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Malfliet, R. A.

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R. A. Malfliet and Y. Karant

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ON THE FRAGMENTATION OF THE PROJECTILE IN RELATIVISTIC HEAVY-ION COLLISIONS*

R.A. Malfliet[†] and Y. Karant*Lawrence Berkeley Laboratory**University of California, Berkeley, Ca 94720*

Abstract

The fragmentation of the projectile in relativistic heavy ion reactions is usually described in terms of a two-step model called abrasion-ablation. The abrasion part is reformulated in the framework of the Boltzmann equation. The results are compared with other approaches and with experiment.

In one of the first experiments with relativistic heavy ions at Berkeley, a very detailed investigation has been performed on the peripheral process of projectile fragmentation. For instance, ^{16}O nuclei accelerated in the Bevelac up to energies of 2.1 GeV per nucleon collide with a Be target and the outgoing fragments are detected in almost the same direction as the incident ^{16}O projectile [1]. This type of experiment has been extended to other bombarding energies and a whole range of projectile-target combinations. The first theoretical analyses considered the reaction as a two-step process in analogy with the conventional cascade-evaporation approach for low energy light ion reactions. In this way Bowman, Swiatecki and Tsang [2] introduced

* This work was supported by the Nuclear Physics Division of the U.S. Department of Energy.

[†] Permanent address : Kernfysisch Versneller Instituut, Groningen, the Netherlands

an abrasion step, in which some of the projectile matter is sheared off by the target, followed by the ablation of the excited fragment in flight. To our knowledge the most refined type of this model has been developed by Hüfner and co-workers [3].

Starting from Glauber's multiple scattering theory and using the so-called coherent approximation they derive a rather simple expression for the cross section for the abrasion of n nucleons. Also, they calculate the average excitation energy after abrasion and the recoil momentum of the fragment. All these quantities can then be used, within a certain model, to calculate the ablation part of the reaction. Based on this model one can understand the main features and obtain qualitative agreement with the experimental data. Although this approach is very attractive, in our opinion it makes some rather drastic assumptions. To illustrate this point we will reconsider the abrasion step in a more general approach based on the Boltzmann equation. We will rederive their simple expression for the abrasion cross section under the same assumptions and furthermore remove most of these simplifications in a new calculation of the abrasion observables.

The Boltzmann equation describes the evolution of the distribution function $N(\vec{r}, \vec{v}, t)$, i.e. the number of particles which are at time t in the phase-space element $d\vec{r} d\vec{v}$, because of collisions of the particles among each other [4]:

$$\left(\frac{\partial}{\partial t} + \vec{v} \cdot \vec{\nabla} \right) N(\vec{r}, \vec{v}, t) = \iiint d\vec{v}'_1 d\vec{v}'_2 d\vec{v}_2 \quad (1)$$

$$\left(N(\vec{r}, \vec{v}'_1, t) N(\vec{r}, \vec{v}'_2, t) W(\vec{v}'_1 \vec{v}'_2 | \vec{v}_2 \vec{v}) - N(\vec{r}, \vec{v}, t) N(\vec{r}, \vec{v}_2, t) W(\vec{v} \vec{v}_2 | \vec{v}'_1 \vec{v}'_2) \right)$$

Here, W denotes the transition probability including momentum and energy conservation, we have neglected the possible presence of an external force field and the equation is written in its non-relativistic form. In order to

be applicable to colliding nuclei the following criteria should be fulfilled. One can treat the system classically (small wavelength) and the nucleons are far apart such that we have a sequence of undisturbed free-space scattering events (long mean free path compared to the nuclear force range). These conditions are reasonably fulfilled at not too high density if one excludes effects like compression and other more collective phenomena (i.e. pion condensation). These may however be present so that we might have to view this approach as a theoretical baseline. The equation (1) as it stands describes, with appropriate initial conditions, the scattering of two nuclei based on individual nucleon-nucleon collisions.

