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Abstract

We consider a scenario in which the dominant quartic coupling for the Higgs doublets arises from the F -term potential, rather than the conventional $SU(2)_L \times U(1)_Y$ D -term potential, in supersymmetric theories. The quartic coupling arises from a superpotential interaction between the two Higgs doublets and a singlet field, but unlike the case in the next-to-minimal supersymmetric standard model the singlet field is not responsible for the generation of the supersymmetric or holomorphic supersymmetry-breaking masses for the Higgs doublets. We find that this naturally leads to a deviation from the conventional picture of top-Yukawa driven electroweak symmetry breaking — electroweak symmetry breaking is triggered by the holomorphic supersymmetry breaking mass for the Higgs doublets (the μB term). This allows a significant improvement for fine-tuning in electroweak symmetry breaking, since the top squarks do not play a major role in raising the Higgs boson mass or in triggering electroweak symmetry breaking and thus can be light. The amount of fine-tuning is given by the squared ratio of the lightest Higgs boson mass to the charged Higgs boson mass, which can be made better than 20%. Solid implications of the scenario include a small value for $\tan \beta$, $\tan \beta \lesssim 3$, and relatively light top squarks.

1 Introduction

One of the most pressing issues in particle theory is to understand the origin of electroweak symmetry breaking. In the standard model, the effect of electroweak symmetry breaking is parameterized by the vacuum expectation value (VEV) of the Higgs field h , which is generated by the potential

$$V = m_h^2 |h|^2 + \frac{\lambda_h}{4} |h|^4, \quad (1)$$

where m_h^2 is negative and λ_h is positive. Since m_h^2 is highly sensitive to the physics at the cutoff scale Λ , however, the standard model cannot adequately describe the origin of electroweak symmetry breaking — the physics of electroweak symmetry breaking is at or beyond Λ .

Weak scale supersymmetry provides an elegant framework to address this issue. Since quadratic divergences to the Higgs boson mass-squared parameters are cut off by masses of superparticles, the physics of electroweak symmetry breaking can be described reliably within the framework of softly broken supersymmetric effective field theory. However, supersymmetry itself does not tell us the origin of the Higgs potential, Eq. (1). Why is m_h^2 negative? What is the origin of λ_h ?

The conventional answer to these questions, which has been dominating over 20 years, is that the restoring force in the potential, λ_h , arises from supersymmetric partners of weak interactions – the $SU(2)_L \times U(1)_Y$ D -term potential – while the mass-squared parameter, m_h^2 , becomes negative because of the logarithmically divergent top-stop loop correction [1]. In fact, this provides an elegant answer to the most naive question in supersymmetric theories: why do the Higgs fields develop VEVs while squarks and sleptons do not? Since the origin of the instability is the top Yukawa coupling, the only fields that could get VEVs are the third generation squarks, \tilde{q}_3 and \tilde{u}_3 , and the Higgs fields. The famous color factor associated with the top-stop loop and/or a small hierarchy between colored and non-colored scalar masses can then explain why the Higgs fields are the only ones that obtain VEVs.

The framework just described, however, has been put in a subtle position after the negative results for the Higgs boson search at the LEP II. To evade the LEP II bound on the physical Higgs boson mass, $M_{\text{Higgs}} \gtrsim 114$ GeV [2], the coupling λ_h must satisfy $\lambda_h \gtrsim 0.43$ because the physical Higgs boson mass is given by

$$M_{\text{Higgs}}^2 = \lambda_h v^2, \quad (2)$$

where $v \equiv \langle h \rangle \simeq 174$ GeV. In the minimal supersymmetric extension of the standard model (MSSM), the $SU(2)_L \times U(1)_Y$ D -terms give the quartic couplings for the Higgs fields $V = (g^2 + g'^2)(|H_u^0|^2 - |H_d^0|^2)^2/8$, where $g \simeq 0.65$ and $g' \simeq 0.36$ are the $SU(2)_L$ and $U(1)_Y$ gauge couplings, and H_u and H_d are the two Higgs doublets giving masses for the up-type and down-type quarks

with the superscript 0 denoting the neutral components. Consider now that the field developing a VEV resides in H_u and H_d as $H_u^0 = h^\dagger \sin \beta$ and $H_d^0 = h \cos \beta$, where $\tan \beta \equiv \langle H_u^0 \rangle / \langle H_d^0 \rangle$. This is a valid approximation if the Higgs bosons arising from the other linear combination are heavier than the excitation of h . In this case we find

$$\lambda_h = \frac{g^2 + g'^2}{2} \cos^2 2\beta. \quad (3)$$

This is too small to give $M_{\text{Higgs}} \gtrsim 114$ GeV. In the MSSM, we avoid this difficulty by invoking a radiative correction from top-stop loop, which gives an extra contribution to λ_h [3]. To evade the LEP II bound, however, we need a rather large mass for the top squarks: $m_{\tilde{t}} \gtrsim (800 \sim 1200)$ GeV for a reasonably small top squark mixing parameter [4]. This in turn gives a large negative contribution to the h mass-squared parameter

$$\delta m_h^2 \simeq -\frac{3m_{\tilde{t}}^2}{4\pi^2 v^2} m_{\tilde{t}}^2 \ln\left(\frac{M_{\text{mess}}}{m_{\tilde{t}}}\right), \quad (4)$$

where M_{mess} is the scale at which the squark and slepton masses are generated. To reproduce the correct scale for electroweak symmetry breaking, we then need to cancel this with other positive contributions to m_h^2 , for example the one from the supersymmetric mass for the Higgs fields. In many supersymmetric models, this requires a cancellation of order a few percent or worse. This problem is often called the supersymmetric fine-tuning problem.

