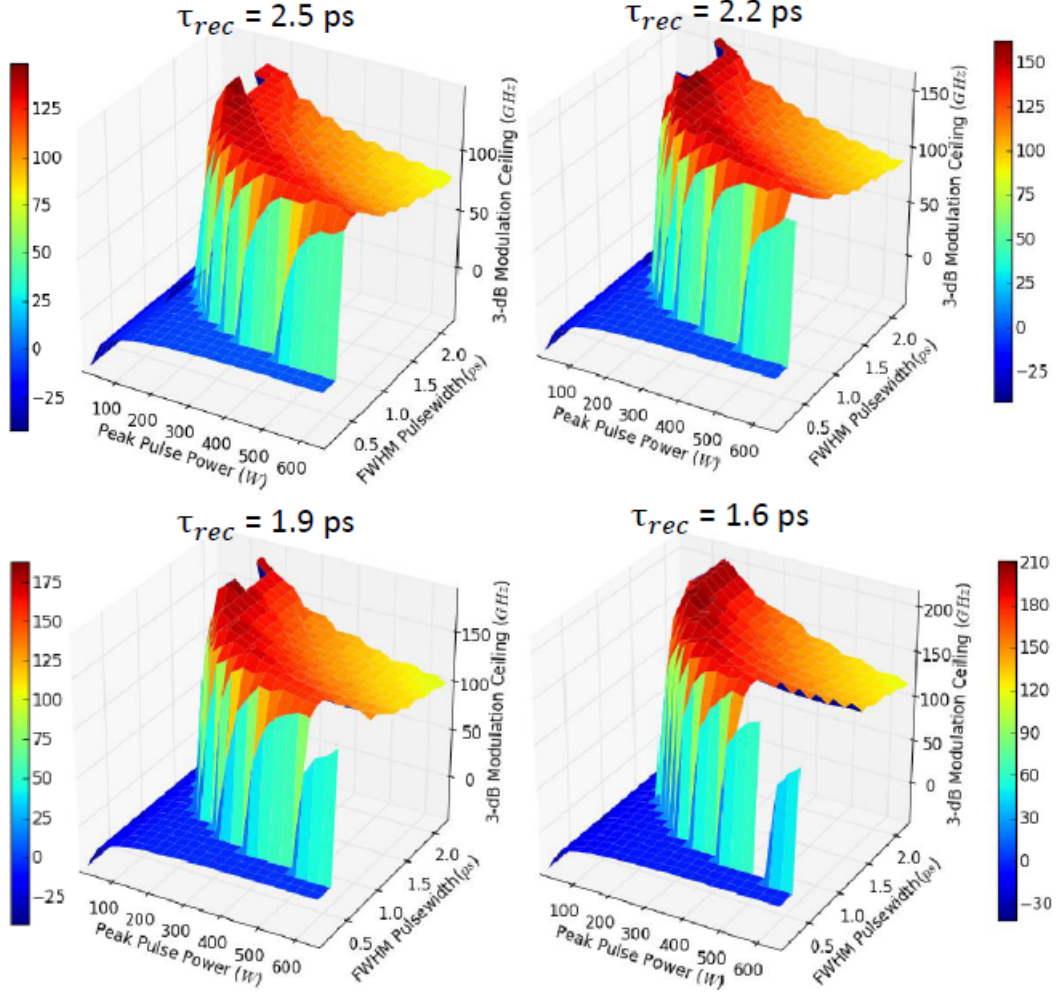


FIG. 26: Device performance surfaces for a selection of additional graphene carrier interband recombination times. Although the surface height (i.e. maximum modulation speed) increases with decreasing recombination time, the shape of the surface remains largely indistinguishable when varying τ_{rec} .

values considered, it is observed that ideal performance is realized for a pump pulse of peak power which diminishes by 30 W for every 100 fs of temporal pulsewidth:

$$P_{peak,ideal} \simeq 600 - 30 t_{width}, \quad (27)$$

where t_{width} gives the pump pulse's temporal FWHM in 100's of femtoseconds, and the