Since we are dealing with peripheral collisions where not too many scattering events take place we neglect the depletion of target and projectile by successive collisions, as is the case in ref. [3]. If we also allow only the "moving" particles (i.e. projectile nucleons and knocked-out target nucleons by previous collisions) to interact with the "frozen" particles in the target we can simplify eq. (1) considerably :

$$\left(\frac{\partial}{\partial t} + \vec{v} \cdot \vec{\nabla}\right) N(\vec{r}, \vec{v}, t) = \int d\vec{v}' \rho_T(\vec{r}) \left[N(\vec{r}, \vec{v}', t) v' \sigma(\vec{v}' | \vec{v}) - N(\vec{r}, \vec{v}, t) v \sigma(\vec{v} | \vec{v}') \right] \quad (2)$$

where $N(\vec{r}, \vec{v}, t)$ represents the distributions fct for the "moving" particles and $\rho_T(\vec{r})$ is the target matter distribution. The symbol $\sigma(\dots)$ stands for the nucleon-nucleon cross section. This linearised Boltzmann-equation can be cast in an integral-equation form :

$$N(\vec{r}, \vec{v}) = \int_0^\infty d\zeta \exp \left\{ - \int_0^\zeta d\zeta' v \sigma_{tot}(v) \rho_T(\vec{r} - \vec{v} \zeta') \right\} \int d\vec{v}' v' \sigma(\vec{v}' | \vec{v}) \rho_T(\vec{r} - \vec{v} \zeta) N(\vec{r} - \vec{v} \zeta, \vec{v}') \quad (3)$$

In this form the equation gives a clear insight in the collision process. The exponential term expresses the probability that there is no collision and an obvious method to calculate the distribution fct is by iteration, which calculates $N^{(n)}(\vec{r}, \vec{v})$ (left-hand side of eq. (3)) in terms of $N^{(n-1)}(\vec{r}, \vec{v})$ (right-hand side of eq. (3)). Under certain assumptions we can simplify even further. As in ref. [3] we are allowing only for zero-degree scattering (i.e. straight-line trajectories along the z-axis like in Glauber theory), we take the cross sections to be energy independent and neglect the explicit time dependence of the collision. Then the iterative solutions of eq. (3) are obtained explicitly in terms of Poisson distributions :

$$N^{(n)}(\vec{b}) = \int dz d\vec{b}' \rho_P(\vec{b}' - \vec{b}, z) \frac{[\chi(\vec{b}')]^n}{n!} \exp \{-\chi(\vec{b}')\}$$

$$\chi(\vec{b}') = \exp \left\{ -\sigma_{tot} \int dz' \rho_T(\vec{b}', z') \right\} \quad (4)$$

$$N(\vec{b}) = \sum_n N^{(n)}(\vec{b}) \quad \mathcal{N}(\vec{b}) = \sum_n 2^n N^{(n)}(\vec{b})$$

$\mathcal{N}(\vec{b})$ denotes the total number of outgoing particles, the factor 2^n arising from the fact that we deal with identical particles. Since under certain conditions (which are fulfilled here) a Poisson distribution can be written as a binomial distribution our result (4) looks very similar to the result of ref. [3]. The main difference occurs in the presence of the projectile density weighting in eq. (4) as an overall external integration while in ref. [3] it appears in the expression for $\chi(\vec{b}')$ itself because of the coherent approximation made there. The iteration index n has to be understood as the *number of collisions*. If we would restrict the "moving" particles to projectile particles only, thus neglecting collisions with the target of particles which

already have been removed from the target, one can identify the index n with the number of abraded particles from the target. In other words each nucleon from the projectile that scatters with the target will abrade a nucleon from the latter making two "moving" nucleons. When these collide again, at the next scattering, they make four, and so on. The total number of abraded particles $D(\bar{b})$ out of the target is then given as :

$$D(\bar{b}) = \sum_n (2^n - 1) N^{(n)}(\bar{b}) \quad (5)$$

However, Hüfner et. al. [3] make the extra assumption mentioned above, and identify n with the number of abraded particles directly. Obviously for $n > 1$ this is incorrect and can even lead to large deviations from the expression (5). Moreover, for $n > 1$ the depletion of the target (the target particles which have been knocked-out should be subtracted from the full density ρ_T) will become an important effect. Also, as more collisions take place the straight-line assumption will break down.