The simplest way to avoid the supersymmetric fine-tuning problem is to make both $m_{\tilde{t}}^2$ and $\ln(M_{\text{mess}}/m_{\tilde{t}})$ small. Suppose that $\ln(M_{\text{mess}}/m_{\tilde{t}})$ is as small as a factor of a few, which is reasonable if the mediation scale of supersymmetry breaking is low. In this case the fine-tuning can be made better than $(20 \sim 30)\%$ if $m_{\tilde{t}} \lesssim (400 \sim 510)(M_{\text{Higgs}}/150 \text{ GeV})(3.5/\ln(M_{\text{mess}}/m_{\tilde{t}}))^{1/2}$ [5, 6]. However, such light top squark masses require some ingredient to be reconciled with the LEP II bound of $M_{\text{Higgs}} \gtrsim 114$ GeV. One of the simplest possibilities then is to introduce an additional contribution to the Higgs quartic coupling other than the $SU(2)_L \times U(1)_Y$ D -terms and the top-stop loop.¹ Suppose now that we actually have such a contribution. We then find that, unless it comes from the coupling of the form $|H_u|^4$, the contribution from the D -terms, Eq. (4), is suppressed, because any other forms of couplings, such as $|H_u H_d|^2$, give powers of $\cos \beta$ when expressed in terms of h , so that $\tan \beta$ is required not to be very large (i.e. $\cos^2 2\beta$ is appreciably smaller than 1) for the new contribution to be effective. This suggests an alternative picture for the origin of the Higgs potential in supersymmetric theories: the dominant source of the Higgs quartic coupling may not be the $SU(2)_L \times U(1)_Y$ D -terms.

In this paper we study a framework in which the *dominant* contribution to λ_h comes from the F -term potential for the Higgs fields, rather than the D -terms of $SU(2)_L \times U(1)_Y$. While such a

¹An alternative possibility is to have a large trilinear scalar coupling for the top squarks. A solution to the supersymmetric fine-tuning problem using this has been presented in [7]. The effect of the trilinear coupling on fine-tuning was also analyzed, e.g., in [8].

contribution arises from coupling the Higgs fields with some other field (a gauge singlet field) in the superpotential, we are not particularly interested in the dynamics of the additional field. In fact, we mainly consider the cases in which the dynamics of this field essentially decouples. We find that this naturally leads us to consider a scenario in which not only the origin of λ_h but also the reason for the instability, $m_h^2 < 0$, deviate from the conventional picture. In this scenario, it is the presence of the holomorphic supersymmetry breaking mass term (μB term) for the Higgs fields that is crucial for electroweak symmetry breaking. The top-stop loop contribution of Eq. (4) also plays a role, but in the absence of the μB term the electroweak symmetry is not broken. In the conventional picture, the top-stop loop itself drives the squared mass of the (up-type) Higgs field negative, triggering electroweak symmetry breaking. The role of the μB term is then to make $\tan\beta$ differ from infinity. In our picture the electroweak symmetry is broken by the presence of the μB term. The role of the top-stop loop contribution then is, besides contributing to electroweak symmetry breaking, to make $\tan\beta$ deviate from 1, which would be the case if the squared masses for the up-type and down-type Higgs fields were the same.

One of the major advantages of the present scenario over the conventional one is that the masses of the top squarks can be small as they do not play a major role in raising the Higgs boson mass nor in triggering electroweak symmetry breaking. This makes it possible to evade severe fine-tuning in electroweak symmetry breaking. We find that the fine-tuning in our scenario is determined essentially by the size of the μB parameter, which in turn is related to the mass of the charged Higgs boson. We find that fine-tuning can be made better than 20% for reasonable values of the lightest Higgs boson mass and the charged Higgs boson mass.

We note here that similar dynamics for the Higgs sector were considered in Ref. [9]. There the possibility of electroweak symmetry breaking by the μB term and the reduction of fine-tuning by large quartic couplings were discussed for generic Higgs potentials in the context of very low ($O(\text{TeV})$) scale fundamental supersymmetry breaking.

In the next section we discuss our scenario. The basic framework is described in section 2.1, and the two particularly interesting cases are studied in more detail in sections 2.2 and 2.3. The simple formulae for the fine-tuning are derived. Some implications of the scenario are discussed in 2.4. Conclusions are given in section 3.

2 μB -driven Electroweak Symmetry Breaking

2.1 The basic framework

Let us consider the superpotential term

$$W = \lambda S H_u H_d, \tag{5}$$

where S is a gauge singlet field and λ is a dimensionless coupling. The $SU(2)_L$ contraction for the Higgs fields is defined by $H_u H_d \equiv H_u^+ H_d^- - H_u^0 H_d^0$. This is the simplest term that can give an F -term contribution to the Higgs quartic coupling, without destroying the successful supersymmetric prediction for gauge coupling unification [10]. The value of the coupling λ at the weak scale is constrained by a Landau pole consideration. Requiring that λ , as well as the gauge and Yukawa couplings, are perturbative up to the unification scale of 10^{16} GeV, the weak scale value of λ is constrained to be $\lambda \lesssim (0.6 \sim 0.8)$, depending on the value of $\tan \beta$ and the assumption on the field content between the weak and the unification scales [11]. A larger value of λ , however, may be obtained if some of the S , H_u and H_d fields are composite states arising at low scales [12] or if the gauge group is extended at the weak scale [13]. As we will see explicitly later, these values of λ can produce a stabilizing force in the Higgs potential (the quartic coupling) stronger than that arising from the $SU(2)_L \times U(1)_Y$ D -terms.

Unlike the case for the next-to-minimal supersymmetric standard model (NMSSM) [14], we are not interested in using the interaction of Eq. (5) to explain the origin of the supersymmetric mass term (μ term) for the Higgs doublets.² Rather, we consider that the μ term, as well as the μB term, arise from some other source. Such a setup can naturally arise, for example, if some of the S , H_u and H_d fields are coupled to the sector that breaks supersymmetry dynamically at a scale of order ($10 \sim 100$) TeV through nearly marginal operators [5, 18]. Alternatively, the μ and μB terms may arise from supergravity effects [19] if the fundamental scale of supersymmetry breaking is at an intermediate scale [20].