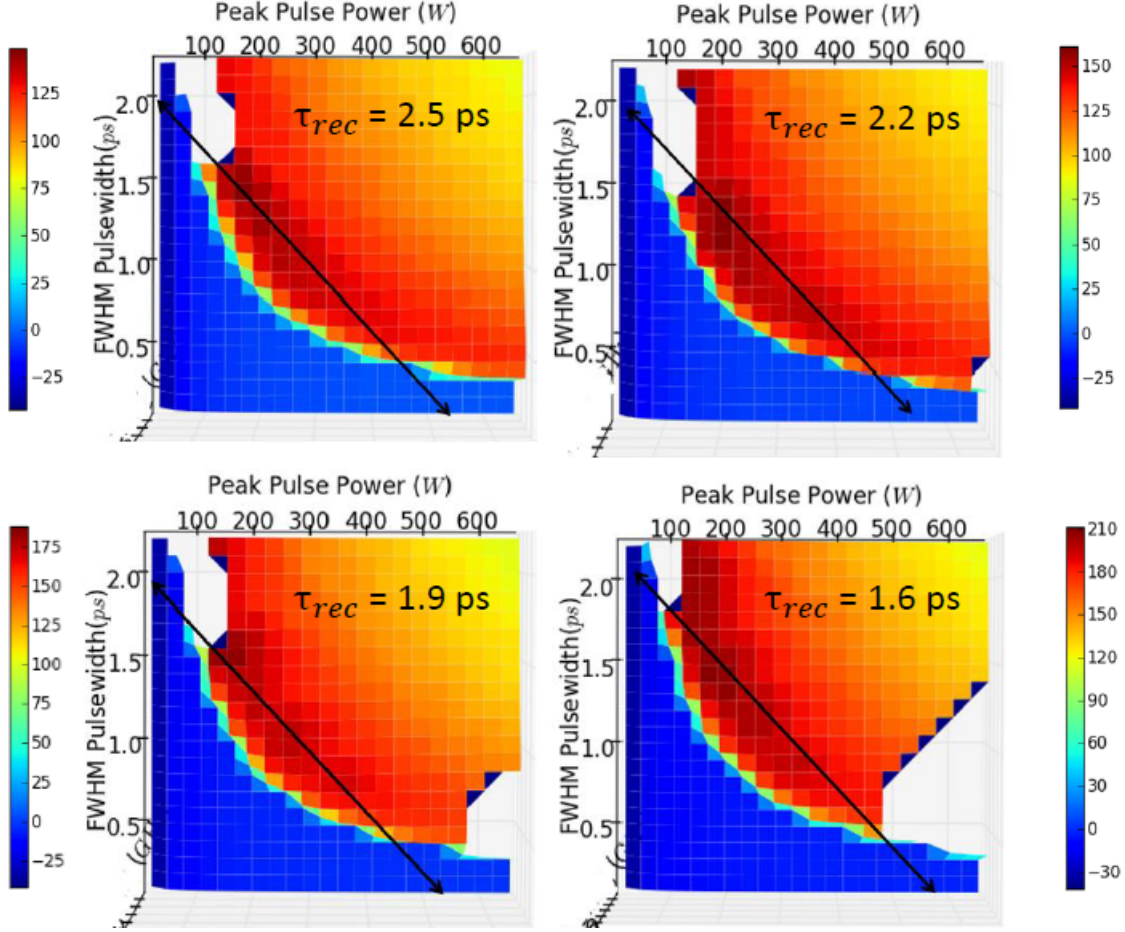


FIG. 27: Top-down view of the surfaces plotted in Figure 26. Notably, the region of each surface corresponding to maximum modulation speed (darkest red) demonstrates a consistent linear dependence of ideal peak power on FWHM pulsewidth, showcasing the applicability of equation (27) across a range of values for τ_{rec} . Notice the significant distinction between the exponential shape of the blue valley's edge versus the much more linear shape of the dark red ridge – excepting $\tau_{rec} = 1.6$ ps (see text).

ideal peak power is returned in Watts. Equation (27) can potentially be used to guide the choice of pump parameters in future high-speed applications.

Of additional interest is the effect on modulation speed had by varying the interband carrier recombination time constant, τ_{rec} , anticipating experimental or environmental factors which may succeed in altering its effective value. Figure 26 shows performance

surfaces within the specified selection of the pump's parameter space for additional values of τ_{rec} . Notably, the surface shape maintains a high degree of consistency across a $\approx 40\%$ variation of recombination times, despite the difference-of-squares dependence in the τ_{rec} terms noted in equations (16) and (17), as well as the highly-nonlinear effects of critical coupling. Moreover, Figure 27 demonstrates that, despite the exponential shape of the intensity threshold (given by the edge of the blue valley), the linear relationship between pump pulsewidth and peak power along the region of maximum modulation speed (darkest red region of surface) is seen to hold for each surface. If there is an exception to this trend, it is seen when $\tau_{rec} = 1.6$ in the lower right of Figure 27, in which the surface maximum continues to linger at low powers for increasing pulsewidths (upper left of surface map). Indeed, it is possible that a comprehensive theoretical examination of a large domain of relevant parameters may reveal a relationship more exponential than linear; however, for a more physically-reasonable subset of the parameter domain as selected herein, the linear relationship is seen to persist. Remarkably, the linear fitting parameters themselves are moreover seen to be constant, rendering equation (27) largely independent of carrier recombination time.

Additionally, the color distributions across the four surfaces of Figure 27 highlight the striking consistency in surface shape, with only the relative height of the surface being altered as a result of a change in carrier recombination time τ_{rec} . Incidentally, the relative height of the surface – determined via its ceiling value, corresponding to the maximum modulation speed realizable under an ideal pump peak-to-width ratio – is itself linearly dependent on τ_{rec} , with the phenomenological result