In order to study all these effects we have to solve the complete Boltzmann-equation (1) for nucleus-nucleus collisions without involving the simplifications discussed above. This is a formidable task and there are in general two methods to do so : a probabilistic one based on Monte-Carlo techniques to evaluate the multi-dimensional integrals and the time-evolution and more deterministic methods usually based on polynomial expansions, solution in terms of moments of the distribution function and iteration techniques. In view of our discussion before we have chosen for the latter. Apart from the physical attractiveness the iteration in terms of order of scatterings should converge rapidly because of the relative long mean free paths in combination with a rapid depletion of projectile and target. The general outline of the method will be discussed extensively elsewhere [5] as well as the application to other aspects of relativistic

heavy-ion reactions (central collisions and composite-particle production). In the following we will only sketch the main ingredient of the approach and focuss on the results of a first calculation.

The non-relativistic Boltzmann-equation is reformulated in a fully covariant way. If we neglect collisions between particles which *both* have already undergone a collision we can construct the iteration scheme in analogy with the one discussed above. We allow for cascades in both projectile and target and take depletion into account. The differential cross section is taken to be isotropic in the center-of-mass $\sigma_{cm}(\Omega) = \frac{\sigma_{tot}}{4\pi}$ where σ_{tot} is the energy-dependent experimental total nucleon-nucleon cross section [6]. Explicit pionic degrees of freedom have, as is the case in ref. [3] not been taken into account. We consider only nucleon-nucleon collisions although we have used the total nucleon-nucleon cross section. The initial projectile and target densities are choosen to be of Gaussian form (in this calculation we consider the $^{16}\text{O} + ^9\text{Be}$ system which are rather light nuclei) with oscillator constants determined from the relation [7]

$$k\omega = 45 A^{-\frac{1}{3}} - 25 A^{-\frac{2}{3}} \quad (6)$$

We have, as a function of impact parameter b , calculated the three important quantities : the number of particles abraded from ^{16}O , the excitation energy of the resulting fragment and its recoil momentum. The first quantity is calculated like in eq. (5) with $N^{(n)}(b)$ obtained directly from our solution of the full Boltzmann-eq. The excitation energy and recoil momentum are obtained as follows from the conservation of total momentum and total energy. The scattering process of an incoming nucleon with a target nucleon takes place in a potential well $V = \epsilon_F + S$ where ϵ_F is the fermi energy [8] and S the average separation energy for a nucleon. The target nucleon has an average fermi energy $\bar{\epsilon}_F = \frac{3}{5} \epsilon_F$.

Since we neglected pionic degrees of freedom only elastic scattering occurs, which means that energy and momentum are conserved. However, since the particles have to surmount an energy-difference $\Delta E = \frac{2}{5} \epsilon_F + S$ in order to escape, which the incoming particles have gained, we obtain an extra energy term and recoil momentum from the momentum-energy balance *outside* the potential well. More explicitly, if we denote by $\mathcal{N}(b, \vec{p})$ the total number of outgoing particles which have momentum $\vec{p}$ we obtain from the conservation laws :

$$\begin{aligned} A_R(b) &= A_T + A_P - \int \mathcal{N}(b, \vec{p}) d\vec{p} \\ Q_R''(b) &= A_P \vec{p}_0 - \int \mathcal{N}(b, \vec{p}) \cos \theta_p [p^2 + \Delta E^2 - 2E\Delta E]^{1/2} d\vec{p} \\ E_R^*(b) &= \left[\int \mathcal{N}(b, \vec{p}) d\vec{p} - A_P \right] \Delta E \\ D(b) &= \int \mathcal{N}(b, \vec{p}) d\vec{p} - A_P \end{aligned} \quad (7)$$

In an obvious notation $A_R(b)$ is the number of particles left in the residual fragment after abrasion, Q_R'' its longitudinal recoil momentum, E_R^* its excitation energy and $D(b)$ is the number of abraded particles out of the target.