The Higgs sector of our framework is then specified by the following superpotential and soft supersymmetry breaking Lagrangian:

$$W = \lambda S H_u H_d + \mu H_u H_d + \hat{W}(S), \quad (6)$$

and

$$\mathcal{L}_{\text{soft}} = -m_{H_u}^2 |H_u|^2 - m_{H_d}^2 |H_d|^2 - (\mu B H_u H_d + \text{h.c.}) + \hat{\mathcal{L}}_{\text{soft}}(S, H_u, H_d), \quad (7)$$

where $\hat{W}(S)$ is a holomorphic function of S , and $\hat{\mathcal{L}}_{\text{soft}}(S, H_u, H_d)$ represents soft supersymmetry breaking terms that contain the S field. In addition, the Higgs potential contains the terms arising from the $SU(2)_L \times U(1)_Y$ D -terms, which we write as

$$V_D = \epsilon \frac{g^2}{2} \sum_{a=1}^3 \left(H_u^\dagger \frac{\sigma^a}{2} H_u + H_d^\dagger \frac{\sigma^a}{2} H_d \right)^2 + \epsilon' \frac{g'^2}{2} \left(\frac{1}{2} H_u^\dagger H_u - \frac{1}{2} H_d^\dagger H_d \right)^2, \quad (8)$$

where σ^a are the Pauli matrices. Here, we have introduced the factors ϵ and ϵ' , which take values $\epsilon, \epsilon' \leq 1$, to take into account potential suppressions of these D -terms. Such suppressions arise, for example, in theories in which the gauginos obtain Dirac masses through supersymmetry

²For discussions of fine-tuning in the NMSSM in various contexts, see e.g. [5, 15 – 17].

breaking operators coupling them with other adjoint fermions [21]. Our electroweak symmetry breaking scenario works even in the case with $\epsilon, \epsilon' \ll 1$, since the existence of the D -term potential is not important. In the absence of the Dirac gaugino masses, as in the usual case, $\epsilon = \epsilon' = 1$. We assume that the parameters appearing in Eqs. (6, 7) are real throughout the paper.

The fact that we do not need to generate the μ or μB term from the interaction of Eq. (5) allows considerable freedoms for $\hat{W}(S)$ and $\hat{\mathcal{L}}_{\text{soft}}(S, H_u, H_d)$. Any constraints that would arise from requiring the appreciable size of the S VEV to be generated are absent here. In fact, this allows very simple forms for $\hat{W}(S)$ and $\hat{\mathcal{L}}_{\text{soft}}(S, H_u, H_d)$ as realistic possibilities. In this paper we particularly focus on the cases in which the dynamics of the S field almost decouples from the physics of electroweak symmetry breaking and the VEV of S is much smaller than the weak scale. The purpose of the interaction of Eq. (5), then, is to provide the source of the Higgs quartic coupling, λ_h , needed to push up the Higgs boson mass. (This interaction also modifies the spectrum in the Higgs sector by mixing the H_u and H_d states with the S states.)

As we will see later, the quartic coupling arising from the interaction of Eq. (5) takes the form of either $|H_u H_d|^2$ or $(|H_u|^2 + |H_d|^2)(H_u H_d + \text{h.c.})$, giving a contribution to λ_h proportional to $\sin^2 2\beta$ or $\sin 2\beta$. This implies that these F -term quartic couplings do not stabilize the direction $|H_u| \rightarrow \infty$ with $H_d = 0$ or $|H_d| \rightarrow \infty$ with $H_u = 0$ in field space. Of course, these directions are eventually stabilized by the quartic couplings arising from the $SU(2)_L \times U(1)_Y$ D -term potential, Eq. (8), but then we are relying on the weak stabilization force from the gauge D -terms and the situation is not very much different from the conventional scenario. Therefore, to use a stronger F -term quartic coupling as the dominant restoring force, we consider the situation in which the directions $H_u H_d = 0$ are stabilized by the *quadratic* terms in the potential. This requires that the squared masses for *both* the up-type and down-type Higgs fields are positive:

$$|\mu|^2 + m_{H_u}^2 > 0 \quad \text{and} \quad |\mu|^2 + m_{H_d}^2 > 0. \quad (9)$$

Electroweak symmetry breaking is then necessarily caused by the μB term which satisfies

$$|\mu B|^2 > (|\mu|^2 + m_{H_u}^2)(|\mu|^2 + m_{H_d}^2), \quad (10)$$

by making one of the eigenvalues in the Higgs mass-squared matrix negative. For this reason, we call the scenario presented here *μB -driven electroweak symmetry breaking*.

In the present scenario, the role of the negative top-stop loop contribution to $m_{H_u}^2$ is, besides facilitating to satisfy Eq. (10), to deviate $\tan\beta$ from 1. If this contribution were absent, the squared masses for the up-type and down-type Higgs fields would be the same so that $\tan\beta$ would exactly be 1, because the theory then possesses a symmetry $H_u \leftrightarrow H_d^\dagger$ (in the limit of

neglecting all the other Yukawa couplings).³ In fact, $\tan\beta$ in this scenario is given by $\tan^2\beta \approx (|\mu|^2 + m_{H_d}^2)/(|\mu|^2 + m_{H_u}^2)$, explicitly showing that $m_{H_u}^2 \neq m_{H_d}^2$ is the source of $\tan\beta \neq 1$. This equation also tells us that $\tan\beta$ is expected to be not very large in the present scenario, e.g. $\tan\beta \lesssim 3$, because a very large $\tan\beta$ requires a fine cancellation between the two terms in the denominator of the right-hand-side. This is consistent with the fact that we use the quartic coupling proportional to $\sin^2 2\beta$ or $\sin 2\beta$, which would vanish in the limit of large $\tan\beta$.

We note that the scenario described here provides an alternative answer to the question: why do only the Higgs fields develop VEVs? The answer is: because they are the only fields that are vector-like under the standard model gauge group. We have seen that electroweak symmetry breaking is triggered by the holomorphic supersymmetry breaking mass term, the μB term, in the Higgs sector. For the squarks and sleptons we cannot write down such terms because they belong to supermultiplets that are chiral under the standard model gauge group. Analogous terms can arise after electroweak symmetry breaking (the left-right mixing term), but they are generically smaller. For the top squarks, however, we may need an additional argument, such as the one that colored particles are somewhat heavier than the others, depending on the model of supersymmetry breaking.

2.2 Scenario I — Supersymmetric Higgs with a “non-supersymmetric” quartic coupling

We now want to discuss the dynamics of electroweak symmetry breaking in more detail. For this purpose we can integrate out the S field at tree level and obtain the effective theory containing only the Higgs doublet fields. In general, the functions $\hat{W}(S)$ and $\hat{\mathcal{L}}_{\text{soft}}(S, H_u, H_d)$ contain arbitrary mass parameters of order the weak scale. The integration of S in the general setup is performed in the Appendix.