$$f_{max} \simeq 335 - 81 \tau_{rec}, \quad (28)$$

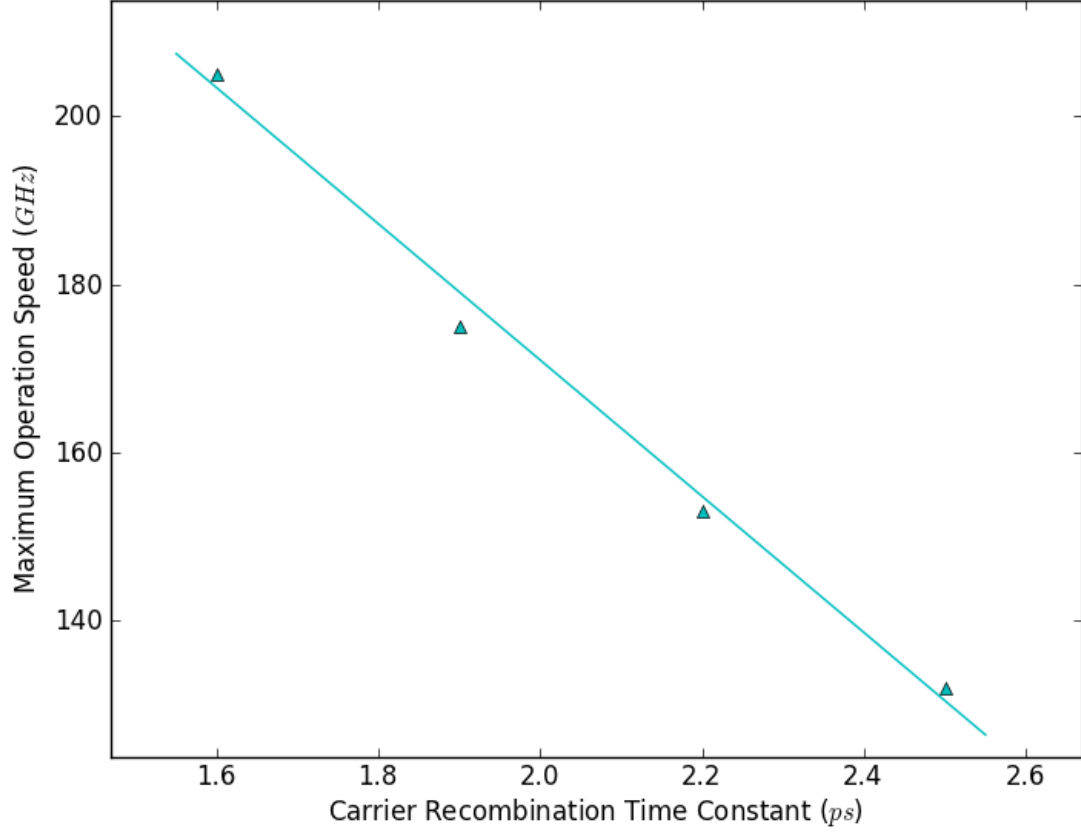


FIG. 28: Dependence of maximum device modulation speed versus carrier recombination time. The maximum modulation ceiling is achieved when equation (27) is satisfied, and the magnitude of this ceiling is subsequently determined by equation (28). The triangles in the above figure represent the peak values of the surfaces in Figure 26, while the solid line is the fit given by equation (28).

where τ_{rec} is the carrier recombination time in picoseconds, and f_{max} is the maximum modulation frequency in gigahertz, realizable when equation (27) is satisfied. As such, while (27) can be used to determine the ideal pump parameters, (28) may be used to predict the maximum modulation speed achievable under such conditions.

7. FUTURE WORK

In further exploring the all-optical amplitude modulation performance limits of the graphene-integrated microring platform developed in this work, it will of course be valuable to conduct laboratory measurements with higher-resolution instrumentation to establish a more precise empirical understanding. Together with the findings of this work, such data may be used to determine more accurately the true limitations on device modulation speed – limits which may be overestimated here. With these limits firmly established, the pump characteristics and other surrounding parameters of the system may be adjusted as outlined here to predict and achieve optimum device performance.

Apart from varying the pump’s characteristics, a noteworthy advantage of the modulator’s physical design is the large degree of flexibility in the off-state transmission in the ring. To decrease the saturation intensity threshold, graphene may be deposited along a shorter arc of the resonator, or may be pre-doped such that fewer transitions are available to begin with. Conversely, to decrease the signal’s base transmission through the chip – which could be used in conjunction with a higher-powered pump to reach greater SNRs – a longer arc of the microresonator may be covered, or additional graphene layers may be stacked one atop the other. The latter approach has exhibited an absorption increasing linearly with the number of sheets in the stack [12, 20]. However, as a tradeoff, the maximum modulation depth has been shown to correspondingly *decrease* linearly with increasing number of graphene layers.

Another key advantage of the microring platform is its natural propensity for multicasting applications, which have recently been investigated in the context of four-wave mixing (FWM) platforms [38–40]. As shown schematically in Figure 29, the traditional optical communications regimes of unicasting and broadcasting allow, respectively, the

