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Publication Date

1994-03-09



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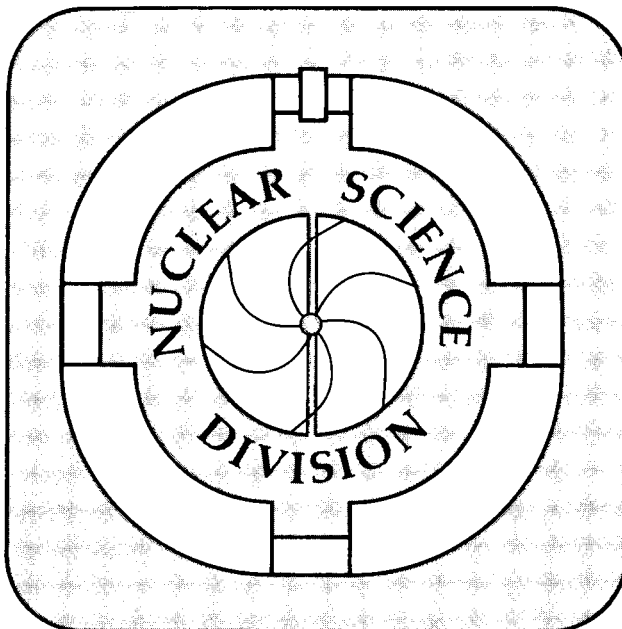
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To be presented at the Fourteenth International Conference on Few Body Problems in Physics, Williamsburg, VA, May 26-31, 1994, and to be published in the Proceedings

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March 1994



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The Spin Observables of Electron-Nucleon Scattering From the Formalism of Nucleon-Nucleon Scattering

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March 9, 1994

This work was supported by the Director, Office of Energy Research Division
of Nuclear Physics of the Office of High Energy and Nuclear Physics of the
U.S. Department of Energy under Contract DE-AC03-76SF00098

THE SPIN OBSERVABLES OF ELECTRON-NUCLEON SCATTERING FROM THE FORMALISM OF NUCLEON-NUCLEON SCATTERING

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ABSTRACT

The eN scattering spin observables are derived from the long established NN scattering formalism. Some conditions on the eN observables emerge more transparently than from the specific eN calculations themselves.

INTRODUCTION

It is useful, and important, to recognize the common role that spin physics plays in the seemingly dissimilar disciplines of nuclear, particle, and electron-scattering physics. That of nuclear physics is the most complicated in the sense that all of the reaction/scattering transition amplitudes that are allowed by parity conservation (P), time-reversal (T) symmetry, and other specific symmetries (e.g., identical particles), are nonvanishing in general. Helicity conservation, a relativistic condition, reduces substantially the number of nonvanishing amplitudes in electron scattering and in particle physics. Thus, these disciplines are spin-physics subsets of that of nuclear reactions. As an illustrative example, the few nonvanishing eN scattering observables are identified through a reduction from the many nonvanishing NN observables.

$$\text{SPIN STRUCTURE } \frac{1}{2} + \frac{1}{2} \rightarrow \frac{1}{2} + \frac{1}{2}$$

For the scattering of two spin-1/2 particles, the corresponding 4 X 4 spin-space matrix of scattering amplitudes can be expanded in terms of direct products of the 2 X 2 Pauli matrices σ_j and σ_k ,

$$M(\theta) = \sum_{j,k} a_{jk}(\theta) \sigma_j \otimes \sigma_k, \quad j,k = 0,x,y,z. \quad (1)$$

In a more compact form, with the 4 X 4 matrix $\sigma_{jk} \equiv \sigma_j \otimes \sigma_k$,

$$M = \sum_{j,k} a_{jk} \sigma_{jk}, \quad (2)$$

and the 16 M-matrix amplitudes,

$$a_{00}, \underline{a_{0x}}, \underline{a_{0y}}, \underline{a_{0z}}, \underline{a_{x0}}, a_{xx}, \underline{a_{xy}}, \underline{a_{xz}}, \\ a_{y0}, \underline{a_{yx}}, a_{yy}, a_{yz}, \underline{a_{z0}}, a_{zx}, \underline{a_{zy}}, a_{zz},$$

can be classified according to their P and/or T symmetries. For example, the eight underlined amplitudes are P odd (with $a_{0x}, a_{x0}, a_{xy}, a_{yx}$ also T odd) and a_{xz} and a_{zx} are T odd in the helicity frame with the z-axis taken along the