We are particularly interested in the situation in which the dynamics of S essentially decouples. We here consider the case in which the decoupling occurs because of a large soft supersymmetry breaking mass of S : $m_S^2 \gg v^2, |M_S|^2, |b_S|$, where M_S and b_S are the supersymmetric mass and the holomorphic supersymmetry breaking mass squared for S , respectively. By large, we mean $(m_S^2)^{1/2}$ of the order of several hundreds of GeV. Since m_S^2 gives a contribution to $m_{H_u}^2$ at the one-loop level, it cannot be larger than about $(8\pi^2/\lambda^2)v^2$.

The effective Higgs potential for the case with $m_S^2 \gg v^2$ is obtained simply by setting $S = 0$ in the potential derived from Eqs. (6, 7, 8), assuming that $\hat{\mathcal{L}}_{\text{soft}}(S, H_u, H_d)$ does not contain a large linear S term. In the absence of the linear term in $\hat{W}(S)$, the potential for the two Higgs

³In general two Higgs doublet potentials, a symmetry $H_u \leftrightarrow H_d^\dagger$ can be spontaneously broken, giving $\tan\beta \neq 1$ [9]. This does not happen in our case because of the supersymmetric relations between the quartic couplings.

doublets is given by

$$V = |\lambda H_u H_d|^2 + (|\mu|^2 + m_{H_u}^2)|H_u|^2 + (|\mu|^2 + m_{H_d}^2)|H_d|^2 + (\mu B H_u H_d + \text{h.c.}) + V_D. \quad (11)$$

(If $\hat{W}(S)$ contains the linear term $L_S^2 S$, μB should be redefined as $\mu B \rightarrow \mu B - \lambda L_S^2$ to absorb the effect of that term.⁴) This potential can, in fact, be obtained by taking the limit $m_S^2 \gg |M_S|^2, |b_S|, |\mu|^2, |a_\lambda|^2$ in the expression in the Appendix. We find that the potential takes the same form as the MSSM potential except that there is a new quartic term, $|\lambda H_u H_d|^2$. Since the S scalar is relatively heavy, it effectively appears at the scale v that a “non-supersymmetric” quartic coupling has been added to the supersymmetric Higgs potential [22].

Approximating that $H_u^0 = h^\dagger \sin \beta$ and $H_d^0 = h \cos \beta$, the Higgs quartic coupling of Eq. (1) is given by

$$\lambda_h = \lambda^2 \sin^2 2\beta + \frac{\epsilon g^2 + \epsilon' g'^2}{2} \cos^2 2\beta. \quad (12)$$

The lightest Higgs boson mass is then given by Eq. (2). This is a good approximation as long as the other Higgs bosons are somewhat heavier, i.e. $2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2 \gtrsim M_{\text{Higgs}}^2$, which we expect to be the case. Our scenario assumes that the dominant contribution to λ_h is the first term in Eq. (12). This is most effective if $\lambda^2 \sin^2 2\beta$ is larger than the largest tree-level MSSM value, $(g^2 + g'^2)/2$, which occurs if $\lambda \gtrsim 0.6$ (0.7) for $\tan \beta \simeq 1.7$ (2.2). (In the case of $\epsilon, \epsilon' \ll 1$, this becomes a necessary condition to reduce fine-tuning.) Such values of λ require deviations from the simplest MSSM at energies above the weak scale, for example, stronger gauge couplings at higher energies [5, 11].

The minimization of the potential, Eq. (11), gives

$$|\mu|^2 + m_{H_u}^2 = \mu B \frac{1}{\tan \beta} - v^2 \left\{ \lambda^2 \cos^2 \beta + \frac{\epsilon g^2 + \epsilon' g'^2}{4} (\sin^2 \beta - \cos^2 \beta) \right\}, \quad (13)$$

$$|\mu|^2 + m_{H_d}^2 = \mu B \tan \beta - v^2 \left\{ \lambda^2 \sin^2 \beta + \frac{\epsilon g^2 + \epsilon' g'^2}{4} (\cos^2 \beta - \sin^2 \beta) \right\}. \quad (14)$$

It is customary to rewrite these equations in the form

$$\frac{\epsilon g^2 + \epsilon' g'^2}{4} v^2 = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2 \beta}{\tan^2 \beta - 1} - |\mu|^2, \quad (15)$$

$$2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2 = \frac{2\mu B}{\sin 2\beta} - \lambda^2 v^2. \quad (16)$$

⁴In addition, if $\hat{\mathcal{L}}_{\text{soft}}(S, H_u, H_d)$ contains a large linear S term, $\hat{\mathcal{L}}_{\text{soft}} = -c_S S + \text{h.c.}$ with $c_S \approx m_S^2 v$, both μ and μB should be redefined to absorb the linear terms: $\mu \rightarrow \mu + \lambda c_S / m_S^2$ and $\mu B \rightarrow \mu B - \lambda L_S^2 + \lambda c_S M_S / m_S^2 - \lambda \kappa c_S^2 / m_S^4 + a_\lambda c_S / m_S^2$, where κ and a_λ are the coefficients of the $S^3/3$ and $-S H_u H_d$ terms in \hat{W} and $\hat{\mathcal{L}}_{\text{soft}}$, respectively. The appropriate redefinitions for the origin of S and parameters in \hat{W} and $\hat{\mathcal{L}}_{\text{soft}}$ are also needed. These redefinitions can generate μ and μB terms even if they are absent before the redefinitions.

In the corresponding equations in the MSSM, the $\lambda^2 v^2$ term in the second equation is absent (and $\epsilon = \epsilon' = 1$ in the first equation). In this case the second equation determines $\tan \beta$ and then the electroweak scale v is determined by the first equation. In our case, however, both v and β appear in the two equations. It is therefore not obvious that the “roles” of these two equations are the same as those in the MSSM. In fact, in our scenario where the dominant restoring force arises from the F -term potential, it is the second equation that determines v . This is easily understood if we take the limit $g, g' \rightarrow 0$ in Eq. (15) (since they are supposed to play negligible roles) — the first equation determines $\tan \beta$ and then v is determined by the second equation.