In fig. 1 we have displayed some of these quantities for the case of $^{16}\text{O} + ^9\text{Be}$ at 2.1 GeV per nucleon bombarding energy. The abrasion cross section is calculated in an obvious way from the $D(b)$ -curve. As indicated by the dotted lines there are two impact parameters b_u and b_L at which $D(b_u) = 0.5$ and $D(b_L) = 1.5$. Therefore the cross section for abrasion of 1 nucleon is given as $\sigma_{abr}(1) = \pi b_u^2 - \pi b_L^2$ and similarly for the cross sections for abrasion of more than one nucleon. The total cross-section for abrasion is then defined as $\sigma_{abr}^{tot} = \pi b_u^2$. The corresponding recoil momentum and excitation energy (per particle) of the residual fragment can be calculated easily from this figure as we have indicated by the dashed lines for the case of abrasion of one nucleon only. The results of this procedure are collected in table I and

compared to the results obtained by Hüfner et. al. [3] for the abrasion-step and the experimental data [1]. The latter include more than just abrasion (we did not specify the decay) and a direct comparison is therefore difficult.

The value b_u , where $D(b_u) = 0.5$ is dictated very much by the geometry of the two colliding nuclei and we believe it to be roughly independent of the specific dynamics of the reaction. It also fits nicely with the experimental value for σ_{abr}^{tot} . The abrasion cross sections for one-nucleon removal etc. are determined by the value of b_L . This will very much depend on the dynamical input or how fast the collision-multiplicity increases. Clearly because of the straight-line assumption and the no-depletion effect the multiplicity for the simple calculation of ref. 3 increases rapidly resulting in abrasion cross sections which are much smaller than in this calculation. As was pointed out by Julius and Rinat [9], the pionic degrees of freedom will change drastically these numbers. Obviously the multiplicity will increase since the pions have rather short mean free paths involving multiple collisions. We expect thus a reduction of $\sigma_{abr}(1)$ etc. because the curve $D(b)$ will rise much faster than in fig. 1. Furthermore we have appreciable excitation energies which will result in a decay of the fragment after abrasion. This should hopefully result in crosssection to find certain fragments which are more in line with the experimental ones.

The recoil momenta are in good agreement with the experimental ones for abrasion of one nucleon. Since these numbers cannot change in the ablation step (if it occurs statistically) they are directly comparable to experiment. The recoil momentum increases rapidly with abrasion of more nucleons and becomes much larger than the corresponding experimental measured recoil momenta. This might indicate that the abrasion of one nucleon is largely responsible, through its ablation step, for most of the observed heavy fragments

Also the high excitation energies for abrasion of more than one nucleon suggest that the heavier fragments are associated with one-nucleon abrasion. These preliminary conclusions should of course be checked in a complete calculation, which must include pions and a model for the final ablation of the fragment. Some work on the ablation has already been initiated [10] by other authors.

A more complete analysis, including some of the effects mentioned above is in progress.

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TABLE 1

NUMBER OF ABRADED PARTICLES	$\sigma_{abr} (mb)$	$\sigma_{abr} (mb)$	$\sigma_{exp} (mb)$	$Q_R'' (MeV/c)$	$Q_R'' (MeV/c)$	$Q_R'' (MeV/c)$	$\epsilon_R^* (MeV)$
	CROSS SECTION			RECOIL MOMENTUM			EXCITATION ENERGY PER PARTIC
	this calc.	ref. [3]	ref. [1]	this calc.	ref. [3]	ref. [1]	this calc.
1	580.	262.	97.1	- 21.	- 20.	- 22.	1.6
2	280.	141.	56.3	- 42.	- 30.	- 27.	3.5
3	158.	100.	47.5	- 58.	- 42.5	- 34.	5.7
4	93.	77.	83.	- 76.	- 41.	- 25.	8.3
TOTAL	1495.		1490.				

Comparison of calculated abrasion observables with experiment (ref. [1]) and the results of ref. [3]. For Q_R'' we have taken the values closest to the $\frac{A}{2} = N = Z$ situation. Our values for Q_R'' and ϵ_R^* are based on a single average separation energy.

Figure caption

Fig. 1 Number of knocked-out nucleons from the projectile $D(b)$, longitudinal recoil momentum $Q_R''(b)$ and excitation energy per nucleon $\epsilon_R^*(b)$ of the remaining fragment, as a function of impact parameter b for the reaction $^{16}\text{O} + ^9\text{Be}$ at 2.1 GeV per nucleon.

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