projectile momentum and the y-axis normal to the scattering plane. The experimental observables, expressed in a general form, are

$$X(jk,lm) = \text{Tr } M \sigma_{jk} M^\dagger \sigma_{lm} / \text{Tr } M M^\dagger, \quad j,k,l,m = o,x,y,z, \quad (3)$$

where j and k label the polarizations of the initial-state particles and l and m label the observed final-state polarizations. For example, the projectile analyzing power $A_y \equiv X(yo,oo)$. Since elastic scattering is its own time-reversed process, the T odd amplitudes a_{xz} and a_{zx} vanish in (2) along with the P odd amplitudes, reducing it to the six term

$$M = a_{oo} + a_{yo} \sigma_{yo} + a_{oy} \sigma_{oy} + a_{xx} \sigma_{xx} + a_{yy} \sigma_{yy} + a_{zz} \sigma_{zz}. \quad (4)$$

With six amplitudes there are $6^2 = 36$ independent observables, of which the following, from (3) and (4), are listed for consideration in electron scattering:

$$\begin{aligned} I &\equiv X(oo,oo) = |a_{oo}|^2 + |a_{yo}|^2 + |a_{oy}|^2 + |a_{xx}|^2 + |a_{yy}|^2 + |a_{zz}|^2 \\ IA_{yo} &\equiv IX(yo,oo) = 2\text{Re}(a_{oo}a_{yo}^* + a_{oy}a_{yy}^*), \\ IA_{oy} &\equiv IX(oy,oo) = 2\text{Re}(a_{oo}a_{oy}^* + a_{yo}a_{yy}^*), \\ IA_{zx} &\equiv IX(zx,oo) = 2\text{Im}(a_{oy}a_{zz}^* - a_{yo}a_{xx}^*), \\ IK_{zx} &\equiv IX(zo,ox) = -2\text{Im}(a_{oy}a_{zz}^* + a_{yo}a_{xx}^*), \\ IA_{zz} &\equiv IX(zz,oo) = 2\text{Re}(a_{oo}a_{zz}^* - a_{xx}a_{yy}^*), \\ IK_{zz} &\equiv IX(zo,oz) = 2\text{Re}(a_{oo}a_{zz}^* + a_{xx}a_{yy}^*). \end{aligned} \quad (5)$$

ELECTRON-NUCLEON SCATTERING

Electron scattering has been used for a long time to probe the electromagnetic structure of nucleons and nuclei. However, except for experiments designed to search for parity nonconserving effects, serious consideration [1,2] of the use of polarized electrons in nuclear and particle physics is a relatively recent development when compared with the use of polarized nucleons and deuterons. This delayed application is easily understood, of course, when it is noted that (in the one-photon exchange approximation) the terms in the cross-section that depend on the transverse polarizations, p_x and p_y , vanish as $1/\gamma = m_e/E$ in the scattering of polarized electrons [3]. This relativistic behavior may be understood, heuristically, from the Lorentz transformation of the electron spin four-vector defined in the rest frame, $S^0 = (0;s) = (0;s_x,s_y,s_z)$. Under a Lorentz boost of β in the z direction,

$$S = L_z(\beta) S^0 = \begin{pmatrix} \gamma & 0 & 0 & \beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \beta\gamma & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} 0 \\ s_x \\ s_y \\ s_z \end{pmatrix} = \gamma \begin{pmatrix} \beta s_z \\ s_x/\gamma \\ s_y/\gamma \\ s_z \end{pmatrix},$$

and, relative to the helicity s_z , the transverse spin components vanish as $1/\gamma$. Thus, at the electron energies of interest in nuclear and particle physics, except for a measurement of the PNC longitudinal (helicity) analyzing power A_z ,

nothing more is learned with polarized electrons (alone) than is available from the scattering of unpolarized electrons. As has become clear, however, during the past two decades of hadronic scattering, there are other (two-spin) observables such as polarization-transfer coefficients, $K_{jm} \equiv X(j_0, 0m)$, and spin-correlation coefficients, $A_{jk} \equiv X(jk, 00)$, which provide information concerning the spin dependence of the interactions that can never be gleaned from unpolarized cross-sections alone. These observables became experimentally accessible with useful precision during that period only through the development of efficient polarimeters and of polarized targets. It is that development, then, that has really made it possible for electron scattering to join, experimentally, the field of spin physics.