We find it useful to rewrite Eqs. (15, 16) in the form

$$\tan^2 \beta = \frac{|\mu|^2 + m_{H_d}^2 + \hat{m}_Z^2/2}{|\mu|^2 + m_{H_u}^2 + \hat{m}_Z^2/2}, \quad (17)$$

$$\lambda^2 v^2 = \frac{2\mu B}{\sin 2\beta} - (2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2), \quad (18)$$

where $\hat{m}_Z^2 \equiv (\epsilon g^2 + \epsilon' g'^2)v^2/2$, which becomes the Z -boson mass squared for $\epsilon = \epsilon' = 1$. This makes it manifest that, in the parameter region of interest $|\mu|^2, |m_{H_u}^2|, |m_{H_d}^2| \gtrsim \hat{m}_Z^2/2$, $\tan \beta$ is mostly determined by the first equation while v by the second equation. Equation (17) also makes it obvious that, as was observed in the previous general discussion, the significant fine-tuning between $|\mu|^2$ and $m_{H_u}^2$ can be avoided for $|\mu|^2 + m_{H_u}^2 > 0$, and in that case $\tan \beta \approx ((|\mu|^2 + m_{H_d}^2)/(|\mu|^2 + m_{H_u}^2))^{1/2}$ is expected to be not large.

The electroweak scale, v , is determined by the second equation, Eq. (18). In the absence of a tuning between $|\mu|^2$ and $m_{H_u}^2$, the only source of a potential tuning is in the right-hand-side of this equation. Now, the combination $2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2$ appearing in the right-hand-side is related to the charged Higgs boson mass, M_{H^\pm} , as

$$M_{H^\pm}^2 = 2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2 + \hat{m}_W^2, \quad (19)$$

where $\hat{m}_W^2 \equiv \epsilon g^2 v^2/2$, which becomes the W -boson mass squared for $\epsilon = 1$. To evade the constraint from the $b \rightarrow s\gamma$ process without relying on excessive cancellations between different contributions, M_{H^\pm} is expected to be somewhat larger than v , e.g. $M_{H^\pm} \gtrsim 250$ GeV [23]. This, therefore, requires some cancellation between the first term and the rest in the right-hand-side of Eq. (18). The amount of the required cancellation is given by the following fine-tuning parameter:

$$\Delta^{-1} \simeq \frac{\lambda^2 v^2}{M_{H^\pm}^2 - \hat{m}_W^2} \approx \frac{M_{\text{Higgs}}^2}{M_{H^\pm}^2}, \quad (20)$$

where the last relation holds approximately for $\tan \beta \lesssim 2$. Consider, for example, the case with $\lambda \simeq 0.8$, which can be obtained consistently with the Landau pole bound without relying on the compositeness of S , H_u or H_d . In this case, the requirement $\Delta^{-1} > 20\%$ (10%) gives

$M_{H^\pm} \lesssim 320$ GeV (450 GeV) for $\epsilon = \epsilon' = 1$ and $M_{H^\pm} \lesssim 310$ GeV (440 GeV) for $\epsilon = \epsilon' = 0$. For $\lambda \simeq 0.7$, the corresponding bound becomes $M_{H^\pm} \lesssim 280$ GeV (390 GeV). For these values of M_{H^\pm} , some amount of a destructive interference between the chargino and charged Higgs boson contributions in the $b \rightarrow s\gamma$ amplitude may be needed, and this prefers the positive sign for the μ parameter.

In the above analysis, the terms $\hat{m}_Z^2/2$ in Eq. (17) were not important. This clearly shows that the $SU(2)_L \times U(1)_Y$ D -term potential is not playing a major role in electroweak symmetry breaking, although it gives a small correction to the Higgs boson mass through the second term of Eq. (12). In fact, this was the dynamics behind the reduction of fine-tuning in the minimally fine-tuned model of Ref. [5], in which the necessary μ and μB terms arise effectively from the linear S terms in $\hat{W}(S)$ and $\hat{\mathcal{L}}_{\text{soft}}(S, H_u, H_d)$. One can check that our Δ^{-1} in Eq. (20) well reproduces the values of the fine-tuning parameter $\tilde{\Delta}^{-1}$, defined in Ref. [5] following [24].

At the leading order in the $1/m_S^2$ expansion, the lightest Higgs boson mass is approximately given by Eq. (2) with Eq. (12), and the charged Higgs boson mass is given by Eq. (19). The masses for the heavier neutral Higgs boson and the pseudo-scalar Higgs boson are given by $M_{H^0}^2 \simeq 2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2 + \lambda^2 v^2 \cos^2 2\beta + \hat{m}_Z^2 \sin^2 2\beta$ and $M_{A^0}^2 = 2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2 + \lambda^2 v^2$, respectively.⁵ These masses, however, are modified by the mixings with the S states except for the mass of the charged Higgs boson. The mixing effects can be taken into account if we use the effective Lagrangian derived using the complete S propagators given in the Appendix. Many of the effects, in fact, can be captured even in the effective potential of Eq. (40) derived using the zero-momentum S propagators. These effects typically cause shifts of the masses at the level of 10%.

2.3 Scenario II — Supersymmetric Higgs with a little seesaw at sub-TeV

We now consider an alternative scenario in which the dynamics of S decouples because of a large supersymmetric mass, $M_S \gg v, (m_S^2)^{1/2}, |b_S|^{1/2}$. Here, by large we mean that M_S is only somewhat larger than v , of the order of several hundreds of GeV. We assume that the linear S terms in $\hat{W}(S)$ and $\hat{\mathcal{L}}_{\text{soft}}(S, H_u, H_d)$ are absent, or their effects are already absorbed in the appropriate redefinitions of parameters.

For $M_S \gg (m_S^2)^{1/2}, |b_S|^{1/2}$, we can supersymmetrically integrate out the S multiplet and obtain the effective superpotential and the Kähler potential for the two Higgs doublets, describing

⁵The precise formulae for the masses of the lightest and heavier Higgs bosons in the S decoupling limit are $M_{h,H^0}^2 = [M_{A^0}^2 + \hat{m}_Z^2 \pm \{(M_{A^0}^2 - \hat{m}_Z^2)^2 \cos^2 2\beta + (M_{A^0}^2 + \hat{m}_Z^2 - 2\lambda^2 v^2)^2 \sin^2 2\beta\}^{1/2}]/2$.

the physics at the scale v . For $\hat{W}(S) = (M_S/2)S^2$, the effective superpotential is given by

$$W = \mu H_u H_d - \frac{\lambda^2}{2M_S} (H_u H_d)^2, \quad (21)$$

while the effective Kähler potential is given by

$$K = |H_u|^2 + |H_d|^2 + \frac{\lambda^2}{M_S^2} |H_u H_d|^2, \quad (22)$$

where the last terms in these equations are obtained using the two supersymmetric terms $(M_S/2)S^2$ and $\lambda S H_u H_d$ through a ‘‘little seesaw mechanism.’’ We drop possible absolute value symbols for the parameters here and below, as they appear only for the squared quantities and all the parameters are assumed to be real.

