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1) Introduction

In recent years much attention has been devoted to elucidating the mechanism of the reaction taking place between two heavy ions undergoing peripheral collisions (1-3). At energies ranging from the Coulomb barrier to ~2 GeV/nucleon the cross section is dominated by fragments with mass inferior to that of the projectile although some pick-up cross-section is also observed (4). Above ~15 MeV/nucleon the cross sections seem to vary little with bombarding energy (5). Fragments with mass close to that of the projectile have angular distributions which are strongly forward peaked. As the mass loss increases the pronounced forward peaking is progressively attenuated and the distributions become flatter.

It is the purpose of this letter to show that all of these facts can be simply understood in terms of two postulates.

a) The mechanism is mediated by the number of nucleon-nucleon collisions taking place along the trajectory describing the relative motion of protectile and target.

b) The deflection of the projectile-like fragment is produced in part by the potential acting between the ions and in part by recoil effects due to the mass change.

The emphasis in the present work is on developing these postulates in order to account for the general features of the reaction mechanism described above. Thus, no detailed comparision with experimental data will be presented. Furthermore, a major approximation is made in which the instantaneous effect of the transfer processes on the trajectory used to describe relative motion is ignored. A similar approximation is inherent in the treatment of reference 12 where the deflection is considered to arise entirely from recoil effects. For the mass transfer itself we will develop a generalised random walk process in which the number of "steps" is equal to the number of nucleon-nucleon collisions. This procedure will be described in the following section. Angular distributions will be discussed in Sect. 3 and in Sect. 4 we make some concluding remarks.

2) Mass Transfer

The first postulate presented in the introduction has already been successfully used by Karol (7) and by Kox et al. (8) to predict total reaction cross-sections. In Karol's formalism the average number of nucleon-nucleon collisions is obtained in the optical limit of Glauber theory (9) as:-

$$T(b) = \langle n \rangle = \bar{\sigma}_{nn} \int_{-\infty}^{\infty} dz \left[\int \rho_1 \rho_2 dv \right] \tag{1}$$

where the integral in square brackets represents the convolution of projectile and target densities which in turn is integrated over the (supposed) straight line trajectory in the beam (z) direction. In eq. 1, b is the impact parameter and $\bar{\sigma}_{nn}$ is the isospin averaged nucleon-nucleon cross-section which depends on incident energy. For a given value of T the probability for exactly n collisions is:-

$$Q_n = \frac{T^n e^{-T}}{n!} \tag{2}$$

and the reaction cross section is given by:-

$$\sigma_R = \int_0^{\infty} 2\pi b.db \; . \; (1-Q_o) \tag{3}$$

Karol further simplified eq. 3 by employing an approximate Gaussian form for the dependence of the projectile-target convolution integral on the radial separation of their mass centres. We will write this as:-

$$\left[\int \rho_1 \rho_2 \, dv \right] = \frac{K^{\circ}}{\sqrt{2\pi\sigma^2}} \; e^{-r^2/2\sigma^2} \tag{4}$$

Equation 1 then yields:-

$$T(b) = K\bar{\sigma}_{nn} \cdot e^{-b^2/2\sigma^2} \qquad (5)$$

This approximation immediately allows a simple calculation of the cross-section for exactly n collisions.

$$\sigma_n = \int_0^\infty Q_n \cdot 2\pi b db = 2\pi\sigma^2 \int_0^{K\bar{\sigma}_{nn}} Q_n(T) \cdot \frac{dT}{T} \qquad (6)$$

which if $n << K\bar{\sigma}_{nn}$ reduces simply to:-

$$\sigma_n \approx \frac{2\pi\sigma^2}{n} \qquad (7)$$

Since $\bar{\sigma}_{nn}$, which is roughly inversely proportional to the beam energy per nucleon consitutes the only energy dependent term in eq. 6 this last result suggests that cross-sections may be largely independent of energy provided that the number of collisions calculated for b=0 remains greater than the projectile mass number.

In order to calculate the cross-section for projectile mass loss we invoke a rather general random walk process in which each nucleon-nucleon collision leads to loss or gain of a piece of mass Δm with probability $P\Delta m$. A simple process along these lines is generated by taking only inelastic scattering and nucleon loss or gain into account ($\Delta m = 0, +/-1$). We will follow this process explicitly since it illustrates some important effects.