The vanishing, then, of the electron transverse-spin amplitudes a_{y0} , a_{xx} , and a_{yy} , reduces (4) to

$$M = a_{00} + a_{0y} \sigma_{0y} + a_{zz} \sigma_{zz} . \quad (6)$$

Thus, the relativistic nature of the electron results in a remarkable simplification of the scattering process. With the number of amplitudes reduced to three, the independent observables are reduced to nine. In the process, all of the amplitudes that flip the electron helicity have vanished, so the electron helicity is conserved. The correlated result from (5) is that

$$A_{y0} = 0, \quad (7)$$

$$A_{zx} = -K_{zx}, \quad A_{zz} = K_{zz} . \quad (8)$$

These results are all known [2], but the relativistic origin of the "turn around" relations (8) had not been noted explicitly. These relations have important experimental consequences, since, with them, a double-scattering experiment to determine a polarization-transfer coefficient K_{jk} can, in principle, always be replaced by a single-scattering experiment to determine the equivalent spin-correlation coefficient A_{jk} . Further simplification results from the dynamical description of the electron scattering process in the one-photon exchange plane-wave Born approximation. One can associate the surviving amplitudes of (6) with components of the hadronic electromagnetic current. The nucleon electromagnetic current, in terms of the two-component nucleon spinors, is [4]

$$J_\mu = X_f^\dagger M^\mu X_i ,$$

with

$$M^0 = 2imGE \sigma_0 ,$$

$$M^1 = iqGM \sigma_y ,$$

$$M^2 = -iqGM \sigma_x ,$$

$$M^3 = 0 . \quad (9)$$

$G_E(q^2) = \sqrt{4\pi} F_L / (1+\tau)$ and $G_M(q^2) = -\sqrt{2\pi} F_T / \sqrt{\tau(1+\tau)}$ are the charge and magnetic form factors of the nucleon, m is the nucleon mass, q is the four-momentum transfer, and $\tau = q^2/4m^2$. Here, the longitudinal, z , coordinate direction is taken along the momentum transfer q , which is effectively the x

coordinate in the helicity frame of (4). Thus, the transformation from the helicity frame to this in-plane transversity frame is given by $\sigma_z \rightarrow \sigma_{x'}$ and $\sigma_x \rightarrow -\sigma_{z'}$, where the primed coordinates refer to this final-state transversity frame, in which (6) becomes

$$M = a_{00} + a_{0y} \sigma_{0y} + a_{zx'} \sigma_{zx'} \quad (10)$$

By inspection, noting that the σ components in (9) are the σ_k of (1), the hadronic-current contributions to these amplitudes are shown explicitly in

$$a_{00} = 2imGE C_{00}(\theta), \quad a_{0y} = qGM C_{0y}(\theta), \quad a_{zx'} = -iqGM C_{zx'}(\theta), \quad (11)$$

where the $C_{jk}(\theta)$ are the electron kinematical factors. Thus, the independent amplitudes are reduced to the two that correspond to G_E and G_M , which has long been established. However, it is interesting to see these results emerge, as they must, from the general formalism of this spin structure, and to see that the one-photon exchange electron-scattering process is a particularly simple example. From (5) and (11), then,

$$\begin{aligned} I &= |a_{00}|^2 + |a_{0y}|^2 + |a_{zx'}|^2 = C_{00}^2 4m^2 G_E^2 + (C_{0y}^2 + C_{zx'}^2) q^2 G_M^2 \\ IA_{0y} &= 2\text{Re } a_{00} a_{0y}^* = 0, \\ IA_{zx'} &= 2\text{Re } a_{00} a_{zx'}^* = -C_{00} C_{zx'} 4mq G_M G_E, \\ IA_{zz'} &= 2\text{Im } a_{0y} a_{zx'}^* = C_{0y} C_{zx'} 2q^2 G_M^2. \end{aligned} \quad (12)$$

In this example of elastic electron scattering as a special case of the general formalism, some results emerge more transparently than from the detailed calculations themselves. Here, one readily sees that the projectile analyzing power $A_{y0} = 0$ from the relativistic electron helicity conservation, while the target analyzing power $A_{0y} = 0$ from the dynamics of the one-photon exchange process. One final consequence of the results (12) is that only two of the three surviving observables are independent. In view of the recognized importance of providing a more accurate determination of the charge form factor of the neutron, (12) shows that $A_{zx'}$ is the observable most sensitive to G_E^n , depending linearly on it [1].

ACKNOWLEDGMENTS

This work was partially supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Nuclear Physics Division of the U.S. Department of Energy under Contract DE-AC03-76SF00098.

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