The dynamics of electroweak symmetry breaking is described by this superpotential and Kähler potential and the soft supersymmetry breaking Lagrangian of Eq. (7) with $\hat{\mathcal{L}}_{\text{soft}}(S, H_u, H_d)$ set to zero. The Higgs potential is then given by

$$V = \frac{\{\mu^2 M_S^2 - \lambda^2 \mu M_S (H_u H_d + \text{h.c.}) + \lambda^4 |H_u H_d|^2\} (|H_u|^2 + |H_d|^2)}{M_S^2 + \lambda^2 (|H_u|^2 + |H_d|^2)} + m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2 + (\mu B H_u H_d + \text{h.c.}) + V_D. \quad (23)$$

In the true $M_S \rightarrow \infty$ limit, the F -term potential in the first line simply gives supersymmetric masses, μ , to H_u and H_d . The appreciable quartic couplings, however, arise if μ/M_S is not very small. The potential of Eq. (23) can also be obtained from Eq. (40) in the Appendix by taking the limit $|M_S|^2 \gg m_S^2, |b_S|, |a_\lambda|^2$ with $|\mu| \gtrsim |a_\lambda/\lambda|$.

Approximating that $H_u^0 = h^\dagger \sin \beta$ and $H_d^0 = h \cos \beta$, the Higgs quartic coupling, λ_h , is given by

$$\lambda_h = \frac{4\lambda^2 \mu}{M_S} \sin 2\beta - \frac{4\lambda^2 \mu^2}{M_S^2} + \frac{\epsilon g^2 + \epsilon' g'^2}{2} \cos^2 2\beta. \quad (24)$$

We then find that the physics associated with the potential Eq. (23) depends strongly on the values of parameters. Let us take the convention that the sign of μB is positive. (This choice can always be made by rotating the phase of $H_u H_d$.) The minimization conditions are given by

$$\mu^2 + m_{H_u}^2 = \mu B \frac{1}{\tan \beta} - v^2 \left\{ \frac{\lambda^2 \mu}{M_S} \left(\sin 2\beta + \frac{1}{\tan \beta} \right) - \frac{2\lambda^2 \mu^2}{M_S^2} - \frac{\epsilon g^2 + \epsilon' g'^2}{4} \cos 2\beta \right\}, \quad (25)$$

$$\mu^2 + m_{H_d}^2 = \mu B \tan \beta - v^2 \left\{ \frac{\lambda^2 \mu}{M_S} \left(\sin 2\beta + \tan \beta \right) - \frac{2\lambda^2 \mu^2}{M_S^2} + \frac{\epsilon g^2 + \epsilon' g'^2}{4} \cos 2\beta \right\}. \quad (26)$$

Subtracting these two equations, we obtain

$$\left(\mu B - \frac{\lambda^2 v^2 \mu}{M_S} \right) \left(\tan \beta - \frac{1}{\tan \beta} \right) = m_{H_d}^2 - m_{H_u}^2 + \frac{\epsilon g^2 + \epsilon' g'^2}{2} v^2 \cos 2\beta. \quad (27)$$

We assume that the right-hand-side of this equation is positive, which is always the case if the difference between $m_{H_u}^2$ and $m_{H_d}^2$ arises from the top-stop loop contribution of Eq. (4) with $\ln(M_{\text{mess}}/m_{\tilde{t}}) \gtrsim 1$ (and $m_{\tilde{t}} \gtrsim 300$ GeV). We then find that for $\mu B - \lambda^2 v^2 \mu / M_S > 0$ (< 0), $\tan 2\beta$ is negative (positive). Let us first consider the case with $\mu B - \lambda^2 v^2 \mu / M_S < 0$. In this case, $\lambda^2 \mu / M_S$ is positive while $\sin 2\beta$ is negative for $|H_u| > |H_d|$, so from Eq. (24) we find that the Higgs potential is not stabilized by the quartic coupling arising from the F -term. This is not good, since then we have to rely on 6-point interactions to stabilize the VEVs, which are too weak to give a phenomenologically acceptable value for the Higgs boson mass. We are thus led to consider the case with $\mu B - \lambda^2 v^2 \mu / M_S > 0$. In this case, the F -term contributions to the quartic coupling can provide a sufficiently strong stabilizing force, as long as the first term in Eq. (24) is positive, i.e. $\lambda^2 \mu / M_S$ is positive. We thus finally obtain the condition for the present scenario to work:

$$\mu B > \frac{\lambda^2 v^2 \mu}{M_S} > 0. \quad (28)$$

With this condition satisfied, the effects from 6-point interactions are small in generic parameter regions, and the lightest Higgs boson mass is given approximately by Eq. (2) with Eq. (24).

Neglecting the contribution from the D -term in the right-hand-side, which is a reasonable approximation in most of the parameter space, Eq. (27) can be used to determine the value of $\tan \beta$. For μB sufficiently larger than $\lambda^2 v^2 \mu / M_S$, we find

$$\tan 2\beta \approx -\frac{2\mu B}{m_{H_d}^2 - m_{H_u}^2}. \quad (29)$$

We are interested in the region where $\tan \beta$ is not large in order for the first term in Eq. (24) to be effective. For $\lambda \simeq 0.8$, for example, we obtain a sufficiently large Higgs boson mass for $\tan \beta \lesssim (2 \sim 3)$, depending on the values of ϵ, ϵ' and the size of the one-loop top-stop correction to the Higgs quartic coupling.