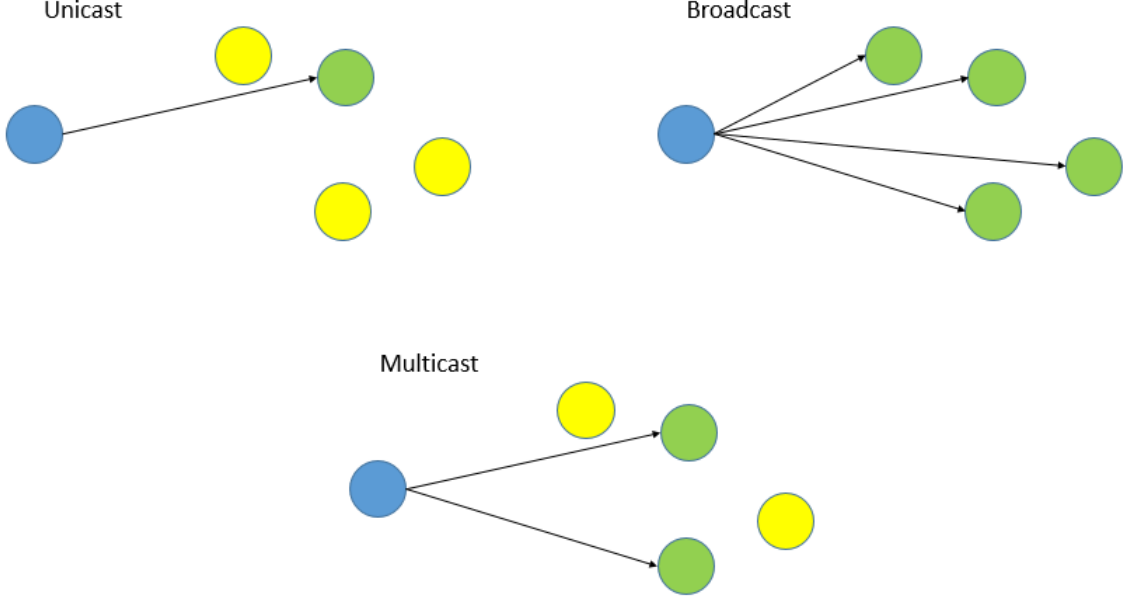


FIG. 29: Communication routing schemes. A unicast limits the data source to choosing and communicating with only one destination location at a time, while a broadcast necessitates that the entire set of possible destinations receive the transmitted data stream. A multicast, by contrast, provides a dedicated and individually-controllable channel between the source and each destination, such that any possible subset of the connected locations may be selected to receive the outgoing data stream.

transmission of a data signal to either *one* or to *all* locations to which the data source has a connected channel. In contrast, multicasting provides a regime by which any given subset of all the possible destinations may be chosen, each receiving the same data signal. Serendipitously, similar ring structures to that used in this work have already demonstrated robust performance as frequency comb generators [41–43], a functionality which may be combined with the results reported here to potentially achieve vigorous multicasting operation: the ring-generated comb will be comprised of distinct frequency lines in the C- and L-bands, the totality of which may be modulated with a pulsed optical pump. This multi-frequency output may then be transmitted to a dense wavelength-division mul-

tiplexer (DWDM) for *demultiplexing*, splitting the signal by frequency, whereupon the desired frequencies may be transmitted or blocked according to their associated destination channel. Each of these transmitting channels would then carry an identical data signal to their associated destination, such that a 50-150 GHz communication link may thereby be selectively established on many channels simultaneously.

8. CONCLUSION

This work has presented a new platform for high-speed all-optical amplitude modulation. Background information has been provided and discussed, and a thorough theoretical analysis of the fundamental physical principles underlying the modulator chip's operation has been given. Measured experimental data has been presented, yielding a device modulation bandwidth of ≥ 50 GHz, with up to ≈ 75 GHz potentially realizable on the same device with higher instrumental resolution, and up to ≈ 100 GHz predicted with a faster BPD. A corresponding numerical study has been detailed, and a framework for predicting and optimizing the performance of similar devices in the future has been given. Finally, the potential impact of the platform has been outlined, including with reference to the availability of increasingly shorter laser pulses, as well as in regard to emerging field applications such as optical multicasting communications.

Appendix: Source Code

Below are included the four separate Python files comprising the numerical computations referenced in this work. “Constants.py” and “Functions.py” must be stored in the same directory as “All-Optical_Modulation.py” and “Modulation_Ceiling.py” in order for the latter two programs to work properly.

```
1 ## Constants.py
2 ## includes the necessary parameters and constants
3 ## relevant to this work's simulations
4 from math import *
5
6 ## Universal Constants
7 # base system of units = g-um-s-e
8 # **note corresponding unit of energy:
9 # 1 H = 1 g*um^2/s^2 = 10^-15 J = 6250 eV
10 h = 1.0531392*10**-19 # reduced Planck's constant, H*s
11 c0 = 3.0*10**14 # vacuum speed of light, um/s
12 c = 3.0*10**14/1.785 # eff. speed of light, um/s
13 e = 1. # electron charge
14 Kb = 1.3806*10**-8 # Boltzman constant, H/K
15 e0 = 344982. # free-space permittivity, e^2/H*um
16
17 ## Graphene Constants
18 vf = 1.1*10**12 # Fermi velocity, um/s
19 af = e**2/(h*c0*4.*e0) # fine structure constant, 1
20
21 ## Design Parameters
22 l = 40. # graphene sheet length, um
```

```

23 area = 1*100. # graphene sheet area, um^2
24 tc = .9954 # coupling coefficient, %
25 Lc = 1570. # cavity length, um
26
27 ## Calculated Quantities
28 tau1 = 150.*10**-15 # thermalization time, s (from Refs 10-11)
29 tau2 = 2.2*10**-12 # recombination rate, s (from Refs 10-11)
30 alpha_0 = 0.11 # lower-power decay constant, dB/um
31
32 ## Variables
33 T = 300. # temperature, K
34 n0 = pi/6.*(Kb*T/(h*vf))**2 # intrinsic carrier density, um^-2
35 n = p = n0
36 nc = pc = n0
37
38 ## Parameters
39 lam_p = 1.54 # wavelength of pump, microns
40 lam_s = 1.6 # wavelength of signal, microns
41 wp = 2*pi*c/lam_p # angular frequency of pump, rad/s
42 ws = 2*pi*c/lam_s # angular frequency of signal, rad/s