The probability for losing mass m is given by:-

$$P_m = \sum_n Q_n \cdot G_{nm} \qquad (8)$$

where Q_n is the probability for exactly n collisions and the G_{nm} factor represents the random walk:-

$$G_{nm} = \sum_{ijk} n! \frac{P_{-1}^{\ i}}{i!} \frac{P_{+1}^{j}}{j!} \frac{P_0^k}{k!} \tag{9}$$

with $n=i+j+k$, $m=i-j$ and $P_{-1}+P_{+1}+P_0 = 1$.

Inserting eqs. 2 and 9 into eq. 8 we obtain:-

$$P_m(T) = \sum_{ijk} \frac{\left(P_{-1}T\right)^i e^{-P_{+1}T}}{i!} \cdot \frac{\left(P_{+1}T\right)^j e^{-P_{+1}T}}{j!} \cdot \frac{\left(P_0T\right)^k e^{-P_0T}}{k!} \tag{10}$$

in which we have replaced the constrained sum $\sum_{ijkn}$ by the unconstrained sum $\sum_{ijk}$. To fix ideas we now assume $P_{-1} > P_{+1}$. Performing the sum over k (which does not involve the constraint) and inserting $j = i - m$ we finally obtain :-

$$P_m(T) = e^{-(P_{-1}+P_{+1})T} \left(\frac{P_{-1}}{P_{+1}}\right)^{M/2} \cdot \mathrm{Im}\ (2T\sqrt{P_{-1}\ P_{+1}}\ ) \tag{11}$$

where Im is a Modified Bessel function of order m. The corresponding cross-section may be obtained as in eq. 5 by integrating over T. In the limit that $K\bar{\sigma}_{nn} >> m$ one obtains:-

$$\begin{aligned} \sigma_m &= \frac{2\pi\sigma^{\ 2}}{|m|} \\ \sigma_{-m} &= \frac{2\pi\sigma^2}{|m|} \left(\frac{P_{+1}}{P_{-1}}\right)^m \end{aligned} \tag{12}$$

where σ_{-m} is the cross section for picking up mass onto the projectile. We note the surprising fact that cross sections for mass loss are independent of the precise values of the probabilities and that the pick-up cross-section exhibits the experimentally observed rapid fall-off.

As a further sophistication of this nucleon transfer model we may take into account a fusion cross section by cutting off the integration over impact parameter at the value obtained from the experimental fusion cross section (σ fusion = πb_F^2). These results are summarized in Fig. 1 which also shows the composition in n, the number of collisions for two mass losses.

Of course the results obtained above may easily be generalised to include pick-up and stripping of fragments other than nucleons. The result of including fragments $\Delta m > 1$ in the calculation is simply to generate an oscillation with period Δm about the nucleon transfer result.

3) Angular Distributions.

Following our second postulate we consider first the angular deflection due to the potential acting between projectile and target ions. If this deflection is small it is given approximately by the change in momentum $\Delta p/p$:-

$$\theta \sim \frac{\Delta p}{p} = \frac{1}{p} \int_{-\infty}^{\infty} Fy.dt \qquad (13)$$

where Fy is the component of the force (nuclear + Coulomb) perpendicular to the direction of motion. Approximating the integral by its value for a straight line trajectory and invoking the fact that fragments usually are observed with velocities close to that of the projectile we write:-

$$\theta \sim \frac{b}{2E} \int_{-\infty}^{\infty} - \frac{1}{r} \frac{dV(r)}{dr} . dz \qquad (14)$$

A simple test of this approximation is furnished by the Coulomb potential $Z_1Z_2e^2/r$ in which case eq. 14 yields:-

$$\theta_{coul.} = \frac{Z_1Z_2e^2}{Eb} \qquad (15)$$

which is identical with the exact result in the limit of small angles ($\tan\theta \sim \theta$). Notice also that in this limit with projectile and target masses A_p, A_t:-

$$\theta_{c.m.} = \frac{A_p + A_t}{A_t} \cdot \theta_{Lab.} \qquad (16)$$

so that the form of the deflection function is the same in both laboratory and centre of mass coordinates. To calculate the nuclear deflection we use the so called double folding potential (10) in which the nucleon-nucleon potential is approximated by a zero range form. Thus:-

$$V_n(r) = J_n \int \rho_1(\bar{r}')\ \rho_2(\bar{r} - \bar{r}')\, d\bar{r}' \qquad (17)$$

where J_n is the volume integral of the nucleon-nucleon potential and the term in square brackets is the convolution of projectile and target densities. Inserting the Gaussian approximation (eq. 4) into eq. 17 and taking into account the Coulomb deflection we easily obtain:-

$$\theta_{Lab} = \frac{Jn\ b\ T}{2E_{Lab}\ \bar{\sigma}_{nn}\ \sigma^2} - \frac{Z_1 Z_2 e^2}{E_{Lab}\ b} \qquad (18)$$