An approximate formula for the fine-tuning parameter can be obtained by inspecting the sum of the two minimization equations, Eqs. (25, 26):

$$v^2 \left(\frac{2\lambda^2 \mu}{M_S} \sin 2\beta + \frac{2\lambda^2 \mu}{M_S \sin 2\beta} - \frac{4\lambda^2 \mu^2}{M_S^2} \right) = \frac{2\mu B}{\sin 2\beta} - (2\mu^2 + m_{H_u}^2 + m_{H_d}^2). \quad (30)$$

The combination $2\mu^2 + m_{H_u}^2 + m_{H_d}^2$ is positive in our scenario, so that the electroweak scale is determined by the balance between this combination and the first term in the right-hand-side. We also note that in the parameter region we are interested, e.g. $\tan \beta \lesssim 2$, the combination inside the parenthesis in the left-hand-side is approximately λ_h . The fine-tuning parameter is then given by the ratio between $\lambda_h v^2$ and $2\mu B / \sin 2\beta$, where the latter quantity is approximately the charged Higgs boson mass squared for μB sufficiently larger than $\lambda^2 v^2 \mu / M_S$. We thus obtain

$$\Delta^{-1} \approx \frac{M_{\text{Higgs}}^2}{M_{H^\pm}^2}, \quad (31)$$

which provides a rough measure for the fine-tuning in the present scenario. Since the formula is approximately the same as that of the previous scenario, Eq. (20), the implications on $M_{H^\pm}^2$ in terms of M_{Higgs} are almost identical.

We note that this scenario is exactly the one employed in Ref. [20] to reduce the fine-tuning, where the supersymmetric masses for S and the Higgs doublets, as well as the holomorphic supersymmetry breaking masses for these fields, are obtained through supergravity effects. In fact, one can check that our approximate formula in Eq. (31) well estimates the values of the fine-tuning parameter $\tilde{\Delta}^{-1}$ calculated in that paper.

2.4 Implications

One of the solid implications of our scenario is that $\tan \beta$ is expected to be small, e.g.

$$\tan \beta \lesssim 3. \quad (32)$$

In both scenarios discussed in the previous two subsections, obtaining a large $\tan \beta$ requires fine-tuning. The small $\tan \beta$ is also required to push up the Higgs boson mass using the quartic coupling arising from the F -term potential.

An important feature of the present scenario is that small top squark masses are allowed because they do not play a major role in raising the Higgs boson mass nor in triggering electroweak symmetry breaking. In fact, in order for our scenario to be relevant for the solution to the supersymmetric fine-tuning problem, the top squarks should be light. How light should they be? Let us look at Eq. (18) for the case with a large m_S^2 . To avoid fine-tuning, each contribution to the right-hand-side must not be much larger than the left-hand-side. Now, for small $\tan \beta$, the quantity in the left-hand-side is roughly the square of the physical Higgs boson mass M_{Higgs}^2 (see Eqs. (2, 12)). We thus have a rough upper bound on the size of the top-Yukawa contribution to the Higgs mass-squared parameter

$$|\delta m_h^2| \lesssim \frac{M_{\text{Higgs}}^2}{\Delta^{-1}}, \quad (33)$$

where Δ^{-1} is the amount of fine-tuning. This in turn gives an upper bound on the top squark masses through Eq. (4). Requiring $\Delta^{-1} \geq 20\%$, for example, we obtain

$$m_{\tilde{t}} \lesssim 720 \text{ GeV} \left(\frac{M_{\text{Higgs}}}{150 \text{ GeV}} \right) \left(\frac{3.5}{\ln(M_{\text{mess}}/m_{\tilde{t}})} \right)^{1/2}, \quad (34)$$

where M_{mess} is the scale at which the squark and slepton masses are generated. This is not an extremely strong bound. (The bound can become even weaker if $\tan \beta$ is appreciably larger than 1; see Eq. (12)). The bound in Eq. (34) implies that our scenario can evade fine-tuning for a broad range of the top-squark masses. In fact, in most cases the limiting factor for fine-tuning

is the mass of the charged Higgs boson, rather than the top squark masses (see e.g. Eq. (20)). The situation is quite similar in the case with a large M_S discussed in section 2.3. We obtain Eqs. (33, 34) from Eqs. (30, 24) in an exactly analogous manner.

It is interesting to compare the expression in Eq. (33) with the corresponding one in the MSSM. In the MSSM, the electroweak VEV is determined by Eq. (15) with ϵ and ϵ' set to 1. For a reasonably large $\tan\beta$, this equation can be approximately written as $M_{\text{Higgs}}^2/2 = -m_{H_u}^2 - |\mu|^2$. We then obtain the bound on the top-Yukawa contribution

$$|\delta m_h^2| \lesssim \frac{M_{\text{Higgs}}^2}{2\Delta^{-1}}, \quad (35)$$

which is a factor 2 tighter than that in Eq. (33). This shows that, even disregarding the issue of the LEP II bound on the Higgs boson mass, the quartic coupling from the F -term has some advantage over those from the $SU(2)_L \times U(1)_Y$ D -terms. For a fixed value of Δ^{-1} , theories with the F -term quartic coupling can accommodate larger values for the top squark masses than theories with the D -term quartic couplings.

3 Conclusions

In this paper we have considered a scenario in which the dominant stabilizing force in the Higgs potential arises from the F -term potential, rather than the conventional $SU(2)_L \times U(1)_Y$ D -terms, in supersymmetric theories. The required quartic coupling arises from the superpotential interaction between the Higgs doublets and a singlet field, but unlike in the case of the next-to-minimal supersymmetric standard model the singlet field is not responsible for generating the μ and μB terms for the Higgs doublets. This allows a larger freedom for the superpotential and supersymmetry-breaking terms for the singlet field than in the case where μ and μB arise from the VEV of the singlet field.

To use the F -term quartic coupling as the dominant stabilizing force, the mass-squared parameters for the up- and down-type Higgs doublets should both be positive. This implies that electroweak symmetry breaking is triggered by the holomorphic supersymmetry breaking mass term for the Higgs doublets, the μB term. The scenario, therefore, requires deviations from the conventional picture not only in the stabilizing force for the Higgs potential but also in the origin of electroweak symmetry breaking. In this scenario, the role of the top-stop loop correction to the up-type Higgs doublet is to make $\tan\beta$ deviate from 1, besides facilitating electroweak symmetry breaking.

One of the advantages of the present scenario is that the masses of the top squarks can be small as they do not play a major role in raising the Higgs boson mass or in triggering electroweak symmetry breaking. This makes it possible to reduce or eliminate the fine-tuning problem in

supersymmetric theories. We have studied electroweak symmetry breaking in the limit where the dynamics of the singlet field essentially decouples. There are two such cases, in which the decoupling occurs either due to the supersymmetry-breaking or supersymmetric mass for the singlet field. We have found that fine-tuning can be significantly reduced in both cases. The fine-tuning parameter Δ^{-1} is given by the squared ratio of the lightest Higgs boson mass to the charged Higgs boson mass, which can be made better than 20% for the charged Higgs boson mass smaller than about 300 GeV.