```

```

1  ## Functions.py
2  ## includes fundamental equations needed
3  ## for this work's simulations
4
5  from math import *
6  from Constants import *
7
8  # Fermi energy dependence on carrier concentration
9  # takes  $\mu m^{-2}$ , returns H
10 def Ef(n):
11     if n >=0.:
12         return h*vf*sqrt(pi*n)
13     else:
14         return -h*vf*sqrt(-pi*n)
15
16 # Fermi-Dirac distribution function:
17 # takes H, returns %
18 def FD(E, Ef=0.):
19     return ((1. + exp((E - Ef)/(Kb*T))))**-1
20
21 # Density of states , near Dirac point:
22 # takes H, returns  $\mu m^{-2}$ 
23 def DOS(E):
24     #return 2.*Ac*abs(E)/(pi*h**2*vf**2) # per unit cell
25     return 2.*abs(E)/(pi*h**2*vf**2) # per  $\mu m^2$ 

```

```

1  ## All-Optical-Modulation.py
2  ## computes all-optical modulation bandwidth;
3  ## used to plot figures 19–23
4
5  from math import *
6  import numpy as np
7  from matplotlib import pyplot as plt
8  from time import clock
9  from Constants import *
10 from Functions import *
11
12 start_time = clock()
13
14
15 single_pulse = False
16 # ^ if true, observe 3-dB modulation ceiling;
17 #   if false, observe single-pulse behavior
18
19 ## initialize arrays to store iteratively-generated data
20 dB_Performance = [] # extinction ratio in dB
21 Mod_Rate = [] # device modulation rate
22 Max_Trans = [] # Maximum chip transmission at given modulation rate
23 Min_Trans = [] # Minimum chip transmission at given modulation rate
24
25 loss = 0.55 # Estimation of intensity coupling to chip after large loss
26 Io = loss*5.0*10**17/area # incident pump intensity, H/um^2*s
27 fwhm = 2.2*10**-12 # pump pulse FWHM
28 tau = fwhm/(2.*sqrt(2*log(2))) # computation of Gaussian tau from fwhm

```

```

29
30 speed_min = speed = 0.01 # minimum modulation speed, normalized by fwhm
31 speed_max = 0.25 # maximum modulation speed
32 ds = (speed_max-speed_min)/50 # increment to modulation speed
33 notify = 0. # counter to update user on program's progress
34
35 if single_pulse:
36     speed_max = speed # only execute loop for low rep-rate
37
38 while speed <= speed_max:
39     ## Initialize arrays to store calculated data
40     I = [] # incident pump intensity
41     IS = [] # scaled intensity, for graphing in range 0-1
42     F_Upper = [] # occ. prob. of upper pump state
43     F_Lower = [] # occ. prob. of lower pump state
44     Time = [] # time
45     Absorption = [] # percentage of signal absorbed
46     Transmission = [] # percentage of signal transmitted
47     Pump_Trans = [] # percentage of signal transmitted
48     N = [] # electron concentration
49     P = [] # hole concentration
50     F_Sig_l = [] # occ. prob. of lower signal state
51     F_Sig_u = [] # occ. prob. of upper signal state
52     EFC = [] # cond. band Fermi energy
53     EFV = [] # val. band Fermi energy
54
55     nc = pc = n0 # initialize carrier counts to equilibrium
56     f_upper = 0. # initialize upper state occ. prob

```

```

57     f_lower = 1. # initialize lower state occ. prob
58
59
60     ## Fill incident intensity array
61
62     rep_period = fwhm/speed # rep period of pump
63     rep_rate = 1/rep_period # rep rate of pump
64     num_steps = 10000 # number of rep rate vals to examine
65     tmax = 3*rep_period # modulate for 3 rep periods
66     if single_pulse:
67         tmax = 10*fwhm # decrease temporal window to observe single-pulse
68         only
69         dt = tmax/num_steps # time increment
70         t = 0.
71         p = t
72         count = 1.
73         notify2 = 0
74         # fill intensity and scaled intensity arrays
75         while t <= tmax:
76             I.append(Io*exp(-(p-fwhm**2)**2/tau**2))
77             IS.append(exp(-(p-fwhm**2)**2/tau**2))
78
79             if (t-dt) <= count*rep_period <= (t+dt):
80                 p = 0.
81                 count += 1
82
83                 t += dt
84                 p += dt

```



```

84
85     t = 0 # reset
86     ## Perform RK4 calculations
87     while t < len(I)-1:
88
89
90         df_upper1 = (pi*af/(DOS(h*wp/2)*h*wp)*I[t]*f_lower
91                     *(1. - f_upper)
92                     - f_upper/tau1)
93         df_lower1 = (-pi*af/(DOS(h*wp/2)*h*wp)*I[t]*f_lower
94                     *(1. - f_upper)
95                     + (1. - f_lower)/tau1)
96
97         df_upper2 = (pi*af/(DOS(h*wp/2)*h*wp)*.5*(I[t]+I[t+1])
98                     *(f_lower+0.5*dt*df_lower1)*
99                     (1. - (f_upper+0.5*dt*df_upper1))
100                    - (f_upper+0.5*dt*df_upper1)/tau1)
101         df_lower2 = (-pi*af/(DOS(h*wp/2)*h*wp)*.5*(I[t]+I[t+1])
102                     *(f_lower+0.5*dt*df_lower1)*
103                     (1. - (f_upper+0.5*dt*df_upper1))
104                    + (1. - (f_lower+0.5*dt*df_lower1))/tau1)
105
106         df_upper3 = (pi*af/(DOS(h*wp/2)*h*wp)*.5*(I[t]+I[t+1])
107                     *(f_lower+0.5*dt*df_lower2)*
108                     (1. - (f_upper+0.5*dt*df_upper2))
109                    - (f_upper+0.5*dt*df_upper2)/tau1)
110         df_lower3 = (-pi*af/(DOS(h*wp/2)*h*wp)*.5*(I[t]+I[t+1])
111                     *(f_lower+0.5*dt*df_lower2)*