Using this simple deflection function we may now readily calculate the angular distribution for exactly n collisions as:-

$$\frac{d\sigma^n}{d\Omega} = \frac{2\pi b\ db\ Q_n(b)}{d\Omega} = \frac{\sigma^2}{\sin\theta} \cdot \frac{1}{T} \cdot \frac{dT}{d\theta} \cdot Q_n(T) \qquad (19)$$

or that for mass loss m from the projectile as:-

$$\frac{d\sigma^m}{d\Omega} = \frac{\sigma^2}{\sin\theta} \cdot \frac{1}{T} \cdot \frac{dT}{d\theta} \cdot P_m(T) \qquad (20)$$

where Pm(T) is given by eq. 11.

The second contribution to the angular distribution is due to recoil effects. A simple prescription for the corresponding angular distribution has been obtained from Goldhaber's model (11,12):-

$$\frac{d\sigma^m}{d\Omega} \sim e^{-P_F^2 \theta_{Lab}^2 / 2\sigma_F^2} \tag{21}$$

in which P_F is the fragment momentum ($P_p A_F/A_p$) and

$$\sigma_F^2 = \sigma_0^2 A_F (A_p-A_F)/(A_p-1) \tag{22}$$

is the width produced by the Fermi motion of the nucleons removed from the projectile. It should be remarked that this simple prescription may be modified when phase space factors for the density of final states are taken into account. However for the moment it will serve to illustrate the effect of nucleon transfer.

The angular distribution obtained when both potential and recoil effects are taken into account is simply the convolution of eqs. 19 and 21. Two examples are shown in Fig. 2 together with angular distributions for individual n-values (eq. 19). It will be seen from the figure that as the mass loss increases higher values of n contribute and produce the characteristic progressive flattening of the angular distributions observed experimentally.

To complete this section we mention one more interesting possibility concerning the kinetic energy loss of the projectile like fragment. If this loss is taken to be proportional to the number of nucleon-nucleon collisions we find immediately that fragments with masses near the projectile mass will have velocities close to that of the projectile whereas smaller masses will show more inelasticity. (The insets in fig. 1 would be identified as energy spectra). Futhermore it would be expected that, in the absence of a strong Coulomb effect, the average velocity for a given fragment decreases smoothly with increasing laboratory angle. Both these simple predictions seem to be substantiated by experiment (4).

4) Summary and Discussion

We have presented a simple model for the reaction mechanism in peripheral heavy ion reactions in which the principle components are the geometrical overlap of projectile and target densities and a random walk process which determines the change in mass of the projectile.

The use of the geometrical overlap to determine the 'strength' of the reaction is the basis of the abrasion-ablation model (3,6). However in this approach all of the mass contained in the overlap volume (of two sharp edged nuclei) is considered to be removed from the projectile and the 'spectator' projectile-like fragment remains on average undeflected. The excitation energy of the fragment is also taken to be a geometrical quantity (proportional to the surface exposed by abrading mass) whereas in the present work we have suggested that the kinetic energy loss of the fragment may be simply related to the number of nucleon-nucleon collisions. In contrast to the result of Hüfner et al (6) the present formalism produces strong localization in impact parameter for the production of a given fragment.

In this sense our approach is closer to that of Harvey (13) whose calculation of yields employs an equation similar to (1) but replaces the random walk with probabilities calculated using Friedman's model (14).

Another interesting comparison can be made with the work of Randrup (1) who solves the master equation for the evolution of the first and second moments of the probability distributions of selected macroscopic variables. The present model employs a much more crude approach to the dynamics of the reaction. (For example the average number of collisions is calculated using projectile and target densities throughout the collision). On the other hand the complete probability distribution for any observable follows from the Poisson distribution for the number of collisions (eq. 2) and the generalized random walk process. Thus the simplified treatment of the dynamics means that the angular distributions or angle integrated cross reactions for any set of observables (inclusive or exclusive) which can be included as constraints in the generalized form of equation 10 (including $\Delta m > 1$) can be immediately written down. One can remark that the number of independent observables (excluding the deflection angle) is equal to the number of terms on the right-hand side of equation 10.

Finally, although we have concentrated on projectile like fragments, it will be clear that observation of target like fragments or simultaneous observation or projectile-like and target-like fragments can be incorporated into the general formalism in a straightforward way.

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Figure Captions.