Our scenario can be combined with any supersymmetry breaking models in which relatively small top squark masses are generated at a low scale, such as the one in [25], to reduce fine-tuning. Examples for such theories are given, for example, in Refs. [5, 20]. Generic predictions for these theories include a small $\tan\beta$, $\tan\beta \lesssim 3$, relatively light top squarks, $m_{\tilde{t}} \lesssim 720 \text{ GeV} (M_{\text{Higgs}}/150 \text{ GeV})(3.5/\ln(M_{\text{mess}}/m_{\tilde{t}}))^{1/2}$, and the existence of the states for a singlet chiral supermultiplet below about a TeV. It will be interesting to further explore possible models in which the present mechanism is incorporated and the fine-tuning of electroweak symmetry breaking is reduced to the level discussed in this paper.

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Appendix

In this appendix, we integrate out the S supermultiplet in the general setup and obtain the effective potential for the two Higgs doublets. Our starting point is the interactions given by Eqs. (6, 7). We assume that the linear S terms in $\hat{W}(S)$ and $\hat{\mathcal{L}}_{\text{soft}}(S, H_u, H_d)$ are absent, since their effects can be absorbed in the appropriate redefinitions of the other parameters and the origin of the S field.

The relevant terms in $\hat{W}(S)$ and $\hat{\mathcal{L}}_{\text{soft}}(S, H_u, H_d)$ are given by

$$\hat{W}(S) = \frac{M_S}{2} S^2, \quad (36)$$

$$\hat{\mathcal{L}}_{\text{soft}}(S, H_u, H_d) = -m_S^2 |S|^2 - \left(\frac{b_S}{2} S^2 + a_\lambda S H_u H_d + \text{h.c.} \right), \quad (37)$$

where we have neglected the S^3 term in $\hat{W}(S)$, which has negligible effects for small $\langle S \rangle$. The interactions between the bosonic fields in S , H_u and H_d are given in components by

$$\mathcal{L}_{\text{int.}} = F_S(\lambda H_u H_d) + S(\lambda F_{H_u} H_d + \lambda H_u F_{H_d} - a_\lambda H_u H_d) + \text{h.c.}, \quad (38)$$

where we have used the same symbol for a chiral superfield and its scalar component. The effective Lagrangian is obtained by integrating out the S and F_S fields using their propagators:

$$G = \left(\begin{array}{cc|cc} G_{F_S F_S^\dagger} & G_{F_S F_S} & G_{F_S S^\dagger} & G_{F_S S} \\ G_{F_S^\dagger F_S^\dagger} & G_{F_S^\dagger F_S} & G_{F_S^\dagger S^\dagger} & G_{F_S^\dagger S} \\ \hline G_{S F_S^\dagger} & G_{S F_S} & G_{S S^\dagger} & G_{S S} \\ G_{S^\dagger F_S^\dagger} & G_{S^\dagger F_S} & G_{S^\dagger S^\dagger} & G_{S^\dagger S} \end{array} \right) \equiv \frac{i}{-(p^2 + |M_S|^2 + m_S^2)^2 + |b_S|^2} \left(\begin{array}{c|c} \hat{G}_{FF} & \hat{G}_{FS} \\ \hline \hat{G}_{FS}^\dagger & \hat{G}_{SS} \end{array} \right), \quad (39)$$

where

$$\begin{aligned} \hat{G}_{FF} &= \begin{pmatrix} -(p^2 + m_S^2)(p^2 + |M_S|^2 + m_S^2) + |b_S|^2 & -b_S M_S^{*2} \\ -b_S^* M_S^2 & -(p^2 + m_S^2)(p^2 + |M_S|^2 + m_S^2) + |b_S|^2 \end{pmatrix}, \\ \hat{G}_{FS} &= \begin{pmatrix} b_S M_S^* & -M_S^*(p^2 + |M_S|^2 + m_S^2) \\ -M_S(p^2 + |M_S|^2 + m_S^2) & b_S^* M_S \end{pmatrix}, \\ \hat{G}_{SS} &= \begin{pmatrix} p^2 + |M_S|^2 + m_S^2 & -b_S^* \\ -b_S & p^2 + |M_S|^2 + m_S^2 \end{pmatrix}. \end{aligned}$$

Here, $p^2 = p^\mu p_\mu$ is the invariant 4-momentum, and the metric is given by $\eta_{\mu\nu} = \text{diag}(-, +, +, +)$.

Neglecting the corrections to the wavefunction renormalizations, the effective potential for the Higgs doublets are obtained using the S and F_S propagators at zero momentum:

$$\begin{aligned} V &= \frac{m_S^2(|M_S|^2 + m_S^2) - |b_S|^2}{(|M_S|^2 + m_S^2)^2 - |b_S|^2} |\lambda H_u H_d|^2 \\ &\quad - \frac{|M_S|^2 + m_S^2}{(|M_S|^2 + m_S^2)^2 - |b_S|^2} |\lambda F_{H_u} H_d + \lambda H_u F_{H_d} - a_\lambda H_u H_d|^2 \\ &\quad + \frac{1}{(|M_S|^2 + m_S^2)^2 - |b_S|^2} \left[\frac{b_S M_S^{*2}}{2} (\lambda H_u H_d)^2 + \frac{b_S^*}{2} (\lambda F_{H_u} H_d + \lambda H_u F_{H_d} - a_\lambda H_u H_d)^2 \right. \\ &\quad + M_S^*(|M_S|^2 + m_S^2) (\lambda H_u H_d) (\lambda F_{H_u} H_d + \lambda H_u F_{H_d} - a_\lambda H_u H_d) \\ &\quad \left. - b_S^* M_S (\lambda H_u H_d)^\dagger (\lambda F_{H_u} H_d + \lambda H_u F_{H_d} - a_\lambda H_u H_d) + \text{h.c.} \right]. \quad (40) \end{aligned}$$

While we do not present the explicit expression, it is also easy to obtain the effective Lagrangian for the Higgs doublets using the full momentum-dependent propagators of Eq. (39).

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