```

```

112         (1. - (f_upper+0.5*dt*df_upper2))
113     + (1. - (f_lower+0.5*dt*df_lower2))/tau1)
114
115 df_upper4 = (pi*af/(DOS(h*wp/2)*h*wp)*I[t]
116             *(f_lower+dt*df_lower3)
117             *(1. - (f_upper+dt*df_upper3))
118             - (f_upper+dt*df_upper3)/tau1)
119 df_lower4 = (-pi*af/(DOS(h*wp/2)*h*wp)*I[t]
120             *(f_lower+dt*df_lower3)
121             *(1. - (f_upper+dt*df_upper3))
122             + (1. - (f_lower+dt*df_lower3))/tau1)
123
124
125 dnc1 = (f_upper*DOS(h*wp/2)/tau1 - (nc-n0)/tau2)
126 dpc1 = ((1 - f_lower)*DOS(h*wp/2)/tau1 - (pc-n0)/tau2)
127
128 dnc2 = (f_upper*DOS(h*wp/2)/tau1
129         - ((nc+0.5*dt*dnc1)-n0)/tau2)
130 dpc2 = ((1 - f_lower)*DOS(h*wp/2)/tau1
131         - ((pc+0.5*dt*dpc1)-n0)/tau2)
132
133 dnc3 = (f_upper*DOS(h*wp/2)/tau1
134         - ((nc+0.5*dt*dnc2)-n0)/tau2)
135 dpc3 = ((1 - f_lower)*DOS(h*wp/2)/tau1
136         - ((pc+0.5*dt*dpc2)-n0)/tau2)
137
138 dnc4 = (f_upper*DOS(h*wp/2)/tau1
139         - ((nc+dt*dnc3)-n0)/tau2)

```

```

140     dpc4 = ((1 - f_lower)*DOS(h*wp/2)/tau1
141            - ((pc+dt*dpc3)-n0)/tau2)
142
143
144     f_upper += (df_upper1 + 2*df_upper2
145               + 2*df_upper3 + df_upper4)*dt/6
146     f_lower += (df_lower1 + 2*df_lower2
147               + 2*df_lower3 + df_lower4)*dt/6
148
149     nc += (dnc1 + 2*dnc2 + 2*dnc3 + dnc4)*dt/6
150     pc += (dpc1 + 2*dpc2 + 2*dpc3 + dpc4)*dt/6
151
152     Efc = Ef(nc+n0)
153     Efv = Ef(-pc-n0)
154
155     # pump dynamics
156     pump_absorption = ((1-exp(-alpha_0*l))
157                       *f_lower*(1-f_upper))
158     pump_transmission = ((pump_absorption - tc)**2
159                          /(1-pump_absorption*tc)**2)
160
161     # signal absorption and subsequent chip transmission:
162     absorption = ((1-exp(-alpha_0*l))
163                  *FD(-h*ws/2,Efv)*(1-FD(h*ws/2,Efc)))
164     transmission = ((absorption - tc)**2
165                     /(1. - absorption*tc)**2)
166
167

```

```

168     # Store loop results
169     F_Upper.append(f_upper)
170     F_Lower.append(f_lower)
171     Time.append(t*dt)
172     N.append(nc)
173     P.append(pc)
174     F_Sig_l.append(FD(-h*ws/2,Efv))
175     F_Sig_u.append(FD(h*ws/2,Efc))
176     EFC.append(Efc*6250)
177     EFV.append(Efv*6250)
178     Pump_Trans.append(pump_transmission)
179     Absorption.append(absorption)
180     Transmission.append(transmission)
181
182     t+= 1
183
184     # ignore early transmission data prior to pump's activation
185     sample = Transmission[int(ceil(fwhm*2/dt)):
186                             int(floor(len(Transmission)))]
187
188     # Store loop results
189     Mod_Rate.append(rep_rate)
190     Max_Trans.append(max(sample))
191     Min_Trans.append(min(sample))
192     dB_Performance.append(10.*log(max(sample)-min(sample),10))
193
194     # update user to program's progress
195     notify += 1.