Figure 1. Mass yields in the nucleon random walk model. The full curve represents equation 12. The decomposition in n for mass losses 5 and 10 is shown in the insets. The effect on the yields of introducing a cut-off in the impact parameter (T) integration is shown by the dashed curves. For this calculation, P_{-1}=.6, P_{+1}=.1, P_0=.3 and σ = 3 fm.

Figure 2. The left part of the figure shows angular distributions for various n values (equation 19). On the right are angular distributions for mass losses 5 and 10 (equation 20). The solid curves are calculations including only potential deflection whereas the dashed curves take account of recoil effects calculated assuming a projectile of mass 40 at 27.5 MeV/nucleon and a value of σ_0 of 100 MeV/C (see equations 21,22).

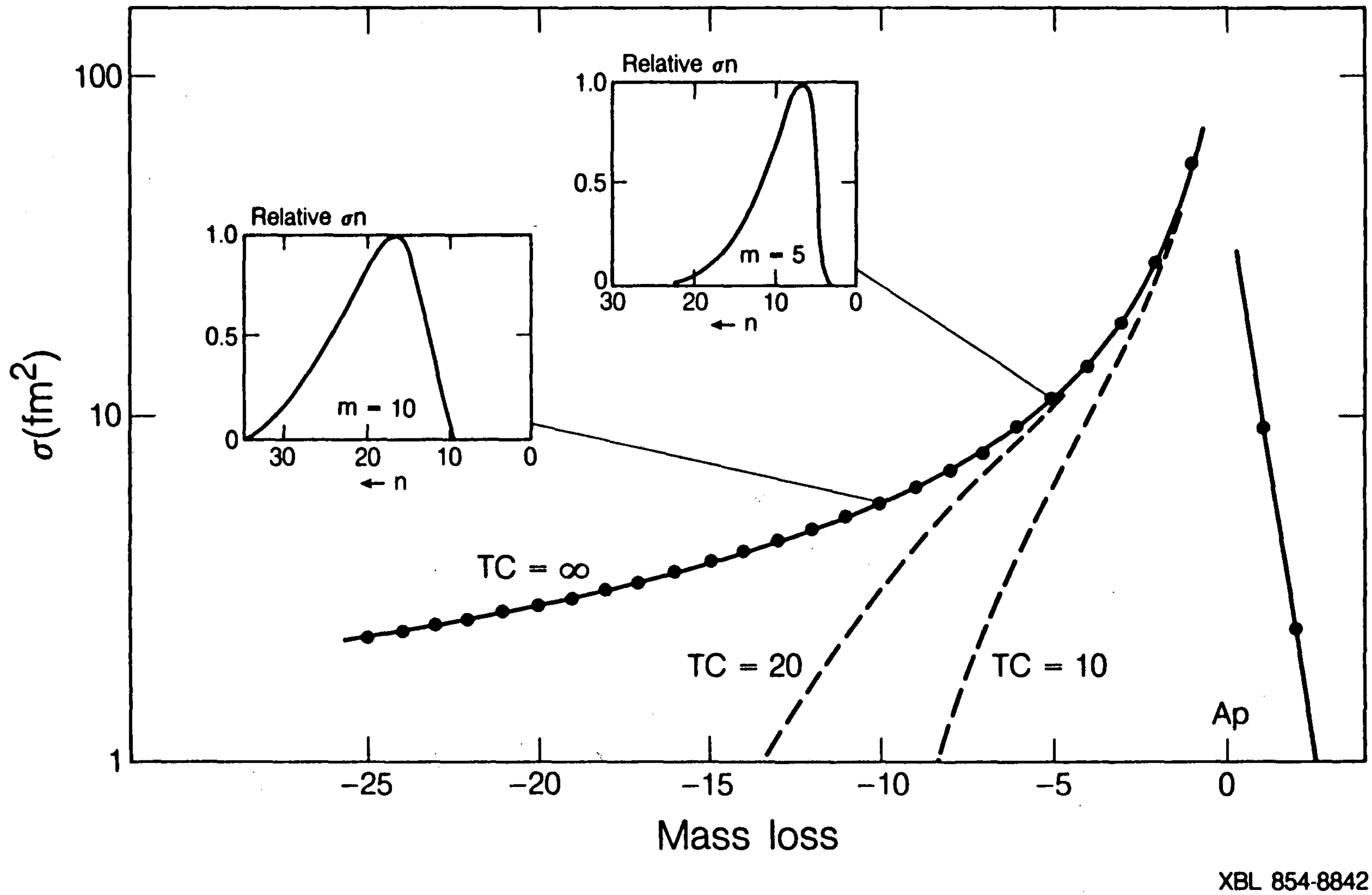
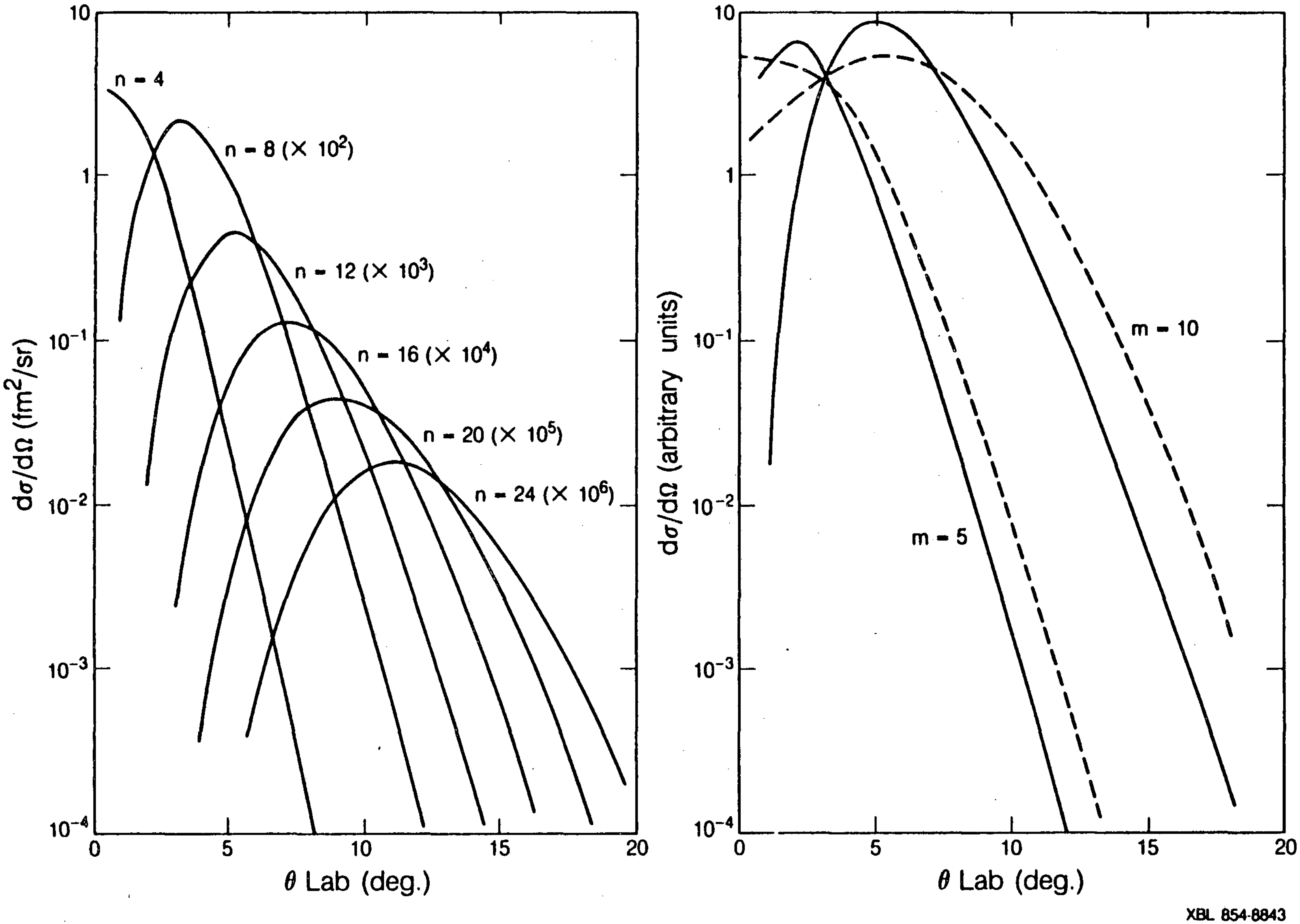

σ(fm²)
100
10
1
Relative σn
1.0
0.5
0
30
20
10
0
← n
m = 10
m = 5
TC = ∞
TC = 20
TC = 10
Ap
−25
−20
−15
−10
−5
0
Mass loss
XBL 854-8842

Fig. 1



n = 4
n = 8 (× 10^2)
n = 12 (× 10^3)
n = 16 (× 10^4)
n = 20 (× 10^5)
n = 24 (× 10^6)
dσ/dΩ (fm^2/sr)
θ Lab (deg.)
m = 10
m = 5
dσ/dΩ (arbitrary units)
θ Lab (deg.)
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Fig. 2

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