```

```

196     if notify % 5 == 0.:
197         print notify, "of", floor((speed_max-speed_min)/ds), "complete"
198
199
200     speed += ds
201
202 # delete un-used final data point
203 I[:] = (i for i in I[0:len(I)-1])
204 IS[:] = (i for i in IS[0:len(IS)-1])
205
206
207 ## determine 3-dB ceiling
208 Mod_Rate[:] = (i*10**-9 for i in Mod_Rate) # Hz --> GHz
209 idx = 0
210 while idx < len(dB_Performance):
211     if dB_Performance[idx] <= -3.:
212         ceiling = Mod_Rate[idx]
213         print "Maximum modulation speed is", ceiling, "GHz"
214         break
215     idx += 1
216
217 # calculate total program run-time
218 elapsed_time = clock() - start_time
219
220 # convert seconds to picoseconds
221 Time[:] = (i*10**12 for i in Time)
222
223 if single_pulse:

```

```

224 plt.plot(#Time, Intensity, 'bo')
225         #Time, F_Upper, 'co',
226         #Time, F_Lower, 'mo')
227     Time, IS, 'b-',
228     #Time, Absorption, 'co')
229     Time, Transmission, 'r-')
230     #Time, Intensity, 'ro')
231     #Time, N, 'co')
232     #Time, F_Sig, 'ko')
233     #Time, EFV, 'ro'
234     #Mod_Rate, dB_Performance, 'b-')
235
236 ## Single Pulse
237 plt.plot(Time, IS, 'b-', label="Pump (incident)")
238 plt.plot(Time, Transmission, 'r-', label="Signal (transmitted)")
239 plt.xlabel("Time (ps)")
240 plt.ylabel("Normalized Intensity (%)")
241 plt.legend(numpoints=1)
242 plt.show()
243
244 else:
245
246     plt.plot(Mod_Rate, dB_Performance, 'b-')
247
248 ## Modulation Ceiling
249 plt.gca().set_xlim([0, Mod_Rate[idx]])
250 plt.gca().set_ylim([-4.0, -0.5])
251 plt.xlabel("Pump Rep Rate (GHz)")

```

```
252     plt.ylabel("Extinction Ratio (dB)")
253     plt.show()
254
255 #####
```

```

1  ## Modulation_Ceiling.py
2  ## computes three-dimensional modulation surfaces
3  ## in pump's parameter space;
4  ## used to plot figures 24–27
5
6  from math import *
7  import numpy as np
8  from matplotlib import pyplot as plt
9  from time import clock
10 from mpl_toolkits.mplot3d import Axes3D
11 from Constants import *
12 from Functions import *
13
14 start_time = clock()
15
16 # input: takes pump peak power in watts,
17 #         pulsewidth in picoseconds
18 def ceiling(watts,ps):
19     disp_red = 0.75 # exaggerated effect of cavity dispersion
20     loss = .65 # coupling loss
21     Io = watts*10**15*loss/area
22     fwhm = ps*10**-12
23
24     tau = 0.5*fwhm/(2.*sqrt(2*log(2)))
25     speed_min = speed = 0.01
26     speed_max = 0.4 # pulse period = 4fwhm, fastest possible
27     ds = (speed_max-speed_min)/25
28

```



```

29  while speed <= speed_max:
30      ## Initialize arrays to store calculated data
31      I = []
32      IS = [] # scaled intensity, for graphing in range 0-1
33      Time = []
34      Absorption = []
35      Transmission = []
36
37      nc = pc = n0
38      f_upper = 0. # init. occ.prob of upper pump state
39      f_lower = 1. # init. occ.prob of lower pump state
40
41
42      ## Fill incident intensity array
43      rep_period = fwhm/speed
44      rep_rate = 1/rep_period
45      num_steps = 3000
46      tmax = 3*rep_period # for modulation bandwidth
47      dt = tmax/num_steps
48      t = 0.
49      p = t
50      count = 1.
51
52      while t <= tmax:
53          I.append(Io*exp(-(p-fwhm*2)**2/tau**2))
54          IS.append(exp(-(p-fwhm*2)**2/tau**2))
55
56          if (t-dt) <= count*rep_period <= (t+dt):

```

```

57         p = 0.
58         count += 1
59
60         t += dt
61         p += dt
62
63     t = 0 # reset
64     ## Perform RK4 calculations
65     while t < len(I)-1:
66
67
68         df_upper1 = (pi*af/(DOS(h*wp/2)*h*wp)*I[t]*f_lower
69                     *(1. - f_upper)
70                     - f_upper/tau1)
71         df_lower1 = (-pi*af/(DOS(h*wp/2)*h*wp)*I[t]*f_lower
72                     *(1. - f_upper)
73                     + (1. - f_lower)/tau1)
74
75         df_upper2 = (pi*af/(DOS(h*wp/2)*h*wp)*.5*(I[t]+I[t+1])
76                     *(f_lower+0.5*dt*df_lower1)*
77                     (1. - (f_upper+0.5*dt*df_upper1))
78                     - (f_upper+0.5*dt*df_upper1)/tau1)
79         df_lower2 = (-pi*af/(DOS(h*wp/2)*h*wp)*.5*(I[t]+I[t+1])
80                     *(f_lower+0.5*dt*df_lower1)*
81                     (1. - (f_upper+0.5*dt*df_upper1))
82                     + (1. - (f_lower+0.5*dt*df_lower1))/tau1)
83
84         df_upper3 = (pi*af/(DOS(h*wp/2)*h*wp)*.5*(I[t]+I[t+1])

```

```

85         *(f_lower+0.5*dt*df_lower2)*
86         (1. - (f_upper+0.5*dt*df_upper2))
87         - (f_upper+0.5*dt*df_upper2)/tau1)
88 df_lower3 = (-pi*af/(DOS(h*wp/2)*h*wp)*.5*(I[t]+I[t+1])
89         *(f_lower+0.5*dt*df_lower2)*
90         (1. - (f_upper+0.5*dt*df_upper2))
91         + (1. - (f_lower+0.5*dt*df_lower2))/tau1)
92
93 df_upper4 = (pi*af/(DOS(h*wp/2)*h*wp)*I[t]
94         *(f_lower+dt*df_lower3)
95         *(1. - (f_upper+dt*df_upper3))
96         - (f_upper+dt*df_upper3)/tau1)
97 df_lower4 = (-pi*af/(DOS(h*wp/2)*h*wp)*I[t]
98         *(f_lower+dt*df_lower3)
99         *(1. - (f_upper+dt*df_upper3))
100         + (1. - (f_lower+dt*df_lower3))/tau1)
101
102
103 dnc1 = (f_upper*DOS(h*wp/2)/tau1 - (nc-n0)/tau2)
104 dpc1 = ((1 - f_lower)*DOS(h*wp/2)/tau1 - (pc-n0)/tau2)
105
106 dnc2 = (f_upper*DOS(h*wp/2)/tau1
107         - ((nc+0.5*dt*dnc1)-n0)/tau2)
108 dpc2 = ((1 - f_lower)*DOS(h*wp/2)/tau1
109         - ((pc+0.5*dt*dpc1)-n0)/tau2)
110
111 dnc3 = (f_upper*DOS(h*wp/2)/tau1
112         - ((nc+0.5*dt*dnc2)-n0)/tau2)

```

```

113     dpc3 = ((1 - f_lower)*DOS(h*wp/2)/tau1
114           - ((pc+0.5*dt*dpc2)-n0)/tau2)
115
116     dnc4 = (f_upper*DOS(h*wp/2)/tau1
117           - ((nc+dt*dnc3)-n0)/tau2)
118     dpc4 = ((1 - f_lower)*DOS(h*wp/2)/tau1
119           - ((pc+dt*dpc3)-n0)/tau2)
120
121
122     f_upper += (df_upper1 + 2*df_upper2
123               + 2*df_upper3 + df_upper4)*dt/6
124     f_lower += (df_lower1 + 2*df_lower2
125               + 2*df_lower3 + df_lower4)*dt/6
126
127     nc += (dnc1 + 2*dnc2 + 2*dnc3 + dnc4)*dt/6
128     pc += (dpc1 + 2*dpc2 + 2*dpc3 + dpc4)*dt/6
129
130     Efc = Ef(nc+n0)
131     Efv = Ef(-pc-n0)
132
133     # signal absorption and subsequent chip transmission:
134     absorption = ((1-exp(-alpha_0*l))
135                 *FD(-h*ws/2,Efv)*(1-FD(h*ws/2,Efc)))
136     transmission = ((absorption - tc)**2
137                   /(1. - absorption*tc)**2)
138
139     # Store loop results for plotting
140     Time.append(t*dt)
141     Absorption.append(absorption)

```

```

141         Transmission.append(transmission)
142
143         t+= 1
144
145         # ignore early transmission data prior to pump's activation
146         sample = Transmission[int(ceil(fwhm*2/dt)):
147                               int(floor(len(Transmission)))]
148
149         # find modulation bandwidth
150         dB_Performance = 10.*log(max(sample)-min(sample),10)
151         if dB_Performance <= -3.:
152             return disp_red*(speed-ds)/fwhm*10**-9 # return prior rep rate
153         speed += ds
154
155
156 # initialize range of pump peak powers
157 power_array = []
158 power = power_min = 20.
159 power_max = 650.
160 dp = 30.
161 while power <= power_max:
162     power_array.append(power)
163     power += dp
164
165 #initialize range of pump pulsewidths
166 width_array = []
167 width = width_min = .100
168 width_max = 2.3

```

```

169 dw = .100

170 while width <= width_max:

171     width_array.append(width)

172     width += dw

173

174 p_len = len(power_array)

175 w_len = len(width_array)

176 data_array = np.zeros((p_len, w_len))

177

178 for p in xrange(0, p_len):

179     for w in xrange(0, w_len):

180         data_array[p, w] = ceiling(power_array[p], width_array[w])

181     print "Completed", p+1, "of", p_len, "iterations"

182

183 # plot data

184 fig = plt.figure("{:.2}".format(tau2) + ' ps')

185 ax = fig.add_subplot(111, projection='3d')

186

187 X, Y = np.meshgrid(power_array, width_array)

188 surf = ax.plot_surface(X, Y, data_array, rstride=1, cstride=1,

189                        cmap=plt.cm.jet, linewidth=0,

190                        vmax=np.nanmax(data_array),

191                        vmin=np.nanmin(data_array))

192 fig.colorbar(surf, shrink=0.75)

193 ax.set_xlabel("Peak Pulse Power ($W$)")

194 ax.set_ylabel("FWHM Pulsewidth ($ps$)")

195 ax.set_zlabel("3-dB Modulation Ceiling ($GHz$)")

196

```

```
197
198
199 time_elapsed = clock() - start_time
200 print "Total time elapsed:", time_elapsed, "seconds"
201
202 # play an audio notification when finished
203 from winsound import Beep as bp
204 bp(262,900)
205 bp(392,900)
206 bp(330,1400)
207
208 plt.show()
```

-
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